

# **Discussion Papers** Collana di E-papers del Dipartimento di Economia e Management – Università di Pisa



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# A product innovation game with managerial delegation

Discussion Paper n. 223 2017

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La presente pubblicazione ottempera agli obblighi previsti dall'art. 1 del decreto legislativo luogotenenziale 31 agosto 1945, n. 660.

Please quote as follows:

Luciano Fanti and Luca Gori (2017), "A product innovation game with managerial delegation", Discussion Papers del Dipartimento di Economia e Management – Università di Pisa, n. 223 (http://www.ec.unipi.it/ricerca/discussion-papers.html).



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This article revisits the works of Lambertini and Rossini (1998) and Bernhofen and Bernhofen (1999) and also extends the analysis to the effects of product innovation in Cournot and Bertrand duopolies with sales delegation.

Keywords Duopoly; Product innovation; Sales delegation

JEL Classification D43; J53; L1

## 1. Introduction

This work contributes to the strand of the industrial organisation literature focused on product differentiation as a device relaxing competition. As is known, investing to reduce product substitutability<sup>1</sup> may be even more important than other related strategies for firms (e.g., the cost-reducing R&D) to cope with the increased competition.<sup>2</sup> Although there exist several articles devoted to showing that (exogenous) product differentiation softens competition, there were a few contributions on the side of the endogenous choice of the extent of product differentiation (Lambertini, 1996; Lambertini and Rossini, 1998; Bernhofen and Bernhofen, 1999; Lorz and Wrede, 2009; Hoefele, 2016). The present article relates more closely to the works of Lambertini and Rossini (1988) and Bernhofen and Bernhofen (1999) (LR and BB henceforth). The main aim of LR is to question whether and how, in a symmetric duopoly with profit maximising (PM) firms, the shape of market competition can affect R&D efforts aimed at reducing product substitutability. The authors show that 1) a prisoner's dilemma at R&D stage may arise due to externalities affecting product innovation, so that firms may be "entrapped" in a competition game with perfect substitutes in both Cournot and Bertrand settings. 2) The less effective innovation investments are, the less likely firms invest in R&D. 3) The likelihood to invest in product differentiation is larger under price competition, thus reversing the established wisdom on the relative incentives of investing in cost-reducing R&D.

The present article revisits this issue by accounting for the separation between ownership and control. Industries with differentiated products, product innovation and managerial delegation are widely observed in actual markets. Since Vickers (1985), the literature on managerial delegation has grown rapidly. However, this stream of literature neglects to account for the strategic use of product innovation. This work aims at filling this gap by considering salesdelegated (S) firms. The main findings are the following. 1) The results of LR with PM-firms are re-examined showing that a prisoner's dilemma with undifferentiated products cannot occur in a Bertrand game. 2) Sales delegation always enhances (resp. reduces) the likelihood of innovation (no innovation) in the case of Cournot (resp. Bertrand) competition.

The rest of the article proceeds as follows. Section 2 briefly sketches the model set up. Section 3 revisits the results of LR with PM-firms. Section 4 studies the case of S-firms. Section 5 concludes.

#### 2. The model set-up

<sup>&</sup>lt;sup>1</sup> In empirical analyses, product innovation is often seen as equivalent to differentiating products.

<sup>&</sup>lt;sup>2</sup> For instance, Scherer and Ross (1990) suggests that three quarters of investment expenditures of US-based firms go into product innovation, and even Japanese firms, which have the lowest share in product innovation, still invested one fifth.

The economy consists of a duopoly where firm i  $(i = \{1,2\})$  produces (a horizontally differentiated) product of variety i. The linear inverse and direct demand functions (Singh and Vives, 1984) of product i  $(i = \{1,2\}, i \neq j)$  are

$$p_i = 1 - q_i - dq_j, \tag{1}$$

and

$$q_i = \frac{1}{1+d} - \frac{p_i}{1-d^2} + \frac{dp_j}{1-d^2},$$
(2)

where  $p_i \ge 0$   $(q_i \ge 0)$  denote firm *i*'s price (resp. quantity) and  $0 \le d \le 1$  is the extent of product substitutability (when d = 1 products are homogeneous). Average and marginal costs are constant and equal to zero so that profits of firm *i* are  $\Pi_i = p_i q_i$ .

By following Vickers (1985), we assume that owner i hires a manager whose pay is  $\omega_i = A_i + B_i u_i \ge 0$ , where  $A_i$  is the fixed salary and  $B_i \ge 0$  is a constant that weights the utility of manager i,  $u_i$ ,<sup>3</sup> which is expressed as:

$$u_i = \Pi_i + b_i q_i \,, \tag{4}$$

where  $b_i$  is the incentive parameter. When  $b_i > 0$  (resp.  $b_i < 0$ ) the owner provides incentives (resp. disincentives) to the manager.

We assume that S-firms non-cooperatively play a three-stage game. Owners choose whether to invest or to do not invest in product innovation (R&D stage) and then set the bonus for the management (contract stage). Managers compete in the product market (market stage). The solution concept is subgame perfection by backward induction.

The strategy set at R&D stage is common knowledge and includes two polar choices: investing a fixed monetary amount (F > 0) or not investing at all (F = 0) in product innovation. We denote this binary strategic choice as I (invest) and NI (not invest). Products are highly heterogeneous ( $d = d^{\circ}$ ) [resp. homogeneous (d = 1)] if both firms invest [resp. do not invest] in R&D. Products are scarcely differentiated ( $d = d^{\circ\circ}$ ) if only one firm invests in R&D ( $0 \le d^{\circ\circ} < 1$ ).

#### 3. LR's results "revisited"

Let us first assume PM  $(b_i = 0)$ . Standard calculations in Cournot (C) and Bertrand (B) models lead to:

$$\Pi_{i,C}^{NI/NI}\Big|_{d=1} = \frac{1}{9}, \quad \Pi_{i,C}^{NI/I} = \frac{1}{(2+d^{\circ\circ})^2},$$
$$\Pi_{i,C}^{I/NI} = \frac{1}{(2+d^{\circ\circ})^2} - F, \quad \Pi_{i,C}^{I/I} = \frac{1}{(2+d^{\circ})^2} - F,$$

and

<sup>&</sup>lt;sup>3</sup> As usual, owners offer a take-it-or-leave-it contract and managers are remunerated at their reserve salary (if the latter is zero then the fixed salary component in manager's compensation will be negative, i.e.  $A_i < 0$ ).

$$\Pi_{i,B}^{NI/NI}\Big|_{d=1} = 0, \quad \Pi_{i,B}^{NI/I} = \frac{1 - d^{\circ\circ}}{(1 + d^{\circ\circ})(2 - d^{\circ\circ})^2},$$
  
$$\Pi_{i,B}^{I/NI} = \frac{1 - d^{\circ\circ}}{(1 + d^{\circ\circ})(2 - d^{\circ\circ})^2} - F, \quad \Pi_{i,B}^{I/I} = \frac{1 - d^{\circ}}{(1 + d^{\circ})(2 - d^{\circ})^2} - F.$$
  
$$\Pi_{i,B}^{I/NI} = \frac{1 - d^{\circ\circ}}{(1 + d^{\circ\circ})(2 - d^{\circ\circ})^2} - F, \quad \Pi_{i,B}^{I/I} = \frac{1 - d^{\circ\circ}}{(1 + d^{\circ})(2 - d^{\circ})^2} - F.$$

Let  $\Delta_1^k := \Pi_i^{I/NI} - \Pi_i^{NI/NI}$ ,  $\Delta_2^k := \Pi_i^{NI/I} - \Pi_i^{I/I}$ ,  $\Delta_3^k := \Pi_i^{NI/NI} - \Pi_i^{I/I}$  be profit differentials and  $F_x^k \Rightarrow \Delta_x^k = 0$  the corresponding threshold curves  $(i = \{1, 2\}, x = \{1, 2, 3\}, k = \{C, B\})$ . In particular,

$$\begin{split} F_1^C &\coloneqq \frac{(5+d^{\circ\circ})(1-d^{\circ\circ})}{9(2+d^{\circ\circ})^2} > 0 \,, \\ F_2^C &\coloneqq \frac{(d^{\circ\circ}-d^{\circ})(4+d^{\circ}+d^{\circ\circ})}{(2+d^{\circ\circ})^2(2+d^{\circ})^2} > 0 \,, \\ F_3^C &\coloneqq \frac{(5+d^{\circ})(1-d^{\circ})}{9(2+d^{\circ})^2} > 0 \,, \\ F_1^B &\coloneqq \frac{1-d^{\circ\circ}}{(1+d^{\circ\circ})(2-d^{\circ\circ})^2} > 0 \,, \\ F_2^B &\coloneqq \frac{(d^{\circ\circ}-d^{\circ})[4-3(d^{\circ\circ}+d^{\circ})+(d^{\circ})^2(1-d^{\circ\circ})+(d^{\circ\circ})^2(1-d^{\circ})]}{(1+d^{\circ})(2-d^{\circ\circ})^2(1+d^{\circ\circ})(2-d^{\circ\circ})^2} > 0 \,, \end{split}$$

and

$$F_3^B := \frac{1 - d^\circ}{(1 + d^\circ)(2 - d^\circ)^2} > 0.$$

Then,

**Lemma 1**. (a) [Cournot]. There exist five regimes in  $(d^{\circ\circ}, F)$  space for any  $d^{\circ}$ .

I.  $\Delta_1^C < 0, \Delta_2^C > 0, \Delta_3^C < 0$ . One (Pareto inefficient) Nash equilibrium (NI,NI). II.  $\Delta_1^C < 0, \Delta_2^C < 0, \Delta_3^C < 0$ . Two Nash equilibria (NI,NI) and (I,I). I payoff dominates NI.

III.  $\Delta_1^C > 0, \Delta_2^C < 0, \Delta_3^C < 0$ . One (Pareto efficient) Nash equilibrium (I,I).

IV.  $\Delta_1^C > 0, \Delta_2^C > 0, \Delta_3^C < 0$ . Two (Pareto efficient) asymmetric Nash equilibria (NI,I) and (I,NI).

V.  $\Delta_1^C < 0, \Delta_2^C > 0, \Delta_3^C > 0$ . One (Pareto efficient) Nash equilibrium (NI,NI).

(b) [Bertrand]. Only regimes III  $(\Delta_1^B > 0, \Delta_2^B < 0, \Delta_3^B < 0)$  and IV  $(\Delta_1^B > 0, \Delta_2^B > 0, \Delta_3^B < 0)$  exist.

**Proof.** Part a) is proved by looking at the sign of profit differentials. Part b) holds as  $\Pi_{i,B}^{NI/NI} = 0$  and  $F_1^B = \Pi_{i,B}^{NI/NI}$ . Then, if  $F_1^B$  were crossed by  $F_2^B$  in  $(d^{\circ\circ}, F)$  space it would mean that the latter curve would enter a region where deviating from I would give negative profits unilaterally. Therefore, (NI,NI) can never emerge in a Bertrand game. **Q.E.D.** 

**Remark 1**. [Cournot]. LR identify with region I+V the likelihood of a prisoner's dilemma. However, when F is sufficiently high (NI,NI) becomes Pareto

efficient (this was already pointed out by BB). In such a case, in fact, an increase in product differentiation does not allow to get an adequate competitive advantage (as the effectiveness of the R&D investment is low) compared to the disadvantage of investing unilaterally in product innovation.

As LR do not account for region V, in Proposition 1 they state "The likelihood of a prisoner's dilemma arising in the R&D stage increases as the effectiveness of investment decreases." (LR, 1998, p. 300). However, by a simple inspection of Figure 1 ( $d^\circ = 0$ ) and Figure 2 ( $d^\circ = 0.5$ ) it is clear that although the area I+V increases when the effectiveness of R&D investments decreases, the likelihood of a prisoner's dilemma correspondingly reduces (region I).



**Figure 1**. Cournot (PM). Profit differentials in  $(d^{\circ\circ}, F)$  space  $(d^{\circ} = 0)$ .



**Figure 2**. Cournot (PM). Profit differentials in  $(d^{\circ\circ}, F)$  space  $(d^{\circ} = 0.5)$ .

**Remark 2**. [Bertrand]. As only regimes III and IV exist (Figure 3), (NI,NI) does never emerge and no prisoner's dilemma arises. In addition, the threshold curves cannot intersect (part (b) of Lemma 1) and thus Figure 1 in LR's work and Figure 1 in BB's work do not hold in a Bertrand setting.

Therefore, Cournot and Bertrand models are not equivalent with regard to the occurrence of a prisoner's dilemma. However, the likelihood of the "good" equilibrium (I,I) is still larger under Bertrand competition.



**Figure 3**. Bertrand (PM). Profit differentials in  $(d^{\circ\circ}, F)$  space  $(d^{\circ} = 0)$ .

Define

$$D_1 := F_1^C - F_1^B = \frac{(1 - d^{\circ \circ})[(d^{\circ \circ})^4 + 2(d^{\circ \circ})^3 - 24(d^{\circ \circ})^2 - 32d^{\circ \circ} - 16]}{9(1 + d^{\circ \circ})(2 + d^{\circ \circ})^2(2 - d^{\circ \circ})^2} < 0,$$

and

$$D_{2} := F_{2}^{C} - F_{2}^{B} = \frac{2(d^{\circ\circ} - d^{\circ})}{(1 + d^{\circ})(1 + d^{\circ\circ})(2 + d^{\circ})^{2}(2 + d^{\circ\circ})^{2}(2 - d^{\circ})^{2}(2 - d^{\circ\circ})^{2}} \times \left\{ -16[d^{\circ}(d^{\circ\circ})^{2} + (d^{\circ})^{2}d^{\circ\circ} + (d^{\circ\circ})^{2} + d^{\circ}d^{\circ\circ} + (d^{\circ})^{2}] + (d^{\circ})^{2}(d^{\circ\circ})^{2}[d^{\circ}(d^{\circ\circ})^{2} + (d^{\circ})^{2}d^{\circ\circ} + d^{\circ}d^{\circ\circ} + 8] \right\} < 0$$

Then,

**Proposition 1**. Under PM, the likelihood of (I,I) is larger higher under Bertrand competition than Cournot competition.

**Proof**. As  $D_1 < 0$  and  $D_2 < 0$ , the result follows. **Q.E.D.** 

# 4. Sales delegation (S)

Profit functions under S are:

$${}^{S}\Pi_{i,C}^{NI/NI}\Big|_{d=1} = \frac{2}{25}, \quad {}^{S}\Pi_{i,C}^{NI/I} = \frac{2[2 - (d^{\circ\circ})^{2}]}{[4 + d^{\circ\circ}(2 - d^{\circ\circ})]^{2}},$$
  
$${}^{S}\Pi_{i,C}^{I/NI} = \frac{2[2 - (d^{\circ\circ})^{2}]}{[4 + d^{\circ\circ}(2 - d^{\circ\circ})]^{2}} - F, \quad {}^{S}\Pi_{i,C}^{I/I} = \frac{2[2 - (d^{\circ})^{2}]}{[4 + d^{\circ}(2 - d^{\circ})]^{2}} - F,$$

and

$${}^{S}\Pi_{i,B}^{NI/NI}\Big|_{d=1} = 0, \quad {}^{S}\Pi_{i,B}^{NI/I} = \frac{2(1-d^{\circ\circ})[2-(d^{\circ\circ})^{2}]}{(1+d^{\circ\circ})[4-d^{\circ\circ}(2+d^{\circ\circ})]^{2}},$$
  
$${}^{S}\Pi_{i,B}^{I/NI} = \frac{2(1-d^{\circ\circ})[2-(d^{\circ\circ})^{2}]}{(1+d^{\circ\circ})[4-d^{\circ\circ}(2+d^{\circ\circ})]^{2}} - F, \quad {}^{S}\Pi_{i,B}^{I/I} = \frac{2(1-d^{\circ})[2-(d^{\circ})^{2}]}{(1+d^{\circ})[4-d^{\circ}(2+d^{\circ})]^{2}} - F.$$

Figure 4 (Cournot) and Figure 5 (Bertrand) show that Lemma 1 holds also under S. The threshold curves  ${}^{s}F_{x}^{k} \Rightarrow {}^{s}\Delta_{x}^{k} = 0$  ( $x = \{1,2,3\}$ ,  $k = \{C,B\}$ ) are not reported to save space but are available on request.



**Figure 4**. Cournot (S). Profit differentials in  $(d^{\circ\circ}, F)$  space  $(d^{\circ} = 0)$ .



**Figure 5**. Bertrand (S). Profit differentials in  $(d^{\circ\circ}, F)$  space  $(d^{\circ}=0)$ .

**Lemma 2**. Under S, the likelihood of (I,I) is larger under Bertrand competition than Cournot competition.

**Proof.** As  ${}^{s}D_{1} := {}^{s}F_{1}^{c} - {}^{s}F_{1}^{B} < 0$  and  ${}^{s}D_{2} := {}^{s}F_{2}^{c} - {}^{s}F_{2}^{B} < 0$ , the result follows. **Q.E.D.** 

Let us now study the effects of S (compared to PM) on product innovation by considering Cournot and Bertrand duopolies separately. Define

$$Z_1^C := {}^{S}F_1^C - F_1^C = \frac{(1 - d^{\circ\circ})[448 + 1344d^{\circ\circ} + 1792(d^{\circ\circ})^2 + 780(d^{\circ\circ})^3 - 7(d^{\circ\circ})^4 - 7(d^{\circ\circ})^5]}{225(2 + d^{\circ\circ})^2[4 + d^{\circ\circ}(2 - d^{\circ\circ})]^2} > 0,$$

$$Z_{2}^{C} := {}^{S}F_{2}^{C} - F_{2}^{C} = \frac{(d^{\circ\circ} - d^{\circ})}{(2 + d^{\circ})^{2}(2 + d^{\circ\circ})^{2}[4 + d^{\circ}(2 - d^{\circ})]^{2}[4 + d^{\circ\circ}(2 - d^{\circ\circ})]^{2}} \times \left\{ 16(d^{\circ\circ})^{3}(d^{\circ})^{3} + 192(d^{\circ\circ})^{3}(d^{\circ})^{2} + 192(d^{\circ\circ})^{2}(d^{\circ})^{3} + 384(d^{\circ\circ})^{3}d^{\circ} + 640(d^{\circ\circ})^{2}(d^{\circ})^{2} + 884d^{\circ\circ}(d^{\circ})^{3} + 192(d^{\circ\circ})^{3} + 704(d^{\circ\circ})^{2}d^{\circ} + 704d^{\circ\circ}(d^{\circ})^{2} + 192(d^{\circ})^{3} + 256(d^{\circ\circ})^{2} + 256d^{\circ\circ}d^{\circ} + 256(d^{\circ})^{2} - 3(d^{\circ\circ})^{5}(d^{\circ})^{4} - 3(d^{\circ\circ})^{4}(d^{\circ})^{5} - 4(d^{\circ\circ})^{5}(d^{\circ})^{3} - 4(d^{\circ\circ})^{4}(d^{\circ})^{4} - 4(d^{\circ\circ})^{3}(d^{\circ})^{5} \right\} > 0$$

$$Z_{3}^{C} := {}^{S}F_{3}^{C} - F_{3}^{C} = \frac{(1 - d^{\circ})[448 + 1344d^{\circ} + 1792(d^{\circ})^{2} + 780(d^{\circ})^{3} - 7(d^{\circ})^{4} - 7(d^{\circ})^{5}]}{225(2 + d^{\circ})^{2}[4 + d^{\circ}(2 - d^{\circ})]^{2}} > 0$$

$$Z_1^B := {}^{S}F_1^B - F_1^B = \frac{(d^{\circ\circ})^3(1 - d^{\circ\circ})(4 - 3d^{\circ\circ})}{(1 + d^{\circ\circ})(2 - d^{\circ\circ})^2[4 + d^{\circ\circ}(2 - d^{\circ\circ})]^2} > 0,$$

$$\begin{split} Z_2^B &:= {}^SF_2^B - F_2^B = \frac{(d^{\circ\circ} - d^{\circ})}{(1 + d^{\circ})(1 + d^{\circ\circ})(2 - d^{\circ})^2(2 - d^{\circ\circ})^2[4 + d^{\circ}(2 - d^{\circ})]^2[4 + d^{\circ\circ}(2 - d^{\circ\circ})]^2} \times \\ &\times \left\{ 3(d^{\circ\circ})^6(d^{\circ})^5 + 3(d^{\circ\circ})^5(d^{\circ})^6 - 7(d^{\circ\circ})^6(d^{\circ})^5 - 4(d^{\circ\circ})^5(d^{\circ})^5 - 7(d^{\circ\circ})^4(d^{\circ})^6 + 4(d^{\circ\circ})^6(d^{\circ})^3 - 3(d^{\circ\circ})^5(d^{\circ})^4 - 3(d^{\circ\circ})^4(d^{\circ})^5 + 4(d^{\circ\circ})^3(d^{\circ})^6 + 4(d^{\circ\circ})^5(d^{\circ})^3 + 4(d^{\circ\circ})^3(d^{\circ})^5 + 116(d^{\circ\circ})^4(d^{\circ})^4 - 304(d^{\circ\circ})^4(d^{\circ})^3 - 304(d^{\circ\circ})^3(d^{\circ})^4 + 192(d^{\circ\circ})^4(d^{\circ})^2 + 752(d^{\circ\circ})^3(d^{\circ})^3 + 192(d^{\circ\circ})^2(d^{\circ})^4 + 192(d^{\circ\circ})^4d^{\circ} - 256(d^{\circ\circ})^3(d^{\circ})^2 - 256(d^{\circ\circ})^2(d^{\circ})^3 - 192(d^{\circ\circ})^4 + 448(d^{\circ\circ})^3 + 448(d^{\circ\circ})^3 - 384(d^{\circ\circ})^2(d^{\circ})^2 - 640d^{\circ\circ}(d^{\circ})^3 - 192(d^{\circ})^4 + 448(d^{\circ\circ})^3 + 448(d^{\circ\circ})^3 - 256(d^{\circ\circ})^2 - 256(d^{\circ\circ})^2 d^{\circ} + 704(d^{\circ\circ})(d^{\circ})^2 \right\} \right\}$$

and

$$Z_3^B := {}^{S}F_3^B - F_3^B = \frac{(d^{\circ})^3(1-d^{\circ})(4-3d^{\circ})}{(1+d^{\circ})(2-d^{\circ})^2[4+d^{\circ}(2-d^{\circ})]^2} > 0.$$

Then,

**Proposition 2**. [Cournot]. S reduces (resp. increases) the likelihood to do not innovate (resp. to innovate).

**Proof.** As  $Z_1^C, Z_2^C > 0$  and knowing that the total area of feasible choices (for a given F > 0) is smaller under S than PM,<sup>4</sup> then area I+V (resp. III) is smaller (resp. larger) under S. **Q.E.D.** 

**Corollary 1**. [Cournot]. As  $Z_3^C > 0$ , then the area in which (NI,NI) is Pareto inefficient is smaller under S than PM. Given Proposition 2, the area of a prisoner's dilemma under S can be smaller or larger than under PM.

**Result 1**. [Bertrand]. S increases the likelihood of investing in product innovation.<sup>5</sup>

As  $Z_1^B > 0$  for any  $0 \le d^\circ < d^{\circ\circ} < 1$  and  $Z_2^B < 0$  for any  $0 \le d^\circ < d^{\circ\circ} < 1$  when  $d^{\circ\circ}$  is sufficiently small or  $Z_2^B \le 0$  if  $d^\circ \le d^* < d^{\circ\circ}$  when  $d^{\circ\circ}$  is sufficiently large,<sup>6</sup> then when R&D investments are highly effective what happens to the size area III is a priori ambiguous. Nevertheless, even in the polar case  $d^\circ = 0$  (maximal effectiveness) area III reduces as the upward shift in  $Z_1^B$  always exceeds the downward shift in  $Z_2^B$  (see Example 1 and Figure 6). Given also that the area of feasible choices for a given F > 0 is increased, Result 1 follows.

**Example 1**. To evaluate the likelihood of (I,I) in  $(d^{\circ\circ}, F)$  space  $(d^\circ = 0)$  in a Bertrand game under S and PM, we compute the corresponding area III as follows:

$${}^{S}A_{III}^{B} := \int_{0}^{0.672} ({}^{S}F_{2}^{B} + F) dd^{\circ\circ} + \int_{0.672}^{1} ({}^{S}F_{1}^{B} + F) dd^{\circ\circ} = 0.038833 + 0.027981 = 0.070958,$$

and

$$A_{III}^{B} := \int_{0}^{0.609} (F_{2}^{B} + F) dd^{\circ\circ} + \int_{0.609}^{1} (F_{1}^{B} + F) dd^{\circ\circ} = 0.044657 + 0.026300 = 0.066814$$

<sup>&</sup>lt;sup>4</sup> A well-established result since Vickers (1985) is that profits under S are smaller (resp. larger) than under PM in Cournot (resp. Bertrand) competition. Thus, the area of feasible choices for a given F > 0 is correspondingly reduced (resp. increased).

<sup>&</sup>lt;sup>5</sup> Since the area of feasible choices for a given F > 0 is increased under Bertrand competition, then the area of (I,I) and the area of (I,NI) and (NI,I) are enlarged by S.

<sup>&</sup>lt;sup>6</sup> The analytical expression of  $d^*$  is not tractable. However, numerical simulations show that  $d^* \in [0, 0.862]$  when  $d^{\circ\circ}$  correspondingly belongs to range [1, 0.87].



**Figure 6**. Bertrand. Area III: S (red) versus PM (black) in  $(d^{\circ\circ}, F)$  space  $(d^{\circ} = 0)$ .

## **5.** Conclusions

This article analysed a product innovation game in Cournot and Bertrand duopolies with S-firms. After reviewing the case of PM-firms early analysed by LR and BB, the work showed that S 1) enhances (resp. reduces) the likelihood to innovate (resp. to do not innovate) in a quantity-setting context, 2) enhances the likelihood to innovate in a price-setting context.

Although in a Cournot game S ends up with a pro-competitive effect at equilibrium, it also causes an anti-competitive outcome as it makes firms more prone to invest in product innovation. In a Bertrand game S is per se pro-collusive: it becomes even more pro-collusive to the extent that it favours investments in product differentiation (i.e., in a Bertrand game with R&D, S is a win-win result for owners).

**Conflict of Interest** The authors declare that they have no conflict of interest.

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