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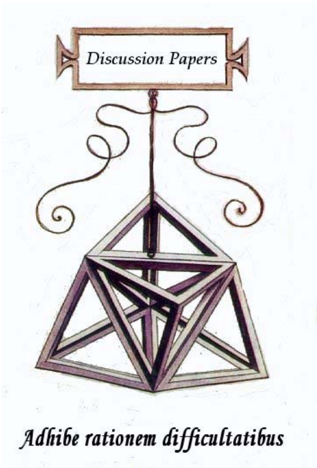

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Nonlinear Economic Growth: Some Theory and Cross-Country Evidence

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Nonlinear Economic Growth: Some Theory and Cross-Country Evidence

Abstract

The paper aims to test the existence of different regimes in the growth process. We propose a simple nonlinear growth model which features different relationships between growth rate and income level. We identify its testable implications in terms of state space dynamics. By estimating Markov transition matrices we show that growth is indeed a nonlinear process. Economic growth proceeds by alternating phases of acceleration and deceleration. We discuss the relevance of these results with respect to the issue of income convergence across countries and models of technological diffusion.

Classificazione JEL: O11, O40, C14, C21.

Keywords: nonlinear growth, distribution dynamics, convergence, structural change, technological diffusion.

Contents

I. Introduction

This paper is a contribution to the debate on the empirical analysis of economic growth, recently surveyed by Durlauf and Quah (1999) and, in particular, focuses on nonlinearities in the growth process.

Different theories suggest that economic growth is nonlinear (see Lewis (1956), Rostow (1960), Mas-Colell and Razin (1973), Murphy et al. (1989), Peretto (1999) and Galor and Weil (2000)). According to this approach, the growth path of an economy displays an initial phase of stagnation, followed by a take-off in which growth rates are increasing, and eventually reaches a regime of steady growth. These different growth regimes, associated to different levels of development, are generated by the structural transformations faced by a growing economy.[1] This also implies that an economy not showing the proper conditions for the take-off can remain trapped in a long period of stagnation. In this framework the influence of international technological spillovers on the growth process of a country is generally negligible, with respect to the internal sources of accumulation.

Differently, another important strand of research on growth focuses on different kinds of interactions which may take place among economies. This literature devoted particular attention to technological spillovers (see e.g. Parente and Prescott (1994) and Basu and Weil (1998)). These contributions analyse the conditions that allow a country, starting its development process, to benefit from the knowledge accumulated by richer countries, and therefore increase its growth rate.[2] Lucas (2000) provides a very simple model of this

[1]For example Peretto (1999) argues that a nonlinear growth process is the result of the transition from growth generated by capital accumulation, subject to decreasing returns to scale, to growth based on knowledge accumulation.

[2]In particular, Basu and Weil (1998) focus on the concept of "appropriate technology". They argue that technological progress can be hampered or slowed down because technology is specific to capital/labour ratios (capital includes both physical and human capital), so that the technology of leader countries cannot quickly diffuse in backward countries. Alternatively, Parente and Prescott (1994) suggest that various type of barriers (e.g. legal) may check the international diffusion of technology.

process.[3] In this setting a nonlinear growth path could be the result of different adoption speeds, a feature not present in Lucas (2000), when the speed increases as a country develops.

In a previous paper (Fiaschi and Lavezzi (2003)) we study the issue of nonlinearity and convergence adopting the distribution dynamics approach. There we follow the current literature and use relative per capita GDP: that is incomes are expressed with respect to world average income. The underlying justification for this normalization is the existence of a world trend of technological progress which benefits all countries. In this paper our aim is to detect the possible nonlinear growth dynamics of a country related to its own development process (i.e. there may exist threshold effects not related to other countries' growth). Hence, the (absolute) level of development of a country is the key variable and in this regard here we consider the level of per capita GDP, without any reference to a world GDP trend.

We show that growth appears indeed as a nonlinear process. We identify three income ranges characterized by a different relation between income and growth rate. At low income levels the relation is negative or flat, at intermediate levels it is positive and, finally, at high income levels it is again negative. In particular, countries in the intermediate income range appear to experience rapid growth (take-off) with increasing growth rates. This contrasts with Barro and Sala-i-Martin (1999)'s claim that no evidence exists of "a middle range of values of k [capital]...for which the growth rate [of capital] γ_k is increasing in k and, hence, in y [income]". However, our description of long-run growth behaviour fits the facts only for a subset of countries, as another subset appears lagging behind. At this stage we cannot discriminate whether this phenomenon is permanent or temporary, even if the number of countries in our sample showing a slow/negative growth in per capita GDP is not negligible.

In the literature on distribution dynamics the state space is defined only in terms of income levels (see, e.g., Quah (1993)), but

[3]Lucas also mentions another reason why poor countries can grow more than rich countries: the flows of resources from the rich to poor countries due to diminishing returns to accumulation.

this fails to reveal the *shape* of the growth process. In contrast, here and in Fiaschi and Lavezzi (2003) we use a novel definition of the state space, which jointly takes into account income levels *and* growth rates. This allows us to capture the presence of nonlinearities. We first present a graphical analysis, providing evidence of a range of income levels where growth rates are increasing. Then we present a growth model with nonlinearities and derive its empirical predictions in terms of *state space dynamics*. Finally, we test for these predictions by estimating Markov transition matrices.

The application of this technique to data on the level of per capita GDP requires particular attention in the interpretation of the results. The definition of income classes with per capita GDP can make the conclusions on the long run questionable if one considers the possibility that in the long run all countries benefit from a common technological trend. However, this fact cannot be taken as granted in our period of observation (many African countries show no growth over a period of 50 years).

Our theoretical model is close in spirit to Romer (1986) and Zilibotti (1995). Although nonlinearities are crucial features of the new growth theory, empirical practice mostly concentrates on linear models (see Durlauf (2001)). Exceptions are Durlauf and Johnson (1995), Liu and Stengos (1999) and Kalaitzidakis et al. (2001), but these papers focus on the nonlinear effects of some explanatory variables. In particular, Liu and Stengos (1999) and Kalaitzidakis et al. (2001) provide some evidence consistent with our result of the existence of an income range where the growth rate is increasing, but their analysis regards initial income levels.

The rest of the paper is organized as follows: Section II. contains the preliminary graphical analysis; Section III. discusses the empirically testable implications of a growth model with nonlinearities; Section IV. reports the quantitative results; Section V. concludes.

II. Graphical Analysis

The basic unit of observation in this paper is the annual per capita GDP level and the corresponding annual growth rate; data are from Maddison (2001) and refer to 122 countries for the period 1950-1998.[4] In Figure 1 we plot the growth rate against per capita GDP for all observations in the sample, and a nonparametric estimation of the relationship between these two variables.[5]

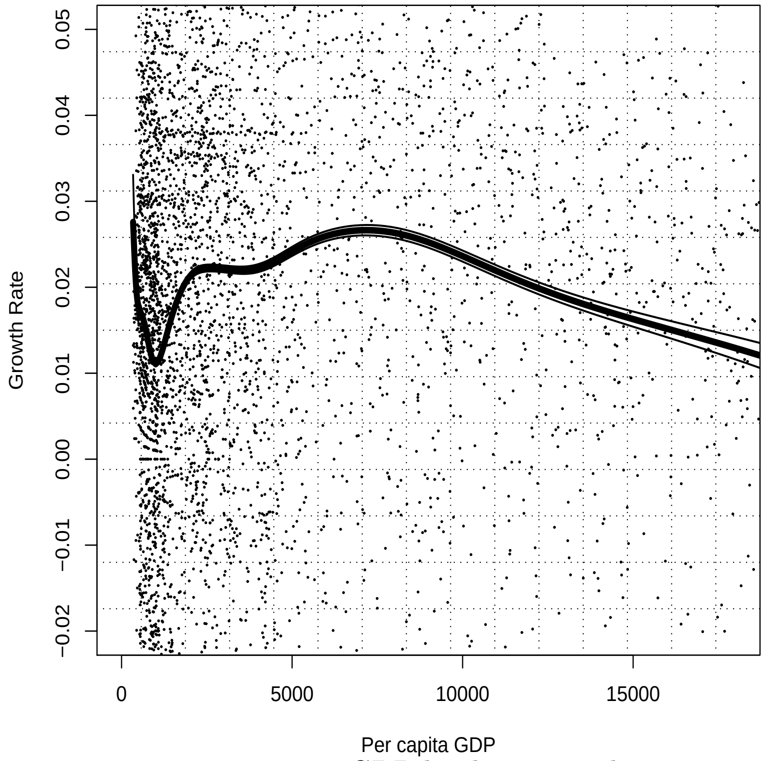

Figure 1: per capita GDP levels vs growth rates

The nonparametric regression in Figure 1 identifies a nonlinear relationship between the level of GDP and the growth rate. In particular, growth is initially high and then decreasing for the lowest

[4]Appendix A contains the country list.

[5]The nonparametric estimate is obtained with the statistical package included in Bowman and Azzalini (1997). We used the standard settings suggested by the authors (i.e. optimal normal bandwidth and weights on observations according to their density). To test the robustness of this estimate, we ran an alternative nonparametric regression by using the plug-in method to calculate the kernel bandwidth, and obtained a similar picture. We refer to Bowman and Azzalini (1997) for more details. Data sets and codes used in the empirical analysis are available on the authors' websites (http://www-dse.ec.unipi.it/fiaschi and http://www-dse.ec.unipi.it/lavezzi).

levels of GDP, quickly reaching a minimum. The subsequent range of GDP is characterized by increasing growth rates that, after reaching a peak, show a tendency to decrease. In the last range of GDP this decreasing growth rate shows a tendency to settle at a steady positive value. It is worth remarking that Figure 1 also suggests the existence of an inverse relationship between growth volatility and the level of GDP.[6]

In Figure 1 we do not control for cross-country heterogeneity. However, Kalaitzidakis et al. (2001) estimate a semiparametric regression, in which they consider as explanatory variables for the GDP growth rate the population growth rate, the investment ratio and the initial income, allowing the latter to enter nonlinearly (they use a smaller set of countries for a shorter time period). They find a similar shape for the relationship between growth rate and income (see their Figure 1).

By looking only at Figure 1 one may conclude that every country tends to grow in the long run. In fact, the estimate always lies above the x-axis. However, this conclusion is questionable if we take into account the growth performances of many African countries, which appear stuck at relatively low GDP levels. The forty-two African countries in the sample had an average growth rate of 1.0% over the period, against an average 2.2% of non-African countries. Within the set of African countries, eight countries had a negative growth rate and eighteen countries had a growth rate lower than 0.5% (half of the group average).

The simple model by Lucas (2000) is compatible with this evidence for what concerns the aggregate picture, but not for the behaviour of individual countries (in general, this remark holds for all models which consider relative backwardness as a possible advantage for a country). In particular, Lucas' model predicts that, when lagging countries start growing, they are expected to jump to a very high growth rate, which eventually converges to the growth rate of the leading countries. For African countries, it seems that

[6] We investigate this aspect in a companion paper, see Fiaschi and Lavezzi (2003b).

the growth process started (overall their income grew by 60% in the period), but their income path is generally very volatile and on average rather flat.

In general, the pooling of cross-country data can mislead the researcher in the identification of actual growth patterns, as it identifies a "representative" growth path. In this regard we will see that the analysis based on transition matrices allows us to avoid such mistakes, since it permits us to keep track of the growth path of each individual country.

III. A simple growth model

In this section we present a simple growth model that can account for the facts emerging from the previous graphical analysis. Then, we derive its empirical implications, which are tested in Section IV..

We consider a simple Solovian growth model with no exogenous technological progress, in which the production function exhibits increasing returns to scale within a certain income range (Appendix C describes the model in more details). Figure 2 depicts this economy under the assumptions that average capital productivity does not decrease so much to generate a poverty trap, and remains sufficiently high for high levels of capital to ensure positive growth in the long run.

In Figure 2 y represents the level of per capita income and $\frac{\dot{y}}{y}$ its growth rate. The model has no equilibria and per capita income tends to grow indefinitely. There exists a region of increasing growth rate with respect to income, i.e. $[\bar{y}_I, \hat{y}]$ and two regions of decreasing growth rates, i.e. $[0, \bar{y}_I]$ and $[\hat{y}, \infty]$. Trajectories A and B indicate alternative growth paths at low income levels. In particular, trajectory B generates a poverty trap in GDP class I. The empirical relevance of these trajectories is discussed in Section IV..

This type of dynamics may be generated by different mechanisms. Traditional development theories (Rosenstein-Rodan (1943), Lewis (1956) and Rostow (1960)) emphasized structural change without a formal analysis, provided more recently by Mas-Colell and Razin

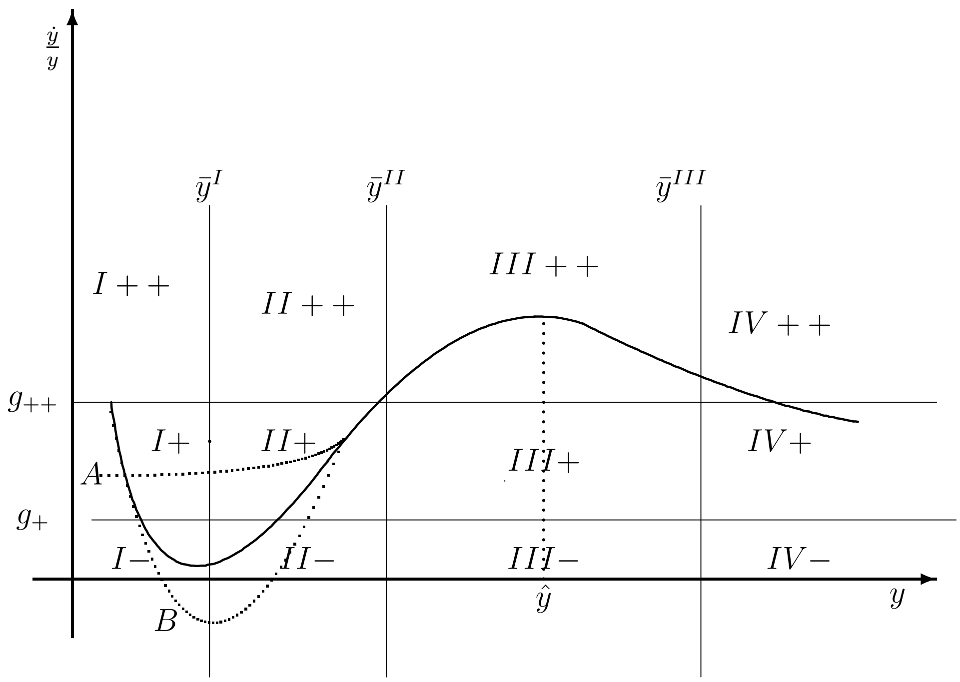

Figure 2: endogenous growth model

(1973) and Murphy et al. (1989). Recent contributions include also Zilibotti (1995), who focuses on the externalities in the capital accumulation process, Peretto (1999), who analyses the change in the growth rate of a country when investing in R&D becomes the main source of growth, and Galor and Weil (2000), who model the interactions between demographic transition and human capital accumulation in the transition from stagnation to growth. All of these models are based on internal factors and do not consider international technological spillovers.

There also exists a literature on technological catch-up which emphasizes the advantage of backward countries to benefit from technological leaders. In Lucas (2000)'s model, the shape of the growth process appears flat for low-income countries which do not use the "leading" technology. When a country starts to benefit from international technological spillovers (and this happens for all countries in Lucas (2000)), its growth rate immediately jumps to a level that is initially greater than the one of rich countries (the difference is a function of the income gap), and eventually converges to the growth rate of the leaders (see also Howitt and Mayer (2002)). Empirically, we should observe a monotonic growth path which follows the initial "jump", and irreversibility of development. In contrast, in our model stagnation is followed by a phase of increasing growth rates. This could be the result of a variable adoption speed, which increases as a country develops. Another difference remains in the behaviour of countries in the initial phase of development: while in the model of technological diffusion development is irreversible, in our model the initial development could lead to a long-run stagnation in presence of a poverty trap (see trajectory B in Figure 2).

In Figure 2 we superimposed on the space $\left(\frac{\dot{y}}{y}, y\right)$ a partition in 12 regions based on three levels of per capita income, $\bar{y}_I$, $\bar{y}_{II}$ and $\bar{y}_{III}$, (which define four income classes), and two levels of growth rates, g_+ and g_{++} (which define three growth rate classes). We choose the growth rate classes in order to include the long-run growth rate in the central class. Since we aim to find empirically testable implications of this model, and the number of available observations is limited, we must maintain the number of possible states as low as possible.

The partition in Figure 2 appears well suited to this purpose. In fact, it identifies regions in the space $\left(\frac{\dot{y}}{y}, y\right)$ characterized by specific relationships between income level and growth rate. In particular, countries in income class I should show decreasing growth rates, while countries in income class II should show a low but increasing growth rates. Countries in income class III should show increasing (at least up to $\hat{y}$) and/or persistently higher growth rates, with respect to all the other income classes. Finally, countries in income class IV should show decreasing growth rates, which tend to settle at a medium level. Moreover, we expect countries in income classes I, II and III to show a tendency to move into income class IV in the long run.

In Figure 2 the trajectory represented by the continuous line indicates the case with no poverty traps. As shown in Appendix C, the model can also include this feature if the growth path cuts the x-axis from above in income class I and from below in income class II (trajectory B in the figure). At first glance, the latter case does not appear to be in agreement with the nonparametric regression in Figure 1. However, it is well-known that nonparametric estimations underestimate the troughs (see Bowman and Azzalini (1997), p. 70). At any rate, the partition in Figure 2 allows for testing the plausibility of the three alternative trajectories. In particular, in presence of a poverty trap the dynamics of the cross-country income distribution would show a tendency to polarize in classes I and IV and this is the crucial point for testing the relevance of trajectory B. Otherwise, all countries would show a tendency to converge in class IV.

IV. Empirical analysis

In this section we discuss our methodology for the empirical investigation and present the results. Following Quah (1993), the growth dynamics of the sample is represented by Markov transition matrices.[7] In the literature on distribution dynamics, the state space has been defined so far only in terms of income levels. In this paper we follow the approach proposed in our previous work (Fiaschi and Lavezzi (2003)) and define the state space in terms of *both* income levels and growth rates. By this definition, it is possible to detect at the same time the presence of two types of heterogeneity: across time and across countries. The presence of time-heterogeneity in the growth path (i.e. nonlinearities) implies that

[7] If it is assumed that the process is stationary and has k states, elements of the unobservable transition matrix p_{ij}, where $i, j \in \{1, ..., k\}$, i.e. the transition probabilities, can be estimated by:

$$\hat{p}_{ij} = \frac{n_{ij}}{n_i}$$

where n_i is the number of observations in state i, and n_{ij} is the number of observed transitions from state i to state j. These estimates, as shown for instance in Norris (1997), pp. 56-57, are the maximum likelihood estimators of the true transition probabilities p_{ij}. See Appendix D for more details on the properties of transition probabilities' estimates.

different income classes are characterized by different growth rate dynamics, whereas cross-sectional heterogeneity can be reflected in the tendency towards polarization of the cross-country distribution. Clearly, these two tendencies can be present at the same time.

A drawback in defining the states in terms of both income level and growth rate is the complex choice of income and growth rate classes, since the number of states tends to be high even for a low number of classes for each one of two variables.

IV.A. Definition of the state space

In Section III. we showed that a definition of 3 growth rate classes and 4 income classes, i.e. a total of 12 states, is sufficient to generate empirically testable implications of the presence of nonlinearities. Moreover, the discussion of the previous Section suggests keeping the number of states as low as possible. This leads us to use exactly 12 states for the empirical analysis. In particular, we adopt the definition of the state space in Table 1.[8]

Income\Growth rate	$< 0.5\%$	$0.5\% - 2.5\%$	$> 2.5\%$
$0 - 1200$	I-	I+	I++
$1200 - 4000$	II-	II+	II++
$4000 - 10000$	III-	II+	III++
> 10000	IV-	IV+	IV++

Table 1: state space definition

The procedure for defining the classes' boundaries consists in two steps. First we set the growth rate classes on the basis of an estimate of the long-run growth rate, and then we set the GDP classes by a comparison of Figure 1 with Figure 2.[9]

As regards the growth rate classes, the theoretical model shows that the class limits should be set in order to obtain a central class which includes the long-run growth rate. According to our model, the identification of such rate concerns the subset of the wealthiest

[8]Slightly different definition of classes' boundaries do not affect our results.

[9]The choice of the number of states and of their boundaries could be based on a more rigorous statistical procedure (see Hansen (2000)). We leave this non-trivial extension for future work.

countries (see Section II.). From an inspection of Figure 1 it seems that the richest countries are converging towards a long-run growth rate of approximately 1.5%. Hence, a range of $\pm 1\%$ should reasonably include this long-run growth rate. The three resulting growth rate classes are consequently defined as $[(-\infty, 0.5\%), (0.5\%, 2.5\%), (2.5\%, +\infty\%)]$.[10] For simplicity we will indicate the three levels of growth rates as "low", "medium" and "high".

The definition of per capita GDP classes directly follows from imposing these growth rate classes on Figure 1, taking into account the representation of the state space in Figure 2.

In the estimate we consider three-year transitions (i.e. from (y_t, g_t) to (y_{t+3}, g_{t+3})) in order to circumvent the possible problem of autocorrelation of shocks. This is particularly relevant for low-income countries, where measurement error can induce serial correlation between growth rates. In Appendix B we report the results with 1-year transitions, which are typically considered in the literature on distribution dynamics, and with 3-year average growth rates, as another way of avoiding the problem of autocorrelation. Overall, our results do not seem to be affected by this phenomenon.

IV.B. Results

Table 2 contains the transition matrix obtained by applying the definition of states of Table 1.[11]
In the transition matrix the first column indicates the number of observations for every state. The number of observations is not equally distributed among states; however, every state appears to have a sufficient number of observations. The ergodic distribution is reported in Table 3.[12]

Table 4 reports the ergodic distribution with respect to a normalization of the distribution's mass in every GDP class. This rep-

[10] Notice the similarity with the classification of countries found in Jones (1997). He defines the growth rate of a country in the post WWII period as "Fast", "Intermediate" or "Slow" according to the following three classes: $[(-\infty\%, 0.4\%), (0.4\%, 2.4\%), (2.4\%, \infty\%)]$. These boundaries are simply obtained by adding and subtracting one percentage point to the average growth of the U.S. productivity on the period considered $(1960-1988)$ equal to 1.4%, according to the idea that the United States have always been on the technological frontier over that

N. Obs	States	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
635	I-	0.48	0.20	0.30	0.01	0	0.01	0	0	0	0	0	0
485	I+	0.30	0.43	0.22	0.01	0.01	0.03	0	0	0	0	0	0
591	I++	0.27	0.18	0.42	0.03	0.02	0.08	0	0	0	0	0	0
631	II-	0.04	0.01	0.01	0.42	0.22	0.28	0.01	0.01	0.01	0	0	0
416	II+	0.01	0	0.01	0.31	0.26	0.37	0.02	0.01	0.01	0	0	0
932	II++	0	0	0	0.20	0.15	0.53	0.03	0.01	0.07	0	0	0
291	III-	0	0	0	0.05	0.02	0.03	0.29	0.23	0.36	0.01	0	0.02
206	III+	0	0	0	0.01	0.01	0	0.29	0.16	0.45	0.01	0.02	0.06
569	III++	0	0	0	0	0	0	0.19	0.15	0.53	0.02	0.02	0.08
216	IV-	0	0	0	0	0	0	0.07	0.01	0.01	0.32	0.26	0.34
190	IV+	0	0	0	0	0	0	0	0	0.01	0.26	0.33	0.41
328	IV++	0	0	0	0	0	0	0	0	0	0.27	0.26	0.46

Table 2: transition matrix

I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
0.02	0.01	0.01	0.03	0.02	0.04	0.06	0.04	0.09	0.20	0.19	0.29

Table 3: ergodic distribution

resentation of the ergodic distribution highlights the growth rate dynamics within each GDP class in steady state.

	-	+	++
I	0.41	0.26	0.34
II	0.34	0.22	0.45
III	0.32	0.19	0.49
IV	0.29	0.28	0.43

Table 4: ergodic distribution normalized for each GDP class

Finally, in Table 5 we report the cross-country income distribution of the first and last year, along with the ergodic distribution in terms of GDP classes only.

Table 5 provides information on the speed of convergence of the

period.

[11]In this matrix and in those in Appendix B, rows may not sum to one due to rounding.

[12]The ergodic distribution represents the long-run, or invariant, distribution. Its existence is generally guaranteed if the process is irreducible, aperiodic and positive persistent. In our case these properties are satisfied. We notice that the ergodic distribution can also include some states with zero mass in the case there exists only one irreducible closed set of positive persistent aperiodic states, and the remaining states are transient (see Isaacson and Madsen (1976), p. 74).

	I	II	III	IV
1950	0.44	0.40	0.13	0.02
1998	0.26	0.27	0.22	0.25
Ergodic	0.04	0.10	0.19	0.67

Table 5: first and last year distribution vs ergodic distribution

actual distribution towards its ergodic limit, and on the tendency to move from one GDP class into another.

In Figure 3 we report the contour plots of kernel density estimations for 3-year transitions of the growth rate within every GDP class (see Durlauf and Quah (1999) for details on stochastic kernels).

The vertical and horizontal axis respectively refer to year t and year $t+3$. We superimpose a grid representing our growth rate classes and a 45° line, which helps to identify the probabilities of acceleration or deceleration of growth rates. This technique complements the estimate of the transition matrix avoiding the problem of the discretization of the income space. The contour plots should be read in the following way: a point on the vertical axis represents the growth rate in year t. A stochastic kernel represents, for every initial growth rate, the probability density of the transition to a growth rate in year $t+3$. The contour plots report the level curves of the stochastic kernel.

Nonlinearities in the growth process For each GDP class we assess whether the predictions from the theoretical model find support.

In GDP class I we expect a deceleration and/or a stagnation of growth. With respect to the other GDP classes we observe that, given a high growth rate, the probability of a low growth rate is the highest ($\hat{p}_{I++,-} = 0.30$ vs $\hat{p}_{II++,-} = 0.23$, $\hat{p}_{III++,-} = 0.21$ and $\hat{p}_{IV++,-} = 0.27$, where $\hat{p}_{ir,w} = \sum_{q \in \{I,...,IV\}} \hat{p}_{ir,qw}$, for $i \in \{I, ..., IV\}$ and for $r, w \in \{-, +, ++\}$),[13] and the probability of another high

[13]These "modified" transition probabilities simply refer to each (stochastic) submatrix corresponding to an income level. E.g. the value $\hat{p}_{I++,-} = 0.30$ is obtained by summing

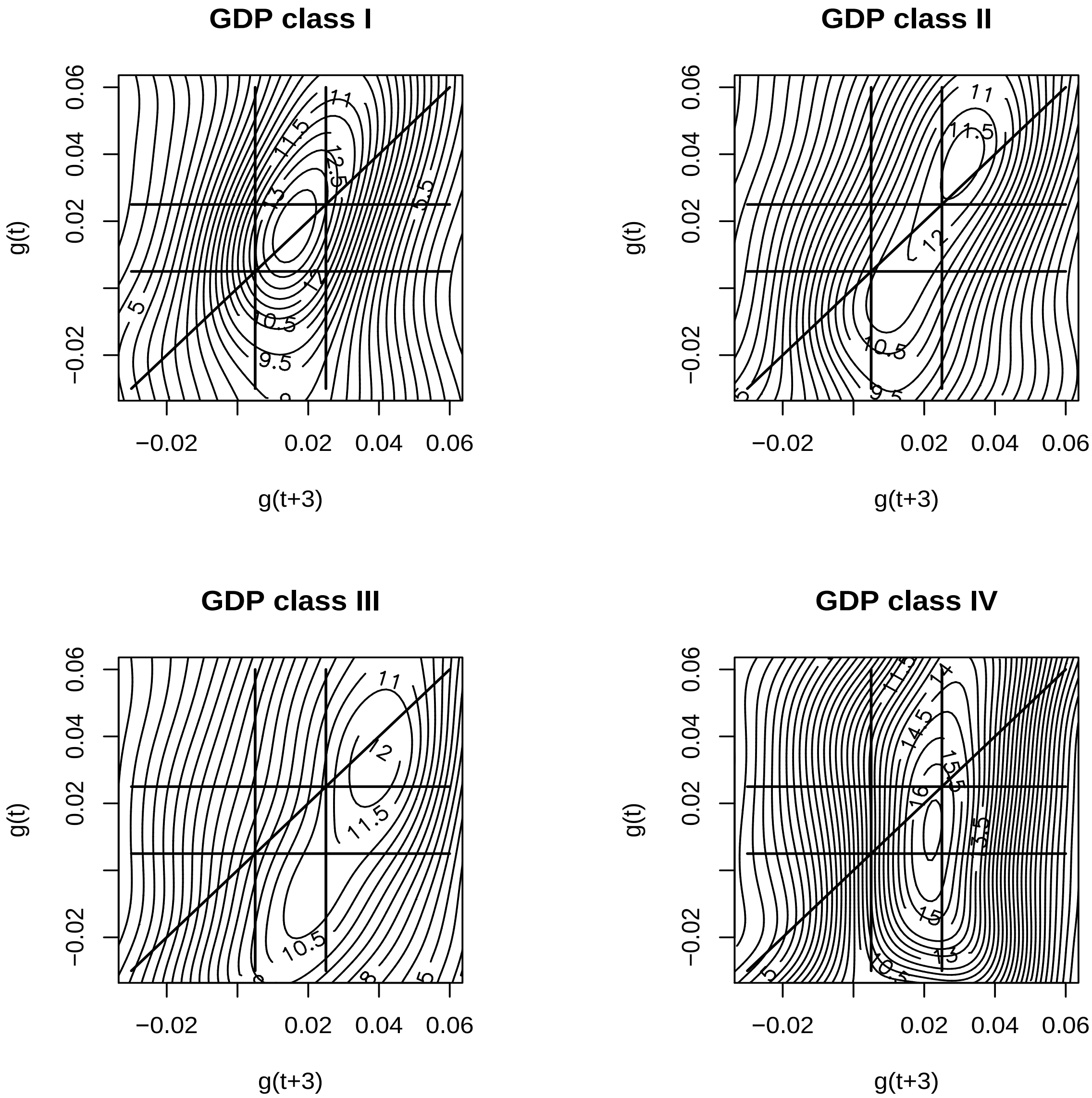

Figure 3: kernel density estimation

growth rate is the lowest after that of GDP class IV ($\hat{p}_{I++,++} = 0.50$ vs $\hat{p}_{II++,++} = 0.60$, $\hat{p}_{III++,++} = 0.61$ and $\hat{p}_{IV++,++} = 0.46$).[14] In addition, given a low growth rate, the probability of another low growth rate is greatest in GDP class I, in particular with respect to GDP classes III and IV ($\hat{p}_{I-,-} = 0.49$ vs $\hat{p}_{II-,-} = 0.47$, $\hat{p}_{III-,-} = 0.35$ and $\hat{p}_{IV-,-} = 0.39$).[15] Finally, given a medium growth rate, the probability of a medium growth rate is the highest ($\hat{p}_{I+,+} = 0.44$ vs $\hat{p}_{II+,+} = 0.27$, $\hat{p}_{III+,+} = 0.19$ and $\hat{p}_{IV+,+} = 0.33$).[16] The observed persistence at medium and low growth rates should imply that, in the long-run, countries in GDP class I should spend a relevant amount of time in this growth rate classes. This insight finds a confirmation in Table 4, first row, where 66% of the mass is in the first two growth rate classes. In Appendix B we report the estimates of transition matrices with 1-year lags and with 3-year average growth rates, showing similar results. Therefore, our findings do not seem to depend on the possible autocorrelation of shocks.

Furthermore, Figure 3 confirms that in GDP class I having a transition to a medium growth rate is the most likely event for almost any initial growth rate, since the peaks of the stochastic kernel are located in correspondence of medium growth rates. This finding provides some empirical support to trajectory A in Figure 2. The comparison of stochastic kernels for the other GDP classes highlights the relatively high persistence at low growth rates, and the relatively low persistence at high growth rates (in particular with respect to GDP classes II and III).

In GDP class II we should observe the beginning of the acceleration phase. Persistence at low growth rate is similar to GDP class

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$\hat{p}_{I++,I-} = 0.27$ and $\hat{p}_{I++,II-} = 0.03$.

[14] Tests of equality between $\hat{p}_{I++,-}$ and, respectively, $\hat{p}_{II++,-}$, $\hat{p}_{III++,-}$ and $\hat{p}_{IV++,-}$ return the following p-values: 0, 0, and 0.17. Tests of equality between $\hat{p}_{I++,++}$ and, respectively, $\hat{p}_{II++,++}$, $\hat{p}_{III++,++}$ and $\hat{p}_{IV++,++}$ return the following p-values: 0, 0, and 0.12 (see Appendix D for more details).

[15] Tests of equality between $\hat{p}_{I-,-}$ and, respectively, $\hat{p}_{II-,-}$, $\hat{p}_{III-,-}$ and $\hat{p}_{IV-,-}$ return the following p-values: 0.23, 0, and 0. Clearly, we cannot reject the hypothesis that $\hat{p}_{I-,-}$ is equal to $\hat{p}_{II-,-}$.

[16] Also in this case the hypothesis of equality between $\hat{p}_{I+,-}$ and the other probabilities can be rejected at 0 level of significance.

I ($\hat{p}_{II-,-} = 0.47$ vs $\hat{p}_{I-,-} = 0.48$), which is in accordance with Figure 2. However, the probability of transition from medium to high growth rate is much higher ($\hat{p}_{II+,++} = 0.39$ vs $\hat{p}_{I+,++} = 0.25$), as well as the probability of persisting at high growth rate ($\hat{p}_{II++,++} = 0.60$ vs $\hat{p}_{I++,++} = 0.48$).[17] Notice also that the mass of probability of high growth rates in the ergodic distribution reported in Table 4 increases from 0.34 to 0.45. Again, transition matrices with 1-year lags and with 3-year average growth rates show similar results. Figure 3 corroborates these findings. In particular, with respect to GDP class I the "ridge" of the stochastic kernel in GDP class II rotates clockwise and therefore the peaks appears to be placed further on the right, especially for the medium/high growth rate.

GDP class III should be characterized by both (i) acceleration of growth and (ii) persistence at high growth rates. In the transition matrix the set of relevant probabilities for point (i) is given by: $\hat{p}_{I+,++}$, $\hat{p}_{II+,++}$, $\hat{p}_{III+,++}$ and $\hat{p}_{IV+,++}$. The estimated values are, respectively: $\hat{p}_{I+,++} = 0.25$, $\hat{p}_{II+,++} = 0.38$, $\hat{p}_{III+,++} = 0.51$ and $\hat{p}_{IV+,++} = 0.42$. Thus, it appears that a country in GDP class III is relatively more likely to show accelerating growth, in accordance with the prediction of the model in Figure 2. More precisely, we find that the probability to increase an already sustained growth rate rises with income for the first three GDP classes, and then decreases in the fourth.[18] As regards point (ii) note that, for a country with a high growth rate, the probability of maintaining such rate is highest in income class III. The relevant value is $\hat{p}_{III++,++} = 0.61$ against $\hat{p}_{I++,++} = 0.50$, $\hat{p}_{II++,++} = 0.60$ and $\hat{p}_{IV++,++} = 0.46$.[19] There is a further increase in the probability of high growth rates in the ergodic distribution reported in Table 4 from 0.45 to 0.49.[20] Again, estimates of transition matrices with 1-year lag and with 3-year

[17]Tests of equality given the following p-values: 0 and 0.

[18]Tests of equality between $\hat{p}_{III+,++}$ and, respectively, $\hat{p}_{I+,++}$, $\hat{p}_{II+,++}$ and $\hat{p}_{IV+,++}$ return the following p-values: 0, 0 and 0.03.

[19]The hypothesis of equality between $\hat{p}_{III++,++}$ and $\hat{p}_{I++,++}$ and $\hat{p}_{IV++,++}$ is rejected. Instead the hypothesis of equality between $\hat{p}_{III++,++}$ and $\hat{p}_{II++,++}$ gives the following p-value: 0.35.

[20]Test of equality gives the following p-value: 0.03.

average growth rates show similar results (see Appendix B). In Figure 3 we observe a rightward shift of the ridge of the kernel. Now the peak of stochastic kernel for medium/high growth rates is more clearly in the high growth rate class.

In GDP class IV deceleration from high growth to medium growth becomes a more likely event ($\hat{p}_{IV++,+} = 0.26$, $\hat{p}_{III++,+} = 0.17$, $\hat{p}_{II++,+} = 0.16$ and $\hat{p}_{I++,+} = 0.20$).[21] In this GDP class there is a relatively high persistence at medium growth rates, in particular with respect to GDP class II and III ($\hat{p}_{IV+,+} = 0.33$, $\hat{p}_{III+,+} = 0.16$ and $\hat{p}_{II+,+} = 0.27$). The ergodic distribution for this GDP class in Table 4 shows the highest value of the probability mass for medium growth rate is in GDP class IV (0.28 vs 0.26, 0.22, 0.19). Estimates with 3-year average growth rates reported in Appendix B, which should reduce at the minimum the possible presence of autocorrelation, show that in GDP class IV the probability of medium growth rate is the highest (0.42 vs 0.23 and 0.36),[22] with respect to the other growth rate classes (it is also the highest with respect to the other GDP classes (0.42 vs 0.35, 0.29 and 0.21).[23] The shape of stochastic kernel in GDP class IV reveals that, indeed, in this class having a medium growth rate is the most likely event starting from any level of growth rate.

Existence of poverty traps A key question for growth empirics is the existence of poverty traps. The ergodic distribution in terms of only GDP classes in Table 5 shows that the proportion of countries in GDP classes I and II strongly tends to decrease in favour of GDP class IV, as is the case in absence of a poverty trap in GDP class I. However, the comparison between the initial, final and ergodic distribution reveals that convergence is very slow.

Table 5 shows that full convergence was far from being achieved

[21] Also in this case the hypothesis of equality between $\hat{p}_{IV++,+}$ and the other probabilities can be rejected. Respectively, the test gives the following p-values: 0, 0 and 0.02.

[22] Tests of equality between the first and the other values return the following p-values: 0 and 0.16.

[23] Tests of equality between the first and the other values return the following p-values: 0.06, 0 and 0.

in 1998. Following Shorrocks (1978), we compute the *asymptotic half life* of this process. It equals about 19 periods (i.e. 57 years), so that at least 114 years from the end of our observation's period should be necessary to have full convergence.[24] A first remark regards the stability of process in the long run. We estimate a transition matrix from observations spanning 48 years, while convergence would occur in more than twice those years; therefore the significance of the ergodic distribution can be questionable.

As noted in the introduction, if all countries follow a common trend, we cannot be sure that the behaviour we identify, e.g. for GDP class $[4000, 10000]$ in the period 1950-1998, may be observed in the same GDP class say in 2050-2098. However, the ergodic distribution can nonetheless provide some insights on the long-run tendency of the cross-country distribution. A slow adjustment process may mean for some countries a long period of very low growth, as it may be the case of African countries. Moreover, even if the distribution dynamics shows a reduction in the weight of GDP classes I-III in favour of IV, in the ergodic distribution a relevant part of mass (0.33) is contained in the first three GDP classes. This implies that there may always exist a set of countries remaining poor in *absolute* terms, even if we do not find unambiguous support of a poverty trap in GDP class I.[25]

The fact that in the ergodic distribution there remains a positive fraction of countries in the first three GDP classes depends on the significant transition probabilities of moving from a GDP class to a class with a lower GDP. This is clearly observable from the transition matrix for GDP classes only (see Table 6).

Similar values are found in the transition matrices with 1-year lags and 3-year average growth rates.[26] This result is in contrast

[24] The asymptotic half life is defined as $h = -log2/log|\lambda_2|$, where λ_2 is the second largest eigenvalue of the transition matrix. In our case $\lambda_2 \simeq 0.9642$. This measure of the speed of convergence is based on the time the process takes from period t to reach half of the distance from its equilibrium level (the ergodic distribution).

[25] Obviously, the members of this set can change over time, since the ergodicity of process means that every country has a positive probability to visit each state.

[26] Tests on the lower off-diagonal elements show that they are statistically different from zero.

Obs.	States	I	II	III	IV
1711	I	0.93	0.07	0	0
1979	II	0.03	0.90	0.07	0
1066	III	0	0.03	0.88	0.09
734	IV	0	0	0.02	0.98

Table 6: transition matrix for GDP classes

with the dynamics predicts by technological diffusion models. For instance in Lucas (2000), once a country leaves stagnation, it can just proceed towards higher income levels.

The overall results seem to support the dynamics depicted in Section III., i.e. the dynamics and the distribution of the probability masses looks coherent with Figure 2.

V. Conclusions

The main result of the paper is the detection of nonlinearities in the growth process. In particular, we find support to the picture of a range of decreasing/low growth rates, followed by a phase of accelerating growth, which eventually decelerates once a country reaches a certain level of per capita GDP. However, this process appears rather slow for low-income income countries, which may mean a long period of a low or zero growth for a relevant number of countries. This is supported by the lack of a tendency for all countries to converge to the same income level due to a non-negligible probability of "reversal of fortune".

In general, contributes analysing cross-country growth dynamics should take into account the nonlinear pattern of the growth rate. In this respect, the paper provides some "stylized facts" which a model aiming to reproduce the development path of a country should match. In particular, if technological diffusion is at the heart of growth of countries, then its features should differ from those presented in Lucas (2000), for instance by allowing for a variable adoption speed of new technologies and reversibility of adoption.[27]

Finally, we believe that traditional development theories still provide interesting insights on the emergence of nonlinearities, as the focus on structural change, that modern growth literature only recently started to explore intensively (see Galor and Weil (2000)). This is the direction of our current research.

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[27]By reversibility of adoption we mean the possibility that a country loses the capability of exploiting technologies previuosly used, due for instance to the depletion of the human capital stock (e.g. for starvations, epidemies, wars, etc.). This would follow the insights of Basu and Weil (1998) on appropriate technologies.

A Country List

AFRICA	1 Algeria	2 Angola	3 Benin	4 Botswana
5 Cameroon	6 Cape Verde	7 Cent. Afr. Rep.	8 Chad	9 Comoros
10 Congo	11 Côte d' Ivoire	12 Djibouti	13 Egypt	14 Gabon
15 Gambia	16 Ghana	17 Kenya	18 Liberia	19 Madagascar
20 Mali	21 Mauritania	22 Mauritius	23 Morocco	24 Mozambique
25 Namibia	26 Niger	27 Nigeria	28 Reunion	29 Rwanda
30 Senegal	31 Seychelles	32 Sierra Leone	33 Somalia	34South Africa
35 Sudan	36 Swaziland	37 Tanzania	38 Togo	39 Tunisia
40 Uganda	41 Zambia	42 Zimbabwe	LATIN AMERICA	43 Argentina
44 Brazil	45 Chile	46 Colombia	47 Mexico	48 Peru
49 Uruguay	50 Venezuela	51 Bolivia	52 Costa Rica	53 Cuba
54 Dominican Rep.	55 Ecuador	56 El Salvador	57 Guatemala	58 Haiti
59 Honduras	60 Jamaica	61 Nicaragua	62 Panama	63 Paraguay
64 Puerto Rico	65 Trin. Tobago	OFF WESTERN	66 Australia	67 New Zealand
68 Canada	69 United States	WEST ASIA	70 Bahrain	71 Iran
72 Iraq	73 Israel	74 Jordan	75 Kuwait	76 Lebanon
77 Oman	78 Qatar	79 Saudi Arabia	80 Syria	81 Turkey
82 UAE	83 Yemen	84 W.Bank Gaza	EAST ASIA	85 China
86 India	87 Indonesia	88 Japan	89 Philippines	90 South Korea
91 Thailand	92 Taiwan	93 Bangladesh	94 Burma	95 Hong Kong
96 Malaysia	97 Nepal	98 Pakistan	99 Singapore	100 Sri Lanka
101 Afghanistan	102 Cambodia	103 Laos	104 Mongolia	105 North Korea
106 Vietnam	EUROPE	107 Austria	108 Belgium	109 Denmark
110 Finland	111 France	112 Germany	113 Italy	114 Netherlands
115 Norway	116 Sweden	117 Switzerland	118 UK	119 Ireland
120 Greece	121 Portugal	122 Spain		

Table 7: country list

B Other estimates

In this appendix we report alternative estimates. We do not provide any discussion of results, except that they support our previous findings.

BA. Estimates with 1-year lags

In this Section we present the transition matrix for 1-year transition, i.e. from (y_t, g_t) to (y_{t+1}, g_{t+1}), along with the tables for the ergodic distribution and the distribution dynamics.

N. Obs	States	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
655	I-	0.54	0.15	0.31	0	0	0	0	0	0	0	0	0
511	I+	0.24	0.54	0.21	0	0.01	0	0	0	0	0	0	0
610	I++	0.28	0.20	0.45	0.01	0.01	0.04	0	0	0	0	0	0
647	II-	0.02	0	0.01	0.48	0.20	0.28	0	0	0	0	0	0
439	II+	0	0	0	0.28	0.37	0.34	0	0	0	0	0	0
962	II++	0	0	0	0.20	0.14	0.61	0.01	0.01	0.04	0	0	0
304	III-	0	0	0	0.03	0.01	0.01	0.47	0.17	0.30	0	0	0
215	III+	0	0	0	0	0	0	0.30	0.26	0.42	0.01	0.01	0.01
597	III++	0	0	0	0	0	0	0.16	0.16	0.63	0.01	0.01	0.04
220	IV-	0	0	0	0	0	0	0.02	0.01	0.01	0.48	0.24	0.23
215	IV+	0	0	0	0	0	0	0	0	0	0.27	0.33	0.40
359	IV++	0	0	0	0	0	0	0	0	0	0.16	0.26	0.58

Table 8: transition matrix

I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
0.03	0.02	0.02	0.05	0.03	0.07	0.07	0.04	0.11	0.16	0.15	0.24

Table 9: ergodic distribution

	-	+	++
I	0.38	0.28	0.34
II	0.33	0.22	0.46
III	0.30	0.20	0.51
IV	0.28	0.28	0.44

Table 10: ergodic distribution normalized for every GDP class

BB. Estimate with 3-year average growth rates

In this section we consider 3-year average growth rates, i.e. from $(y_t, \bar{g}_{t,t+2})$ to $(y_{t+3}, \bar{g}_{t+3,t+5})$, where $\bar{g}_{t,t+2}$ is the average annual growth rate from period t to period $t+2$ and $\bar{g}_{t+3,t+5}$ is is the average annual

	I	II	III	IV
1950	0.44	0.40	0.13	0.02
1998	0.26	0.27	0.22	0.25
Ergodic distr.	0.07	0.15	0.23	0.56

Table 11: distribution of the first and last year vs ergodic distribution for only GDP classes

growth rate from period $t+3$ to period $t+5$. In the following we report the usual Tables:

N. Obs	States	I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
202	I-	0.50	0.30	0.20	0	0	0	0	0	0	0	0	0
207	I+	0.28	0.48	0.21	0.01	0.02	0	0	0	0	0	0	0
169	I++	0.19	0.21	0.41	0.04	0.03	0.12	0	0	0	0	0	0
186	II-	0.06	0.01	0.01	0.48	0.20	0.23	0	0	0	0	0	0
185	II+	0	0	0	0.25	0.42	0.31	0	0.01	0.02	0	0	0
292	II++	0	0	0	0.12	0.20	0.54	0.01	0.01	0.08	0	0	0
83	III-	0	0	0	0.04	0.05	0.05	0.35	0.18	0.34	0	0	0
73	III+	0	0	0	0	0	0	0.21	0.27	0.47	0	0.04	0.01
197	III++	0	0	0	0	0	0	0.13	0.17	0.56	0.03	0.04	0.09
55	IV-	0	0	0	0	0	0	0.07	0.02	0.02	0.36	0.29	0.24
93	IV+	0	0	0	0	0	0	0	0	0	0.20	0.47	0.32
88	IV++	0	0	0	0	0	0	0	0	0	0.16	0.41	0.43

Table 12: transition matrix

I-	I+	I++	II-	II+	II++	III-	III+	III++	IV-	IV+	IV++
0.02	0.01	0.01	0.03	0.03	0.04	0.05	0.04	0.10	0.15	0.28	0.24

Table 13: ergodic distribution

C Analytical model

This appendix presents the analytical model depicted in Figure 2. Consider an economy with the following typical Solovian capital accumulation equation:

$$\dot{k} = sf(k) - (\delta + n)\, k, \tag{1}$$

	-	+	++
I	0.38	0.35	0.27
II	0.29	0.29	0.42
III	0.27	0.21	0.51
IV	0.23	0.42	0.36

Table 14: ergodic distribution normalized for every GDP class

	I	II	III	IV
1960	0.44	0.40	0.13	0.02
1998	0.27	0.28	0.20	0.25
Ergodic distr.	0.04	0.10	0.19	0.67

Table 15: distribution of the first and last year vs ergodic distribution for only GDP classes

where k is capital per capita, s is the constant saving rate, f is the production function, δ is the depreciation rate of capital, n is the growth rate of population. Under the following assumptions:

- $f(0) = 0$;
- $f' > 0 \; \forall k > 0$, $\lim_{k \to 0} f' > \frac{n+\delta}{s}$ and $\lim_{k \to +\infty} f' = a > \frac{n+\delta}{s}$;
- $f'' > 0 \; \forall k \in \left[\check{k}, \hat{k}\right]$ and $f'' < 0 \; \forall k \in \left[0, \check{k}\right] \smile \left[\hat{k}, \infty\right)$.

the most interesting cases are two.

In the first case every country, independent of its initial level of capital, has a long-run growth rate of capital per capita equal to $sa - n - \delta$. This happens if $\frac{f(k)}{k} > \frac{n+\delta}{s} \forall k \in [0, \infty)$. The proof is straightforward from equation (1), since $\dot{k} > 0$ for $k = 0$ and $\dot{k} > 0$ for $k > 0$. The change in concavity is the cause of the nonlinear pattern of the growth rate (which depends on average capital productivity).

In the second case there are two equilibria: $\bar{k}_1 < \bar{k}_2$. This happen if $\exists k \in (0, \infty)$ such that $\frac{f(k)}{k} < \frac{n+\delta}{s}$. The first equilibrium $\bar{k}_1$ is an attractor: in fact, $\dot{k} > 0$ for $k \in \left[0, \bar{k}_1\right)$ and $\dot{k} < 0$ for $k \in \left(\bar{k}_1, \bar{k}_2\right)$ (this directly derives from the shape of f). The second equilibrium, $\bar{k}_2$, is unstable: in fact, $\dot{k} < 0$ for $k \in \left(\bar{k}_1, \bar{k}_2\right)$ and $\dot{k} > 0$ for $k \in \left[\bar{k}_2, \infty\right)$. This implies that a country will converge to the equilibrium with a lower level of capital if its initial level of capital is lower than $\bar{k}_2$, while it will have a positive long-run growth equal to $sa - n - \delta$ if its initial level of capital is greater than $\bar{k}_2$. Also in this case the growth rate can follow a nonlinear path.

Finally, restating the above results in term of per capita income we have:

$$\frac{\dot{y}}{y} = \frac{f'(k)}{\frac{f(k)}{k}} \frac{\dot{k}}{k} = \frac{f'(k)}{\frac{f(k)}{k}} \left[s\frac{f(k)}{k} - n - \delta \right].$$

Thus, also $\frac{\dot{y}}{y}$ has a non monotonic path and $\lim_{k \to +\infty} \frac{\dot{k}}{k} = \frac{\dot{y}}{y} = sa - n - \delta$ since $\lim_{k \to +\infty} \frac{f'(k)}{\frac{f(k)}{k}} = 1$ for the assumption on f for $k \to +\infty$. Figure 2 reports the relationship between the growth rate and the level of income: the growth path represented by a solid line and Trajectories A refer to the first case, while Trajectory B to the second case.

D Inference on Markov transition matrices

In this appendix we illustrate a procedure to make inference on the elements of a Markov transition matrix.

DA. Basic notation

Suppose that the observations of a process with k states, i.e. with state space $S = 1, ..., k$, are collected for more than one period. Let n_{ij} be the number of observations in the sample corresponding to transitions from state i to state j, $n_i = \Sigma_{j=1}^{k} n_{ij}$ the total number of observations in state i, and $\mathbf{n}_i = (n_{i1}, ..., n_{ik})$ the vector collecting all n_{ij}, $i \in S$ and $j = 1, ..., k$; hence $n = \Sigma_{i=1}^{k} n_i$ is the total number of observations.

Let $\mathbf{P}$ be the $(k \times k)$ transition matrix. The element p_{ij} represents the transition probability from state i to state j, so that $\Sigma_{j=1}^{k} p_{ij} = 1$ and $0 \leq p_{ij} \leq 1$. Moreover, let p_i be the fraction of observations in initial state i, i.e. $p_i = \frac{n_i}{n}$.

Suppose the ergodic distribution for this process exists. The ergodic distribution is defined as:

$$\pi = \pi \mathbf{P} \tag{2}$$

under the constraint:

$$\pi \mathbf{u}' = 1,$$

where $\mathbf{u}$ is the sum vector. From another point of view π corresponds to a row of the matrix $\mathbf{P}^t$ for $t \to \infty$.

DB. Inference

In the following we assume that the rows of $\mathbf{P}$ are independent.

DB.i. Consistent estimators

The maximum likelihood (ML) estimator of $\mathbf{P}$, $\hat{\mathbf{P}}$, is given by:

$$\hat{\mathbf{P}} = [\hat{p}_{ij}] = \left[\frac{n_{ij}}{n_i} \right], \tag{3}$$

where $n_i = \Sigma_{j=1}^{n} n_{ij}$ (for a proof see e.g. Norris (1997), pp. 55-56). $\hat{\mathbf{P}}$ being the ML estimator, these estimates are consistent.

In general, take $\mathbf{P}$ and a function M such that $M : \mathbf{P} \to \Re$. Since $\mathbf{P}$ is unknown, then $M(\mathbf{P})$ is unknown as well. A natural estimator is $\hat{M} = M\left(\hat{\mathbf{P}}\right)$, which, in turn, is consistent (see Trede (1999)). M can represent any function (linear and non-linear), e.g. the function which associates the transition matrix to an element of its ergodic distribution (when it exists).

DB.ii. Distribution of estimates

Stuart and Ord (1994), p. 260, show that the distribution of $\mathbf{n}_i$ converges to a n-variate normal distribution, with means $n_i p_{ij}$, variances $n_i p_{ij} (1 - p_{ij})$ and covariances $cov\,(n_{ij}, n_{iq}) = -n_i p_{ij} p_{iq}$. Thus $\sqrt{n_i}\,(\hat{p}_{ij} - p_{ij})$ tends towards the normal distribution $N\,(0; p_{ij}\,(1 - p_{ij}))$. Notice that, defining $p_i\left(\underline{\mathrm{k}}, \bar{k}\right) = \sum_{k=\underline{\mathrm{k}}}^{\bar{k}} p_{ik}$, then $\sqrt{n_i}(\hat{p}_i\left(\underline{\mathrm{k}}, \bar{k}\right) -$

$p_i\left(\underline{\mathrm{k}}, \bar{k}\right))$ tends towards the normal distribution $N(0; p_i\left(\underline{\mathrm{k}}, \bar{k}\right)(1 - p_i\left(\underline{\mathrm{k}}, \bar{k}\right)))$.

The asymptotic distribution of $\hat{M}$ can be derived by the delta method (DM) (see Trede (1999)). Consider the first order Taylor series expansion of $M\left(\hat{\mathbf{P}}\right)$ around $M(\mathbf{P})$:

$$M\left(\hat{\mathbf{P}}\right) = M(\mathbf{P}) + DM(\mathbf{P})\left(\operatorname{vec}\left(\hat{\mathbf{P}}' - \mathbf{P}'\right)\right),$$

where

$$DM(\mathbf{P}) = \frac{\partial M(\mathbf{P})}{\partial \operatorname{vec}(\mathbf{P}')'} \tag{4}$$

is a $1 \times k^2$ vector, which contains the first derivatives of M with respect to each element of $\mathbf{P}$.

Since the rows of $\mathbf{P}$ are independent and each row tends towards a n-variate normal distribution, we have

$$\sqrt{n}\left(\operatorname{vec}\left(\hat{\mathbf{P}}' - \mathbf{P}'\right)\right) \xrightarrow{d} N(0, \mathbf{V}),$$

where

$$\mathbf{V} = \begin{bmatrix} \mathbf{V}_1 & & \\ & \cdots & \\ & & \mathbf{V}_k \end{bmatrix} \tag{5}$$

is a block diagonal with

$$\mathbf{V}_m = [v_{m,ij}] = \begin{cases} \frac{p_{mi}(1-p_{mi})}{p_m} & \text{for } i = j \\ -\frac{p_{mi}p_{mj}}{p_m} & \text{for } i \neq j \end{cases}$$

for $m = 1, ..., k$ and 0 elsewhere.

Therefore the asymptotic distribution of M is given by:

$$\sqrt{n}\left(M\left(\hat{\mathbf{P}}\right) - M(\mathbf{P})\right) \xrightarrow{d} N\left(0, \sigma_M^2\right), \tag{6}$$

where

$$\sigma_M^2 = (DM(\mathbf{P}))\,\mathbf{V}\,(DM(\mathbf{P}))'. \tag{7}$$

Since both $DM(\mathbf{P})$ and $\mathbf{V}$ are unknown, they are estimated by $DM\left(\hat{\mathbf{P}}\right)$ and $\hat{\mathbf{V}}$ calculated on the basis of (the elements of) $\hat{\mathbf{P}}$. As

$\hat{\mathbf{P}}$ is a ML-estimator, then $DM\left(\hat{\mathbf{P}}\right)$ and $\hat{\mathbf{V}}$ are consistent too and therefore the estimate of the variance of M is given by:

$$\hat{\sigma}_M^2 = \left(DM\left(\hat{\mathbf{P}}\right)\right)\hat{\mathbf{V}}\left(DM\left(\hat{\mathbf{P}}\right)\right)'. \tag{8}$$

Since $M(\mathbf{P})$ is normally distributed, then the $(1-\alpha)$-confidence interval for $M\left(\hat{\mathbf{P}}\right)$ is

$$M\left(\hat{\mathbf{P}}\right) \pm c\frac{\hat{\sigma}_M}{\sqrt{n}}, \tag{9}$$

where c is the $\left(1-\frac{\alpha}{2}\right)$-quantile of the $N(0,1)$. Alternatively,

$$s = \frac{M\left(\hat{\mathbf{P}}\right) - M(\mathbf{P})}{\frac{\hat{\sigma}_M}{\sqrt{n}}} \tag{10}$$

converges towards a Gaussian distribution under the null hypothesis $M\left(\hat{\mathbf{P}}\right) = M(\mathbf{P})$.

DB.iii. Testing

The Delta Method provides the most general procedure of testing; however, for the simpler tests on the elements of $\mathbf{P}$ we use a more direct way: we focus on the comparison of two elements of the transition matrix and of two elements of ergodic distribution.

Tests on elements of P

Comparison of two elements of different rows The first test regards the difference between two transition probabilities belonging to different rows. Under the assumption of independence among the rows of $\mathbf{P}$, $s = \frac{\hat{p}_{ij} - \hat{p}_{mq}}{\sqrt{\frac{\hat{\sigma}_{ij}^2}{n_i} + \frac{\hat{\sigma}_{mq}^2}{n_m}}}$ converges to a Gaussian distribution under the null hypothesis $\hat{p}_{ij} = \hat{p}_{mq}$, where $i \neq m$ and $\hat{\sigma}_{ij}^2 = \hat{p}_{ij}(1-\hat{p}_{ij})$. The proof is straightforward given the normality of the asymptotic distribution of $\mathbf{P}$ and the assumption of independence among the rows of $\mathbf{P}$.

Comparison of two elements of the same row A second test regards the difference between two transition probabilities belonging to the same row. Then $s = \frac{\hat{p}_{ij}-\hat{p}_{iq}}{\sqrt{\frac{\hat{\sigma}^2_{ij}}{n_i}+\frac{\hat{\sigma}^2_{iq}}{n_i}-\frac{2cov(\hat{p}_{ij},\hat{p}_{iq})}{n_i}}}$ converges to a Gaussian distribution under the null hypothesis of an identical value of $\hat{p}_{ij}$ and $\hat{p}_{iq}$, where $j, q \in \{1, ..., n\}$ and $cov\left(\hat{p}_{ij}, \hat{p}_{iq}\right) = -\hat{p}_{ij}\hat{p}_{iq}$. Also in this case the proof is straightforward.

Starting from these two types of tests we can test all possible combinations among elements of $\mathbf{P}$.

Tests on elements of π

Comparison between single elements of ergodic distribution To test the difference between elements of ergodic distribution requires the application of the DM. First we have to calculate the derivatives of the function M. In this case M is a function calculating the difference between any two elements of this distribution. Let $ED\left(\mathbf{P}\right)$ be this function:

$$ED\left(\mathbf{P}\right) = \pi_q - \pi_m, \text{ where } m, q \in \{1, ..., k\}.$$

To calculate σ^2_{ED} we need to know the analytical derivatives of the ergodic distribution with respect to the elements of the transition matrix. Conlisk (1985) provides an analytical formulation. Assume that the increase in the element j in row i, p_{ij}, is absorbed by a decrease in the element of the last column k of row i, p_{ik} (the row sum must sum to one). Thus, the derivative of the $q-th$ element of the ergodic distribution is defined as follows:

$$\frac{\partial \pi_q}{\partial p_{ij}} = \pi_i\left(z_{jq} - z_{kq}\right) \forall i, j, q \in \{1, ..., k\},$$

where z_{jq} is an element of fundamental matrix $\mathbf{Z} = \left(\mathbf{I} - \mathbf{P} - \mathbf{b}\mathbf{u}'\right)^{-1}$ and $\mathbf{b}$ is any $1 \times k$ row vector such that $\mathbf{b}'\mathbf{u} \neq 0$.

Then:

$$\frac{\partial ED\left(\mathbf{P}\right)}{\partial p_{ij}} = \pi_i\left(z_{jq} - z_{kq}\right) - \pi_i\left(z_{jm} - z_{km}\right) = \pi_i\left[\left(z_{jq} - z_{kq}\right) - \left(z_{jm} - z_{km}\right)\right],$$

from which we can calculate $\hat{\sigma}^2_{ED}$. Applying (9) we obtain the confidence interval for $ED = \pi_q - \pi_m$ and/or by (10) we can test the null hypothesis $\pi_q = \pi_m$.

Comparison between elements of the ergodic distribution normalized with respect to different subsets of states Consider the following function of the transition matrix:

$$EDN\left(\mathbf{P}\right) = \frac{\pi_{q_1}}{\Sigma_{m=s_1}^{e_1}\pi_m} - \frac{\pi_{q_2}}{\Sigma_{m=s_2}^{e_2}\pi_m},$$

where $q_1 \in \{s_1, ..., e_1\}$ and $q_2 \in \{s_2, ..., e_2\}$. This represents the difference in the elements of the ergodic distribution, normalized within two subsets of states; in our case these two subsets of states are $(s_1, ..., e_1)$ and $(s_2, ..., e_2)$.

The first step consists in calculating the first derivative of $EDN\left(\mathbf{P}\right)$ with respect to the elements of $\mathbf{P}$. Given the analytical derivative of an element of the ergodic distribution with respect to an element of $\mathbf{P}$, when the last column k absorbs any positive perturbation, we obtain:

$$\begin{aligned}
\frac{\partial EDN\left(\mathbf{P}\right)}{\partial p_{ij}} &= \\
&= \frac{\frac{\partial \pi_{q_1}}{\partial p_{ij}}\left(\Sigma_{m=s_1}^{e_1}\pi_m\right) - \pi_{q_1}\left(\Sigma_{m=s_1}^{e_1}\frac{\partial \pi_m}{\partial p_{ij}}\right)}{\left(\Sigma_{m=s_1}^{e_1}\pi_m\right)^2} - \frac{\frac{\partial \pi_{q_2}}{\partial p_{ij}}\left(\Sigma_{m=s_2}^{e_2}\pi_m\right) - \pi_{q_2}\Big(\Sigma_{m=s}^{e_2}}{\left(\Sigma_{m=s_2}^{e_2}\pi_m\right)^2} \\
&= \frac{\pi_i\left[\left(z_{jq_1} - z_{kq_1}\right)\left(\Sigma_{m=s_1}^{e_1}\pi_m\right) - \pi_{q_1}\Sigma_{m=s_1}^{e_1}\left(z_{jm} - z_{km}\right)\right]}{\left(\Sigma_{m=s_1}^{e_1}\pi_m\right)^2} + \\
&\quad - \frac{\pi_i\left[\left(z_{jq_2} - z_{kq_2}\right)\left(\Sigma_{m=s_2}^{e_2}\pi_m\right) - \pi_{q_2}\Sigma_{m=s_2}^{e_2}\left(z_{jm} - z_{km}\right)\right]}{\left(\Sigma_{m=s_2}^{e_2}\pi_m\right)^2}.
\end{aligned}$$

Then, by (8) and (9) we can construct the confidence interval for $EDN\left(\hat{\mathbf{P}}\right)$ and/or by (10) we can test the null hypothesis $\frac{\pi_{q_1}}{\Sigma_{m=s_1}^{e_1}\pi_m} = \frac{\pi_{q_2}}{\Sigma_{m=s_2}^{e_2}\pi_m}$.

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