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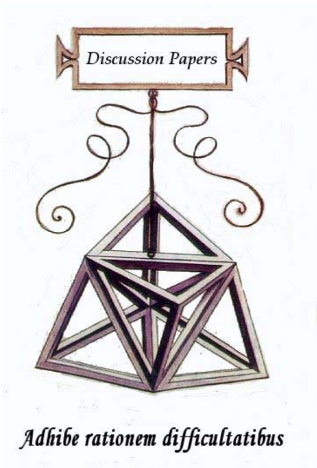

Davide Fiaschi - Pier Mario Pacini

Growth and coalition formation

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Abstract

In this paper we analyse a growth model where agents have different factors'endowments and form coalitions to produce output. Economic growth is the result of accumulation of human capital. The latter is a by-product of production activity within a coalition. The grand coalition corresponds to the maximum efficient agents'allocation. However, due to heterogeneous endowments rich agents could not be an incentive to form a coalition with poor agents if rule governing the division of coalition output states an equal sharing among all members of coalitions. Rich agents tend to form coalitions among themselves and poor agents cannot benefit of positive externalities of coalescing with richer agents. This determines both a lower output and a lower long-run growth rate.

Classificazione JEL: C72, D31, O12, O15
Keywords: coalition formation, growth, stratification, inequality

Contents

I. Introduction

Traditionally growth models consider an economy with one representative agent and one representative firm, so that output depends only on the efficient use of the aggregate supply of factors. However production is generally distributed among a large number of different firms and their output is the result of the joint work of many individuals with heterogeneous endowments. To tackle this aspect the present paper analyses a growth model where agents have different factors' endowments and production is performed within different groups of agents (*coalition* in the sequel), that coalesce conferring their endowments to the group production activity. Thus coalition formation becomes a relevant issue for growth theory (see Aghion, Caroli, and Garcia-Penalosa (1999)).

In this model economic growth is the result of the accumulation of human capital. The latter is a by-product of the production activity within a coalition, i.e. human capital is accumulated via learning by doing. The aim is to catch the positive externality generated by working in a team, where different individual working experiences are mixed together producing in turn new knowledge (*distribution and cross-fertilization effects*). We assume that the diffusion of knowledge is limited to the members of a coalition; thus the *grand coalition* corresponds to the maximum efficient allocation for this economy, i.e. it implies both the maximum output and the maximum growth rate. However, due to heterogeneous endowments rich agents could not have an incentive to form a coalition with poor agents if the division of coalitional output is based on an equal sharing rule among all the members of a coalition. In this case agents' allocation is generally not a Pareto optimum.

Social stratification is a result of our model (see Durlauf (1996)). Rich agents tend to form coalitions among themselves, and poor agents are excluded by the positive externality of coalescing with richer agents. This determines both a lower output in every period and, because diffusion of new knowledge is limited by the coalition structure, a lower long-run growth rate (see for a similar phenomenon Durlauf (1993) and Bènabou (1996)). An economy starting with a more unequal distribution of initial resources tends to persist in that state. More equal societies tends to have a greater long-run growth rate, which agrees with empirical evidence (see Aghion, Caroli, and Garcia-Penalosa (1999)). Numerical simulations show that this finding holds for a wide combination of parameters of the model.

Stratification is a common result in coalition formation games where agents have heterogeneous endowments (see e.g. Fiaschi and Pacini (2001), Gravel and Thoron (2003) and Fiaschi and Pacini (2003)). However this literature is basically concentrated on static equilibrium analysis. To our knowledge this is the first attempt to analyse the dynamic effect of coalition formation with heterogeneous agents. The effect of distribution on long-run growth rate is the subject of a large number of contributes (see e.g. Quah (1997) and Aghion, Caroli, and Garcia-Penalosa (1999)). Quah (1997) presents a setting similar to ours, but applied to coalition formation among countries. Our novelty is to take explicitly into account the issues of the creation and diffusion of knowledge among agents, by providing a new channel by which inequality and growth can interact. Durlauf (1996) is the closed reference to our approach. Finally, Kremer and Maskin (1996) are also closed in spirit to our approach, by focusing on the segregation of workers generated by heterogeneous individual skills.

The paper is organized as follows: Section II. presents the model; Sections III. describes the coalition formation process; Section IV. present the dynamics of the economy. Finally Section V. reports the results of the numerical simulations. Section VI. concludes.

II. The basic model

Consider an economy with a population $\Im$ of N agents indexed by i. At any point in time t agents produce, consume out of their production and from the activity of consumption they get a payoff. The details of the distributive rules, payoffs functions and of the technology available for production will be described later, but the basic idea of the model is that the output of a coalition only depends on the *human capital stocks* of its members. Therefore we abstract from the role of physical capital and we assume that what is relevant for production are inputs such as individual abilities, skills, talents, that agents own and that can be applied to the production process. This kind of capital is accumulated through time out of the same production activity. Individual abilities mix in production and in this way new knowledge is obtained. This increases the amount of human capital available in the following period, according to a standard model of learning by doing (see Arrow (1962)).

Before turning to the explicit description of the behavioural and dynamical aspects of the model let us assume that in any period t any agent is endowed with a certain quantity k_t^i of human capital. Let $k_t = \left\{k_t^1, \dots, k_t^N\right\}$ be the distributions of human capital endowments in any given period t.

II.A. Production and coalitional output

Production is performed by agents coalesced in groups and let us denote by S a single coalition of agents ($|S|$ will denote the cardinality of S). The vector of human capital endowments of the members of S in period t is denoted by $k_t^S = \left\{k_t^i\right\}_{i \in S}$; finally K^S denotes the aggregate input of human capital in a coalition S. Each agent can provide his endowment k_t^i to the coalition S which he is participating into in period t; we assume that an agent cannot be member of more than one coalition in any single period and that his physical and human capital endowments cannot be split and contributed to coalitions different from the one he is participating into.

Given individual contributions by the members of a coalition, we assume that the coalitional output is given by the following production function:

$$Y^S = \left[\sum_{i \in S} \left(k_t^i \right)^\alpha \right]^{\frac{\beta}{\alpha}} \text{ with } \alpha, \beta > 0. \tag{1}$$

The parameter α measures the elasticity of substitution among the human capital endowments of different agents. The higher α the higher is this elasticity. A plausible assumption is $\alpha < 1$, which means that, taken as fixed the aggregate stock of human capital, a more equal distribution of the individual stocks determines higher output[1]. In other words we are assuming that a pair of agents with a Bachelor's degree is more productive than a pair of agents in which one has a doctoral degree and the other has a secondary education degree. Parameter β measures the returns to scale: we assume $\beta > 1$ in order to provide agents the proper incentives to coalesce.

Given the production function (1), we are implicitly assuming that coalitional output depends not only on the aggregate amount of human capital of coalition, but also on the distribution of individual endowments of the coalition's members. In fact, denote $\nu_t^i = \frac{k_t^i}{K_t^S}$ the share of human capital contribution of agent i over the aggregate coalitional input, we observe that the coalitional output can be written as:

$$Y_t^S = \left(K_t^S \right)^\beta \left[\sum_{i \in S} \left(\nu_t^i \right)^\alpha \right]^{\frac{\beta}{\alpha}}, \tag{2}$$

which shows that the coalitional output is an increasing function of the aggregate coalitional input (K_t^S) and depends on the distribution of the human capital within coalition $\left\{ v_t^i \right\}_{i \in S}$. In particular, given K^S, the coalitional output is greater in coalitions with a lower dispersion in the distribution of endowments.[2] This fact is a first

[1] In more technical terms $\alpha < 1$ is the condition that guarantees the convexity of the isoquants of the coalitional production function.

[2] A rigorous proof is in Shorrocks and Foster (1987), where this property of the concave transformation of different distributions is applied to social choice theory.

source of (static) inefficiency when the distribution of resources is unequal. In Section IV. we will show that, in addition to this static effect there exists a dynamical effect of inequality, which tends to lower the growth rate of economy.

II.B. Distribution of coalitional output and agents' preferences

Once output is produced in a coalition S, it is distributed among the participating members. There are many alternatives on how to model the division of coalition output, but, for the sake of simplicity we assume that the so called *equal-sharing* rule holds, i.e. every agent in a coalition receives the same amount of output independently of his endowment[3], that is

$$y_t^i = \frac{Y_t^S}{|S|}. \tag{3}$$

The rule governing the division of coalitional output crucially affects the individual incentives to aggregate. Heuristically, agents with greater human capital endowment have less incentives to coalesce with poor agents when the sharing rule is based on the number of agents instead of individual endowments. Therefore we are considering the worst case for the formation of a coalition including rich and poor agents.

We further assume that the individual payoff function is identical across agents and is linear in the share of the coalitional output that

[3]Such a strong assumption needs some justification. First of all we have to observe that the marginalistic rule could not be applied due to the presence of increasing returns to scale. In general it would seem plausible to assume that the share of coalitional output assigned to an agent should refer to the individual human capital endowment that he conferred to the group production, so that a general formulation of the distributive rule would be as follows:

$$y_t^i = Y_t^S \cdot \left[\frac{\left(k_t^i\right)^\eta}{\Sigma_{i\in S} \left(k_t^i\right)^\eta} \right], \quad \eta \in [0, \infty)$$

so that the higher η the higher is the share of coalitional output assigned to highly endowed agents. The assumption of an exact value of η would need an explicit bargaining foundation, which is beyond the scope of this paper (we refer to Fiaschi, Garrido, and Pacini (2001) for an exploration in this direction), so that for simplicitly purposes we stick to the commonly examined case of $\eta = 0$ i.e. the *equal sharing rule*.

an agent receives out of his participation in a coalition S. Thus the payoff of the agent i, when he participates in a coalition S, is given by:

$$U_i\left(S\right)=\frac{Y_t^S}{\left|S\right|}. \tag{4}$$

III. Coalition formation

In a static setting without human capital accumulation, Fiaschi and Pacini (2001) characterize the strong Nash equilibrium of a coalition formation game similar to the one presented above[4]. This static equilibrium coalition structure shows the so-called *consecutiveness* property (as defined in Fiaschi and Pacini (2001)), that is a coalition with rich and poor agents must also contain agents with average endowments. This implies that there exists a hierarchy among coalitions in terms of average endowments in equilibrium. In a dynamic setting Durlauf (1996) calls this phenomenon *stratification*. However, as we discussed in Fiaschi and Pacini (2001), these results can be little informative on the effective process of coalition formation in which information and coordination problems must be taken into account explicitly.[5] Instead here we model coalition formation as a adaptive dynamic process in which in every period agents decide to coalesce or abandon formed coalitions in order to maximize their individual payoffs. In particular we assume that the coalition structure is the result of two distinct steps:

The exclusion step: in any period t, an agent $i\in S_{t-1}$ (or more than one) can be excluded by S_{t-1} if all other members of S_{t-1} get a strict Pareto improvement by leaving aside i and producing among themselves. Of course this can be interpreted as a decision by the agents in $S_{t-1}/\left\{i\right\}$ to sign a contract for the production and

[4]The most relevant difference is the individual possibility of defecting from cooperation within a formed coalition. We exclude this possibility in the present model.

[5]It is worth remarking that human capital accumulation does not affect the equilibrium, since accumulation is a by product of production activity. Therefore the equilibrium coalition structure just described also represents the dynamic equilibrium of the game.

distribution in the next period which does not include i, thus giving rise to a coalition $S_t = S_{t-1}/\{i\}$. More precisely the exclusion of an agent i from a coalition S_{t-1} takes place if

$$U_h(S_{t-1}/\{i\}) > U_h(S_{t-1}) \;\; \forall h \in S_{t-1}/\{i\} \tag{5}$$

or, given the assumption of an equal sharing distributive rule, if

$$\frac{\left[\sum_{j\in S_{t-1}/\{i\}}\left(k_t^j\right)^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S_{t-1}|-1} > \frac{\left[\sum_{j\in S_{t-1}}\left(k_{t-1}^j\right)^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S_{t-1}|} \;\; \forall h \in S_{t-1}/\{i\}\,. \tag{6}$$

The shrinking effect on the cardinality of a coalition of excluding unprofitable agents is however counteracted by the decision to eventually accept new agents in the surviving coalition if these latter are willing to enter.

The inclusion steps: in order to formalize this point we assume that, in any period t, every agent can decide whether to remain in the coalition S_{t-1}^i which he was a member of or to move to another coalition S_{t-1} if $U_i(\{S_{t-1}\cup i\}) > U_i\left(S_{t-1}^i\right)$. Both decisions can be implemented without costs, but the last one is subject to the approval of the members of the receiving coalition S_{t-1}, who will accept the incoming agent only if their payoffs do not decrease by accepting him. All other agents remain stick to their previous decisions. Of course an agent is always free to leave a coalition and form a new one in which he is the unique member, if this is profitable for him.[6] More precisely, given a coalition S_{t-1} and an agent $i \notin S_{t-1}$, a new coalition $S_t = \{S_{t-1}\cup i\}$ will form if

$$U_h(S_t) \geq U_h(S_{t-1}) \;\; \forall h \in S_{t-1} \text{ and } U_i(S_t) > U_i\left(S_{t-1}^i\right). \tag{7}$$

III.A. The formation of the grand coalition and the equilibrium coalition structure

The *grand coalition* is the most efficient coalition structure in terms of aggregate output. In general, given the conditions for ag-

[6]Since the timing of agents'choices could affect the final coalition structure, the order according to which agents make their choice is randomly decided.

gregation as in (7), the grand coalition will be eventually accepted provided that the payoff that anyone will receive in it is at least equal to the payoff he gets in any possible smaller coalition S, i.e.

$$U_i(\Im) \geq U_i(S) \;\; \forall i \in \Im \text{ and } \forall S \subset \Im,\, i \in S \tag{8}$$

with strict inequality holding for at least an i. Given the equal sharing distributive rule, condition (8) becomes:

$$\frac{\left[\sum_{i=1}^{N}\left(k^i\right)^{\alpha}\right]^{\frac{\beta}{\alpha}}}{N} \geq \frac{\left[\sum_{i\in S}\left(k^i\right)^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S|} \Rightarrow \frac{\sum_{i=1}^{N}\left(k^i\right)^{\alpha}}{\sum_{i\in S}\left(k^i\right)^{\alpha}} \geq \left[\frac{N}{|S|}\right]^{\frac{\alpha}{\beta}} \;\forall S \subset \Im. \tag{9}$$

In terms of ν^i, i.e. the individual share of the total stock of human capital of coalition, condition (9) can be rewritten as

$$\left[\frac{K^N}{K^S}\right]^{\alpha} \frac{\sum_{i\in N}\left(\nu_i\right)^{\alpha}}{\sum_{i\in S}\left(\nu_i\right)^{\alpha}} \geq \left[\frac{N}{|S|}\right]^{\frac{\alpha}{\beta}} \;\forall S \subset \Im. \tag{10}$$

Condition (10) makes clear that, given the number of agents N and the aggregate stock of human capital K^N, the distribution of resources is a key factor in the possibility of forming the grand coalition (e.g. if an agent has no human capital then grand coalition never form).

An equilibrium for our economy is a coalition structure σ, i.e. a partition of $\Im$ in coalitions S, such as no agent (or group of agents) has an incentive to change the coalition which he belongs to, i.e. no new coalition can form such that (7) holds and no one can be profitably excluded by a coalition, i.e. for no existing coalition condition (5) applies. Proposition 1 (for a similar result in a sligthly different setting see Fiaschi and Pacini (2001)) confirms that grand coalition is an equilibrium when resources are evenly distributed:

Proposition 1 *When the distribution of individual stocks of human capital is even and the inclusion and exclusion from coalitions respectively follow the rules (7) and (5), then the grand coalition is an equilibrium coalition structure.*

Proof. See Appendix A. ∎

Proposition 1 says that the most efficient allocation of agents in coalitions is obtained when human capital stocks are evenly distributed. However, we remark that the process of coalition formation we described above does not ensure that this will take place in just one period, and indeed more periods may be needed. In this latter case, even starting from an equal society, the accumulation of human capital may introduce differences in agents' endowments so that we cannot grant that grand coalition will form.

IV. Dynamics

Dynamics is driven by the accumulation of human capital. In this section we describe the properties of such accumulation process, based on the insights of Arrow (1962).

IV.A. Individual human capital accumulation

Human capital accumulation is the result of the interaction among agents in the production activity within a coalition. We assume that new knowledge and (and its conversion into human capital) are produced in every period mixing the competencies of the different agents participating into a coalition: *learning by doing* is the source of growth in our economy (see Arrow (1962)).[7] Moreover, we assume that the diffusion of knowledge is limited to the members of a coalition.[8] This assumption is crucial for our results since it makes the growth of the economy to depend on the coalition structure. In particular, we assume that:

[7] We could consider an alternative formulation, where time not spent in production is devoted to accumulation of human capital, like in Lucas (1988); however this would imply to set up an intertemporal framework; on the contrary here learning by doing is only a by-product of production activity.

[8] This can happen either because knowledge is excludible, or because the newly created knowledge is of an implicit and personal type that cannot be appropriated by others (e.g. capacity of performing tasks with greater speed).

$$k_{t+1}^i - k_t^i = \frac{A\left[\sum_{j\in S}\left(k_t^j\right)^\theta\right]^{\frac{\gamma}{\theta}}}{|S|} - \delta k_t^i, \tag{11}$$

where δ is the depreciation rate of human capital; we assume that $A \geq \delta \geq 0$. $A\left[\sum_{j\in S}\left(k_t^j\right)^\theta\right]^{\frac{\gamma}{\theta}} / |S|$ represents the new knowledge produced in the coalition, which increases the human capital stocks of its members. Equation (11) implies that two agents with different endowments belonging to the same coalition will see their human capital stocks converging in the long-run.[9]

The analysis of the production function of new human capital, which has properties similar to the coalitional production function of Section II.A., provides us with some restrictions on the parameters' values. In particular:

- parameter γ measures the returns to scale of the technology producing new human capital by means of human capital; in the following we will assume $\gamma = 1$ in order to have long-run equilibrium where human capital can grow at a constant rate. The intuition is the following: from equation (11) and under the assumption $\gamma = 1$, we have:

  $$\frac{k_{t+1}^i - k_t^i}{k_t^i} = g_t^i = \frac{A\left[\sum_{j\in S}\left(k_t^j\right)^\theta\right]^{\frac{1}{\theta}}}{k_t^i\,|S|} - \delta;$$

  in the case of an even distribution of resources, i.e. every member of the coalition S has the same human capital stock $\bar{k}_t^S$, the growth rate of individual human capital stock is given by:

  $$g_t = A\,|S|^{\frac{1}{\theta}-1} - \delta, \tag{12}$$

  which is constant, given the size of the coalition S.

[9]In this respect the formation of coalitions tends to decrease inequality.

- The distribution of the human capital endowments within a coalition affects the level of production of new knowledge. The effect of this diversity is determined by the parameter θ. For example, taken a coalition S with two agents, i and j, where $k^i > k^j$; then the production of new knowledge under the assumption $\gamma = 1$ is given by:

$$\frac{K^S A \left[\left(v^i\right)^\theta + \left(1 - v^i\right)^\theta \right]^{\frac{1}{\theta}}}{2},$$

  which is decreasing (increasing) in v^i (remember that $v^i > 1/2$) when θ is lower (greater) than 1.[10] We term this as the *distribution effect*. The most plausible economic scenario is that agents with more equal human capital stocks, taken as given the aggregate stock, produces an higher amount of new knowledge. Therefore we assume that $0 < \theta < 1$.

- The number of members of a coalition affects the level of new produced knowledge. Consider a coalition S, whose members have the same human capital stock $\bar{k}^S$, then the growth rate of human capital, under the assumption $\gamma = 1$ is given by:

$$g_t = A \left| S \right|^{\frac{1}{\theta} - 1} - \delta.$$

  Under the assumption $0 < \theta < 1$ we have that $\frac{1}{\theta} > 1$, therefore the produced new knowledge and the growth rate of human capital stock is positively related with the number of the members of coalition. We term this as the *cross-fertilization effect*. In economic terms this means that every agent, independent of his level of human capital stock, has a different type of human capital. The higher the number of these different types the higher is the production of new knowledge.

We notice that the distribution and the cross-fertilization effects are positively related; in particular, the lower is θ the higher are both effects; relaxing the assumption $\gamma = 1$ would allow for more complex relationships.

[10] The simple derivative with respect to v^i proves this statement.

IV.B. Economy with an even distribution of resources

The interaction between the process of coalition formation described in Section III. and the accumulation of human capital makes the analytic characterization of the dynamics of this economy particularly difficult. For this reason we investigate the dynamic properties by numerical simulations. However, it can be useful to characterize the dynamics in the case of an even distribution of resources, which leads to the formation of grand coalition. In fact, as we will see, in this case the economy can reach both the maximum production in every period (static efficiency) and the maximum growth rate (dynamic efficiency). This will be the benchmark case for our numerical results.

Assume that all agents have the same initial endowment of human capital, i.e.

$$k_0^i = \bar{k} \; \forall i \in \Im.$$

We know that the equilibrium coalition structure in this case is the grand coalition (see Proposition 1). Therefore the per-capita output of the grand coalition (equal to agents' payoff) is given by:

$$y^N = N^{\frac{\beta}{\alpha}-1}\bar{k}^{\beta}, \tag{13}$$

which is positively related to N ($\beta > \alpha$ by assumption). The growth rate of human capital of every agent is given by:

$$g = AN^{\frac{1}{\theta}-1} - \delta, \tag{14}$$

which is positively related to N ($\theta < 1$ for hypothesis). Since every agent will have always the same endowment of human capital, i.e. $\bar{k}_t^i = \bar{k}(1+g)^t \; \forall i$, the growth rate of average output is given by:

$$g^y = (g+1)^{\beta} - 1 = \left[AN^{\frac{1}{\theta}-1} + (1-\delta)\right]^{\beta} - 1. \tag{15}$$

It is to remark that the positive relationship between g^y (and g) and N is due to the assumptions that we made about the creation of new knowledge (11); the greater is N the greater is the diversity of competences available in society and hence the greater is also the amount of new knowledge produced by mixing those competences. This is a common result in growth literature (see Jones (2003)).

Equation (13) is the maximum average output for an economy with an average endowment of human capital equal to $\bar{k}^{\beta}$, while equation (15) is the maximum growth rate of the average output for an economy with N agents.

V. Numerical simulations

Numerical simulations is the approach we follow to investigate the properties of this economy. This decision is motivated by the difficulties to get an analytical characterization of the dynamics of this economy (apart from the case of an even distribution of resources). Since agents change, disrupt and form new coalitions depending on the incentives that they receives in terms of payoffs, human capital accumulation gives rise to different trajectories depending on the coalition that different agents may be visiting during the evolution of the system. Path-dependence is therefore likely in this economy. This continuous change in the individual endowments may change the incentives that an agent receives to remain in a coalition or rather to move to other groups; this, in turn, may change the rate of growth of the human capital endowment of the entering individual, if he is accepted. As a consequence we cannot predict when the coalition formation process evolves toward a less unequal (and more performing) economy.

V.A. The steps of numerical simulations

Before presenting the results of the numerical simulations it can be useful to sum up the steps occurring in any period t:

1. given a certain coalitional structure $\sigma_{t-1} = \left(S_{t-1}^{1}, \ldots, S_{t-1}^{M}\right)$ formed in the last period, in every coalition S_{t-1}^{m} $(m = 1, \ldots M)$ members decide if to exclude some agents. We remember that an agent $i \in S_{t-1}^{m}$ can be excluded if all other members receives a positive net benefit in terms of payoff from his exclusion.

Typically the candidates to the exclusion will be the lowest endowed people in a coalition. Once excluded from a coalition, an agent forms temporarily a singleton coalition until the next step takes place. The structure of society after exclusion is common knowledge.

2. Given the current situation all agents, in a random sequence, formulate a proposal to adher to a coalition in order to improve their own position. Every agent chooses to move to the coalition that grants him the highest payoff among those that are willing to accept him.[11]

3. The current period coalitional structure σ_t forms and every agent receives his payoff on the basis of the produced coalitional output as in (1) and of the ruling distributive rule as in (3).

4. The human capital of every agent modifies according to (11) and period t is finished.

V.B. Common parameters to all numerical simulations

The following values of some basic parameters are kept constant throughout all numerical simulations: The caridinality N of the population is set equal to 100 agents. All initial distributions of endowments are drawn by a normal distribution with mean 30.[12] Every simulation runs for 50 periods. Moreover, for every configuration of parameters we run 20 different simulations in order to eliminate possible random disturbances. Finally, we set $A = \delta = 0.02$, so that agents that perform in isolation do not have any change in their stocks of human capital.

[11] We assume that if an agent cannot find a coalition in which his payoff is greater than the one currently received, then he does not formulate any proposal and remains in the coalition which he belonged to.

[12] Once the mean of the initial distributions of endowments is kept fixed, a change in the standard deviation directly reflects a change in the inequality of the distribution of initial endowments.

V.C. Results

The results of numerical simulations are summarized in three main groups. The first concerns the effects on the long-run outcomes of different parametrizations of the coalitional production function. The second group examines how the same long run outcomes are affected by different parametrizations of the law governing the accumulation of human capital. Finally, the third group analyses the effects on the long run outcomes of different initial distributions of endowments.

V.C.i. Increasing returns to scale vs elasticity of substitution in the coalitional production function

In this section we analyse the effects of different values of α and β (see equation (1)). In particular, we set $\theta = 0.7692$ and consider 400 possible combinations of α and β, with $\alpha \in [0.525, 1]$ and $\beta \in [1.005, 1.1]$ for three different values of the standard deviation of the initial distribution of resources, namely 2, 5 and 8. Figure 1 reports the results as to (i) the average growth rate of human capital stocks (first column), (ii) the Gini index of the distribution of endowments (second column) and (iii) the number of coalitions (third column), all computed in the last period of the simulation.
The first row of Figure 1 reports the results when the initial distributions of endowments is generated from a normal distribution with standard deviation equal to 2, the second row when the standard deviation is set equal to 5 and the third row when the standard deviation is set equal to 8. The Gini index for the three initial distributions are equal to 0.0387, 0.0966 and 0.1544 respectively; the maximum attainable growth rate of human capital (obtained when the grand coalition forms) is the same for all simulations and equal to 0.0596 (see equation (14)).

The value of the parameter α seems to have a strong impact on the long-run outcomes. An increase in the elasticity of substitution, given the magnitude of the returns to scale β, decreases the incentive to coalesce. This determines an increase in the inequality

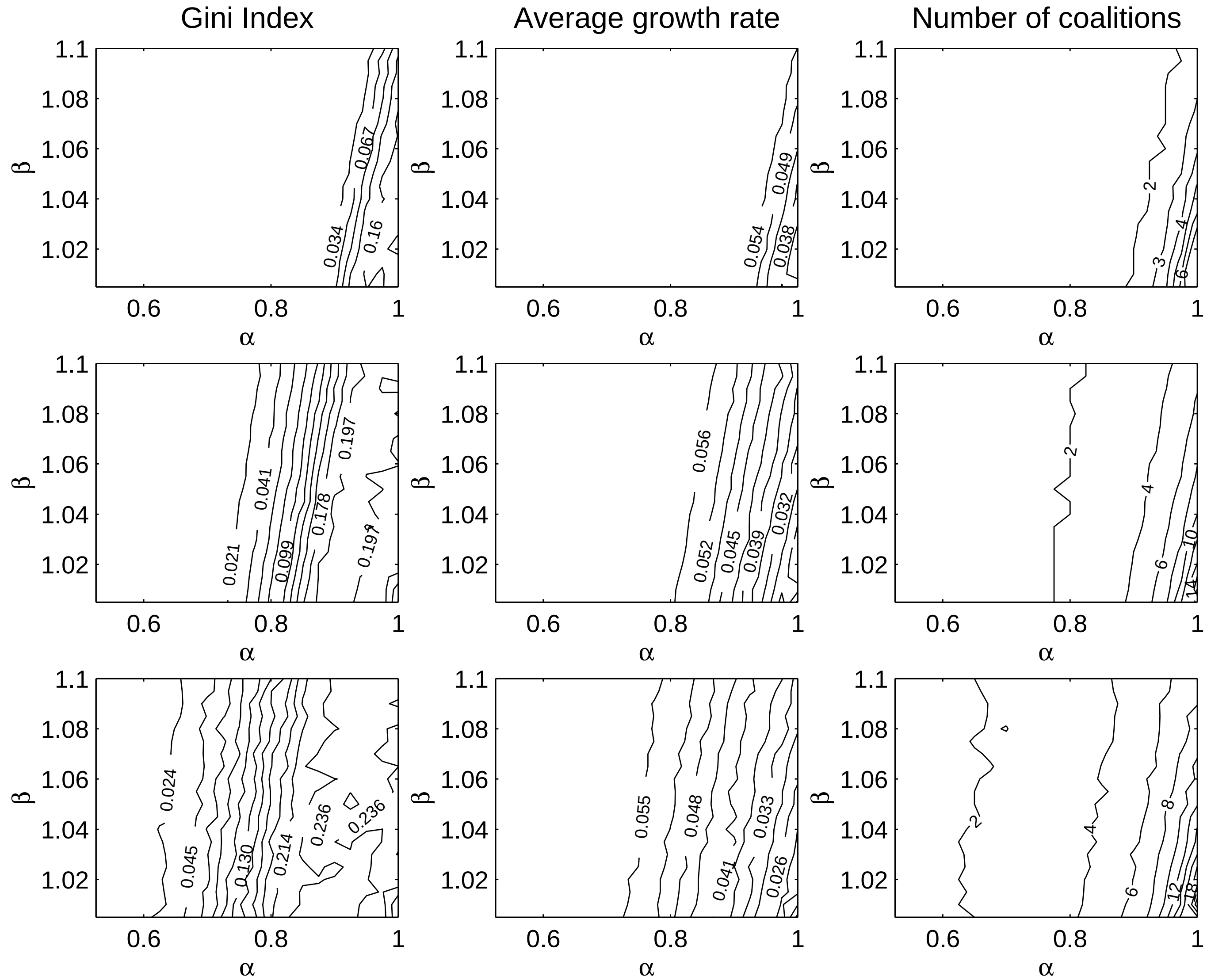

Figure 1: Increasing returns to scale vs elasticity of substitution

of the final distribution of endowments (measured by the Gini index), a decrease in the growth rate and an increase in the number of coalitions in the coalition structure of the last period. Of course, these three aspects are related: the higher the number of coalitions the lower is the efficiency in the accumulation and the higher is the final inequality.

On the other hand, given a high value of α, an increase in the returns to scale β, favouring the aggregation among agents, tends to decrease inequality and to increase the efficiency of economy, which is reflected in a higher growth rate of human capital and in a lower number of coalitions.

The comparison of the results for different values of the standard deviation in the initial distributions of endowments shows that an higher initial inequality causes an enlargement of the region in the parameters' space (α, β) in which allocation is inefficient and, given the same combination of parameters, a lower efficiency. This suggests that similar economies, i.e. with the same parameters' values, but different initial distributions of resources, can show very different long-run behaviours. The dynamics of our economy appears to be very sensitive to initial condition, i.e. there is path dependence.[13]

Finally, the comparison between the initial and final values of the Gini index for the distribution of endowments shows that the process of coalition formation can lead to an increase in inequality for a wide range of parameters (high α and low β). Since a higher inequality means a lower efficiency, this result can justify the public intervention, for example in the education sector in order to improve the long-run outcome (see Durlauf (1996)).

V.C.ii. Accumulation of human capital

In this section we analyse the interaction between α and θ (the parameters measuring the distribution and cross-fertility effects). We consider three alternative values of α (namely 0.7, 0.83 and

[13] In the growth's literature there are many models where long-run equilibrium strongly depends on the initial condition, see Aghion, Caroli, and Garcia-Penalosa (1999).

0.975) and 35 values of θ evenly distributed in the range $[0.66, 1]$. We recall that to every θ corresponds a different maximum attainable growth rate, and, given the assumed values of θ the latter ones are in the range $[0, 0.1944]$ (the higher is θ the lower is the maximum growth rate). For all simulations we set the standard deviation of initial distributions equal to 5 (corresponding to a Gini index of 0.0966). Figure 2 reports the results.

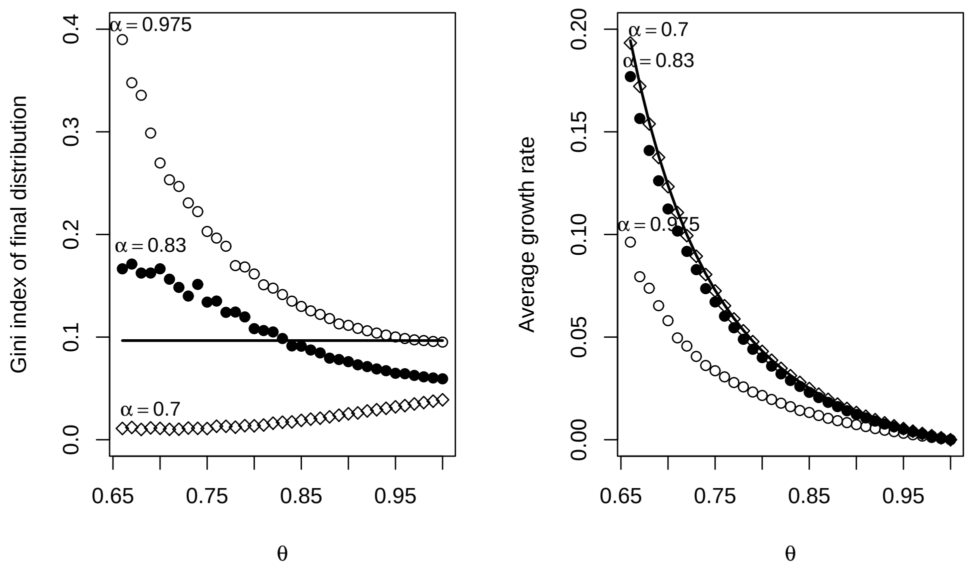

Figure 2: Accumulation of human capital

The pictures on the left and on the right respectively show the Gini index of the final distribution of endowments and the average growth rates for the three different values of α, all computed in the last period. The horizontal solid line in the picture on the left reports the Gini index of initial distribution equal to 0.0966, while the solid line in the picture on the right reports the maximum growth rate for every θ. For low values of α (i.e. 0.70) the incentives to coalesce are so strong that human capital accumulation tends to equalize all individual endowments reducing inequality below the level characterizing the initial distribution of endowments. The grand coalition is always the long run outcome and the economy growths at the maximum rate.

As α increases the effects of the accumulation process becomes ambiguous. For low values of θ, i.e. for high values of the distribution and cross-fertilization effects, the initial inequality tends to raise. Indeed for this value of α the incentives to coalesce between rich and poor agents are low, so that the aggregation in coalitions mostly regards rich agents. The latter benefit of the high distribution and cross-fertilization effects, which causes a further increase in the inequality. In turn, this higher inequality means still less future incentive to social aggregation. The average growth rate of the last period reflects this dynamics. Indeed the growth rate is far from the maximum attainable growth rate and the distance is decreasing in the value of θ (see in particular the case $\alpha = 0.975$).

Therefore an increase in the distribution and cross-fertilization effects can have both positive and negative effects on the long-run outcome, depending on the strength of the incentives to coalesce. An economy whose potential growth rate is high (low θ) could show lower growth rates with respect an economy whose potential growth rate is low (high θ) if the incentives to coalesce in the first economy are lower (α of the first economy is greater).

V.C.iii. Initial distribution of endowments and long-run equilibrium

The previous findings suggest that there is an inverse relation between the inequality in the initial distribution of endowments and the long-run growth rate. To test this hypothesis we take $\alpha = 0.85$, $\beta = 1.045$ and $\theta = 0.7692$ and consider 51 different initial distributions, whose standard deviations are evenly distributed in the range $[0, 10]$ (the Gini indices of these distributions are in the range $[0, 0.19249]$). The maximum attainable growth rate of human capital is always the same for all the simulations and is equal to 0.0596. Figure 3 reports the results.

In the picture on the left we report the Gini index of the initial distribution of endowments versus the Gini index of the final distribution (the solid line is the bisector). The process of coalition formation can have two effects on inequality: for low initial level of

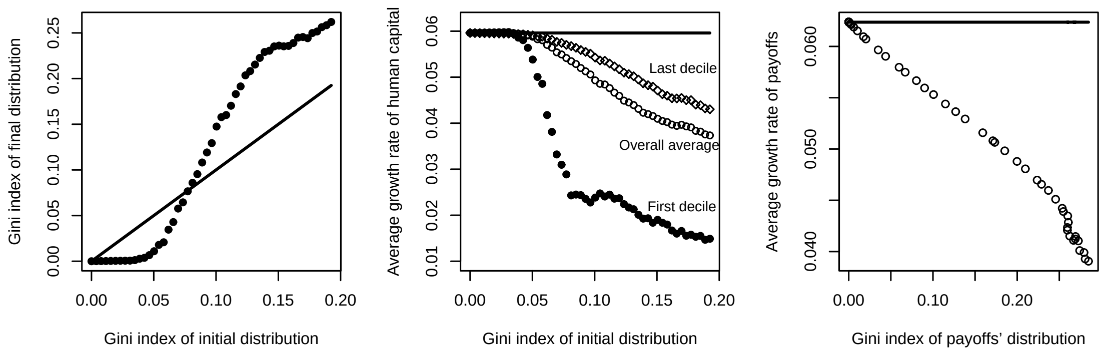

Figure 3: Initial distribution of endowments and long-run equilibrium

inequality it leads to a decrease, while the opposite occurs for a high level of inequality in the initial distribution. The intuition behind this phenomenon is the following; when initial endowments are fairly distributed among agents there are no obstacles to the formation of coalitions among rich and poor agents: therefore the economy shows a further decrease in inequality. However, when initial inequality is sufficiently high then only rich agents form coalitions and benefit of the distribution and cross-fertilization effect. This is evident from the central picture in Figure 3. In the last period the endowments of the last decile of the population (the richest agents) grow at a rate which is twice the rate of the first decile of the population (the poorest agents). This affects the overall efficiency, which worsens as initial inequality increases. In the central picture the solid line reports the maximum attainable growth rate equal to 0.0596; the difference between the latter and the average growth is increasing as Gini index increases.

Finally, in the picture on the right we report the Gini index of the final distribution of payoffs versus the average growth rate of payoffs in the last period. Solid line reports the maximum attainable growth rate of payoffs equal to 0.0624 (see equation (15)). We observe a strong inverse relationship, in accordance with theoretical results and empirical evidence in the growth literature (see Aghion, Caroli, and Garcia-Penalosa (1999)).

VI. Concluding remarks

A greater inequality in the initial distribution of resources determines a lower long-run growth rate of economy: this is the main finding of the paper. A greater income (payoff) inequality is related to a lower growth rate of income (payoffs). This result is not new in the growth literature (see Aghion, Caroli, and Garcia-Penalosa (1999)), but we propose a new explanation: *the exclusion of poor agents from coalitions including rich agents.* This fact limits the diffusion of new knowledge and impede poor agents to increase their human capital. The process of coalition formation can lead to the emergence of a stratified society. We find that an economy characterized by a high level of inequality in the initial distribution of productive resources has poor performances with respect to more equal societies. This phenomenon appears to be persistent and only public intervention can allow an economy to escape from this trap.

This model is a first step in the analysis of the importance of coalition formation in the development process. Future extensions should include the analysis of alternative distributive rule, as well as the introduction in the model of other productive factors, as physical capital. A change in the institutional setting which governs the distribution of coalitional output in favour of more skilled agents has two competing effects: on the one hand it increases the incentives to coalesce in the period, so increasing static efficiency; on the other hand it can increase inequality, decreasing the future incentive to coalesce and therefore the long run growth. The presence of many productive factors can shed light on the effects on growth of the possible complementarities/substituibilities among factors, as well as on which factors a country can base its development (e.g. the accumulation of physical and/or of human capital).

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Appendix

A Proof of Proposition 1

Let $\bar{k} = K^N/N \; \forall i$ be the distribution of endowments and suppose that σ be an equilibrium coalition structure, where there are two coalitions S and S' such that $N > |S| \geq |S'| > 0$. In the following we show that (a) the exclusion condition (6) never applies to any S, (b) any agent in S' has an incentive to move to S and (c) no agent in S has an incentive to object to his entrance in S. Therefore σ is not an equilibrium coalition structure. This holds for every possible coalition structure with at least two coalitions; therefore grand coalition is the only equilibrium coalition structure.

In order to prove the statement (a) is sufficient to prove that the shrinking of a coalition is never profitable for the remaining agents. Indeed suppose to take a subcoalition $G \subset S$: then agents in G would be excluded if

$$\frac{\left[\sum_{j \in S/\{G\}} \bar{k}^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S| - |G|} = (|S| - |G|)^{\frac{\beta}{\alpha}-1} \bar{k}^{\beta} > \frac{\left[\sum_{h \in S} \bar{k}^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S|} = |S|^{\frac{\beta}{\alpha}-1} \bar{k}^{\beta},$$

which cannot be satisfied for $\beta > 1 > \alpha$.

To prove point (b) take an agent $i \in S'$: he will move toward S' if

$$\frac{\left[\sum_{j \in \{S \cup i\}} \bar{k}^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S| + 1} = (|S| + 1)^{\frac{\beta}{\alpha}-1} \bar{k}^{\beta} > \frac{\left[\sum_{j \in S'} \bar{k}^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S'|} = |S'|^{\frac{\beta}{\alpha}-1} \bar{k}^{\beta},$$

which is always satisfied provided $\beta > 1 > \alpha$.

Next we show point (c), i.e. no agent in S has an incentive to object to the new entrant i. Indeed there would be an objection if for some $j \in S$ the following would verify

$$\frac{\left[\sum_{j \in \{S \cup i\}} \bar{k}^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S| + 1} = (|S| + 1)^{\frac{\beta}{\alpha}-1} \bar{k}^{\beta} < \frac{\left[\sum_{j \in S} \bar{k}^{\alpha}\right]^{\frac{\beta}{\alpha}}}{|S|} = |S|^{\frac{\beta}{\alpha}-1} \bar{k}^{\beta};$$

however this is impossible when $\beta > 1 > \alpha$.

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