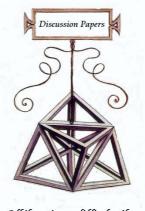


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Gaetano Alfredo Minerva

How Do Cost (or Demand) Asymmetries and Competitive Pressure Shape Trade Patterns and Location?

Discussion Paper n. 40

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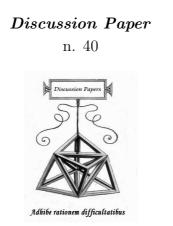
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Gaetano Alfredo Minerva

How Do Cost (or Demand) Asymmetries and Competitive Pressure Shape Trade Patterns and Location?

Abstract

A two sectors, two regions economy, where one sector is perfectly competitive and the other is monopolistically competitive, is considered. The region hosting more firms in the monopolistic sector produces at a lower marginal cost (or equivalently produces varieties more intensely demanded by consumers). We show how different trade patterns arise in this sector as a function, among the others, of overall competitive pressure. If capital is mobile between regions in the long run, we characterize when full agglomeration in the more productive region is the equilibrium. Finally, some numerical examples show how structural changes in trade patterns originate from changes in the parameters of the model.

JEL classification: D43; F12; L13.

Keywords: Industrial location; monopolistic competition; intraindustry trade; cost advantage; demand intensity.

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I. Introduction

Much of the debate about international and interregional trade issues aims at pointing out the effects of the opening of trade when the trading regions are heterogeneous in some respect. This paper tries to answer this question as well, in a specific framework. Economic space is made of two regions hosting two sectors which differ in the underlying market structure. The first is monopolistically competitive and produces an array of horizontally differentiated varieties, while the other is a residual sector, characterized by perfect competition, representing the rest of the economy. I assess the emergence of different patterns of trade in the monopolistic sector as a consequence of: a) a differential in marginal cost among firms according to the region they belong to; b) a differential in the degree of competition in each region, due to the difference in the number of firms in each local market; c) different degrees of overall competitive pressure, measured by the total number of firms in the economy; d) the abandoning of the CES (Dixit-Stiglitz) monopolistic competition model, in favour of a linear demand specification. Due to the linearity of the demand for differentiated varieties, we will show that point a) is amenable to an interpretation in terms of different intensities of demands for the differentiated products, according to the region where they are manufactured.

The perfectly competitive sector is characterized by constant returns to scale. The monopolistically competitive sector is modelled according to the Vives (1990) specification. This sector can be thought to be manufacturing or services: it is particularly suited to capture market structure prevailing in traditional sectors as textiles, clothing, and food processing. Vives model has been cast in an economic geography core-periphery setting by Ottaviano, Tabuchi, and Thisse (2002).

After having derived short-run trade equilibria, we make the hypothesis that capital is mobile between regions in the long run, analytically deriving conditions ensuring complete agglomeration of manufacturing in the more productive region. Moreover, if only partial agglomeration of manufacturing is attained, we study how the spatial allocation changes as a function of trade costs.

The basic set up employed in this paper follows what is known in the economic geography literature under the headings of *footloose capital* model (Martin and Rogers, 1995, Ottaviano, 2001), FC model hereafter. The differences between core-periphery and FC models are thoroughly analyzed in Baldwin et al. (2003). For our purposes it suffices to stress the following. In a core-periphery setting the mobile factor is labour, so that its migration induces also an expenditure shifting in the region where migration occurs, because people spend their earnings where they live. On the contrary, in a FC model the mobile factor is capital, whose rewards are repatriated to capital owners who are immobile in the two regions. Due to this fundamental difference, the FC model does not show circular causality (self-reinforcement) in agglomeration, and is more tractable analytically.

As already shown in Behrens (2003), the linear demand model gives rise to asymmetric trade patterns when the two regions differ in the number of firms located, with one region hosting significantly more firms than the other. The possibility of asymmetric trade patterns is a realistic feature being inevitably lost under the CES Dixit-Stiglitz specification. Our introducing of cost (or demand) asymmetries among the two regions strengthen the tendency of asymmetric patterns to arise: in addition to toughness of competition, firms in the high cost region have to overcome this problem as well. This makes the range of two-way trade smaller.

Our framework is very similar to Behrens (2003, 2004) modelling, apart from the fact that his paper is a full-fledged NEG model (what is mobile there are workers that locate where indirect utility is higher, while here we are assuming the mobility of capital towards locations where the rental rate is higher). The main contribution of the present paper is to add to the picture asymmetries between the two regions. We model them as stemming either from a cost advantage of producing in region A over region B, or from a demand premium that varieties manufactured in A enjoys with respect to *B* products. The linearity of the demand functions then ensures that, from the point of view of the individual firm, profits functions in the two circumstances are analytically equivalent, giving to our problem a twofold interpretation. That these features play a role in explaining unilateral trade patterns is a well established stylized fact. Moreover, I argue that, in Behrens' model, price competition in the last stage of the game *and* a special assumption he makes about prices of non-traded commodities are indeed equivalent to assume quantity competition. All subsequent calculations are derived consistently.

The idea of studying industrial clustering due to technological externalities in a monopolistic competition framework with linear demands appears already in Belleflamme, Picard, and Thisse (2000). Contrary to their approach, we assume that the productivity advantage of one region over the other is constant whatever the number of local firms is. We extend the analysis to all possible levels of transport costs, i.e. not just those allowing two-way trade (this special case is sometimes referred to as *market overlapping* hypothesis). We focus also on what happens either when trade is not possible at all (autarchy) or when the direction of trade is unilateral, from the more productive region to the less productive one (one-way trade). Moreover, not normalizing the total number of firms in the economy to some convenient value (Belleflamme *et al.* set it to 1), and leaving it to be an exogenous parameter, we show that trade patterns are also affected by the "thickness" of the monopolistic sector.

The paper is organized as follows. In section II. we present the model. In section III. we compute the short-run equilibrium of the economy (for a fixed spatial distribution of firms) distinguishing among different trade patterns. Afterwards (section IV.) we let capital going where the rental rate is higher. In some cases, which we analytically characterize, full agglomeration of the manufacturing sector in one region will be the long-run equilibrium. Under other circumstances, capital movements can cause endogenously a no-trade outcome.

II. The model

The model developed in this paper is in various manners linked to other works in the economic geography literature. The closest relatives are Belleflamme et al. (2000), Ottaviano et al. (2002), and Behrens (2003, 2004). The economy is made of two regions s = $\{A, B\}$ of equal size, with consumers having identical preferences, and two sectors: a monopolistically competitive sector, producing an array of differentiated goods and a perfectly competitive sector producing a homogeneous good 0, which we may think as the rest of economy.

II.A. Consumer's behaviour

Consumers' preferences over the Chamberlinian monopolistically competitive industry are specified according to Vives $(1990)^1$. Representative consumers in the two regions share the same preferences and maximize the following utility function:

$$U(q_0, x(j)) = \xi \int_{j \in N} x(j) dj - \frac{1 - \omega}{2} \int_{j \in N} [x(j)]^2 dj - \frac{\omega}{2} \left[\int_{j \in N} x(j) dj \right]^2 + q_0$$
(1)

Parameters in the utility function are $\xi > 0$ and $0 \le \omega \le 1$. The set of varieties is $S = \{j | j \in [0, N]\}$. They are uniformly distributed on [0, N], with N being the total mass of the monopolistic sector. The parameter ξ is a proxy for the intensity of preference for the differentiated good. The higher ξ , the higher this preference. The parameter ω represents the degree of product differentiation among varieties. When ω approaches zero varieties are so much differentiated that they can be thought to belong to completely different sectors (total utility is simply additive in the utility derived from each good), while ω equal to 1 represents perfectly homogeneous products.

¹In Vives (1990) and Belleflamme et al. (2000) the total mass of the monopolistically competitive sector N is normalized to 1. We do not use this normalization because N will turn to be one of the key parameters of the model.

We assume that the representative consumer in region s is endowed with K_s units of capital and L units of labour, with labour supply L being equal in the two regions. Income comes from the rental rate of capital and wage.

The budget constraint of the representative individual in region A can be written as

$$\int_{j \in n_A} p(j)x(j)dj + \int_{j \in n_B} p(j)x(j)dj + p_0q_0 = w_A L + r_A K_A \quad (2)$$

where p(j) is the price of a variety, x(j) is the quantity demanded, w_A is wage in region A, r_A is the rental rate of capital in region A. We distinguish between varieties produced in region A (whose mass is n_A), and varieties produced in region B (whose mass in n_B). The quasilinear structure of $U(\cdot)$ implies that consumption of commodity 0 is the residual of what is spent on the monopolistic sector. Consequently, provided income is high enough so to allow a positive consumption of good 0 in equilibrium, every further increase in income corresponds to an equal increase in the consumption of the agricultural commodity, not affecting the demand for the differentiated varieties in manufacturing.

After having plugged the budget constraint in the utility function, maximization yields inverse demand functions. Inverse demand for a variety $j \in n_A$ produced in A and sold in A is

$$\frac{p_{AA}(j)}{p_0} = \xi - (1 - \omega) x_{AA}(j) - \omega X_A$$
(3)

where x_{AA} is demand for variety j and

$$X_A = \int_{j \in n_A} x_{AA}(j) dj + \int_{j \in n_B} x_{AB}(j) dj$$

is total demand for the monopolistic sector from consumers located in region A, consisting of varieties manufactured both in region Aand in region B. A variety $j \in n_B$ produced in B but sold in A has an inverse demand equal to

$$\frac{p_{AB}(j)}{p_0} = \xi - (1 - \omega) x_{AB}(j) - \omega X_A$$
(4)

where variables have the same interpretation as above. Similar expressions can be derived for products sold in market B.

II.B. Labour market

L is employed as a variable input either in manufacturing or in agriculture, and the supply of labour is perfectly elastic between sectors. In manufacturing, c_s units of labour are needed for each unit of output, and this labour requirement differs in the two regions, that is $c_A \neq c_B$. This assumption wants to capture the fact that there are locations where productivity of labour in manufacturing is higher. Turning to the production of the homogeneous good, it is carried out under constant returns to scale, with unit labour requirement set equal to one by an appropriate choice of scale. Labour market clearing requires that

$$Q_0 + \int_{j \in n_A} c_A x(j) dj = L$$

where Q_0 is total economy's numeraire production. A positive amount of labour is employed in sector 0 because labour supply is assumed to be high enough so to cover all input requirements of the differentiated commodity sector. Constant returns to scale ensures that wage in sector 0 is equal to the exogenously fixed price p_0 . Since labour market is assumed to be in equilibrium, $w_A = p_0$. If $w_A \ge p_0$ workers would move from one sector to the other until equality in the wage rates is reached due to perfect elasticity of supply.

II.C. Firms' behaviour

Firms play a two-stages game. In the first stage they establish in each region, up to the point clearing capital market. By an appropriate choice of scale, each plant requires one unit of capital to start up production (entry cost). The mass of firms will exactly equal the mass of capital available in that region at a given moment². In the second stage there is market competition. As said earlier, c_s units of labour are needed to produce one unit of the differentiated output, and this marginal cost differs across regions. Markets are segmented, so that each firms sets the strategic variable (price or quantity) in each regional market in which it operates. Notice that exporting in the foreign region requires t units of good 0 for each unit of output. Total profits of a representative A firm are then:

$$\frac{\Phi_A(i)}{p_0} = \left[\frac{p_{AA}(i)}{p_0} - c_A\right] x_{AA}(i) + \left[\frac{p_{BA}(i)}{p_0} - c_A - t\right] x_{BA}(i) - \frac{r_A}{p_0}$$

We substitute inverse demands (3) and (4) in the profit function. We do so because we assume that firms maximize profits with respect to quantities. As claimed by Vives (1990), in a model of monopolistic competition maximization with respect to prices or quantities brings the same results, since the individual firm behaves as a monopolist on the residual demand. Actually, what in a Cournot oligopoly is a wrong conjecture (the fact that all other firms' sales are unaffected by a change in the quantity sold by one of them) turns out to be true in a model of monopolistic competition, because each firm taken in isolation is negligible. The same applies to competition in prices, since the price index is unaffected by a change in the price of an isolated firm.

The equivalency could possibly fail in a trade model like ours if trade does not take place actually, because transport costs are too high, and a positive foreign demand does not correspond to a price greater or equal to costs. In this case firms may be thought to set a *fictional* price abroad, even if demand at this price is zero, and this could prevent the equivalency. In Behrens (2004), when a firm does not sell in the foreign market, because transport costs are too high, it will "set the lowest possible price for which this [foreign] demand

²This is the same assumption made in Martin and Rogers (1995). As they do, we will introduce two different time horizons. In the short run capital available in each region equals the capital endowment of the representative consumer in that region $(K_s = n_s)$. In the long run capital flows freely from one region to the other, so that the only equality that has to hold is $K_A + K_B = N$.

is zero^{"3}. In Appendix A. I show how, employing this pricing rule, price and quantity setting are perfectly equivalent.

Solving the model with respect to quantities, and substituting inverse demand functions, profits for a firm in A are:

$$\frac{\Phi_A(i)}{p_0} = \left[\xi - (1-\omega)x_{AA}(i) - \omega X_A - c_A\right] x_{AA}(i) + \left[\xi - (1-\omega)x_{BA}(i) - \omega X_B - c_A - t\right] x_{BA}(i) - \frac{r_A}{p_0}$$
(5)

As was mentioned earlier, given that each firm is negligible with respect to aggregate quantities, a change in output in one of them leaves unchanged the output index X_A . A similar expression can be derived for profits of a firm in B.

III. Short-run equilibrium

In the short run capital is immobile so that the number of firms located in each region is fixed and equal to the capital endowment of residents. In the first stage of the game, free entry and exit imply that there is a bidding process for available capital in both regions so that the rental rate equalizes operating profits.

Assumption 1 Throughout the paper, $c_A < c_B$, and $n_A > n_B$.

Region A hosts a greater number of firms, and is more productive, because it requires less input to produce one unit of output. We think this could be justified in several ways. In the first instance, it could be the outcome of Marshallian (or network) externalities, so that input requirement is smaller where more firms are located. However the intensity of the externality is exogenous, time-invariant, and does not depend on the mass of firms located in A^4 . In the second instance, since in the short run the number of firms is equal to the endowment of capital of each region, our

³See Appendix A in Behrens (2004).

⁴In some respects this is a simplification with respect to Belleflamme et al. (2000) where the cost reduction depends proportionally on the share of firms located in the region.

assumption amounts to saying that the region (or country) being relatively capital abundant is also more productive. This is clearly the case when we compare regions at different stages of economic development. Moreover, assuming that low cost region A hosts less firms than B would not be interesting because producing in A would be unconditionally more profitable. Under mild conditions, every long-run dynamics such that capital goes where its remuneration is higher would converge again to a distribution of firms so that Assumption 1 be satisfied. The interesting point we want to study in this paper is how tougher competition $(n_A > n_B)$ and lower costs of production $(c_A < c_B)$ balance each other, and shape industrial location and trade.

In addition, as mentioned earlier, there is another interpretation to our set up. Due to the linearity of demand, the cost disadvantage of region B is equivalent to an upward shift of the intercept of the demand function for region A products with respect to region B, with firms in both regions incurring the same marginal cost of production, as an inspection of the objective function (5) shows. Recovering the underlying preference structure, it is

$$U(q_0, x(j)) = \xi \int_{j \in n_A} x(j) dj + (\xi - \theta) \int_{j \in n_B} x(j) dj - \frac{1 - \omega}{2} \int_{j \in N} [x(j)]^2 dj - \frac{\omega}{2} \left[\int_{j \in N} x(j) dj \right]^2 + q_0$$

Asymmetric trade patterns are then the by-product of an asymmetry in tastes, with the region having a stronger preference for its products hosting at the same time more firms.

Remark 1 In the profit function (5), the cost differential θ is analytically equivalent to an upward shift equal to θ of the intercept of demand functions for A varieties with respect to B.

Without loss of generality, we make the following substitutions:

$$\eta \equiv \xi - c_A, \qquad \theta \equiv c_B - c_A$$

so that $\eta - \theta \equiv \xi - c_B$. Firms in A maximize profits with respect to $x_{AA}(j)$ and $x_{BA}(j)$, taking as given quantity indices X_A and X_B .

The adoption of the Nash equilibrium solution implies that each firm takes as given individual output of rival firms and consequently total market output X_s . Equilibrium quantity of a firm located in A and selling to consumers in A is

$$x_{AA}^* = \frac{\eta - \omega X_A}{2(1 - \omega)} \tag{6}$$

As to the quantity sold by firms located in A to consumers in B we have

$$x_{BA}^* = \frac{\eta - t - \omega X_B}{2(1 - \omega)} \tag{7}$$

Expressions pertaining to firms in B can be derived similarly.

Let us initially consider the case where the two markets overlap, with firms in A exporting to B, and firms in B exporting to A (twoway trade). Following Ottaviano et al. (2002) we solve for the Nash equilibrium, but we depart from them since we require that the conjecture made on the *output indices* X_s be consistent (instead of price indices). As was mentioned earlier, this is irrelevant for equilibrium unless firms set a fictional price for varieties not traded in equilibrium. Moreover, when computing a regional price index, one has to face the dilemma of whether to include or not prices of varieties not traded in equilibrium⁵. Using a quantity index the problem is naturally ruled out, since the output of a firm enters the index only if it is a strictly positive quantity (i.e. only if the variety is effectively traded).

Total output in market A at equilibrium is:

$$X_A = n_A x_{AA}^* + n_B x_{AB}^*$$

where we used the symmetry of the model to say that

$$\begin{aligned} x_{AA}(i) &= x_{AA}(j) \quad \forall i, j \in n_A, \quad i \neq j \\ x_{AB}(i) &= x_{AB}(j) \quad \forall i, j \in n_B, \quad i \neq j \end{aligned}$$

⁵Behrens (2004) includes all prices in the determination of the price index. Tabuchi and Thisse (2002) only include prices set by firms that are effectively selling a variety in the region.

The equilibrium values are:

$$x_{AA}^{*} = \frac{2\eta(1-\omega) + \omega n_B(\theta+t)}{2(1-\omega)(2-2\omega+\omega N)}$$
$$x_{AB}^{*} = \frac{2(\eta-\theta-t)(1-\omega) - \omega n_A(\theta+t)}{2(1-\omega)(2-2\omega+\omega N)}$$

Quantities sold in market B are at equilibrium:

$$x_{BB}^* = \frac{2(\eta - \theta)(1 - \omega) - \omega n_A(\theta - t)}{2(1 - \omega)(2 - 2\omega + \omega N)}$$
$$x_{BA}^* = \frac{2(\eta - t)(1 - \omega) + \omega n_B(\theta - t)}{2(1 - \omega)(2 - 2\omega + \omega N)}$$

We see the role of the comparative cost (or demand) advantage and competitive pressure (proxied by n_A and n_B) computing the relative share of export from a firm in A with respect to a firm in B,

$$\frac{x_{BA}^*}{x_{AB}^*} = \frac{2(\eta - t)(1 - \omega) + \omega n_B(\theta - t)}{2(\eta - \theta - t)(1 - \omega) - \omega n_A(\theta + t)} > 1$$

meaning that firms locate in region A export more than their counterparts in region B. There are two elements bringing this result: the cost advantage, and the fact that $n_A > n_B$. Let us consider the case where $\theta = 0$, and $n_A = n_B$. The ratio x_{BA}^*/x_{AB}^* would be then equal to 1. If $n_A > n_B$, a firm in A exports more: even if firms in A have no a priori advantage, they face less competition when exporting to market B because less firms are located there⁶. As soon as there is a comparative advantage of A firms (i.e. $\theta > 0$), the ratio x_{BA}^*/x_{AB}^* gets even bigger: products manufactured in Aenjoy more favourable conditions (either on the demand side, or on the production side, with the two cases being formally equivalent in terms of firms maximization problem).

⁶When a firm located in A exports to B it has to face a greater number of competitors from the same region $(n_A > n_B)$ than a firm in B which exports to market A. However, due to transport costs, the pressure exerted by domestic competitors in the export region (i.e. B) is relatively more important in determining firms' profitability.

Focusing on the ratio of domestic production of each firm in the two regions,

$$\frac{x_{AA}^*}{x_{BB}^*} = \frac{2\eta(1-\omega) + \omega n_B(\theta+t)}{2(\eta-\theta)(1-\omega) - \omega n_A(\theta-t)}$$

we have that this ratio could be higher, lower, or equal to 1 according to the parameters of the model. When $\theta = 0$, and $n_A = n_B$, then $x_{AA}^*/x_{BB}^* = 1$. If $n_A > n_B$, and $\theta = 0$ then $x_{AA}^*/x_{BB}^* < 1$ because firms in A face more domestic competitors than firms in B. The same holds if θ is sufficiently small. On the contrary, when θ is high enough, the competition effect is offset by the technological (or demand) effect, and domestic output of a firm in A will be higher than output of a firm in B.

III.A. Trade patterns

We now determine the different trade patterns that arise in our linear demand monopolistic competition model. The analysis of trade patterns other than two-way trade has already been carried out in a core-periphery setting by Behrens (2003, 2004), without asymmetries. The aim of this section is to show the role played by the total mass of the monopolistic sector N, which is an exogenous parameter of the economy, and proxies overall competitive pressure. The higher the total mass, the tougher competition will be. Since the fixed capital requirement of firms is normalized to 1, a higher value for N stands for less capital required to set up a firm, so that increasing returns to scale are less intense. Our analysis then amounts to a comparative statics exercise on the intensity of increasing returns to scale.

We proceed imposing the non-negativity of equilibrium quantities x_{AA}^* , x_{BA}^* , x_{BB}^* , and x_{AB}^* , and then we consider all possible configurations to get the full characterization of short-run trade patterns.

Firms in A always produce a positive quantity of the monopolistic good for their domestic market, because it is always true that $x_{AA}^* > 0$.

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We concentrate now on exports of A firms to B. If $t > \eta$, $x_{BA}^* < 0$ for every λ . When $\theta < t < \eta$, $x_{BA}^* > 0$ if the share of B firms is small enough relatively to the total:

$$\lambda > 1 - \frac{2(\eta - t)(1 - \omega)}{\omega N(t - \theta)} \equiv \nu_{BA} \tag{8}$$

When the share of firms in B is high enough, export to region B is blockaded. By assumption the number of firms in region A is greater than the number of firms in B, so that admissible values are $\lambda \in (1/2, 1)$. Hence ν_{BA} will be binding if it belongs to (1/2, 1]. This is verified when the total mass N is

$$N > \frac{4(\eta - t)(1 - \omega)}{\omega(t - \theta)} \equiv N_{BA}$$

On the contrary, for $N < N_{BA}$, the threshold ν_{BA} is less than 1/2, it is not binding, and export to B is always possible under our assumptions. Finally, when $t < \theta$, A firms export for every λ .

We pass on to deriving conditions for firms located in region B. They always sell a positive quantity in their domestic market if $t > \theta$. If transport costs are greater than the cost differential, firms in B, producing under less favourable terms, will be protected in their domestic market, because the disadvantage they have is more than offset by barriers to trade. If $t < \theta$, $x_{BB}^* > 0$ if

$$\lambda < \frac{2(\eta - \theta)(1 - \omega)}{\omega N(\theta - t)} \equiv \nu_{BB} \tag{9}$$

that is relatively few firms are located in A. The intuition is that when many competitors have a cost advantage (many firms are located in A) the competitive pressure exerted by A firms hinders domestic production in region B. Conversely, if λ is small, there are few competitors producing at a lower cost. The threshold ν_{BB} belongs to the interval (1/2, 1) when the total mass of firms is

$$N_{BB} < N < 2N_{BB}$$

where

$$N_{BB} \equiv \frac{2(\eta - \theta)(1 - \omega)}{\omega(\theta - t)}$$

with $\nu_{BB} > 1$ for $N < N_{BB}$ ($\nu_{BB} > 1/2$ for $N < 2N_{BB}$). If the total mass of firms N is less than N_{BB} , competitive pressure is softened, allowing a positive (i.e. profitable) production of B firms in their domestic market whatever the spatial distribution is. If N exceeds $2N_{BB}$, the fact that x_{BB} be positive is not compatible with the assumption that $\lambda > 1/2$.

Firms in *B* cannot export a positive quantity to *A* as long as $t > \eta - \theta$. If $t < \eta - \theta$, $x_{AB}^* > 0$ provided

$$\lambda < \frac{2(\eta - \theta - t)(1 - \omega)}{\omega N(\theta + t)} \equiv \nu_{AB}$$
(10)

We have that $\nu_{AB} \in (1/2, 1)$ when

$$N_{AB} < N < 2N_{AB}$$

where

$$N_{AB} \equiv \frac{2(\eta - \theta - t)(1 - \omega)}{\omega(\theta + t)}$$

with $\nu_{AB} > 1$ for $N < N_{AB}$ ($\nu_{AB} > 1/2$ for $N < 2N_{AB}$).

All the above conditions constitute an extension of Behrens (2003) to a setting with regional heterogeneity in marginal costs (or in the intensity of preference for the differentiated varieties according to the region of production). Interestingly, contrary to Behrens, a measure of overall competitive pressure in the industry matters in shaping trade patterns. Two cases should be distinguished at this point: the cost advantage of region A could be high (respectively low), if $\theta > \eta - \theta$ (respectively $\theta < \eta - \theta$). In terms of asymmetries of the demand functions, the intercept of the demand for B products $\eta - \theta$ could be smaller (respectively bigger) than the difference θ between the two intercepts. In what follows we stick to the following assumption.

Assumption 2 The cost advantage θ is such that $\theta < \eta - \theta$.

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The analysis could be carried out without problems for the case $\theta > \eta - \theta$ as well, but for simplicity it is carried out only in one case. Moreover, if we interpret the model as one featuring taste asymmetries, it is preferable to assume that the difference between the intercepts of the demand functions (θ) be smaller than the smallest intercept ($\eta - \theta$).

First we consider autarchy, the case involving no-trade among the two regions.

Lemma 2 Autarchy constitutes the short-run equilibrium if one of the following conditions is satisfied:

i) $t > \eta$; ii) $\theta < t < \eta$, with $N > N_{BA}$ and $\lambda \leq \nu_{BA}$.

Proof. Point *i*) is easily derived. As to point *ii*), autarchy is the short-run equilibrium only if $N > N_{BA}$, that is only when x_{BA}^* could be zero. Both for $\theta < \eta - \theta < t < \eta$, and for $\theta < t < \eta - \theta < \eta$, this is true if $\lambda \leq \nu_{BA}$ and $N > N_{AB}$. Since $2N_{AB} < N_{BA}$, when $N > N_{BA} x_{AB}^*$ is zero.

Under one-way trade firms in A supply domestic and foreign markets, while firms in B supply their domestic market only. There is an asymmetry in trade relations.

Lemma 3 One-way trade constitutes the short-run equilibrium if one of the following conditions is satisfied:

i) for $\eta - \theta < t < \eta$, $N < N_{BA}$; or $N > N_{BA}$ and $\lambda > \nu_{BA}$;

ii) for $\theta < t < \eta - \theta$, $N_{AB} < N < 2N_{AB}$ and $\lambda \ge \nu_{AB}$; or $2N_{AB} < N < N_{BA}$; or $N > N_{BA}$ and $\lambda > \nu_{BA}$;

iii) for $t < \theta$, $N_{AB} < N < 2N_{AB}$ and $\nu_{AB} \leq \lambda < \nu_{BB}$; or $2N_{AB} < N < 2N_{BB}$ and $\lambda < \nu_{BB}$.

Proof. Let us start from $t > \theta$. Remember again that $N_{BA} > 2N_{AB}$. Then simply consider all the combinations of N and λ ensuring that $x_{BA}^* > 0$ and $x_{AB}^* = 0$.

When $t < \theta$, it is possible to show that $N_{BB} > N_{AB}$. Nothing can be said about the ordering among N_{BB} and $2N_{AB}$ and the threshold

 ν_{BB} becomes redundant when it is greater than 1 (think for example to a case where $2N_{AB} < N < N_{BB}$).

We now characterize two-way trade, when both regions trade with each other.

Lemma 4 Two-way trade constitutes the short-run equilibrium for $t < \eta - \theta$, and $N < N_{AB}$; or $N_{AB} < N < 2N_{AB}$ and $\lambda < \nu_{AB}$.

Proof. We derive conditions making the quantity sold abroad by *B* firms, x_{AB}^* , positive either for $\theta < t < \eta - \theta$, or $t < \theta < \eta - \theta$. Two-way trade is possible independently of the share λ when $N < N_{AB}$., it has to be $N_{AB} < N < 2N_{AB}$ and $\lambda < \nu_{AB}$. As to the quantity x_{BB}^* , it will be always positive under the conditions stated in the proposition: it suffices to remind that $N_{AB} < N_{BB}$ (implying trivially $2N_{AB} < 2N_{BB}$) and $\nu_{AB} < \nu_{BB}$.

As mentioned earlier, the role played by N has often been neglected in the literature. Notice that when $N_{AB} < N < 2N_{AB}$, the share of firms located in A should not exceed the threshold ν_{AB} if we want two-way trade to be feasible. If this were not the case, then it would be prohibitive to export into region A for B firms, due to toughness of competition. If the total mass of the monopolistic sector exceeds $2N_{AB}$, two-way trade is impossible, since profits margins will be compressed by the large number of firms, and B firms will not be able to export (or even to produce for their domestic market, when $N \geq 2N_{BB}$). Belleflamme et al. (2000) in their paper restrict attention to two-way trade. The only condition imposed concerns the level of transport costs t, that should be sufficiently low, and they normalize the total mass N to 1. By Lemma 4, the relative share λ could be ignored only if $N < N_{AB}$ (which corresponds to $N_{AB} > 1$ in their setting). If $N_{AB} < N < 2N_{AB}$, as the agglomeration process of firms in region A unfolds, when λ reaches ν_{AB} two-way trade is no longer sustainable, and the short-run equilibrium consists of one-way trade. Even if our model is not completely equivalent to Belleflamme *et al.*, because we assume that the cost differential is fixed, what they do is hence to assume implicitly that

 $N < N_{AB},$

$$N_{AB} > 1 \Leftrightarrow 2(\eta - \theta - t)(1 - \omega) > \omega(\theta + t)$$

which is an additional parameters' restrictions that should be indicated explicitly.

We say that in region B a process of deindustrialization has occurred when it is not possible for a firm operating in B to make non-negative profits.

Lemma 5 Short-run equilibrium involves deindustrialization of region B for $t < \theta$, $N_{BB} < N < 2N_{BB}$ and $\lambda \ge \nu_{BB}$; or $N \ge 2N_{BB}$.

To appreciate the economic meaning, let us focus on the cost of products that could be sold in market B. The fact that $t < \theta$, means that the total cost for A firms $(c_A + t)$ of a product sold in region B is lower than the cost incurred by B firms themselves (c_B) . If $N \ge 2N_{BB}$ only A firms are capable of being in the market, assuming a spatial distribution $\lambda \in (1/2, 1)$.

Equilibrium prices can be derived simply substituting equilibrium quantities in (3) and (4). Conditions for the non-negativity of mark-ups $(p_{AA}^* - c_A)$ and $(p_{BA}^* - c_A - t)$ coincide with those for equilibrium quantities x_{AA}^* , x_{BA}^* so that non-negativity of mark-ups is implied by non-negativity of quantities (the same apply for firms in B).

IV. Long-run equilibrium

Bidding for available capital determines the equality between operating profits and the rental rate in the short run, $r_s^*/p_0 = \Pi_s^*$, for a given spatial distribution of firms⁷. In the long run capital is mobile between regions so that the spatial distribution of firms is no longer equal to the initial endowment of capital in A and B. Capital flows occur in response to the differential in the equilibrium rental rate

⁷Competition in capital market drives the rental rate down to operating profits because of free entry of firms hiring capital.

 $r_A^*(\lambda) - r_B^*(\lambda)$, determined in the short-run. When the differential is positive capital goes from region A to region B. Viceversa, when the differential is negative, capital flows out of A into B.

For every trade pattern we identified, we argue about existence, uniqueness and convergence to the equilibrium distribution λ^* , so that $r_A(\lambda^*) = r_B(\lambda^*)$, in the sections below. If the equilibrium is reached, operating profits in the two regions will be equalized as well. In other terms, individuals look first for investment opportunities in the local market (which fixes the rental rate at operating profits in that region), then they look abroad, causing the exit of firms in the local market and the subsequent entry in the foreign one if the rental rate obtained there is higher (which determines equality of rental rates across regions). The model can be further extended assuming that capital moves between regions before local rental rate equals local profits. Individuals opt for lending their capital abroad, if the remuneration they obtain is higher, even if local capital market has not reached equilibrium yet. It can be argued that the final spatial distribution will be the same, irrespectively of the adjustment process, because for each trade pattern we prove that whenever a spatial equilibrium exists it is unique and determined by the primitive parameters of the model.

Remark 6 The unique equilibrium is reached irrespectively of the following adjustment processes in capital market:

i) Local rental rate first equals local operating profits; interregional capital flows then ensures the equalization of rental rates in the two regions;

ii) Entrepreneurs in a region bid simultaneously for domestic and foreign capital, and capital can flow before equalization of local operating profits to local rental rate is established.

In what follows we compare operating profits in the two regions under different trade patterns. Our goal is to establish which firms are performing better, whether those located in A or in B, as a function of the total mass of the monopolistic sector N, and the spatial distribution λ . Once having obtained these results, it is possible to determine the long-run outcome of the economy, under the assumption that capital is invested where interest rate is higher.

IV.A. Autarchy

Under autarchy, operating profits of a variety $i \in [0, n_s]$ produced in $s \in \{A, B\}$ are

$$\Pi_s(i) = [\xi - (1 - \omega)x_s(i) - \omega X_s - c_s]x_s(i)$$

where total output under autarchy in region s is equal to

$$X_s = \int_{j \in n_s} x(j) dj$$

Equilibrium output x_s^* is:

$$x_{s}^{*} = \frac{\xi - c_{s}}{2(1 - \omega) + \omega n_{s}}$$
(11)

and correspondingly equilibrium price is

$$\frac{p_s^*}{p_0} = \frac{(1-\omega)(\xi+c_s) + c_s n_s \omega}{2(1-\omega) + \omega n_s}$$

Profits are finally

$$\Pi_s^* = \frac{(1-\omega)(\xi - c_s)^2}{[2(1-\omega) + \omega n_s]^2}$$

Proposition 7 When differentiated varieties are not tradeable (see Lemma 2), $\Pi_A^* > \Pi_B^*$ if:

i) $N < N_U$, where

$$N_U \equiv \frac{2\theta(1-\omega)}{\omega(\eta-\theta)}$$

ii) $N > N_U$ and $\lambda < \lambda_U$, where

$$\lambda_U \equiv \frac{2\theta(1-\omega) + \omega\eta N}{\omega N(2\eta - \theta)}$$

Proof. Solving the inequality $\Pi_A^* > \Pi_B^*$ leads to the condition

$$\omega[(\eta - \theta)n_A - \eta n_B] < 2\theta(1 - \omega)$$

that, after having substituted $n_A \equiv \lambda N$, and $n_B \equiv (1 - \lambda)N$, is equivalent to

$$\lambda < \frac{2\theta(1-\omega) + \omega\eta N}{\omega N(2\eta - \theta)} \equiv \lambda_U \tag{12}$$

Assuming that $N < N_U$, where

$$N_U = \frac{2\theta(1-\omega)}{\omega(\eta-\theta)}$$

makes λ_U bigger than 1, so that $\Pi_A^* > \Pi_B^*$ for every λ .

The economic intuition of this result is the following. When the total mass of firms N is small, firms in A will make higher profits for every admissible λ , because of the cost advantage θ : region A is sufficiently attractive to host the whole manufacturing sector. On the other hand, if N is large, then the whole manufacturing sector could not locate entirely in A and still doing better than an isolated firm in B. In this case the actual spatial distribution of firms will matter for profitability, and the fraction of firms in A should be small enough to get $\Pi_A^* > \Pi_B^*$. The bigger the cost advantage θ , the larger the values of λ for which short-run profits in A are greater than in B, because λ_U increases as θ rises.

Two components related to the degree of competition affect profitability: the first is overall competitive pressure, measured by N; the second is local competitive pressure, measured by λ . Only when the total mass of firms is thick $(N > N_U)$ it makes a difference for firms with a cost or taste advantage being located in a crowded region or not. The same kind of reasoning applies to other trade regimes.

Given the monotonicity of $\Pi_s^*(\lambda)$ in λ , the value λ_U is also the stable long-run equilibrium distribution of the economy, provided one of the conditions of Lemma 2 is satisfied.

IV.B. One-way trade

Under one-way trade, firms located in region B do not make positive export to A. The output index in region A is then $X_A = \int_{j \in n_A} x_{AA}(j) dj$. Substituting in (6), and employing the symmetry of the model, the equilibrium quantity x_{AA}^* is

$$x_{AA}^* = \frac{\eta}{2(1-\omega) + \omega n_A}$$

equal to (11), the quantity sold in region A under autarchy. As in a situation without trade at all, firms in A are protected against competition coming from foreign firms, and they behave in the same way of autarchy in the local market. This makes the home component of profits equal to autarchy profits. Profits of A firms are also made of a component coming from abroad, making total profits equal to:

$$\Pi_A^* = \Pi_A^h + \Pi_A^f = \frac{(1-\omega)\eta^2}{[2(1-\omega)+\omega n_A]^2} + \frac{[2(\eta-t)(1-\omega)+\omega n_B(\theta-t)]^2}{4(1-\omega)(2-2\omega+\omega N)^2}$$
(13)

This is the sum of home profits (Π_A^h) and foreign profits (Π_A^f) . Profits of firms in B are

$$\Pi_B^* = \Pi_B^h = \frac{[2(\eta - \theta)(1 - \omega) - \omega n_A(\theta - t)]^2}{4(1 - \omega)(2 - 2\omega + \omega N)^2}$$
(14)

corresponding just to the home component.

A sufficient condition for A profits to be greater than B profits is $t < \theta$, which turns to be true under case *iii*) of Lemma 3. If the cost (or demand intensity) advantage of region A is greater than transport costs, markets are relatively well integrated and location in A allows higher profits regardless of the spatial distribution.

When $t > \theta$, I am not able to provide a closed-form solution for λ_O , the value of λ such that $\Pi_A^*(\lambda_O) = \Pi_B^*(\lambda_O)$, and $\Pi_A^*(\lambda) \ge \Pi_B^*(\lambda)$ for every $\lambda \le \lambda_O$. Nonetheless in Appendix B. I prove that, whenever this value exists, it is unique. Results are summarized in the proposition that follows. **Proposition 8** When one-way trade is established (see Lemma 3), $\Pi_A^* > \Pi_B^*$ if $t < \theta$. If $t > \theta$, we have one of the following cases: i) If $N < N_O$, where

$$N_O \equiv \frac{2(1-\omega)}{\omega(t-\theta)} \left(\sqrt{\eta^2 + (\eta-t)^2} - \eta + \theta\right)$$

 $\Pi_A^* > \Pi_B^*$ for every admissible λ .

ii) If $N > N_O$, and λ_O exists, then it is unique, and $\Pi_A^* \ge \Pi_B^*$ for $\lambda \le \lambda_O$.

iii) If $N > N_O$, and λ_O does not exist, then $\Pi_A^* < \Pi_B^*$.

Proof. See Appendix B..

Under one-way trade and $t > \theta$, there are several possible configurations, with firms in A performing better than firms in B, or, viceversa, firms in B doing better than in A. If λ_O does not exist, one-way trade cannot be a stable long-run outcome of the economy: either we have full agglomeration (point i) in the proposition) or we have transition to another trade pattern (point iii).

IV.C. Two-way trade

From Lemma 4, two-way trade is possible only if $t < \eta - \theta$. At the same time, the total number of firms in the economy has to satisfy the conditions $N < N_{AB}$; or $N_{AB} < N < 2N_{AB}$, and $\lambda < \nu_{AB}$. Profits of firms in A are

$$\Pi_{A}^{*} = \Pi_{A}^{h} + \Pi_{A}^{f} = \frac{[2\eta(1-\omega) + \omega n_{B}(\theta+t)]^{2} + [2(\eta-t)(1-\omega) + \omega n_{B}(\theta-t)]^{2}}{4(1-\omega)(2-2\omega+\omega N)^{2}} \quad (15)$$

while profits of firms in B are

$$\Pi_{B}^{*} = \Pi_{B}^{h} + \Pi_{B}^{f} = \frac{[2(\eta - \theta)(1 - \omega) - \omega n_{A}(\theta - t)]^{2} + [2(\eta - \theta - t)(1 - \omega) - \omega n_{A}(\theta + t)]^{2}}{4(1 - \omega)(2 - 2\omega + \omega N)^{2}}$$
(16)

made up of a home component and a foreign component. The following proposition explains the relative profitability of firms in the two regions as a function of the total mass N and the share λ .

Proposition 9 When two-way trade is established (see Lemma 4), $\Pi_A^* > \Pi_B^*$ if $t < \theta$. If $\theta < t < \eta - \theta$ one of the following conditions has to be satisfied:

i) $N < N_T$, where

$$N_T = \frac{2\theta(1-\omega)(2\eta-t-\theta)}{\omega(\theta^2+t^2)}$$

ii) $N > N_T$ and $\lambda < \lambda_T$, where

$$\lambda_T \equiv \frac{1}{2} + \frac{1}{2} \frac{N_T}{N}$$

Proof. If $t < \theta$, it is easy to see that $\Pi_A^* > \Pi_B^*$. If $\theta < t < \eta - \theta$, comparing (15) and (16), A profits are greater than B profits if

$$n_A - n_B < \frac{2\theta(1-\omega)(2\eta - t - \theta)}{\omega(\theta^2 + t^2)} \equiv N_T \tag{17}$$

which could be expressed in terms of λ and N as

$$\lambda < \frac{1}{2} + \frac{1}{2} \frac{N_T}{N} \equiv \lambda_T$$

The threshold λ_T is lower than one as long as $N > N_T$. That is, if the total mass of firms is big enough, this guarantees the existence of a spatial distribution making better off firms in B in the short run. The long-run behaviour of the economy depends as usual on the assumption that capital flows where the interest rate is higher, with the interest rate equal to operating profits. Given the monotonicity of profits in λ under two-way trade, λ_T turns out to be the long-run equilibrium distribution.

IV.D. Full vs. partial agglomeration in the long run

We now characterize in terms of the parameters' values, and in terms of the total mass of the monopolistically competitive sector the emergence of full agglomeration of manufacturing in region A. We give conditions so that, starting from a short-run equilibrium involving a positive share of firms in B, capital eventually becomes employed solely in region A.

Proposition 10 Full agglomeration of the manufacturing sector in region A is the long-run equilibrium of the economy whenever one of the following conditions is met:

i) $t > \eta$ and $N < N_U$;

ii) $\eta - \theta < t < \eta$, and $N < N_O$;

iii) $\theta < t < \eta - \theta$, and $N < N_{AB}$; or $N_{AB} < N < N_T < 2N_{AB}$ (equivalently $N_{AB} < N < 2N_{AB} < N_T$), and $N < N_O$; or $\lambda_T > \nu_{AB}$ if $N_T < N < 2N_{AB}$, and $N < N_O$; iv) $t < \theta$.

Proof. See Appendix C.. ■

Summarizing the results, we can say that when $t > \theta$, that is transport costs are greater than the cost advantage of A, full agglomeration in the long run of the manufacturing sector requires that the total mass of firms in the economy be sufficiently small. If this is not the case then the long-run equilibrium of the economy involves partial agglomeration only. Analytically this requires that N be less than N_U and N_O under autarchy and one-way trade respectively. When transport costs allow the emergence of two-way trade (point *iii*)) the conditions are more elaborated, essentially due to the fact that full agglomeration can be reached either directly $(N < N_{AB})$ or transiting across one-way trade first. In the latter case, the conditions in the proposition guarantee two things: that two-way trade be not the long-run equilibrium of the economy, and that once one-way trade is reached as a result of migration of firms to region A it cannot be a long-run equilibrium either (which is the case if $N < N_O$).

Finally, if the cost advantage of region B is greater than transport costs, full agglomeration of the manufacturing sector will be the long-run equilibrium whatever the total mass of the monopolistically competitive sector is.

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A clear-cut implication this paper shares with other economic geography models is that sufficiently low transport costs foster full agglomeration of the monopolistically competitive sector in the more productive region $(\partial N_O/\partial t < 0 \text{ and } \partial N_{AB}/\partial t < 0)$.

When the long run involves partial agglomeration, in the following proposition we show how λ_O , and λ_T react to changes in transport costs (the value of spatial equilibrium under autarchy λ_U does not depend on transport costs).

Proposition 11 The interior equilibria λ_O and λ_U are decreasing in the level of transport costs t.

Proof. When λ_O exists it will be decreasing in t. This is a consequence of the fact that, when $t > \theta$, $\Pi_A^*(\lambda, t)$ is decreasing in t, $\Pi_B^*(\lambda, t)$ is increasing, and properties P1 and P2 from Appendix B. hold.

As to λ_T , since N_T is decreasing in t, it will be decreasing in t as well. Consequently we get the familiar result that shrinking barriers to trade foster agglomeration of manufacturing in the more productive region.

IV.E. Numerical examples

The following numerical examples represent some structural changes in trade patterns and location equilibria that take place under the dynamic process we have assumed in the paper (capital flows where interest rate is higher). In Example 1 the changes are originated by declining transport costs. In Example 2 a decrease in the cost differential (or, alternatively, an upward shift of the demand function for B products) has an inhibiting effect on trade.

IV.E.i. Example 1: Decreasing trade costs fosters agglomeration in A

Let us set N = 300, $\eta = 100$, $\theta = 6$, $\omega = 0.1$. The initial distribution of firms is $n_A = 200$ and $n_B = 100$, so that $\lambda = 2/3$.

For $t > \eta = 100$, equilibrium trivially involves autarchy in the short run: transport costs are too high for interregional trade to occur. In the long run we do not have full agglomeration, because $N > N_U = 1.15$, and the long-run equilibrium share of firms in Aequals to $\lambda_U = 0.52$.

From Lemma 2 we know that autarchy is the short-run equilibrium also for 0.4 < t < 100 provided $N > N_{BA}$ and $\lambda \leq \nu_{BA}$. This is the case, for instance, when t = 40, because $N > N_{BA} = 64$ and $\lambda < \nu_{BA} = 0.89$. In alternative, when $\lambda > \nu_{BA}$, one-way trade emerges, with A firms exporting to B, and B firms producing for their local market only. If the initial distribution is $\lambda = 2/3$, as soon as t is less than 20.34 firms in A start exporting to region B. What would happen in the long run? At t = 16 one-way trade is sustainable because $N > N_O = 66$, and the long-run equilibrium distribution of the manufacturing sector would be $\lambda_{Q} = 0.52$. This result differs sharply from Behrens (2003), where as soon as one-way trade in manufacturing arises there does not exist any more a spatial long-run equilibrium with partial agglomeration. Conversely, in our framework, even if there is a cost (or taste) advantage for products manufactured in A this is not enough to get complete agglomeration and partial agglomeration can be a stable equilibrium if $N > N_O$, the mass of the monopolistic sector being thick. The strength of agglomeration forces is different in a core-periphery setting and a FC setting⁸. In the core-periphery model of Behrens (2003) migration of workers generates both *demand-linked* circular causality (migration generates expenditure shifting by workers, which generates in turn production shifting, and this determines more migration to fulfill firms' fixed costs requirements) and *cost-linked* circular causality (a higher mass of differentiated products is available where production is concentrated, and workers find more convenient to locate there to save on trade costs), whereas in our FC model, these effects are not present.

If transport costs shrink to t = 7, N_O would rise so that N < 1

⁸See Chapter 3 in Baldwin *et al.* (2003).

 $N_O = 766$, and the long-run outcome of the economy would be full agglomeration.

Let us imagine that transport costs are t = 2 and $\lambda = 2/3$, implying the following relations, $207 = N_{AB} < N < 2N_{AB} = 414$ and $\lambda < \nu_{AB} = 0.69$. This parameters' configuration corresponds to two-way trade. Since $t < \theta$, Π_A^* is greater than Π_B^* so that the share of firms in A increases, until $\lambda = \nu_{AB}$. At this share, the short-run equilibrium involves one-way trade again. Still A firms are doing better than those in B, so that location in A continues until full agglomeration of manufacturing is reached.

IV.E.ii. Example 2: Decreasing the cost (or taste) differential inhibits trade

Let us now consider a case where N = 300, $\eta = 100$, t = 33, $\omega = 0.1$, and an initial distribution of firms $\lambda = 0.65$. Let us assume that the cost (or taste) differential is $\theta = 26$.

For this value of θ , we have one-way short-run equilibrium, as $2N_{AB} < N < N_{BA}$ (see Lemma 3). This share turns to be also the long-run equilibrium (a long-run one-way trade equilibrium exists because $N > N_O$).

We now make the following thought experiment. Region B managed in some way to narrow the gap in terms of costs with A products, or it has improved consumers' perception for its varieties, as the distance between the intercepts of the two demand functions has become smaller. Assume that now $\theta = 6$. The decrease in the differential between varieties produced in the two regions causes a structural change in the trade pattern. Now $N > N_{BA}$, and λ gets close to ν_{BA} , since firms in B are still doing better than firms in A. As more firms move to region B (and λ shrinks), it becomes tougher for A firms to export in B as competition there increases. Finally, when λ is equal to ν_{BA} one-way trade is inhibited, and the economy gets back to the no-trade equilibrium, with a long-run distribution equal to $\lambda_U = 0.52$.

This example shows that some short-run asymmetric trade pat-

terns are not sustainable in the long-run. As in the example above, one-way trade is not conducive of more agglomeration in the exporting region A. The vanishing of one-way trade in the long run originates in this example by a decrease in the differential θ between firms located in the two regions. The result is no trade and a more homogeneous spatial distribution (from $\lambda_O = 0.65$ to $\lambda_U = 0.52$).

V. Concluding remarks

This work focused on cost and competitive asymmetries among regions shaping different trade patterns at various levels of transport costs. We derived analytical conditions ensuring full agglomeration in the long run of the monopolistically competitive sector in the location where production is cheaper or, alternatively, where more intensely demanded varieties are manufactured. Our main finding was to show that if competitive pressure is not strong, a single region may host the whole sector, because firms take advantage of the better production conditions while not suffering excessively from the presence of the other competitors. When competitive pressure is strong (i.e. the monopolistic sector is thick), both regions host a positive share of the industry.

Our numerical examples focused on the way trade patterns and long-run location equilibria are affected by a change in barriers to trade and a change in the cost (or demand) differential θ . While a decrease in transport costs fosters agglomeration in the better equipped region (though the intensity of agglomeration forces is less intense than in the usual core-periphery model), a decrease in the asymmetry of the two regions could make the short-run one-way trade equilibrium not sustainable in the long run: as firms' distribution becomes more homogenous due to the decrease in the asymmetry, the economy gets back to autarchy. This neatly shows that a cost (or demand) advantage may not guarantee the persistence of trade relationships in the long run (also in a unilateral fashion), since the establishment of firms in the less productive region makes invariantly exports tougher.

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Appendix

A. Equivalency of quantity and price setting under the pricing rule in Behrens (2004)

In Appendix A in Beherens (2004) it is shown that in order to achieve the equivalency between the *perceived* demand function and the *realized* demand function in the linear demand model it is sufficient to assume a particular pricing rule for varieties not traded in equilibrium.

The *perceived* demand function is the solution to maximization of utility function (1) under the budget constraint, that we call maximization problem \mathcal{P}_Q :

$$(\mathcal{P}_Q) \begin{cases} \max & U(q_0, x(j)) \\ \text{s.t.} & \int_0^N p(j) x(j) dj + q_0 = \phi_0 \end{cases}$$

Substituting directly the equality constraint and computing the first order conditions yield the following system of equations for the differentiated varieties:

$$\xi - (1 - \omega)x(i) - \omega \int_0^N x(j)dj - p(i) = 0, \quad i \in [0, N].$$
(18)

The system is solved giving

$$x^{*}(i) \equiv \frac{\xi}{1+\omega(N-1)} - \frac{1}{1-\omega}p(i) + \frac{\omega}{(1-\omega)[1+(N-1)\omega]} \int_{0}^{N} p(j)dj$$
(19)

It is apparent that if p(i) is too high, the term $x^*(i)$ could be negative. Since, the quantity demanded of a variety is non-negative by definition, we get that the *perceived* demand function of variety $i, \tilde{x}^*(i)$, is $\tilde{x}^*(i) = \max\{0, x^*(i)\}$. Demand is indeed zero when $x^*(i) \leq 0$, that is

$$p(i) > \bar{p}(i) \equiv \frac{\xi(1-\omega)}{1+(N-1)\omega} + \frac{\omega}{1+(N-1)\omega} \int_0^N p(j)dj$$

and $\tilde{x}^*(i) = x^*(i) = 0$ for $p(i) = \bar{p}(i)$. Hence $\bar{p}(i)$ is the smallest price making the *perceived* demand equal to zero.

It can be shown that the solution to \mathcal{P}_Q is in general different from the solution to the following maximization problem, taking into account explicitly non-negativity constraints on the quantity consumed of each variety:

$$(\mathcal{P}_{QE}) \begin{cases} \max & U(q_0, x(j)) \\ \text{s.t.} & \int_0^N p(j) x(j) dj + q_0 = \phi_0, \quad x(i) \ge 0, \quad i \in [0, N] \end{cases}$$

Following Behrens (2004), the lagrangian associated to this optimization problem gives the Karush-Kuhn-Tucker conditions

$$\xi - (1 - \omega)x(i) - \omega \int_0^N x(i)di - p(i) + \mu(i) = 0, \quad i \in [0, N] \quad (20)$$

$$\mu(i) \ge 0, \quad x(i) \ge 0, \quad i \in [0, N] \quad \text{and} \quad \int_0^N \mu(i)x(i)di = 0 \quad (21)$$

and *realized* demand functions are then

$$x^{*}(i) = \frac{\xi}{1 + \omega(N-1)} - \frac{1}{1 - \omega} [p(i) - \mu(i)] + \frac{\omega}{(1 - \omega)[1 + (N-1)\omega]} \int_{0}^{N} [p(j) - \mu(j)] dj$$

where $\mu(i)$ are the multipliers.

Behrens demonstrates that if firms set $\bar{p}(i)$ abroad whenever they do not export, perceived and realized demands coincide. Our purpose it to show that this equivalency could be achieved directly making the behavioural assumption that firms set quantities. Actually in this case the dependent variable in the demand functions has to be p(i). The *perceived* demand is

$$p(i) = \xi - (1 - \omega)x(i) - \omega \int_0^N x(i)di$$

while the *realized* demand is

$$p(i) = \xi - (1 - \omega)x(i) - \omega \int_0^N x(i)di + \mu(i)$$

It is straightforward to see that the two demand functions always coincide as long as $\mu \equiv 0$, which is a necessary condition to get equivalency between the two optimization problems (if $\mu(i) \neq 0$ for some *i* the equivalency never holds). Moreover when $\mu \equiv 0$ conditions (20) and (21) reduces to (18).

B. Proof of Proposition 8

Step 1 (Non-monotonicity of $\Pi_A^*(\lambda)$). We substitute in (13) and (14) the expressions $n_A \equiv \lambda N$, and $n_B \equiv (1 - \lambda)N$. First of all we determine whether, under one-way trade and $t > \theta$, $\Pi_A^*(\lambda)$ and $\Pi_B^*(\lambda)$ are strictly monotonic in λ . It is easy to see that Π_B^* is increasing in λ .

The function $\Pi_A^*(\lambda)$ is non-monotone. First of all notice that $\partial \Pi_A^h/\partial \lambda < 0$. Then we have that $\partial \Pi_A^f/\partial \lambda \geq 0$ (both quantity x_{BA}^* and price p_{BA}^* are non-decreasing in λ). Moreover $\partial \Pi_A^f(\lambda, t)/\partial \lambda = 0$ when $t = \{\theta, t_{\sup}\}$, where t_{\sup} is the maximum value of transport costs compatible with one-way trade for a given λ (derived making explicit in (8) transport costs t). The function $\partial \Pi_A^f(\lambda, t)/\partial \lambda$ has a unique maximum in t, computed equalizing to zero its derivative, let it be t_{\max} . Then if

$$\frac{\partial \Pi_A^h(\lambda)}{\partial \lambda} + \left. \frac{\partial \Pi_A^f(\lambda, t)}{\partial \lambda} \right|_{t=t_{\max}} < 0, \tag{22}$$

 $\partial \Pi_A^*(\lambda, t) / \partial \lambda < 0$ for every admissible t. Actually it turns out that (22) is less than zero if and only if the following condition is verified:

$$\frac{\eta^2}{[2(1-\omega)+\omega\lambda N]^3} > \frac{(\eta-\theta)^2}{4(2-2\omega+\omega N)^2[2-2\omega+\omega(1-\lambda)N]}$$
(23)

Consequently (23) does not hold when λ is sufficiently close to one and θ is small. In such a case profits of firms located in region A increase as the share of firms in A increases, because the rise in profits coming from the foreign region more than offset the fall in the home component. The function Π_A^* can be first decreasing and then increasing in λ as it tends to 1, provided that t is in a neighborhood of t_{max} .

Step 2 (Uniqueness of λ_O). We demonstrate the following two properties. They turn to be useful when dealing with existence and uniqueness of λ_O .

Property P1. The first is that

$$\frac{\partial \Pi_B^*(\lambda, t)}{\partial \lambda} \bigg|_{\lambda=1} - \frac{\partial \Pi_A^*(\lambda, t)}{\partial \lambda} \bigg|_{\lambda=1} > 0.$$
 (24)

Computing (24), we get the following condition:

$$\frac{N\omega\phi(\omega)}{2(1-\omega)(2-2\omega+\omega N)^3} > 0,$$

where $\phi(\omega)$ is a parabola with upward concavity and imaginary roots, so that it is always positive.

Property P2. The second property we are interested in is that $\partial^2 \Pi_A^* / \partial \lambda \partial \lambda > 0$, meaning that Π_A^* is a convex function.

Taken together, these two properties ensure that whenever $\Pi_B^*(\lambda)$ crosses $\Pi_A^*(\lambda)$ it will do it only once: provided λ_O exists in an admissible range of λ , it will be unique.

Step 3 (Cases of non-existence of λ_O). $\Pi_A^*(\lambda)$ and $\Pi_B^*(\lambda)$ are continuous functions on $\lambda \in (1/2, 1]$, but existence of λ_O is not always guaranteed. The first case of non-existence is when $\Pi_A^*(1) > \Pi_B^*(1)$. Given properties in Step 2, this is also a necessary and sufficient condition for $\Pi_A^*(\lambda)$ to be greater than $\Pi_B^*(\lambda)$ for every admissible λ . Solving the inequality, $\Pi_A^*(1) > \Pi_B^*(1)$ if $N < N_O$, where

$$N_O \equiv \frac{2(1-\omega)}{\omega(t-\theta)} \left(\sqrt{\eta^2 + (\eta-t)^2} - \eta + \theta \right).$$

Other cases of non-existence are when $\Pi_B^*(\lambda)$ lies above $\Pi_A^*(\lambda)$ for every admissible λ . In particular, it could be the case that, even though $\Pi_A^*(\lambda)$ and $\Pi_B^*(\lambda)$ intersect at some $\lambda \in (1/2, 1]$, this point does not satisfy constraints ν_{AB} , or ν_{BA} under points *i*) and *ii*) in Lemma 3. When this is the case, in the long run we have transition from one-way trade to two-way trade ($\lambda < \nu_{AB}$) or autarchy ($\lambda \leq \nu_{BA}$).

C. Proof of Proposition 10

I prove separately each point in the statement of the proposition.

Point i). The proof descends from Lemma 2 and Proposition 7 and corresponds to full agglomeration with non-tradeable varieties.

Point ii). If $\theta < t < \eta$ and we are in the short-run autarchic equilibrium (point *ii*) in Lemma 2), full agglomeration is not possible because no-trade requires that $\lambda \leq \nu_{BA} < 1$, while full agglomeration obviously entails $\lambda = 1$. Full agglomeration cannot be reached unless transiting across the one-way trade short-run equilibrium.

With one-way trade and $\eta - \theta < t < \eta$ (point *i*) of Lemma 3), Proposition 8 requires that $N < N_O$. Notice that $N_O < N_{BA}$. This can be checked solving the corresponding inequality, and arriving at a point where it is straightforward to see that

$$\sqrt{\eta^2 + (\eta - t)^2} < 2\eta - t < 3\eta - t - \theta$$

When the total mass of firms is less than N_O then full agglomeration takes place.

Point iii). When $\theta < t < \eta - \theta$, we could be either in a one-way or a two-way short-run equilibrium. Two-way short-run equilibrium occurs under conditions in Lemma 4. By Proposition 9, if $N < N_T$ then $\Pi_A^* > \Pi_B^*$ for every λ . If $N < N_{AB}$, two-way trade is the short-run equilibrium for every share λ . We prove that whenever $N < N_{AB}$ we get full agglomeration, since $N_T - N_{AB} > 0$. Solving this inequality is equivalent to solve $f(\theta) > 0$, where $f(\theta)$ is equal to

$$f(\theta) = (\eta - t)\theta^2 + 2t\eta\theta - t^2(\eta - t)$$
(25)

The function $f(\theta)$ is a parabola in θ with upward concavity, with two negative roots. Since all admissible values of θ are greater than zero, $f(\theta)$ will be positive in this range, implying that $N_T - N_{AB} > 0$.

For $N_{AB} < N < 2N_{AB}$, two-way trade arises only if $\lambda < \nu_{AB}$, and full agglomeration in the long run can be reached only through transition to short-run one-way trade equilibrium ($\lambda \ge \nu_{AB}$, see point ii) in Lemma 3). With one-way trade, we recall that a necessary condition for complete agglomeration is $N < N_O$.

Transition to one-way trade happens if two-way trade is not a long-run equilibrium, which turns to be true in the following cases. The first case is when $N_{AB} < N < N_T < 2N_{AB}$ (equivalently $N_{AB} < N < 2N_{AB} < N_T$), this making profits in A greater than in B for every λ under two-way trade. Consequently, λ rises until the economy experiences one-way trade. The second case is when $N_{AB} < N_T < N < 2N_{AB}$, so that an equilibrium distribution λ_T exists. In such a case two-way trade equilibrium is impossible only if $\lambda_T \geq \nu_{AB}$. Again, there will be a switching to one-way trade before the equilibrium share λ_T could be reached.

Point iv). When $t < \theta$, the short-run equilibrium depends on the total mass of firms N. Let us first consider short-run one-way trade (*iii*) in Lemma 3). Profits in A are higher than in B by Proposition 8. As capital moves to region A (λ rises) we have deindustrialization of B provided $N > N_{BB}$. Actually when λ becomes greater or equal to ν_{BB} profits of firms in B are non-positive. Consequently all the residual capital in B is suddenly diverted towards region A, and this ensures complete agglomeration in A. If $N < N_{BB}$, firms in B make positive profits for every λ but profits made in A are higher and full agglomeration is attained again.

Consider short-run two-way trade of Lemma 4. If $N_{AB} < N < 2N_{AB}$, as the fraction of firms in A rises, we go back to one-way trade ($\nu_{AB} \leq \lambda < \nu_{BB}$), and so it applies what we said earlier, that is full agglomeration is always the long-run outcome. If $N < N_{AB}$, being in A is always more profitable, and in the long-run the whole manufacturing sector will be concentrated in this region.

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