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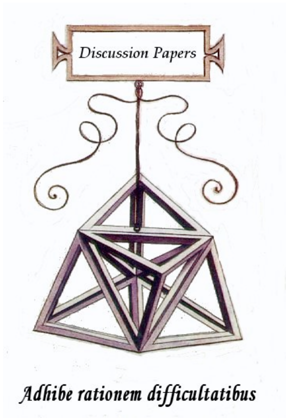

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How to compete? Cournot vs. Bertrand in a vertical structure with an integrated input supplier

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Abstract

We study whether a quantity or a price contract is chosen at equilibrium by one integrated firm and its retail competitor in a differentiated duopoly. Using a similar vertical structure, Arya et al. (2008) showed that Bertrand competition is more profitable than Cournot competition, which contrasts with conventional wisdom. In this paper, we first demonstrate that such a result is robust to the endogenous determination of the type of contract. Second, by introducing managerial incentives in the model, we find that delegation to managers entails conflicting choices of the strategic variable by the two firms as long as products are sufficiently differentiated, causing non-existence of equilibrium in pure strategies. Significantly high product substitutability reconciles firms' objectives under delegation, leading unique or multiple equilibria with symmetric types of contracts to arise.

JEL codes: D43, L13, L21

Keywords: Upstream monopolist, outsourcing, price competition, quantity competition, managerial delegation.

1 Introduction

Theory of Industrial Organization has often challenged the result brought out in the seminal paper by Singh and Vives (1984) that quantity (price) competition is more profitable than price (quantity) competition when goods are substitute (complement). Such a result, also shown by the authors to hold in a general demand framework and under cost asymmetries,[1] has been proved in subsequent research not to be robust to a wider range of cost and demand asymmetries (Zanchettin, 2006) and, moreover, not to generalize to an oligopoly under substantial quality differences between firms (Häckner, 2000), to a managerial delegation context with negative network externalities (Pal, 2015), and to a mixed market (Ghosh and Mitra, 2010; Haraguchi and Matsumura, 2016).[2] While the Cournot vs. Bertrand debate has been addressed, among others, in the above-mentioned works assuming perfectly competitive input markets, it has been getting some renewed attention in more recent literature on vertically related markets with upstream market power. In the strand of literature introducing upstream suppliers in the form of unions, it is shown that the result of Singh and Vives (1984) may not hold when two downstream independent firms bargain over the input price (wage) with their labor unions and labour returns are constant (Correa-López and Naylor, 2004), or in the presence of fixed wages determined by monopolistic unions and labour decreasing returns (Fanti and Meccheri, 2011). A reversal result is also found by Fanti and Meccheri (2015) in a duopoly with managerial delegation, under both decentralized and centralized unionization. Furthermore, whether firms' profits are higher under Cournot or Bertrand is investigated in a vertical structure by Mukherjee et al. (2012), which point out the Bertrand dominance from an aggregate profits standpoint under both uniform or discriminatory pricing by an upstream monopolist, and by Alipranti et al. (2014), which show that the equilibrium upstream profits are higher in Bertrand in a framework with bargaining through non-linear two-part tariffs. More related to the present work is the work by Arya et al. (2008) which focuses on the case of substitutes under a linear wholesale price contract to reveal that, when an integrated firm is the monopoly supplier of an essential input to a non-integrated retail rival, Bertrand is more profitable than Cournot.[3]

Starting from the early contribution of Singh and Vives (1984), the strategic decision between price and quantity has been further investigated in IO literature.[4] In that work, by developing a two-stage game where firms commit to

[1] Robustness of the Singh and Vives (1984)'s result is further confirmed by Tanaka (2001a) and Tanaka (2001b) tackling the case with substitutes respectively in an oligopoly and a duopoly with vertical differentiation, and by Tasnádi (2000) in which the price-setting scenario is a Bertrand-Edgeworth-type game.

[2] See Vives (1985), Qiu (1997), Cellini et al. (2004) and Mukherjee (2011), for further analysis related to the efficiency conditions in Cournot vs. Bertrand.

[3] In addition, Arya et al. (2008) find that consumer surplus and social welfare are both lower under Bertrand competition than under Cournot competition.

[4] Whether firm compete with respect to quantities or prices is an issue of controversy in economic literature dating back to Cournot (1838) and Bertrand (1883). For the main arguments, see the Introduction in Vives (1989). Moreover, refer to Reisinger and Ressner

a type of contract (price or quantity) and compete accordingly on the product market, the circumstances leading the Cournot (Bertrand) profits to exceed the Bertrand (Cournot) profits are shown to sustain Cournot (Bertrand) as an equilibrium in dominant strategies.[5] In most recent years, studies that identify the crucial factors affecting the endogenous choice of the strategic market variable have increasingly spread. In what follows we mention most notable works, among others, which employ the same game structure as Singh and Vives (1984) to show several circumstances under which the non-cooperative firms' choice of the competition regime emerges as part of a subgame perfect equilibrium. The latter has been shown to depend on the relative magnitude of demand and the degree of substitutability (Reisinger and Ressner, 2009), on whether firms select cooperatively or non-cooperatively the market variable in supergames (Lambertini, 1997), on the optimal policy - subsidy policy or trade liberalization - implemented by governments in a strategic trade setting (Choi et al., 2016), on welfare concern of a public firm (Matsumura and Ogawa, 2012), the number of private firms (Haraguchi and Matsumura, 2016), firms' subsidization (Scrimitore, 2013) and unionization (Choi, 2012) in mixed markets. Moreover, the question of how firms should compete in a duopoly has also received a good deal of attention in strategic delegation literature, to which this paper is closely related. Such a literature suggests that the choice between price and quantity competition, which provides an equivalence result under relative performance incentives (Miller and Pazgal, 2001), is crucially affected by the type of managerial contract and the intensity of network effects (Chirco and Scrimitore, 2013; Pal, 2015), as well as by the presence of both unions and managerial delegation (Fanti and Meccheri, 2015).

Further theoretical analysis in this field has been performed by assuming that firms are vertically related in an input-output chain. Recent research, indeed, has demonstrated that the endogenous choice of the strategic variable on the downstream market is sensitive to the structure of the upstream market for the input by and the vertical contracting procedure. In this regard, Correa-López (2007) finds that, for both the cases of unions and profit-maximizing firms as input suppliers, firms choose to compete either in the quantity space or in the price space, depending upon the degree of product differentiation and the distribution of bargaining power over the input price. These factors, conversely, are shown by Manasakis and Vlassis (2014) not to affect the equilibrium strategy, which turns out be quantity (price) under substitutes (complements), when it is determined as a renegotiation-proof contract between each downstream producer and its exclusive input supplier. Finally, Basak and Wang (2016) attribute to two-part-tariff pricing in the input market, determined through centralized Nash bargaining, a key role in leading firms to choose a price contract.

The present study reconsiders the relative profitability of Cournot and Bertrand

(2009, pp. 1157-1159), and literature cited therein, for evidence on whether firms are more likely to engage in quantity or price competition.

[5] Such an endogenous choice can be interpreted as the choice of a price or a quantity contract, along the lines of Singh and Vives (1984), or as the choice of the strategic variable to play on the product market.

in a vertical market.[6] In particular, we assume a duopoly in which the production of a key inputs is outsourced by an independent firm to a vertically integrated retail competitor which is a monopolist on the upstream market. Given this framework, we endogenize the choice of the strategic variable (price or quantity) made by both firms at the preplay stage of a game, thus prior to the input price setting stage and the retail competition stage. By doing so, we revisit Arya et al. (2008), checking whether their conclusion that competing à la Bertrand yields higher profits than competing à la Cournot when the mode of competition is exogenously given, still holds when it is endogenously determined.[7] We demonstrate that choosing a price contract is the dominant strategy for both firms, which leads symmetric Bertrand to arise as a subgame perfect equilibrium and to satisfy, according to the findings of Arya et al. (2008), Pareto optimality with respect to a symmetric Cournot game. However, our result draws attention on the unilateral incentives driving one firm's choice of price as a strategic variable, which is independent of the rival's choice. Indeed, committing to a price contract enables the vertically integrated producer (VIP) to enjoy its monopolistic position on the upstream market by charging a relatively high input price and, at the same time, by favoring the rival's output expansion.[8] Through the strategic choice of price, moreover, the independent firm exploits the advantage of the output expansion induced by the rival to keep its retail price higher. This result implies that the argument that Cournot is less competitive than Bertrand, thus resulting in firm's lower output and higher mark-up, which justifies its occurrence as the equilibrium strategic choice in a standard duopoly, cannot applies in a context of vertical relationships such as ours. Our work points out that, in the presence of outsourcing to an integrated rival, the choice of a Bertrand contract ensures higher profitability than a Cournot contract to both the integrated firm, which gains from a higher input price charged on a higher demand of inputs on the upstream channel, and the independent firm, which benefits from the higher output induced by the rival and more relaxed price competition on the retail market. Thus, by highlighting the forces shaping one firm's unilateral incentives towards Cournot vs. Bertrand in a duopoly with

[6] By assuming the presence of a firm outsourcing input supplies from its downstream integrated rival, we capture a quite spread real-world phenomenon. The latter characterizes regulated industries such as telecommunications, energy and transportations, where access to the network infrastructure is provided by a vertically integrated incumbent to retail competitors (see interesting examples of such sectors in Bourreau et al., 2011). Outsourcing to retail competitors is frequently observed also in markets with unregulated prices that better fit the scenario described in this paper. For instance, BMW supplied diesel engines to Toyota, as well as Volkswagen and Fiat did to Suzuki, Sony sources its LCD panels from Samsung and Sharp. Finally, Gazal Corporation, a leading Australian clothing company, distributes its products through its own major retailers, as well as through a range of independent retail outlets. The phenomenon has recently increased due to the growth of e-commerce which has brought manufacturers into direct competition with their retail counterparts. This is, among others, the case of Foxconn, a Taiwanese electronics manufacturer operating as a major supplier of Apple, Acer, Microsoft and Dell, that moved into the online retail market in China.

[7] Other works have tackled the issue of outsourcing to a vertically integrated rival in a theoretical perspective. See Arya et al (2008, p. 2) for some related studies quoted therein.

[8] This is also consistent with the findings of Moresi and Schwartz (2017).

a supply relationship, our work contributes to a better understanding of firms' strategic motives in vertical markets.

Moreover, this work extends the analysis carried out in the baseline model outlined above to a framework with managerial delegation by both firms, following the approach pioneered by Vickers (1985), Fersthman and Judd (1987) and Sklivas (1987).[9] Introducing strategic delegation to managers in our model is of particular interest since it shapes firms' interactions along the chain, affecting both the competitive toughness of retailers' conduct and the VIP's ability to commit to a relatively high input price, in that strategically orienting the choice of the retail market variable. While a large body of research has examined the strategic effects of delegating control to managers in markets without vertical relationships,[10] to the best of our knowledge there are no studies considering strategic delegation in a vertical oligopoly with an imperfectly competitive upstream market, with the exception of Park (2002).[11] By showing the circumstances under which managerial incentives affect the choice of the competition variable to play on the retail market under outsourcing of one firm to the rival, this paper enriches existing knowledge on the strategic implications of competition in vertically related markets. In particular, we find that delegation to sales-interested managers can lead to non-existence of equilibrium in pure strategies or to uniqueness or multiplicity of symmetric equilibria with both firms acting as price setters or quantity setters, depending on the degree of product substitutability. We put forward an argument maintaining that managerial incentives, by allowing firms to strategically orient downstream aggressiveness, modifies the firm's incentives to offer a price or a quantity contract and may cause an equilibrium non-existence problem. This occurs when product substitutability is sufficiently low, case in which the presence of the integrated firm acting as a price (quantity) setter makes the choice of a quantity (price) contract optimal for the independent firm, while the VIP always chooses to mimic the rival's choice. Conversely, sufficiently high product substitutability alters the incentives of the independent firm by making its foreclosure more likely and retail price competition fiercer, which respectively enhances its concern for higher market shares and the need to soften retail price competition, leading symmetric Bertrand and symmetric Cournot to arise at equilibrium. Our results contribute to the growing literature on managerial incentives pointing out how, in a vertical differentiated market with delegation by both an independent firm and its integrated rival/supplier, the extent of product substitutability provides

[9] Indeed, literature on managerial delegation focuses on the separation of ownership from control in large companies to bring out the idea that firms competing in oligopolistic markets, through incentive contracts appropriately designed by stockholders to their managers, credibly commit to pursue other objectives than profit maximization, e.g., sales maximization (Vickers, 1985), or revenues maximization (Fersthman and Judd, 1987; Sklivas, 1987) for strategic purposes.

[10] See Sengul et al. (2012) and Lambertini (2017) to capture the mechanism at work in the basic models and for a survey of related literature.

[11] In Park (2002) managerial contracts are aimed to control for the vertical externality effect caused by the presence of an upstream monopolist, in that leading pure profit maximization to arise at equilibrium.

conditions for the existence and uniqueness of equilibria.[12]

This paper is organized as follows. Section 2 develops the basic model which is extended to managerial delegation in Section 3. Finally, Section 4 presents concluding remarks.

2 The baseline model

Following the baseline model of duopoly competition by Arya et al. (2008), we assume one vertical integrated producer (VIP) and one independent firm, respectively firm 1 and firm 2, offering differentiated products. Firm 1 operates as an unregulated monopolist on the upstream market, supplying of a critical input both its downstream affiliate and the downstream rival, the latter being charged a per-unit wholesale price z for the input. Firms are endowed with a technology relying on perfect vertical complementarity (i.e., one unit of input is embodied in each unit of output) and enabling firm 1 and firm 2 to produce the retail output at constant marginal costs c_1 and c_2, respectively (with $c_1 \leq c_2$, as commonly assumed in this literature). Firm 1's costs to produce the input are normalized to zero, for the sake of simplicity, while no fixed costs and no capacity constraints are assumed.

The demand side on the downstream market is a simplified version of Singh and Vives (1984), the inverse demand function being:

$$p_i = a - \gamma q_j - q_i \tag{1}$$

where p_i and q_i are, respectively, the retail price and the retail output of variety i ($i = 1, 2$), $a > 0$ (with $a > c_2 \geq c_1$) is the reservation price and γ measures the degree of substitutability between the two varieties (i.e., goods are regarded as almost unrelated when $\gamma \to 0$ and almost homogeneous when $\gamma \to 1$). More precisely, we consider the interval of the product substitutability parameter that ensures the non-foreclosure condition for firm 2 recovered throughout the paper, i.e., $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$ with $\frac{a-c_2}{a-c_1} \leq 1$, and that coincides with the unit-interval of imperfect product substitutability only in the absence of cost differences between the two firms.

Given the above assumptions, firm 1's profits, i.e. the sum of its upstream and retail profits, are:

$$\pi_1 = zq_2 + (p_1 - c_1)\, q_1 \tag{2}$$

while firm 2's retail profits are as follows:

$$\pi_2 = (p_2 - z - c_2)\, q_2 \tag{3}$$

We first analyze a non-cooperative multi-stage game in which at the first stage

[12] The existence of multiple equilibria is a key feature of strategic delegation literature. In this regard, see the results achieved by Chirco and Scrimitore (2013) and Pal (2015).

each firm non-cooperatively chooses the type of contract, price or quantity,[13] while at the second stage firm 1 decides upon the input wholesale price it charges to firm 2. The last stage of the game describes competition on the retail market, with each firm acting as a price setter or a quantity setter according to the choice made at the first stage. The subgame perfect Nash equilibrium (SPNE) of the game is found by backward induction, which requires to work out the equilibrium outcomes of the following subgames:
- (*qq*), where firms play a symmetric Cournot game, both behaving as quantity takers;
- (*pq*), where firm 1 acts as a price setter (and a quantity taker) and firm 2 as a quantity setter (and a price taker);
- (*qp*), where firm 1 acts as a quantity setter (and a price taker) and firm 2 as a price setter (and a quantity taker);
- (*pp*), where firms play a symmetric Bertrand game, both behaving as price takers.

2.1 The qq subgame

We consider the subgame at which both firms play à la Cournot facing the inverse demand in (1).[14] At the retail market stage, maximization of the profit functions in (2) and (3), respectively for firm 1 and firm 2, with respect to quantities yields the following reaction functions exhibiting standard strategic substitutability (Bulow et al., 1985).

$$\begin{aligned} q_1 &= \frac{a-\gamma q_2 - c_1}{2} \\ q_2 &= \frac{a-\gamma q_1 - c_2 - z}{2} \end{aligned}$$

The solution of the system given by the two reaction functions gives the optimal quantities as functions of the input price z

$$q_1 = \frac{2\left(a-c_1\right)-\gamma\left(a-c_2-z\right)}{4-\gamma^2} \tag{4}$$

$$q_2 = \frac{2\left(a-c_2-z\right)-\gamma\left(a-c_1\right)}{4-\gamma^2} \tag{5}$$

Notice that an increase of the wholesale price z raises q_1 and lowers q_2.
At the previous stage of the game, the integrated firm maximizes (2), after incorporating the optimal quantities in (4) and (5). This allows us to identify the optimal wholesale price charged to the retailer:

$$z^{qq} = \frac{\gamma^3\left(a-c_1\right)+4\left(2-\gamma^2\right)\left(a-c_2\right)}{2\left(8-3\gamma^2\right)}$$

[13] This amounts to assuming that each firm makes a binding contract with final buyers. If a firm chooses a price contract, it is committed to supply the amount customers demand at a predetermined price, irrespective of whether the rival firm chooses a price or a quantity contract. Likewise, if a firm chooses a quantity contract, this implies a commitment to supply a predetermined quantity, independently of the strategy selected by its competitor.
[14] The analysis can be also resumed from Arya et al. (2008).

At equilibrium, the retail output of the two firms is as follows:[15]

$$q_1^{qq} = \frac{\left(8-\gamma^2\right)(a-c_1)-2\gamma(a-c_2)}{2\left(8-3\gamma^2\right)}$$
$$q_2^{qq} = \frac{2\left(a-c_2-\gamma(a-c_1)\right)}{8-3\gamma^2}$$

The expressions of the equilibrium profits are included in Appendix A (eqts A1-A2).

2.2 The pq subgame

We assume that firm 1 acts as a price setter and firm 2 as a quantity setter, so that we respectively consider the direct demand function and the inverse demand function:

$$q_1 = a-p_1-\gamma q_2 \tag{6}$$
$$p_2 = a(1-\gamma)+\gamma p_1-q_2\left(1-\gamma^2\right) \tag{7}$$

At the retail market stage, profit maximization of (2) and (3) with respect to p_1 and q_2 yields the following reaction functions:

$$p_1 = \frac{a-\gamma q_2+c_1}{2}$$
$$q_2 = \frac{a(1-\gamma)+\gamma p_1-c_2-z}{2\left(1-\gamma^2\right)}$$

which reveals that q_2 is a strategic substitute for firm 1, while p_1 is a strategic complement for firm 2 (Bulow et al., 1985).
The solutions of the above system are:

$$p_1 = \frac{a\left(2-\gamma^2\right)+2c_1\left(1-\gamma^2\right)-(a-c_2)\gamma+\gamma z}{4-3\gamma^2} \tag{8}$$
$$q_2 = \frac{2(a-c_2)-\gamma(a-c_1)-2z}{4-3\gamma^2} \tag{9}$$

Notice that p_1 is increasing and q_2 decreasing in z.
At the wholesale price setting stage, firm 1 maximizes (2) with respect to z, after incorporating (8) and (9). We thus obtain:

$$z^{pq} = \frac{8\left(1-\gamma^2\right)(a-c_2)+\gamma^3(a-c_1)}{2\left(8-7\gamma^2\right)}$$

[15] The condition $\gamma \leq \frac{a-c_2}{a-c_1}$ we assumed above clearly ensures that the non-foreclosure condition for firm 2 (i.e., $q_2 \geq 0$) applies to this setting as well as to the other subgames; therefore it is kept throughout the paper.

which also gives the following retail outcome at equilibrium:

$$\begin{aligned} p_1^{pq} &= \frac{a\left(8-5\gamma^2\right)-2\left(a-c_2\right)\gamma-c_1\left(9\gamma^2-8\right)}{2\left(8-7\gamma^2\right)} \\ q_2^{pq} &= \frac{2\left(a-c_2-\gamma\left(a-c_1\right)\right)}{8-7\gamma^2} \end{aligned}$$

The expressions of the equilibrium profits are included in Appendix A (eqts A3-A4).

2.3 The qp subgame

Here we assume that firm 1 acts as a quantity setter, while firm 2 acts as a price setter. This lets us consider the following inverse and the direct demand functions, respectively for firm 1 and firm 2:

$$p_1 = a\left(1-\gamma\right)+\gamma p_2-q_1\left(1-\gamma^2\right) \quad (10)$$

$$q_2 = a-p_2-\gamma q_1 \quad (11)$$

At the retail market stage, profit maximization of (2) and (3) with respect to q_1 and p_2 yields the following reaction functions:

$$\begin{aligned} q_1 &= \frac{a-c_1-\gamma\left(a-p_2+z\right)}{2\left(1-\gamma^2\right)} \\ p_2 &= \frac{a+c_2-\gamma q_1+z}{2} \end{aligned}$$

showing that p_2 is a strategic complement for firm 1, while q_1 is a strategic substitute for firm 2. The system of the reaction functions gives the following solutions:

$$q_1 = \frac{2\left(a-c_1\right)-\left(a-c_2\right)\gamma-\gamma z}{4-3\gamma^2} \quad (12)$$

$$p_2 = \frac{\left(2-\gamma^2\right)\left(a+z\right)+2c_2\left(1-\gamma^2\right)-\left(a-c_1\right)\gamma}{4-3\gamma^2} \quad (13)$$

showing that q_1 is decreasing and p_2 is increasing in z.
At the previous stage, after incorporating (12) and (13), firm 1 optimally chooses the following wholesale price:

$$z^{qp} = \frac{4\left(1-\gamma^2\right)\left(2-\gamma^2\right)\left(a-c_2\right)+\gamma^3\left(a-c_1\right)}{2\left(8+4\gamma^4-11\gamma^2\right)}$$

so that, the equilibrium retail outcome is:

$$\begin{aligned} q_1^{qp} &= \frac{8\left(a-c_1\right)+4\left(a-c_2\right)\gamma^3-\gamma\left(6\left(a-c_2\right)+5\gamma\left(a-c_1\right)\right)}{2\left(8+4\gamma^4-11\gamma^2\right)} \\ p_2^{qp} &= \frac{4\left(a+c_2\right)\gamma^4-2\left(7a+4c_2\right)\gamma^2+4\left(3a+c_2\right)-\gamma\left(4-3\gamma^2\right)\left(a-c_1\right)}{2\left(8+4\gamma^4-11\gamma^2\right)} \end{aligned}$$

The expressions of the equilibrium profits are included in Appendix A (eqts A5-A6).

2.4 The pp subgame

We now consider the case in which both firms play à la Bertrand on the downstream market, also tackled by Arya et al. (2008). We run the model using the following direct demand functions:

$$q_1 = \frac{a(1-\gamma) - p_1 + \gamma p_2}{(1-\gamma^2)} \tag{14}$$

$$q_2 = \frac{a(1-\gamma) - p_2 + \gamma p_1}{(1-\gamma^2)} \tag{15}$$

Standard profit maximization at the last stage of the game yields the following reaction functions:

$$p_1 = \frac{a(1-\gamma) + c_1 + \gamma p_2 + z\gamma}{2}$$

$$p_2 = \frac{a(1-\gamma) + c_2 + \gamma p_1 + z}{2}$$

which exhibit strategic complementarity. Then, we obtain the solutions of the price stage:

$$p_1 = \frac{a(2+\gamma)(1-\gamma) + \gamma c_2 + 2c_1 + 3z\gamma}{4-\gamma^2}$$

$$p_2 = \frac{a(2+\gamma)(1-\gamma) + \gamma c_1 + 2c_2 + z\left(2+\gamma^2\right)}{4-\gamma^2}$$

It is easy to check that p_1 and p_2 are increasing in z.
Firm 1's profit maximization with respect to z at the upstream market stage leads to this equilibrium solution:

$$z^{pp} = \frac{8(a-c_2) + \gamma^3(a-c_1)}{2(8+\gamma^2)}$$

which allows us to calculate the equilibrium prices which are as follows:

$$p_1^{pp} = \frac{8(a+c_1) + 2\gamma(a-c_2) - \gamma^2(a-3c_1)}{2(8+\gamma^2)}$$

$$p_2^{pp} = \frac{2a\gamma^2 + 4(3a+c_2) - \gamma\left(4+\gamma^2\right)(a-c_1)}{2(8+\gamma^2)}$$

The expressions of the equilibrium profits are included in Appendix A (eqts A7-A8).

2.5 The equilibrium of the strategic game

Before turning to derive the optimal choice between price and quantity made by both firms at the first stage of the game, we compare the equilibrium variables across the above four subgames. For this purpose we introduce the following lemma.

Lemma 1 *The following rankings regarding the equilibrium market variables (i.e., individual output, wholesale and retail prices) across the four subgames described above apply as long* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$*:*

- $z^{qp} \leq z^{pq} \leq z^{qq} \leq z^{pp}$
- $q_1^{pp} \leq q_1^{qp} \leq q_1^{pq} \leq q_1^{qq}$
- $p_1^{pq} \leq p_1^{qq} \leq p_1^{pp} \leq p_1^{qp}$
- $q_2^{qq} \leq q_2^{qp} \leq q_2^{pq} \leq q_2^{pp}$
- $p_2^{pq} \leq p_2^{pp} \leq p_2^{qq} \leq p_2^{qp}$

We now search for the subgame perfect Nash equilibrium of the game described in the following 2 x 2 normal-form game, where (q) and (p) are the strategies available to both players and the pay-offs are the equilibrium profits derived in the above subgames and included in Appendix A (eqts. A1–A8).

1/2	q	p
q	$\pi_1^{qq};\ \pi_2^{qq}$	$\pi_1^{qp};\ \pi_2^{qp}$
p	$\pi_1^{pq};\ \pi_2^{pq}$	$\pi_1^{pp};\ \pi_2^{pp}$

Figure 1 The pay-off matrix of the strategic game

The comparison of the payoffs in the above game allows us to prove the following proposition.

Proposition 1. *Both the vertically integrated producer (firm 1) and the independent firm (firm 2) choose to compete as price setters on the retail market, irrespective of the rival's action. That is, price is a dominant strategy for each firm and (pp) arises as the unique subgame perfect Nash equilibrium (SPNE) of the game, regardless of the degree of product differentiation* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$ *and the relative efficiency between the two firms.*

Proof.

Let us consider the following profit differentials:

- $\pi_1^{qp} - \pi_1^{pp} = -\frac{\gamma^2\left(4-\gamma^2\left(2+\gamma^2\right)\right)((a-c_2)-\gamma(a-c_1))^2}{(4\gamma^4-11\gamma^2+8)(1-\gamma^2)(8+\gamma^2)} < 0$
- $\pi_1^{pq} - \pi_1^{qq} = \frac{4\gamma^2((a-c_2)-\gamma(a-c_1))^2}{(8-7\gamma^2)(8-3\gamma^2)} > 0$
- $\pi_2^{pp} - \pi_2^{pq} = \frac{\gamma^2\left(8-5\gamma^2\right)\left(32-9\gamma^4-20\gamma^2\right)((a-c_2)-\gamma(a-c_1))^2}{(1-\gamma^2)(8+\gamma^2)^2(8-7\gamma^2)^2} > 0$
- $\pi_2^{qp} - \pi_2^{qq} = \frac{4\gamma^4\left(8-9\gamma^2\right)((a-c_2)-\gamma(a-c_1))^2}{(8-7\gamma^2)^2(8-3\gamma^2)^2} > 0$

The above inequalities prove that (p) is a dominant strategy for either firm 1 or firm 2.

Corollary 1 *As already highlighted by Arya et al. (2008), the symmetric Bertrand configuration (pp) arising as the unique subgame perfect Nash equilibrium of the strategic game in Figure 1 Pareto-dominates the symmetric Cournot outcome (qq), for any* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$.

*Proof***.**
- $\pi_1^{pp} - \pi_1^{qq} = \frac{\gamma^2\left(4+\gamma^2\right)((a-c_2)-\gamma(a-c_1))^2}{(1-\gamma^2)(8+\gamma^2)(8-3\gamma^2)} > 0$
- $\pi_2^{pp} - \pi_2^{qq} = \frac{\gamma^2\left(256-32\gamma^2-8\gamma^4+9\gamma^6\right)((a-c_2)-\gamma(a-c_1))^2}{(1-\gamma^2)(8+\gamma^2)^2(8-3\gamma^2)^2} > 0$

In order to illustrate the mechanism at work in this setting, we investigate the market variables' pattern included in Lemma 1, focusing on the asymmetric market configurations as compared to the symmetric ones.[16] In QQ and PQ the VIP takes the output of its rival as given, thus ignoring the impact on its profits on the upstream channel when it chooses both price and quantity as a strategic variable (i.e., firm 1's reaction functions at the market stage in both games do not depend on z). This induces firm 1 to sell too much on the retail market in QQ, thus keeping the rival's output - thus its demand of inputs - low. In PQ, conversely, firm 2 acts as a high producing firm by behaving as a price cutter (Singh and Vives, 1984, p. 550), so that its output q_2 is larger than in QQ, which further lowers p_1. These patterns, also consistent with a reduction of z in PQ with respect to QQ (i.e., $\partial p_1/\partial z > 0$ and $\partial q_2/\partial z < 0$), contribute to lowering q_1 and p_2, thus strengthening the toughness of retail competition.

By comparing the outcomes in QP and PP, however, we should consider that in these cases the VIP is aware that any action aimed at gaining an advantage over its rival on the downstream market - a reduction of its retail price or an expansion of its output - reduces the demand for its input. Therefore, both QP and PP are characterized by a lower incentive of the VIP to cut its own retail price or expand its output compared to QQ and PQ, respectively. It derives that in QP the VIP, acting as a price cutter, is more induced to keep p_2 relatively higher in order not to excessively lower q_2 through its own price reduction. This causes q_1 to be inevitably larger and q_2 lower than in PP, with both p_1 and p_2 being very high at equilibrium. Notice that the signs of the derivatives $\partial q_1/\partial z < 0$ and $\partial p_2/\partial z > 0$ confirm this pattern of the strategic variables, since z decreases in QP as compared to PP. The result is that the toughness of retail competition is very low under QP.

The above discussion contributes to explaining the result in Proposition 1 and allows us to underline the differences in firms' incentives between the framework as in Singh and Vives (1984) under substitutes and ours. In the

[16] We recall that the asymmetric market configurations (pq) and (qp) are characterized by the price setter and the quantity setter being respectively on their Cournot and Bertrand reaction functions (Singh and Vives, 1984, p. 550).

former, indeed, one firm's incentive is to undercut its rival in order to gain a competitive advantage due to higher market shares. Such an incentive leads quantity to be the dominant strategy since it allows each firm to act as a price cutter when the rival chooses price and to avoid to face a price cutter when the rival chooses quantity. Conversely, the presence of a vertical supply link between two downstream competing rivals as in our framework leads price to be the dominant strategy or each firm. Indeed, in such circumstances a price contract allows the VIP to exploit its monopoly power on the upstream market, raising the input demand from the rival and charging a higher wholesale price on it (for this purpose deviating from QQ on PQ, and from QP to PP), and the independent firm to enjoy the advantage from softening retail price competition on the larger output induced by the rival (for this purpose deviating from PQ on PP, and from QQ to QP). It turns out that PP represents the unique subgame perfect Nash equilibrium of the game, which Pareto-dominates QQ.

3 Price vs. quantity under managerial delegation

In this section we investigate the mechanism through which delegation to managers interacts with the incentives driving the choices of the strategic contract made by the vertical integrated producer and the independent retailer. We keep the assumptions and notations from the previous section and, moreover, make standard hypotheses in strategic delegation literature regarding managerial behavior. Following Vickers (1985), we define firm i's managerial objective function as follows:

$$U_i = \pi_i + \lambda_i q_i \tag{16}$$

where λ_i $(i = 1, 2)$, i.e., the incentive parameter, is the weight put on sales, which identifies the degree of discretion assigned by firm's owner to her risk-neutral manager.[17] The weight λ_i is non-cooperatively and simultaneously chosen by firms' owners on a profit-maximizing basis and is such that we impose no *a priori* restriction on its value. Clearly, when $\lambda_i > 0$ ($\lambda_i < 0$), the manager is rewarded (penalized) for sales maximization and behaves more (less) aggressively on the product market as compared to the profit-maximizing case, which is finally recovered when $\lambda_i = 0$.[18]

By denoting market configurations as (QQ), (PQ), (QP) and (PP), we search backwards for the subgame perfect Nash equilibrium supporting price or quantity as the optimal choice for each firm. Under the assumption that managerial incentives affects decisions made on the retail market, a delegation

[17] The specification of the managerial objective function à la Vickers (1985) in this setting is formally equivalent to that defined by Fershtman and Judd (1987) as a linear combination of firm's profits and revenues.

[18] As common in this literature, it is asserted that $\lambda_i < 0$ implies overcompensation for profits. See Fershtman and Judd (1987, pp. 937-938) for an interpretation of the mechanism behind overcompensation as a owner's tax imposed on sales.

stage preceding the last stage of downstream competition is added to the basic model described in the previous section.

3.1 The QQ subgame

At the last stage of the game, each manager maximizes the objective function in (16) choosing the final output level. This dentifies the following reaction functions, respectively for firm 1 and firm 2:

$$
\begin{aligned}
q_1 &= \frac{a - c_1 - \gamma q_2 + \lambda_1}{2} \\
q_2 &= \frac{a - c_2 - \gamma q_1 + \lambda_2 - z}{2}
\end{aligned}
$$

clearly exhibiting strategic substitutability. Solving the system of the two reaction functions, we obtain the optimal quantities as functions of the input price z and the incentive parameters λ_1 and λ_2:

$$
q_1 = \frac{2(a - c_1) - \gamma(a - c_2) + \gamma(z - \lambda_2) + 2\lambda_1}{4 - \gamma^2} \tag{17}
$$

$$
q_2 = \frac{2(a - c_2) - \gamma(a - c_1) - \gamma\lambda_1 - 2(z - \lambda_2)}{4 - \gamma^2} \tag{18}
$$

It is easy to prove that:
- $\frac{\partial q_1}{\partial \lambda_1} > 0$; $\frac{\partial q_1}{\partial \lambda_2} < 0$;
- $\frac{\partial q_2}{\partial \lambda_2} > 0$; $\frac{\partial q_2}{\partial \lambda_1} < 0$;

which show that one firm's higher aggressiveness raises its own retail output and reduces that of the rival.
At the third stage of the game, i.e., the delegation stage, owner i $(i = 1, 2)$ maximizes with respect to λ_i her own profits obtained after substituting (17) and (18) in (2) and (3), thus choosing the optimal extent of delegation to assign each manager. We get the following reaction functions for the two firms:

$$
\begin{aligned}
\lambda_1 &= \frac{\gamma\left(2\gamma(a - c_1) - \gamma^2(a - c_2) - 2z\left(2 - \gamma^2\right) - \gamma^2\lambda_2\right)}{4(2 - \gamma^2)} \\
\lambda_2 &= \frac{\gamma^2\left(2(a - c_2) - \gamma(a - c_1) - 2z - \gamma\lambda_1\right)}{4(2 - \gamma^2)}
\end{aligned}
$$

Notice that $\frac{\partial \lambda_1}{\partial \lambda_2} < 0$ and $\frac{\partial \lambda_2}{\partial \lambda_1} < 0$, which imply strategic substitutability.
The solutions of the delegation stage are:

$$
\begin{aligned}
\lambda_1 &= \frac{\gamma\left(\gamma\left(4 - \gamma^2\right)(a - c_1) - 2\gamma^2(a - c_2) - 2z\left(4 - 3\gamma^2\right)\right)}{(\gamma^4 - 12\gamma^2 + 16)} \\
\lambda_2 &= \frac{\gamma^2\left(\left(4 - \gamma^2\right)(a - c_2) - 2\gamma(a - c_1) - 2z\left(2 - \gamma^2\right)\right)}{(\gamma^4 - 12\gamma^2 + 16)}
\end{aligned}
$$

In this regard, it is important to consider that $\frac{\partial\lambda_1}{\partial z} \leq 0$ and $\frac{\partial\lambda_2}{\partial z} \leq 0$ in the given interval, which reveal that a higher input price lowers both firms' aggressiveness on the retail market.
At the second stage of the game, firm 1, by maximizing with respect to z its own profits calculated at the optimal quantities and the optimal delegation parameters, decides upon the wholesale price which is as follows at equilibrium:

$$z^{QQ} = \frac{16\left(2-\gamma^2\right)\left(4-3\gamma^2\right)(a-c_2)+\gamma^7(a-c_1)}{2\left(128+48\gamma^4+\gamma^6-160\gamma^2\right)}$$

Therefore, the incentive parameters at the subgame perfect Nash equilibrium are:

$$\begin{aligned}
\lambda_1^{QQ} &= -\frac{2\gamma\left(4-\gamma^2+2\gamma\right)\left(4-\gamma^2-2\gamma\right)(a-c_2-(a-c_1)\gamma)}{128+48\gamma^4+\gamma^6-160\gamma^2} \\
\lambda_2^{QQ} &= \frac{\gamma^2\left(16-\gamma^4-8\gamma^2\right)(a-c_2-(a-c_1)\gamma)}{128+48\gamma^4+\gamma^6-160\gamma^2}
\end{aligned}$$

It can be easily checked that $\lambda_1^{QQ} \leq 0$ and $\lambda_2^{QQ} \geq 0$, for any $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$, which proves that firm 1's manager is penalized for sales at equilibrium, while firm 2's manager is allowed to consider sales to some extent. Finally, the output levels at equilibrium are:

$$\begin{aligned}
q_1^{QQ} &= \frac{\left(128+\gamma^6+8\gamma^4-96\gamma^2\right)(a-c_1)-8\gamma\left(8-5\gamma^2\right)(a-c_2)}{2\left(48\gamma^4+128+\gamma^6-160\gamma^2\right)} \\
q_2^{QQ} &= \frac{2\left(16-\gamma^4-8\gamma^2\right)(a-c_2-\gamma(a-c_1))}{128+48\gamma^4+\gamma^6-160\gamma^2}
\end{aligned}$$

See Appendix B (eqts B1-B2) for the equilibrium profits.

3.2 The PQ subgame

Using the demand functions in (6) and (7) and the same procedure as in the above setting, we solve the subgame in which firm 1 is a managerial price-setting firm and firm 2 is a managerial quantity-setting firm.
The following reaction functions at the retail market stage:

$$\begin{aligned}
p_1 &= \frac{a-\gamma q_2+c_1-\lambda_1}{2} \\
q_2 &= \frac{a(1-\gamma)+\gamma p_1-c_2+\lambda_2-z}{2\left(1-\gamma^2\right)}
\end{aligned}$$

reveal that q_2 is a strategic substitute for firm 1, while p_1 is a strategic complement for firm 2 and yield the following solutions:

$$\begin{aligned}
p_1 &= \frac{a\left(2-\gamma^2\right)+2\left(1-\gamma^2\right)(c_1-\lambda_1)-(a-c_2)\gamma-\gamma(\lambda_2-z)}{4-3\gamma^2} \\
q_2 &= \frac{2(a-c_2)-\gamma(a-c_1)+2(\lambda_2-z)-\gamma\lambda_1}{4-3\gamma^2}
\end{aligned}$$

It is easy to prove that:
- $\frac{\partial p_1}{\partial \lambda_1} < 0$; $\frac{\partial p_1}{\partial \lambda_2} < 0$;
- $\frac{\partial q_2}{\partial \lambda_2} > 0$; $\frac{\partial q_2}{\partial \lambda_1} < 0$;

showing that higher aggressiveness of firm 1 reduces its own retail price and the rival's output, while higher aggressiveness of firm 2 raises its own output and reduce the rival's price.

The reaction functions at the delegation stage are:

$$\begin{aligned} \lambda_1 &= \frac{\gamma\left(\gamma\left(2-\gamma^2\right)(a-c_1)-\gamma^2(a-c_2)-4z\left(1-\gamma^2\right)-\gamma^2\lambda_2\right)}{4\left(2+\gamma^4-3\gamma^2\right)} \\ \lambda_2 &= \frac{\gamma^2\left(\gamma(a-c_1)-2(a-c_2)+2z+\gamma\lambda_1\right)}{4\left(2-\gamma^2\right)} \end{aligned}$$

which are respectively characterized by strategic substitutability (i.e., $\frac{\partial \lambda_1}{\partial \lambda_2} < 0$) and strategic complementarity (i.e., $\frac{\partial \lambda_2}{\partial \lambda_1} > 0$). The solutions of this stage are the following:

$$\begin{aligned} \lambda_1 &= \frac{\gamma\left(\gamma\left(4-\gamma^2\right)(a-c_1)-2\gamma^2(a-c_2)-\left(8-6\gamma^2\right)z\right)}{5\gamma^4-20\gamma^2+16} \\ \lambda_2 &= \frac{\gamma^2\left(\gamma\left(2-\gamma^2\right)(a-c_1)-\left(4-3\gamma^2\right)(a-c_2)+4z\left(1-\gamma^2\right)\right)}{5\gamma^4-20\gamma^2+16} \end{aligned}$$

Since $\frac{\partial \lambda_1}{\partial z} \leq 0$ and $\frac{\partial \lambda_2}{\partial z} \geq 0$, we can assert that a higher input price lowers firm 1's aggressiveness and raises firm 2's aggressiveness on the retail market.

At equilibrium we get:

$$z^{PQ} = \frac{\gamma^7(a-c_1)+16(1-\gamma)(1+\gamma)\left(2-\gamma^2\right)\left(4-3\gamma^2\right)(a-c_2)}{2\left(128-47\gamma^6-288\gamma^2+208\gamma^4\right)}$$

$$\begin{aligned} \lambda_1^{PQ} &= -\frac{2\gamma\left(5\gamma^4-20\gamma^2+16\right)(a-c_2-(a-c_1)\gamma)}{128-47\gamma^6-288\gamma^2+208\gamma^4} \\ \lambda_2^{PQ} &= -\frac{\gamma^2\left(4-3\gamma^2\right)^2(a-c_2-(a-c_1)\gamma)}{128-47\gamma^6-288\gamma^2+208\gamma^4} \end{aligned}$$

Clearly, $\lambda_1^{PQ} \leq 0$ and $\lambda_2^{PQ} \leq 0$ for any $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$, which proves that both firms are penalized for sales (namely, for their aggressiveness) at equilibrium.

$$\begin{aligned} p_1^{PQ} &= \frac{32\left(4-9\gamma^2\right)(a+c_1)+8\gamma^3\left(1-\gamma^2\right)(a-c_2)-(39a+55c_1)\gamma^6+8(25a+27c_1)\gamma^4}{2\left(128-47\gamma^6-288\gamma^2+208\gamma^4\right)} \\ q_2^{PQ} &= \frac{2\left(4-3\gamma^2\right)^2(a(1-\gamma)-c_2+c_1\gamma)}{128-47\gamma^6-288\gamma^2+208\gamma^4} \end{aligned}$$

See Appendix B (eqts B3-B4) for the equilibrium profits.

3.3 The QP subgame

Using the demand functions in (10) and (11) and by standard procedure, we solve the game in which firm 1 is a managerial quantity-setting firm and firm 2 is a managerial price-setting firm. The reaction functions at the retail market stage are:

$$q_1 = \frac{a(1-\gamma) + \gamma p_2 - c_1 + \lambda_1 - z\gamma}{2(1-\gamma^2)}$$

$$p_2 = \frac{a - \gamma q_1 + c_2 - \lambda_2 + z}{2}$$

which show that p_2 is a strategic complement for firm 1, while q_1 is a strategic substitute for firm 2.
At this stage we obtain the following solutions:

$$q_1 = \frac{2(a-c_1) - \gamma(a-c_2) + 2\lambda_1 - \lambda_2\gamma - z\gamma}{4-3\gamma^2}$$

$$p_2 = \frac{\left(2-\gamma^2\right)(a+z) - \gamma(a-c_1) + 2\left(1-\gamma^2\right)(c_2-\lambda_2) - \gamma\lambda_1}{4-3\gamma^2}$$

It is easy to prove that:
- $\frac{\partial q_1}{\partial \lambda_1} > 0$; $\frac{\partial q_1}{\partial \lambda_2} < 0$;
- $\frac{\partial p_2}{\partial \lambda_2} < 0$; $\frac{\partial p_2}{\partial \lambda_1} < 0$;

showing that higher aggressiveness of firm 1 raises its own retail output and reduces the rival's price, while higher aggressiveness of firm 2 reduces both its own price and the rival's output.
The reaction functions at the delegation stage:

$$\lambda_1 = \frac{\gamma\left((a-c_2)\gamma^2 - 2\gamma(a-c_1) + 2z\left(2-\gamma^2\right) + \gamma^2\lambda_2\right)}{4(2-\gamma^2)}$$

$$\lambda_2 = \frac{\gamma^2\left(\left(2-\gamma^2\right)(a-c_2) - \gamma(a-c_1) - 2z\left(1-\gamma^2\right) - \gamma\lambda_1\right)}{4(2+\gamma^4-3\gamma^2)}$$

respectively exhibit strategic complementarity (i.e., $\frac{\partial \lambda_1}{\partial \lambda_2} > 0$) and strategic substitutability (i.e., $\frac{\partial \lambda_2}{\partial \lambda_1} < 0$) and, moreover, identify the following solutions:

$$\lambda_1 = \frac{\gamma\left(\gamma^2\left(2-\gamma^2\right)(a-c_2) - \gamma\left(4-3\gamma^2\right)(a-c_1) + 2z\left(1-\gamma^2\right)\left(4-\gamma^2\right)\right)}{5\gamma^4 - 20\gamma^2 + 16}$$

$$\lambda_2 = \frac{\gamma^2\left(\left(4-\gamma^2\right)(a-c_2) - 2\gamma(a-c_1) - 2z\left(2-\gamma^2\right)\right)}{5\gamma^4 - 20\gamma^2 + 16}$$

Notice the sign of the following derivatives, $\frac{\partial \lambda_1}{\partial z} \geq 0$ and $\frac{\partial \lambda_2}{\partial z} \leq 0$, which show that a higher input price raises firm 1's aggressiveness and limits firm 2's aggressiveness on the retail market.

At the wholesale price setting stage, we get:

$$z^{PP} = \frac{8\left(1-\gamma^2\right)\left(4-\gamma^2\right)\left(2-\gamma^2\right)^2(a-c_2)+(a-c_1)\gamma^7}{2\left(128-71\gamma^6+8\gamma^8-288\gamma^2+224\gamma^4\right)}$$

which yields the following equilibrium delegation parameters:

$$\begin{aligned}
\lambda_1^{QP} &= \frac{\gamma\left(2-\gamma^2\right)\left(5\gamma^4-20\gamma^2+16\right)(a-c_2-(a-c_1)\gamma)}{128-71\gamma^6+8\gamma^8-288\gamma^2+224\gamma^4} \\
\lambda_2^{QP} &= \frac{\gamma^2\left(4-\gamma^2\right)\left(4-3\gamma^2\right)(a-c_2-(a-c_1)\gamma)}{128-71\gamma^6+8\gamma^8-288\gamma^2+224\gamma^4}
\end{aligned}$$

where $\lambda_1^{QP} \geq 0$ and $\lambda_2^{QP} \geq 0$ for any $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$, that is, both managers are instructed at equilibrium to care about sales, namely they are rewarded for aggressiveness. The equilibrium retail outcome is as follows:

$$\begin{aligned}
q_1^{QP} &= \frac{\left(128-19\gamma^6+120\gamma^4-224\gamma^2\right)(a-c_1)-4\gamma\left(2-\gamma^2\right)\left(8-\gamma^2\left(9-2\gamma^2\right)\right)(a-c_2)}{2(128-71\gamma^6+8\gamma^8-288\gamma^2+224\gamma^4)} \\
p_2^{QP} &= \frac{4a\left(1-\gamma^2\right)\left(3-2\gamma^2\right)(2-\gamma)^2(2+\gamma)^2+2c_2\left(2-\gamma^2\right)\left(16-4\gamma^6+21\gamma^4-32\gamma^2\right)-\gamma\left(64-13\gamma^6+76\gamma^4-128\gamma^2\right)(a-c_1)}{2(128-71\gamma^6+8\gamma^8-288\gamma^2+224\gamma^4)}
\end{aligned}$$

See Appendix B (eqts B5-B6) for the equilibrium profits.

3.4 The PP subgame

By maximizing the objective function in (16) and using demand functions in (14) and (15), each manager chooses the optimal price at the retail market stage. We therefore obtain the following reaction functions:

$$p_1 = \frac{a(1-\gamma)+c_1+\gamma p_2-\lambda_1+z\gamma}{2} \tag{19}$$

$$p_2 = \frac{a(1-\gamma)+c_2+\gamma p_1-\lambda_2+z}{2} \tag{20}$$

which exhibit strategic complementarity, and the following solutions of the price stage:

$$\begin{aligned}
p_1 &= \frac{a(2+\gamma)(1-\gamma)+\gamma(c_2-\lambda_2)+2(c_1-\lambda_1)+3z\gamma}{4-\gamma^2} \\
p_2 &= \frac{a(2+\gamma)(1-\gamma)+\gamma(c_1-\lambda_1)+2(c_2-\lambda_2)+z\left(2+\gamma^2\right)}{4-\gamma^2}
\end{aligned}$$

It is easy to prove that:

- $\frac{\partial p_1}{\partial \lambda_1} < 0$; $\frac{\partial p_1}{\partial \lambda_2} < 0$;
- $\frac{\partial p_2}{\partial \lambda_2} < 0$; $\frac{\partial p_2}{\partial \lambda_1} < 0$;

showing that one firm's higher aggressiveness reduces both its own retail price and that of the rival.

At the delegation stage, profit maximization by each owner gives the following reaction functions:

$$\lambda_1 = \frac{\gamma\left(\gamma^2(a-c_2)-\gamma\left(2-\gamma^2\right)(a-c_1)+\gamma^2\lambda_2+4z\left(1-\gamma^2\right)\right)}{4\left(2-\gamma^2\right)}$$

$$\lambda_2 = \frac{\gamma^2\left(\gamma(a-c_1)-\left(2-\gamma^2\right)(a-c_2)+\gamma\lambda_1+2z\left(1-\gamma^2\right)\right)}{4\left(2-\gamma^2\right)}$$

Notice that $\frac{\partial\lambda_1}{\partial\lambda_2}>0$ and $\frac{\partial\lambda_2}{\partial\lambda_1}>0$, i.e., the reaction functions exhibit strategic complementarity.
The solutions of the delegation stage are the following:

$$\lambda_1 = -\frac{\gamma\left(a\gamma(1-\gamma)\left(4-\gamma^2+2\gamma\right)-c_1\gamma\left(4-3\gamma^2\right)+c_2\gamma^2\left(2-\gamma^2\right)-2z\left(1-\gamma^2\right)(2-\gamma)(2+\gamma)\right)}{(4-\gamma^2+2\gamma)(4-\gamma^2-2\gamma)}$$

$$\lambda_2 = -\frac{\gamma^2\left(a(1-\gamma)\left(4-\gamma^2+2\gamma\right)+c_2\left(3\gamma^2-4\right)+c_1\gamma\left(2-\gamma^2\right)-4z\left(1-\gamma^2\right)\right)}{(4-\gamma^2+2\gamma)(4-\gamma^2-2\gamma)}$$

Also notice that $\frac{\partial\lambda_1}{\partial z}\geq 0$ and $\frac{\partial\lambda_2}{\partial z}\geq 0$, which reveals that a higher input price raises both firms' aggressiveness on the retail market.
Finally, by maximizing firm 1's profits in (1) calculated at the optimal prices and the optimal delegation parameters with respect to z, we solve for the equilibrium wholesale price charged to firm 2 in this setting :

$$z^{PP} = \frac{\gamma^7(a-c_1)+8\left(4-\gamma^2\right)\left(2-\gamma^2\right)^2(a-c_2)}{2\left(128+64\gamma^4-7\gamma^6-160\gamma^2\right)}$$

At equilibrium, the delegation parameters are:

$$\lambda_1^{PP} = \frac{\gamma\left(2-\gamma^2\right)\left(4-\gamma^2+2\gamma\right)\left(4-\gamma^2-2\gamma\right)(a-c_2-(a-c_1)\gamma)}{128+64\gamma^4-7\gamma^6-160\gamma^2}$$

$$\lambda_2^{PP} = -\frac{\gamma^2\left(5\gamma^4+16\left(1-\gamma^2\right)\right)(a-c_2-(a-c_1)\gamma)}{128+64\gamma^4-7\gamma^6-160\gamma^2}$$

where $\lambda_1^{PP}\geq 0$ and $\lambda_2^{PP}\leq 0$, for any $\gamma\in\left(0,\frac{a-c_2}{a-c_1}\right)$, which shows that at equilibrium firm 1's manager is instructed to care about sales, to some extent, while firm 2's manager is penalized for sales. The equilibrium prices are:

$$p_1^{PP} = \frac{a\left(4-\gamma^2+2\gamma\right)\left(3\gamma^4+10\gamma^3-24\gamma^2-16\gamma+32\right)-4c_2\gamma^3\left(2-\gamma^2\right)+c_1\left(128-11\gamma^6+72\gamma^4-160\gamma^2\right)}{2(128+64\gamma^4-7\gamma^6-160\gamma^2)}$$

$$p_2^{PP} = \frac{4a\left(48-2\gamma^6+21\gamma^4-56\gamma^2\right)+2c_2\left(4-\gamma^2\right)\left(2-\gamma^2\right)\left(4-3\gamma^2\right)-\gamma\left(64\left(1-\gamma^2\right)+\gamma^4\left(20-\gamma^2\right)\right)(a-c_1)}{2(128+64\gamma^4-7\gamma^6-160\gamma^2)}$$

See Appendix B (eqts B7-B8) for the equilibrium profits.

3.5 The equilibrium of the strategic game under managerial delegation

In this section, we determine the SPNE of the game by moving backwards to the first stage of the game and searching for the optimal choice, (Q) or (P), of each firm, as described in the following matrix:

1/2	Q	P
Q	$\pi_1^{QQ};\ \pi_2^{QQ}$	$\pi_1^{QP};\ \pi_2^{QP}$
P	$\pi_1^{PQ};\ \pi_2^{PQ}$	$\pi_1^{PP};\ \pi_2^{PP}$

Figure 2 The pay-off matrix of the game under managerial delegation

where the pay-offs are the equilibrium profits of the above subgames, which are included in Appendix B (eqts. B1–B8).
Before comparing the firms' profits across the four considered subgames, we introduce the following lemmas that allow us to identify the forces driving the optimal choice between (Q) and (P) by each firm.

Lemma 2 *By running the model under managerial delegation, we get the following rankings regarding the equilibrium market variables (i.e., individual output, wholesale and retail prices) which apply as long* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$*:*
- $z^{PQ} \leq z^{QP} \leq z^{QQ} \leq z^{PP}$
- $q_1^{PP} \leq q_1^{PQ} \leq q_1^{QP} \leq q_1^{QQ}$
- $p_1^{QP} \leq p_1^{QQ} \leq p_1^{PP} \leq p_1^{PQ}$
- $q_2^{QQ} \leq q_2^{PQ} \leq q_2^{QP} \leq q_2^{PP}$
- $p_2^{QP} \leq p_2^{PP} \leq p_2^{QQ} \leq p_2^{PQ}$

Lemma 3 *Managerial delegation dampens the VIP's aggressiveness when it acts as a price setter rather than a quantity setter, regardless on whether the rival's strategy is price or quantity. Indeed, it is easy to check that* $\lambda_1^{PQ} \leq \lambda_1^{QQ} \leq 0 \leq \lambda_1^{PP} \leq \lambda_1^{QP}$*, independently of* γ *in the interval ensuring firm 2's non-foreclosure. In the same interval of* γ*, we obtain that* $\lambda_2^{PQ} \leq \lambda_2^{PP} \leq 0 \leq \lambda_2^{QQ} \leq \lambda_2^{QP}$ *which reveals that delegation acts in the opposite direction when the independent firm is considered, that is, it dampens firm's aggressiveness when playing as a quantity setter rather than a price setter, regardless of the rival's strategy.*

Lemma 2-3 can be explained in the light of the intuition given to Proposition 1, which is concerned with the no-delegation case. We focus on the incentive of the VIP to enhance the demand of inputs from the rival and that of the independent rival to reduce toughness of retail price competition to explain the pattern of the incentive parameters across the four subgames. Indeed, when the rival plays quantity (price), firm 1 uses managerial incentives to affect downstream competition and exploit strategic complementarity (substitutability) of q_2 with respect to p_1, which leads q_2 to increase. This will induce firm 1 by to reduce its own aggressiveness under price with respect to quantity competition (i.e., $\lambda_1^{PQ} \leq \lambda_1^{QQ}$ and $\lambda_1^{PP} \leq \lambda_1^{QP}$), regardless of the choice of firms 2. Likewise, firm 2 uses managerial incentives to soften downstream competition under quantity

rather than price competition (i.e., $\lambda_2^{PQ} \leq \lambda_2^{PP}$ and $\lambda_2^{QQ} \leq \lambda_2^{QP}$), regardless of the choice of the rival, causing both p_1 and p_2 to increase.
We can now state the following proposition.

Proposition 2. *In a scenario in which the VIP and the independent firm are managerial and choose whether to act as a quantity setter or a price setter, their optimal strategy profile depends on the degree of product differentiation over the interval* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$ *and their relative efficiency. When there are no or negligible cost differences between the two firms, so that* $0.959 < \frac{a-c_2}{a-c_1} \leq 1$*: a) no equilibrium in pure strategies to exist as long as* $\gamma \in (0, 0.913)$*; b)* (PP) *arises as a unique SPNE when* $\gamma \in (0.913, 0.959)$*, while* (PP) *coexists with* (QQ) *as multiple equilibria when* $\gamma \in \left(0.959, \frac{a-c_2}{a-c_1}\right)$*. Moreover,* (PP) *emerges as a unique equilibrium of the game in the interval* $\gamma \in \left(0.913, \frac{a-c_2}{a-c_1}\right)$ *in the presence of more relevant cost differences* (i.e., when $0.913 < \frac{a-c_2}{a-c_1} \leq 0.959$). *When cost differences are substantial* (i.e., $\frac{a-c_2}{a-c_1} \leq 0.913$), no equilibrium in pure strategies exist over the entire interval $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$.

*Proof***.**

Consider the following inequalities:

- $\pi_1^{QP} - \pi_1^{PP} = -\frac{2\gamma^8\left(2-\gamma^2\right)^3(B-A\gamma)^2}{\Phi\Psi(1-\gamma^2)} < 0;$
- $\pi_1^{PQ} - \pi_1^{QQ} = -\frac{8\gamma^8\left(2-\gamma^2\right)(B-A\gamma)^2}{\Theta\Omega} < 0;$
- $\pi_2^{PP} - \pi_2^{PQ} = -\frac{2\gamma^8\varphi\left(2-\gamma^2\right)(B-A\gamma)^2}{\Psi^2\Omega^2(1-\gamma^2)} > 0 \Rightarrow 0.913 < \gamma < 1;$
- $\pi_2^{QP} - \pi_2^{QQ} = \frac{2\gamma^8\phi\left(2-\gamma^2\right)(B-A\gamma)^2}{\Phi^2\Theta^2} > 0 \Rightarrow 0 < \gamma < 0.959$

where:
$A = a - c_1$, $B = a - c_2$, $\Phi = 128 + 8\gamma^8 - 71\gamma^6 + 224\gamma^4 - 288\gamma^2$,
$\Psi = 128 - 7\gamma^6 + 64\gamma^4 - 160\gamma^2$, $\Theta = 128 + \gamma^6 + 48\gamma^4 - 160\gamma^2$.
$\Omega = 128 - 47\gamma^6 + 208\gamma^4 - 288\gamma^2$, $\varphi = 65\,536 - 3969\gamma^{14} + 42488\gamma^{12} - 194128\gamma^{10} + 489920\gamma^8 - 735\,744\gamma^6 + 655360\gamma^4 - 319488\gamma^2$, $\phi = 65\,536 - 64\gamma^{16} + 103\gamma^{14} + 6744\gamma^{12} - 57\,072\gamma^{10} + 208\,448\gamma^8 - 412\,160\gamma^6 + 458\,752\gamma^4 - 270\,336\gamma^2$,

with Φ, Ψ, Θ and Ω strictly positive in the given interval, and φ and ϕ having a zero at $\gamma = 0.913$ and $\gamma = 0.959$, respectively.

The above inequalities prove the above proposition as follows. Let us consider the case with cost symmetry between the two firms. In the interval $\gamma \in (0, 0.913)$ firm 1 plays Q (P) when the rival plays Q (P), thus choosing the same strategic variable as the rival does, while firm 2 plays P (Q) when the rival plays Q (P), that is, it chooses the strategy that is opposite to the rival's. This implies

that no equilibrium in pure strategies exists over the given interval. When $\gamma \in (0.913, 1)$, while firm 1 still chooses the same strategic variable as that of the rival regardless of γ, firm 2 plays (P) as a dominant strategy as long as $\gamma \in (0.913, 0.959)$, which implies that (PP) is the unique equilibrium arising in this interval, whereas it chooses the same strategic variable as the rival's as long as $\gamma \in (0.959, 1)$, which leads both (PP) and (QQ) to coexist as subgame perfect equilibria.
The comparison of firms' profits at the symmetric market configurations (PP) and (QQ) introduces the following corollary.

Corollary 2 *The symmetric Bertrand outcome (PP) Pareto-dominates the symmetric Cournot outcome (QQ), independently of the degree of product differentiation.*

Proof.

- $\pi_1^{PP} - \pi_1^{QQ} = \frac{2\Lambda\gamma^4\left(2-\gamma^2\right)(B-A\gamma)^2}{\Psi\Theta(1-\gamma^2)} > 0$
- $\pi_2^{PP} - \pi_2^{QQ} = \frac{2F\gamma^4\left(2-\gamma^2\right)(B-A\gamma)^2}{\Psi^2\Theta^2(1-\gamma^2)\Psi^2\Theta^2} > 0$

where F is a polynomial of degree 18 (we omit the formula for brevity) and $\Lambda = 64 + \gamma^6 + 16\gamma^4 - 64\gamma^2$, with no zero over the given interval.

Proposition 2 can be explained in detail in what follows. From Lemmas 2-3 we know that in the PQ game managerial delegation, by limiting the aggressiveness of both firms with respect to any other market structure, makes the upstream market less profitable (indeed, z assumes the lowest value in PQ), while it raises profitability of the downstream market due to the highest retail prices. This causes, as long as product differentiation is high enough, the choice of a quantity contract rather a price contract to be optimal for both the VIP when the rival chooses a quantity contract (indeed, QQ is preferred to PQ by firm 1) and the independent firm when the rival chooses price (indeed, PQ is preferred to PP by firm 2), which contrasts with the no-delegation case. More precisely, firm 1 chooses a quantity contract in QQ to exploit its competitive advantage on the downstream segment, rather than market power on the upstream channel, while firm 2 chooses PQ to gain from softened retail price competition rather than from an output expansion. Furthermore, as highlighted in Lemma 3, delegation lets QP entail the highest aggressiveness by both firms, thus the lowest retail prices, which reduces retail market profitability as compared to that of the upstream market. However, this does not alter the firms' optimal choice with respect to the no-delegation case when product substitutability is sufficiently low. Indeed, QP is still dis-preferred to PP by firm 1, which aims at exploiting the upstream segment profitability, thus reducing its aggressiveness downstream, by choosing price. Also, QP is still preferred to QQ by firm 2, since a price contract allows it to behave more aggressively downstream, namely to sell a higher output, thus exploiting its retail market competitive advantage. Non-existence of equilibrium clearly arises under such circumstances.

By contrast, when products are strongly substitutes, firms' incentives turn out to get aligned with each other, yielding the existence of unique or multiple subgame perfect symmetric equilibria. Indeed, by making foreclosure of the independent firm more likely, high product substitutability leads this firm to switch from quantity to price when the rival chooses price, with the aim to gain from increased aggressiveness of both firms, and thus from higher retail output, in PP with respect to PQ. This causes symmetric Bertrand to arise as a pure strategy Nash equilibrium, being price the dominant strategy for firm 2 in the interval $\gamma \in \left(0.913, \frac{a-c_2}{a-c_1}\right)$ (with $0.913 < \frac{a-c_2}{a-c_1} \leq 0.959$). Moreover, high product substitutability makes retail price competition very fierce, so that the independent firm is induced to switch from price to quantity when the rival chooses quantity, in the attempt to gain from reduced aggressiveness of both firms, and thus from higher retail prices, in QQ with respect to QP. This yields symmetric Cournot as a pure strategy Nash equilibrium of the game, which coexists with the symmetric Bertrand equilibrium in the interval $\gamma \in \left(0.959, \frac{a-c_2}{a-c_1}\right)$, as long as $0.959 < \frac{a-c_2}{a-c_1} \leq 1$).

The occurrence of non-existence of equilibrium and multiplicity of equilibria under certain conditions induces us to search for the equilibria in mixed strategies. The following proposition is then introduced.

Proposition 3. *Let us assume that there are no or negligible cost differences between the two firms, so that* $0.959 < \frac{a-c_2}{a-c_1} \leq 1$*. Then, a Nash equilibrium in mixed strategies exists both in the interval* $\gamma \in (0, 0.913)$ *where no pure strategy equilibrium exists, and in the interval* $\gamma \in \left(0.959, \frac{a-c_2}{a-c_1}\right)$ *where multiplicity of equilibria in pure strategies arises. Conversely, no equilibrium in mixed strategies exists in the interval* $\gamma \in (0.913, 0.959)$ *where firm 2 plays* (P) *as a strictly dominant strategy. In the presence of substantial cost heterogeneity, the upper intervals progressively shrink, determining the existence of a unique mixed strategy equilibrium over the interval* $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$ *when* $\frac{a-c_2}{a-c_1} \leq 0.913$.

See Appendix C for the proof.

4 Concluding remarks

In this work we have examined the endogenous choice of the strategic variable, price or quantity, in a differentiated duopoly characterized by a vertical relationship between one independent firm outsourcing input supply to an integrated retail competitor. We have shown that price competition emerges as a subgame perfect equilibrium in dominant strategies, regardless of the degree of product substitutability. This proves that the result of Arya et al. (2008) that Bertrand is more profitable than Cournot is robust to the endogenous determination of the equilibrium mode of competition. The strategic perspective of our analysis contributes to underlining that such a result relies on the fact that Bertrand allows the VIP to exploit its monopolistic position on the upstream market by inducing a higher demand of inputs from the rival and the independent firm to gain from a higher intensive margin by relaxing retail price competition. By identifying the effects of such incentives of firms' equilibrium choices, we have been able to explain the results obtained when the basic model is extended to managerial delegation. The latter has been found to dramatically alter the result of the no-delegation setting, causing non-existence of equilibrium in pure strategies as long as product differentiation is high enough. Such a non-existence result derives from the following conflicting firms' objectives. On the one hand, delegation induces the VIP to exploit its market power on the upstream segment under symmetric price competition and its competitive advantage on the downstream market under symmetric quantity competition. On the other hand, delegation pushes the independent firm to deviate from any symmetric market configuration in order to gain from a higher intensive (extensive) margin when the rival chooses price (quantity). However, very high product substitutability, by defining the conditions under which the independent firm is induced to expand its own output and to reduce the toughness of price competition when the VIP plays respectively price and quantity, aligns the objectives of the independent firm with those of the VIP. This has been shown to cause the existence of at least one symmetric equilibrium entailing price or quantity competition and, moreover, the uniqueness vs. multiplicity of equilibria. Finally, we have highlighted the existence of a mixed strategy equilibrium in the cases in which the model may either yield multiple or no equilibrium in pure strategies.

An immediate implication of our analysis is that delegation may provide incentives for an integrated firm to exploit its competitive advantage downstream rather than its monopoly power on the upstream market, which contrasts with previous literature dealing with distribution channel's profitability under linear pricing (Moresi and Schwartz, 2017). Future research could investigate price vs. quantity competition under non-linear wholesale price contracts, both in the no-delegation and in the delegation case, with the aim to provide new insights to both literature on vertical relationships and that on managerial incentives. Further research should also aim at performing the same analysis as in this paper under a different timing regarding the design of managerial incentives, namely by assuming that they can affect both retail competition and the wholesale price setting stage. Both these aims are included in our research agenda.

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Appendix A

Profits of firm 1 and firm 2 in the four subgames of Section 2 are as follows:

$$\pi_1^{qq} = \frac{\left(8+\gamma^2\right)A^2+4B^2-8AB\gamma}{4\left(8-3\gamma^2\right)} \tag{A1}$$

$$\pi_2^{qq} = \frac{4\left(B-A\gamma\right)^2}{\left(8-3\gamma^2\right)^2} \tag{A2}$$

$$\pi_1^{pq} = \frac{12a^2-8Cc_1-4Dc_2-\gamma A\left(8B+3\gamma A\right)}{4\left(8-7\gamma^2\right)} \tag{A3}$$

$$\pi_2^{pq} = \frac{4\left(1-\gamma^2\right)\left(B-A\gamma\right)^2}{\left(8-7\gamma^2\right)^2} \tag{A4}$$

$$\pi_1^{qp} = \frac{12a^2-8Cc_1-4Dc_2-\left(11a^2-7Cc_1-4Dc_2\right)\gamma^2-8\gamma AB\left(1-\gamma^2\right)}{4(4\gamma^4-11\gamma^2+8)} \tag{A5}$$

$$\pi_2^{qp} = \frac{\left(2-\gamma^2\right)^2 (B-A\gamma)^2}{\left(4\gamma^4-11\gamma^2+8\right)^2} \tag{A6}$$

$$\pi_1^{pp} = \frac{12a^2-8Cc_1-4Dc_2-\gamma A\left(A\gamma^3+3A\gamma+8B\right)}{4\left(1-\gamma^2\right)\left(8+\gamma^2\right)} \tag{A7}$$

$$\pi_2^{pp} = \frac{\left(2+\gamma^2\right)^2 (B-A\gamma)^2}{\left(1-\gamma^2\right)\left(8+\gamma^2\right)^2} \tag{A8}$$

where $A=a-c_1$; $B=a-c_2$; $C=2a-c_1$; $D=2a-c_2$.

Appendix B

Profits of firm 1 and firm 2 in the four subgames of Section 3 are as follows:

$$\pi_1^{QQ} = \frac{\widehat{\Phi}-32\left(4a^2-3Cc_1-Dc_2\right)\gamma^2+A\gamma\left(A\gamma^3\left(16+\gamma^2\right)-64B\left(2-\gamma^2\right)\right)}{4\Theta} \tag{B1}$$

$$\pi_2^{QQ} = \frac{2\left(2-\gamma^2\right)\left(16-\gamma^2\left(8+\gamma^2\right)\right)^2 (B-A\gamma)^2}{\Theta^2} \tag{B2}$$

$$\pi_1^{PQ} = \frac{\widehat{\Phi}+16\left(9a^2-7Cc_1-2Dc_2\right)\gamma^4-32\widehat{\Psi}\gamma^2-A\gamma\left(15A\gamma^5+64B\left(1-\gamma^2\right)\left(2-\gamma^2\right)\right)}{4\Omega} \tag{B3}$$

$$\pi_2^{PQ} = \frac{2\left(2-\gamma^2\right)\left(4-3\gamma^2\right)^4 (B-A\gamma)^2}{\Omega^2} \tag{B4}$$

$$\pi_1^{QP} = \frac{\widehat{\Phi}-16AB\gamma\left(2-\gamma^2\right)^3-\widehat{\Theta}\gamma^6+16\widehat{\Omega}\gamma^4-32\widehat{\Psi}\gamma^2}{4\Phi} \tag{B5}$$

$$\pi_2^{QP} = \frac{2\left(1-\gamma^2\right)\left(2-\gamma^2\right)\left(4-\gamma^2\right)^2\left(4-3\gamma^2\right)^2 (B-A\gamma)^2}{\Phi^2} \tag{B6}$$

$$\pi_1^{PP} = \frac{\widehat{\Phi}-\widehat{\Theta}\gamma^6+16\widehat{\Omega}\gamma^4-32\widehat{\Psi}\gamma^2-\gamma(a-c_1)\left(A\gamma^7+16B\left(2-\gamma^2\right)^3\right)}{4\Psi(1-\gamma^2)} \tag{B7}$$

$$\pi_2^{PP} = \frac{2\left(2-\gamma^2\right)\left(16\left(1-\gamma^2\right)+5\gamma^4\right)^2 (B-A\gamma)^2}{\Psi^2\left(1-\gamma^2\right)} \tag{B8}$$

where:
$A=a-c_1$, $B=a-c_2$, $C=2a-c_1$, $D=2a-c_2$,
$\Phi=128+8\gamma^8-71\gamma^6+224\gamma^4-288\gamma^2$, $\Psi=128-7\gamma^6+64\gamma^4-160\gamma^2$,
$\Theta=128+\gamma^6+48\gamma^4-160\gamma^2$, $\Omega=128-47\gamma^6+208\gamma^4-288\gamma^2$,

with Φ, Ψ, Θ and Ω strictly positive over the interval $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$. Moreover, we pose:

$\widehat{\Phi} = 192a^2 - 128Cc_1 - 64Dc_2$, $\widehat{\Psi} = 10a^2 - 7Cc_1 - 3Dc_2$,
$\widehat{\Theta} = 31a^2 - 23Cc_1 - 8Dc_2$, $\widehat{\Omega} = 11a^2 - 8Cc_1 - 3Dc_2$.

Appendix C

Let us denote z $(1-z)$ the probability that firm 1 chooses P (Q) and w $(1-w)$ the probability that firm 2 chooses P (Q). At the mixed strategy equilibria in the intervals $\gamma \in (0, 0.913)$ and $\gamma \in (0.959, 1)$, firm 2's owner chooses to compete in terms of price with probability $w^*(\gamma)$, while firm 1's owner chooses to compete in terms of price with probability $z^*(\gamma)$, where such probability functions are as follows:

$$\begin{aligned} w^*(\gamma) &= \frac{4\Phi\Psi\left(1-\gamma^2\right)}{\tau} \\ z^*(\gamma) &= \frac{\phi\Psi^2\Omega^2\left(1-\gamma^2\right)}{\Upsilon Z\left(2-\gamma^2\right)} \end{aligned}$$

where:
$\Phi = 128 + 8\gamma^8 - 71\gamma^6 + 224\gamma^4 - 288\gamma^2$, $\Psi = 128 - 7\gamma^6 + 64\gamma^4 - 160\gamma^2$,
$\Omega = 128 - 47\gamma^6 + 208\gamma^4 - 288\gamma^2$, $\Upsilon = 128 - 19\gamma^6 + 120\gamma^4 - 224\gamma^2$,
$\tau = 131\,072 + 177\gamma^{16} - 6120\gamma^{14} + 58\,824\gamma^{12} - 274\,560\gamma^{10} + 731\,648\gamma^8 - 1173\,504\gamma^6 + 1122\,304\gamma^4 - 589\,824\gamma^2$
$\phi = 65\,536 - 64\gamma^{16} + 103\gamma^{14} + 6744\gamma^{12} - 57\,072\gamma^{10} + 208\,448\gamma^8 - 412\,160\gamma^6 + 458\,752\gamma^4 - 270\,336\gamma^2$

with Φ, Ψ, Ω, Υ, τ strictly positive over the interval $\gamma \in \left(0, \frac{a-c_2}{a-c_1}\right)$, while ϕ has a zero at $\gamma = 0.959$ and Z is a polynomial of degree 34, the formula of which is omitted for brevity, with a zero at $\gamma = 0.925$ in the given interval.
Probabilities $w^*(\gamma)$ and $z^*(\gamma)$ are depicted in Figure 3a and Figure 3b respectively. Notice continuity of $w^*(\gamma)$, while $z^*(\gamma)$ has an asymptotic behavior around $\gamma = 0.925$.

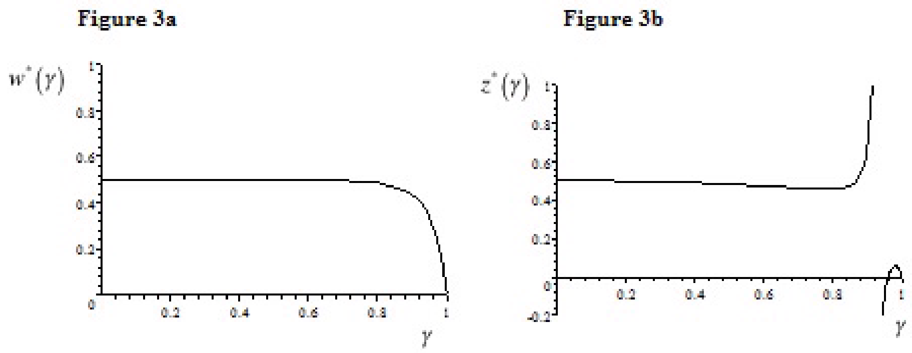

Figure 3 The probability distributions $w^*(\gamma)$ and $z^*(\gamma)$ at the mixed strategy Nash equilibrium.

Moreover, it can be verified that $0 < w^* (\gamma) < 1/2$ in the entire given interval (i.e., firm 1 randomizes its own choice of the strategic variable, provided that firm 2 chooses (P) with probability lower than 1/2). Conversely, $0 \leq z^* (\gamma) < 1/2$ (i.e., firm 2 randomizes its own choice of the strategic variable, provided that firm 1 chooses (P) with probability lower than 1/2), provided that γ lies in the interval of non-existence of an equilibrium in pure strategies, i.e., when $\gamma \in (0, 0.913)$, and in the interval of existence of multiple equilibria in pure strategies, i.e., when $\gamma \in \left(0.959, \frac{a-c_2}{a-c_1}\right)$. In the latter, $z^* (\gamma)$ approaches zero at both the lower bound, at which $\phi = 0$, and the upper bound. The asymptotic pattern of $z^* (\gamma)$, finally, reveals the non existence of a mixed strategy in the interval $\gamma \in (0.913, 0.959)$ where (P) arises as a strictly dominant strategy for firm 2.