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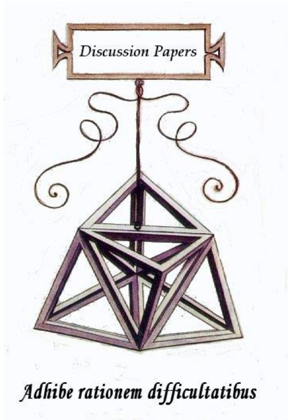

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Local Environmental Quality and Heterogeneity in an OLG Agent-Based Model with Network Externalities

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Keywords: Agent-Based Models; Overlapping Generations; Local Environment; Network Externalities.

JEL: C63; D62; O13; Q2.

Local Environmental Quality and Heterogeneity in a OLG Agent-Based Model with Network Externalities

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Abstract

Most of the theoretical contributions on the relationship between economy and environment assumes the environment as a good distributed homogeneously among the agents. The aim of this work is to weaken this hypothesis and to consider that the environment can have a local character even if conditioned through externalities by the choices made at global level. In particular, adapting the classical framework introduced in John and Pecchenino (1994) to analyze the dynamic relationship between environment and economic process, in this paper we propose an OLG agent-based model where the agents may have different initial environmental endowments, or may be heterogeneous in their preferences. What emerges is that, despite the attention devoted to local environmental aspects, the network externalities (determined through the scheme of Moore neighbourhoods) play a fundamental role in defining environmental dynamics and they may induce the emergence of chaotic dynamics. On the other hand, the heterogeneity of preferences and/or initial conditions plays an ambiguous role. In fact, depending on the weight of network externalities and the impact of consumption and/or defensive expenditures, heterogeneity may stabilize or destabilize the system.

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1 Introduction

In recent years environmental issues have assumed an increasing relevance both in the sphere of economic research and in political agenda. From a theoretical point of view, what emerges is the presence of modelling approaches characterized by profound differences. On the one hand, there is the traditional vision, in which the environmental problem is solved centrally by a benevolent social planner. Leaving aside the various specific differences within these works, what characterizes this approach is that the social planner has a perfect knowledge of the evolution (or of the evolutionary mechanism) of the environment, and he can optimally allocate resources between economic activity and environmental protection (see Van Der Ploeg and Withagen, 1991). Clearly, the resulting allocation (or path, in dynamic models) is Pareto efficient. This kind of result, even if extremely important from a regulatory point of view, characterizing the "what it should be" (Brock and Xepapadeas, 2010), clashes with the problem of defining a set of incentives, if any, such as to generate this allocation on the market where, given their wealth and constraints, agents can decide autonomously. From a different theoretical point of view, a part of the research has focused on studying the decentralized solution of such a problem in which individuals do not consider the possibility to tackle directly environmental quality, and therefore, due to lack of adequate investment, environmental quality stands out (or evolves) at a not optimal level (the so called *tragedy of commons*). What it turns out is that economic agents, in order to react to environmental degradation, tend to make self-protective choices that can exacerbate environmental problems (see Antoci et al., 2007, 2019).[1]

While this analysis allows capturing many stylized facts, it is also true that there are individual behaviours that are not compatible with this approach. Indeed, empirical studies show that several individuals seem to internalize environmental problems, adopting environmentally friendly consumption styles (see Bodur and Sarigöllü, 2005; Morrison and Beer, 2017). From a theoretical point, this is a puzzle. Why do agents internalize environmental problems overcoming the free-rinding problem? For example Prieur and Bréchet (2013) try to answer this question by assuming that environmental awareness is positively correlated to human capital, but the correlation between education and eco-sustainable behaviour does not seem very strong (see Kinda, 2010). Furthermore, if the initial and direct effect of the emergence of a generalized environmental awareness is positive for the environmental problem, an overestimation of this phenomenon can have dramatic consequences. Indeed, this can lead policymakers to downgrade the environmental problem to a secondary problem that can be managed in a decentralized way through the market. But what could happen if this policy is implemented? The questions we pose, albeit in a very abstract and theoretical context, are the following: is it possible to identify a bottom-up mechanism that motivates agents to deal with environmental problems? If so, what is the role of the initial environmental endowments? What happens if the agents are heterogeneous? What kind of dynamics are generated?

In order to face these problems, we have adopted and partially adapted the model structure proposed by John and Pecchenino (1994), in the specification studied by Zhang (1999). In particular, we consider an overlapping generations model in which agents live 2 periods and have to make decisions on how to use their wage between 2 alternative activities: saving to consume a private

[1] This phenomenon is called as *maladaptation*. Barnett and O'Neill (2010) introduces the definition of maladaptation by describing that series of actions aimed at apparently reducing environmental impact but which end up generating further negative effects on the environment.

good when they are old or spending for the environment. Differently from the original framework, we assume that decisions are not made by a representative agent (actually in the John and Pecchenino framework the allocative problem is solved by a short lived social planner) who internalizes intra-temporal externalities, but are made by individual agents. As already noted in Naimzada and Sodini (2010), this approach tends, for a sufficiently large number of individuals, to generate a solution in which the agents decide to do not contribute for the maintenance or improvement of the environmental quality (free riding). In order to overcome this modelling problem, in partial agreement with the approach used by La Torre et al. (2019), in this paper we assume that the agents live, without the possibility of migration, on a cell of a two-dimensional grid and make decisions based on the environmental level that is experienced at that cell. In other words, with this approach, we move from considering the environment as a good distributed homogeneously among the agents to being instead a multidimensional reality that assumes, potentially, different values for each agent. The underlying economic idea is that the decisions on environmental protection taken by an individual living in a very polluted place are certainly very different from those made by an individual living in a tropical paradise. If this is what happens locally, what makes the problem non-trivial is that the environmental and consumption choices of nearby agents affect each other through externalities that cross the boundaries of the cell (a concept similar to the transboundary externalities introduced in La Torre et al. (2019), but studied among the agents). Specifically, we analyze an overlapping generations agent-based model with local interactions. The structure of the model makes it impossible to obtain results in the form of theorems. However, starting from particular cases in which we are able to characterize the properties of the dynamic system generated by the model and through some numerical simulations, what emerges is that, given the externalities between the cells, the decentralized solution is not Pareto efficient, not even from the point of view of the single generation. Furthermore, the decentralized decision system creates conditions prone to the emergence of chaotic dynamics: agents have to react to changes induced by other decision makers who live in cells characterized by a different environmental quality. From an economic point of view, this means that a decentralized system of decisions on environmental issues is difficult to be analyzed in terms of environmental dynamics. The initial environmental conditions, even in the presence of homogeneous agents, lead to differentiated dynamics for economic agents (then, *history matters*). Indeed, the decentralised decision-making system is unable to exploit the interconnection between the economy and the environment. On the other hand, from a dynamic point of view, heterogeneity in preferences and/or initial conditions play an ambiguous role. In fact, depending on the extent of the interactions and the impact of consumption and/or defensive expenditures, heterogeneity may stabilize or destabilize the system: concerning the environmental issue, whose dynamics are not perfectly foreseen (we assume that agents form their expectations on the behaviour of their neighbours in a *naive* form), the presence of several nonlinearities in the decision problem and multiple interactions make the results complex to be characterized.

The remainder of the paper is organized as follows: Section 2 describes the structure of the OLG agent-based model; Section 3 provides and discuss the results of simulations; finally, Section 4 concludes.

2 The model

We consider an overlapping generations economy where two generations, the *young* and the *old*, coexist at every discrete time period $t = 1, ..., +\infty$.[2] The number of economic agents belonging to each generation is assumed to be constant and fixed to N^2. In particular, each agent lives, without the possibility of migration[3], in a cell of a $N \times N$ lattice L and we indicate him as agent $\{i, j\}$ where $i = 1..N, j = 1..N$. In the specification of the model, we further assume that L is wrapped to create a *torus* structure within which it is possible to define a neighbourhood for every cell.[4]

By following John and Pecchenino (1994) and Naimzada and Sodini (2010), every agent $\{i, j\}$ born at time t has preferences towards the consumption in the old age, $c_{t+1}^{i,j}$, and a positive index of the local environmental quality in the old age, $E_{t+1}^{i,j}$. Then, we assume the following logarithmic utility function

$$U^{i,j}(c_{t+1}^{i,j}, E_{t+1}^{i,j}) = \ln c_{t+1}^{i,j} + \eta^{i,j} \ln E_{t+1}^{i,j} \tag{1}$$

where the parameter $\eta^{i,j}$ represents the positive individual elasticity of substitution between consumption and environmental good. During his youth, each agent $\{i, j\}$ supplies inelastically his time endowment (which is normalized to 1) to the productive sector receiving a wage w_t.[5] Then, the individual in the cell $\{i, j\}$ on the lattice allocates the wage between savings $s_t^{i,j}$ for old age consumption $c_{t+1}^{i,j}$ and environmental defensive expenditures $m_t^{i,j}$, intended to improve environmental quality at $t + 1$.

The consumption good is produced by Z identical firms which operate competitively. Then, the output Y is produced according to the Cobb-Douglas technology

$$Y = Af(K_t) = A\, K_t^{\alpha}(N^2)^{1-\alpha} \tag{2}$$

where K_t is the physical capital, A is a positive parameter representing the technological progress and $\alpha \in (0, 1)$ represents the elasticity of capital. It follows that the (homogenous) capital per worker is defined as $k_t = \frac{K_t}{N^2}$.

Concerning the index of local environmental quality, we assume that each agent

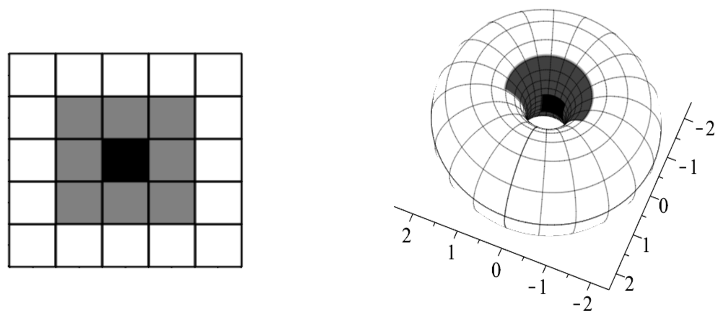
[2] At $t = 0$ there exists a unique generation of young agents.

[3] Migration is a topic closely related to the environmental issues we deal with (see, for example Marchiori and Schumacher, 2011) and has also been included in several ABMs, such as the well-known *Sugarscape Model* proposed by Epstein and Axtell (1996). However, in the present model, in order to avoid an excessively complicate analytical structure, we assume that the life of the agent unfolds entirely in the cell in which he was born.

[4] The torus avoids the problem of considering neighbourhoods for the cells at the boundary of the grid.

[5] The wage is assumed to be homogeneous among agents.

receives externalities from the actions of his contemporaries positioned on his *immediate* neighbourhood. Specifically, we use a *Moore neighbourhoods* scheme (see Shiflet and Shiflet, 2014), in which any neighbourhood is composed by nine cells: the central one indexed with $\{i, j\}$ and the eight cells which surround it (see Figure 1).



(a) (b)

Figure 1: (a) A sketch of the Moore neighbourhood scheme. The black cell in the grid represents the $\{i, j\}$ agent, while the dark grey cells describe the boundary of the neighbourhood, that is agents $\{l, v\}$ for every $l = i-1, ..., i+1$ and $v = j-1, ..., j+1$ with $\{l, v\} \neq \{i, j\}$. The white cells define agents that do not belong in the immediate neighbourhood of the agent in the black cell. (b) Folding of L and view of Moore neighborhood in the torus structure.

Following the specification employed in John and Pecchenino (1994) and Zhang (1999), the index of *local* environmental quality evolves according to

$$E_{t+1}^{i,j} = (1-b)E_t^{i,j} + b\overline{E}^{i,j} + \left[\gamma^{i,j}\, m_t^{i,j} + \sum_{l=i-1}^{i+1} \sum_{v=j-1}^{j+1} \gamma_o^{l,v}\, m_t^{l,v}\right] - \left[\beta^{i,j}\, c_t^{i,j} + \sum_{l=i-1}^{i+1} \sum_{v=j-1}^{j+1} \beta_o^{l,v}\, c_t^{l,v}\right] \tag{3}$$

with $\{l, v\} \neq \{i, j\}$. The parameter $\overline{E}^{i,j} > 0$ represents the value towards the index tends when consumption and environmental expenditures are null, $b \in [0, 1]$ measures the speed of such an adjustment and $E_t^{i,j} > 0$ is the current level of the index.[6] The terms enclosed in the square brackets concern instead the impact of agents decisions on the future level of environmental quality experienced in the cell $\{i, j\}$: (i) $\gamma^{i,j}\, m_t^{i,j}$ and $\beta^{i,j}\, c_t^{i,j}$ measure, respectively, the impact of environmental contribution of agent $\{i, j\}$ and the degradation

[6]For the sake of clarity, the term $E_t^{i,j}$ could be interpreted as (i) the index suggesting the current level of environmental amenities or as (ii) the stock of the free access environmental good at t.

provoked by his comsumption choices; (ii) $\sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1}\gamma_o^{l,v}\, m_t^{l,v}$ defines the aggregate environmental improvement on the produced by defensive choices of $\{i,j\}$'s neighbors while $\sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1}\beta_o^{l,v}\, c_t^{l,v}$ represents the aggregate environmental degradation caused by their consumption choices. Assuming that one's choices always have a higher impact on one's local environment than others', we consider both $\gamma^{i,j} \geq \gamma_o^{l,v}$ and $\beta^{i,j} \geq \beta_o^{l,v}$ for every $\{i,j\}$ and $\{l,v\} \neq \{i,j\}$. Furthermore, we assume that the individual saving $s_t^{i,j}$ gives rise to the gross return $(1+r_{t+1}-\delta)$ where r_{t+1} is the real interest rate and δ is the depreciation rate of capital. Then, each individual $\{i,j\}$ faces the following life-cycle budget constraints:

$$c_{t+1}^{i,j} = (1 + r_{t+1} - \delta)s_t^{i,j}; \tag{4}$$

$$w_t^{i,j} = s_t^{i,j} + m_t^{i,j}; \tag{5}$$

$$c_{t+1}^{i,j} > 0, m_t^{i,j}, s_t^{i,j} \geq 0. \tag{6}$$

At each period t, firms maximize their profit, and then the following equilibrium equations for wage and interest rate are obtained:

$$w_t = A(1-\alpha)k_t^{\alpha}; \tag{7}$$

$$r_t = A\alpha\, k_t^{\alpha-1}. \tag{8}$$

In order to make his own decisions in terms of $m_t^{i,j}$ and $s_t^{i,j}$, each agent $\{i,j\}$ in L takes w_t and r_{t+1} as given. In addition, at the beginning of t, agent $\{i,j\}$ observes his local environmental quality level $E_t^{i,j}$ and the aggregate consumption in his neighbourhood $\sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1} c_t^{l,v}$ and makes *expectations* on the environmental defensive expenditures of his eight neighbors $m_t^{l,v}$ (with $l = i-1, ..., i+1$ and $v = j-1, ..., j+1$, $\{l,v\} \neq \{i,j\}$). Therefore, individual $\{i,j\}$ maximizes the objective function

$$U_t^{i,j}(c_{t+1}^{i,j}, E_{t+1}^{e,i,j}) \tag{9}$$

under constraints (4)-(5)-(6), with respect to the choice variables $s_t^{i,j}$, $m_t^{i,j}$. The expected future environmental quality at $t+1$ for the agent $\{i,j\}$ is given by

$$E_{t+1}^{e,i,j} = (1-b)E_t^{i,j} + b\overline{E}^{i,j} + \left[\gamma^{i,j}\, m_t^{i,j} + \sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1}\gamma_o^{l,v}\,(m_t^{l,v})^e\right] - \left[\beta^{i,j}\, c_t^{i,j} - \sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1}\beta_o^{l,v}\, c_t^{l,v}\right], \tag{10}$$

with $\{l,v\} \neq \{i,j\}$.[7] The term $\sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1}\gamma_o^{l,v}\,(m_t^{l,v})^e$ refers to the aggregate environemntal improvement triggered by the expectations of the agent

[7]Since (i) the consumption of agents in old age is generated by the saving defined at time $t-1$ (i.e. an observable historical data at time t) and (ii) the real interest rate is given for agents, we assume that the consumption of old individuals in t is known to young agents.

$\{i,j\}$ on the defensive expenditures of his neighbours. In particular, we assume that every agent expects his neighbours continue to behave as in the previous period (*naive expectations*), that is $(m_t^{l,v})^e = m_{t-1}^{l,v}$ for every $l = i-1,..,i+1$, $v = j-1,..,j+1$, $\{l,v\} \neq \{i,j\}$ and for every $t \geq 1$.[8]
As suggested by the condition in (6), we allow for the possibility that each agent $\{i,j\}$ decides not to be involved in environmental maintenance, i.e. we consider the corner solution of the agent's maximization problem $m_t^{i,j} = 0$. The interior solution of the problem is instead characterized by the first order condition

$$-U_1^{i,j}(c_{t+1}, E_{t+1}^{e,i,j})(1+r_{t+1}-\delta)+\gamma^{i,j}U_2^{i,j}(c_{t+1}^{i,j}, E_{t+1}^{e,i,j}) = 0 \qquad i = 1,..,N;\ j = 1,..,N. \tag{11}$$

Considering the occurrence of corner solutions and the condition in (11), we derive the following individual optimal choices $(s_t^{i,j})^*$ and $(m_t^{i,j})^*$

$$\begin{aligned}(s_t^{i,j})^* &= \min\left(\frac{E_{t+1}^{e,i,j}}{\gamma^{i,j}\eta^{i,j}}, w_t\right) = \\ &= \min\left(\frac{(1-b)E_t^{i,j} + b\overline{E}^{i,j} + \gamma_{i,j}\, w_t + \Lambda - \beta^{i,j}\,(c_t^{i,j})^* - \Psi}{\gamma^{i,j}(\eta^{i,j}+1)}, w_t\right);^{9}\end{aligned} \tag{12}$$

$$(m_t^{i,j})^* = \max\left(0, w_t - (s_t^{i,j})^*\right); \tag{13}$$

where $(c_t^{i,j})^* = (1 + r_{t+1} - \delta)(s_{t-1}^{i,j})^*$ for every $i = 1,...,N; j = 1,...,N$, $\Lambda = \sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1}\gamma_o^{l,v}\,(m_{t-1}^{l,v})^*$ and $\Psi = \sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1}\beta_o^{l,v}\,(c_t^{l,v})^*$, with $\{l,v\} \neq \{i,j\}$.

In the young age, every agent supplies his saving $s_t^{i,j}$ to firms to be invested in physical capital. Then, the capital accumulation dynamics of the entire economy is defined by

$$N^2\, k_{t+1} = \sum_{i=1}^{N}\sum_{j=1}^{N} s_t^{i,j}. \tag{14}$$

Recalling the the solutions of the maximization problem for the agent $\{i,j\}$ in (12)-(13), the equilibrium expressions for wage and real interest rate in (7)-(8) and equations (3)-(14), then the equilibrium dynamics for every $\{i,j\}$ are defined by the following system of first order difference equations:

$$\begin{cases} k_{t+1} = \min\left(\frac{(1-b)E_t^{i,j} + b\overline{E}^{i,j} + \gamma_{i,j}\, A(1-\alpha)k_t^{\alpha} + \Lambda - \beta^{i,j}\, c_t^{i,j} - \Psi}{\gamma^{i,j}(\eta^{i,j}+1)}, A(1-\alpha)k_t^{\alpha}\right) \\ E_{t+1}^{i,j} = (1-b)E_t^{i,j} + b\overline{E}^{i,j} + \left[\gamma^{i,j}\,(m_t^{i,j})^* + \Lambda^*\right] - \left[\beta^{i,j}\, c_t^{i,j} + \Psi\right] \end{cases} \tag{15}$$

[8] At $t = 0$, i.e. at the period in which the first generation of individuals born, agents expect that there are no externalities arising from the behaviour of others.

where $\Lambda^* = \sum_{l=i-1}^{i+1} \sum_{v=j-1}^{j+1} \gamma_o^{l,v} \left(m_t^{l,v}\right)^*$, with $\{l,v\} \neq \{i,j\}$.

The high level of heterogeneity introduced in the model does not allow to study in depth its analytical properties. For this reason, the results generated by the model are analyzed through numerical simulations, discussed in the following section.

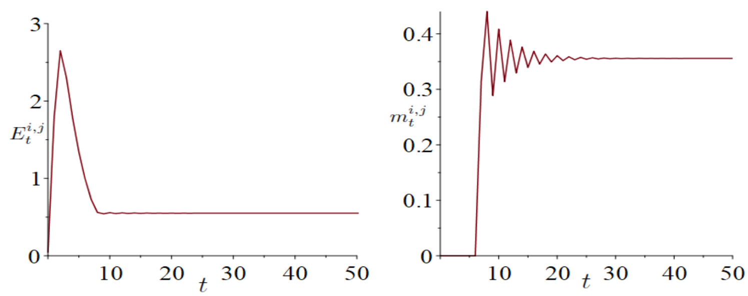
3 Simulation Results

In this section, we present and discuss some relevant simulation results. Because of the linear specification of environmental quality with respect to $c_t^{i,j}$ and $m_t^{i,j}$, it is possible that, for some parametric configurations, the trajectories generate negative values of the *actual* environmental indexes or of the expected ones. If it occurs, then the system becomes not well defined. Analytical conditions that preserve the non negativity of $E_t^{i,j}$ and $E_t^{e,i,j}$ can be formulated in terms of $\overline{E}_{i,j}$. In particular, enough large values of $\overline{E}_{i,j}$ are sufficient to make the system well defined for any t.
Starting from a condition in which (i) agents are isolated (i.e. no externalities produced by neighbours decisions) and (ii) homogeneity in preferences and the same initial values of environmental quality are assumed, we discuss the role of neighbourhood interaction when externalities are assumed in presence of heterogeneous initial levels of environmental (see subsection 3.1) or heterogeneity in preferences (see subsection 3.2).

3.1 Neighbourhood Interaction and Heterogeneous initial conditions: *history matters*

In the first two numerical simulations, we focus on the role played by interactions among individuals (in the same Moore neighbourhood) in destabilizing the local environment dynamics in each cell of L. We assume that all the agents included in L are identical. It follows that $\eta^{i,j} = \eta$, $\forall\{i,j\}$, and that both the magnitude parameters $\beta^{i,j}$ and $\gamma^{i,j}$ are equal in every cell, i.e. $\beta^{i,j} = \beta$ and $\gamma^{i,j} = \gamma$, $\forall\{i,j\}$. In these numerical exercises, we fix $N = 70$ (that is, 4900 individuals in L) and apply the following parameter set: $\alpha = 0.12, \beta = 0.3, \gamma = 0.138, \delta = 0.016, \eta = 0.8, A = 5, b = 0.22, \overline{E} = 8$.
Assuming that the externalities produced by the agents' decisions are null ($\beta_o^{l,v} = \gamma_o^{l,v} = 0$ for every $\{l = i-1..i+1; v = j-1..j+1\}$ with $\{l,v\} \neq \{i,j\}$) and the initial condition $(E_0, k_0, c_0, m_0) = (0.041, 0.0001, 0, 0)$, homogeneous within L, we have that all the agents are synchronized and the system converges towards a stationary state. Panel (a) in Figure 2 displays the dynamics of $E_t^{i,j}$, whereas Panel (b) in Figure 2 shows how, at the stationary state, all the agents invest a fixed amount of their wage in the environment.

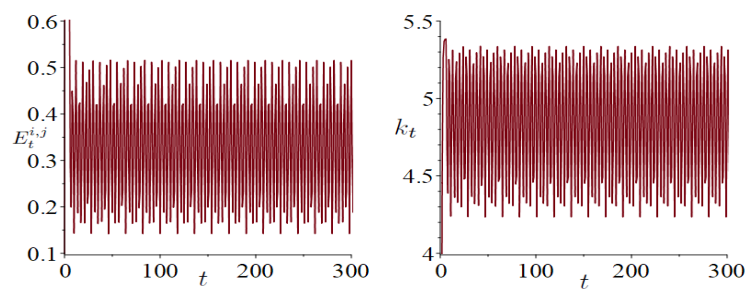
If we consider network externalities in each Moore neighbourhood (that is, $\beta_o^{l,v}$ and $\gamma_o^{l,v}$ positive for every $\{l = i-1..i+1; v = j-1..j+1\}$ with $\{l,v\} \neq \{i,j\}$), we notice that the resulting interactions may change the dynamics of



(a) (b)

Figure 2: (a) Dynamics of environmental quality in every cell $\{i, j\}$, converging to the stationary value $E^* \simeq 0.550597$. (b) Associated dynamics of defensive expenditures: after the first iterations in whom each agent decides for not contributing, $m_t^{i,j}$ becomes positive and finally converges to the stationary value $m^* > 0$.

the system. In particular, if all agents in L start from the same initial condition $(E_0, k_0, c_0, m_0) = (0.041, 0.0001, 0, 0)$, the dynamics of the system are chaotic in each cell (see Figure 3). Specifically, agents continue to switch in their decisions, but all agents in L behave the same manner.



(a) (b)

Figure 3: (a) Oscillating dynamics of environmental quality in every cell $\{i, j\}$ when $\beta_o^{l,v} = 0.0051$ and $\gamma_o^{l,v} = 0.031$. (b) Associated oscillating dynamics of capital.

We now move to explore the role played by heterogeneity in the initial condition $E_0^{i,j}$.

In this numerical experiment, we consider that all the agents in the lattice experience the same initial environmental level $E_0^{i,j} = 0.041$ except those positioned on a row $\bar{i}$ of the grid (see Figure 4), where agents have an initial level $E_0^{\bar{i},j} = 0.033$. Applying the same parameter set introduced above, both the environmental quality levels and the defensive expenditures exhibit oscillations in each cell, but in this case, the coordination between the agents is partial. According to the initial conditions, indeed, two groups are created in the long term, coordinated within but different from each other.[10] From an economic point of view, interactions are not able to override the different initial conditions, even if the potential characteristics of environment and economic sector are the same. Therefore, the results are clustered with respect to the initial conditions.

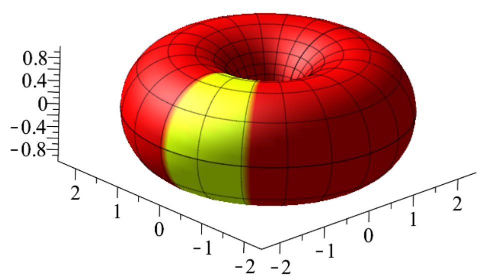


Figure 4: The figure colors in red the section of the torus with agents that have an initial condition $E_0^{i,j} = 0.041$ and in yellow those that have an initial condition $E_0^{i,j} = 0.033$. These different initial conditions also determine a different evolution of the environmental levels.

[10]The same phenomenon may be obtained considering a homogeneous initial condition for all the agents except those positioned on the same column of the grid or, more generally, on more rows, diagonals and/or columns of L.

Considering a wider form of heterogeneity, we now assume that the initial conditions on the environmental quality are heterogeneous among all agents, namely, that they are randomly extracted from a numerical range. In this case, the dynamics of all agents are oscillating but there is no coordination among them. This is shown in Figure 5 where we display the oscillating dynamics in different cells of L and the comparison among them.

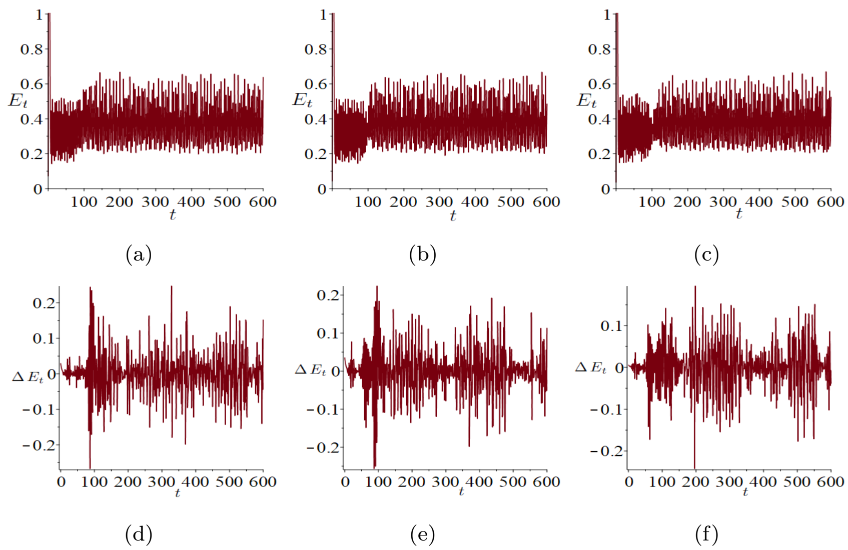
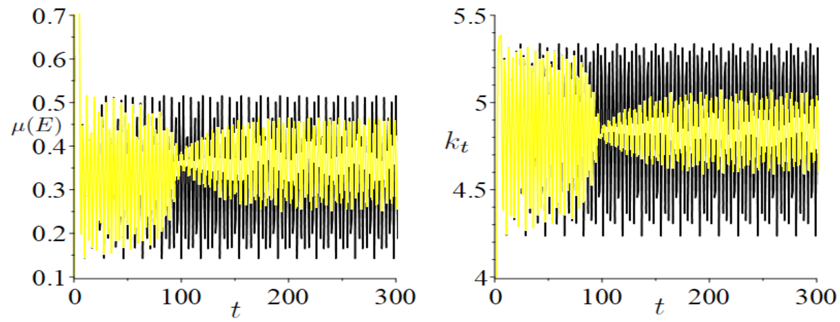


Figure 5: Time series in Panels (a), (b) and (c) describe the dynamics of $E_t^{i,j}$ in the cells $\{1,1\}$, $\{34,55\}$ and $\{68,41\}$. Panels (d), (e) annd (f) show the differences ΔE_t versus time highlighting the non coordinated dynamics in these cells (precisely, (d) $E_t^{1,1} - E_t^{34,55}$; (e) $E_t^{1,1} - E_t^{68,41}$; (f) $E_t^{34,55} - E_t^{68,41}$). In this exercise the initial conditions $E_0^{i,j}$ are randomly extracted in the range $[0.031, 0.085]$.

It is interesting to notice that, compared with the oscillations displayed in figure 3 (where $E_0^{i,j}$ is homogeneous within L), the oscillations in the latter case are reduced. Therefore, the presence of agents who are all equal in terms of preferences (η) and initial wealth (k_0), but living on areas characterized by different environmental conditions, may reduce, at least in part, the oscillations both in the environmental quality level and in the process of capital accumulation. In this regards, Panel (a) in Figure 6 allows to compare the oscillations in the average level assumed by the environment in the cells of L, $\mu(E)$, when heterogeneous initial conditions E_0 are assumed (coloured in yellow) with those obtained in the case of homogeneous initial conditions (coloured in black); while Panel (b) in Figure 6 compares the oscillations in the value assumed by k_t. The reason for this phenomenon is that agents with the same initial environmental level have coordinated behaviour and, by acting all in the same way, the effects of their decisions on every cell of L are high. On the other hand, agents living in cells characterized by an (initially) different environmental level react heterogeneously, making their decisions according to the specific environmental quality level of the cell. This mitigates the results of network decisions, and the resulting oscillations in the environmental quality level.



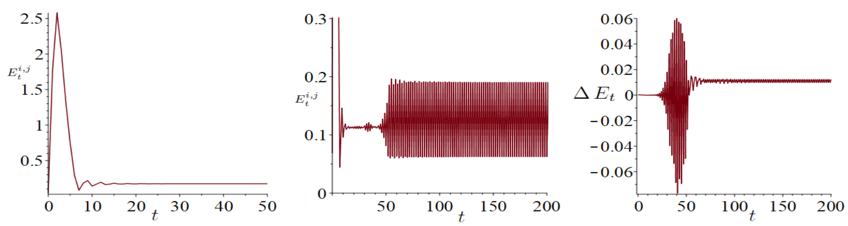
(a)

(b)

Figure 6: (a) Oscillations in $\mu(E)$ when $E_0^{i,j}$ is homogeneous (depicted in black) versus oscillations in $\mu(E)$ when $E_0^{i,j}$ is heterogeneous (depicted in yellow). (b) Comparison on oscillations of k_t, with the same colors legends applied in Panel (a).

Comparing these numerical exercises we can therefore conclude that, although the agents are identical to each other (i.e. they have the same preferences on consumption and environmental quality), (i) the presence of network externalities affects the defensive choices of each agent $\{i, j\}$ and may destabilize the long-term (synchronised) dynamics of the local environment and (ii) the assumption of heterogeneous initial conditions $E_0^{i,j}$ within L generates non coordinated behaviours and may induce the mitigation of oscillating dynamics. This means that the *history matters* and that even identical agents will have to experience different environmental conditions because of the different (better or worse) environmental quality levels "inherited" at the beginning of the world ($t = 0$).

We note that the result on the role played by heterogeneous initial conditions $E_0^{i,j}$ in defining the complexity of the dynamics cannot be generalized to all the parametric specifications of the model. Indeed, this result depends in a crucial and nonlinear way on the values assumed by $\beta^{i,j}$, $\gamma^{i,j}$, $\beta_o^{l,v}$ and $\gamma_o^{l,v}$, that is by parameters that regulate the choice of agents and the externalities among the cells. To explain this point, we notice that an higher initial condition $E_0^{i,j}$ for an individual produces, as a direct effect, a decrease of $m_t^{i,j}$ and an increase of $c_t^{i,j}$. Depending on the weight of the externalities produced $m_t^{i,j}$ and $c_t^{i,j}$, this fact may induce an improvement or worsening of environmental quality experienced in his neighbours. At this point, the neighbours will take decisions influenced by what happened in the initial cell. It, in turn, will produce a new feedback in the initial cell, and such feedback may have a reinforcing effect on the agent's decisions or move in the opposite direction. Specifically, applying the parameter set $\alpha = 0.12, \beta = 0.3, \gamma = 0.138, \delta = 0.016, \eta = 0.8, \beta_o = 0.0051, \gamma_o = 0.0031, A = 5, b = 0.22, \overline{E} = 8$ with the homogeneous initial environmental quality level $E_0^{i,j} = 0.041$, we have that all the agents are synchronized and the system converges towards a stationary state (see Panel (a) in Figure 7).[11] If we consider the same parameter configuration but assume that each $E_0^{i,j}$ is extracted in the numeric range $[0.031, 0.085]$, we observe oscillating environmental dynamics (see Panel (b) in Figure 7) and non coordinated behaviours among agents (see Panel (c) in Figure 7).



(a)

(b)

(c)

Figure 7: (a) Convergent dynamics of $E_t^{i,j}$ when homogeneous $E_0^{i,j}$ are assumed. (b) Oscillating dynamics of $E_t^{i,j}$ in the cell $\{1,1\}$ when heterogeneous $E_0^{i,j}$ are assumed. (c) The differences $\Delta\, E_t$ versus time highlighting the non coordinated dynamics in cells $\{1,1\}$ and $\{3,4\}$.

Therefore, Figure 7 shows how heterogeneity in the initial environmental quality endowment within L may also have a destabilizing role and induce oscillations in the dynamics of $E_t^{i,j}$.
Regardless of dynamic outcomes, what seems to robustly emerge from the model is that the utility experienced by cohorts in the case of the homogeneous initial conditions among agents is, on average ($\mu(U)$), higher than the utility experienced in the case of heterogeneous initial conditions. An example is shown in Figure 8, which compares the average utility $\mu(U)$ when $E_0^{i,j}$ is homogeneous (trajectory in black) with the oscillations in $\mu(U)$ when $E_0^{i,j}$ is heterogeneous (trajectory in yellow).

[11] Notice that the value of γ_o is lower than the one applied in Figure 3.

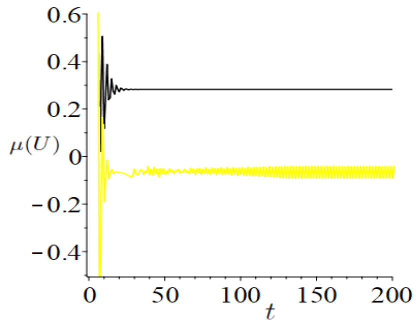


Figure 8: Parameter set: $\alpha = 0.12, \beta = 0.3, \gamma = 0.138, \delta = 0.016, \eta = 0.8, \beta_o = 0.0051, \gamma_o = 0.0031, A = 5, b = 0.22, \overline{E} = 8$. Dynamics of $\mu(U)$ when $E_0^{i,j}$ is homogeneous (depicted in black) versus dynamics of $\mu(U)$ when $E_0^{i,j}$ is heterogeneous (depicted in yellow).

3.2 Heterogeneous preferences

In this paragraph, we focus on the analysis of the role played by agents' preferences and in particular we discuss the dynamic consequences of assuming them as heterogeneous. In the first numerical simulation, assuming the existence of network externalities in each Moore neighbourhood, individuals with homogeneous preferences ($\eta^{i,j} = \eta$ for every $\{i,j\}$) and homogeneous initial environmental levels (i.e. $E_0^{i,j} = E_0$ for every $\{i,j\}$), we observe synchronised dynamics among all the cells, converging to the stationary state E^*. Specifically, applying the parameter set $\alpha = 0.12, \beta = 0.3, \beta_o = 0.0051, \gamma = 0.138, \gamma_o = 0.031, \delta = 0.016, \eta = 2.1, A = 5, b = 0.22, \overline{E} = 8$ and the initial condition $(E_0, k_0, c_0, m_0) = (0.041, 0.0001, 0, 0)$, the dynamic scenario depicted in Panel (a) and (b) of Figure 9 occurs.

Reducing the elasticity of substitution between consumption and environment ($\eta^{i,j}$), homogeneous within L, we can notice the onset of oscillating dynamics. In particular, setting $\eta^{i,j} = \eta = 0.855$, we continue to observe synchronized dynamics, but a continuous switching in the values assumed by the level of environmental quality and capital (see Panel (c) and (d) in Figure 9).

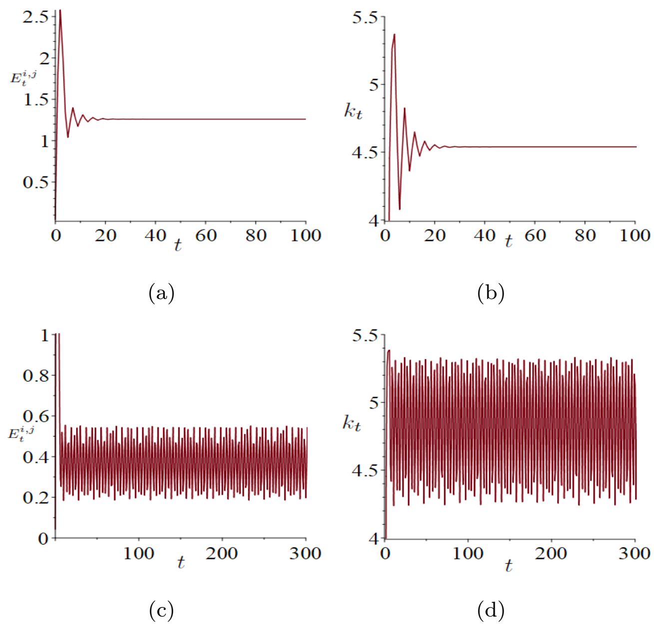
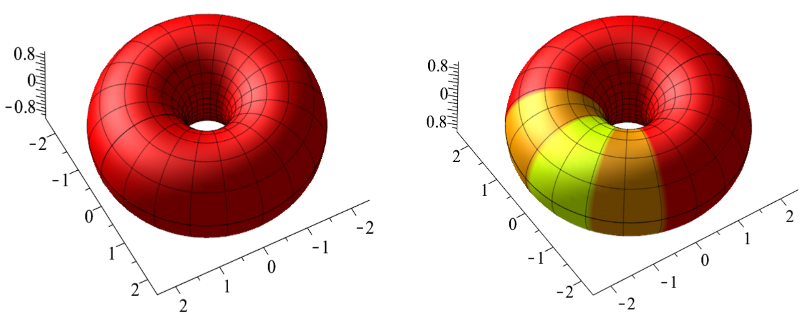


Figure 9: (a) Convergent dynamics of the local environmental quality $E^{i,j}$. (b) Associated convergent dynamics of the capital k. (c) Chaotic dynamics of $E^{i,j}$ for $\eta^{i,j} = \eta = 0.855$. (d) Associated chaotic dynamics for k.

Figure 9 therefore allows to conclude how, if on the one hand the interaction between agents that give a high weight in their preferences to the environmental variable can be a vehicle of coordinated and convergent behaviours (therefore, the need of the agents to obtain acceptable environmental levels dominates the possible oscillations caused by the interaction), on the other hand if all individuals give a relatively low weight to the environment in their preferences, the network externalities may determine the emergence of complex dynamics. After exploring the role played on environmental dynamics by homogeneous preferences within L, we focus on analysing the long-term effects of decisions and interaction among agents with heterogeneous preferences. First, we consider that all the agents in L have the same preferences (that is, the same $\eta^{i,j}$) except those positioned on a row $\bar{i}$ of the grid. In particular, consider the parameter set $\alpha = 0.12, \beta = 0.3, \beta_o = 0.0051, \gamma = 0.138, \gamma_o = 0.031, \delta = 0.016, A = 5, b = 0.22, \overline{E} = 8$ and a homogeneous initial environmental level $E_0 = 0.041$. If we fix $\eta^{i,j} = 2.1$ (for every $i \neq \bar{i}$) while $\eta^{\bar{i},j} = 2.2$ we obtain that both environmental quality levels and defensive expenditures approximate to a stationary state, but in this case the coordination between agents is partial. Indeed, starting from the same initial environmental level (see Panel (a) in Figure 10), the partition into two groups in terms of preferences dumps in the environmental dynamics, resulting in (i) the coordination among the agents in the $\bar{i}$ (which converge at the same, highest, stationary level of environmental quality), which have a "pulling" effect on the coordinated dynamics of the agents closest to them (which converge at an intermediate level of environmental quality), and (ii) the coordination towards the lowest value of environmental quality among the cells furthest from those in row $\bar{i}$.[12] Panel (b) in Figure 10 shows the final distribution of environmental quality levels among the torus.



(a) (b)

Figure 10: (a) Initial homogeneous configuration of the local environmental quality towards the torus (every cell is colored in red because the initial environmental level is the same). (b) The section depicted in yellow describes the cells on the row $\bar{i}$ where the highest environmental quality level is reached, the sections in orange define the areas affected by the positive externalities of the most "environment-oriented" agents, where intermediate values of environmental quality are obtained and (iii) the section in red defines the remaining portion of the torus where the lowest stationary state is reached.

Second, we consider that the preferences are heterogeneous among all agents, namely, that $\eta^{i,j}$ are randomly extracted from a numerical range. In this case, considering a homogeneous initial condition $E_0^{i,j} = E_0 = 0.041$ and extracting every $\eta^{i,j}$ in the numeric range $[1.8, 2.4]$, the dynamics of all agents are convergent but there is no coordination among them. This is shown in Figure 11 where we display the final distribution of environmental quality level among the different cells of the torus.

[12]The same phenomenon may be obtained considering homogeneous η for all the agents except those positioned on the same column of the grid or, more generally, on more rows, diagonals and/or columns of L.

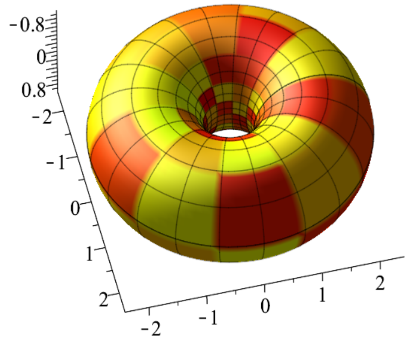
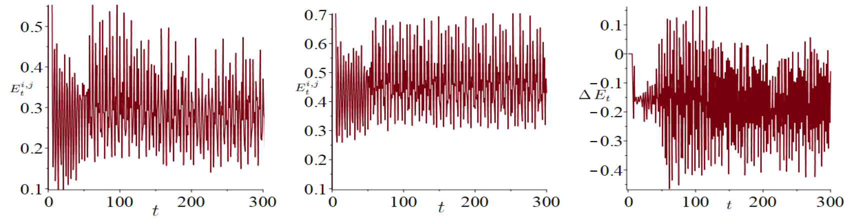


Figure 11: The figure colours with different shades from red to yellow the different cells of the torus in which the environmental quality converges to different final levels.

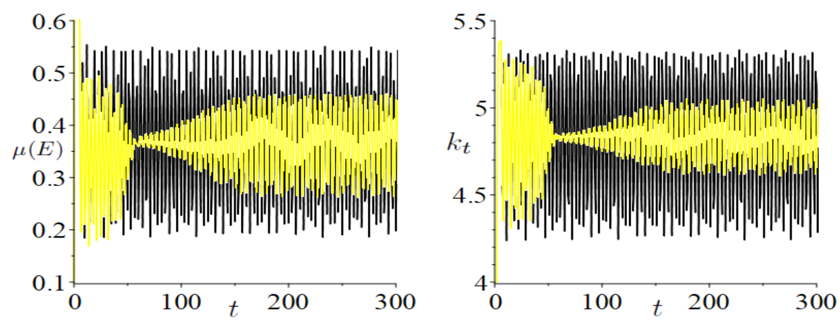
It is interesting to notice how the neighbourhood interaction between heterogeneous agents has a damping effect on the variability of behaviours assumed in the cells. In fact, in the numerical simulation we have carried out, the variance of the normalized stationary environmental levels $\widetilde{E}^{*,i,j}$, $\sigma^2(\widetilde{E}^*)$, is lower than the variance of the normalized elasticities of substitution $\widetilde{\eta}^{i,j}$, $\sigma^2(\widetilde{\eta})$. This means that agents heterogeneously affect each other in the decision-making process, mitigating the results of decisions on the network, and therefore bringing the stationary values reached in each cell closer to the average value assumed by the environmental quality in L.
If we consider extracting the heterogeneous $\eta^{i,j}$ in a numeric range composed of low values (in the numerical exercise we use the range $[0.65, 1.05]$), the non-synchronized dynamics of all agents becomes oscillating (see Figure 12).



(a) (b) (c)

Figure 12: Time series in Panels (a) and (b) describe the oscillating dynamics of $E_t^{i,j}$ in two different cells (specifically, cells $\{1,1\}$ and $\{63,40\}$). Panels (c) show the difference $\Delta E_t = E_t^{1,1} - E_t^{63,40}$ versus time highlighting the non coordinated dynamics in such cells.

Furthermore, comparing the dynamics displayed in Panel (c) of Figure 9 with those ones depicted in Figure 12, we can notice that the oscillations in the latter case are reduced. Then, assuming cells characterized by the same initial condition, but agents that have different degrees of priority with respect to the environment may reduce, at least in part, the oscillations both in environmental levels and in capital. Figure 13 allows to compare both (i) the oscillations in the average level $\mu(E)$ when $\eta^{i,j}$ are heterogeneous with those obtained in the case of homogeneous $\eta^{i,j}$ (see Panel (a)) and (ii) the oscillations in the value assumed by k_t in the two occurrences (see Panel (b)).



(a) (b)

Figure 13: (a) Oscillations in $\mu(E)$ when $\eta^{i,j}$ is homogeneous (depicted in black) versus oscillations in $\mu(E)$ when $\eta^{i,j}$ are heterogeneous (depicted in yellow). (b) Comparison on oscillations of k_t, with the same colors legends applied in Panel (a).

An explanation of such a phenomenon is that agents with identical preferences have coordinated behaviours and, acting all in the same manner, their decisions generate high reflections on the dynamics within L. In contrast, agents defined by heterogeneous preferences react heterogeneously, responding to the environmental level according to their own (different) preferences (consumption or environment oriented). This mitigates the results of network decisions and the resulting fluctuations in E_t and k_t.
Similarly to the previous paragraph, the results of the simulations significantly depend on the configuration of the parameters. In fact, by decreasing the weight of the externalities among cells, the change from a scenario with homogeneous agents to one with heterogeneous ones can produce opposite results to those previously described. In particular, considering the (i) homogeneous initial environmental quality level $E_0^{i,j} = 0.041$, (ii) the parameter set $\alpha = 0.12, \beta = 0.3, \gamma = 0.138, \delta = 0.016, \beta_o = 0.0051, \gamma_o = 0.01, A = 5, b = 0.22, \overline{E} = 8$ and (iii) homogeneous preferences among agents ($\eta^{i,j} = 0.855$), we have that all the agents are synchronized and the system converges towards a stationary state (see Panel (a) in Figure 14).[13] If we consider the same parameter configuration but assume that each $\eta^{i,j}$ is extracted in the numeric range $[0.65, 1.05]$, we observe oscillating environmental dynamics (see Panel (b) in Figure 14) and non coordinated behaviours among agents (see Panel (c) in Figure 14).

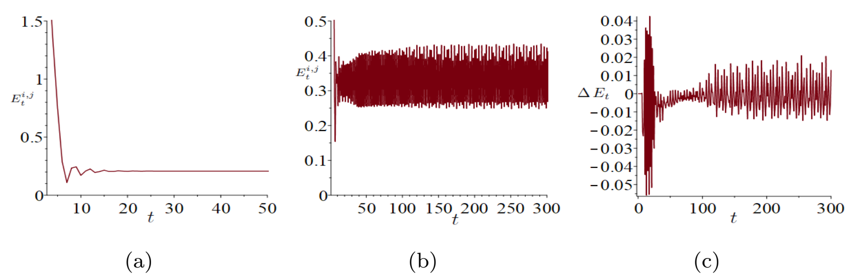


Figure 14: Convergent dynamics of $E_t^{i,j}$ when $\eta^{i,j}$ is homogeneous. (b) Oscillating dynamics of $E_t^{i,j}$ in the cell $\{1, 8\}$ when $\eta^{i,j}$ is heterogeneous. (c) The differences $\Delta\, E_t$ versus time highlighting the non coordinated dynamics in cells $\{1, 8\}$ and $\{9, 4\}$.

Figure 14 then shows how heterogeneous preferences among agents in L may also have a destabilizing role and induce the emergence of oscillating dynamics for $E_t^{i,j}$.

[13]Notice that the value of γ_o is lower than the one applied in Figure 9.

3.3 The centralized case

Differently from what shown in the previous paragraphs (where a decentralized solution of the local environment problem is considered), in this paragraph we describe the result of assuming the existence of a short lived social planner (see Pecchenino, 1995) that maximizes the utility of the single cohort. Therefore, taking into account the network externalities created among the cells, the maximization problem is defined and follows:

$$\max_{m_t^{i,j}, s_t^{i,j}} \quad (\sum_{i=1}^{N}\sum_{j=1}^{N} U^{i,j}(c_{t+1}^{i,j}, E_{t+1}^{i,j}) = \sum_{i=1}^{N}\sum_{j=1}^{N} \ln c_{t+1}^{i,j} + \eta^{i,j} \ln E_{t+1}^{i,j})$$

subject to

$$c_{t+1}^{i,j} = (1 + r_{t+1} - \delta) s_t^{i,j} \qquad \forall i = 1, .., N; \forall j = 1, .., N$$

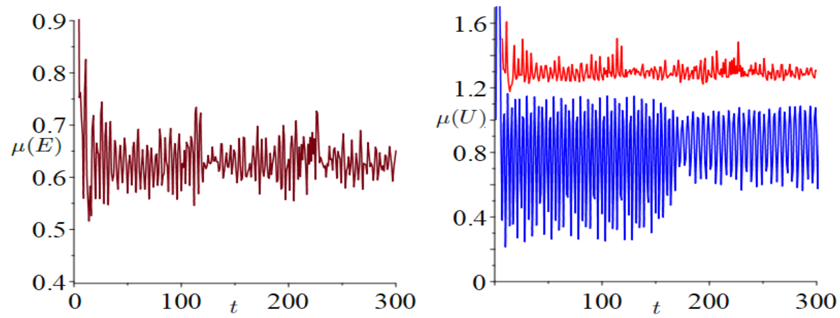
$$w_t^{i,j} = s_t^{i,j} + m_t^{i,j} \qquad \forall i = 1, .., N; \forall j = 1, .., N$$

$$E_{t+1}^{i,j} = (1-b)E_t^{i,j} + b\overline{E}^{i,j} + \left[\gamma^{i,j}\, m_t^{i,j} + \sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1} \gamma_o^{l,v}\, m_t^{l,v}\right] - \left[\beta^{i,j}\, c_t^{i,j} + \sum_{l=i-1}^{i+1}\sum_{v=j-1}^{j+1} \beta_o^{l,v}\, c_t^{l,v}\right]$$

$$\forall i = 1, .., N; \forall j = 1, .., N \text{ (with } \{l, v\} \neq \{i, j\})$$

$$c_{t+1}^{i,j} > 0, m_t^{i,j}, s_t^{i,j} \geq 0 \qquad \forall i = 1, .., N; \forall j = 1, .., N.$$

If the social planner internalizes the environmental problem, we notice that centralized decisions allow the achievement, on average, of higher environmental quality levels within L, compared with the levels obtained in the case of decentralized ones. Besides, leaving the decisions to the social planner allows for higher levels of agents' utility than in the decentralized case. In particular, by assuming heterogeneous initial environmental levels $E_0^{i,j}$ (extracted in the numeric range $[0.031, 0.085]$) and applying the parameter set $\alpha = 0.12, \beta = 0.3, \gamma = 0.138, \beta_o = 0.0051, \gamma_o = 0.031, \delta = 0.016, \eta = 0.8, A = 5, b = 0.22, \overline{E} = 8$, we notice that the average environmental quality within L, $\mu(E)$, oscillate among higher values compared with the oscillations observed in Panel (a) of Figure 6, where a decentralized solution is considered (see Panel (a) in Figure 15). Concerning the average utility $\mu(U)$ experienced within the lattice, in the case of centralized decisions it assumes higher values than those obtained with the same parameter set in the decentralized case (see Panel (b) in Figure 15).

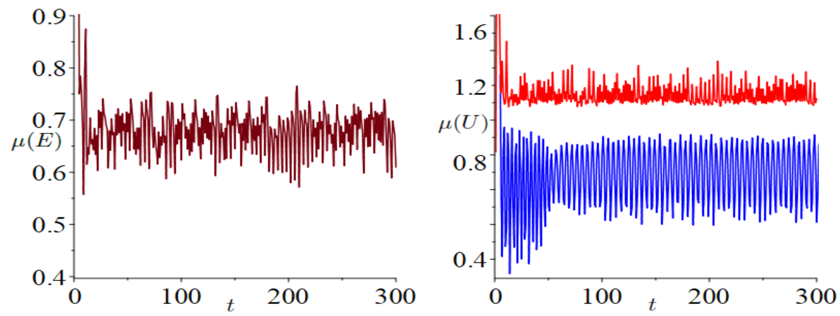


(a) (b)

Figure 15: (a) Oscillations in $\mu(E)$ in the case of centralized solution. (b) Oscillations in $\mu(U)$ when decisions are decentralized (coloured in blue) versus oscillations in $\mu(U)$ when decisions are centralized (coloured in red).

Therefore, the results in this comparison allow confirming that the decentralized solution discussed in previous paragraphs is not Pareto efficient, not even for of view of the single generation.

The same comparison can be made in the case of heterogeneous preferences. Even in this case, the existence of a social planner that maximizes the utility of the generation induces both (i) the achievement of higher average levels of environmental quality than the case with decentralized decisions and (ii) higher average levels of the agents' utility. In particular, considering the (i) homogeneous initial environmental quality level $E_0^{i,j} = 0.041$, (ii) the parameter set $\alpha = 0.12, \beta = 0.3, \gamma = 0.138, \delta = 0.016, \beta_o = 0.0051, \gamma_o = 0.01, A = 5, b = 0.22, \overline{E} = 8$ and (iii) heterogeneous preferences among agents (each $\eta^{i,j}$ extracted in the numeric range $[0.65, 1.05]$), we notice that the average environmental quality within $\mu(E)$ oscillate among higher values compared with the oscillations observed in Panel (a) of Figure 13, where a decentralized solution is considered (see Panel (a) in Figure 16). Furthermore, the average utility $\mu(U)$ assumes higher values than those obtained with the same parameter set in the decentralized case (see Panel (b) in Figure 16).



(a) (b)

Figure 16: (a) Oscillations in $\mu(E)$ in the case of centralized solution. (b) Oscillations in $\mu(U)$ when decisions are decentralized (coloured in blue) versus oscillations in $\mu(U)$ when decisions are centralized (coloured in red).

4 Conclusions

In this article, adapting the modelling framework introduced by John and Pecchenino (1994) to analyze the dynamic relationship between the environment and economic activity, we focus on the local dimension of the environment, i.e. we remove the assumption, common in literature, in which the environmental resource is treated as a good homogeneously distributed among agents. To this end, we have proposed an OLG agent-based model in which we consider a *lattice* structure populated by overlapping generations of heterogeneous agents, which may have different initial environmental endowments or may be heterogeneous in their preferences. The numerical experiments proposed have allowed noticing that, despite the attention devoted to local environmental aspects, the network externalities (determined through the scheme of Moore neighbourhoods) play a fundamental role in defining environmental dynamics. Indeed, when externalities between the cells are assumed, the decentralized decision system reveals to be prone for the emergence of chaotic dynamics: agents react to changes induced by his neighbours' decisions, characterized by a different environmental quality level. From an economic point of view, this means that a decentralized system of decisions on environmental issues is difficult to be analyzed in terms of environmental dynamics. Concerning the different initial environmental quality endowments within the lattice, even in the presence of homogeneous agents, they lead to differentiated dynamics for economic agents, underlining how environmental *history matters* in defining long run dynamics. On the other hand, the heterogeneity of preferences and/or initial conditions plays an ambiguous role. In fact, depending on the weight of network externalities and the impact of consumption and/or defensive expenditures, heterogeneity may stabilize or destabilize the system.

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