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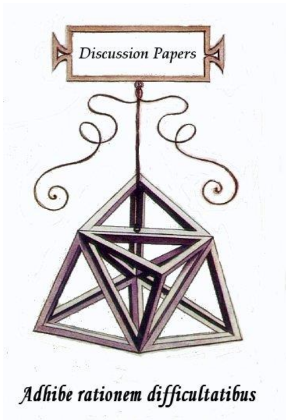

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Monopoly strategy with purchase-dependent preferences and endogenous preference change

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Discussion Paper
n. 271

Stéphane Lemarié and Cecilia Vergari

Monopoly strategy with purchase-dependent preferences and endogenous preference change

Abstract

We study the optimal monopoly pricing when consumers may buy one or two units of a good over two successive periods and their preferences evolve depending on past purchases and preference persistence.
If the monopolist cannot discriminate between past and new buyers, he refrains from intertemporal price discrimination. Selling over two periods improves the monopoly profit as compared to the benchmark one-period equilibrium only for high preference persistence.
Conversely, if the monopoly can discriminate between second period buyers, behavior-based price discrimination becomes profitable only for low preference persistence.

Keywords: Repeated purchases, monopoly dynamic pricing, vertical differentiation

JEL: L10, Q10.

1 Introduction

Purchasing a good or a service answers a need from the buyer. The most standard case studied in the literature is that where the need fully re-arises in each period. The literature on durable goods studies the other polar case where the need fully disappears after one purchase. We are interested here in the intermediate case where the need for re-purchasing the good in the future is more or less persistent. In this situation, a monopolist facing rational consumers may adapt its pricing to preference persistence.

Pesticides used by farmers is one example that illustrates this situation. Pesticides are used to limit losses caused by pest and they may be used several times. Farmers' interest for pesticides increases with the magnitude of the pest population they control for. Using a pesticide decreases the pest population and consequently decreases the preference for a second purchase. Audit services provides another illustration. An audit will improve a firm's organization and knowledge in such a way that future audit will be less needed, but will still have value. The product may also be a process innovation that may be bought and combined over time. Buying a first cost-reducing innovation may decrease the firm's willingness to pay for a second innovation. The persistence of preferences may also be related to the limited lifetime of a good. In such a case, the higher is the risk that the good breaks up or is no longer adapted to new technical standards, the higher is the persistence of preferences for this good.

The evolution of preferences based on past purchase has been addressed in some specific literature. For instance, Becker and Murphy (1988) develop a dynamic model of addictive consumption to represent addiction in the realm of rational behaviors and derive demand functions for addictive goods. Their analysis, however, is not addressed in an IO framework where the supply side can define dynamic pricing strategically. On the other hand, dynamic pricing has been extensively addressed in the IO literature but without considering the possibility for such evolution of preferences. In the literature on behavior-based price discrimination, that is the most closely related to our paper, preferences are independent of past purchases: they are either stable over time or they evolve randomly.[1]

We address this gap in the dynamic pricing literature by analyzing the pricing strategy of a monopoly supplier in a two-period model. We consider a discrete choice model where heterogeneous consumers choose to buy (or not) one unit of the product at each of two successive periods. The first-period decision affects second-period preferences: using the product in the first period decreases the second-period willingness and this decrease becomes more drastic the weaker is preference persistence.

We first consider the case where the monopoly does not have access to the consumers' purchase history, so that pricing is uniform in each period. If the monopoly wants to sell its product at both periods, the

[1] We provide a review of the related literature in the next section.

first-period purchases impact the second-period pricing and this is fully anticipated by the consumers. Thus, the consumers who only buy once can make an arbitrage between buying at the first or at the second period. The anticipation of the second-period equilibrium by the consumers influences their first-period decision, which, in turn, constrains the monopoly. As a consequence, the monopoly cannot price discriminate over time. If the preferences are weakly persistent this strategy with sales at both periods leads to very low profits for the monopoly: the monopoly is then better off selling the product in only one period.

We also consider the case where the monopolist can observe and track the purchasing history of the consumers and set a specific price that depends on this history. Hence, in the second period, there is price discrimination between consumers who already bought the good ("past buyers") and those who did not ("new buyers"). If preferences are persistent enough we get results similar to Fudenberg and Villas-Boas (2007) where preferences are independent of the purchasing history. Consumers who buy in the first period signal their high valuation for the good, leading to a higher price for past buyers compared to new buyers. As a consequence, consumers with high valuation for the good refrain from buying in the first period, leading to a decrease of the first period demand. This mechanisms reflects a ratchet effect: these high-valuation consumers anticipate that buying in the first period signals their type, and that this may lead to more rent extraction later, via a higher price. In such a case, behavior-based price discrimination (BPD) is not valuable for the monopolist, so that he prefers to set uniform prices in the second period. Conversely, if preferences persist only weakly, then the consumers who purchase at the first period have a low willingness to pay for the good in the second period, leading to a lower second-period price for past buyers compared to new buyers. Buying at the first period becomes attractive because the consumer then benefits from a discount at the second period. In this situation, the BPD strategy is beneficial for the monopolist compared to the case with uniform prices in the second period. This result is particularly original since, contrary to usual discrimination models, high-valuation consumers have an incentive to signal their type in the first period.

At last, two additional results are established. First, we show that, for the monopoly, behavior-based price discrimination is equivalent to pure bundling in the first period. Either strategy is welfare improving for low preference persistence levels, as it leads to an increase of the quantity sold over the two periods. Second, if preference persistence is negatively related to product quality, we show that quality has two contrary effects: a positive effect on the current period profit but a negative effect on the next period profit because it decreases preference persistence and, thus, demand. The negative effect can dominate the positive one when the quality is high enough, leading to situations where the profit decreases with the quality.

The structure of the paper is as follows. We review the related literature in Section 2. Next, we set up the 2-period baseline model with uniform prices at each period (Section 3), that we solve backwards (Sections 4 and 5). In Section 6, we consider the possibility for the monopolist to implement BPD or pure bundling in the first period. Finally, in Section 7, we analyze the extended version of the model where preference persistence is related to product quality.

2 Literature review

The literature on dynamic pricing of *non durable goods* considers the possibility to price discriminate among consumers depending on their past purchases and it analyzes whether this BPD is profitable for firms in a context where past purchases do not affect consumers' preferences.

Fudenberg and Villas-Boas (2007), that has already been mentioned, provides an extensive survey of this literature. They point out that firms may face a commitment problem: the seller may prefer to be able to commit to ignore information about the buyer's past decisions because rational consumers may anticipate this behavior based price discrimination, and so alter their initial purchases. This arises in the monopoly case where, in a two-period model with a continuum of heterogeneous consumers, they find that it is never profitable to implement BPD. Rather, the seller would prefer to set the same monopoly price in both periods, with no price variation through time. Acquisti and Varian (2005) also consider a two-period monopoly but focusing on two consumer types. They point out that, in spite of the rapid advance in information technology, consumers can be able to protect their privacy via "anonymizing technologies". They thus approach the seller's problem from the perspective of mechanism design and they study the conditions under which conditioning prices on consumers' past history may be profitable, that is the present values of the prices charged to the high-value and low-value persons differ. They also find that a flat price in each period is the best strategy in the presence of rational consumers. However, they point out that in some circumstances conditioning prices on consumers' past history may be profitable. This may occur when the value of the second unit of consumption is larger than the value of the first unit of consumption, that is, when the seller can offer some benefits, like personalized enhanced services, to induce consumers to reveal their identities. Note that this result is somewhat in line with what we find for low levels of preference persistence. In our case, however, the monopolist optimally rewards past buyers with lower prices. So that our model generalizes this intuition by considering a continuum of types and preferences that evolve over time.

Departing from these models, Battaglini (2005) characterizes the optimal long-term contract offered in an infinitely repeated setting by a monopolist to a consumer whose preferences evolve following a Markov process. In each period the agent can have either a high or a low marginal valuation for the good and types are correlated. He shows that, contrary to what happens with constant types, for the seller it is profitable to use the information acquired along the dynamic interaction.

Fudenberg and Villas-Boas (2007) also review some oligopoly models. Among others, Fudenberg and Tirole (2000) consider a two-period Hotelling duopoly with two firms producing exogenously horizontally differentiated goods. When consumers' preferences are constant over the two periods, they find that firms have the incentive to conquer their rivals' customers with "special introductory prices" with the result that more information may induce tougher competition in the market and, in turn, reduce industry profits. Moreover, the switching of a fraction of consumers to the supplier they like less is socially inefficient. Long-term contracts may increase efficiency. Results change in the opposite case of preferences that are independent over time: knowledge of consumers' past purchase does not provide information about second-period preferences so that the two-period equilibrium is simply two repetitions of the static equilibrium, and it is socially efficient. Chen and Pearcy (2010) extends the model by Fudenberg and Tirole (2000) introducing a copula to model a more general dependence relation of brand preferences across periods and show that, instead, in some cases the practice of inducing brand switching may increase industry profit. More recently, Jing (2017) investigates how BPD affects endogenous product quality differentiation in a two-period vertical duopoly.

Some of the economic effects arising in our model are also related to the literature on dynamic pricing of *durable goods*, that are goods that yield utility over a number of periods. Consumers need to buy a durable good only once, then they choose whether and in which period to adopt the product. Indeed, in our model, the extreme case in which preferences are not persistent at all (or they are very low), resembles a durable good setting as there is no interest for a second purchase. Note, however, that our framework does not formally fit a durable good problem because consumers' benefits from purchasing one good last only one period. Nevertheless, we find that whenever the persistence of preferences is low enough, selling over two periods does not improve the monopoly profit as compared to the benchmark one-period equilibrium. This result traces the well known two-period durable good problem with rational consumers where the monopolist would prefer to be able to commit to sell only in one period (see, Bulow, 1982). The intuition, that we share with this literature, is that the monopolist has to deal with a cannibalization problem because he faces competition from his own product in the second period. Extending the model to many periods that become sufficiently small, the Coase conjeture applies: the monopolist loses its market power and in each period the competitive outcome arises (see, Stokey, 1981). Commitment can improve this outcome. Stokey (1979) considers a monopoly seller who can commit to future prices in a continuous time setting. She shows that intertemporal price discrimination may be optimal in the presence of positive production costs that decline over time; whereas in the absence of production costs, the firm's profit-maximizing strategy is to forego the opportunity to price discriminate, and instead to sell the product only at the initial instant. More recently, Besanko and Winston (1990), in a finite horizon monopoly setting, show that decreasing the product price over time is the optimal strategy because it enables the seller to take advantage of differences in consumers' valuations of products in order to extract consumers' surplus. This strategy is referred to as intertemporal price skimming. Liu and Zhang (2013) extend Besanko and Winston's framework to dynamic price competition introducing intratemporal demand competition, on top of intertemporal demand substitution.[2]

Lastly, we mention some macroeconomic literature on household dynamic consumption and savings choices. Indeed, our assumption about the evolution of preferences may recall the habit persistence models of, among others, Abel (1990) and Campbell and Cochrane (1999), representative-agent consumption-based asset pricing models relying on the idea that the "repetition of a stimulus diminishes the perception of the stimulus and responses to it" (Campbell and Cochrane, 1999, p. 208). More recently, Hai et al. (2020), introduce the notion of memorable goods for analyzing households' choices. Memorable goods are such that "a consumer draws utility from her past consumption experience, that is, through memory", (Hai et al., 2020, p.1). These authors show that memorable goods creates incentives to optimally consume in spikes rather than to smooth consumption, capturing important empirical features for goods such as vacation and entertainment. This literature has some link with the model we propose here. Indeed, if a consumer buys a memorable good, this good provides utility during the periods following its consumption, so that the interest of the consumer for a new purchase decreases, which means that his preferences for this good are less persistent. The analysis by Hai et al. (2020), however, does not address the pricing strategy on the supply side, as we do here.

From the modeling point of view, we are related to the vertical differentiation literature. As far as we know most papers in this literature consider that consumption occurs only during one period: heterogeneous consumers either buy or not one unit of a good. A notable exception is Gabszewicz and Wauthy (2003) that add the "joint purchase option" to the classical duopoly model of vertical differentiation. We depart from Gabszewicz and Wauthy (2003) in that we consider pricing policy by a monopoly over two periods, rather than duopoly competition over one period. As already mentioned, a cannibalization problem occurs in our model with a monopoly selling one product over two periods. Moreover, prices are defined simultaneously by the two competing firms in Gabszewicz and Wauthy (2003) while, in our framework, prices would be defined sequentially if we considered a duopoly between two firms selling one good each at two successive periods.

[2]Dynamic pricing is also addressed in the particular context with limited quantities and fixed cost related to limited capacity like passenger transportation tickets. These goods, that can be ordered in advance, are perishable in the sense that they are consumed only once but, as in the durable good case, consumers can purchase at most one item. Dynamic pricing by firms comes from the fact that they control prices contingent on time and present market conditions. See, among others, Levin et al. (2009a) and Levin et al. (2009b).

3 The model

A consumer has the opportunity to buy (or not) one unit of a product during two successive periods ($t = 1, 2$). x_t is the decision variable of the consumer which is equal to 1 if he buys and uses the product at period t and 0 otherwise. We assume that consumers do not benefit from using more than one unit in each period; that is, the product is non storable and cannot be resold.

θ_t is the state variable of the consumer at period t that defines his preference. The preference at period 2 (θ_2) is related to the preference and decision of the consumer at period 1 (θ_1 and x_1, respectively). More precisely, we suppose that the preference of the consumer stays the same if the consumer does not use the product at period 1: $\theta_2 = \theta_1$ if $x_1 = 0$. Conversely, if the consumer uses the product at period 1, then its preference is only a fraction $\alpha \in [0, 1]$ of its preference at the first period: $\theta_2 = \alpha\theta_1$ if $x_1 = 1$. We can summarize the evolution of preferences by the following equation:

$$\theta_2 = (1 - x_1)\theta_1 + x_1\alpha\theta_1 = \theta_1(1 - x_1(1 - \alpha)) \tag{1}$$

Parameter α denotes the preference persistence of the consumer that already bought the product.

At each period, the benefit of the consumer is $u_t(x_t, \theta_t) = x_t(\theta_t q - p_t)$ where p_t is the price of the product at period t and q is the quality of this product. We suppose that there is no discounting, so that consumer cumulative utility is

$$\begin{aligned} v(x_1, x_2, \theta_1) &= u_1(x_1, \theta_1) + u_2(x_2, \theta_2) \\ &= x_1\theta_1 q + x_2\theta_1 q - x_1 x_2(1 - \alpha)\theta_1 q - x_1 p_1 - x_2 p_2 \end{aligned} \tag{2}$$

If $\alpha = 0$, there is no longer any preference for the product once the consumer buys it in the first period. In other words, the consumer has no interest for a second purchase. Note that this particular case resembles but it does not fit a durable good problem because the consumer gets only the utility over one period ($u_1 > 0$ and $u_2 = 0$ in this case). If instead $\alpha = 1$, then the preference fully persists: we are in a standard vertically differentiated model that is just repeated twice, with stable preferences over time. We suppose that consumers are heterogeneous with respect to the initial preference for the product (θ_1). More precisely, we suppose that θ_1 is uniformly distributed between 0 and 1.

The supply is made by a monopoly who maximizes its profit over the two periods, that is, its cumulated profit. The marginal product cost is normalized to 0.

The timing of the game is the following: at the first period (i) the monopoly chooses p_1, and then (ii) each consumer chooses x_1 and first period sales occur; at the second period (iii) the monopoly chooses p_2, and then (iv) each consumer chooses x_2, second period sales occur and each economic actor gains its profit. The game is solved backward. We suppose that all players fully anticipate the consequences of their decisions on the equilibrium of the following periods.

After solving this baseline model (Sections 4 and 5), we will consider a situation where the monopoly can price discriminate at the second period between past and new buyers (Section 6). More precisely, consumers who buy the product at the first period face a different price at the second period compared to those who do not. We will show also that this extension is equivalent to the case where the monopoly sells, at the first period, a bundle of two units of product for consumers who wish to use the product at both periods. Notice that α and q are supposed to be independent, that is, the preference persistence for one product is not related to the quality of this product. This assumption will be relaxed in one extension of the model (see Section 7).

Standard one period model. Before proceeding with the solution of the model, as a benchmark, we briefly recall the equilibrium with the standard one-period model. The consumer can either buy the product ($x = 1$) or not ($x = 0$). Its utility is defined as:

$$u(x, \theta) = x\theta q - xp.$$

Let $\tilde{\theta}$ denote the consumer type which is indifferent between buying or not, that is:

$$\tilde{\theta} : \theta q - p = 0 \quad \Leftrightarrow \quad \tilde{\theta} = \frac{p}{q}$$

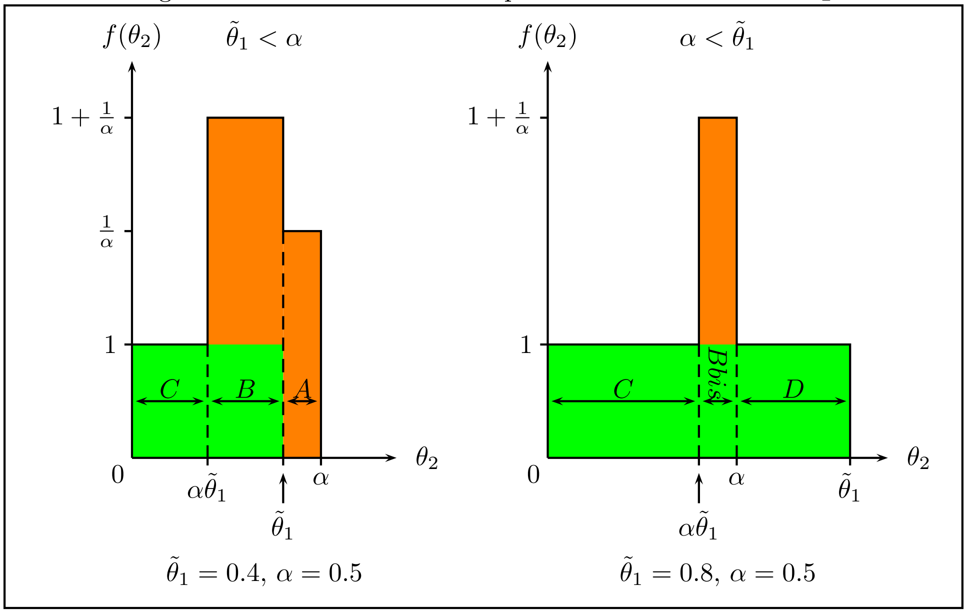
The demand for the product is then $D = 1 - \tilde{\theta}$, with $p \leq q$ and monopoly profit is $\pi = p(1 - p/q)$. The one-period equilibrium is then:

$$p^* = \frac{q}{2}, \quad D^* = \frac{1}{2}, \quad \pi^* = \frac{q}{4}. \tag{3}$$

4 Second-period subgame equilibrium

In order to solve the second-period subgame, we first characterize the second-period demand for the product, that is the purchasing decision of consumers in period 2 for any given price p_2. Then, we solve for the monopoly pricing decision.

Figure 1: Two illustrative examples of the distribution of θ_2



4.1 Consumer decision and consumers' demand

At period 2, each consumer maximizes its second period utility $u_2(x_2, \theta_2)$. Consumer type θ_2 has an interest to buy the product if and only if:

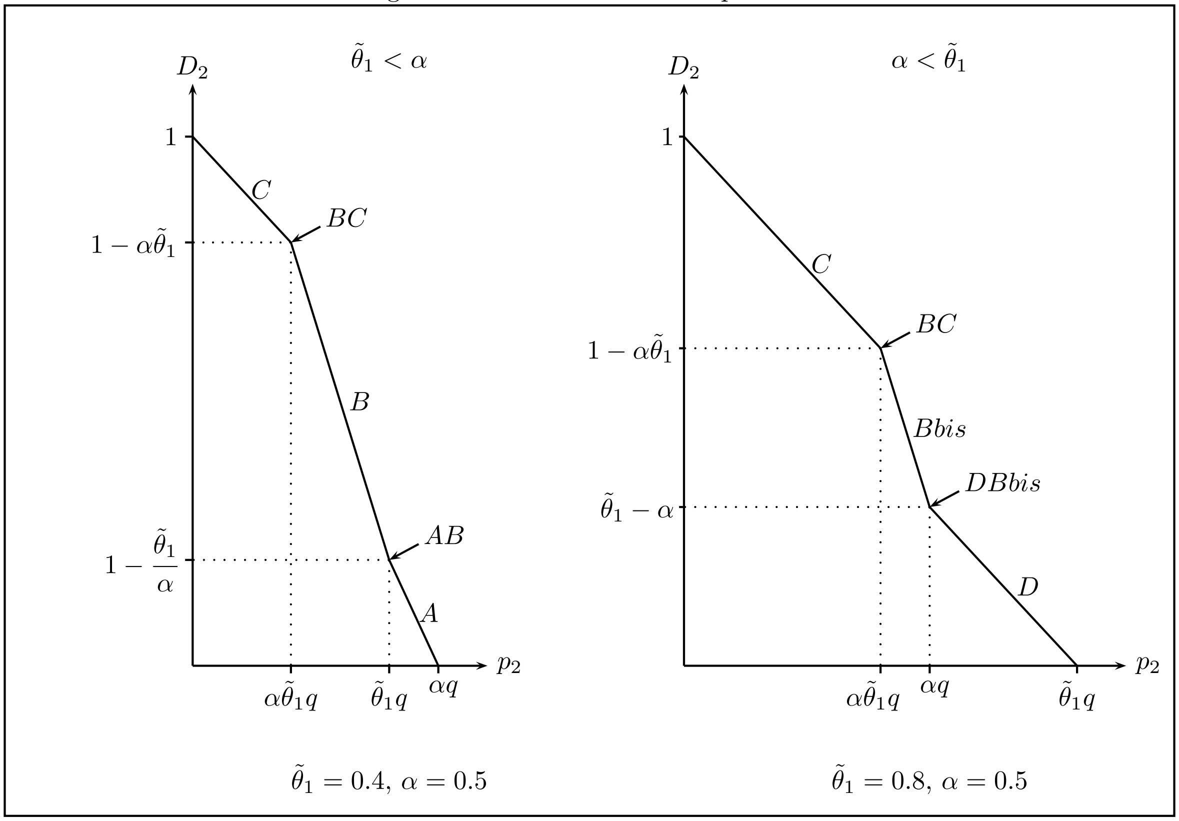
$$u_2(1, \theta_2) > u_2(0, \theta_2) \quad \Leftrightarrow \quad \theta_2 > \tilde{\theta}_2 \quad \text{with } \tilde{\theta}_2 = \frac{p_2}{q}. \tag{4}$$

This result is identical compared to a standard one period model. The decision at period 2 only depends on the preference parameter θ_2 that captures the initial preference (θ_1) and the decision at the first period (x_1).

The demand is however different compared to the standard one-period model because the distribution of θ_2 is not uniform. Let $\tilde{\theta}_1 \in [0, 1]$ be the preference of the indifferent consumer between buying and not buying in the first period. Hence all consumers such that $\theta_1 \in [\tilde{\theta}_1, 1]$ decide $x_1 = 1$ so that their preference becomes $\theta_2 = \alpha\theta_1$. Conversely, all consumers such that $\theta_1 \in [0, \tilde{\theta}_1]$ decide $x_1 = 0$ so that their preference is unchanged ($\theta_2 = \theta_1$). The distribution of θ_2 results from the aggregation of the distributions of two types of consumers.

Figure 1 illustrates the two possible distributions of θ_2. If $\tilde{\theta}_1 < \alpha$, the consumer with the highest θ_2 is a consumer who uses the product at period 1 and is such that $\theta_1 = 1$. Hence, the consumers with the highest willingness to pay for the product at period 2 are those who already use the product at period 1. They are represented in the zone A of Figure 1. Conversely, if $\alpha < \tilde{\theta}_1$, the consumer with the highest θ_2 is a consumer

Figure 2: Demand function at period 2



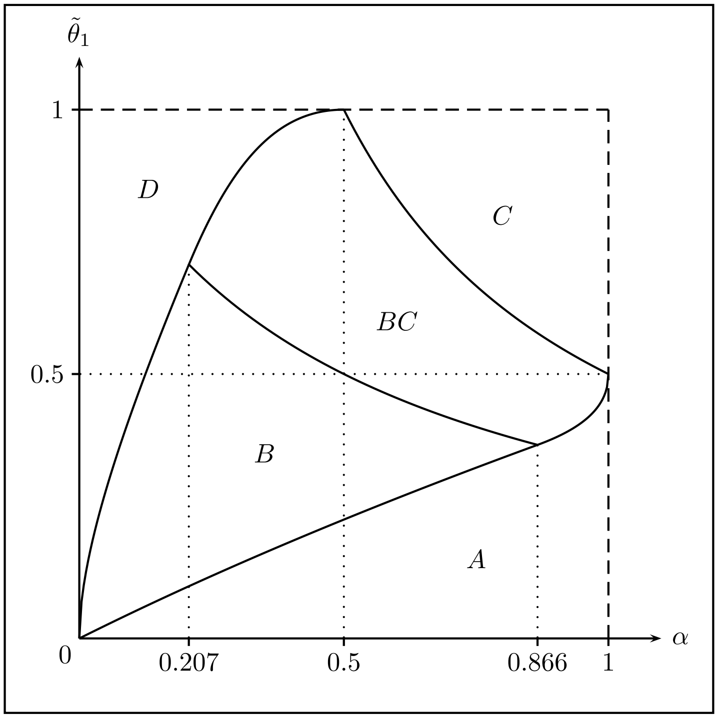
who did not use the product at period 1 and is such that $\theta_1 = \tilde{\theta}_1$. In that case, the consumers with the highest willingness to pay for the product at period 2 are those who did not buy the product at the first period. They are represented in the zone D of Figure 1.

The distribution of θ_2 leads to a kinked demand function such as the ones illustrated in Figure 2. We define D_2^i the demand on the portion $i \in \{A, B, Bbis, C, D\}$. All the D_2^i are linear and decreasing in p_2. The detailed expression of D_2^i is defined in Appendix A.1.

4.2 Monopoly pricing decision (p_2)

The monopoly defines p_2 to maximize its second period profit $\pi_2^i = D_2^i \cdot p_2$. A specific compilation is required to deal with the particular form of the demand function. We proceed as follows. We first maximize the profit with each fraction i of the demand function. In each case, the optimal price may be interior to the range of prices leading to D_2^i or it may correspond to one bound of this range. For example, when considering D_2^B, the price has to be between the two bounds p_2^{BC} and p_2^{AB}. Hence the optimal price is either interior (p_2^B) or equal to one of the two bounds. The optimal prices for all the fractions i are equilibrium candidates. Appendix A.2 details all these candidates and the set of values of $\tilde{\theta}_1$ and α where they can be considered.

Figure 3: Second period equilibrium



Note that candidates B and $Bbis$ are identical, so that we no longer use $Bbis$ and only consider B.

The equilibrium price can then be compiled by comparing the monopoly profit with the different candidates (see Appendix A.3 for more details). Figure 3 displays the second period equilibrium for any value of $\tilde{\theta}_1$ and α. We call $\hat{\theta}_1^{Y/Z}$ the threshold values that define the limits between candidates Y and Z. The thresholds are all defined in Table 1.

If the equilibrium corresponds to the case A, the consumers who buy in period 2 are among the consumers who already buy in period 1 (more precisely, those with the highest θ_1). In cases B, BC and C, some of the consumers who buy in period 2 already buy in period 1, but there are also consumers who buy in period 2 and do not buy in period 1. Cases BC and C differ from B by the fact that (in cases C and BC) all those who buy at period 1 also buy at period 2. In case D, the consumers who buy at period 2 are among those who do not buy at period 1. Hence, in that case, no consumer buys at both periods: consumers either buy once (either at period 1 or 2) or do not buy.

Table 2 synthesizes the price candidates and the corresponding monopoly profits. The following proposition summarizes some properties of this equilibrium:

Proposition 1 *The monopoly profit and price at the second period subgame equilibrium are: (i) increasing with the quality of the product (q); (ii) independent or increasing with the preference persistence (α); (iii)*

Table 1: Definition of the thresholds $\hat{\theta}_1$

$\hat{\theta}_1^{A/B} = \sqrt{1+\alpha} - 1$	$\hat{\theta}_1^{BC/C} = \dfrac{1}{2\alpha}$
$\hat{\theta}_1^{A/BC} = \dfrac{1-\sqrt{1-\alpha}}{2\alpha}$	$\hat{\theta}_1^{D/B} = \alpha + \sqrt{\alpha(1+\alpha)}$
$\hat{\theta}_1^{B/BC} = \dfrac{1}{1+2\alpha}$	$\hat{\theta}_1^{D/BC} = \dfrac{4\alpha}{1+4\alpha^2}$

Table 2: Second period price equilibrium and corresponding profit

Case	Price	Period 2 monopoly profit
A	$\dfrac{q\alpha}{2}$	$\dfrac{q\alpha}{4}$
B	$\dfrac{q\alpha(1+\tilde{\theta}_1)}{2(1+\alpha)}$	$\dfrac{q\alpha(1+\tilde{\theta}_1)^2}{4(1+\alpha)}$
BC	$q\alpha\tilde{\theta}_1$	$q\alpha\tilde{\theta}_1(1-\alpha\tilde{\theta}_1)$
C	$\dfrac{q}{2}$	$\dfrac{q}{4}$
D	$\dfrac{q\tilde{\theta}_1}{2}$	$\dfrac{q\tilde{\theta}_1^2}{4}$

independent or decreasing with the quantity of product bought at period 1 ($1 - \tilde{\theta}_1$).

These properties can be checked rather easily from the expressions of the second period profits and prices (detailed compilation is presented at the end of Appendix A.3). The property (i) is standard and observed in the benchmark case recalled above. It states that, for a given demand at period 2 (i.e. given values of α and $\tilde{\theta}_1$) the monopoly profit and price increase with the quality of the product. The property (ii) comes from the fact that, with a given value of $\tilde{\theta}_1$, the second period demand always increases if preferences are more persistent. Indeed, if a consumer uses the first period product, the higher is α, the higher is θ_2 for this consumer and consequently its willingness to pay for the product at the second period. The property (iii) comes also from the fact that the second period demand decreases as the quantity of product used at period 1 increases. Suppose that $\tilde{\theta}_1$ decreases by δ, then all consumers between $\tilde{\theta}_1 - \delta$ and $\tilde{\theta}_1$ (additional demand at period 1) have lower willingness to pay for the product at the second period, and this leads to a decrease of the demand at period 2.

5 First-period equilibrium

At period 1, the monopoly chooses the price p_1 and then consumers decide whether to buy or not the product. These decisions affect $\tilde{\theta}_1$ and consequently the equilibrium at the second period. Hence both the monopoly and the consumers have to anticipate the impact of their decisions at period 1 on the equilibrium at period 2. The period 1 equilibrium is solved backward. We first present the consumer demand, and then analyze the pricing equilibrium at period 1.

5.1 Consumer demand equilibrium

The period 1 consumer decision is defined as follows: all consumers such that $\theta_1 < \tilde{\theta}_1$ do not use the product, while all consumers such that $\theta_1 \geq \tilde{\theta}_1$ do use the product. Hence $\tilde{\theta}_1$ synthesizes all the consumers' decisions at period 1. $\tilde{\theta}_1$ is an equilibrium if, knowing p_1, α, q and the subgame equilibrium at period 2, each consumer prefers not to change its decision at period 1. We first analyze the choice of each consumer depending on θ_1. We then consider the choices of all the consumers (such that $\theta_1 \in [0, 1]$) depending on $\tilde{\theta}_1$ and define the equilibrium value of $\tilde{\theta}_1$ such that no consumer would like to modify its first period decision. This equilibrium value of $\tilde{\theta}_1$ depends on p_1 and this enables us to establish the first period demand.

Consider first the choice of each consumer depending on his preference parameter θ_1. The consumer compares its profit with $x_1 = 1$ *vs* $x_1 = 0$. Several comparisons are possible depending on the consequences of these two possible decisions on period 2. If $\theta_1 < \tilde{\theta}_2$ then the consumer chooses $x_2 = 0$ whatever x_1. The

Table 3: Individual consumer decision depending on his preference parameter (θ_1)

Condition on θ_1	Comparison to be made	Condition for … $x_1 = 1$	$x_1 = 0$
$\theta_1 < \tilde{\theta}_2$	$(0,0)$ *vs* $(1,0)$	$\theta_1 \geq p_1/q$	$\theta_1 \leq p_1/q$
$\theta_1 \in [\tilde{\theta}_2, \tilde{\theta}_2/\alpha]$	$(0,1)$ *vs* $(1,0)$	$p_1 \leq p_2$	$p_1 \geq p_2$
$\theta_1 > \tilde{\theta}_2/\alpha$	$(0,1)$ *vs* $(1,1)$	$\theta_1 \geq p_1/(q\alpha)$	$\theta_1 \leq p_1/(q\alpha)$

consumer compares the alternatives $(0,0)$ and $(1,0)$. If $\theta_1 \in [\tilde{\theta}_2, \tilde{\theta}_2/\alpha]$ then the consumer chooses $x_2 = 1$ if $x_1 = 0$ and $x_2 = 0$ if $x_1 = 1$. Hence, he compares $(0,1)$ and $(1,0)$. At last, if $\theta_1 > \tilde{\theta}_2/\alpha$, the consumer chooses $x_2 = 1$ whatever x_1, leading him to compare $(0,1)$ and $(1,1)$. The comparison of profit in each of these cases leads to:

$$
\begin{aligned}
v(1,0,\theta_1) > v(0,0,\theta_1) \quad &\Leftrightarrow \quad \theta_1 > \frac{p_1}{q} \\
v(1,0,\theta_1) > v(0,1,\theta_1) \quad &\Leftrightarrow \quad p_1 < p_2 \\
v(1,1,\theta_1) > v(0,1,\theta_1) \quad &\Leftrightarrow \quad \theta_1 > \frac{p_1}{\alpha q}
\end{aligned}
$$

All these results are synthesized in Table 3.

We now consider the choices of all consumers for $\theta_1 \in [0,1]$ to derive the equilibrium value of $\tilde{\theta}_1$. The compilation of the equilibrium value of $\tilde{\theta}_1$ is synthesized here but more details and illustrations are given in Appendix B.1. As it can be observed in Table 3 the consumers' decisions depend on θ_1, $\tilde{\theta}_2$, q, α, p_1 and p_2. The second-period (subgame) equilibrium values of $\tilde{\theta}_2$ and p_2 depend on $\tilde{\theta}_1$ and q (see Figure 3 and Table 2). Hence, for any value of $\tilde{\theta}_1$, and given q, α and p_1 it is possible to define all the conditions that need to be satisfied for $\tilde{\theta}_1$ to be an equilibrium value.

Suppose for example that $\tilde{\theta}_1$ and α are such that the period 2 equilibrium corresponds to the case A. We have $\tilde{\theta}_2^* = \alpha/2$ and we know that $\tilde{\theta}_1 < \tilde{\theta}_2^*$. All consumers such that $\theta_1 < \tilde{\theta}_1$ do not buy the product at period 2 ($\theta_2 < \tilde{\theta}_2^*$ whatever x_1), and they thus compare $(0,0)$ and $(1,0)$ and have to prefer $(0,0)$, because $\theta_1 < \tilde{\theta}_1$. All consumers such that $\theta_1 \in [\tilde{\theta}_1, \alpha/2]$ still do not buy the product at period 2 and make the same comparison between $(0,0)$ and $(1,0)$, but these ones have to prefer $(1,0)$ because $\theta_1 > \tilde{\theta}_1$. For the consumers such that $\theta_1 \in [\alpha/2, 1/2]$, we have $\theta_2 < \tilde{\theta}_2^*$ if $x_1 = 1$ and $\theta_2 > \tilde{\theta}_2^*$ if $x_1 = 0$. Hence, they have to compare $(1,0)$ with $(0,1)$ and to prefer $(1,0)$, because $\theta_1 > \tilde{\theta}_1$. At last, all consumers such that $\theta_1 > 1/2$ do buy the product at period 2 ($\theta_2 > \tilde{\theta}_2^*$ whatever x_1) so that they compare $(1,1)$ and $(0,1)$ and have to prefer $(1,1)$, because $\theta_1 > \tilde{\theta}_1$. For a given value of p_1 and if the period 2 subgame corresponds to case A, there is only one value of $\tilde{\theta}_1$ such that all these conditions are satisfied, that is, no consumer regrets its first period decision.

Table 4: Constraints that the first period equilibrium demand must fulfill

Period 2	$\tilde{\theta}_2^*$		Condition depending on θ_1
A	$\alpha/2$	*	$(0,0) \succ (1,0)$ for $\theta_1 \in [0, \tilde{\theta}_1]$
		*	$(1,0) \succ (0,0)$ for $\theta_1 \in [\tilde{\theta}_1, \alpha/2]$
			$(1,0) \succ (0,1)$ for $\theta_1 \in [\alpha/2, 1/2]$
			$(1,1) \succ (0,1)$ for $\theta_1 \in [1/2, 1]$
B	$\frac{\alpha(1+\tilde{\theta}_1)}{2(1+\alpha)}$		$(0,0) \succ (1,0)$ for $\theta_1 \in [0, \tilde{\theta}_2^*]$
		*	$(0,1) \succ (1,0)$ for $\theta_1 \in [\tilde{\theta}_2^*, \tilde{\theta}_1]$
		*	$(1,0) \succ (0,1)$ for $\theta_1 \in [\tilde{\theta}_1, \tilde{\theta}_2^*/\alpha]$
			$(1,1) \succ (0,1)$ for $\theta_1 \in [\tilde{\theta}_2^*/\alpha, 1]$
BC	$\alpha\tilde{\theta}_1$		$(0,0) \succ (1,0)$ for $\theta_1 \in [0, \alpha\tilde{\theta}_1]$
		*	$(0,1) \succ (1,0)$ for $\theta_1 \in [\alpha\tilde{\theta}_1, \tilde{\theta}_1]$
		*	$(1,1) \succ (0,1)$ for $\theta_1 \in [\tilde{\theta}_1, 1]$
C	$1/2$		$(0,0) \succ (1,0)$ for $\theta_1 \in [0, 1/2]$
			$(0,1) \succ (1,0)$ for $\theta_1 \in [1/2, 1/(2\alpha)]$
		*	$(0,1) \succ (1,1)$ for $\theta_1 \in [1/(2\alpha), \tilde{\theta}_1]$
		*	$(1,1) \succ (0,1)$ for $\theta_1 \in [\tilde{\theta}_1, 1]$
D	$\tilde{\theta}_1/2$		$(0,0) \succ (1,0)$ for $\theta_1 \in [0, \tilde{\theta}_1/2]$
		*	$(0,1) \succ (1,0)$ for $\theta_1 \in [\tilde{\theta}_1/2, \tilde{\theta}_1]$
		*	$(1,0) \succ (0,1)$ for $\theta_1 \in [\tilde{\theta}_1, 1]$

** Most constraining conditions that define $\tilde{\theta}_1$ as a function of p_1.*

We can observe actually that, among all the conditions corresponding to the different consumers, the most constraining ones are those for θ_1 close to $\tilde{\theta}_1$. In the case considered here, we need to have $(0,0) \succ (1,0)$ for $\theta_1 < \tilde{\theta}_1$ and $(1,0) \succ (0,0)$ for $\theta_1 \in [\tilde{\theta}_1, \alpha/2]$. Hence, we require both $\theta_1 \geq p_1/q$ for $\theta_1 \leq \tilde{\theta}_1$ and $\theta_1 \leq p_1/q$ for $\theta_1 \geq \tilde{\theta}_1$. This is possible only if $\tilde{\theta}_1 = p_1/q$, and this value also fulfills the conditions for $\theta_1 < \alpha/2$. In summary, $\tilde{\theta}_1^A = p_1/q$ defines the first period demand equilibrium leading to case A at the second period.

Table 4 synthesizes the constraints that must be checked for $\tilde{\theta}_1$ to be an equilibrium at period 1, with all the possible equilibria at period 2. Proceeding like for the case A, the equilibrium value of $\tilde{\theta}_1$ can be defined, by focusing on the constraints that are just below and above $\tilde{\theta}_1$ (see Appendix B.1 for a more detailed resolution):

In case BC, we have $\tilde{\theta}_1^{BC} = p_1/(\alpha q)$ because $(0,1) \succ (1,0)$ is equivalent to $(0,1) \succ (1,1)$ since $p_2^{BC} = \tilde{\theta}_1 \alpha q$. Hence we need to have both $(0,1) \succ (1,1)$ and $(1,1) \succ (0,1)$, which lead to $\tilde{\theta}_1^{BC} = p_1/(\alpha q)$.

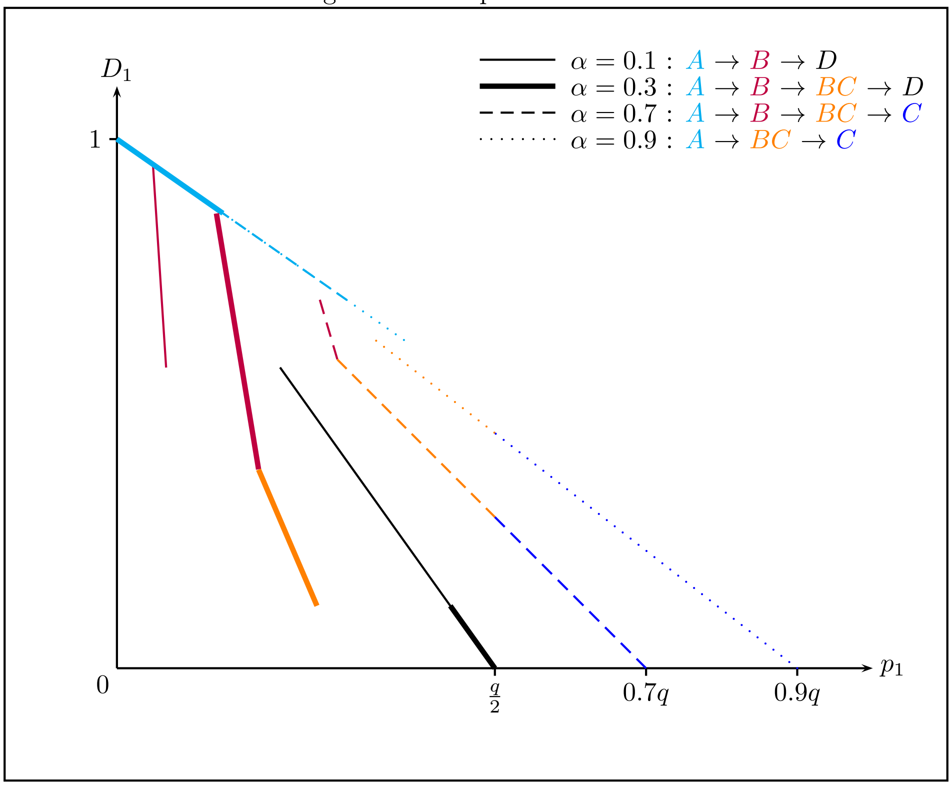
In case C, we have $\tilde{\theta}_1^C = p_1/(\alpha q)$ because we need to have both $(0,1) \succ (1,1)$ and $(1,1) \succ (0,1)$.

At last in cases B and D, we have $p_1 = p_2$ because we need to have both $(0,1) \succ (1,0)$ and $(1,0) \succ (0,1)$. As p_2 is a function of $\tilde{\theta}_1$ in both cases, by inverting this function we get the equilibrium value of $\tilde{\theta}_1$ as

Table 5: Equilibrium decision of consumers

Case	A	B	BC or C	D
$\tilde{\theta}_1$	$\dfrac{p_1}{q}$	$\dfrac{2(1+\alpha)p_1}{\alpha q} - 1$	$\dfrac{p_1}{\alpha q}$	$\dfrac{2p_1}{q}$

Figure 4: First-period demand



function of p_1. More precisely, we get

$$p_1 = p_2^B = \frac{q\alpha(1-q)(1+\tilde{\theta}_1)}{2(1+\alpha)} \quad \Leftrightarrow \quad \tilde{\theta}_1^B = \frac{2(1+\alpha)p_1}{\alpha q} - 1$$

$$p_1 = p_2^C = \frac{q\tilde{\theta}_1}{2} \quad \Leftrightarrow \quad \tilde{\theta}_1^D = \frac{2p_1}{q}$$

Table 5 synthesizes the equilibrium values of $\tilde{\theta}_1$ depending on the equilibrium at period 2. Each of these values are valid over the ranges defined in Figure 3. These ranges can be translated as ranges of values over p_1. For example, in the case A with $\alpha < 0.866$ we have:

$$0 \leq \tilde{\theta}_1^A \leq \hat{\theta}_1^{A/B} \quad \Leftrightarrow \quad 0 < p_1 < q(\sqrt{1+\alpha} - 1)$$

All conditions on p_1 are summarized in Appendix B.1.2 and Figure 4 illustrates the first period demand with different preference persistence levels. Notice that the four levels considered in this figure covers all possible forms of first period demand.[3]

Several observations are worth making on Figure 4. Whatever the range of price, the first period demand decreases with p_1 but with different slopes and intercepts. This corresponds to the fact that equilibrium values of $\tilde{\theta}_1$ are increasing with p_1. Second, all the conditions on p_1 defining the different parts of the demand function are linear in q. As a consequence, a change in q only affects the scale on the horizontal axis of Figure 4. Third, when the first period demand is close to $1-\hat{\theta}_1^{A/B}$ (for $\alpha < 0.866$) or $1-\hat{\theta}_1^{A/BC}$ (for $\alpha > 0.866$), there is a range of prices where there are two demand levels for a given price: one level leading to the case A at period 2, and one level leading to the case B or BC at period 2. For this range of prices, we have two demand equilibria at period 1. Fourth, for $\alpha < 0.5$, there is a discontinuity in the range of prices leading to a first period demand close to $1-\hat{\theta}_1^{D/B}$ (for $\alpha < 0.207$) or $1-\hat{\theta}_1^{D/BC}$ (for $\alpha > 0.207$). In other words, there is a range of intermediary prices where the demand is not defined.

At last the following proposition summarizes the impact of q and α on the demand.

Proposition 2 *The first period demand is (i) increasing with the quality, and (ii) independent or increasing with the persistence of the consumers' preference.*

Remind that the demand is $1-\tilde{\theta}_1$. We can indeed check that the values of $\tilde{\theta}_1$ summarized in Table 5 are decreasing with q and independent or decreasing with α. Figure 4 illustrates this property.

5.2 Monopoly pricing

At the first period, the monopoly chooses the price that maximizes its cumulated profit over the two periods. Because of the particular form of the first-period demand function (see Figure 4), we first compile the equilibrium candidate for each portion of the demand function and then define the equilibrium by comparing the monopoly profit between these candidates. To better present the properties of the model, we first consider the candidate when no consumer buys the product twice, and then the candidate when some consumers buy twice and, at last, compare these two candidates.

- Monopoly pricing if no consumer buys the product twice. In this case D, the cumulated profit of the monopoly is:

$$\Pi = p_1(1-\tilde{\theta}_1) + \frac{q\tilde{\theta}_1^2}{4}$$

[3] More precisely, with $\alpha < 0.207$ the first period demand is $1-\tilde{\theta}_1^A$ for $\tilde{\theta}_1 \in [0, \hat{\theta}_1^{A/B}]$, $1-\tilde{\theta}_1^B$ for $\tilde{\theta}_1 \in [\hat{\theta}_1^{A/B}, \hat{\theta}_1^{D/B}]$, $1-\tilde{\theta}_1^D$ for $\tilde{\theta}_1 \in [\hat{\theta}_1^{D/B}, 1]$. In other words, the sequence is $A \to B \to D$. As the persistence increases this sequence becomes $A \to B \to BC \to D$ for $\alpha \in [0.207, 0.5]$, $A \to B \to BC \to C$ for $\alpha \in [0.5, 0.866]$ and $A \to BC \to C$ for $\alpha \in [0.866, 1]$.

With $\tilde{\theta}_1 = 2p_1/q$, we get

$$\Pi = p_1\left(1 - \frac{2p_1}{q}\right) + \frac{p_1^2}{q}$$

This profit is concave in p_1 and maximum for $p_1 = q/2$ leading to $\tilde{\theta}_1 = 1$ and $\Pi^D = q/4$.

Remind that, in this case, the first period demand equilibrium requires that $p_1 = p_2$. The cumulated profit is equal to the total quantity times this price. Total quantity is equal to $(1-\tilde{\theta}_1)+(\tilde{\theta}_1-\tilde{\theta}_1/2) = 1-\tilde{\theta}_1/2$ and this is maximum for the lowest value of $\tilde{\theta}_1$ which is $\hat{\theta}_1^{D/B}$ or $\hat{\theta}_1^{D/BC}$. However, reaching this quantity requires to define low price $p_1 = q\tilde{\theta}_1/2$: low $\tilde{\theta}_1$ implies low p_1 and p_2. Hence we have a price and a quantity effect that goes in opposite directions, and compilation shows that the price effect dominates the quantity effect. The best choice is to choose the lowest quantity sold at period 1 (i.e. $\tilde{\theta}_1 = 1$), a situation which enables the monopoly to charge rather high prices.

Note that in this extreme case where $\tilde{\theta}_1 = 1$, the last condition presented in Table 4 ($(1,0) \succ (0,1)$) does not need to be checked. Hence the equilibrium pricing is $p_1 \geq q/2$ leading to $\tilde{\theta}_1 = 1$ and $p_2 = q/2$ leading to $\tilde{\theta}_2 = 1/2$. Finally, no sale occurs at period 1 and period 2 corresponds to the standard one period problem. This result can be summarized with the following proposition.

Proposition 3 *The monopoly does not earn more by selling over two periods in the case where no consumers buy the product twice.*

- Monopoly pricing if some consumers buy the product twice. Candidates A, B, BC and C are possible in this case. For each case, we compile an optimal price that maximizes the monopoly cumulated profit and the value of $\tilde{\theta}_1$ that corresponds to this price and, at last, we check whether this value is in the zone specific to the case that is considered (cf. Figure 3) The detailed compilation is provided in Appendix B.2. For example in the case A optimal (unconstrained) price is $q/2$ leading to $\tilde{\theta}_1 = 1/2$ which is out of the zone where the equilibrium at period 2 corresponds to A. Hence, in this case, the monopoly is constrained to choose $p_1 = q\hat{\theta}_1^{A/BC}$ or $p_1 = q\hat{\theta}_1^{A/B}$ leading to $\tilde{\theta}_1 = \hat{\theta}_1$. Table 6 details all the pricing candidates and the corresponding values of $\tilde{\theta}_1$. Figure 5 synthesizes the value of $\tilde{\theta}_1$ that corresponds to each equilibrium candidate. We can see from this figure that, in the cases A and B, the monopoly is constrained to choose the maximum p_1 leading to the maximum value of $\tilde{\theta}_1$ in this zone. Conversely, in the case C the monopoly is constrained to use the minimum value of p_1 leading to the minimum value of $\tilde{\theta}_1$ in this zone. The case BC is the only one where the equilibrium candidate corresponds to an interior solution (as long as $\alpha > 0.25$).

Table 6 also reports the comparison of the optimal prices at the first and second period in each case. In the case A, the period 1 price is lower compared to the period 2 price. However this result is only due to

Table 6: First period equilibrium candidates

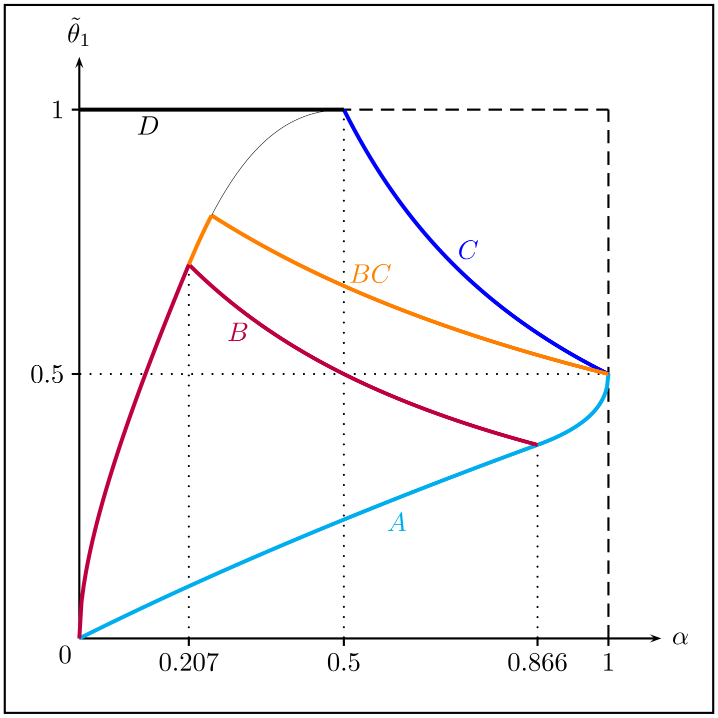
Case	Condition	p_1	$p_1 \lesseqqgtr p_2$	$\tilde{\theta}_1$
$A.1$	$\alpha \in [0, 0.866]$	$q\hat{\theta}_1^{A/B}$	$p_1 < p_2$	$\hat{\theta}_1^{A/B}$
$A.2$	$\alpha \in [0.866, 1]$	$q\hat{\theta}_1^{A/BC}$	$p_1 < p_2$	$\hat{\theta}_1^{A/BC}$
$B.1$	$\alpha \in [0, 0.207]$	*see Appendix B.2*	$p_1 = p_2$	$\hat{\theta}_1^{B/D}$
$B.2$	$\alpha \in [0.207, 0.866]$	$\dfrac{\alpha q}{1-2\alpha}$	$p_1 = p_2$	$\hat{\theta}_1^{B/BC}$
$BC.1$	$\alpha \in [0.207, 0.25]$	*see Appendix B.2*	$p_1 = p_2$	$\hat{\theta}_1^{D/BC}$
$BC.2$	$\alpha \in [0.25, 1]$	$\dfrac{\alpha q}{1+\alpha}$	$p_1 = p_2$	$\dfrac{1}{1+\alpha}$
C	$\alpha \in [0.5, 1]$	$\dfrac{q}{2}$	$p_1 = p_2$	$\hat{\theta}_1^{BC/C}$

the constraint $p_1 < q\hat{\theta}_1$. Indeed without constraint, the monopoly would actually choose $p_1 > p_2^A$. Hence the monopoly has to strongly distort its first period price to respect the condition leading to the case A at the second period. In all the other cases, the prices are identical at the two periods. In cases B and BC we have observed before that this constraint is necessary to have an equilibrium on the demand side. In case C this result is related to the fact that the monopoly is constrained to choose a minimum value for p_1, and this minimum value actually corresponds to p_2.

At last we can search for the optimal price by comparing the profit with each equilibrium candidate. For the time being, we limit this comparison to the cases where some consumers buy the product twice. It can be observed that, once p_1 is introduced in the profit function, the profit with each candidate is linear with q with a coefficient that only depends on α. Figure 6 graphs the profit level with each of the equilibrium candidate, as a function of α. With each candidate, the profit is increasing with α. This profit tends to 0 as α goes to 0. As α is close to 0, A and B are the only possible cases where some consumers are buying. The cumulated profit tends to 0 because these cases require a very low value of $\tilde{\theta}_1$ (see Figure 5) if α goes to 0. As a consequence, the monopoly profit is very close to 0 because p_1 is constrained to be close to 0 and the second-period profit is also close to 0 because the willingness to pay of consumers at the second period is very low is cases A and B if α is close to 0. Conversely, if α equals 1, preferences are fully persistent and the consumption at period 1 does not affect the second-period preference. The benchmark one period model is repeated twice and the cumulated monopoly profit is twice the profit over one period.

Figure 6 helps comparing equilibrium candidates. We can observe directly that, if the monopoly wants to have some consumers buying twice, its best pricing is p_1^{BC} as long as the persistence level enables to reach the case BC at the second period ($\alpha \in [0.207, 1]$). Otherwise the monopoly chooses p_1^B which can be considered as a second best. The interest for BC can be explained by two complementary arguments. Firstly, at the second period, BC corresponds to a kink of the demand curve (see Figure 2) where the

Figure 5: Synthesis of local optima at period 1



demand becomes much less elastic if the monopoly choose a price below p_2^{BC}. The marginal increase of quantity from decreasing the price becomes much smaller below p_2^{BC}. Secondly, at the first period, BC is the only case where the price candidate can be an interior solution as illustrated in Figure 5. Conversely, there is no interior solution in the cases A, B and C. Cases A and B require $\tilde{\theta}_1$ (and thus p_1) to be lower than the interior solution and the converse holds for the case C.

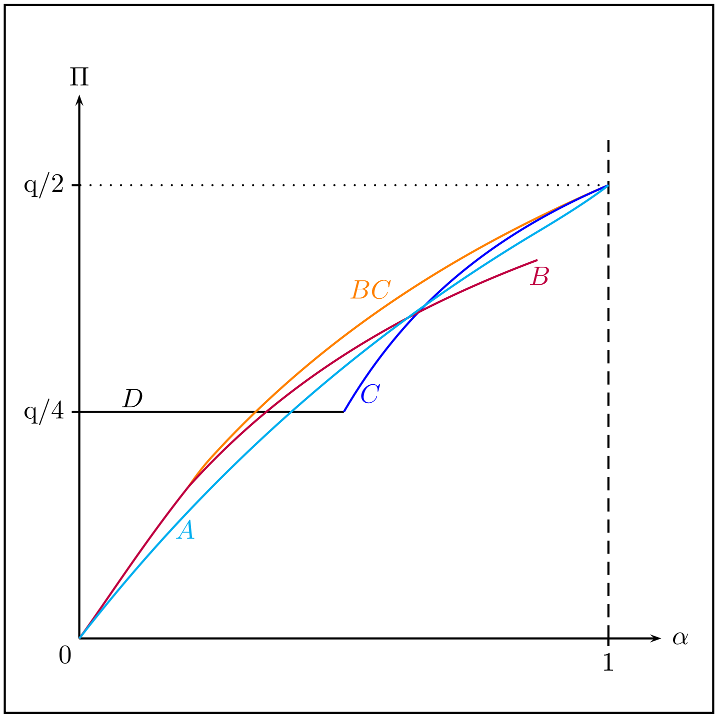
The following proposition summarizes the property of the price.

Proposition 4 *If, at period 2, the monopoly sells part of its products to consumers that already buy at period 1, then the best strategy is to define the same price at the two periods.*

The monopoly does not price discriminate over time because consumers fully anticipate the second period equilibrium. Indeed, rational consumers can make an arbitrage between buying at period 1 and 2. If p_2 is expected to be higher than p_1, part of the consumers switch their purchase from the second to the first period, and this drives down p_2 to the point where it equals p_1 (cf. proposition 1). Note that this result would not hold with myopic consumers. If consumers were not able to anticipate the consequences of their first-period decision on the second-period equilibrium, then intertemporal price discrimination would indeed take place.[4]

[4]The detailed compilation of the equilibrium with myopic consumers is relegated to a Supplementary Appendix available

Figure 6: Cumulated profit with each local optimum



- Comparison of pricing candidates. We finally compare the equilibrium in the case where no consumer buys at both periods (case D, which corresponds to the benchmark) and the case where some consumers buy at both periods (case BC or B). We can directly see from Figure 6 that, at equilibrium, some consumers buy twice only if the persistence of preferences is high enough. The detailed comparison is reported in Appendix B.2.2. Remind also that we observed multiple period 1 demand equilibria for some range of p_1 values. Appendix B.2.3 shows that the equilibrium price (p_1^{BC} or p_1^D depending on α) does not fall in this range.

Proposition 5 *If $\alpha > 1/3$ the monopoly increases its profit (compared to the benchmark) by selling over two periods at the same price. This price is a limit price such that all consumers that buy in the first period also buy in the second period ($\tilde{\theta}_2 = \alpha\tilde{\theta}_1$).*

Summarizing, for $\alpha \geq 1/3$, the equilibrium is interior, that is BC, whereas for $\alpha < 1/3$, the equilibrium is D. Looking at the latter corner equilibrium D, we observe that nobody buys in period 1, the monopolist prefers to serve half of the consumers only in period 2. Note that this equilibrium takes place when preference persistence is low, so it is quite intuitive that consumers use the product only once.

from the authors upon request.

In the interior equilibrium BC, we observe that consumers that buy the product in period 1 also buy it in period 2, and there are few additional consumers that only buy in period 2. The monopoly does not price discriminate over time because consumers fully anticipate the second period equilibrium, as we explain before.

6 Second-period price discrimination and product bundling

We consider here the possibility for the monopoly to price discriminate at period 2 between consumers who already buy the product (called *past* buyers) and those who do not (called *new* buyers). We next introduce the possibility for the monopoly to sell a bundle of two units of product at period 1. In both cases we rule out resale.

Regarding second-period price discrimination, two cases can emerge. In the first case, the monopolist cannot track a consumer's behavior in the first period individually and set a specific price to him. As a consequence, there is an incentive constraint such that the monopoly cannot charge a higher price to the past consumer because the past consumer would then show up as a new consumer. This case appears for example if a coupon is given to the past consumers, this coupon offering only a discount. Alternatively, we can think that the monopolist can track the purchasing history of the consumers and set a specific price that depends on this history. In this case there is no incentive constraint on the prices set in the second period for price discrimination to take place.

The resolution is detailed in Appendix C. At period 2, the monopoly faces two separate demand functions corresponding to past and new buyers. These two demands depend on $\tilde{\theta}_1$ and α. Price discrimination such that the monopolist is constrained to reward the past buyers is possible only for low enough values of the persistence parameter ($\alpha < min\left\{0.5, \tilde{\theta}_1\right\}$). In contrast, for high levels of α, the monopolist can implement price discrimination only if he knows the consumers' past history. We also check that the equilibrium at period 2 leads to higher profit for the monopoly compared to what he gets in the baseline model. The period 1 is solved using the same method as the one we used in the baseline model: both consumers and monopoly anticipate the consequences of their first-period decisions on the second-period equilibrium and maximize their utility or profit over the two periods. Resolution shows that the equilibrium has similar properties compared to the equilibrium BC in the baseline model: all past buyers buy at period 2 and the price for the new buyers at period 2 is equal to the price at period 1.

Under price discrimination the monopoly cumulated profit is higher compared to the baseline model if and only if $\alpha < 0.5$, that is, if the price paid by past buyers is lower than the one paid by new buyers. In particular, with $\alpha < 0.33$ price discrimination leads the monopoly to sell at the two periods (with a strategy rather equivalent to BC) instead of selling only during one period (equilibrium D). In this last case (with $\alpha < 0.33$) more product is sold, leading to an increase of the total welfare over the two periods. Conversely, if $\alpha \in [0.33, 0.5]$ less product is sold with price discrimination (compared to BC in the baseline model), leading to a decrease of the total welfare.

At last we introduce the possibility for the monopoly to sell a bundle of two units of product at period 1. One consumer buying this bundle will use one unit at period 1 and one unit at period 2. This consumer leaves the market after period 1 and does not need to be considered by the monopoly when defining the second period price. Formal details (see Appendix D) show that only pure bundling, i.e. selling only the bundle at period 1, can be profitable for the monopoly, whereas there is no interest for mixed bundling, i.e. selling both the bundle and a single unit of the product at period 1. The equilibrium is equivalent to the one with price discrimination: the price of the bundle equals the sum of the price at period 1 plus the price paid by past buyers at period 2; and all consumers who buy at period 1 also use the product at the second period.

These results are summarized in the following proposition:

Proposition 6 *Price discrimination at the second period or pure bundling at period 1 are equivalent. These strategies enable the monopoly to increase its profit if the persistence of preferences is low enough ($\alpha < 0.5$). Either strategy leads to a welfare gain if it enables the monopoly to sell its product at two rather than only one period ($\alpha < 0.33$). Otherwise (if $\alpha \in [0.33, 0.5]$) it leads to a welfare loss.*

A remark is worthwhile concerning price discrimination in the second period. Namely, for $\alpha > 1/2$ this strategy is not profitable for the monopolist. Note that in this case the change in preferences induced by the purchase in the first period is quite low, so that we are close to the classic result by Fudenberg and Villas-Boas (2007) where preferences are stable over time so that a high valuation of the good in the first period implies a high valuation also in the second period. In particular, for $\alpha > 1/2$, the price set for new buyers in the second period is lower than the price set for the past buyers. The consequence is that some consumers that would buy under non price discrimination refrain from buying in the first period because they foresee they can get a lower price in the second period (as they will be identified as low valuation consumers). This induces a lower demand (and price) in the first period with respect to the non price discrimination scenario. The larger demand in $t = 2$ due to the presence of new buyers (that are absent under non price-discrimination) does not compensate the lower profit in $t = 1$ as these new buyers buy the product at a lower price than the past buyers.

This result overturns for low values of the persistence of preferences ($\alpha < 1/2$): the monopolist has incentive to reward past buyers with a lower price in $t = 2$ with respect to the price set for the new buyers.

This result is new with respect to the previous literature and it is due to the fact that a low persistence of preferences implies a large drop in the willingness to pay of past buyers in the second period. Anticipating that they will be rewarded by a lower price in the second period, some consumers that would not buy under non price-discrimination decide to buy in the first period. That is, the demand (as well as the price) in the first period is larger than under non price discrimination. As our result states, conditioning the price on purchasing history is profitable for the monopolist.

7 Extension: relating preference persistence with product quality

Until now, we have supposed that preference persistence was independent from product quality. However, we can expect these two parameters to be related, and more particularly to be negatively related. Indeed the consumption of a product with higher quality may provide a high enough utility which would make a second purchase useless. The argument can also be turned the other way by saying that the quality of a product could be considered as higher if the second purchase of the product is less interesting.

Pesticide used by farmers illustrates this case. Pesticides are used in order to limit the loss caused by pests, and high quality pesticide leads to a more important reduction of this loss by inducing a drastic decrease of the size of the pest population. A farmer can apply a pesticide once or several times over the production period. Several applications are necessary when the size of the pest population remains important, which is the case with low quality pesticide. Hence the persistence of farmers' preference for buying a pesticide a second time is negatively related to the quality of this product.

We analyze an extended version of the model where we suppose that $\alpha = 1 - q$ with $q \in [0, 1]$.[5] Most of the results obtained with the baseline version of the model are maintained. In particular, at the equilibrium, the first and second period prices are still identical (Proposition 4) and behavior based price discrimination is still interesting with low preference persistence, i.e. with high quality (proposition 6).

The main difference comes from the fact that the product quality has now two contrary effects when the product is bought twice (i.e. all cases except those leading to D at period 2). On the one hand, quality has the classic positive effect on utility, demand at the current period and supplier profit. However, on the other hand, quality now decreases utility persistence, leading to a decrease of the second period utility, second period demand and second period profit. The first effect dominates the second one for low quality levels and the converse holds with high quality levels. As a consequence, in cases where the farmer buys the product twice, both the second period and the cumulated monopoly profit are concave with the quality.

[5] The detailed resolution of this version of the model is relegated to a Supplementary Appendix available from the authors upon request.

The threshold over which these profits are decreasing with the quality are different depending on the cases, but they are generally close to 0.5.

8 Conclusion

We have investigated the price discrimination incentives over time by a monopolist in a framework where consumers may buy one or two units of a good during two successive periods. The novelty of our analysis comes from the assumption that consumers' preferences are affected by past purchases, so that they endogenously adjust over time.

We have distinguished two cases: uniform pricing across consumers' types in each period and behavior-based price discrimination (BPD). Interestingly, from the formal point of view, the uniform pricing case reveals to be more complex than the BPD scenario. Indeed, in the former the distribution of preferences in the second period results from the aggregation of the distributions of two types of consumers, new and past buyers; in contrast, in the latter the monopolist faces two perfectly separated markets, so that the derivation of the demand in the second period is standard.

We have shown that in case of uniform pricing across consumers' types in each period, the monopolist has incentives to make some consumers buy the product twice only if the preference persistence is sufficiently high. This allows the monopolist to increase its profit compared to the standard one-period monopoly. In contrast, if the preference persistence is low, the best strategy is to sell its product only in the second period, leading to the one-period market equilibrium. In any case, the monopolist does not discriminate over time.

Comparing this baseline model to the case where the monopoly can implement BPD, or sell a bundle of two units of product at period 1 (pure bundling), we have shown that these two strategies are equivalent and they are profitable only if the preference persistence is low enough: they enable the monopoly to escape from the one-period equilibrium and, in this case, they are welfare improving.

To the best of our knowledge, the contributions in the related IO literature assume that the preferences of consumers are not affected by past purchases. There are however many goods that might be purchased more than once affecting consumers' willingness to pay over time. Our monopoly framework is a first step in understanding the pricing dynamics for these types of products. There are at least two interesting extensions that are worth mentioning. The first one regards supply side strategies that could affect the persistence of preferences through, for instance, the design of the product, the choice of the quality that could be related to the product life duration, or advertising. The second extension is related to the introduction of duopoly competition. This could be either in the form of a sequential duopoly (where the first firm supplies the product in the first period and the second firm supplies the product at the second period), or considering the two firms competing in both periods. In the latter case both intra-temporal and inter-temporal competition would take place.

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Appendix

A Period 2 demand and monopoly pricing

A.1 Period 2 demand function

Figure 1 synthesizes the distribution of θ_2.

Consider first the case where $\tilde{\theta}_1 < \alpha$. In order to define the demand in period 2, D_2, we need to distinguish three possibilities according to the position of $\tilde{\theta}_2 = p_2/q$ with respect to $\tilde{\theta}_1$:

A $\tilde{\theta}_2 \in [\tilde{\theta}_1, \alpha]$, that is equivalent to $p_2 \in [q\tilde{\theta}_1, \alpha q]$: the demand is then $D_2^A = \frac{\alpha - \tilde{\theta}_2}{\alpha} = 1 - \frac{p_2}{\alpha q}$.

B $\tilde{\theta}_2 \in [\tilde{\theta}_1 \alpha, \tilde{\theta}_1]$, that is equivalent to $p_2 \in [\alpha q\tilde{\theta}_1, q\tilde{\theta}_1]$: the demand is then $D_2^B = \frac{\alpha - \tilde{\theta}_1}{\alpha} + (\tilde{\theta}_1 - \tilde{\theta}_2)(1 + \frac{1}{\alpha}) = 1 + \tilde{\theta}_1 - \frac{p_2(1+\alpha)}{\alpha q}$.

C $\tilde{\theta}_2 \in [0, \alpha\tilde{\theta}_1]$, that is equivalent to $p_2 \in [0, \alpha q\tilde{\theta}_1]$: the demand is then $D_2^C = \frac{\alpha - \tilde{\theta}_1}{\alpha} + (\tilde{\theta}_1 - \alpha\tilde{\theta}_1)(1 + \frac{1}{\alpha}) + \alpha\tilde{\theta}_1 - \tilde{\theta}_2 = 1 - \frac{p_2}{q}$.

Consider now the case where $\alpha < \tilde{\theta}_1$.

D $\tilde{\theta}_2 \in [\alpha, \tilde{\theta}_1]$, that is equivalent to $p_2 \in [\alpha q, q\tilde{\theta}_1]$: the demand is then $D_2^D = \tilde{\theta}_1 - \tilde{\theta}_2 = \tilde{\theta}_1 - \frac{p_2}{q}$.

Bbis $\tilde{\theta}_2 \in [\alpha\tilde{\theta}_1, \alpha]$, that is equivalent to $p_2 \in [\alpha q\tilde{\theta}_1, \alpha q]$: the demand is then $D_2^{Bbis} = \tilde{\theta}_1 - \alpha + (\alpha - \tilde{\theta}_2)(1 + \frac{1}{\alpha}) = 1 + \tilde{\theta}_1 - \frac{p_2(1+\alpha)}{\alpha q}$.

C $\tilde{\theta}_2 \in [0, \alpha\tilde{\theta}_1]$. The demand is identical to D_2^C defined above. Indeed we have: $D_2^{Cbis} = \tilde{\theta}_1 - \alpha + (\alpha - \tilde{\theta}_2)(1 + \frac{1}{\alpha}) + \alpha\tilde{\theta}_1 - \tilde{\theta}_2 = 1 - \tilde{\theta}_2 = 1 - \frac{p_2}{q}$.

We can observe that $D_2^B = D_2^{Bbis}$ except that the range of values for p_2 is different for these two parts of the demand function. This is the reason why we keep the distinction between these two cases in the resolution of period 2 equilibrium.

A.2 Pricing candidates at period 2

Period 2 monopoly profit writes as $\pi_2^i = D_2^i \cdot p_2$, with $i \in \{A, B, Bbis, C, D\}$. For each of the five cases, we maximize the period 2 profit with the specific demand expression. The result provides an equilibrium candidate. We also characterize the parameters' restrictions under which each candidate leads indeed to the corresponding demand expression. Once each equilibrium candidate is compiled, we compare the corresponding monopoly profit to find the optimal price. Note that both equilibrium candidates and optimal prices will depend on $\tilde{\theta}_1$, q and α.

- **Case** A**.** For $\tilde{\theta}_2 \in [\tilde{\theta}_1, \alpha]$. Maximization of $\pi_2^A = D_2^A \cdot p_2$ with respect to p_2 leads to the following price candidate

$$p_2^A = \frac{\alpha q}{2}$$

This price candidate implies that $\tilde{\theta}_2 = \frac{\alpha}{2}$. We can check that $\tilde{\theta}_2 < \alpha$. However $\tilde{\theta}_2 > \tilde{\theta}_1$ only if $\tilde{\theta}_1 < \frac{\alpha}{2}$. If, instead, $\tilde{\theta}_1 > \frac{\alpha}{2}$ the candidate is the corner solution leading to the minimum value of θ_2 in the region A, that is:

$$\tilde{\theta}_2 = \tilde{\theta}_1 \quad \Leftrightarrow \quad p_2^{AB} = q\tilde{\theta}_1$$

- **Case** B**.** For $\tilde{\theta}_2 \in [\alpha\tilde{\theta}_1, \tilde{\theta}_1]$. The interior equilibrium price candidate is

$$p_2^B = \frac{\alpha q(1+\tilde{\theta}_1)}{2(1+\alpha)}$$

This candidate implies that $\tilde{\theta}_2 = \frac{\alpha(1+\tilde{\theta}_1)}{2(1+\alpha)}$. This value is included in $[\alpha\tilde{\theta}_1, \tilde{\theta}_1]$ iff $\tilde{\theta}_1 \in \left[\frac{\alpha}{2+\alpha}, \frac{1}{1+2\alpha}\right]$. If, instead, $\tilde{\theta}_1 < \frac{\alpha}{2+\alpha}$ the candidate is the corner equilibrium price p_2^{AB} leading to $\tilde{\theta}_2 = \tilde{\theta}_1$. Finally, if $\tilde{\theta}_1 > \frac{1}{1+2\alpha}$ the candidate is the corner equilibrium price such that $\tilde{\theta}_2 = \alpha\tilde{\theta}_1$

$$\tilde{\theta}_2 = \alpha\tilde{\theta}_1 \quad \Leftrightarrow \quad p_2^{BC} = \alpha q\tilde{\theta}_1.$$

- **Case** $Bbis$**.** For $\tilde{\theta}_2 \in [\alpha\tilde{\theta}_1, \alpha]$. The interior equilibrium price candidate is the same as the one defined in the case B ($p_2^{Bbis} = p_2^B$). This candidate implies that $\tilde{\theta}_2 = \frac{\alpha(1+\tilde{\theta}_1)}{2(1+\alpha)}$. We can observe that we always have $\frac{\alpha(1+\tilde{\theta}_1)}{2(1+\alpha)} < \alpha$. Moreover, as observed above in the case B we have $\frac{\alpha(1+\tilde{\theta}_1)}{2(1+\alpha)} > \alpha\tilde{\theta}_1$ only if $\tilde{\theta}_1 < \frac{1}{1+2\alpha}$. If, instead, $\tilde{\theta}_1 > \frac{1}{1+2\alpha}$ the candidate is the corner equilibrium price defined in p_2^{BC}.

We can observe that this candidate $Bbis$ corresponds to the candidate B extended to the case where $\tilde{\theta}_1 > \alpha$. Indeed, if we consider the candidate B with $\tilde{\theta}_1 > \alpha$: (i) we always have $\tilde{\theta}_1 > \frac{\alpha}{2+\alpha}$; (ii) if $\tilde{\theta}_1 < \frac{1}{1+2\alpha}$ the price is p_2^B (like for the candidate $Bbis$), (iii) if $\tilde{\theta}_1 > \frac{1}{1+2\alpha}$ the price is p_2^{BC} (like for the candidate $Bbis$). Hence, we no longer need to distinguish B and $Bbis$, and we only consider the candidate B.

- **Case** C**.** For $\tilde{\theta}_2 \in [0, \alpha\tilde{\theta}_1]$. The interior equilibrium price candidate is

$$p_2^C = \frac{q}{2}$$

This candidate implies that $\tilde{\theta}_2 = 1/2$. We have $\tilde{\theta}_2 < \alpha\tilde{\theta}_1$ for $\tilde{\theta}_1 > \frac{1}{2\alpha}$ which requires that $\alpha > 1/2$. If, instead, $\tilde{\theta}_1 < \frac{1}{2\alpha}$ the candidate is the corner equilibrium price p_2^{BC} defined before.

This candidate implies that $\tilde{\theta}_2 = 1/2$. We have $\tilde{\theta}_2 < \alpha\tilde{\theta}_1$ for $\tilde{\theta}_1 > \frac{1}{2\alpha}$ which requires that $\alpha > 1/2$.

Table 7: Synthesis of equilibrium candidates

Case	Price	Period 2 monopoly profit
A	$\dfrac{\alpha q}{2}$	$\dfrac{\alpha q}{4}$
AB	$q\tilde{\theta}_1$	$\dfrac{q\tilde{\theta}_1(\alpha-\tilde{\theta}_1)}{\alpha}$
B	$\dfrac{\alpha q(1+\tilde{\theta}_1)}{2(1+\alpha)}$	$\dfrac{\alpha q(1+\tilde{\theta}_1)^2}{4(1+\alpha)}$
BC	$\alpha q\tilde{\theta}_1$	$\alpha q\tilde{\theta}_1(1-\alpha\tilde{\theta}_1)$
C	$\dfrac{q}{2}$	$\dfrac{q}{4}$
D	$\dfrac{q\tilde{\theta}_1}{2}$	$\dfrac{q\tilde{\theta}_1^2}{4}$
DB	αq	$\alpha q(\tilde{\theta}_1-\alpha)$

- Case D. For $\tilde{\theta}_2 \in (\alpha, \tilde{\theta}_1)$, the interior equilibrium price candidate is

$$p_2^D = \frac{q\tilde{\theta}_1}{2}.$$

This price candidate implies that $\tilde{\theta}_2 = \tilde{\theta}_1/2$ and it holds for $\tilde{\theta}_1 > 2\alpha$, possible only for $\alpha < 1/2$. If, instead, $\tilde{\theta}_1 < 2\alpha$ the candidate is the corner equilibrium price such that $\tilde{\theta}_2 = \alpha$

$$\tilde{\theta}_2 = \alpha \quad \Leftrightarrow \quad p_2^{DB} = \alpha q.$$

- Synthesis. Table 7 synthesizes the price candidates and the corresponding monopoly profits. Figure 7 synthesizes the different threshold values of $\tilde{\theta}_1$. Eleven different zones can be distinguished in this figure and Table 8 presents (with "X"' or "(X)") the equilibrium candidates that can be implemented in each zone. For example, if $\tilde{\theta}_1 < \frac{\alpha}{2+\alpha}$ we are in zone (1) (cf. Figure 7) and we can see from Table 8 that the equilibrium candidates p_2^A, p_2^{AB}, p_2^{BC} can be implemented.

A.3 Comparison of monopoly profit with alternative pricing candidates

It can be observed from Table 8 that at least two candidates can be implemented in each zone of Figure 7. The optimal price at period 2 necessitates to compare the period 2 monopoly profit between different candidates. More precisely we make pairwise comparisons between equilibrium candidates. Indeed for a

Figure 7: Zones with alternative pricing candidates

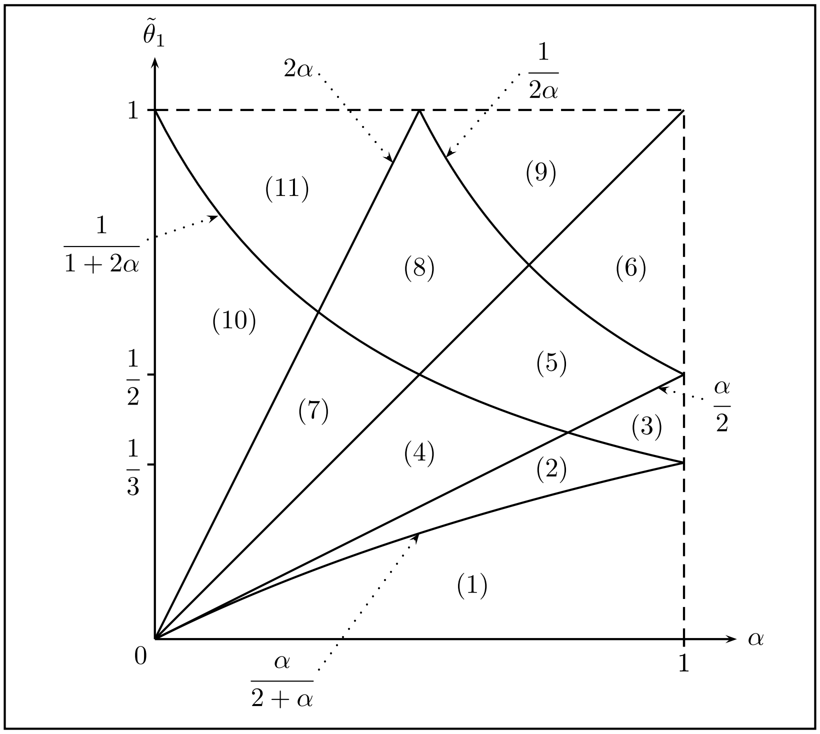


Table 8: Equilibrium candidates

	$\tilde{\theta}_1 < \alpha$						$\tilde{\theta}_1 > \alpha$				
	A	AB	B	BC	C		D	DB	B	BC	C
(1)	X	(X)		(X)		(7)		(X)	X	(X)	
(2)	X		X	(X)		(8)		(X)		X	
(3)	X			X		(9)		(X)		(X)	X
(4)		(X)	X	(X)		(10)	X		X	(X)	
(5)		(X)		X		(11)	X			X	
(6)		(X)		(X)	X						

candidate to be a second period equilibrium in a given $(q, \tilde{\theta}_1)$ zone, we need to check that there is no incentive to deviate from one case to the other.

It can easily be checked that $\pi_2^A > \pi_2^{AB}$, $\pi_2^B > \pi_2^{AB}$, $\pi_2^B > \pi_2^{BC}$, $\pi_2^C > \pi_2^{BC}$, $\pi_2^D > \pi_2^{DB}$ and $\pi_2^B > \pi_2^{DB}$. For the nine other cases, we need to compare the profits between two local optima.

- Comparison of π_2^A and π_2^B. This comparison needs to be made in the case (2). We have:

$$\pi_2^A - \pi_2^B = \frac{\alpha q}{4(1+\alpha)} \cdot \left(-\tilde{\theta}_1^2 - 2\tilde{\theta}_1 + \alpha\right)$$

This difference is quadratic and concave in $\tilde{\theta}_1$. For $q \in [0, 1]$, the lowest root is negative and the highest root is:

$$\hat{\theta}_1^{A/B} = \sqrt{1+\alpha} - 1$$

It can be observed that $\hat{\theta}_1^{A/B}$ is in the zone (2). We always have $\alpha/(2+\alpha) < \hat{\theta}_1^{A/B} < \alpha/2$ and $\hat{\theta}_1^{A/B} < 1/(1+2\alpha)$ for $q < 0.866$.

- Comparison of π_2^A and π_2^{BC}. This comparison needs to be made in the cases (1) and (3) (because BC is dominated by B in zone (2)). Note that $\tilde{\theta}_1 < 1/2$ in these zones. We have:

$$\pi_2^A - \pi_2^{BC} = \frac{\alpha q}{4} \cdot \left(4\alpha\tilde{\theta}_1^2 - 4\tilde{\theta}_1 + 1\right)$$

This difference is quadratic and convex in $\tilde{\theta}_1$. For $q \in [0, 1]$, the highest root is greater than $1/2$ and the lowest root is:

$$\hat{\theta}_1^{A/BC} = \frac{1 - \sqrt{1-\alpha}}{2\alpha}$$

Note that for any $q \in [0, 1]$ we have $\hat{\theta}_1^{A/BC} > \alpha/(2+\alpha)$. Hence $\hat{\theta}_1^{A/BC}$ is above the zone (1). As a consequence, in this zone (1), we always have $\pi_2^A > \pi_2^{BC}$.

Let us now consider the zone (3). It can be observed that $\hat{\theta}_1^{A/BC}$ is in this zone. More precisely, $\hat{\theta}_1^{A/BC} = 1/2$ for $\alpha = 1$, and reach the lower bound of the zone (3) (i.e. $1/(1+2\alpha)$) for $q = 0.866$. Hence, in the zone (3), we have $\pi_2^A > \pi_2^{BC}$ only if $\alpha \geq 0.866$ and $1/(1+2\alpha) < \tilde{\theta}_1 < \hat{\theta}_1^{A/BC}$. Otherwise, we have $\pi_2^A < \pi_2^{BC}$.

- Comparison of π_2^{AB} and π_2^{BC}. This comparison needs to be made in the case (5). We have:

$$\pi_2^{AB} - \pi_2^{BC} = \frac{q(1-\alpha)\tilde{\theta}_1}{\alpha} \cdot \Big(\alpha - (1+\alpha+\alpha^2)\tilde{\theta}_1\Big)$$

This difference is quadratic and concave in $\tilde{\theta}_1$. The lowest root is 0. For $\alpha \in [0,1]$, the highest root is lower than $1/(2+\alpha)$. Hence $\tilde{\theta}_1$ is always greater than the two roots when it is in the zone (5). As a consequence, the difference is negative and we always have $\pi_2^{BC} > \pi_2^{AB}$ in this zone (5).

- Comparison of π_2^{AB} and π_2^{C}. This comparison needs to be made in the case (6). We have:

$$\pi_2^{C} - \pi_2^{AB} = \frac{q}{4\alpha} \cdot \Big(4\tilde{\theta}_1^2 - 4\alpha\tilde{\theta}_1 + \alpha\Big)$$

This difference is quadratic and convex in $\tilde{\theta}_1$, and has no root for $\alpha \in [0,1]$. Hence, the difference is positive and we always have $\pi_2^{C} > \pi_2^{AB}$ in the zone (6).

- Comparison of π_2^{DB} and π_2^{BC}. This comparison needs to be made in the case (8). We have:

$$\pi_2^{DB} - \pi_2^{BC} = -q\alpha^2(1-\tilde{\theta}_1^2) < 0$$

we always have $\pi_2^{BC} > \pi_2^{DB}$ in this zone (8).

- Comparison of π_2^{DB} and π_2^{C}. This comparison needs to be made in the case (9). We have:

$$\pi_2^{DB} - \pi_2^{C} = \frac{q}{4}\Big(4\alpha\tilde{\theta}_1 - (1+4\alpha^2)\Big)$$

This difference is linear and increasing in $\tilde{\theta}_1$ and the root is greater than 1. Hence, the difference is negative and we always have $\pi_2^{C} > \pi_2^{DB}$ in the zone (9).

- Comparison of π_2^{D} and π_2^{B}. This comparison needs to be made in the case (10). We have:

$$\pi_2^{D} - \pi_2^{B} = \frac{q}{4(1+\alpha)} \cdot \Big(\tilde{\theta}_1^2 - 2\alpha\tilde{\theta}_1 - \alpha\Big)$$

This difference is quadratic and convex in $\tilde{\theta}_1$. The lowest root is negative for $\alpha \in [0,1]$. The highest root is

$$\hat{\theta}_1^{D/B} = \alpha + \sqrt{\alpha(1+\alpha)}$$

This root is in the zone (10) for $\alpha < 0.207$. Hence, in this zone (10), for $\alpha < 0.207$ and $\tilde{\theta}_1 > \hat{\theta}_1^{D/B}$ we have $\pi_2^D > \pi_2^B$. Otherwise, we have $\pi_2^B > \pi_2^D$.

- Comparison of π_2^D and π_2^{BC}. This comparison needs to be made in the case (11). We have:

$$\pi_2^D - \pi_2^{BC} = \frac{q}{4} \cdot \tilde{\theta}_1 \cdot \Big((1+4\alpha^2)\tilde{\theta}_1 - 4\alpha\Big)$$

This difference is quadratic and convex in $\tilde{\theta}_1$. The lowest root is equal to 0. The highest root is

$$\hat{\theta}_1^{D/BC} = \frac{4\alpha}{1+4\alpha^2}$$

This root is in the zone (11) for $\alpha \in [0.207, 0.5]$. Hence, in this zone (11), for $\alpha \in [0.207, 0.5]$ and $\tilde{\theta}_1 > \hat{\theta}_1^{D/BC}$ we have $\pi_2^D > \pi_2^{BC}$. Otherwise, we have $\pi_2^{BC} > \pi_2^D$.

- Synthesis. Table 8 synthesizes the equilibrium candidates that can be implemented in each of the 11 zones of Figure 7. In some cases, one candidate is dominated by the other at least in some zone. These cases are reported in Table 8 by a "(X)". For example, the monopoly profit is always higher with p_2^A compared to p_2^{AB}, p_2^{BC}. Hence, AB and BC are dominated in zone (1) so that the optimal price in zone (1) is p_2^A. In some other cases, we have defined the threshold values $\hat{\theta}_1$ such that the ranking of candidates reverse around this threshold.

All these conditions are synthesized in Figure 3 that displays the second-period equilibria for any value of $\tilde{\theta}_1$ and α. One can observe that the equilibrium candidates AB and DB are always dominated.

We also use the notation $\hat{\theta}_1^{B/BC}$ as the threshold value above which the candidate BC is implemented instead of B, and $\hat{\theta}_1^{BC/C}$ as the threshold value under which the candidate BC is implemented instead of C.

B Equilibrium at period 1

The period 1 decisions depend on the period 2 equilibrium. We first define the first period demand for any case where the period 2 equilibrium can be. We then compile the optimal first period price for the monopoly.

B.1 First period demand equilibrium

B.1.1 Value of $\tilde{\theta}_1$ corresponding to an equilibrium

- Case leading to A at period 2 Figure 8 provides a basis for the analysis in the case leading to A at period 2. Here we are in a configuration where $\tilde{\theta}_1$ is small. For a given value of $\tilde{\theta}_1$ all consumers have no interest to change their decisions if the four following conditions are fulfilled:

1. $\theta_1 \leq p_1/q$ for the consumers such that $\theta_1 < \tilde{\theta}_1$. Indeed, these consumers choose $x_2 = 0$ whatever his choice at period 1. Hence, they have no interest to change their decision if, for each of them, we have $(0,0) \succ (1,0)$ which is equivalent to $\theta_1 \leq p_1/q$ (cf. table 3). Since $\theta_1 < \tilde{\theta}_1$ this condition requires $\tilde{\theta}_1 \leq p_1/q$.

2. $\theta_1 \geq p_1/q$ for $\theta_1 \in [\tilde{\theta}_1, \alpha/2]$, which requires that $\tilde{\theta}_1 \geq p_1/q$.

3. $p_1 \leq p_2^A = \alpha q/2$ for $\theta_1 \in [\alpha/2, 1/2]$.

4. $\theta_1 \geq p_1/(\alpha q)$ for $\theta_1 > 1/2$, which requires that $1/2 > p_1/(\alpha q)$ or equivalently $p_1 < \alpha q/2$.

Combining these four conditions, we can conclude that if $\tilde{\theta}_1 = p_1/q$ and $p_1 \leq \alpha q/2 = p_2^A$ then no consumer has an incentive to change its decision at period 1, which means that this situation is an equilibrium.

- Case leading to B at period 2. Figure 9 provides a basis for the analysis in the case leading to B at period 2. For a given value of $\tilde{\theta}_1$ all consumers have no interest to change their decision if the four following conditions are fulfilled:

1. $\theta_1 \leq p_1/q$ for $\theta_1 < \tilde{\theta}_2$, which requires that $\tilde{\theta}_2 \leq \frac{p_1}{q}$ which is equivalent to $\tilde{\theta}_1 \leq \frac{2(1+\alpha)p_1}{\alpha q} - 1$.

2. $p_1 \geq p_2^B$ for $\theta_1 \in (\tilde{\theta}_2, \tilde{\theta}_1)$.

3. $p_1 \leq p_2^B = \frac{q\alpha(1+\tilde{\theta}_1)}{2(1+\alpha)}$ for $\theta_1 \in [\tilde{\theta}_1, \tilde{\theta}_2/\alpha]$, which requires that $\tilde{\theta}_1 \geq \frac{2(1+\alpha)p_1}{\alpha q} - 1$.

4. $\theta_1 \geq p_1/(\alpha q)$ for $\theta_1 > \tilde{\theta}_2/\alpha$, which requires that $\tilde{\theta}_2/\alpha > p_1/(\alpha q)$ which is equivalent to $\tilde{\theta}_1 \geq \frac{2(1+\alpha)p_1}{\alpha q} - 1$.

Combining the first and both the third and fourth conditions leads to $\tilde{\theta}_1 = \frac{2(1+\alpha)p_1}{\alpha q} - 1$. In addition, combining the second and the third conditions leads to $p_1 = p_2$. Note however that $\tilde{\theta}_1 = \frac{2(1+\alpha)p_1}{\alpha q} - 1$ is equivalent to $p_1 = \frac{\alpha q(1+\tilde{\theta}_1)}{2(1+\alpha)}$ which is indeed equal to p_2.

- Case leading to BC at period 2. We next suppose that the period 1 decision leads to situation BC at the period 2 subgame. Figure 10 provides a basis for the analysis. Depending on θ_1 the consumers have different alternatives to compare. For a given value of $\tilde{\theta}_1$ all consumers have no interest to change their decision if the four following conditions are fulfilled:

Figure 8: Case A

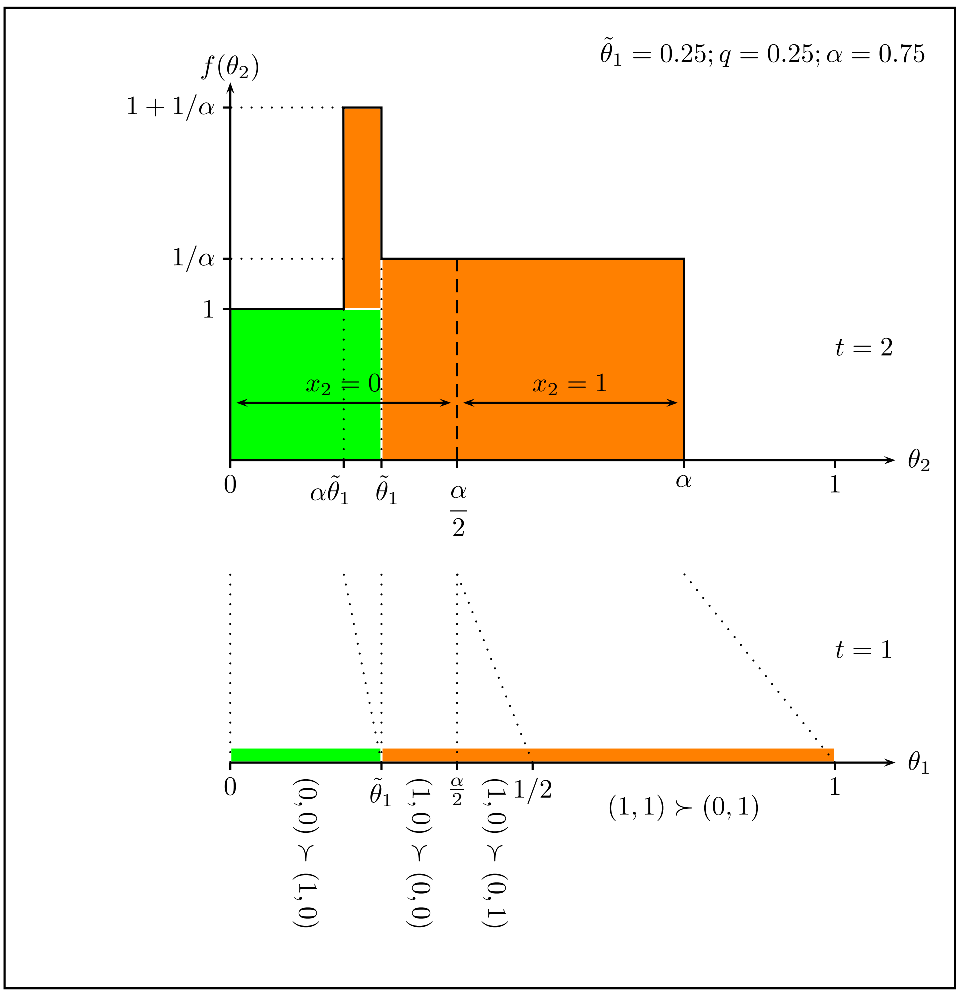


Figure 9: Case B

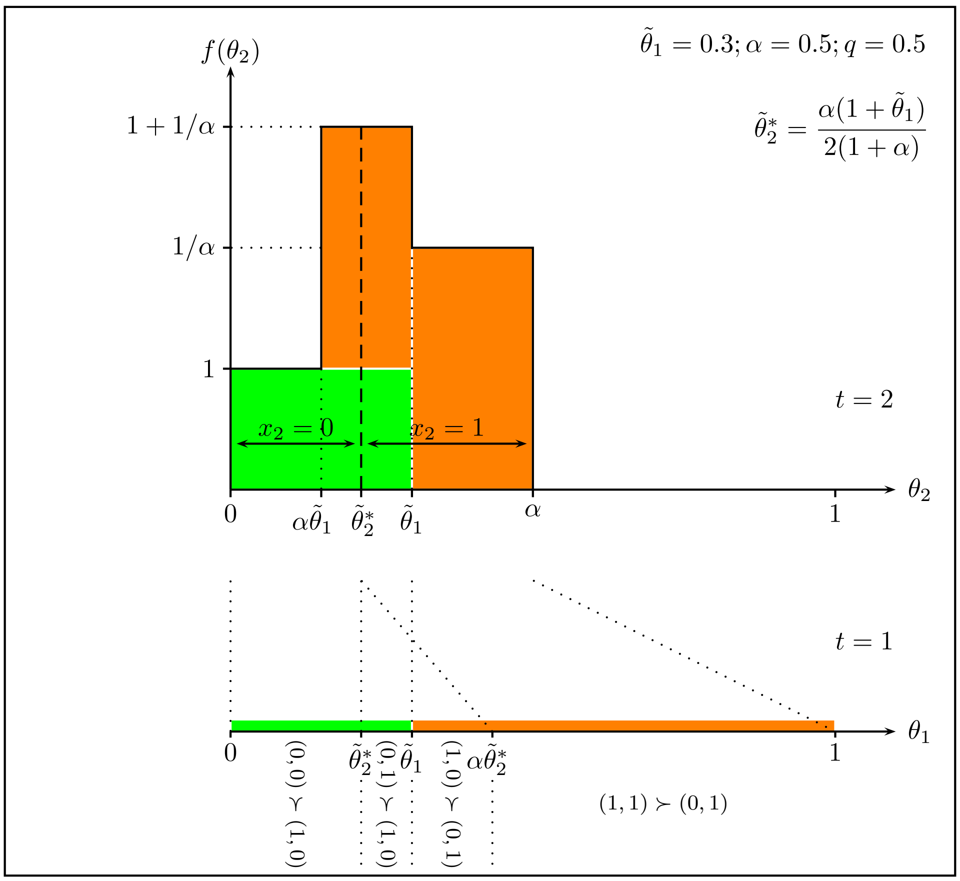
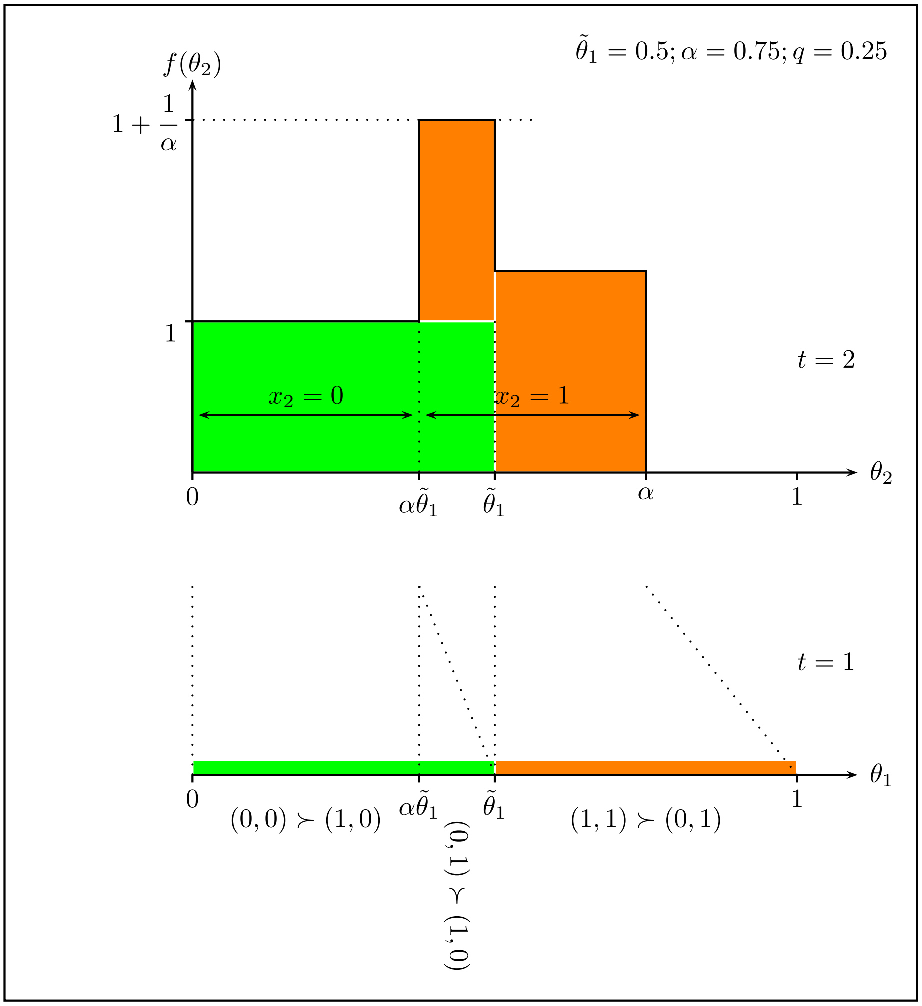


Figure 10: Case BC



1. $\theta_1 \leq p_1/q$ for $\theta_1 < \alpha\tilde{\theta}_1$, which requires that $\alpha\tilde{\theta}_1 \leq p_1/q$, which is equivalent to $\tilde{\theta}_1 \leq p_1/(\alpha q)$

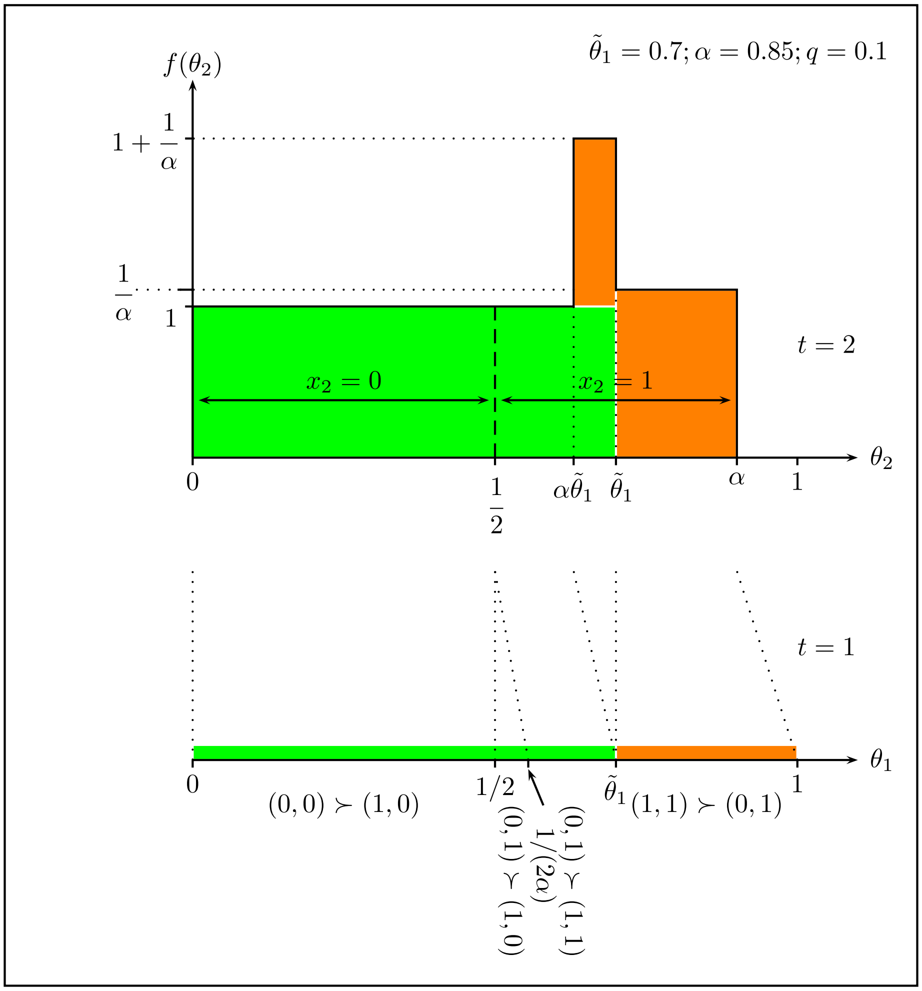
2. $p_1 \geq p_2^{BC} = \tilde{\theta}_1 \alpha q$ for $\theta_1 \in [\alpha\tilde{\theta}_1, \tilde{\theta}_1]$. This condition is equivalent to $\tilde{\theta}_1 \leq p_1/(\alpha q)$.

3. $\theta_1 \geq p_1/(\alpha q)$ for $\theta_1 > \tilde{\theta}_1$, which requires that $\tilde{\theta}_1 \geq p_1/(\alpha q)$.

Combining these three conditions, we can conclude that if $\tilde{\theta}_1 = p_1/(\alpha q)$ then no consumer has an incentive to change its decision at period 1, which means that this situation is an equilibrium.

- Case leading to C at period 2. We next suppose that the period 1 decision leads to a situation C at the period 2 subgame. Figure 11 provides a basis for the analysis in these cases. Depending on θ_1 the consumers have different alternatives to compare (cf. bottom of both figures). For a given value of $\tilde{\theta}_1$ all consumers have no interest to change their decision if the four following conditions are fulfilled:

Figure 11: Case C



1. $\theta_1 \leq p_1/q$ for $\theta_1 < 1/2$, which requires that $\frac{1}{2} \leq \frac{p_1}{q} \Leftrightarrow p_1 \geq \frac{q}{2}$.

2. $p_1 \geq p_2^C = q/2$ for $\theta_1 \in [1/2, 1/(2\alpha)]$.

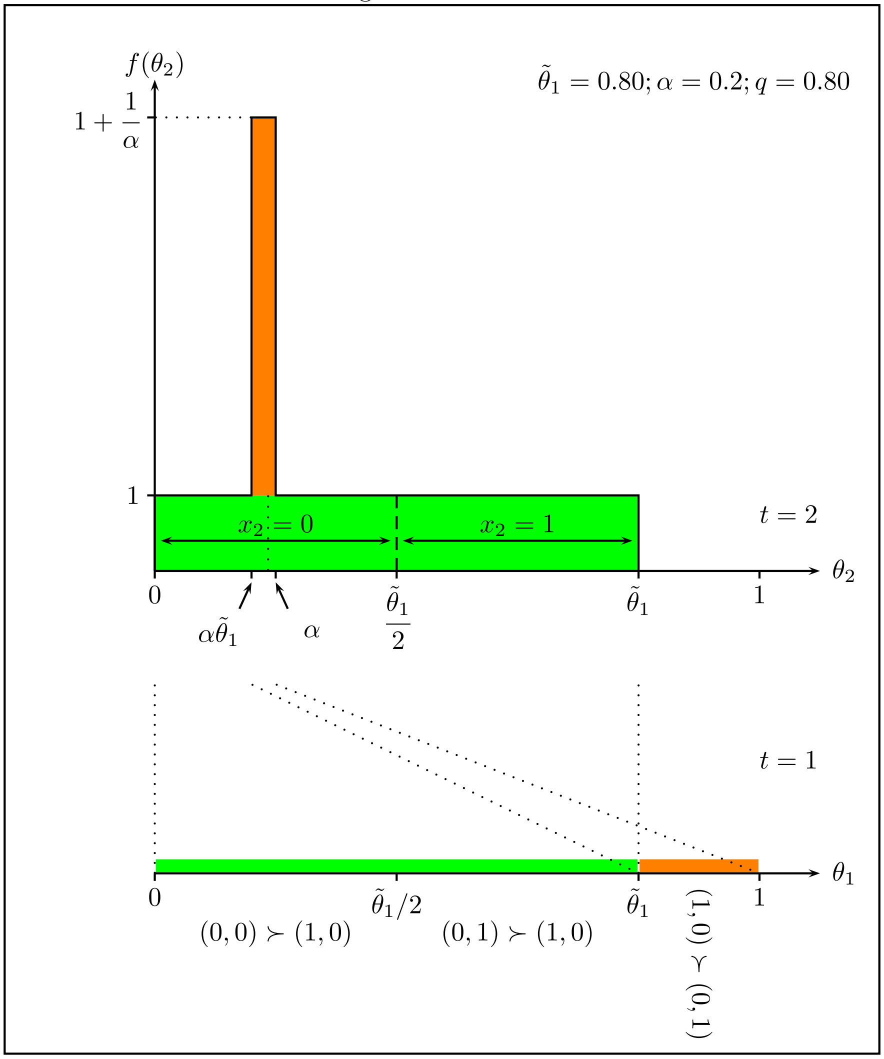
3. $\theta_1 \leq p_1/(\alpha q)$ for $\theta_1 \in [1/(2\alpha), \tilde{\theta}_1]$, which requires that $\tilde{\theta}_1 \leq p_1/(\alpha q)$.

4. $\theta_1 \geq p_1/(\alpha q)$ for $\theta_1 > \tilde{\theta}_1$, which requires that $\tilde{\theta}_1 \geq p_1/(\alpha q)$.

Combining these four conditions, we can conclude that if $\tilde{\theta}_1 = p_1/(\alpha q)$ and $p_1 \geq q/2$ then no consumer has an incentive to change its decision at period 1, which means that this situation is an equilibrium.

- Case leading to D at period 2. Figure 12 provides a basis for the analysis in the case leading to D at period 2. For a given value of $\tilde{\theta}_1$ all consumers have no interest to change their decisions if the three following conditions are fulfilled:

Figure 12: Case D



1. $\theta_1 \leq p_1/q$ for $\theta_1 < \tilde{\theta}_1/2$, which requires that $\tilde{\theta}_1/2 \leq p_1/q$ or equivalently $\tilde{\theta}_1 \leq 2p_1/q$.

2. $p_2 \leq p_1$ for $\theta_1 \in [\tilde{\theta}_1/2, \tilde{\theta}_1]$ in order to have $(0,1) \succ (1,0)$.

3. $p_1 \leq p_2$ for $\theta_1 > \tilde{\theta}_1$, in order to have $(1,0) \succ (0,1)$.

Combining the second and the third conditions leads to $p_1 = p_2$. Remind that $p_2 = q\tilde{\theta}_1/2$, hence $p_1 = q\tilde{\theta}_1/2$. Note that the first condition is checked with this particular value of p_1. In summary $\tilde{\theta}_1 = 2p_1/q$ is an equilibrium.

B.1.2 Synthesis of value of equilibrium values of $\tilde{\theta}_1$

	$\alpha \in [0, 0.207]$	$\alpha \in [0.207, 0.5]$	$\alpha \in [0.5, 0.866]$	$\alpha \in [0.866, 1]$
$\tilde{\theta}_1^A$	$[0; \hat{\theta}_1^{A/B}]$	$[0; \hat{\theta}_1^{A/B}]$	$[0; \hat{\theta}_1^{A/B}]$	$[0; \hat{\theta}_1^{A/BC}]$
$\tilde{\theta}_1^B$	$[\hat{\theta}_1^{A/B}; \hat{\theta}_1^{D/B}]$	$[\hat{\theta}_1^{A/B}; \hat{\theta}_1^{B/BC}]$	$[\hat{\theta}_1^{A/B}; \hat{\theta}_1^{B/BC}]$	-
$\tilde{\theta}_1^{BC}$	-	$[\hat{\theta}_1^{B/BC}; \hat{\theta}_1^{D/B}]$	$[\hat{\theta}_1^{B/BC}; \hat{\theta}_1^{BC/C}]$	$[\hat{\theta}_1^{A/BC}; \hat{\theta}_1^{BC/C}]$
$\tilde{\theta}_1^C$	-	-	$[\hat{\theta}_1^{BC/C}; 1]$	$[\hat{\theta}_1^{BC/C}; 1]$
$\tilde{\theta}_1^D$	$[\hat{\theta}_1^{D/B}; 1]$	$[\hat{\theta}_1^{D/BC}; 1]$	-	$q \in [0.866, 1]$

$$\begin{aligned}
\tilde{\theta}_1^A \geq 0 \quad &\Leftrightarrow \quad p_1 \geq 0 \\
\tilde{\theta}_1^A \leq \hat{\theta}_1^{A/B} \quad &\Leftrightarrow \quad p_1 \leq q(\sqrt{1+\alpha} - 1) \\
\tilde{\theta}_1^A \leq \hat{\theta}_1^{A/BC} \quad &\Leftrightarrow \quad p_1 \leq \frac{q(1-\sqrt{1-\alpha})}{2\alpha}
\end{aligned}$$

$$\begin{aligned}
\tilde{\theta}_1^B \geq \hat{\theta}_1^{A/B} \quad &\Leftrightarrow \quad p_1 \geq \frac{q\alpha}{2\sqrt{1+\alpha}} \\
\tilde{\theta}_1^B \leq \hat{\theta}_1^{D/B} \quad &\Leftrightarrow \quad p_1 \leq \frac{q\alpha(1+\alpha\sqrt{\alpha(1+\alpha)})}{2(1+\alpha)} \\
\tilde{\theta}_1^B \leq \hat{\theta}_1^{B/BC} \quad &\Leftrightarrow \quad p_1 \leq \frac{q\alpha}{1+2\alpha}
\end{aligned}$$

$$\begin{aligned}
\tilde{\theta}_1^{BC} \geq \hat{\theta}_1^{B/BC} \quad &\Leftrightarrow \quad p_1 \geq \frac{q\alpha}{1+2\alpha} \\
\tilde{\theta}_1^{BC} \geq \hat{\theta}_1^{A/BC} \quad &\Leftrightarrow \quad p_1 \geq \frac{q(1-\sqrt{1-\alpha})}{2} \\
\tilde{\theta}_1^{BC} \leq \hat{\theta}_1^{D/BC} \quad &\Leftrightarrow \quad p_1 \leq \frac{4q\alpha^2}{1+4\alpha^2} \\
\tilde{\theta}_1^{BC} \leq \hat{\theta}_1^{BC/C} \quad &\Leftrightarrow \quad p_1 \leq \frac{q}{2}
\end{aligned}$$

$$\begin{aligned}
\tilde{\theta}_1^C \geq \hat{\theta}_1^{BC/C} \quad &\Leftrightarrow \quad p_1 \geq \frac{q\alpha}{1+2\alpha} \\
\tilde{\theta}_1^C \leq 1 \quad &\Leftrightarrow \quad p_1 \leq q\alpha
\end{aligned}$$

$$
\begin{aligned}
\tilde{\theta}_1^D \geq \hat{\theta}_1^{D/B} \quad &\Leftrightarrow \quad p_1 \geq \frac{q(\alpha + \sqrt{\alpha(1+\alpha)})}{2} \\
\tilde{\theta}_1^D \geq \hat{\theta}_1^{D/BC} \quad &\Leftrightarrow \quad p_1 \geq \frac{2q\alpha}{1+4\alpha^2} \\
\tilde{\theta}_1^D \leq 1 \quad &\Leftrightarrow \quad p_1 \leq \frac{q}{2}
\end{aligned}
$$

B.2 Monopoly pricing equilibrium

This appendix presents only the equilibrium candidates where some consumers buy the product twice (A, B, BC or C).

B.2.1 Equilibrium candidates

- Case leading to A at period 2. Remind that in the case A, the period 2 profit is $\alpha q/4$, which is independent from $\tilde{\theta}_1$. Hence the monopoly can decide its period 1 price without taking care of the effect on the period 2 profit. The period 1 profit is :

$$\pi_1^A(p_1) = p_1 \cdot (1 - \tilde{\theta}_1^A) = p_1 \cdot \left(1 - \frac{p_1}{q}\right)$$

This function is concave in p_1 and have a maximum for $p_1 = q/2$. However this maximum does not respect the condition for having this equilibrium because we then have $\tilde{\theta}_1 = 1/2$ which is greater than both $\hat{\theta}_1^{A/B}$ and $\hat{\theta}_1^{A/BC}$.

Hence the optimal price is constrained to be $q\hat{\theta}_1^{A/B}$ (for $\alpha \in [0, 0.866]$) or $q\hat{\theta}_1^{A/BC}$ (for $\alpha \in [0.866, 1]$).

The profit over the two periods is:

$$\Pi = \pi_1^* + \pi_2^A = q\hat{\theta}_1(1 - \hat{\theta}_1) + \frac{q(1-q)}{4}$$

For $\alpha \in [0, 0.866]$, remind that $\hat{\theta}_1^{A/B} = \sqrt{1+\alpha} - 1$ so that

$$\Pi^{A1} = q \cdot \left((\sqrt{1+\alpha} - 1)(2 - \sqrt{1+\alpha}) + \frac{\alpha}{4}\right)$$

For $\alpha \in [0.866, 1]$, remind that $\hat{\theta}_1^{A/BC} = (1 - \sqrt{1-\alpha})/(2\alpha)$ so that

$$\Pi^{A2} = q \cdot \left(\frac{(1 - \sqrt{1-\alpha})(2\alpha - 1 + \sqrt{1-\alpha})}{4\alpha^2} + \frac{\alpha}{4}\right)$$

It can be shown that the optimal price at period 1 is lower compared to the optimal price at period 2. However, this property is related to the fact that the monopoly cannot charge a very high price. Indeed, if the monopoly charges a price higher than $q\hat{\theta}_1$ then $\tilde{\theta}_1$ is no longer in the zone leading to subgame equilibrium A at period 2. Without this constraint, the monopoly would charge a price $q/2$ that would actually be higher than p_2. In summary, the constraint $\tilde{\theta}_1 < \hat{\theta}_1$ forces the monopoly not to charge a high price, and actually the maximum price he can charge ($q\hat{\theta}_1$) is lower than the price at period 2. Without this constraint, the monopoly would prefer to charge a price at a period 1 higher than the price at period 2. Hence the constraint $\tilde{\theta}_1 < \hat{\theta}_1$ forces the monopoly to strongly distort its price and to price discriminate in the converse way compared to what he would do without this constraint (i.e. $p_1 < p_2$ instead to $p_1 > p_2$).

- Case leading to B at period 2. The cumulated profit of the monopoly is

$$\Pi^B = p_1(1-\tilde{\theta}_1) + \frac{q\alpha(1+\tilde{\theta}_1)^2}{4(1+\alpha)}$$

This is concave in p_1, and maximum for $p_1 = \frac{\alpha q}{1+\alpha}$, leading to $\tilde{\theta}_1 = 1$ which is out of the zone leading to the equilibrium B at period 2.

For $\alpha \in [0, 0.207]$ we will have $\tilde{\theta}_1 = \hat{\theta}_1^{D/B} = \alpha + \sqrt{\alpha(1+\alpha)}$. The corresponding value of p_1 is

$$p_1^{B1} = \frac{q\alpha\left(1+\alpha+\sqrt{\alpha(1+\alpha)}\right)}{2(1+\alpha)}$$

leading to:

$$\Pi^{B1} = q \cdot \frac{\alpha\left(3-2\alpha-\sqrt{\alpha(1+\alpha)}\right)\left(1+\alpha+\sqrt{\alpha(1+\alpha)}\right)}{4(1+\alpha)}$$

For $\alpha \in [0.207, 0.866]$ we will have $\tilde{\theta}_1 = \hat{\theta}_1^{B/BC} = 1/(1+2\alpha)$. The corresponding value of p_1 is $p_1^{B2} = \alpha q/(1+2\alpha)$ leading to:

$$\Pi^{B2} = q \cdot \frac{\alpha(1+2\alpha)}{(1+2\alpha)^2}$$

- Case leading to BC at period 2. The monopoly profit is

$$\begin{aligned}\Pi^{BC} &= (1-\tilde{\theta}_1^{BC})p_1 + q\alpha\tilde{\theta}_1^{BC}\left(1-\alpha\tilde{\theta}_1^{BC}\right) \\ &= \left(1-\frac{p_1}{\alpha q}\right)p_1 + \left(1-\frac{p_1}{q}\right)p_1\end{aligned}$$

This function is concave in p_1 and maximum for $p_1 = \alpha q/(1+\alpha)$, leading to $\tilde{\theta}_1 = 1/(1+\alpha)$.

Note that $\tilde{\theta}_1 = 1/2$ for $\alpha = 1$. When α increases from 0.25 to 1, $\tilde{\theta}_1 = 1/(1+\alpha)$ decreases but is always interior to the zone leading either to BC. More precisely we always have $\tilde{\theta}_1 > 1/2$, $\tilde{\theta}_1 > 1/(1+2\alpha)$ and $\tilde{\theta}_1 < 1/(2\alpha)$. Hence, for $q \in [0.25, 1]$, $p_1^{BC2} = \alpha q/(1+\alpha)$ in an interior solution. The profit is then $\Pi^{BC2} = q\alpha/(1+\alpha)$

For $\alpha \in [0, 0.25]$, we have $1/(1+\alpha) > \hat{\theta}_1^{D/BCbis}$ so that the monopoly is constrained to choose p_1 leading to $\hat{\theta}_1^{D/BC}$:

$$\frac{p_1}{\alpha q} = \hat{\theta}_1^{D/BC} = \frac{4\alpha}{1+4\alpha^2} \qquad \Leftrightarrow \qquad p_1^{BC1} = \frac{4q\alpha^2}{1+4\alpha^2}$$

The profit is then

$$\Pi^{BC1} = q \cdot \frac{8\alpha^2(1-2\alpha(1-\alpha))}{(1-4\alpha^2)^2}$$

Note that the price is the same in both periods.

- Case leading to C at period 2. Cumulated profit writes as:

$$\Pi^C = p_1\left(1 - \frac{p_1}{\alpha q}\right) + \frac{q}{4}$$

This objective is decreasing in p_1 if $p_1 > \alpha q/2$, or equivalently $\tilde{\theta}_1 > 1/2$, which is always the case. Hence the profit is decreasing in p_1, so that the cumulated profit is maximal at $p_1 = q/2$, leading to $\tilde{\theta}_1 = 1/(2\alpha)$ and cumulated profit is $\Pi^C = q \cdot \frac{2\alpha-1}{4\alpha}$. Note that the price is the same in both periods: $p_1^C = p_2^C = q/2$.

B.2.2 Comparison of equilibrium candidates

Six different comparisons need to be made.

- D *vs* BC for $\alpha \in [0.207, 0.5]$. If $\alpha \in [0.207, 0.25]$, we have:

$$\Pi^D - \Pi^{BC1} = q \cdot \frac{(1-2\alpha)^3(1+6\alpha)}{4(1+4\alpha^2)^2} > 0$$

If $\alpha \in [0.25, 0.5]$, we have:

$$\Pi^D - \Pi^{BC2} = q \cdot \frac{1-3\alpha}{2(1+\alpha)}$$

This last difference is positive iff $\alpha < 1/3$. We can conclude that D dominates BC if $\alpha \in [0.207, 0.333]$ and the converse if $\alpha \in [0.333, 0.5]$.

- D *vs* B. This comparison does not need to be made if D is dominated by BC. Hence BC or B are compared for $\alpha \in [0, 0.333]$.

If $\alpha \in [0, 0.207]$, we have:

$$\Pi^{D} - \Pi^{B1} = q \cdot \frac{(1-\alpha)(1-\alpha-2\alpha^2-2\alpha\sqrt{\alpha(1+\alpha)})}{4(1+4)} > 0 \qquad \text{(for } \alpha < 0.207\text{)}$$

If $\alpha \in [0.207, 0.333]$, we have:

$$\Pi^{D} - \Pi^{B2} = q \cdot \frac{1-8\alpha^2}{4(1+2\alpha)^2} > 0 \qquad \text{(for } \alpha < 0.333\text{)}$$

- ***D vs A*.** This comparison needs to be made only when D dominates either BC or B (i.e. if $\alpha \in [0, 0.333]$). We then have:

$$\Pi^{D} - \Pi^{A1} = q \cdot \frac{13+3\alpha-12\sqrt{1+\alpha}}{4} > 0 \qquad \text{(for } \alpha < 0.333\text{)}$$

- ***BC vs C*** for $\alpha \in [0.5, 1]$. We have:

$$\Pi^{BC2} - \Pi^{C} = q \cdot \frac{(1-\alpha)^2}{4\alpha(1+\alpha)} > 0$$

- ***BC vs B*.** This comparison needs to be made only when BC dominates D (i.e. if $\alpha \in [0.333, 0.866]$). We have:

$$\Pi^{BC2} - \Pi^{B2} = q \cdot \frac{\alpha^3}{(1+\alpha)(1+2\alpha)^2} > 0$$

- ***BC vs A*.** This comparison needs to be made only when BC dominates D (i.e. if $\alpha \in [0.333, 1]$). If $\alpha \in [0.333, 0.866]$ we have:

$$\Pi^{BC2} - \Pi^{A1} = q \cdot \frac{12+19\alpha+3\alpha^2-12(1+\alpha)\sqrt{1+\alpha}}{4(1+\alpha)} > 0$$

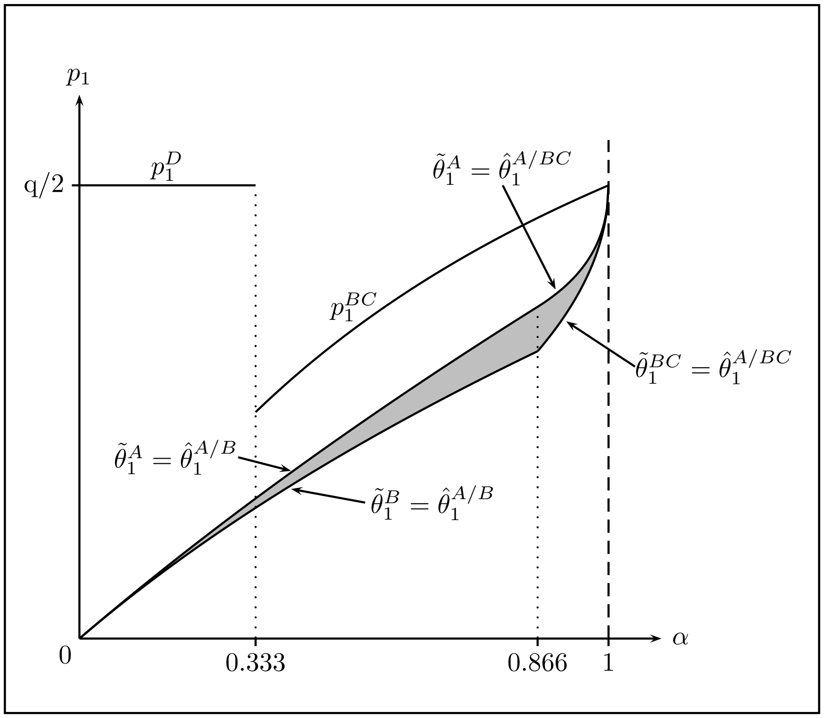
If $\alpha \in [0.866, 1]$ we have:

$$\Pi^{BC2} - \Pi^{A2} = q \cdot \frac{(1-\alpha)\left(2(1-\alpha^2)+\alpha(1+\alpha^2)-2(1+\alpha)\sqrt{1-\alpha}\right)}{4\alpha^2(1+\alpha)}$$

This difference is positive for $\alpha > 0.866$.

- Synthesis. D is the equilibrium for $\alpha \in [0, 0.333]$ because it dominates A and B for $\alpha \in [0, 0.333]$ and BC for $\alpha \in [0.207, 0.333]$

Figure 13: Comparison between price equilibrium and prices leading to multiple first period demand equilibrium



BC is the equilibrium for $\alpha \in [0.333, 1]$ because it dominates A for $\alpha \in [0.333, 1]$, B for $\alpha \in [0.333, 0.866]$, C for $\alpha \in [0.5, 1]$, and D for $\alpha \in [0.333, 0.5]$,

B.2.3 Issues related to multiplicity of equilibrium on the demand side

As observed in Section 5.1 and illustrated in Figure 4, there is a range of prices where we have multiple demand equilibria. This range is illustrated by the gray zone in Figure 13.

For $\alpha < 0.866$, this range is such that $\tilde{\theta}_1$ is close to $\hat{\theta}_1^{A/B}$. This range is more precisely such that $\tilde{\theta}_1^A \leq \hat{\theta}_1^{A/B}$ and $\tilde{\theta}_1^B \geq \hat{\theta}_1^{A/B}$ (see detailed condition in appendix B.1.2). We can check that the price equilibrium does not fall in this range. Indeed, the highest value of this range is $p_1 = q(\sqrt{1+\alpha} - 1)$, and both p_1^D (for $\alpha < 0.333$) and p_1^{BC} are higher than this highest value.

For $\alpha > 0.866$, this range is such that $\tilde{\theta}_1$ is close to $\hat{\theta}_1^{A/BC}$. This range is more precisely such that $\tilde{\theta}_1^A \leq \hat{\theta}_1^{A/BC}$ and $\tilde{\theta}_1^{BC} \geq \hat{\theta}_1^{A/BC}$. We can check that the price equilibrium does not fall in this range. Indeed, the highest value of this range is $p_1 = q(1 - \sqrt{1-\alpha})/(2\alpha)$, and and p_1^{BC} is higher than this highest value.

C Price discrimination at period two

C.1 Period 2 monopoly pricing

p_2^N is the price paid at period 2 by new buyers and p_2^P is the price paid at period 2 by past buyers. There is no incentive constraint, we can have both $p_2^P < p_2^N$ or $p_2^P > p_2^N$.

We first define the two optimal prices for the monopoly in $t = 2$.

Note that, as in the basic model, the indifferent consumer is located in $\tilde{\theta}_2 = p_2/q$.

- Optimal price for new buyers (p_2^N). New buyers are uniformly distributed along $[0, \tilde{\theta}_1]$ with density 1. The period 2 demand from these buyers is $D_2^N = \tilde{\theta}_1 - \tilde{\theta}_2$. The profit is

$$\pi_2^N = p_2^N \cdot D_2^N = p_2 \cdot \left(\tilde{\theta}_1 - \frac{p_2^N}{q} \right)$$

This function is concave in p_2^N and maximum for $p_2^N = q\tilde{\theta}_1/2$. We then have $\tilde{\theta}_2^N = \tilde{\theta}_1/2$ which is indeed always interior.

- Optimal price for past buyers (p_2^P). Past buyers are uniformly distributed along $[\alpha\tilde{\theta}_1, \alpha]$ with density $1/\alpha$. The period 2 demand from these buyers is $D_2^P = 1 - \tilde{\theta}_2/\alpha$. The profit is

$$\pi_2^P = p_2^P \cdot D_2^P = p_2 \cdot \left(1 - \frac{p_2^P}{q\alpha} \right)$$

This function is concave in p_2^P and maximum for $p_2^P = q\alpha/2$, leading to $\tilde{\theta}_2^P = \alpha/2$.

This value of $\tilde{\theta}_2^P$ is lower than the higher bound α but not necessarily higher than the lower bound. More precisely we have $\alpha/2 > \alpha\tilde{\theta}_1$ if only if $\tilde{\theta}_1 < 1/2$. Hence the interior solution can be implemented only for $\tilde{\theta}_1 < 1/2$.

If $\tilde{\theta}_1 > 1/2$ then the best price for the monopoly is to choose p_2^{PC} such that $\tilde{\theta}_2^{PC} = \alpha\tilde{\theta}_1$. Hence the monopoly chooses $p_2^{PC} = q\alpha\tilde{\theta}_1$. In turn, the profit is $\pi_2^{PC} = p_2^{PC} \cdot (1 - \tilde{\theta}_1)$. In such a case all the past buyers buy again the product at the second period.

If $\tilde{\theta}_1 < 1/2$, the monopoly can choose the interior solution p_2^P. Note that

$$p_2^P < p_2^N \quad \Leftrightarrow \quad \frac{q\alpha}{2} < \frac{q\tilde{\theta}_1}{2} \quad \Leftrightarrow \quad \tilde{\theta}_1 > \alpha$$

If $\tilde{\theta}_1 > 1/2$, the monopoly is constrained to choose the corner solution p_2^{PC}. Note that

$$p_2^{PC} < p_2^N \quad \Leftrightarrow \quad q\alpha\tilde{\theta}_1 < \frac{q\tilde{\theta}_1}{2} \quad \Leftrightarrow \quad \alpha < \frac{1}{2}$$

The second period profit in case of price discrimination is then

for $\tilde{\theta}_1 < 1/2$, $\pi_2^{NP} = \pi_2^N + \pi_2^P = \frac{q}{4}(\alpha + \tilde{\theta}_1^2)$;

for $\tilde{\theta}_1 > 1/2$, $\pi_2^{NPC} = \pi_2^N + \pi_2^{PC} = \frac{q}{4}\tilde{\theta}_1(4\alpha(1-\tilde{\theta}_1) + \tilde{\theta}_1)$.

Whenever α is low enough, namely $\alpha < min\left\{0.5, \tilde{\theta}_1\right\}$, we get that the price set for the past buyers is lower than the price set for the new buyers ($p_2^P < p_2^N$ and $p_2^{PC} < p_2^N$), that is, the only way to induce consumers that already bought in $t = 1$ to buy again in $t = 2$ is by setting a lower price for them. In contrast, when α is high, the monopolist cannot implement such a price discrimination: the incentive is to set in $t = 2$ a price for the past buyers that is higher than the price for the new buyers, $p_2^P > p_2^N$ and $p_2^{PC} > p_2^N$.

The two equilibria presented above are candidates and we should compare the profit with the case where the monopoly does not price discriminate.

Formally, in order to find the second period equilibrium, we need to compare:

for $\tilde{\theta}_1 < 1/2$, candidate (p_2^P, p_2^N) with candidates A, B, BC and D;

for $\tilde{\theta}_1 > 1/2$, candidate (p_2^{PC}, p_2^N) with candidates C, B, BC and D.

Remark 1 *The proper comparisons lead us to conclude that price discrimination always prevails. Indeed, if* $\tilde{\theta}_1 < 1/2$, $\pi_2^{NP} > \pi_2^A$, $\pi_2^{NP} > \pi_2^B$, $\pi_2^{NP} > \pi_2^{BC}$ *and* $\pi_2^{NP} > \pi_2^D$*; if* $\tilde{\theta}_1 > 1/2$, $\pi_2^{NPC} > \pi_2^{BC}$, $\pi_2^{NPC} > \pi_2^C$, $\pi_2^{NPC} > \pi_2^B$ *and* $\pi_2^{NPC} > \pi_2^D$.

Proof. Five out of the eight comparisons gives directly the result:

$$\pi_2^{NP} - \pi_2^A = \frac{q\tilde{\theta}_1^2}{4} > 0;$$
$$\pi_2^{NP} - \pi_2^B = \frac{q(\alpha - \tilde{\theta}_1)^2}{4(1+\alpha)} > 0;$$
$$\pi_2^{NP} - \pi_2^D = \frac{q\alpha}{4} > 0;$$
$$\pi_2^{NPC} - \pi_2^{BC} = \frac{q(1-2\alpha)^2\tilde{\theta}_1^2}{4} > 0;$$
$$\pi_2^{NPC} - \pi_2^D = q\tilde{\theta}_1\alpha(1-\tilde{\theta}_1) > 0.$$

The comparison of π_2^{NPC} and π_2^B requires more detailed analysis. We have:

$$\pi_2^{NPC} - \pi_2^B = \frac{-q}{4(1+\alpha)} \cdot \left(\alpha + (-2\alpha - 4\alpha^2)\tilde{\theta}_1 + (-1 + 4\alpha + 4\alpha^2)\tilde{\theta}_1^2\right)$$

So that $\pi_2^{NPC} - \pi_2^B > 0$ if and only if $\alpha + (-2\alpha - 4\alpha^2)\tilde{\theta}_1 + (-1+4\alpha+4\alpha^2)\tilde{\theta}_1^2 < 0$. Note that $-1+4\alpha+4\alpha^2 > 0 \iff \alpha > 0.207$ and the two $\tilde{\theta}_1$-roots of the above polynomial are:

$$\tilde{\theta}_1^a = \frac{\alpha + 2\alpha^2 - \sqrt{\alpha - 3\alpha^2 + 4\alpha^4}}{-1 + 4\alpha + 4\alpha^2},$$
$$\tilde{\theta}_1^b = \frac{\alpha + 2\alpha^2 + \sqrt{\alpha - 3\alpha^2 + 4\alpha^4}}{-1 + 4\alpha + 4\alpha^2}$$

Recall that the above comparison is relevant for $\tilde{\theta}_1 > 1/2$, and for $\tilde{\theta}_1 < 1/(1+2\alpha)$ if $\alpha \in (0.207, 0.866)$ and for $\tilde{\theta}_1 < \hat{\theta}_1^{D|Bbis}$ if $\alpha < 0.207$. For $\alpha > 0.207$, the polynomial of interest, $\alpha + (-2\alpha - 4\alpha^2)\tilde{\theta}_1 + (-1 + 4\alpha + 4\alpha^2)\tilde{\theta}_1^2$ is convex and it is negative because $\tilde{\theta}_1^a < 1/2$ and $\tilde{\theta}_1^b > 1/(1+2\alpha)$, that is, the set of values of $\tilde{\theta}_1$ we need to consider are between the two roots. This implies that $\pi_2^{NPC} - \pi_2^B > 0$. For $\alpha < 0.207$, $\alpha + (-2\alpha - 4\alpha^2)\tilde{\theta}_1 + (-1 + 4\alpha + 4\alpha^2)\tilde{\theta}_1^2$, is concave and it is negative because $\tilde{\theta}_1^a < 1/2$ and $\tilde{\theta}_1^b < 0$, that is, the set of values of $\tilde{\theta}_1$ we need to consider are larger than the two roots. This implies that $\pi_2^{NPC} - \pi_2^B > 0$.

Also the comparison of π_2^{NP} and π_2^{BC} requires more detailed analysis.

$$\pi_2^{NP} - \pi_2^{BC} = \frac{q(\alpha - 4\alpha\tilde{\theta}_1 + \tilde{\theta}_1^2 + 4\alpha^2\tilde{\theta}_1^2)}{4}$$

Recall that the above comparison is relevant for $\tilde{\theta}_1 < 1/2$, and for $\alpha > 0.207$. The above polynomial has two $\tilde{\theta}_1$ roots: $\frac{2\alpha - \sqrt{-\alpha + 4\alpha^2 - 4\alpha^3}}{1+4\alpha^2}$ and $\frac{2\alpha + \sqrt{-\alpha + 4\alpha^2 - 4\alpha^3}}{1+4\alpha^2}$. These roots are not real numbers for $\alpha > 0.207$, because the term under square root is negative ($-\alpha + 4\alpha^2 - 4\alpha^3 < 0$). As this polynomial is positive in $\tilde{\theta}_1 = 0$, it means that it is positive for any real value of $\tilde{\theta}_1$, and, in turn, $\pi_2^{NP} > \pi_2^{BC}$.

Finally,

$$\pi_2^{NPC} - \pi_2^C = \frac{q(1 - \tilde{\theta}_1)((4\alpha - 1)\tilde{\theta}_1 - 1)}{4}$$

This difference is positive because $\tilde{\theta}_1 > 1/(4\alpha - 1)$ in our range of interest as we are in $\tilde{\theta}_1 > 1/(2\alpha) > 1/(4\alpha - 1)$. ■

C.2 Equilibrium at period 1

At period 1, $\tilde{\theta}_1$ leading to price discrimination in $t = 2$ is an equilibrium if, given p_1 and given the subgame equilibrium at period 2, both the new buyers and the past buyers prefer not to change their decisions at

$t = 1$. In the following, we consider in turn the subgame equilibrium NPC and the subgame equilibrium NP.

C.2.1 Case leading to NPC at period 2

In the case leading to NPC at period 2, we are in a configuration where $\tilde{\theta}_1 > 1/2$ and $p_2^{PC} > p_2^N \iff \alpha > 1/2$. Also recall that $p_2^{PC} = q\alpha\tilde{\theta}_1$, $p_2^N = q\tilde{\theta}_1/2$, and $\tilde{\theta}_2^N = \tilde{\theta}_1/2 > \tilde{\theta}_2^{PC} = \alpha\tilde{\theta}_1 \iff \alpha > 1/2$.

- Demand from consumers. For any value of $\tilde{\theta}_1$, we can distinguish three sets of consumers depending on θ_1, each set corresponding to a specific trade off. For each set, we analyze below the conditions that lead each consumer not to change his first period decision (i.e. choosing $x_1 = 0$ if $\theta < \tilde{\theta}_1$ and $x_1 = 0$ if $\theta > \tilde{\theta}_1$):

> If $\theta_1 \in [0, \tilde{\theta}_1/2]$, the consumer chooses $x_2 = 0$ whatever his choice at period 1. Hence we need to have $(0,0) \succ (1,0)$ which is equivalent to $\theta_1 \leq p_1/q$. Since $\theta_1 < \tilde{\theta}_1/2$, the condition $(0,0) \succ (1,0)$ requires to have $\tilde{\theta}_1 \leq 2p_1/q$.
>
> If $\theta_1 \in [\tilde{\theta}_1/2, \tilde{\theta}_1]$, the consumer chooses $x_2 = 1$ if $x_1 = 0$ but $x_2 = 0$ if $x_1 = 1$. Hence we need to have $(0,1) \succ (1,0)$. This is equivalent to $\theta_1 q - p_2^N > \theta_1 q - p_1 \iff p_1 > p_2^N = q\tilde{\theta}_1/2 \iff \tilde{\theta}_1 \leq 2p_1/q$.
>
> If $\theta_1 \in [\tilde{\theta}_1, 1]$, the consumer chooses $x_2 = 1$ whatever his choice at period 2. Hence we need to have $(1,1) \succ (0,1)$ which is equivalent to $\theta_1 \geq \frac{p_1 + p_2^{PC} - p_2^N}{q\alpha}$. Since $\theta_1 > \tilde{\theta}_1$, the condition $(1,1) \succ (0,1)$ requires that $\tilde{\theta}_1 \geq \frac{p_1 + p_2^{PC} - p_2^N}{q\alpha} \iff \tilde{\theta}_1 \geq \frac{2p_1}{q}$.

Combining the above three conditions, we can conclude that if $\tilde{\theta}_1 = 2p_1/q$ then no consumer has an incentive to change its decision at period 1, which means that this situation is an equilibrium.

- Monopoly pricing decision. At the first period, the monopoly chooses the price that maximizes its cumulated profit over the two periods which is $p_1 \cdot (1 - \tilde{\theta}_1) + \pi_2^{NPC}$. After introducing $\tilde{\theta}_1 = 2p_1/q$ we get:

$$\Pi^{NPC} = \frac{-p_1(p_1 - q + 4p_1\alpha - 2q\alpha)}{q}$$

This function is concave in p_1 and has a maximum for $p_1^{NPC} = \frac{q(1+2\alpha)}{2(1+4\alpha)}$. The profit over the two periods is then:

$$\Pi^{NPC*} = \frac{q(1+2\alpha)^2}{4(1+4\alpha)}.$$

It can be easily verified that at this candidate equilibrium $\tilde{\theta}_1 > 1/2$ and the optimal price at period 1 is such that: $p_1^{NPC} = p_2^N < p_2^{PC} \iff \alpha > 1/2$.

C.2.2 Case leading to NP at period 2

In the case leading to NP at period 2, we are in a configuration $\tilde{\theta}_1 < 1/2$. Also recall that $p_2^P = q\alpha/2$, $p_2^N = q\tilde{\theta}_1/2$, $\tilde{\theta}_2^N = \tilde{\theta}_1/2 < \tilde{\theta}_2^P = \alpha/2 \iff \tilde{\theta}_1 < \alpha$.

- Demand from consumers. For any value of $\tilde{\theta}_1$, we can distinguish four sets of consumers depending on θ_1, each set corresponding to as specific trade off. For each set, we analyze below the conditions that lead each consumer not to change his first period decision (i.e. choosing $x_1 = 0$ if $\theta < \tilde{\theta}_1$ and $x_1 = 0$ if $\theta > \tilde{\theta}_1$):

If $\theta_1 \in [0, \tilde{\theta}_1/2]$, the consumer chooses $x_2 = 0$ whatever his choice at period 2. Hence we need to have $(0, 0) \succ (1, 0)$ which is equivalent to $\theta_1 \leq p_1/q$. Since $\theta_1 < \tilde{\theta}_1/2$, the condition $(0, 0) \succ (1, 0)$ requires to have $\tilde{\theta}_1 \leq 2p_1/q$.

If $\theta_1 \in [\tilde{\theta}_1/2, \tilde{\theta}_1]$, the consumer chooses $x_2 = 1$ if $x_1 = 0$ but $x_2 = 0$ if $x_1 = 1$. Hence we need to have $(1, 0) \succ (0, 1)$. This is equivalent to $\theta_1 q - p_2^N > \theta_1 q - p_1 \iff p_1 > p_2^N = q\tilde{\theta}_1/2 \iff \tilde{\theta}_1 \leq 2p_1/q$.

If $\theta_1 \in [\tilde{\theta}_1, 1/2]$, the consumer chooses $x_2 = 0$ if $x_1 = 1$ (because $\theta_2 < \theta_2^P$ in this case) and $x_2 = 1$ if $x_1 = 0$. Hence we need to have $(1, 0) \succ (0, 1)$ which is equivalent to $\theta_1 \geq 2p_1/q$. As $\theta_1 > \tilde{\theta}_1$, the condition $(1, 0) \succ (0, 1)$ requires that $\tilde{\theta}_1 > 2p_1/q$.

If $\theta_1 \in [1/2, 1]$, the consumer chooses $x_2 = 1$ whatever his choice at period 2. Hence we need to have $(1, 1) \succ (0, 1)$ which is equivalent to $\theta_1 \geq \frac{p_1 + p_2^P - p_2^N}{q\alpha}$. As we have $\theta_1 \geq 1/2$, the condition $(1, 1) \succ (0, 1)$ requires that $1/2 \geq \frac{p_1 + p_2^P - p_2^N}{q\alpha} \iff \tilde{\theta}_1 \geq 2p_1/q$.

Combining the above four conditions, we can conclude that if $\tilde{\theta}_1 = 2p_1/q$, then no consumer has an incentive to change its decision at period 1, which means that this situation is an equilibrium.

- Monopoly pricing decision At the first period, the monopoly chooses the price that maximizes its cumulated profit over the two periods which is $p_1 \cdot (1 - \tilde{\theta}_1) + \pi_2^{NP}$. After introducing $\tilde{\theta}_1 = 2p_1/q$ we get:

$$\Pi^{NP} = \frac{-4p_1^2 + 4p_1 q + q^2\alpha}{4q}$$

This function is concave in p_1 and has a maximum for $p_1 = q/2$ which is outside the interval of interest as it would imply $\tilde{\theta}_1 = 1$, while we need here to have $\tilde{\theta}_1 < 1/2$. This means that $\tilde{\theta}_1 = 1/2$, and in turn, $p_1^{NP} = q/4$. The profit over the two periods is then:

$$\Pi^{NP*} = \frac{q(3 + 4\alpha)}{16}.$$

It can be easily shown that the optimal price at period 1 is such that: $p_1^{NP} = p_2^N < p_2^P \iff \alpha > 1/2$.

C.2.3 Period 1 price equilibrium with discrimination

p_1^{NPC} and p_1^{NP} are two price candidates, and the monopoly chooses the one leading to the highest cumulated profit for him. We have:

$$\Pi^{NPC*} - \Pi^{NP*} = \frac{q(1+2\alpha)^2}{4(1+4\alpha)} - \frac{q(3+4\alpha)}{16} - = \frac{q}{16+64\alpha)} > 0$$

We conclude that the optimal price is p_1^{NPC}.

Direct comparisons of cumulated profits in each local optimum show that the equilibrium is NPC for $\alpha \leq 1/2$, whereas it stays BC for $\alpha > 1/2$. Indeed, for $\alpha < 1/3$, we need to compare:

$$\Pi^{NPC*} - \Pi^{D} = \frac{q(1+2\alpha)^2}{4(1+4\alpha)} - \frac{q}{4} = \frac{q\alpha^2}{1+4\alpha} > 0;$$

For $\alpha > 1/3$ we need to compare:

$$\Pi^{NPC*} - \Pi^{BC} = \frac{q(1+2\alpha)^2}{4(1+4\alpha)} - \frac{q\alpha}{1+\alpha} = \frac{q(-1+2\alpha)(-1-3\alpha+2\alpha^2)}{4(1+\alpha)(1+4\alpha)}.$$

The above difference is positive iff $\alpha < 1/2$ because $-1-3\alpha+2\alpha^2 < 0$ for $\alpha \in (1/3, 1)$. Figure 14 displays this result by adding Π^{NPC*} (bold olive line) in addition to the candidates with no price discrimination.

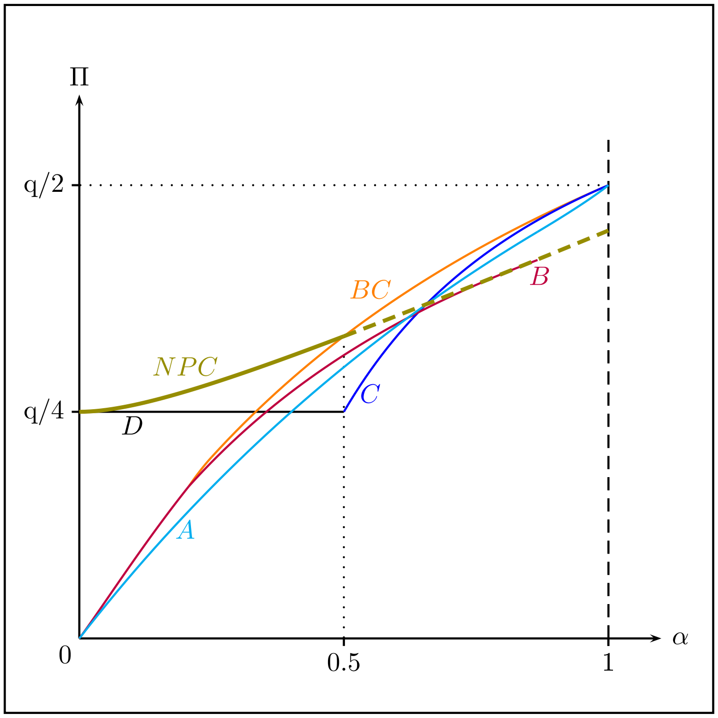
Two results are worth pointing out. First, price discrimination at period 2 eliminates the case where no consumer buys twice (equilibrium D). Second, it is not always beneficial for the monopolist to be able to condition his second-period pricing to purchasing history. Indeed, this strategy for $\alpha > 1/2$ is dominated by the non-discrimination strategy (BC in this range of α). Namely, comparing strategies NPC and BC we find that in both cases, it is optimal to induce all past consumers to buy also in t=2, that is: $\tilde{\theta}_2^{PC} = \alpha\tilde{\theta}_1^{NPC}$ and $\tilde{\theta}_2^{BC} = \alpha\tilde{\theta}_1^{BC}$. $p_2^P = q\alpha\tilde{\theta}_1^{NPC}$ and $p_2^{BC} = q\alpha\tilde{\theta}_1^{BC}$. As for $t = 1$ we get that:

$$\tilde{\theta}_1^{NPC} = \frac{1+2\alpha}{1+4\alpha} > \tilde{\theta}_1^{BC} = \frac{1}{1+\alpha} \iff \alpha > 1/2$$

which implies that $\tilde{\theta}_2^{PC} > \tilde{\theta}_2^{BC} \iff \alpha > 1/2$ and $p_2^P > p_2^{BC} \iff \alpha > 1/2$.

$$p_1^{NPC} = \frac{q(1+2\alpha)}{2(1+4\alpha)} < p_1^{BC} = \frac{q\alpha}{1+\alpha} \iff \alpha > 1/2$$

Figure 14: Comparison of cumulated profit with and without price discrimination



In words, for $\alpha > 1/2$, both the demand and the price in $t = 1$ under price discrimination are lower than under non price-discrimination. As for the second period, where $p_1^{NPC} = p_2^N < p_2^P \iff \alpha > 1/2$ and $p_1^{BC} = p_2^{BC} > p_2^N \iff \alpha > 1/2$, we have a larger overall demand under price discrimination (for any α) but a lower price for the new buyers and a higher price for the past buyers (wrt non price-discrimination): the larger demand in $t = 2$ due to the presence of new buyers (that are absent under non price-discrimination) does not compensate the lower profit in $t = 1$ as these new buyers buy the product at a lower price than the past buyers.

C.3 Welfare analysis

We compare total welfare with and without price discrimination. Welfare generated by each consumer is 0 for consumers with low θ_1 that never buy the product, $q\theta_1$ for consumers with intermediary values of θ_1 that buy the product only once, and $q\theta_1(1 + \alpha)$ for consumers with the higher values of θ_1 that buy the product twice. Prices are ignored because they correspond to a transfer to the monopoly.

This welfare comparison is made for $\alpha < 1/2$: we need to compare BC and NPC for $\alpha \in [1/3, 1/2]$ and D and NPC for $\alpha < 1/3$. We first compile the total welfare in the cases BC, D and NPC and then make the comparisons.

- Case BC**.** Remind that $\tilde{\theta}_1^{BC} = 1/(1+\alpha)$. All consumers such that $\theta_1 > \tilde{\theta}_1^{BC}$ buy at both periods. At period 2, all consumers such that $\theta_1 > \alpha\tilde{\theta}_1^{BC}$ buy the product. Hence all consumer with $\theta_1 \in [\alpha\tilde{\theta}_1^{BC}, \tilde{\theta}_1^{BC}]$ buy one unit of product at period 2.

$$\begin{aligned} W^{BC} &= \int_0^{\alpha\tilde{\theta}_1^{BC}} [0]\, d\theta_1 + \int_{\alpha\tilde{\theta}_1^{BC}}^{\tilde{\theta}_1^{BC}} [q\theta_1]\, d\theta_1 + \int_{\tilde{\theta}_1^{BC}}^{1} [q\theta_1(1+\alpha)]\, d\theta_1 \\ &= \frac{q(1+\alpha+\alpha^2)}{2(1+\alpha)} \end{aligned}$$

- Case D**.** Remind that this case leads to the same situation compared to the benchmark where the product is sold only during one period. More precisely, no consumer buy at period 1 and half of the consumer buy at period 2.

$$W^D = \int_0^{1/2} [0]\, d\theta_1 + \int_{1/2}^{1} [q\theta_1]\, d\theta_1 = \frac{3q}{8}$$

- Case NPC**.** Remind that $\tilde{\theta}_1^{NPC} = (1+2\alpha)/(1+4\alpha)$. All consumers such that $\theta_1 > \tilde{\theta}_1^{NPC}$ buy at both periods. Indeed, in this case NPC, all past buyers buy at period 2. At period 2, all consumers with $\theta_1 \in [\tilde{\theta}_1^{NPC}/2, \tilde{\theta}_1^{NPC}]$ are new buyers and buy at period 2.

$$\begin{aligned} W^{NPC} &= \int_0^{\tilde{\theta}_1^{NPC}/2} [0]\, d\theta_1 + \int_{\tilde{\theta}_1^{NPC}/2}^{\tilde{\theta}_1^{NPC}} [q\theta_1]\, d\theta_1 + \int_{\tilde{\theta}_1^{NPC}}^{1} [q\theta_1(1+\alpha)]\, d\theta_1 \\ &= \frac{q(3+4\alpha(4+3\alpha))}{8(1+4\alpha)}. \end{aligned}$$

When comparing the three welfare levels we can observe that:

$$W^{BC} - W^{NPC} = \frac{q(1-2\alpha)(1+3\alpha-2\alpha^2)}{8(1+\alpha)(1+4\alpha)} > 0$$

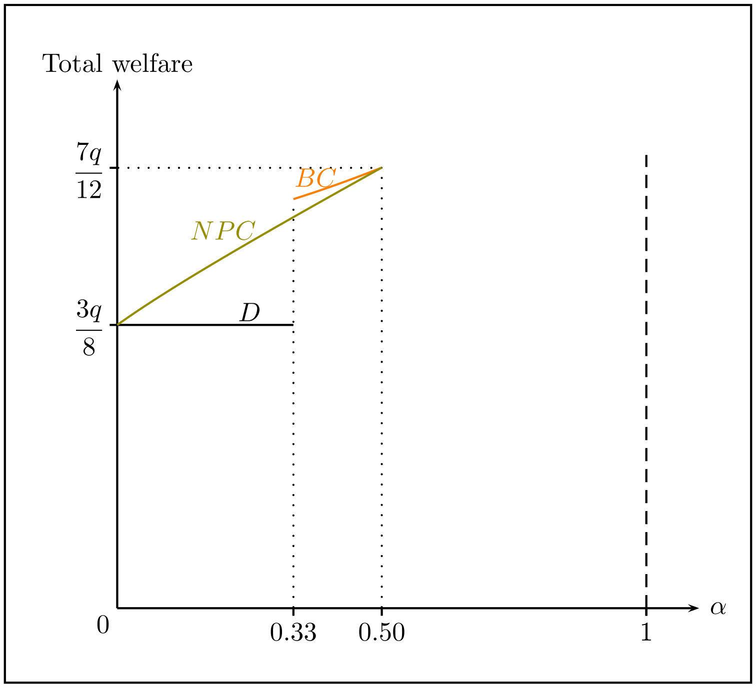
$$W^{NPC} - W^{D} = \frac{q\alpha(1+3\alpha)}{2(1+4\alpha)} > 0.$$

This result is illustrated in Figure 15. From this comparison we can see that if, at the equilibrium, some product can be sold twice, then price discrimination is welfare reducing. Conversely, if no product is sold twice at the equilibrium, then price discrimination is welfare enhancing.

A more precise look at the threshold on θ_1 helps understanding this result.

When comparing BC and NPC, we can see that $\tilde{\theta}_1^{NPC} < \tilde{\theta}_1^{BC}$ but $\tilde{\theta}_1^{NPC}/2 > \alpha\tilde{\theta}_1^{BC}$. With price discrimination, more consumers consume twice but less consumers consume once. The extra number

Figure 15: Comparison of total welfare with and without price discrimination



of consumers consuming twice is lower compared to the number of consumers that no longer consume once. Indeed, total demand over the two periods is equal to 1 with BC and is lower than 1 with NPC (total demand with NPC is equal to $(1 + 6\alpha + 4\alpha^2)/(2(1 + 4\alpha))$). This explains why price discrimination leads to a welfare loss. Price discrimination enables to define specific prices for past and new buyers with lower prices for the former. Hence, compared to unique second period pricing, price discrimination leads to extra consumers buying twice (consumers that are past buyers at period 2) and less consumers buying once (consumers that are new buyers at period 2).

When comparing D and NPC, we can see that $\theta_1^{NPC}/2 < 1/2$. Price discrimination leads to extra consumers buying at least once. In addition, among these consumers, some are buying twice with price discrimination while they are all buying only once in the case D. Having an option for a second purchase is welfare increasing. Without price discrimination, this second purchase option forces the monopoly to strongly distort its price leading to a profit loss for him. Hence the monopoly prefers to avoid this price distortion and sell the product at only one period. Price discrimination at period 2 enables to escape from this distortion, so that it is preferred by the monopoly and leads to higher total welfare.

D Monopoly selling a bundle of two units of product at period 1

- Mixed bundling strategy. At period 1, the consumer can either buy nothing, one unit of product at p_1 or two units of product at p_B. At period 2, the consumer can either buy nothing or one unit of product at p_2. As before the consumer can fully anticipate the price at period 2. A consumer with high value of θ generally buys two units of product: he buys the bundle if $p_B < p_1 + p_2$ or the two products separately if $p_1 + p_2 < p_B$. This arbitrage opportunity on the consumer side makes the mixed bundling strategy useless for the monopoly.

- Pure bundling strategy. Here, at period 1, the consumer can buy either the bundle of two units of products (at price p_B) or nothing. Consumers such that $\theta_1 > \bar{\theta}_1$ buy the bundle. These consumers exit the market at period 2. Then, at period 2, all consumers such that $\theta_1 < \bar{\theta}_1$ can buy either one unit of product (at price p_2) or nothing.

The demand at period 2 is $\bar{\theta}_1 - p_2/q$. The monopoly maximizes $p_2(\bar{\theta}_1 - p_2/q)$. The optimal price and corresponding profit are $p_2^* = q\bar{\theta}_1/2$ and $\pi_2^* = q\bar{\theta}_1^2/4$. All consumers such that $\theta_1 > \bar{\theta}_1/2$ buy one unit of product at period 2.

We now consider the demand at period 1. We have three conditions:

At period 1, consumers such that $\theta_1 > \bar{\theta}_1$ choose the bundle if $q\theta_1(1+\alpha) - p_B > q\theta_1 - p_2^*$ which is equivalent to $\theta_1 > \frac{2p_B - q\bar{\theta}_1}{2q\alpha}$. Hence this requires that $\bar{\theta}_1 > \frac{2p_B - q\bar{\theta}_1}{2q\alpha}$ which is equivalent to $\bar{\theta}_1 > \frac{2p_B}{q(1+2\alpha)}$.

Consumers such that $\theta_1 \in [\bar{\theta}_1/2, \bar{\theta}_1]$ choose to buy in $t = 2$ rather than buying the bundle if $q\theta_1 - p_2^* > q\theta_1(1+\alpha) - p_B$ which is equivalent to $\theta_1 < \frac{2p_B - q\bar{\theta}_1}{2q\alpha}$. Hence this requires that $\bar{\theta}_1 < \frac{2p_B - q\bar{\theta}_1}{2q\alpha}$ which is equivalent to $\bar{\theta}_1 < \frac{2p_B}{q(1+2\alpha)}$.

Consumers such that $\theta_1 < \bar{\theta}_1/2$ choose not to buy anything rather than buying the bundle if $0 > q\theta_1(1+\alpha) - p_B$ which is equivalent to $\theta_1 < \frac{p_B}{q(1+\alpha)}$. Hence this requires that $\bar{\theta}_1 < \frac{2p_B}{q(1+\alpha)}$.

The first and second conditions lead to $\bar{\theta}_1 = \frac{2p_B}{q(1+2\alpha)}$. This value fulfills the third condition: $\frac{2p_B}{q(1+2\alpha)} < \frac{2p_B}{q(1+\alpha)}$.

The monopoly cumulated profit is:

$$\Pi = \left(1 - \frac{2p_B}{q(1+2\alpha)}\right) p_B + \frac{p_B^2}{q(1+2\alpha)^2}$$

This profit is concave in p_B. The optimal price and the corresponding cumulated profit are:

$$p_B^{PB} = \frac{q(1+2\alpha)^2}{2(1+4\alpha)} \qquad \text{and} \qquad \Pi^{PB} = \frac{q(1+2\alpha)^2}{4(1+4\alpha)}$$

This bundle price leads to $\bar{\theta}_1^{PB} = \frac{1+2\alpha}{1+4\alpha}$ and $p_2^{PB} = \frac{q(1+2\alpha)}{2(1+4\alpha)}$.

- Pure bundling and price discrimination are equivalent. Recall that, with price discrimination at period 2, the equilibrium is NPC, which means that all past buyers buy again the product at period 2. Hence with price discrimination or pure bundling, we have three sets of consumers:

Consumers with low values of θ_1 that never buy the product. They are such that $\theta_1 \in [0, \tilde{\theta}_1^{NPC}/2]$ with price discrimination and such that $\theta_1 \in [0, \bar{\theta}_1^{PB}/2]$ with pure bundling.

Consumers with intermediate values of θ_1 that buy the product only at period 2. They pay p_2^N with price discrimination and p_2^{PB} with pure bundling. They are such that $\theta_1 \in [\tilde{\theta}_1^{NPC}/2, \tilde{\theta}_1^{NPC}]$ with price discrimination and such that $\theta_1 \in [\bar{\theta}_1^{PB}/2, \bar{\theta}_1^{PB}]$ with pure bundling.

Consumers with high value of θ that buy the product twice (or the bundle). They pay $p_1^{NPC} + p_2^P$ with price discrimination and p_B^{PB} with pure bundling. They are such that $\theta_1 \in [\tilde{\theta}_1^{NPC}, 1]$ with price discrimination and such that $\theta_1 \in [\bar{\theta}_1^{PB}, 1]$ with pure bundling.

Because of this equivalence between the two strategies, we obtain the same equilibrium. Indeed we can check that $\Pi^{PB} = \Pi^{NPC}$ and:

$$\bar{\theta}_1^{PB} = \tilde{\theta}_1^{NPC} = \frac{1+2\alpha}{1+4\alpha}$$

$$p_B^{PB} = p_1^{NPC} + p_2^{PC} = \frac{q(1+2\alpha)^2}{2(1+4\alpha)}$$

$$p_2^{PB} = p_2^N = \frac{q(1+2\alpha)}{2(1+4\alpha)}$$

The profit and welfare represented in Figures 14 and 15 are valid with pure bundling. Note that this equivalence holds for any value of $\alpha \in [0, 1]$. However, for $\alpha \in [0.5, 1]$ pure bundling (or price discrimination) leads to a lower monopoly profit and leads to lower total welfare (compared to the profit with no price discrimination).[6] Hence, nothing interesting emerge from pure bundling with $\alpha \in [0.5, 1]$.

[6]The total welfare with pure bundling for $\alpha \in [0.5, 1]$ is the extension of the curve NPC represented in Figure 15 and this curve is below the welfare level with BC.