Financial Constraints and Firm Dynamics

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Abstract

The short run effects of financial constraints (FCs) on the expected growth rate of firms and their long-term implications on the evolution of the firm size distribution have been recently investigated by several scholars. In this paper we extend the analysis to a wider and largely unexplored range of possible FCs effects, including the autoregressive and heteroskedastic structure of the firm growth process and the degree of asymmetry in the distribution of growth shocks. We measure FCs with an official credit rating index which directly captures the borrowers' opinion on a firm's financial soundness and, consequently, the availability and cost of external resources. Our investigations reveal that FCs operate through several channels. In the short term, FCs reduce the average firm growth rate, induce anti-correlation in growth shocks and reduce the dependence of growth rates volatility on size. Financing constraints also operate through asymmetric threshold effects, both preventing potentially fast growing firms from enjoying attractive growth opportunities, and further deteriorating the growth prospects of already slow growing firms. The sub-diffusive nature of the growth process of constrained firms is compatible with the distinctive properties of their size distribution.

JEL codes: C14, D20, G30, L11

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1 Introduction

Firms' ability to access external financial resources represents a factor influencing several dimensions of firm dynamics, as the links between financial and operational activities of firms involve many types of decisions, pertaining, for instance, investment strategies, the ability to enter or survive in a market, job creation and destruction, innovative activity, and internationalization patterns.

Within the vast body of literature focusing on the relationships between finance and firms' dynamics, a well developed tradition of empirical studies has sought to identify the effect of financing problems on the size-growth trajectories of firms (for reviews, see Whited, 2006; Fagiolo and Luzzi, 2006; Oliveira and Fortunato, 2006). A first major problem in this identification rests in the intrinsically difficult task of measuring financial constraints (FCs). In fact, FCs are not directly observable, as it is not possible to know whether banks or other financial institutions refuse a loan or if particularly high interest rates are charged to a given firm. To overcome this difficulty, researchers have proposed different approaches seeking to classify firms into financially constrained and unconstrained categories. The debate about which particular measure to use, originating in the literature on financing constraints to firms' investment (Fazzari et al., 1988; Kaplan and Zingales, 2000), is still open. A first strategy is to sort firms into constrained or unconstrained groups according to their relative ranking in the distribution of some variable which is supposed to be related with the need, availability and cost of external finance. A few examples include age, size, cash flow, leverage, availability of collateral, interest coverage, payout ratios, cash flow sensitivity of cash. Alternatively, a multivariate approach can be followed, through the construction of index measures of FCs which summarize several aspects of firm financial structure into a single indicator. This approach allows to capture different degrees of FCs, avoiding a simple binary categorization(Kaplan and Zingales, 1997; Whited and Wu, 2006; Musso and Schiavo, 2008). In the same spirit, several classifications have been proposed which rely on some kind of credit rating measure (of specific bonds or commercial papers, or of the overall debt position of a firm). These measures have the advantage to judge access to credit on the basis of financial markets' evaluation of the credit quality of a firm (Whited, 1992; Almeida et al., 2004). All of these approaches measure FCs by resorting to "hard" data, i.e. exploiting information available through business registers. The most common alternative consists of classifications based on survey data. Surveys typically involve managers or entrepreneurs, who are asked to make a self-assessment of whether firms have been rationed or not, whether the cost and the amount of granted loans were in line with their expectations and needs, and, more generally, about the difficulties they have faced in accessing financing from banks or other institutions (Winker, 1999; Angelini and Generale, 2008; Campello et al., 2009).

None of the proposed approaches are without their pitfalls, and there is no clear consensus on how different ways of measuring FCs can impact the obtained results. On the one hand, both univariate or multivariate proxies derived from business registers inevitably give an indirect measure of FCs, as they implicitly assume that the poor records of firms with respect to the chosen variables get translated into a bank's unwillingness to grant credit. This assumption appears particularly problematic when the analysis is exclusively based on totally exogenous variables, like age, or structural and extremely persistent variables, like availability of collateral. On the other hand survey based measures, which are seemingly closer to answering the question as to whether a firm has actually been constrained or not, are well known to suffer from misreporting and sample selection bias, whose effect is difficult to quantify. Moreover, by collecting the opinion of the credit seeker about their own financing conditions, survey data look at the demand side of credit relations. Rather, given the strong informational asymmetries characterizing capital markets, it is the opinion of the credit supplier on the credit seeker that plays the crucial role in determining credit conditions.

Once a measure of FCs has been selected for the analysis, the standard approach in the literature has been to check the significance of this measure in a standard firm growth regression, either by directly including the chosen FCs proxy among the regressors, or by modeling FCs as dummy variables indicating that a firm belongs to some specific FCs category. The generally accepted finding is that FCs negatively affect firm growth, and that this effect is stronger for younger and smaller firms (see Angelini and Generale, 2008). These findings are in line with the recent theoretical literature on financing and growth models Cooley and Quadrini (2001); Clementi and Hopenhayn (2006), largely based on the models of industrial dynamics in Jovanovic (1982) and Hopenhayn (1992).

A major limitation of these studies is however that the kind of specifications employed can only identify location-shift effects in the conditional distribution of growth rates. The shift is accounted for by a statistically significant correlation of average growth rates with the FC proxy, or by observed deviations in expected growth rates between the classes of constrained and unconstrained firms. Although there is a general agreement that FCs downplay growth prospects of firms, there is no clear reason why this reduction should exclusively translate into a negative shift of the average growth rate. In fact, there are various pieces of evidence that make the shift hypothesis rather simplistic. Firstly, the evidence that FCs problems affect several dimensions of firms' behavior and strategies, such as investment/divestment in fixed capital (Fazzari et al., 1988; Devereux and Schiantarelli, 1990; Bond et al., 2003) and in working capital (Fazzari and Petersen, 1993), cash management policies (Campello et al., 2009), inventory demand (Kashyap et al., 1994), or R&D and innovation strategies (Hall, 2002; Brown et al., 2009), clearly suggests that the role played by FCs is likely to be complex and structured. Secondly, recent qualitative evidence on firms' reactions to the current financial crisis (see Campello et al., 2009) suggests that firms undertake heterogeneous responses to FC problems: there are firms that tend to abandon some investment projects, despite their potential, while other firms, especially those which are already experiencing poor growth dynamics, tend to display a much higher propensity to sell off productive assets as a way to generate funds. Heterogeneous responses can induce different effects in different quantiles of the (conditional) growth rates distribution.

To account for the many possible channels through which FCs can affect firm growth, in this paper we extend the usual autoregressive growth model. We introduce a parametric specification of the heteroskedasticity of growth rates and we allow for asymmetries in growth shocks across firms subject to different strength of FCs. The first extension is motivated by the robust empirical observation that smaller firms experience more volatile growth patterns (among others, see Hymer and Pashigian, 1962; Amaral et al., 1997; Bottazzi and Secchi, 2005). Such heteroskedasticity is typically viewed as a factor to wash away in obtaining consistent estimates (Hall, 1987; Evans, 1987; Dunne et al., 1988). Conversely, we consider it as part of the phenomenon under study, and we want to understand if FCs have a role in explaining the relationship between volatility of growth and size. Our second extension, that is the assessment of possible asymmetries, is pursued by investigating the extent to which FCs affect the overall shape of growth rates distribution, a topic so far largely neglected (see Fagiolo and Luzzi, 2006, for the only exception we are aware of). Our specification enables us to reconcile the effects of FCs on firm growth dynamics with the observed differences in the firm size distribution (FSD) of constrained and non-constrained firms. Exploring such differences is of recent interest and the evidence is both scant and controversial. Cabral and Mata (2003) found that the evolution of the FSD is determined by firms ceasing to be financially constrained, while Fagiolo and Luzzi (2006) and Angelini and Generale (2008) concluded that FCs are not the main determinant of FSD evolution. At least part of the explanation for such seemingly contrasting evidence may come from the different proxies of FCs employed in these studies. Indeed, Cabral and Mata (2003) measure FCs with age, assuming that younger firms are more constrained, while Fagiolo and Luzzi (2006) and Angelini and Generale (2008) adopt reported cash flow and survey-based measure of FCs, respectively.

We perform our analysis using a measure of FCs based on an official credit rating. Credit ratings, by their very definition, are similar to the multivariate indicators of FCs derived from hard data. Similarly to those measures, they do not suffer from the biases inherently affecting survey measures and offer the opportunity to account for different degrees of exposure to FCs. The official source, the high reliability and the widespread use of the specific rating adopted in our study strongly suggest that it is used as an actual benchmark for the lending decisions of banks and financial institutions. In this respect, our rating does not only summarize a wide range of potential sources of financial problems. It also captures the actual expectations of credit suppliers on the ability of firms to meet obligations, thus getting closer to measuring whether or not credit is granted to a particular firm.

Using a large panel of Italian manufacturing firms, our analysis reveals that FCs do affect the process of firm growth through multiple channels. In the short term, FCs reduce the average firm growth rate, induce anti-correlation in growth shocks and reduce the strength of the dependence of volatility of growth rates on size. In addition, we also find asymmetric effect on growth rates distribution. On the one hand, FCs prevent attractive growth opportunities from being seized by constrained but yet potentially fast growing firms. This effect is particularly strong for younger firms. At the same time, and especially among older firms, FCs tend to be associated with a further depression in the growth prospects of already slow-growing firms. These effects are consistent with the distinctive features of the size distribution of more severely constrained firms obtained through a snapshot analysis on cross-sectional data.

The paper is organized as follows. Section 2 describes the data, introduces our FC measure and provides a first descriptive account of the relevance of the FC phenomenon. Section 3 analyzes the role of FCs in affecting the age profile of the FSD. In Section 4 we develop our baseline framework and derive the hypotheses guiding our empirical investigations and the interpretation of results. Section 5 presents the main results of our analysis of FC effects on the patterns of firm growth, also investigating the effects of FCs on the firm growth rates distribution. Section 6 tests the robustness of the findings with respect to a set of potentially relevant determinants of size-growth dynamics and firms' financing decisions. In Section 7 we summarize our findings and conclude.

2 Financing constraints: definition and basic facts

We employ a large database of Italian firms maintained by the Italian Company Account Data Service (Centrale dei Bilanci, CeBi). CeBi was founded as a joint agency of the Bank of Italy and the Italian Banking Association in the early 1980s to assist in supervising risk exposure of the Italian banking system. Today CeBi is a private company owned by major Italian banks, which continue to exploit its services in gathering and sharing information about firms. The long term institutional role of CeBi ensures high levels of data reliability, substantially limiting measurement errors. The dataset is of a business register type, collecting annual reports for virtually all *limited liability* firms. The data available for the present study follow approximately 200,000 firms active in manufacturing over the period 1999-2003. Considering this sector, our data account for about 45% of total employment and about 65% of aggregate value added over the years of observation.[1] Moreover, the data replicate pretty well the distributional profile of firm size in the overall population of Italian manufacturing.[2] For each firm, we were able to access a subset of the original list of variables included in the annual reports. We derive the Age of the firm from its foundation year, and we proxy firm size through real Total Sales. The decision to prefer Total Sales over Number of Employees as a measure of size is because in our data, consistently with the Italian accounting system, employment figures are reported

[1] These shares are computed with respect to National Accounts data by sector of activity, as reported by Eurostat. Pistaferri et al. (2010) report similar figures. They also report that the CeBi database contains approximately the 7% of all Italian manufacturing firms.

[2] For 2003, the annual report of the Italian Statistical Office (ISTAT, 2005) provides the following distribution: 82% of firms has less then 10 employees; 15% has 10-to-49 employees; 2% has 50-to-249 employees, and 1% has more than 250 employees. In our data there is a very mild overrepresentation of medium-larger firms: 78% of firms has less then 10 employees; 13% is in the 10-249 size class; 8% has 50-to-249 employees and 1% has more than 250 employees.

in the notes accompanying financial statements, and are therefore likely to be affected by less reliable updates. For small firms especially, a mistake of even few units of personnel in employment reports may produce a huge error in the measurement of employment growth rates.[3]

As a measure of FCs we adopt the credit rating index that CeBi produces for all the firms included in the dataset. In fact, credit ratings account for the "opinion [of credit suppliers] on the future obligor's capacity to meet its financial obligations"(Crouhy et al., 2001). CeBi ratings enjoy several qualities identified as desirable for a measure of financial constraints (Cleary, 1999; Lamont et al., 2001). First, they result from a multivariate score, thus summarizing a wide range of dimensions of firm performance. Second, they are updated in every year, thus allowing for the identification of time effects. Third, they do not force the researcher to work with a binary categorization of constrained versus non-constrained firms. Indeed, the graduation of scores attributed by credit ratings to the different firms allow to distinguish among different degrees of difficulty in accessing external funds. These features are common to CeBi ratings and ratings issued by international agencies like Moody's or Standard & Poor's. However, CeBi ratings enjoy three specific advantages. First, they give an assessment of the overall quality of a firm, rather than imply a judgment about the quality of a single liability of a company. Second, they are available for all the firms included in the dataset, while credit files from international rating institutions bias the scope of analysis towards a much less representative sub-sample of firms. Third, the CeBi index is perceived as an official rating, due to the tight link established between CeBi and major Italian banks. This justifies the heavy reliance of banks on CeBi ratings: it is generally true that a firm with very poor rating is not likely to receive credit.[4]

The CeBi index is a score ranking firms in 9 categories of creditworthiness: 1-high reliability, 2-reliability, 3-ample solvency, 4-solvency, 5-vulnerability, 6-high vulnerability, 7-risk, 8-high risk, and 9-extremely high risk. The ranking is purely ordinal. We define three classes of firms subject to different degrees of financial constraints: Non Financially Constrained (NFC), Mildly Financially Constrained (MFC) and Highly Financially Constrained (HFC), corresponding respectively to firms rated from 1 to 4, 5 to 7, and 8 to 9. Since the CeBi index is updated at the end of each year, it is the

[3]Nominal sales are deflated via 3-digit sectoral production price indexes made available by the Italian Statistical office, base year 2000. A basic cleaning procedure to remove a few outlying observations is applied (see the appendix for details). Reported results refer to 2000-2003 as one year is obviously lost in the computation of growth rates.

[4]See also Pistaferri et al. (2010) for a similar use of CeBi ratings as a proxy of firms' access to credit market.

Table 1: FINANCIAL CONSTRAINTS BY AGE CLASSES

	Whole Sample		Non Financially Constrained		Mildly Financially Constrained		Highly Financially constrained	
Firm's age (years)	Number of firms	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)	Number of firms (percentage of age class)	Size: mean (median)
0-4	38,020	1.795	10,356	1.804	20,408	1.970	7,256	1.293
		(0.606)	(27.2)	(0.525)	(53.7)	(0.719)	(19.1)	(0.449)
5-10	52,150	3.369	18,269	4.115	27,862	3.248	6,019	1.666
		(0.860)	(35.0)	(0.844)	(53.4)	(0.995)	(11.5)	(0.439)
11-20	62,977	7.093	29,130	8.210	29,408	6.400	4,439	4.354
		(1.522)	(55.9)	(1.606)	(46.7)	(1.663)	(7.0)	(0.525)
21-30	35,579	10.139	18,966	11.147	15,080	9.544	1,533	3.520
		(2.674)	(53.3)	(2.719)	(42.4)	(2.921)	(4.3)	(0.696)
31-∞	20,645	25.917	11,374	26.600	8,213	22.157	1,058	47.760
		(4.516)	(55.1)	(4.919)	(39.8)	(4.764)	(5.1)	(1.345)
Total	209,371	7.577	88,095	9.614	100,971	6.386	20,305	4.662
		(1.301)	(42.1)	(1.548)	(48.2)	(1.371)	(9.7)	(0.494)

Size as real sales, millions of euro.

rating in $t-1$ that is relevant for credit suppliers when they have to decide whether to provide credit in year t. Therefore, the assignment to the three classes is based on one-period lagged values of the ratings. Together, this choice also reduces the simultaneity issue potentially arising in regression analysis.[5]

Table 1 shows descriptive statistics. According to our definition financing problems appear to represent a significant phenomenon: about 10% of the whole sample is affected by severe difficulties in raising external resources (i.e. are HFC firms), while almost half of our sample (48%, cfr. the MFC class) faces less severe, but still significant problems. This is in partial contrast with a result reported in Angelini and Generale (2008) on a smaller Italian dataset. Secondly, FCs are a pervasive phenomenon, affecting firms of different sizes and ages: more than 5% of old firms are in the HFC class, and the mean size of HFC firms is comparable with the mean size of the other two classes.[6] However, confirming a robust finding in the literature, FCs seem more relevant among young and small firms: 20% of young firms are HFC, against a 5% of HFC firms found in the group of older firms and, moreover, the median size of HFC firms is, in all age classes, almost one third smaller as compared to the other FC classes.

3 Financing constraints and age profile of the FSD

Figure 1 reports kernel estimates of the empirical density of real sales by age.[7] Results broadly confirm the basic stylized facts observed in previous studies, where size is proxied with employment: the FSD is right-skewed and both the mode and the width of the distribution increase with age. This visual impression is confirmed by a Fligner-Policello test for stochastic dominance. The FSD of older firms dominates those of younger firms, meaning that a firm randomly drawn from the group

[5] In order to check the sensitivity of our results to the adopted classification, we also considered two alternative assignment procedures. In the first procedure, firms were assigned to FC classes on the basis of the worst rating displayed over the sample period. In the second procedure, we restricted the analysis to firms that did never change their financial status over the whole time window (i.e., based on their ratings in the different years, they always fell in the same FC class). Our main conclusions were not affected by the choice of the assignment procedure, though. All the results are available upon request.

[6] The very high mean found within HFC old firms, 47.760, is explained by the presence of quite large firm (actually the largest in the dataset) which is old and HFC over the sample period. The mean size falls to 18,415 if we exclude this firm from the sample.

[7] Here as well as throughout the work, estimates of densities are obtained using the Epanenchnikov kernel with the bandwidth set using the simple heuristic described in Silverman (1986).

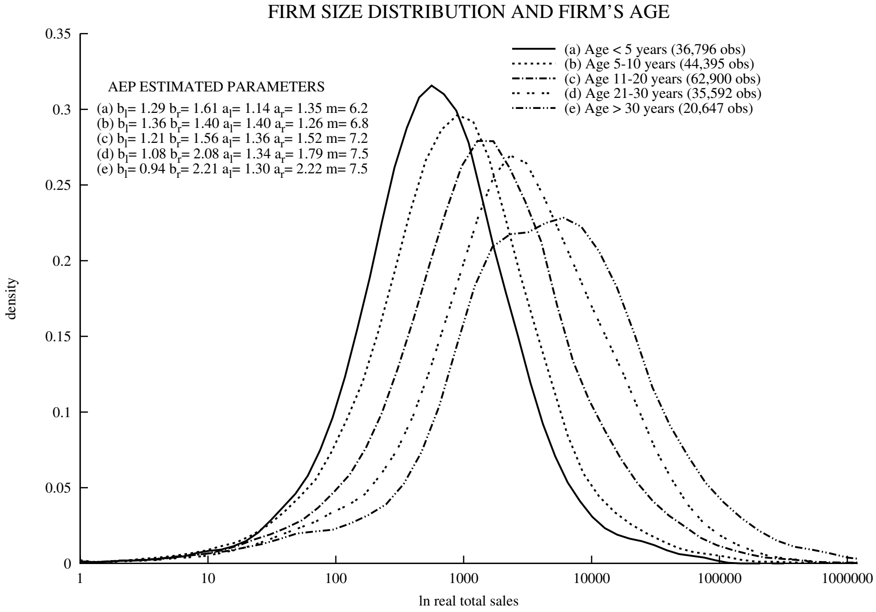

Figure 1: FSD and Age. Pooled data over 2000-2003.

of older firms is, with a probability significantly higher than 50%, bigger than a firm randomly extracted from the group of younger firms.[8]

However, from the graphical analysis alone it is difficult to provide a precise statement on the validity of a second common piece of evidence reported in the literature, i.e. that the degree of FSD skewness diminishes with age. Available studies tend to agree on this point, although Angelini and Generale (2008) report that the FSD appears to be more symmetric when using sales, instead of number of employees. To provide a quantitative assessment of this issue, we consider the Asymmetric Exponential Power (AEP) distribution. This family copes with asymmetries and leptokurtosis, at the same time allowing for a continuous variation from non-normality to normality. The AEP density

$$f_{\mathrm{AEP}}(x;\mathbf{p}) = \frac{1}{C}\; e^{-\left(\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}\theta(m-x)+\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}\theta(x-m)\right)}\quad, \tag{1}$$

where $\mathbf{p} = (b_l, b_r, a_l, a_r, m)$, $\theta(x)$ is the Heaviside theta function and $C = a_l A_0(b_l) + a_r A_0(b_r)$ with $A_k(x) = x^{\frac{k+1}{x}-1}\;\Gamma\left(\frac{k+1}{x}\right)$, is characterized by 5 parameters. Two positive shape parameters, b_r and b_l, describe the tail behavior in the upper and lower tail, respectively. Two positive scale parameters,

[8]This test is presented in Fligner and Policello (1981) and can be interpreted as a test of stochastic dominance in the case of asymmetric samples. A pair-wise comparison of the distribution in Figure 1 confirms significant differences, with negligible p-scores (less than 10^{-6} in all cases).

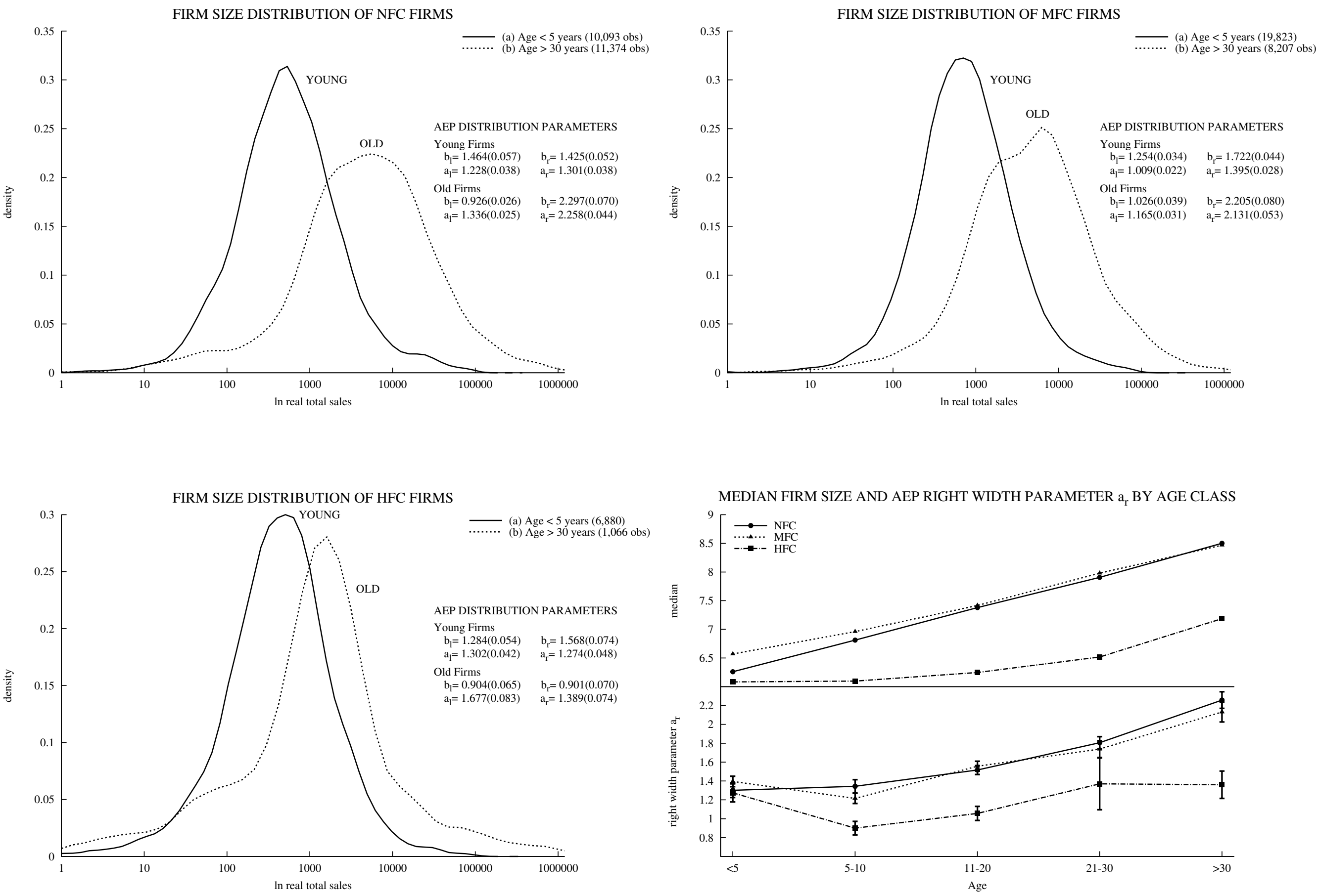

Figure 2: FSD, Age and Financial Constraints. Pooled data over 2000-2003.

a_r and a_l, are associated with the width of the distribution above and below the modal value, which is captured through the location parameter m.

Maximum Likelihood estimates of the AEP parameters are reported in Figure 1 (corresponding standard errors are always smaller than 0.05). They reveal two different patterns in the degree of FSD skewness, arising respectively in the right- and left-hand side of the distribution. The left tail becomes fatter as age increases (b_l decreases while a_l is approximately stable) so that among relatively smaller firms size differences are bigger among older firms. In the right-hand side of the distribution, as we move from younger to older firms, there is a shift in probability mass from the tail to the central part of the distribution (b_r increases with age) together with an overall increase in the width of support (a_r increasing with age).[9]

We then ask whether disaggregation into FC classes can help explaining the asymmetric effect

[9]Notice that the Extended Generalized Gamma distribution applied in Cabral and Mata (2003), which possesses only one shape parameter, would not have allowed to independently account for the different behaviors observed in the two tails.

that age seems to exert on the properties of the FSD. Figure 2 reports kernel estimates of the FSD for firms in the different FC classes, directly comparing young (less than 5 years) and old (more than 30 years) firms in each class.[10]. Since we cannot follow cohorts of firms in our data, a comparison across firms of different age is the only way to have a clue on the relationship between size, age and financial constraints. The results (top left and right, and bottom left panels) suggest that the size-age profile of NFC and MFC firms share similar distributional properties, while the FSD of HFC firms display distinctive features. The difference essentially concerns the intensity of the effect that age exerts on the location and variance of the size distribution. When comparing young and old firms within each class, we observe that the increase in both location and variance induced by firm aging is much milder among HFC firms than in the other two classes. This is confirmed by the results in the bottom-right plot. Here we proxy location and width of the FSD, respectively, with the median size and the estimates of the right width AEP parameter, a_r, and then report how these two indicators vary by age and FC class. Both measures are very similar across all the FC classes when we consider young firms. Then, as age increases, it is possible to identify two diverging trends, one common to NFC and MFC firms, and a second specific to the group of HFC firms. The median size of NFC and MFC firms increases more than tenfold from young to old firms, while the median size of HFC firms increases only by a factor of 5. Similarly, the estimates of a_r reveal that FSD dispersion increases significantly with age for NFC and MFC firms, while the increase is much more modest for HFC firms. The existence of such diverging patterns is also supported by the estimates of b_r, the parameter describing the right tail behavior. Indeed from very similar values for young firms ($\sim$1.4, $\sim$1.7 and $\sim$1.6 for the NFC, MFC and HFC classes respectively), the estimated coefficients diverge when old firms are considered: old NFC and old MFC firms display values of b_r close to 2, and hence approximately consistent with a Gaussian distribution, while for old HFC firms the estimated b_r drops from 1.6 to 0.9.

In summary, the dependence of the aggregate FSD on firms' age found in Figure 1 results from a mixture of the FSD of financially constrained firms, which are responsible for the fat left tail observed in any age class, and of the FSD of non-constrained firms, which accounts for the tendency toward a Gaussian behavior observed in the right tail of older firms. While the distribution of young

[10] Other age classes are not reported for the sake of clarity.

firms is similar across different FC classes, clear-cut differences appear when older firms are considered. This fact suggests a certain degree of persistence in FC classes.[11] Indeed if the probability of a firm to belong to a FC class at a given age were independent of its past growth process, the FC decomposition of the FSD would not reveal stronger differences among the older firms than among the younger ones. Moreover, the effect exerted by financial conditions seems to extend beyond a simple shift in the mean, as testified by the age profile of the estimated parameters. In the next section we propose a framework designed to capture the different effects plausibly at the basis of the interaction between age, financial constraints and firm growth, which has been revealed by the snapshot analysis of the size distribution.

4 Analytical Framework

We start from the phenomenological model of industrial dynamics based on the classical work by Gibrat (1931). Let s_t be the logarithm of firm size at time t. The simple integrated process

$$s_t = s_{t-1} + \epsilon_t \tag{2}$$

with *iid* distributed shocks ϵ_t, often referred to as the "Law of Proportionate Effect", has been shown to yield a good first order description of the observed dynamics of firm size (see among others Mansfield, 1962; Kumar, 1985; Hall, 1987). In order to account for the various effects of FCs on firm growth dynamics we consider a generalized version of the model, at the same time allowing for FC-class specific values in the relevant coefficients

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\; s_{t-1} + \sigma_{FC}(s_{t-1})\epsilon_{FC,t}\ , \tag{3}$$

where λ captures an autoregressive component in the (log) levels of firm size, σ is a function describing the heteroskedastic structure of the process and ϵ are assumed to be independent of size. The

[11]Due to the short time window of our database we cannot directly test the persistence in firm financial conditions over long span of times. The analysis of transition matrix between FC classes reveals a significant persistence. The average 1-year probability to remain in the same class is 83.72% for NFC firms, 77.47% for MFC firms, and 55.90% for HFC firms.

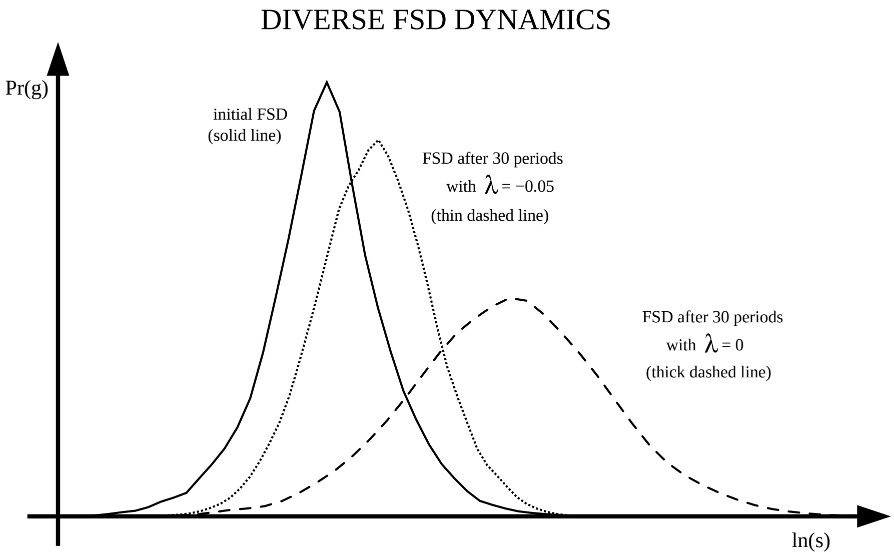

Figure 3: Evolution of the FSD for two different values of the autoregressive coefficient λ in equation (3). In the simulations we consider $15,000$ firms and we set $M_\epsilon = 0.25$ and $V_\epsilon = 0.5$.

inclusion of an AR(1) structure accounts for the fact that smaller firms are often reported to grow faster (see Lotti et al., 2003, for an in-depth review of the empirical literature).[12] The function σ introduces a dependence of the standard deviation of growth shocks on size, which has been reported in a large number of empirical studies. The common finding is that volatility is higher for smaller firms, and that the relationship displays an exponential decrease (see the discussion and references in Bottazzi and Secchi, 2005).

By allowing for FC class specific coefficients, the model in equation (3) allows FCs to produce an effect through four different channels: on the drift term c, on the autoregressive term λ, on the heteroskedasticity term $\sigma(s_{t-1})$ and on the properties of the distribution of growth shocks ϵ. Let us outline the economic interpretation of these channels, and the predictions that can be made.

The coefficient λ is related to the long-term dynamics of the evolution of size. Too see how let us neglect, for the sake of simplicity, the FC subscript and the heteroskedasticity correction, and let the mean and variance of the size distribution at time t be M_{s_t} and V_{s_t}, respectively. Under the

[12]The AR(1) specification can be replaced with a more general linear model. For the present discussion the 1-lag structure is sufficient, as we checked that the inclusion of further lags does not generate significant modifications in the estimates of λ.

hypothesis of a constant λ, their evolution from $t = 0$ to $t = T$ is given by

$$M_{s_T} = (1+\lambda)^T M_{s_0} + \frac{(1+\lambda)^T - 1}{\lambda} M_\epsilon \,, \quad V_{s_T} = (1+\lambda)^{2T} V_{s_0} + \frac{(1+\lambda)^{2T} - 1}{(1+\lambda)^2 - 1} V_\epsilon$$

where M_ϵ and V_ϵ are the mean and variance of the shocks ϵ.[13] When $\lambda = 0$, as in the benchmark Gibrat's model, we have a diffusion process: the time evolution of s_T follows a unit root process (discrete Brownian motion) asymptotically diverging to a log-normal FSD with indefinitely increasing variance and zero mean. Conversely, when $\lambda < 0$ the process is sub-diffusive and the FSD converges in probability to a stationary distribution with finite variance $V_\epsilon/(1-(1+\lambda)^2)$. The analysis in Section 3 suggests that $\lambda < 0$ may be the case for more severely constrained firms. Figure 3 shows that even small differences in the value of λ, can quickly produce significantly different FSD shapes.

Next, differences in c across FC classes provide information on the effect of FCs on the central tendency of the distributions, i.e. on the aforementioned location-shift effects across constrained or non-constrained firms. This is the kind of effect captured by the standard growth regression models traditionally proposed in the literature. Under the plausible conjecture that FCs reduce the set or the amount of growth opportunities seizable by constrained firms, then the prediction of the model is that the group of most severely constrained firms has the lowest estimated c.

Furthermore, differences in σ across FC classes captures an heteroskedasticity effect due to FCs, revealing that FCs also produce changes in the way the variability of growth rates depends on size. The often found reduction of growth rate volatility with size has been interpreted as a portfolio effect (Bottazzi and Secchi, 2005): since larger firms are typically more diversified than small firms (in terms of products, lines of business, plants,…) they can balance negative and positive shocks hitting their single branches (at least if the various activities are weakly correlated). According to this interpretation, we can conjecture that FCs, by reducing the range of attainable new growth opportunities, also reduce the diversification advantage of bigger firms. We therefore expect to observe weaker heteroskedasticity effects within the group of the most severely constrained firms.

Finally, concerning the possible effects of FC on the empirical distribution of growth shocks,

[13] See the Appendix for a formal derivation.

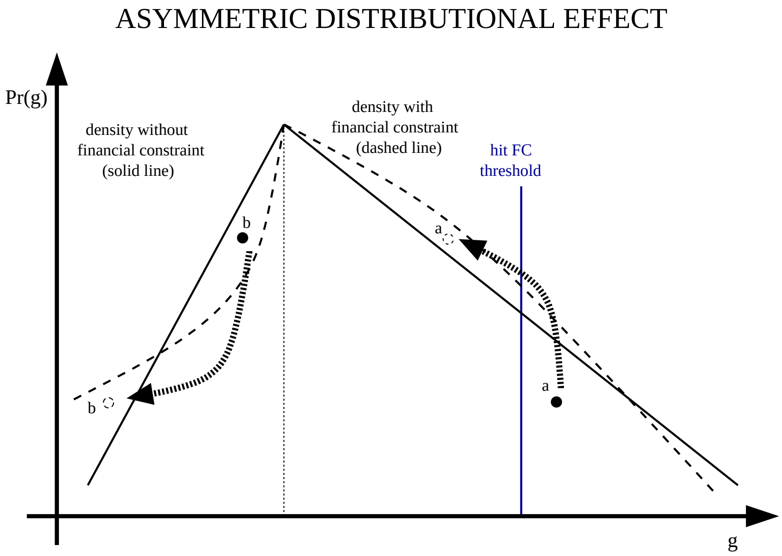

Figure 4: Possible effects of financing constraints on the growth rates distribution.

we can sketch some predictions based on the qualitative findings in Campello et al. (2009). In Figure 4 the solid line corresponds to a Laplace distribution of growth shocks (a "tent" on a log-scale) which represents the benchmark for non-constrained firms. The Laplace distribution has been chosen because invariably observed in empirical data across different countries and at different levels of sectoral aggregation (cfr. Stanley et al., 1996; Bottazzi and Secchi, 2006).[14] The dashed line describes the possible distributional effects that could plausibly emerge under the influence of binding FCs. One effect is a "pinioning the wings" effect: FCs prevent firms that face potentially good growth opportunities from actually seizing some of them (beyond a certain 'hit FC' threshold), thus forcing these firms to abandon or postpone some profitable investment projects. Although positive growth is still attainable in the presence of FCs, these firms would have enjoyed much higher growth records, if not hit by FCs. Such an effect would imply a slimming down of the right tail of the growth shocks distribution (cfr. 'case a' in Figure 4). Another possibility is that FC are responsible for a "loss reinforcing" effect. This predicts that firms who are already facing losses in market shares will experience a further deterioration in their poor growth rates in the presence of credit constraints problems, for example because they are forced to sell productive assets and divest activities, thus ultimately facing a reduction in revenues. This effect would be reflected in a shift of mass from the left-hand part of the density towards the bottom extreme, generating a fatter left tail (cfr. 'case b' in

[14] A first attempt to explain the emergence of this stylized fact, based on the idea of dynamic increasing returns, is presented in Bottazzi and Secchi (2006).

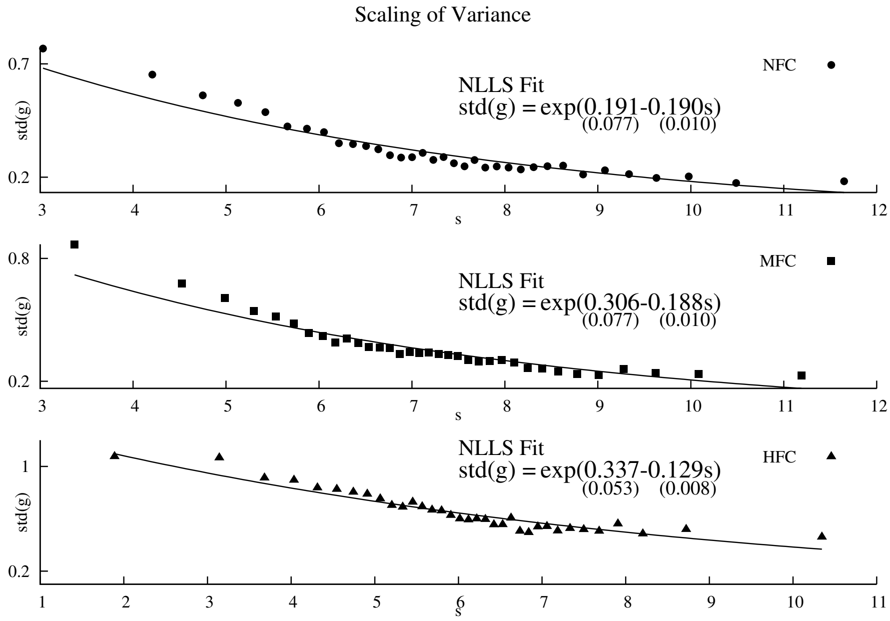

Figure 5: Empirical relation between the standard deviation of growth and firm size, by FC classes.

Figure 4).

5 Main results

A preliminary step in estimating equation (3) involves modeling heteroskedasticity. We characterize $\sigma_{FC}(s_{t-1})$ starting from the data. We consider the standard definition of growth rates in terms of log-differences of size

$$g_{i,t} = s_{i,t} - s_{i,t-1} \quad , \tag{4}$$

and then, for each FC class, we plot the standard deviation of g computed within different bins (quantiles) of the log-size distribution against the average log-size of the bin. Figure 5 reports results obtained with 35 size bins. The whole procedure is very robust in terms of choice of the number of bins. Scatter plots of the data tend to agree with previous studies, finding that the relationship displays an exponential decrease. This is confirmed, for all FC classes, by the Non-Linear Least Squares estimates reported in the graphs. It is also worth noticing that the relationship does not depend on age. In fact, within each FC class, we do not observe any statistically significant difference in the estimated relation when considering young versus old firms.[15]

[15]Results available upon request.

Taking this evidence into account, we insert an explicit exponential heteroskedasticity term $\sigma_{FC}(s_{t-1}) = \exp(\gamma_{FC} \cdot s_{t-1})$ in our baseline model to obtain

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\ s_{t-1} + \exp(\gamma_{FC} \cdot s_{t-1})\epsilon_{FC,t} \quad . \tag{5}$$

A further important modeling issue concerns an appropriate treatment of the distribution of residuals. As mentioned, previous studies have documented that the distribution of growth shocks, once heteroskedasticity has been properly modeled, is well approximated by a Laplace distribution. A first choice would therefore be to allow for Laplacian residuals, via Least Absolute Deviation (LAD) estimates. However, following the discussion in Section 4, we are also interested into possible asymmetries in the distribution of growth shocks, and therefore, we perform Maximum Likelihood estimates of equation (5) where we assume an Asymmetric Laplace distribution (ALAD) of the residuals.[16]

Table 2 presents the results (cfr. Model 1) obtained in each FC class. A first notable finding concerns the cross-class patterns in the autoregressive components. The estimated λ is not significant for NFC firms, while it is significant but practically zero in the MFC class. This suggests that an integrated process can represent a good approximation for the evolution of size in these two classes. Conversely, the estimate of λ is significantly negative for HFC firms (about -0.02, roughly three times bigger than in the other classes, in absolute value). This reveals that strong FCs give rise to sizeable deviations from the Gibrat's benchmark.[17]

The patterns in the constant terms are in line with expectations: average growth rate is positive for non constrained firms, while statistically equal to zero in the other two classes. Confirming intuition and standard results in the literature, FC problems reduce the average growth rate.

The estimates of the γ coefficients, confirming the graphical investigation reported in Figure 5, reveal the clear-cut role of FCs in explaining the heteroskedasticity of growth shocks. For NFC and MFC firms the estimated value is very close to -0.20 (which is strikingly similar to those reported

[16]This corresponds to assume that the error term follows an AEP distribution with $b_l = b_r = 1$, and with a_l and a_r estimated from data.

[17]If one is ready to accept the persistence in financial conditions over relatively long spans of time that we have indirectly inferred is Section 3, this result is sufficient to explain the lack of Gaussianization in the right tail of the FSD observed among the HFC firms (cf. Figure 2 above).

Table 2: REGRESSION ANALYSIS[a]

		Main Estimates	Robustness checks	
	FC CLASS	Model 1	Model 2A	Model 2B
	NFC			
γ		-0.200*(0.001)	-0.194*(0.001)	-0.193*(0.0010)
constant		0.019*(0.001)	0.022*(0.001)	0.024*(0.0024)
ln($S_{i,t-1}$)		-0.0001(0.0003)	-0.007*(0.001)	-0.008*(0.0007)
ln($Age_{i,t}$)			-0.025*(0.001)	-0.026*(0.0008)
ln($Assets^{b}_{i,t-1}$)			0.011*(0.001)	0.011*(0.0005)
ln($GOM^{b}_{i,t-1}$)			0.0001(0.0005)	0.0004(0.0005)
a_l, a_r		0.201, 0.176	0.197, 0.171	0.198, 0.170
Number of observations		89344	85382	85382
	MFC			
γ		-0.204*(0.001)	-0.195*(0.001)	-0.195*(0.001)
constant		-0.002(0.001)	0.0004(0.0003)	-0.002(0.001)
ln($S_{i,t-1}$)		-0.0063*(0.0004)	-0.017*(0.001)	-0.017*(0.001)
ln($Age_{i,t}$)			-0.041*(0.001)	-0.041*(0.001)
ln($Assets^{b}_{i,t-1}$)			0.015*(0.001)	0.014*(0.001)
ln($GOM^{b}_{i,t-1}$)			0.005*(0.0004)	0.005*(0.0004)
a_l, a_r		0.231, 0.224	0.224, 0.216	0.223, 0.216
Number of observations		102321	97437	97437
	HFC			
γ		-0.164*(0.002)	-0.152*(0.0026)	-0.151*(0.003)
constant		0.006(0.003)	0.024*(0.003)	0.016*(0.004)
ln($S_{i,t-1}$)		-0.019*(0.002)	-0.046*(0.002)	-0.046*(0.002)
ln($Age_{i,t}$)			-0.106*(0.003)	-0.108*(0.003)
ln($Assets^{b}_{i,t-1}$)			0.037*(0.002)	0.036(0.002)
ln($GOM^{b}_{i,t-1}$)			0.006*(0.001)	0.007*(0.001)
a_l, a_r		0.448, 0.425	0.431, 0.395	0.430, 0.395
Number of observations		20911	18834	18834

[a] ALAD estimates, standard errors in parenthesis.
[b] Assets is proxied with Net Tangible Assets. Gross Operating Margin(GOM) has been transformed to avoid negative numbers.
[*] Significantly different from zero at 1% level.

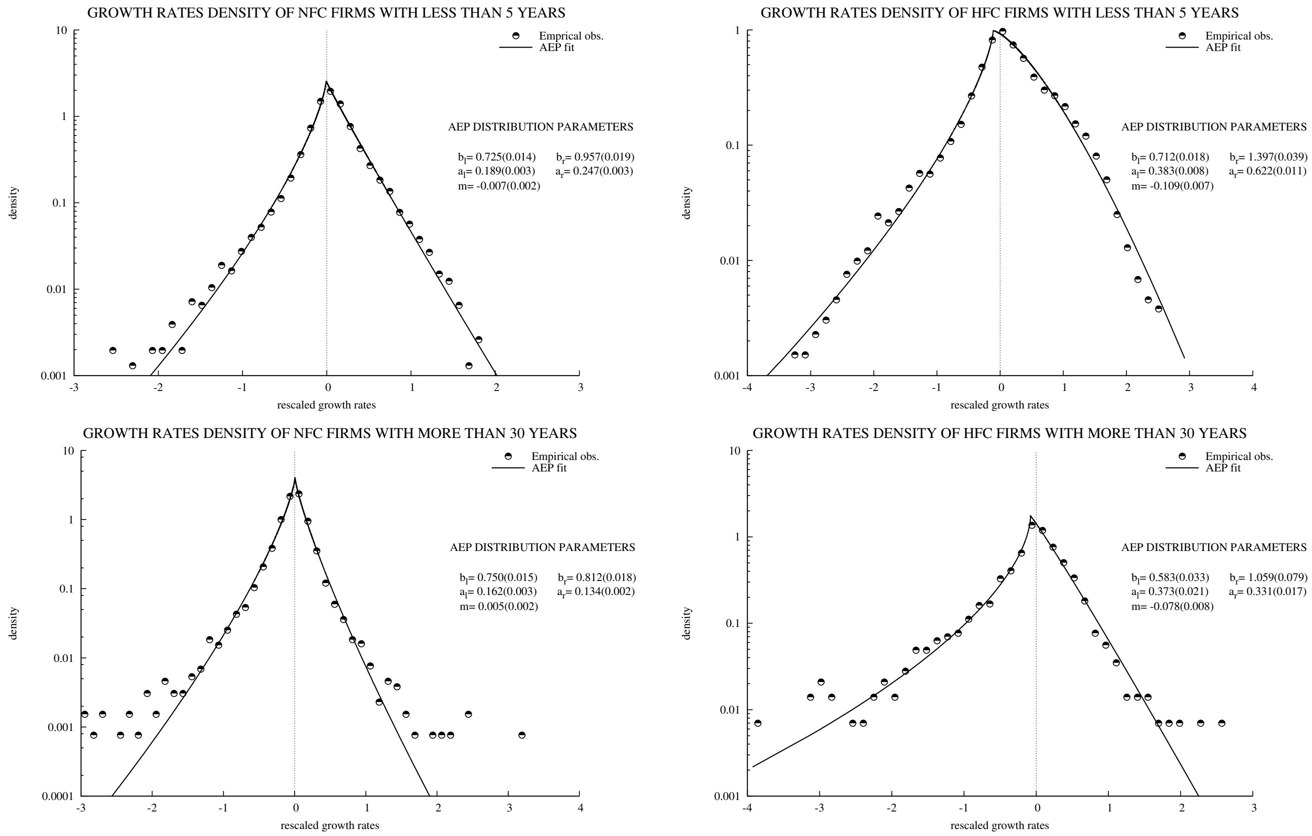

Figure 6: Growth rates distributions and financial constraints. Pooled data over 2000-2003.

in other studies on different data). This means that, in these two classes, the standard deviation of growth rates among largest firms (say, those firms with $s_{t-1} \simeq 10$), is approximately three times smaller than the standard deviation among small firms (say, those firms with $s_{t-1} \simeq 4$). Instead, within HFC firms the estimated γ is about -0.16, implying a smaller reduction in growth dispersion when moving from small to big firms, as compared to the other two classes (growth dispersion among larger firms is only about twice smaller than among smaller firms). This is once again in accordance with the intuition that FCs create a threshold effect, reducing the span of growth opportunities that constrained firms can access. According to the aforementioned "portfolio theory" interpretation, the implication is that the diversification advantage of bigger firms is considerably reduced by the effect of FCs.

Finally, the estimates of a_l and a_r suggest a relatively symmetric distribution of residuals. However, the ALAD estimation assumes an exact Laplace shape (i.e., b_l=b_r=1). In order to provide a more general assessment of the possible presence of asymmetry it is worthwhile investigating the structure of the residuals, also with respect to different age classes. This is done in Figure 6 where we show kernel estimates of the empirical distributions of the residuals for young-NFC firms (top-left), young-HFC firms (top-right), old-NFC firms (bottom-left), and old-HFC firms (bottom-right).[18] The estimates of the AEP coefficients b_l, b_r, a_l, a_r are reported in each panel, and differences in tail behavior are quantified by an AEP fit (solid line). A comparison across the estimates confirms the tent-shape approximation. However, the age-class disaggregation shows that FCs produce apparent differences in the shape of the shocks distributions. The very presence of such a sizeable effect is an interesting finding *per se*. Recall that in fact location-shift and variance-shift effects due to FCs are already captured in the regression through c and σ, respectively. Thus, what remains in the residuals is only the result of asymmetric tail effects induced by FCs. Let us first focus on young firms (compare the two top panels in Figure 6). If we move from NFC to HFC firms, we observe a clear-cut slimming down of the right tail: there is a leftward shift in probability mass from the right tail to the central part of the distribution (b_r increases from about 0.96 for NFC firms, and to almost 1.40 for the HFC class). Correspondingly, the right width parameter a_r also shows a clear-cut increase (from about 0.25 to about 0.62). In contrast, the left tails of the two distributions do not display any significant difference (both a_l and b_l are quite similar across NFC and HFC firms). The picture changes completely when we consider old firms (see the bottom panels). In this case the differences between NCF and HFC firms are stronger in the left tail. HFC firms have a fatter left tail, suggesting that FCs produce a shift in probability mass towards the left tail: b_l decreases from 0.75 to almost 0.58.[19] Overall, these findings are in line with the existence of two types of FC effects described in Section 4, and also suggest that such effects operate differently on different age classes. The "pinioning the wings" effect of FCs mainly affects young firms, while older firms are those mostly affected by the "loss reinforcing" effect of FCs.

[18]The distributions of MFC firms are not presented here to keep the figures more readable. The results (available upon request), substantially mimicking the findings obtained for NFC firms, do not affect the main conclusions of our reasoning.

[19]There is also an effect on the right side of the supports, qualitatively similar to that noted across young firms, and resulting in a fatter right tail for NFC firms. For old firms, however, the effect is very mild.

6 Robustness checks

Our baseline framework in equation (5) clearly leaves out important factors that are likely to play a role in size-growth dynamics. In this respect, we have seen that age can be a major candidate, exerting interesting effects on the distributional properties of residuals. Of course, there could also be others. In this section we explore the robustness of the FC effects collected so far, by enlarging the set of explanatory variables considered.

The relatively short time dimension of the data does not allow to perform reliable panel estimates, which would help to control for unobserved firm-specific heterogeneity. However, we can extend the set of regressors to control for the potentially relevant factors which we can observe. Firstly, the inclusion of firm age is mandatory, given the high correlation of age with size, and the significant effects that age has on the distributional properties of both size and growth. Secondly, there are two dimensions that need to be controlled for, namely availability of internally generated resources and availability of collateral. These are crucial factors, since they interact with external FCs in determining the overall amount of financial resources available to a firm. The rationale behind the inclusion of a proxy for collateral is that, as predicted by theory and confirmed by evidence (Angelini and Generale, 2008), the availability of hard capital can ease the access to external financing. We measure the availability of collateral using the stock of Net Tangible Assets (labeled ASSETS). Further, we proxy internal resources with the logarithm of Gross Operating Margin (GOM, equivalent to the EBIDTA), thus yielding a measure of the profit margin generated by the operational activities of a firm.[20] Given the relatively high frequency of negative GOM in the sample (about 30%), negative GOM values were transformed to 1 before taking logs. In fact, for the purposes of our analysis, negative and null operating revenues can be considered equivalent, as in both cases there is a need for the firm to completely rely on external resources in financing the operations.[21]

We run a preliminary Granger causality test between firm growth rates and FC. We estimate two regression models. In the first model we use dummy variables distinguishing whether firms belong

[20]The use of GOM implies, by definition, that we do not consider the cash flow generated by non operating earnings and losses. These should not be very relevant, however, since we are working with manufacturing firms. Moreover, due to the limited data availability, we cannot consider the cash flows absorbed by taxes. Assuming, as a first approximation, a constant tax rate, this would amount to a constant shift in the value of our regressor.

[21]As done for size, both GOM and ASSETS were deflated with appropriate sectoral price indexes, at the 3-digit level of industry disaggregation.

to HFC class or not. In the second model we directly use the risk-rating values as reported in the database. Both models are augmented by the controls discussed above (age, GOM and ASSET, plus lagged size). In both specifications, pooling over all the sample, we find that while past FC status Granger-causes growth, past growth does not Granger-causes FC status. This result confirms our choice to use lagged values of ratings as proxy for FC to growth.

Then, we move to our main robustness analysis, by adding the controls to our baseline specification. We first perform Maximum Likelihood ALAD estimates of the following extended model

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\ s_{t-1} + \beta_{1_{FC}}\ \ln(age_t) + \beta_{2_{FC}}\ \ln(GOM_{t-1}) + \\ \beta_{3_{FC}}\ \ln(ASSETS_{t-1}) + \exp(\gamma_{FC}\ s_{t-1})\epsilon_{t_{FC}} \quad (6)$$

where both GOM and ASSETS enter with a 1-period lag, at least partially accounting for simultaneity issues concerning these variables, and we again model heteroskedasticity via an exponential correction.[22] Results are reported in Table 2 under the heading "Model 2A". The most notable change induced by the inclusion of controls is that deviations from the Gibrat's benchmark of $\lambda = 0$ are now observed in all the FC classes. As frequently reported in studies exploring augmented Gibrat's regression, additional regressors absorb part of the size coefficient. However, the estimates of λ across the FC classes reproduce the pattern previously obtained from our baseline model: the autoregressive coefficient has a much lower value for the HFC class, thus confirming that the negative impact of size on growth rates is stronger for financially constrained firms. Estimates of the heteroskedasticity parameter γ are basically unaffected by the addition of further regressors and confirm the patterns emerging from the simplest specification.

In general, the effects exerted by the added covariates present interesting cross-class differences. Age displays a negative and significant coefficient in all classes, in agreement with the expectation that on average older firms grow less than younger firms. The magnitude increases with the strength of FCs, however, thus revealing that the detrimental effect of age is stronger among HFC firms. It should also be noted that age is the regressor with the strongest effect (highest coefficient in absolute value). Next, concerning the role of ASSETS, we find a positive and significant effect,

[22]Concerning the use of a GMM-SYS estimator, standard Sargan/Hansen tests confirm that the time span of the database is too short to identify a valid set of instruments among past levels and past differences of the covariates.

stronger for HFC firms: the availability of hard capital as collateral becomes more beneficial for growth when FC are stronger. Similarly, the availability of internal resources has beneficial effects on growth only when some degree of FCs is present, while internal resources do not seem to be crucial for unconstrained firms (GOM is not significant for NFC, positive and significant for MFC and HFC). However, even when significant, the magnitudes of GOM coefficients are negligible in practical terms, suggesting that internal resources play (if any) a second order role compared to other regressors.

A further check that we perform concerns the possible role of sector-specific dynamics. It is well known that a firm's dependence on external financing varies across industrial sectors (Rajan and Zingales, 1998), so that it is likely that firms operating in different industries would display, on average, a different degree of exposure to FC problems. There is also evidence (Hall, 2002) that such sectoral differences in modes of financing, and thus differential exposure to FCs, are very likely to vary depending on the sources and procedures of innovation activity of firms. In order to control for these industry-wide factors, we re-estimate equation (6) adding dummy variables which corresponds to the classical Pavitt taxonomy of sectoral patterns of innovation (Pavitt, 1984). The results (cfr. Model 2B in Table 2) are clearly in line with previous estimates: all the coefficients remain unchanged in practical terms.[23]

Finally, we also investigate whether the distributional properties of growth shocks are affected by the inclusion of the new regressors. To this purpose we perform AEP estimates of the empirical distribution of residuals of Model 2B, by FC classes and separately for young and old firms. Note that location-shift effects due to age are captured by the age coefficient in the regression, and also recall that (as shown in Section 5) age does not have any residual effect on the variance of growth rates, once controlling for size. Therefore, distributional differences in the residuals of Model 2B across age classes point toward additional effects of age in the tails. The estimates of AEP parameters, reported in Table 3, are not significantly different from those obtained with the simplest model

[23]We also explored a further specification considering 2-period lags of size, ASSETS and GOM. This allows for a check of varying effects over time, and provides a further control for possible endogeneity of covariates at $t-1$. The estimates of λ retain their signs and magnitudes, again displaying negligible values for NFC firms and then increasingly negative as FCs become stronger. Second lag coefficients of GOM and ASSETS absorb part of the first lag effects of these variables. The most noticeable difference compared to the estimates presented in Table 2 is a significant reduction in the age coefficient, whose magnitude becomes comparable with that of the other regressors, and also comparable across FC classes. The results are available upon request.

Table 3: Growth Rates Distributions – Robustness checks

	AEP Parameters			
	b_l	b_r	a_l	a_r
YOUNG (age < 5)				
NFC	0.729(0.0143)	0.975(0.0199)	0.188(0.0028)	0.244(0.0034)
HFC	0.713(0.0185)	1.436(0.0405)	0.374(0.0076)	0.602(0.0104)
OLD (age > 30)				
NFC	0.751(0.0155)	0.823(0.0189)	0.159(0.0025)	0.134(0.0022)
HFC	0.717(0.0465)	0.988(0.0813)	0.384(0.0197)	0.314(0.0177)

[a] AEP fit of residuals from Equation (6), Pavitt class dummies also included. Standard errors in parenthesis.

specification (apart from a small increase in the b_l parameter for HFC firms).

Overall, our main conclusions remain the same even with the inclusion of other relevant determinants of size-growth dynamics, such firm age and the availability of internal financial resources or collateral, and remain unchanged when we also control for differences in sectoral patterns of innovation.

7 Conclusion

CeBi credit ratings represent a good measure of a firm's access to external resources. They summarize several dimensions of a firm's financial conditions and allow to measure the different degree of credit problems, thus improving upon the rather strict binary distinction between constrained versus non-constrained firms often adopted in the literature. Moreover, they are heavily relied upon by banks and investors in granting and pricing credit lines, thus representing an important benchmark or a key ingredient in lending decisions. Using CeBi ratings to build a proxy for financial constraints, we extended the typical autoregressive linear model of size-growth dynamics by including a parametric description of heteroskedasticity and by providing a more flexible and robust characterization of growth shocks. Our results shows that the effects of FC on firm growth are sizeable and operate through several channels. Firstly, FCs magnify the negative effect of size on expected growth rates:

the lower average growth rate that typically characterizes large versus small firms becomes even lower when FCs are presents. This is consistent with the age profile of the firm size distribution of financially constrained and non-constrained firms. For older firms, the FSD of non constrained firms possesses a Gaussian shape, while the FSD of financially constrained firms is more peaked. This is the typical signature of the sub-diffusive nature of the growth process associated with a negative autoregressive coefficient. Since our measure of FCs varies over time, the fact that we identify significant differences in the size distribution of different FC classes suggests a relatively high degree of persistence across the different groups. This is an interesting aspect of the FC phenomenon, which we cannot however test directly, given the relatively short temporal span of our data.

A further effect of FCs is on the relationship between firm size and variance of growth rates. Larger firms are well known to generally display a lower variability in their growth rates. This observation has been related to a portfolio effect: larger firms tend to be more diversified, and thus, to the extent that the different activities are weakly related, diversification produces a lower volatility in aggregate growth rates. FCs seem to reduce the ability of larger firms to exploit their diversified structure. Indeed for more severely constrained firms, the negative relationship between growth rates variability and size is weaker than for unconstrained firms.

Furthermore, once the autoregressive structure and the heteroskedasticity effects are controlled for, our model reveals that FCs have an additional, asymmetric effect on the tails of the growth rates distribution. We are able to identify a loss reinforcing effect: firms who are already witnessing a reduction in sales, see their performance worsened in the presence of FCs. This is plausibly the results of activity dismissal and divestment. At the same time, however, firms experiencing positive growth rates, if hit by FCs, are likely to see their growth potentials depressed. In fact, credit problems generate a ”pinioning the wings” effect which prevents constrained firms from fully seizing the available growth opportunities. The economic consequences of these two effects are different. While the loss reinforcing effect can be seen as a natural market selection mechanism, generating, at least in the long run, a more efficient reallocation of productive resources, the pinioning effect plausibly translates into a net loss of growth opportunities. The fact that the pinioning mechanism is more common across younger firms is not unexpected and is compatible with the presence of frictions and inefficiencies in the capital market.

According to our credit-rating based measure of financial constraints, the problem of credit rationing is widespread and affects a much larger population of Italian manufacturing firms than what suggested by previous predictions obtained from survey-based measures (Angelini and Generale, 2008, see). This difference can be explained either by a self-selection bias in the population of respondents which is known to often affect survey data, or by admitting the possibility that not all firms with poor credit ratings were actually to be considered financially rationed. However, this consideration does not weaken the conclusions of our analysis. On the contrary, the fact that we still observe significant differences among the FC classes, notwithstanding the possible use of a somewhat loose proxy of FCs, represents a strong proof of the existence of a real economic effect. The adoption of a more stringent measure of FC would change the results in the direction of an even cleaner identification of this effect.

Finally, it is worth asking if our measure of FCs can also be considered as a proxy for the overall availability of financial resources, capturing at the same time difficulties in accessing external finance as well as shortage of internal financial resources. We tend to believe it can, as indeed internal resources constitute the best guarantee to potential lenders that firms are able to sustain the due interest payments. As a result, firms with sound financial conditions and reasonable levels of profits are almost automatically assigned high ratings, while the shortage of internal resources, whether generated by poor operating performances or by unsound financial conditions, is very likely to be punished with bad ratings. In any case, our conclusions are still valid even when we explicitly add a control for the availability of internal resources. Indeed, while profit margins are associated with produce a positive shift in the average growth rate, both the pinioning and loss reinforcing effects of FCs remain unchanged, as does the reduced ability of larger and financially constrained firms to exploit diversification economies.

In summary, we have shown that FC problems do have relevant effects on the operating activities of firms. In order to identify these effects, however, one has to do more work than just relying upon standard linear regression framework. FC effects are indeed manifold and impact on several aspects of firm growth dynamics, ranging well beyond a shift in the expected growth rates.

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8 APPENDIX

8.1 Cleaning anomalous observations

We removed a few anomalous data from our sample. Cleaning was performed using Total Sales as a reference variable. For each firm, a missing value was inserted, in the place of the original value of

Total Sales, when the latter lay outside the interval

$$[Median(TS_i)/10; Median(TS_i) * 10] \quad , \tag{7}$$

where the median is computed over the years for which data are available for firm i. Table 4 shows yearly descriptive statistics computed before and after the cleaning. It is apparent that the procedure does not introduce any relevant change to the data.

Table 4: TOTAL SALES[a] DESCRIPTIVE STATISTICS

				BEFORE CLEANING FILTER				
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5700.82	1014.00	48730.09	57.89	4894.16	1.00	5634948.00	109689.00
2001	5972.90	1011.00	73679.67	141.82	29897.12	1.00	17547260.00	113405.00
2002	5804.92	973.00	67304.35	146.66	32359.62	1.00	16484840.00	116084.00
2003	5639.77	953.00	64724.22	147.42	32317.38	1.00	15803760.00	115777.00
				AFTER CLEANING FILTER				
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5754.55	1046.00	47700.57	58.99	5192.76	1.00	5634948.00	107250.00
2001	5878.64	1025.00	69435.93	159.48	37224.24	1.00	17547260.00	112036.00
2002	5806.96	992.00	67093.95	150.02	33371.72	1.00	16484840.00	113849.00
2003	5688.46	981.00	65417.79	147.67	32063.94	1.00	15803760.00	111810.00

[a] Nominal Total Sales in thousands of Euro.

8.2 Asymptotic behavior of the autoregressive process

Start from the model of firm size evolution as described in (3), where the shocks ϵ are independent and identically distributed according to a probability density f with mean c. Let s_0 be the initial size of the firm. By dropping the heteroskedastic term (i.e. setting $\sigma(s_t) = 1$) for simplicity, and by recursive application of (3), the size after T time steps, s_T, can be written as the weighted sum of T

independent random variables

$$s_T = (1+\lambda)^T\, s_0 + \sum_{\tau=0}^{T-1} (1+\lambda)^\tau\, \epsilon_{t-\tau}\ .$$

Consider the cumulant generating function of the size at time T, $\tilde{g}_{s_T}$, defined as the logarithm of the Fourier transform of the unconditional distribution

$$\tilde{g}_{s_T}(k) = \log \mathrm{E}[e^{iks_T}]\ .$$

Due to the i.i.d. nature of the shocks it is immediate to see that

$$\tilde{g}_{s_T}(k) = \tilde{g}_{s_0}((1+\lambda)^T k) + \sum_{\tau=0}^{T-1} \tilde{f}((1+\lambda)^\tau k)$$

where $\tilde{g}_{s_0}$ and $\tilde{f}$ are the cumulants of the initial size distribution and of the shocks distribution, respectively. As a consequence, if the initial size distribution and the shocks distribution possess the cumulant of order n, C^n, then the size distribution at time T also possesses it, and thus, with obvious notation

$$C^n_{s_T} = \left. \frac{d^m}{dk^m}\, \tilde{g}_{s_T}(k) \right|_{k=0} = (1+\lambda)^{mT}\, C^n_{s_0} + \frac{(1+\lambda)^{mT} - 1}{(1+\lambda)^m - 1}\, C^n_\epsilon\ .$$

Equation (4) in Section 4 directly follows by noting that the mean and the variance are the first and second cumulants, respectively: $M = C^1$ and $V = C^2$.