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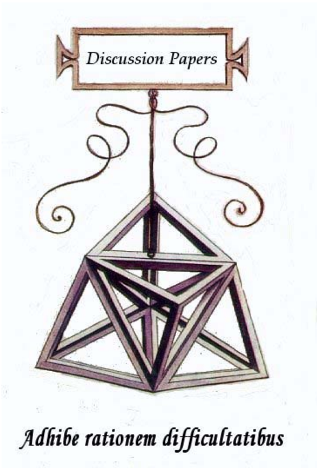

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WELFARE IMPLICATIONS OF NATIONAL DEBT IN AN OLG MODEL WITH ENDOGENOUS FERTILITY

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Abstract

In the present work we show that, when one allows for endogenous fertility in Diamond's (1965) OLG model, public debt plays still a clear-cut role on dynamic inefficiency (DI): for correcting DI, national debt must be increased. DI is more likely to occur when the economy capital income share and the preference for children are sufficiently low and the degree of patience is sufficiently high. However, differently from Diamond's case, DI turns out to be a necessary but not a sufficient condition for welfare improvements to obtain via debt increases, since, in presence of endogenous fertility, the optimal level of debt is typically lower than the one associated to the traditional Golden Rule. Hence, not taking fertility choices into account would lead policymakers to overshoot the target, i.e. to issue too high a level of national debt. Finally, a sensitivity analysis shows that the optimal level of debt is higher the lower the capital share, the higher individuals' degree of patience, the bigger the child rearing cost and the lower the preference for children. On policy grounds we argue that for debt tightening policies to be optimal in the long run, it is necessary that the cost of rearing children does not increase (or, if anything, reduces).

J.E.L. classification: D91, E62, H63, J13.

Keywords: Overlapping Generations, endogenous fertility, dynamic inefficiency, debt.

1 Introduction

As well known, OLG economies can undergo dynamic inefficiency (DI), that is overaccumulation of capital relative to the level which would maximize the social welfare (see Samuelson (1958) and Diamond (1965))[1]. This situation invalidates the First Theorem of Welfare for perfectly competitive OLG economies and may well apply even to the simplest scenario in which only savings are endogenous and government is absent. However, the introduction of public debt, as pointed out by Diamond (1965), can correct this situation by crowding out the steady state level of savings and, thus, of capital[2].

Up to now most works have usually assumed the population growth rate as exogenous[3]; however, this rate is crucial in determining DI, so that, in this work, we relax such a traditional assumption by endogeneizing the individual's choice of the number of children. By doing so we aim at answering

[1] Such level of consumption and capital accumulation path are usually referred to as "Golden Rule" values. When the maximization problem concerns a "Social Welfare function", rather than steady state consumption, the solutions are called the "Modified Golden Rule" allocations. As for the most recent empirical evidence on such issue, see the contributions by Abel et al. (1989) and Anderson (1993): in fact, while the first work confirms that dynamic efficiency has been satisfied by the U.S. economy and other six developed countries, the second study casts doubt on such conclusion as for the U.S., Canada and Great Britain.

[2] We acknowledge that in order to approach a long run equilibrium in a OLG model a hard conceptual apparatus must be used, as Samuelson himself admits: "...in order to define an equilibrium path of interest in a perfect capital market endowed with perfect certainty, you have to determine all interest rates between now and the end of time; every finite time period points beyond itself!." (Samuelson (1958), p. 467). Moreover a further caveat is suggested by the possible different time adjustments needed to approach market equilibria which might require an infinite time and therefore the equilibrium should be defined only as "potentially dynamically efficient" (Anderson (1993), pp. 345-46). We thank an anonymous referee to have suggested the considerations and the citations mentioned above.

[3] Notable exceptions have concerned, in particular, the analysis of endogenous fertility choices on the (optimality of) Social Security systems. On this point see, among others, Zang and Nishimura (1992 and 1993), Cigno (1995), Rosati (1996) and Lagerlöf (1997). Another ample field of investigation of the endogenous fertility in a OLG framework is that focusing upon the so-called demographic transition (e.g. Galor and Weil (1996)), but also in such a field the issue of the public debt has been neglected.

the following questions: 1) Under which conditions a level of optimal[4] debt (that is correcting DI) does exist? 2) What are the determinants and the characteristics of such optimal level? 3) How do Diamond's rules on debt management change in presence of endogenous fertility?

The paper is organized as follows: in section 2 we lay out the basic framework and then we focus on the effect of the introduction of debt upon the steady state level of capital in a decentralized economy. Next, we analyze the welfare effects of debt variations, and, after showing the possible failure of Diamond's rule, we characterize the optimal level of debt. A sensitivity analysis on the main parameters and conclusions will end the paper.

2 The model set up

2.1 Individuals

Following a standard way to endogeneize fertility in an OLG framework (e.g. Strulik (1999) and (2003)) life is divided into three periods: childhood, young adulthood, and old–age. During childhood, individuals do not make any decisions. Young adult individuals belonging to generation, say, $t-1$, have

[4] We evaluate the optimality exactly as Diamond (1965) or Samuelson (1975) in terms of the utility of the representative individual in stationary state. We are aware of the fact that normative criteria to evaluate policy changes commonly used in static problems are not well-defined in intertemporal models of endogenous population in that, for example, the Pareto criterion requires that the number and identity of individuals is unaffected by the choice of policy. In order to better understand this problem several authors (e.g. Blackorby-Donaldson (1984), Dasgupta (1994), Razin and Sadka (1995) and Golosov, Jones and Tertilt (2004)) developed different attempts to deal with this problem; they argued that the need to assign ethical rights to potential (unborn) persons whose preferences do not exist implies a reconsideration of the individual orderings of alternatives of this generation, in that an evaluation of preferences of unborn persons is logically based on future interests of the generation currently alive. However, in our frame of stationary state and representative individual and for our purposes the representative individual level of welfare is still valid (as for example in Abìo (2003) who extends - by endogeneizing the fertility rate - Samuelson (1975)) to evaluate the welfare effects of debt increases or decreases and so on. We thank an anonymous referee to have raised the issue of Pareto efficiency in presence of endogenous fertility.

an utility function U, defined over c_{1t}, c_{2t+1}, n_t, that is consumption in the first and second period of adulthood and number of children, respectively: thus, in such a period of life agents, who receive a working income w_t, choose their optimal intertemporal allocations of consumption/savings and fertility. By assuming for simplicity that every single young adult can have children, the population at the steady state will be stationary or increasing if n is equal or bigger than 1 respectively (with $n-1$ being the long run growth rate of the economy as well). Moreover, rearing a child requires a fixed cost, e[5].

2.2 Firms

Firms own a constant returns to scale production technology $F(K_t, L_t)$ by which they transform physical capital (K_t) and labor (L_t), into the consumption good. Since we assume a perfect competitive market, firms hire capital and labor by remunerating them according to their marginal productivity. Moreover, due to the homogeneity of degree one of F, it follows that $w_t = f(k_t) - f'_k(k_t)\, k_t$ and $r_t = f'_k(k_t)$ (in the case of absence of depreciation) or $r_t = f'_k(k_t) - 1$ (in the case of full depreciation), where low letters (apart from factor prices) indicate variables expressed in per worker terms and the subscript of the derivative function f' indicates the derivation variable. Incidentally, it is worth noting that, as we will show later, in our model the steady state level of capital is independent of the assumptions on the production side and on the market structure.

2.3 Government

Following Diamond (1965), suppose that the government at each date t issues an amount B_t of national debt and levies lump sum taxes upon the young adults, according to the ordinary dynamic equation: $B_{t+1} = B_t(1+r_t) - \tau_{1t}N_{t-1}$ (where τ_{1t} is the lump sum tax) which, by reckoning that $N_{t-1} = L_t$, in per worker terms is:

[5] This assumption departs from Strulik ((1999) and (2003)) who assumes the rearing cost as a fixed fraction of w.

$$b_{t+1}n_t = b_t(1 + r_t) - \tau_{1t}; \tag{1}$$

finally, again by following Diamond (1965), we assume that the debt in per worker terms is constant, so that the level of taxes, τ_{1t}, is equal to $b(1 + r_t - n_t)$.

2.4 Decentralized solution

The young adults are supposed to face the following maximization problem:

$$\max U(c_{1t}, c_{2t+1}, n_t) = b_1 \log(c_{1t}) + b_2 \log(c_{2t+1}) + b_3 \log(n_t)$$

where

$$c_{1t} = w_t - \tau_{1t} - en_t - s_t$$

$$c_{2t+1} = (1 + r_{t+1})s_t.$$

Now two assumptions can be made as for the individuals' behavior with respect to the choice of the number of children: 1) they are ultra-perfect foresighted, so that they account for the effect of their choice of children on the level of taxes; 2) they are atomistic and because each individual's choice cannot influence the aggregate rate of growth of population, they take n_t as exogenous in the term $b(1 + r_t - n_t)$ while solving their maximization problem with respect to n_t. Notice that the difference between the two assumptions is crucial in determining the difference between the choice of a benevolent planner and the decentralized solution: that is, in case hypothesis 2) is assumed, the "social" optimal choice of savings and population is different from the (typically, suboptimal) decentralized solution. For simplicity, in this paper we focus only on the second case, in which the steady state solutions are:

$$s^* = \frac{b_2 e(w - b(1 + r))}{e\beta - bb_3} \tag{2}$$

$$n^* = \frac{b_3(w - b(1+r))}{e\beta - bb_3}, \tag{3}$$

where $\beta = b_1 + b_2 + b_3$; necessary and sufficient conditions for $s, n > 0$ are alternatively: i) $b > max\left[\frac{w}{1+r}, Q\right]$; ii) $b < min\left[\frac{w}{1+r}, Q\right]$, where $Q = \frac{eb_2}{b_3}$. By passing, we note that, by eq. (3), in this simple standard OLG frame the population growth depends positively on the wage, in line with a classical view à la Malthus.

Given the market clearing equation $s_t N_{t-1} = K_{t+1} + B_{t+1}$, or, equivalently, $s_t = (k_{t+1} + b)\, n_t$ and the solutions for s, n above, the market clearing condition boils down to the following long run per worker capital:

$$k^* = \frac{eb_2}{b_3} - b. \tag{4}$$

It is interesting to note that at the equilibrium there is a complete "crowding out" effect of the public debt upon the stock of capital, that is, a one to one correspondence between them. Hence, similarly to Diamond's results, also in our frame national debt should be increased in order to correct DI.

Moreover, the condition for a positive long run capital stock is the following:

$$b < Q.$$

In particular, by assuming the usual CD production function in per worker terms

$$y = Bk^a, \tag{5}$$

where B is a constant index of technology, and full depreciation of capital (that is $aBk^{a-1} = 1 + r$), necessary and sufficient conditions for $s, n > 0$ are alternatively: i) $b > max\,[(1-a)Q, Q]$; *ii)* $b < min\,[(1-a)Q, Q]$. Hence, the overall conditions for the positivity of the long run values of s, n, k simply boil down to the ii) above mentioned, which, recalling that $a < 1$, is $b < (1-a)\,Q$. In other words public debt must be sufficiently low, especially when rearing costs and the degree of patience are low and preference for children is high.

Finally, by inspection of eq. (4) two remarks are worth making:

Remark 1 *As expected, the long run per worker capital is inversely linked with the factors increasing the population growth and, thus, depends positively on the rearing cost* (e) *and on the preferences for children* (b_3) *and positively linked with the factor increasing accumulation, that is with the degree of patience* (b_2).

Remark 2 *Rather interestingly, the long run per worker capital is independent of the technology: for whatever technology (CD, CES, Leontief and so on) the long run per worker capital is the same. But this also means that it is independent of all the usual assumptions on the side of firms, typical of the OLG framework: i.e. the constant returns to scale and competitive market.*

To put it in a different way, the content of Remark 1 can be summarized as follows: i) the higher the preference for children the less saving will be accumulated for older age; ii) when, for given preferences for children, the cost of rearing them is lower, more children will be grown and less saving accumulated. Note that these results appear to be at all coherent with the empirical evidence[6].

As for Remark 2, when fertility is endogenously chosen by individuals with CD preferences and constant rearing costs, the steady-state capital depends only on the preferences and not on the technology[7].

[6] We thank an anonymous referee for pointing us to this coherence of the results.

[7] Such an independence is intuitive if we think that savings and population growth depend in a similar way on the production technology; this a standard result also in other OLG models, where, for instance, the human capital rather than the number of children is endogeneized (see Michel and Vidal (2000)). On the other hand, the rate of growth of income g^* is, as expected, dependent on the type of technology as follows: $g^* = n^* - 1 = \frac{b_3\left[(1-a)Bk^{*a} + b(1-aBk^{*a-1})\right] - e\beta}{e\beta - bb_3}$.

3 The welfare effects of debt variations and the optimal level of debt

Following Diamond (1965), in this section we investigate the effects of debt variations on the steady state utility level of a representative individual.

At the steady state, the individual budget constraint has the form:

$$c_1 + \frac{c_2}{1+r} + en = w + b\left(\overline{n} - r - 1\right), \tag{6}$$

where $\overline{n}$ is the economy fertility gross rate, which individuals take as given. Then, by differentiating it with respect to b we get:

$$\frac{dc_1}{db} + \frac{dc_2}{db}\frac{1}{(1+r)} + e\frac{dn}{db} = \frac{c_2}{(1+r)^2}\frac{dr}{db} + \frac{dw}{db} + (\overline{n} - r) - b\frac{dr}{db} + b\frac{d\overline{n}}{db}. \tag{7}$$

Next, by assuming that the policymaker is benevolent and that all individuals have the same lifetime consumption/leisure pattern, by exploiting the Envelope Theorem the following equality holds:

$$\frac{dU}{db} = U_1\frac{dc_1}{db} + U_2\frac{dc_2}{db} + U_3\frac{dn}{db} = U_1\left[\frac{dc_1}{db} + \frac{dc_2}{db}\frac{1}{1+r} + e\frac{dn}{db}\right]; \tag{8}$$

finally, by reckoning that $c_2 = s\,(1+r)$, $\frac{s}{n} = k + b$, $\frac{dw}{dr} = -k$, and exploiting equation (7) it follows that:

$$\frac{dU}{db} = U_1\left\{\underbrace{(\overline{n} - r - 1)\left[1 + \frac{k+b}{1+r}\frac{dr}{db}\right]}_{D} + \underbrace{b\frac{d\overline{n}}{db}}_{P}\right\}. \tag{9}$$

Note that, the equation above amends Diamond's formula (that is, eq. 29, page 1142, corresponding to part D of our eq. (9)), with an extra term $\left(b\frac{d\overline{n}}{db}\text{ or } P \text{ factor}\right)$, which represents the novelty of the present work. In fact Diamond concludes that *"utility is decreased in the efficient case and* *increased in the inefficient case"*[8]; from inspection of equation (9), instead, we get the following proposition, which generalizes Diamond's rule:

Proposition 1 *Under Cobb-Douglas preferences, and with lump sum taxation, increasing the level of debt is always welfare worsening (improving) if and only if* $(\overline{n}-r-1)\left[1+\frac{k+b}{1+r}\frac{dr}{db}\right]+b\frac{d\overline{n}}{db}<(>)\,0$.

From the proposition above the following corollary descends:

Corollary 1: $\overline{n}-r-1>0$ *is a necessary but not a sufficient condition for debt issuing to be welfare improving.*

Proof. Preliminarly, note that, by eq (3) and by the proporties of the CD production function, $\frac{\partial r}{\partial b}=\frac{\partial r}{\partial k}\frac{\partial k}{\partial b}>0$ and, from eqs. (3) and (4), $\frac{\partial \overline{n}}{\partial b}=\frac{\partial \overline{n}}{\partial k}\frac{\partial k}{\partial b}<0$. Then, when $r+1<\overline{n}$, $\frac{dU}{db}$ can be negative if $(\overline{n}-r-1)\left[1+\frac{k+b}{1+r}\frac{dr}{db}\right]+b\frac{d\overline{n}}{db}<0$; on the other hand, if $r+1>\overline{n}$, then, $\frac{dU}{db}=(\overline{n}-r-1)\left[1+\frac{k+b}{1+r}\frac{dr}{db}\right]+b\frac{d\overline{n}}{db}$ is unambigously negative. ■

Note that the such result, again, is due to the fact that the sign of $\frac{d\overline{n}}{db}$ brings about an ambiguity on the overall effect on the steady state utility level. In particular, the higher the sentivitity of fertility choices to debt variations, the more likely a debt increase will be welfare worsening, even in presence of DI. As a consequence, the corollary above can revert Diamond's conclusions: in the "inefficient case", utility can be decreased rather than increased by an increase of the long run stock of public debt.

Moreover, it is possible to show that

Proposition 2 *Increasing debt is welfare improving until a threshold level is reached; debt increases beyond that threshold level become welfare worsening.*

Proof. Although we do not have any explicit solution for $\frac{dU}{db}=0$, after some calculus we get that eq. (9), after subsituting the decentralized equilibrium expression for all the variables, can be written as: $E\left[b_2\left(b_2b_3+e^2a^2\beta\left(\beta+b_2\right)\right)+\right.$

[8] We recall that an economy is referred to as "dynamically inefficient" when the productivity of capital is lower than the economy growth rate (in our notation $1+r<n$). Note that in our model DI still implies overaccumulation, due to the fact that $\frac{\partial r}{\partial k}<0$ and $\frac{\partial n}{\partial k}>0$ so that $1+r<n$ implies $k>k^{GR}$.

$bb_3aeH + b^2b_3^2(\beta + b_2)(1-a)\big]$,where $E = \frac{\left(\left(\frac{1}{b_3}(eb_2 - bb_3)\right)^a\right)eBb_3}{(e\beta - bb_3)^2(bb_3 - eb_2)^2}$, which is positive if $k > 0$ and $H = 6b_1b_2 - 3b_1b_2/a + b_3^2 - 3b_2b_3/a - 4b_2^2/a + 2b_1b_3 + b_1^2 + 6b_2b_3 + 7b_2^2 - ab_1b_2 - ab_2b_3 - 2ab_2^2$. The expression in brackets is quadratic form in b, which has at most two possible positive roots and only the smaller one is a candidate (local) maximum for the U function, which we call b^O. ■

Note that the result is in line with Diamond's findings, although in an at all different. Moreover, we can further qualify our result as follows

Proposition 3 *The level of public debt which maximises the welfare of individuals is lower than that satisfying* $D = 0$.

Proof. Let us call b^D the level of debt such that $D = 0$ (that is $r + 1 = \overline{n}$). In fact, the latter equality implies that $b^D = \frac{e}{b_3(1-a)}(b_2(1-a) - a\beta)$; when substituting such expression into $\frac{d\overline{n}}{db}$, one gets: $\frac{d\overline{n}}{db} = -(b_1 + ab_2 + b_3)^{-1}\beta^{-2}a^{-1}e^{-2}(a-1)^4\left(\left(\frac{ea\beta}{b_3(1-a)}\right)^a\right)b_2Bb_3^2$, which is negative, so that $\frac{dU}{db}\big|_{b^D} < 0$; hence, we obatain that $b^O < b^D$. ■

Note that we can take b^D value as an approximation of the level of debt which would be optimal in the absence of endogenous fertility (in fact such a level would maximize the utility of the representative agent in case $\frac{d\overline{n}}{db} = 0$)[9]. Thus, the following remark holds:

Remark 3 *The endogenous behavior of fertility implies that a benevolent government should issue a lower amount of debt, relative to the case of exogenous population growh rate. Indeed, a positive preference for children always reduces the amount of resources that the individuals transfer through savings to their old age and therefore decreases the accumulation of capital. As a consequence the risk of overaccumulation is lower or, equivalently, the optimal level of debt is lower when fertility choices are accounted for.*

Finally, note that the condition $b^O < b^D$ means that $(\overline{n} - r - 1) > 0$, that is, $k^O > k^D$.

[9] However, it is worth recalling that, as shown by Lagerlöf [10], in such a case fertility rates would be negative and, hence, fertility should be subsidized.

4 Sensitivity analysis

In this section we perform a comparative statistics analysis so as to assess the sensitivity of the optimal level of debt to the parameters of the models. Figure 1 in Appendix A shows a numerical example for a set of preferences and technological parameters[10]. Both this Figure and Table 1 confirm the statement contained in Proposition 3, in that the optimal level of debt is always higher than that maximizing the representative agent's utility when the aggregate fertility rate does not vary with debt. As for the sensitivity analysis[11], let us start with the role of the share of capital. Looking at Table 1, we can see that by doubling the share of capital (from 0.1 to 0.2, first and second rows of Table 1 respectively), and other things equal, the associated value of b^O decreases by 50%, from 0.45 to 0.21: intuitively, when a rises, savings tend to decrease due to the (negative) effect on the wage (in our specification of preferences the interest rate effect on savings and fertility is zero); moreover, such a negative change more than offsets the opposite effect on the per worker level of capital, generated by the reduction of $\overline{n}$; as a consequence, since the equilibrium level of per worker capital decreases, it is less likely for the economy to undergo the risk of overaccumulation, so that the optimal level of debt tends to decrease.

The third row of Table 1 shows that, as expected, the higher the degree of patience, the higher the optimal level of debt, since individuals tend to (over)accumulate for securing a relatively higher amount of consumption

[10] Several numerical simulations have shown that, for realistic values of parameters, the $\frac{dU}{db}$ locus has only one positive root in the existence set of the problem analyzed here.

[11] The choice of the set of values for our simulations may be so briefly illustrated: i) values of the capital share of about 0.1 and 0.2 might mimic the technology respectively of an underdeveloped and of a developing country; ii) values of the intertemporal discount factor $\mathrm{b}_2 = \frac{1}{1+z} = (0.9, 0.5)$, where z is the rate of time preference, imply, taking account of the time span of one generation, two values of z about 0.5% and 2.5 % per year, respectively; as for the values chosen for the other parameters, after several trials, we report only some of those implying plausible values of the steady state equilibrium variables: for example a) the cost of rearing one child in percentage of earned income (e/w) ranges from 38% to 72%; b) the emerging long run ratio debt/GDP ranges from 16% to 90% which is largely coherent with the evidence of many countries. Other specifications of the parameters, not reported here, did not change qualitatively our results.

during their elderly.

Turning to the parameters specific to our work, in the fourth row the effects of a rise in the cost of rearing children are depicted: namely, such change generates a proportional increase of both the optimal level of debt and the debt-to-national-income-ratio. In fact, when rearing a child becomes more costly, the aggregate fertility rate gets lower and tends to increase the level of per worker capital. Analogously, the row at the bottom of Table 1 shows that an increase in the preference for children (b_3), leads to an increase of the aggregate fertility rate, so that a lower level of debt (the reduction of which is of the same order of the b_3 change) is necessary to correct the DI of the economy. It is worth noting that, while the optimal level of debt is in general significantly sensitive to the parameter specification, the ratio e/w is relatively stable; however, both values react substantially when the cost of rearing children varies.

Finally, the Figures 2 to 4 in Appendix A represent the $\frac{dU}{db} = 0$ locus (i.e., the locus identifying the optimal level of debt) in the a, b space, for different values of b_2, e and b_3 respectively: these Figures confirm the unambiguous role played by the parameters dicussed above for a reasonably large range of parameter values: hence, we can be confident about the robustness of our findings.

5 Conclusions

This paper extends the traditional OLG framework a là Diamond (1965) by allowing for endogenous fertility choices. Under this scenario we address the issue of the existence and the characterization of the optimal level of debt, that is the level which is necessary to correct overaccumulation (or dynamic inefficiency -DI). Our results can be summarized as follows:

1) the steady state level of capital emerging from the market equilibrium depends only on the preferences side but neither on the technological one nor on the supply side conditions, while the growth rate is positively linked to the wage;

2) the existence of dynamic inefficiency (and, thus, the necessity for

issuing national debt so as to correct it) is favoured by a small capital income share, on the technological side, a sufficiently high degree of patience and a sufficiently low preference for children, on preferences grounds;

3) similarly to Diamond's result, public debt plays a clear-cut role on the dynamic inefficiency: in particular, debt must be increased (decreased) when the economy is overaccumulating (underaccumulating);

4) however, Diamond's (1965) rule turns out to be only a necessary but not a sufficient condition for welfare improvement to obtain in the presence of endogenous fertility: we find that increasing debt can be welfare worsening even though DI (i.e. $n > 1+r$) is being experienced. In fact, we show that a debt increase is welfare improving until a threshold level is reached, but not necessarily does such value satisify the equality between the productivity of capital and the population growth rate; further increases of debt beyond this level bring upon a reduction of welfare, without DI being completely corrected;

5) a comparative statics analysis shows that the optimal level of debt is higher the bigger the child rearing cost, the lower the capital share, the higher individuals' degree of patience and the lower the preference for children. In other words, the endogenous behavior of fertility implies that a benevolent government should issue a lower amount of debt, relative to the case of exogenous population growh rate. This is due to the fact that individuals, coeteris paribus, save less for their old age; since this reduces the risk of overaccumulation of capital, it follows that the optimal level of debt necessary for correcting DI is lower in the presence of endogenous fertility. As a consequence, policymakers disregarding the effects of debt management policies on fertility choices are more prone to the risk of "overshooting" the target, that is of issuing too high a stock of national debt.

Another policy implication of our analysis is the following: the reduction of the cost of rearing children appears to be crucial for the current debt-tightening policies undertaken by several European countries to generate optimal redistributions among generations. In fact, countries like Italy, which have experienced an increase of the capital share and a decrease of the propensity to save over the last three decades, are likely to be moving in the right direction provided that they accompany the current reduction of the public debt stock with policies designed to keep low (or, better, reduce) the costs of rearing children, so as to secure a welfare improvement in the long run.

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A Figures and Tables

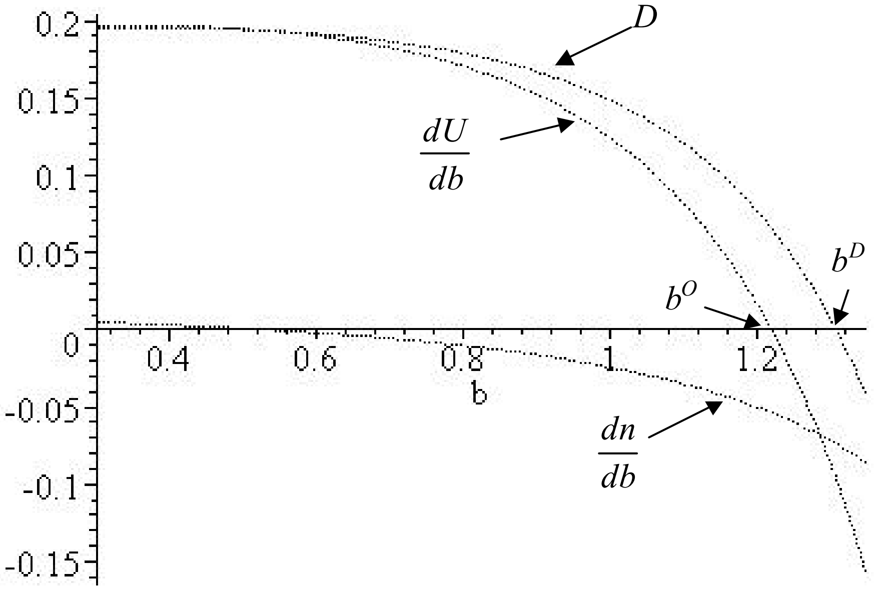

Figure 1: D, $\frac{dU}{db}$ and $\frac{dn}{db}$ loci. Parameters: $b_1 = 1$, $b_2 = 0.9$, $b_3 = 0.9$, $e = 2$, $B = 1$, $a = 0.1$.

Table 1: *Sensitivity of the optimal level and of other economic indicators to parameter variations*

Parameters				Variables			
e	b_2	b_3	a	b^D	b^O	$\frac{b^O}{f}$	$\frac{e}{w}$
0.3	0.9	0.4	0.1	0.48	0.45	0.52	0.38
0.3	0.9	0.4	0.2	0.24	0.21	0.25	0.43
0.3	0.5	0.4	0.1	0.22	0.20	0.24	0.40
0.6	0.9	0.4	0.1	0.97	0.90	0.97	0.72
0.3	0.9	1	0.1	0.17	0.16	0.20	0.42

Other parameters imposed: $b_1 = 1$, $B = 1$.

$\frac{b^O}{f}$ is the optimal debt to GDP ratio.

$\frac{e}{w}$ is the ratio of the cost of rearing a chind and the wage.

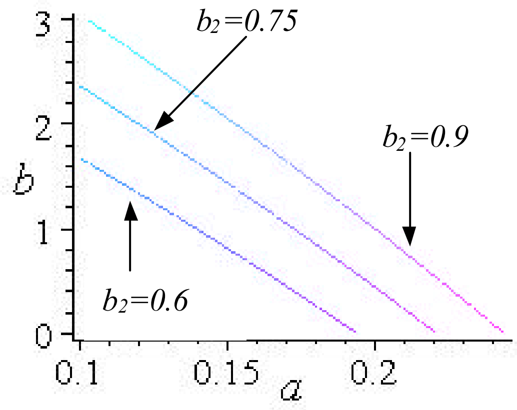

Figure 2: $\frac{dU}{db} = 0$ locus in the a, b space, for different values of b_2.Other values imposed: $B = 1$, $b_1 = 1$, $e = 5$, $b_3 = 0.9$.

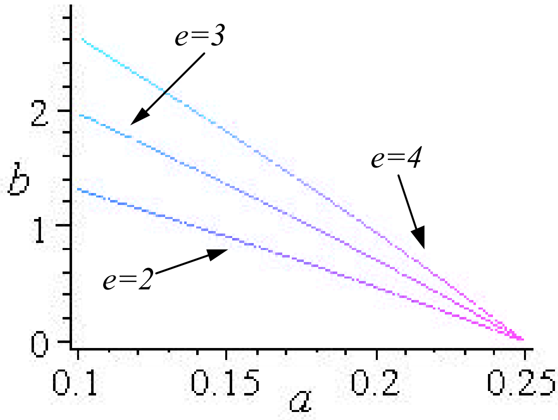

Figure 3: $\frac{dU}{db} = 0$ locus in the a, b space, for different values of e.Other values imposed: $B = 1$, $b_1 = 1$, $b_2 = 0.95$, $b_3 = 0.9$.

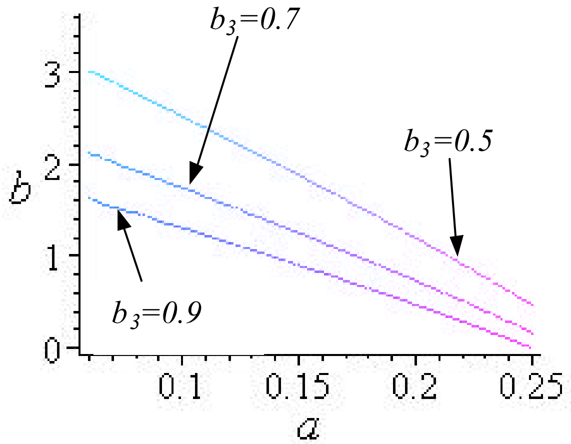

Figure 4: $\frac{dU}{db} = 0$ locus in the a, b space, for different values of b_3.Other values imposed: $e = 2$, $B = 1$, $b_1 = 1$, $b_2 = 0.95$.

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