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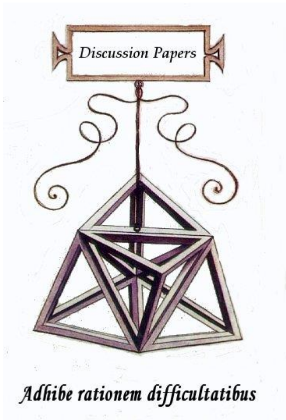

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Strategic product compatibility in network industries

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Abstract

The degree of compatibility is a crucial feature of network goods. This article considers the degree of product compatibility as a strategic variable in a Cournot duopoly with network consumption externalities. For doing this, it develops a non-cooperative "compatibility decision game" (CDG) in which choosing whether letting products being (in)compatible occurs at the first stage. When compatibility is costless, the unique (Pareto efficient) sub-game perfect Nash equilibrium (SPNE) of the CDG predicts that both firms choose product compatibility. When there exist quasi-fixed costs of compatibility, the spectrum of SPNE of the CDG greatly increases. The equilibrium outcomes change depending on whether the extent of product compatibility is endogenous (i.e., it is a profit-maximising variable) or exogenous (e.g., it is given by technical constraints) on the firm side. In the former case, the emerging SPNE implies ranging from a unique regime to multiple regimes of (in)compatibility allowing the emergence of different scenarios: deadlock, prisoner's dilemma and coordination game. In the latter case, the CDG can also become an anti-coordination game. This allows us to provide a novel explanation for the well-known and widespread case according to which Apple products in the computer market (based on macOS) can also be used with Windows OS, but Microsoft products (based on Windows OS) cannot be used with macOS. The article also pinpoints the welfare outcomes corresponding to the SPNE.

Keywords Network externality; Product compatibility; Cournot duopoly; Quasi-fixed costs

JEL Classification L1, L3, D4

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1. Introduction

Network products are of increasing importance in the market, as shown by the development of telecommunication services, internet-related activities, software and so on. This has a special concern in strategic competitive markets, as pinpointed by the pioneering article by Katz and Shapiro (1985), but also under R&D innovation (Cabral, 1990) and in a monopoly market (Cabral et al., 1999). These issues are well-surveyed by Shy (2011). A relevant question of network markets concerns the compatibility of products. The present article tackles this issue and innovates in the literature by using a game-theoretic approach to study whether the production of in(compatible) products emerges as a sub-game perfect Nash equilibrium (SPNE) in a non-cooperative compatibility decision game (CDG) with complete information, in which the production of compatible goods require sustaining fixed costs of compatibility (Katz and Shapiro, 1985). Indeed, the existence of an interior optimal degree of compatibility (partial compatibility), e.g., Stadler et al., 2022, or a corner solution implying full compatibility does not mean that the production of compatible products emerges as an endogenous SPNE of a non-cooperative compatibility decision game with complete information. The emergence of this kind of equilibrium depends on the firms' incentives at the first (decision) stage of the game.

A consumption externality arises when the utility that a consumer derives from the consumption of a good depends on the number of other users who belong to the same network. However, different sizes of the network can occur depending on specific features of the network market.[1] Specifically, Katz and Shapiro (1985, p. 424) offer a clear example – regarding hardware and software markets – of the importance of the degree of compatibility for the size of the network and thus of the consumption externalities: "If two brands of hardware are able to work with the same software (e.g. an operating system), then the two brands are said to be compatible. Therefore, depending on how and whether software produced for working on one brand of hardware may also work on another brand of hardware, for the consumer of one brand of hardware the relevant network is the set of users who buy other brands of hardware which are totally (or partially) compatible with her brand".

Since early noted by Katz and Shapiro (1985), firms can choose whether to manufacture compatible products and, through this choice, they establish whether only its products or aggregate market sales are the relevant ones in the formation of the network dimension, which in turn affect the size of network consumption externalities. Therefore, the compatibility decision must be properly analysed, and the crucial question is "whether firms will have proper incentives to produce compatible goods or

[1] As exemplified by Katz and Shapiro (1985, p. 424) "The scope of the network that gives rise to the consumption externalities will vary across markets. In some cases, such as the automobile example, the sales of only one firm will constitute the relevant network. In other cases, the relevant network will comprise the outputs of all firms producing the good. For example, the number of stereo phonographs of any one brand is not a determinant of the supply of records that a consumer can play on his or her stereo."

services" (Katz and Shapiro, 1985, p. 425). We argue that the question early posed by Katz and Shapiro should be studied in an appropriate game-theoretic setting.

Despite the importance of oligopolistic markets with network externalities and the crucial role of the firms' decisions about compatibility, a small number of works has been done to give a robust game-theoretic answer to the compatibility decision issue[2] in the standard oligopolistic Cournot framework.

This article aims at filling the gaps discussed so far. For doing this, it conducts an analysis based on a non-cooperative game-theoretic setting in a Cournot duopoly and then develops a strategic two-stage non-cooperative game with complete information, in which two firms simultaneously choose: in the first stage, whether to produce (in)compatible network goods[3], and in the second stage, to compete on quantity in the product market.[4] In this regards, the article develops the CDG and then appropriately endogenizes the choice of compatibility (K) versus non-compatibility (NK) by investigating the occurrence of the SPNE of the game and the corresponding efficiency properties and the related welfare outcomes.

Following Katz and Shapiro (1985, 418)[5], network firms can incur two types of costs: production costs and compatibility costs. The former (fixed and variable) represent the standard costs related to the existing technology of production (to simplify the analysis, we assume that both are equal to zero without loss of generality). The latter represents the costs of achieving compatibility, which is assumed to be fixed in this article, i.e., independent of the scale of production; this, in turn, implies that compatible and incompatible products have the same (zero) marginal production costs.

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[2] For the sake of precision, Katz and Shapiro (1985) distinguish two basic technologies by which compatibility can be achieved: (i) through the joint adoption of a product standard (which requires a common effort and decision by firms, acting together to make their products compatible); this means that the decision for compatibility should be appropriately analysed through a cooperative game, and (ii) through the construction of an adapter (in a general sense, not necessarily a physical device), where a single firm can act unilaterally to make its product compatible with those of another firm; this means that the decision for compatibility should be appropriately analysed through a non-cooperative game. In this respect, we may say that our model implicitly assumes that compatibility can be achieved – by using the terminology of Katz and Shapiro (1985) – through the construction of an adapter.

[3] The present article discusses the CDG in a duopolistic quantity competition setting by considering endogenous and exogenous compatibility on the firm side (as pinpointed at the beginning of Section 2). In the former case, it compares full compatibility (when the product or the components produced by all firms are compatible with each other) versus incompatibility (when the product or no components produced by different firms are compatible), as is standard in the literature on the compatibility between competing products (e.g., Matutes and Regibeau, 1988; Economides, 1989). A Recent exception is Stadler et al. (2022), which considers endogenous partial compatibility but does not analyse the CDG. In the latter case, it studies the possible complete characterisation of SPNE emerging for any degree of compatibility.

[4] Our approach considers a static game. As the issue of compatibility has been also considered under a dynamic setting (e.g., Katz and Shapiro, 1986a; Heinrich, 2017), we may consider an appropriate dynamic game as a possible direction of future research on this issue.

[5] Early case studies of compatibility decisions have been conducted by Brock (1975) in the U.S. mainframe computer and Kurdle (1975) in the farm machinery industries.

Examples of fixed costs of compatibility include “costs of developing and designing a compatible product, the costs of negotiating to select a standard, and the costs of introducing a new, compatible product” (Katz and Shapiro, 1985, p 427).

If firms choose to produce compatible goods, it is possible to distinguish two cases: (a) the degree of compatibility is endogenous on the firm side, i.e., it is chosen as a profit maximising variable at an intermediate stage of the game; (b) the degree of compatibility is exogenously given, for example, by technical constraints, so that each firm has does not have the opportunity to modify the extent of compatibility of its products, which is therefore determined by the state of the art of the existing technology.

The convenience to produce compatible or incompatible products emerges from the interaction between two counterbalancing effects on profits in the CDG (in both cases of the endogenous and exogenous degree of compatibility). On one hand, there exists a positive effect exerted by the degree of compatibility and the intensity of the network effect (both affecting output). On the other hand, there exists a negative effect due to the fixed cost of compatibility (which does not affect output).

If the degree of compatibility is endogenous on the firm side, the CDG shows a rich spectrum of SPNE outcomes depending on the network size and the extent of the fixed cost of compatibility (there exists a one-to-one relationship between these two variables in determining the relevant SPNE). This happens although profits are always increasing in the degree of compatibility. For any given value of the extent of the network effect, the SPNE of the CDG changes from the Pareto efficient (K, K) to the Pareto efficient (NK, NK), in which the game is a deadlock with no conflict between self-interest and mutual benefit of in(compatibility), as long as the fixed cost of compatibility increases, passing through intermediate results, in which there exists a conflict between self-interest and mutual benefit of compatibility or there is indeterminacy (coordination game), in which the firm’s payoff that can be obtained by playing NK is larger than those that can be obtained by playing K.

As consumer surplus under the compatibility regime can be higher or lower than the corresponding value of the incompatibility regime, the article also pinpoints some relevant social welfare outcomes corresponding to the SPNE, in which it is possible to have a win-win result for society, in which both firms and consumers are better off, by producing incompatible products.

If the degree of compatibility is exogenous on the firm side, the CDG confirms the SPNE outcomes obtained when the degree of compatibility is endogenous with one relevant exception: the emergence of an anti-coordination game in which only one firm produces compatible goods. Specifically, if the network effect is sufficiently low (resp. high) and, a fortiori, the degree of compatibility is sufficiently high (resp. low), the SPNE allows for a compatibility regime, which is Pareto efficient for both firms (resp. Pareto inefficient for both firms). This is because the positive profit effect of the network and the compatibility on profits overweighs (resp. does not overweigh) the negative effect of the fixed cost of compatibility. For intermediate values of both the network size and the degree of compatibility, the quasi-balance between the positive and the negative effect on profits induces the occurrence of an equilibrium in which only one firm chooses to be compatible at an exogenously given degree, while the rival chooses to produce non-compatible goods. Finally, when the degree of compatibility is very high (e.g., close to the full compatibility regime), but the network effect is sufficiently low, multiple symmetric Nash equilibria emerge. This is because the strong positive effect of the high compatibility is sufficient to induce both firms to unilaterally produce compatible goods, but this effect is not enough magnified due to the low intensity of the network effect to an extent that it prevents both firms to deviate unilaterally by the compatibility regime. Then, both compatibility and incompatibility are SPNE, although for both firms the common choice to be non-compatible would lead to being better off.

Definitively, one result is worthwhile to be stressed here. As known, the compatibility between two products may run in only one direction (for example, because some components of one firm can be used together with those of the rival, but not vice versa).[6] A recent paradigmatic illustrative case is that of the compatibility between the operating systems of Apple and Microsoft: indeed, Windows OS can be used in Mac computers, which allows for dual booting, but Mac OS cannot be used with the PC.[7] So far, one explanation for this type of apparently puzzling situation typically has been ascribed to the fact that the two firms might have different levels of network externalities or they are asymmetrically located: for instance, (i) larger firms are more likely to prefer incompatibility than smaller firms (Katz and Shapiro, 1985), (ii) Asian computer firms are more likely to prefer intra-technology than inter-technology competition (Ferguson and Morris, 1993), (iii) firms with asymmetric strategic positions with respect to quality or costs may have different preferences about compatibility (Einhorn, 1992; Farrell et al., 1993). The present article has shown that one-way compatibility can endogenously emerge as a sub-game perfect Nash equilibrium of the CGD due to the existence of a fixed cost of compatibility.

This result provides a novel, theoretically robust, explanation emerging in an appropriate game-theoretic setting, for the stylised fact of the one-way compatibility, in which also the firms' history plays a crucial role. Through the unilateral choice of compatibility, one firm increases its market share. This is because its output increases and that of the rival decreases. In this sense, the degree of compatibility can be used as a strategic device by one firm to gain market share. This works is like other well-studied devices such as managerial delegation or corporate social responsibility. Unlike them, however, which generally constitute a unique SPNE where firms are entrapped in a prisoner's dilemma, the choice of compatibility can represent an efficient SPNE (deadlock or anti-prisoner's dilemma).

These endogenously determined results are novel to the previous literature about compatibility. Although drawing policy conclusions is not the object of this article, the

[6] According to Besen and Farrel (1994), who analysed the case of one-way compatibility, the firm wishing to join its rival's network, when the latter preferd to maintain its technology as a propriety standard, is defined as the Pesky Little Brother, because in such a case the firms' game is similar to the game between a big brother who wants to be left alone and a pesky little brother who wants to be with his big brother.

[7] This illustrative case of one-way compatibility is also mentioned by Kim and Choi (2015, p. 115).

model and the related results show issues in which public policy can have an important role, for instance by affecting (e.g., through patents, copyright, subsidies, etc.) the fixed cost for compatibility with a vast range of privately and socially oriented outcomes.

The rest of the article proceeds as follows. Section 2 presents the basic ingredients of the model, showing the details of a Cournot duopoly in which firms can produce compatible (K) or incompatible (NK) products by pinpointing the timing of the CDG under the assumptions of the endogenous and exogenous degree of compatibility (k). Section 3 studies the endogenous market configuration emerging in the CDG when k is endogenous (i.e., it is chosen as a profit-maximising variable) on the firm side. Section 4 studies the endogenous market configuration emerging in the CDG when k is exogenous (i.e., it is given by technological constraints) on the firm side. Both sections also consider the social welfare outcomes corresponding to the SPNE. Section 5 outlines the main conclusions and some possible related policy issues.

2. The model

The model presented in this section directly departs from Katz and Shapiro (1985), who were first in studying network externality and product compatibility in a quantity-setting strategic competitive market. That work was followed by two other relevant articles (Katz and Shapiro, 1986a, 1986b) on the topic of product compatibility in oligopolistic industries. In these contributions, the degree of compatibility was exogenously given. The industrial organisation literature has also concentrated on the case of endogenous compatibility by letting firms choose the extent of the degree of compatibility with aim at maximising profits (e.g., Economides, 1989; Kim and Choi 2015; Stadler et al., 2022). The present article deals with both approaches (exogenous and endogenous compatibility) and adds to the literature the compatibility decision stage to a Cournot duopoly, in which each firm strategically chooses whether its products should be compatible with those of its rival. This will allow us to pinpoint a complete set of sub-game perfect Nash equilibria (SPNE) emerging in the compatibility decision game and then study whether product compatibility can endogenously be a market outcome in a strategic context.

Whether the degree of product compatibility can be considered an exogenous or endogenous variable for each firm depends on the main characteristics of the existing technology. If the technology does (not) allow to choose product compatibility, then the extent of the degree of compatibility is endogenous on the firm side and chosen to maximise profits (is exogenous on the firm side and it is taken as given according to the state of the art of the existing technology).

The timing of the non-cooperative compatibility decision game with complete information depends on whether the degree of compatibility is endogenous or exogenous on the firm side.

If the degree of compatibility is *endogenous*, at the first (decision) stage each firm chooses to let product being compatible or incompatible. At the second (intermediate) stage, each firm chooses the extent of product compatibility to maximise profits. At the third (market) stage, each firm competes à la Cournot in the product market and then chooses the quantity to maximise profits.

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The section now outlines the main features of the model. Consider a Cournot duopoly in which firms produce homogeneous network goods (Katz and Shapiro, 1985). The network effect (consumption externality) can be positive (e.g., mobile communications, software, internet-related activities, online social networks, fashion, etc) or negative (e.g., traffic congestion or network congestion over limited bandwidth). Under a positive (resp. negative) externality an increasing number of users increases (resp. reduces) the individual utility and thus the value of the goods for each consumer thus causing the so-called bandwagon (resp. snob) effect. To tackle the issue of strategic product compatibility we follow the main narrative of Katz and Shapiro (1985) and assume that firms are unable to commit themselves to a given output level and thus consumers form their expectations on total sales, which are fulfilled at equilibrium according to the standard rational expectations hypothesis. The microeconomic foundations of the (linear) market demand with network externality and product compatibility follow the recent article by Buccella et al. (2022).[8] More formally, the (normalised) inverse market demand of firm i is (see, e.g., Hoernig 2012; Chirco and Scrimitore, 2013; Bhattacharjee and Pal, 2014; Fanti and Buccella, 2016; Buccella et al., 2022):

$$p_i = 1 - q_i - q_j + n(y_i + k_i y_j), \tag{1}$$

where p_i is the marginal willingness to pay towards products of network i $(i, j = \{1,2\},\ i \neq j)$ q_i denotes the quantity of the goods produced by firm i and $n \in (-1,1)$ is the strength of the network effect (the higher the absolute value of n, the stronger the effect of network goods). Positive (resp. negative) values of n refers to the case of positive (resp. negative) network goods capturing the bandwagon (resp. snob) effect. The parameter $k_i \in [0,1]$ measures the degree of compatibility of the network of product j towards the network of product i. Pairwise, considering the normalised inverse market demand of firm j (not shown) the parameter $k_j \in [0,1]$ measures the degree of compatibility of the network of product i with the network of product j. In addition, y_i $(i, j = \{1,2\},\ i \neq j)$ denotes the consumers' expectations about the equilibrium output produced by firm i. For analytical tractability, and without loss of generality, we will consider henceforth the case of symmetric compatibility $k_i = k_j = k$, which resembles the case of common standardisation (Stadler et al., 2022). Although this hypothesis may seem simplistic, it nevertheless allows us to capture the emergence of, amongst other paradigms, an anti-coordination game, in which one firm chooses to let its products being compatible and the rival chooses to let its product being fully incompatible.

The generic firm i's profit function is given by:

[8] See also Naskar and Pal (2020) and Shrivastav (2021).

$$\Pi_i = (p_i - w)q_i - Z, \tag{2}$$

where $0 \leq w < 1$ is the constant average and marginal cost (i.e., the technology has constant returns to scale), which is set to zero henceforth without loss of generality as our aim is to deal with the most parsimonious modelling structure possible, and $Z > 0$ represents a quasi-fixed cost of compatibility. This assumption directly follows the original article by Katz and Shapiro (1985) and in line with a more recent contribution by Planer-Friedrich and Sahm (2021), who assumed quasi-fixed costs CSR with a similar narrative. Considering a cost function of product compatibility dependent on k, $Z(k) = xk$ (constant returns to scale), where $x > 0$ is a parameter measuring the efficiency of the product compatibility technology, allows to get a value of k that minimises profits at the second stage of the game and this, in turn, implies that a corner solution is optimal: $k = 1$ when x is relatively low and $k = 0$ when x is relatively high. Alternatively, by assuming a cost function with decreasing returns to scale $Z(k) = (x/2)k^2$ does not allow to solve the compatibility decision game in closed form, although a value of k that maximises profits (allowing partial compatibility), computed through numerical simulations, at the second stage of the game exists.[9]

3. Endogenous product compatibility

This section considers the case of endogenous product compatibility and then considers the possibility of a technology allowing to choose the value of k to maximise profits at an intermediate stage of the game.

3.1. The symmetric sub-game K/K

Consider the symmetric sub-game in which both firms choose to let their products being compatible with the products of the rival (K/K). Given the expression in (2), the equilibrium output at the third stage of the game must satisfy the first-order condition:

$$\frac{\partial \Pi_i}{\partial q_i} = 0 \Leftrightarrow 1 - 2q_i - q_j + n(y_i + ky_j) = 0. \tag{3}$$

Eq. (3) allows us to obtain the firm i's reaction function, that is:

$$q_i(q_j, y_i, y_j) = \frac{1 - q_j + n(y_i + ky_j)}{2}. \tag{4}$$

[9] We note that once we relax the assumption that moving to compatibility has no impact on marginal costs, things may change. As early highlighted by Katz and Shapiro (1985, p. 438), since marginal costs (different from fixed costs) affect the equilibrium output level, then it may occur that when the increase in marginal costs for compatibility is sufficiently large relative to the network effects, total output will be lower under complete compatibility than under incompatibility, with a subsequent reduction in the consumer surplus due to moving towards complete compatibility. Moreover, the incentive for choosing compatibility may no longer exert its positive effect on the network consumption externality, but, in a context of cooperative decision and oligopolistic competition, that of jointly increasing costs to reduce the total market sales because this may increase profits by raising revenues more than the corresponding increase in costs. However, an exhaustive numerical analysis of the case with variable non-linear cost of compatibility is beyond the scope of the present article but would be a worthwhile extension left for future research.

From Eq. (4), the reaction function of firm i (and the symmetric counterpart of firm j) are negatively sloped. This implies that products are perceived by firms as strategic substitutes (i.e., network effects and product compatibility do not affect the standard slope of the reaction functions).

By imposing the usual "rational expectation condition" such that $y_i = q_i$ and $y_j = q_j$ and solving the system of output reaction functions composed by (4) and its counterpart for firm j, the output and profits as a function of k in the symmetric sub-game K/K are respectively the following:

$$q_i^{K/K} = \frac{1}{3-n(1+k)}, \tag{5}$$

and

$$\Pi_i^{K/K} = \frac{1}{[3-n(1+k)]^2} - Z, \tag{6}$$

where $Z < \frac{1}{[3-n(1+k)]^2} \coloneqq Z_{TH}^{K/K}(n,k)$ is the condition that must be fulfilled to guarantee that profits in the sub-game K/K are positive.

At the second (intermediate) stage of the game, firm i chooses the degree of product compatibility to maximise profits. Therefore,

$$\frac{\partial \Pi_i^{K/K}}{\partial k} = \frac{2n}{[3-n(1+k)]^3}. \tag{7}$$

The expression in (7) allows us to conclude that when the network externality is positive (resp. negative) the sign of $\frac{\partial \Pi_i^{K/K}}{\partial k}$ is positive (resp. negative) for any $k \in [0,1]$. Therefore, in the case of positive (resp. negative) network effects the optimal value of the degree of product compatibility is in the sub-game K/K a corner solution given by $k = 1$ (resp. $k = 0$) implying full compatibility (resp. no compatibility) between the products of the two networks.

Then, if $n > 0$ the Nash equilibrium values of output and profits in the sub-game K/K are respectively given by the following expressions:

$$q_i^{*K/K}(n) = \frac{1}{3-2n}, \tag{8}$$

and

$$\Pi_i^{*K/K}(n) = \frac{1}{(3-2n)^2} - Z, \tag{9}$$

where the threshold that must be satisfied to get positive profits becomes $Z < \frac{1}{(3-2n)^2} \coloneqq Z_{TH}^{K/K}(n)$. The equilibrium consumer surplus and the equilibrium social welfare function in this sub-game are respectively the following:

$$CS^{*K/K}(n) = \frac{2(1-n)}{(3-2n)^2}, \tag{10}$$

and

$$W^{*K/K}(n) = CS^{*K/K}(n) + 2\Pi_i^{*K/K}(n) = \frac{2(2-n)}{(3-2n)^2} - 2Z, \tag{11}$$

The condition that guarantees positive values of social welfare in the sub-game K/K is fulfilled for any $Z < Z_{TH}^{K/K}(n)$.

If $n < 0$ the Nash equilibrium values of output and profits in the sub-game K/K coincides with those of the sub-game NK/NK.

3.2. The symmetric sub-game NK/NK

Consider now the symmetric sub-game in which both firms produce incompatible products (NK/NK), i.e., $k = 0$ for each firm, thus avoiding the quasi-fixed costs of product compatibility ($Z = 0$). This implies that profits of firm i becomes $\Pi_i = p_i q_i$, where $p_i = 1 - q_i - q_j + n y_i$. Given these expressions, the equilibrium output at the third stage of the game must satisfy the first-order condition:

$$\frac{\partial \Pi_i}{\partial q_i} = 0 \Leftrightarrow 1 - 2q_i - q_j + n y_i = 0. \tag{12}$$

Eq. (12) allows us to obtain the firm i's reaction function, that is:

$$q_i(q_j, y_i) = \frac{1 - q_j + n y_i}{2}. \tag{13}$$

By using Eq. (13) together with the symmetric counterpart of firm j allows to get the Nash equilibrium values of output and profits in the sub-game NK/NK are respectively the following:

$$q_i^{*NK/NK}(n) = \frac{1}{3-n}, \tag{14}$$

and

$$\Pi_i^{*NK/NK}(n) = \frac{1}{(3-n)^2}, \tag{15}$$

The equilibrium consumer surplus and the equilibrium social welfare function in this sub-game are respectively the following:

$$CS^{*NK/NK}(n) = \frac{2-n}{(3-n)^2}, \tag{16}$$

and

$$W^{*NK/NK}(n) = CS^{*NK/NK}(n) + 2\Pi_i^{*NK/NK}(n) = \frac{4-n}{(3-n)^2}, \tag{17}$$

3.3. The asymmetric sub-game K/NK

Let now us consider the case in which firm i produces compatible goods and the rival, firm j, produces incompatible goods. This implies that the profit functions of firm i and firm j can respectively be written as follows:

$$\Pi_i = p_i q_i - Z, \tag{18}$$

and

$$\Pi_j = p_j q_j, \tag{19}$$

where $p_i = 1 - q_i - q_j + n(y_i + k y_j)$ and $p_j = 1 - q_j - q_i + n y_j$.

Knowing the marginal willingness to pay of consumers of firm i and firm j, the maximisation of the expressions in (18) with respect to q_i and the maximisation of the expression in (19) with respect to q_j at the third stage of the game allows us to get the reaction functions of firm i and firm j in the sub-game K/NK, which are respectively given by Eqs. (4) and (13). By assuming $y_i = q_i$ and $y_j = q_j$ and solving the system of asymmetric output reaction functions, one gets:

$$q_i^{K/NK} = \frac{1 - n(1-k)}{(3-n)(1-n) + nk}, \tag{20}$$

$$q_j^{K/NK} = \frac{1-n}{(3-n)(1-n) + nk}, \tag{21}$$

where $q_i^{K/NK} > q_j^{K/NK}$ and

$$\Pi_i^{K/NK} = \frac{[1-n(1-k)]^2}{[(3-n)(1-n)+nk]^2} - Z, \tag{22}$$

$$\Pi_j^{K/NK} = \frac{(1-n)^2}{[(3-n)(1-n)+nk]^2}. \tag{23}$$

From (22), the feasibility condition that guarantees the positivity of profits of firm i in the asymmetric sub-game K/NK is $Z < \frac{[1-n(1-k)]^2}{[(3-n)(1-n)+nk]^2} \coloneqq Z_{TH}^{K/NK}(n,k)$.

At the second (intermediate) stage of the game, the K-firm i chooses the degree of product compatibility to maximise profits. Therefore,

$$\frac{\partial \Pi_i^{K/NK}}{\partial k} = \frac{2n(1-n)(2-n)[1-n(1-k)]}{[(3-n)(1-n)+nk]^3}. \tag{24}$$

The expression in (24) allows us to conclude that when the network externality is positive (resp. negative) the sign of $\frac{\partial \Pi_i^{K/NK}}{\partial k}$ is positive (resp. negative) for any $k \in [0,1]$. Therefore, in the case of positive (resp. negative) network effects the optimal value of the degree of product compatibility chosen by the K-firm in the sub-game K/NK a corner solution given by $k = 1$ (resp. $k = 0$) implying full compatibility (resp. no compatibility) between the products of the K-firm and those of the NK-firm.

Then, if $n > 0$ the Nash equilibrium values of output and profits in the sub-game K/NK are respectively given by the following expressions:

$$q_i^{*K/NK}(n) = \frac{1}{3(1-n)+n^2}, \tag{25}$$

$$q_j^{*K/NK}(n) = \frac{1-n}{3(1-n)+n^2}, \tag{26}$$

and

$$\Pi_i^{*K/NK}(n) = \frac{1}{[3(1-n)+n^2]^2} - Z, \tag{27}$$

$$\Pi_j^{*K/NK}(n) = \frac{(1-n)^2}{[3(1-n)+n^2]^2}, \tag{28}$$

where the threshold that must be satisfied to get positive profits for the K-firm becomes $Z < \frac{1}{[3(1-n)+n^2]^2} \coloneqq Z_{TH}^{K/NK}(n)$. The equilibrium consumer surplus and the equilibrium social welfare function in this sub-game are respectively the following:

$$CS^{*K/NK}(n) = \frac{(1-n)[3(1-n)+1+n^2]}{2[3(1-n)+n^2]^2}, \tag{29}$$

and

$$W^{*K/NK}(n) = CS^{*K/NK}(n) + \Pi_i^{*K/NK}(n) + \Pi_j^{*K/NK}(n) = \frac{2-n}{3(1-n)+n^2} - Z, \tag{30}$$

The condition that guarantees positive values of social welfare in the sub-game K/NK is fulfilled for any $Z < Z_{TH}^{K/NK}(n)$.

If $n < 0$ the Nash equilibrium values of output and profits in the sub-game K/NK coincides with those of the sub-game NK/NK.

3.4. Endogenous product compatibility and endogenous market outcomes

This section examines the first stage of the game, in which each firm chooses whether to let their products become compatible (or incompatible) compared to those of the rival in a non-cooperative quantity-setting environment with network externalities and endogenous product compatibility. The main variables of the problem are summarised in Table 1 (optimal values of k) and Table 2 (payoff matrix), in which the equilibrium profit functions are given by the expressions (9), (15), (25) and (26).

Table 1. Optimal (profit maximising) values of k.

Firm j $\rightarrow$ Firm i $\downarrow$	K	NK
K	1, 1	1, 0
NK	0, 1	0, 0

Table 2. The compatibility decision game (payoff matrix) when k is endogenous.

Firm j $\rightarrow$ Firm i $\downarrow$	K	NK
K	$\Pi_i^{*K/K}(n), \Pi_j^{*K/K}(n)$	$\Pi_i^{*K/NK}(n), \Pi_j^{*K/NK}(n)$
NK	$\Pi_i^{*NK/K}(n), \Pi_j^{*NK/K}(n)$	$\Pi_i^{*NK/NK}(n), \Pi_j^{*NK/NK}(n)$

The technical restrictions that must be satisfied to have well-defined equilibria in pure strategies for every strategic profile (one for each player) are $Z < Z_{TH}^{K/K}(n)$ and $Z < Z_{TH}^{K/NK}(n)$, where $Z_{TH}^{K/NK}(n) > Z_{TH}^{K/K}(n)$ for any $0 < n < 1$. Interestingly, therefore, the unique feasibility condition that must hold to guarantee meaningful Nash equilibrium and the corresponding welfare outcomes is $Z < Z_{TH}^{K/K}(n)$. Then, to derive all the possible SPNE of this non-cooperative game, one must study the sign of the profit differentials for $i = \{1,2\}, i \neq j$, that is:

$$\Delta\Pi_A(n) \coloneqq \Pi_i^{*K/NK}(n) - \Pi_i^{*NK/NK}(n), \tag{31}$$

$$\Delta\Pi_B(n) \coloneqq \Pi_i^{*NK/K}(n) - \Pi_i^{*K/K}(n), \tag{32}$$

and

$$\Delta\Pi_C(n) \coloneqq \Pi_i^{*NK/NK}(n) - \Pi_i^{*K/K}(n). \tag{33}$$

The first threshold defines the incentive of firm i to deviate from K to NK when its sign is negative (and vice versa when its sign is positive) when the rival, firm j, is playing NK. The second threshold defines the incentive of firm i to deviate from NK to K when its sign is negative (and vice versa when its sign is positive) when the rival, firm j, is playing K. The third threshold determines the Pareto efficiency/inefficiency of a symmetric SPNE.

From (31), the sign of $\Delta\Pi_A(n)$ is positive (resp. negative) if $Z < Z_A(n)$ (resp. $Z > Z_A(n)$), where

$$Z_A(n) \coloneqq \frac{n(2-n)(n^2-4n+6)}{[3(1-n)+n^2]^2(3-n)^2}, \tag{34}$$

is the threshold value of the quasi-fixed cost of compatibility (as a function of the extent of the network externality) such that $\Delta\Pi_A(n) = 0$.

From (32), the sign of $\Delta\Pi_B(n)$ is positive (resp. negative) if $Z < Z_B(n)$ (resp. $Z > Z_B(n)$), where

$$Z_B(n) \coloneqq \frac{n(2-n)(3n^2-8n+6)}{[3(1-n)+n^2]^2(3-2n)^2}, \tag{35}$$

is the threshold value of the quasi-fixed cost of compatibility (as a function of the extent of the network externality) such that $\Delta\Pi_B(n) = 0$.

From (33), the sign of $\Delta\Pi_C(n)$ is positive (resp. negative) if $Z < Z_C(n)$ (resp. $Z > Z_C(n)$), where

$$Z_C(n) \coloneqq \frac{3n(2-n)}{(3-n)^2(3-2n)^2}, \tag{36}$$

is the threshold value of the quasi-fixed cost of compatibility (as a function of the extent of the network externality) such that $\Delta\Pi_C(n) = 0$. In addition, we note that $Z_{TH}^{K/K}(n) > Z_B(n) > Z_A(n) > Z_C(n)$ for any $n > 0$, and $Z_B(n) = Z_A(n) = Z_C(n) = 0$ and $Z_{TH}^{K/K}(n) = 1/9$ if $n = 0$.

Then, the following proposition holds:

Proposition 1. If the network externality is positive, the endogenous market structure of the compatibility decision game if k is endogenous is the following.

[1] If $0 \leq Z < Z_C(n)$ then (K, K) is the unique Pareto efficient SPNE and the CDG is an anti-prisoner's dilemma.

[2] If $Z_C(n) < Z < Z_A(n)$ then (K, K) is the unique Pareto inefficient SPNE and the CDG is a prisoner's dilemma.

[3] If $Z_A(n) < Z < Z_B(n)$ then (K, K) and (NK, NK) are two symmetric SPNE (NK payoff dominates K) and the CDG is a coordination game.

[4] If $Z_B(n) < Z < Z_{TH}^{K/K}(n)$ then (NK, NK) is the unique pareto efficient SPNE and the CDG is an anti-prisoner's dilemma.

Proof. If $0 \leq Z < Z_C(n)$ then $\Delta\Pi_A(n) > 0$, $\Delta\Pi_B(n) < 0$ and $\Delta\Pi_C(n) < 0$ for any $0 \leq n < 1$, so that Point [1] holds. If $Z_C(n) < Z < Z_A(n)$ then $\Delta\Pi_A(n) > 0$, $\Delta\Pi_B(n) < 0$ and $\Delta\Pi_C(n) > 0$ for any $0 \leq n < 1$, so that Point [2] holds. If $Z_A(n) < Z < Z_B(n)$ then $\Delta\Pi_A(n) < 0$, $\Delta\Pi_B(n) < 0$ and $\Delta\Pi_C(n) > 0$ for any $0 \leq n < 1$, so that Point [3] holds. If $Z_B(n) < Z < Z_{TH}^{K/K}(n)$ then $\Delta\Pi_A(n) < 0$, $\Delta\Pi_B(n) > 0$ and $\Delta\Pi_C(n) > 0$ for any $0 \leq n < 1$, so that Point [4] holds. **Q.E.D.**

The results summarised in Proposition 1 are clear and driven exclusively by the relative size of the quasi-fixed cost, and therefore how this cost affects profits of the firms that decide to make their products compatible. These results go in an unexpected direction than those already pinpointed by the existing literature on the optimal product compatibility (Economides, 1989; Kim and Choi 2015; Stadler et al., 2022). Although there exists an optimal value of the extent of product compatibility, studying the incentives of selfish firms that in a non-cooperative game must choose (by considering a game theoretic approach) whether to produce compatible goods (incurring a quasi-fixed cost) or non-compatible goods (not incurring a quasi-fixed cost), allows us to reveal the existence of different paradigms (see Figure 1 for a geometrical representation of Proposition 1). If the quasi-fixed is zero or low, each firm has a dominant strategy represented by K, the K-firm produces more than the NK-firm, which in turn leads to a Pareto efficient outcome (there is no conflict between self-interest and mutual benefit of product compatibility). If the quasi-fixed cost becomes larger, each firm continues to have K as a dominant strategy, but profits of the K-firms markedly reduces resulting in a Pareto inefficient outcome (there is a conflict between self-interest and mutual benefit of product compatibility). If the quasi-fixed cost increases further, there is no more a dominant strategy, and the incentive of each firm is to play the same strategy as its rival. This is because the quasi-fixed cost makes profits of the K-firm when the rival plays NK lower than those that can be obtained by playing NK. This eventually leads to a coordination game in which NK payoff dominates K. Other increases in the quasi-fixed cost modify the incentive of each firm to have NK as a dominant strategy by reducing profits of the K-firm r below those of the NK-firms. These results holds under positive network externality (bandwagon effect). Unlike this case, under negative network externality (snob effect) the emerging SPNE of the CDG is trivial: no firm has an incentive to let its products become compatible and any deviations from NK leads each firm to be worse off. Therefore, the unique Pareto efficient SPNE emerging for any $n < 0$ is (NK, NK) and the CDG is a deadlock in which there is no conflict between self-interest and mutual benefits of producing incompatible goods.

Interestingly, Proposition 1 (and Figure 1) predicts that for any value of the quasi-fixed cost of compatibility, the CDG changes paradigm for increasing values of the extent of the (positive) consumption externality: 1) when the externality is low, self-interest and mutual benefit of *product incompatibility do not conflict* and the game is a deadlock; 2) if the extent of the network externality becomes larger, there is indeterminacy and multiple Nash equilibria emerge, but there is an incentive for firms to play NK, which payoff dominates K; 3) when the externality increases further, self-interest and mutual benefit of *product compatibility conflict* and the game is a prisoner's dilemma; 4) when the externality is high, self-interest and mutual benefit of *product compatibility do not conflict* and the game is a deadlock. Therefore, the network externality favours product compatibility by increasing firms' profits.

As the only feasibility constraint is $Z < Z_{TH}^{K/K}$, as also shown in Figure 1, there is a one-to-one correspondence between Z and n such that for each value of Z (resp. n) there exists a corresponding value of n (resp. Z) above (resp. below) which the CDG is meaningful, and the higher the quasi-fixed cost of compatibility, the higher the strength of the network effect required for feasibility (i.e., the network externality should be high enough to adequately sustain the market demand and total revenues to avoid negative profits). This is useful to tackle the issue of the social welfare outcomes corresponding to the Nash equilibrium outcomes when the extent of the network externality varies. In this regard, in what follows we will consider three different values of the quasi-fixed cost of compatibility $Z = 0.1$ (Figure 2), $Z = 0.25$ (Figure 3) and $Z = 0.5$ (Figure 4) and study the shape of the prevailing values of consumer surplus and social welfare for meaningful values of n. The first relevant result that should be pinpointed in the inverted U-shaped behaviour of the consumer surplus under K/K when n increases. This is because an increase in the extent of the (positive) network effect shifts outward the market demand, in turn, increasing quantity and price that consumers are willing to pay. On one hand, the increase in the quantity allows consumers to be better off. On the other hand, the increase in the market price makes consumers worse off. When n is high enough, the latter effect dominates the former and the consumer surplus reduces with n. This outcome is indeed exacerbated under full compatibility. Therefore, consumers can be better off under no compatibility than under full compatibility. The second relevant result is the existence of a unique value of n, given Z, below (resp. above) which social welfare under K/K is smaller (resp. larger) than social welfare under NK/NK. When $Z = 0.1$ (resp. $Z = 0.25$) [resp. $Z = 0.5$], this value is $n = 0.513$ (resp. $n = 0.767$) [$n = 0.947$]. In both scenarios, the social welfare function monotonically increase with n. Consumer welfare and social welfare outcomes depend on the prevailing SPNE and then on the mutual relationship between the quasi-fixed cost of compatibility and the extent of the network externality. Figures 2-4 (Panel B) reveal that the existence of a trade-off between the consumers' interest and the firms' interest. Low (resp. high) values of the quasi-fixed cost of compatibility favour the full compatibility (resp. the no compatibility) scenario. When Z is low, there exists a small range of intermediate values of n such that both consumers and firms are better off under full compatibility than under no compatibility so that endogenous full compatibility represents a win-win outcome for the society. For intermediate values of Z there are win-win results for the society irrespective of the value of n. When Z is high, there exists a small range of relatively low values of n such that both consumers and firms are better off under no compatibility than under full compatibility so that endogenous no compatibility represents a win-win outcome for the society!

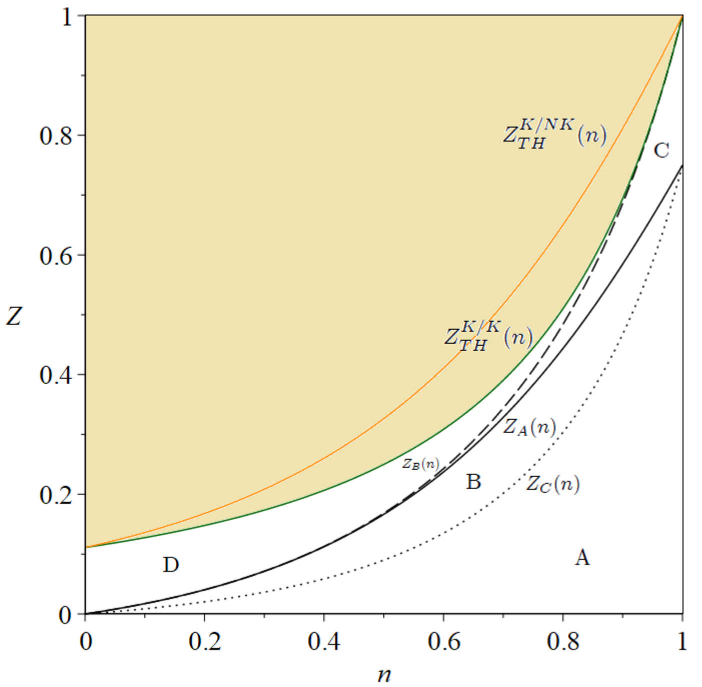
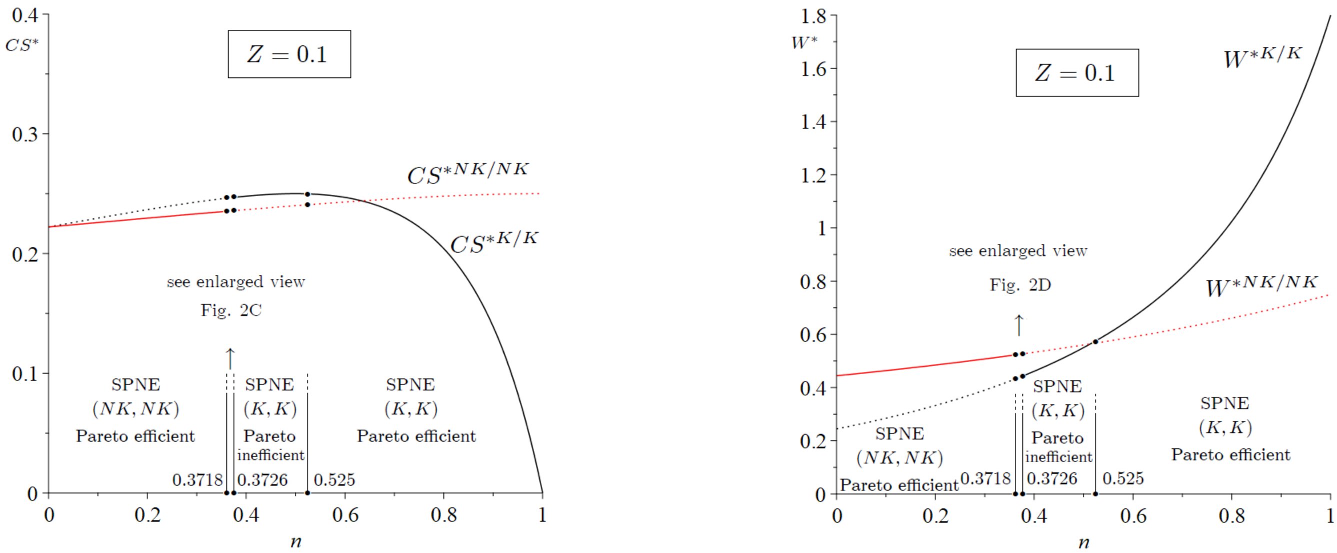


Figure 1. The compatibility decision game when k is endogenous: SPNE in the space (n, Z). The sand-coloured region represents unfeasibility. Area A: (K, K) is the unique Pareto efficient SPNE. Area B: (K, K) is the unique Pareto inefficient SPNE. Area C: (K, K) and (NK, NK) are two symmetric SPNE (NK payoff dominates K). Area D: (NK, NK) is the unique Pareto efficient SPNE.



(A) (B)

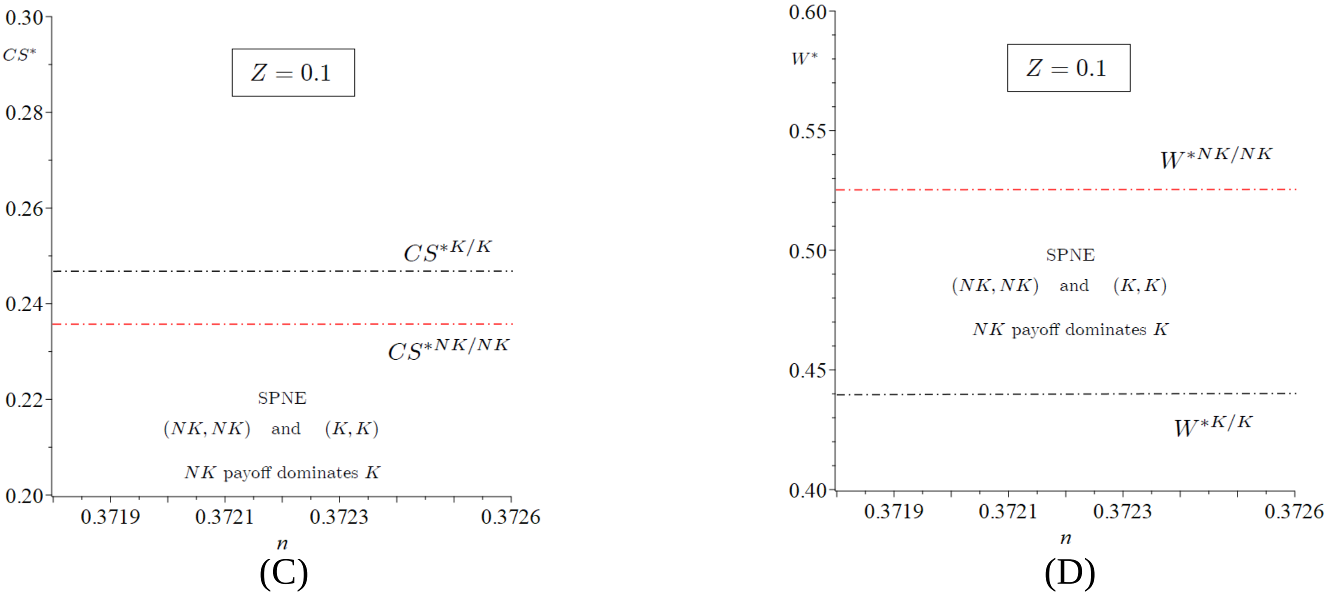
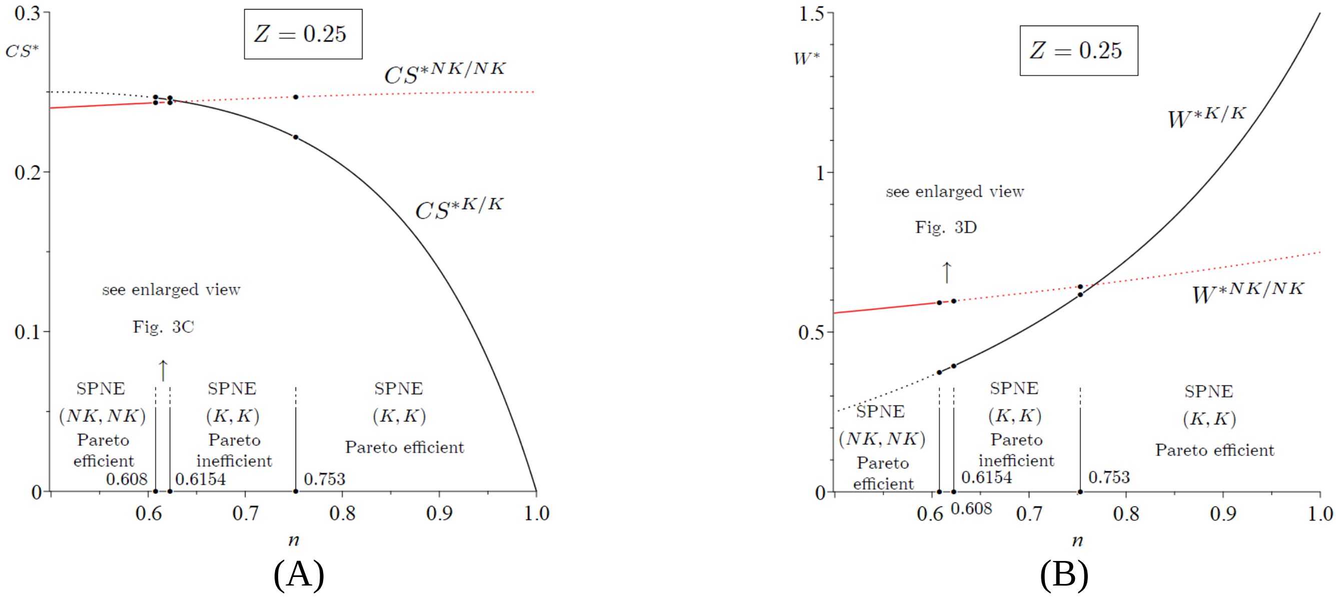
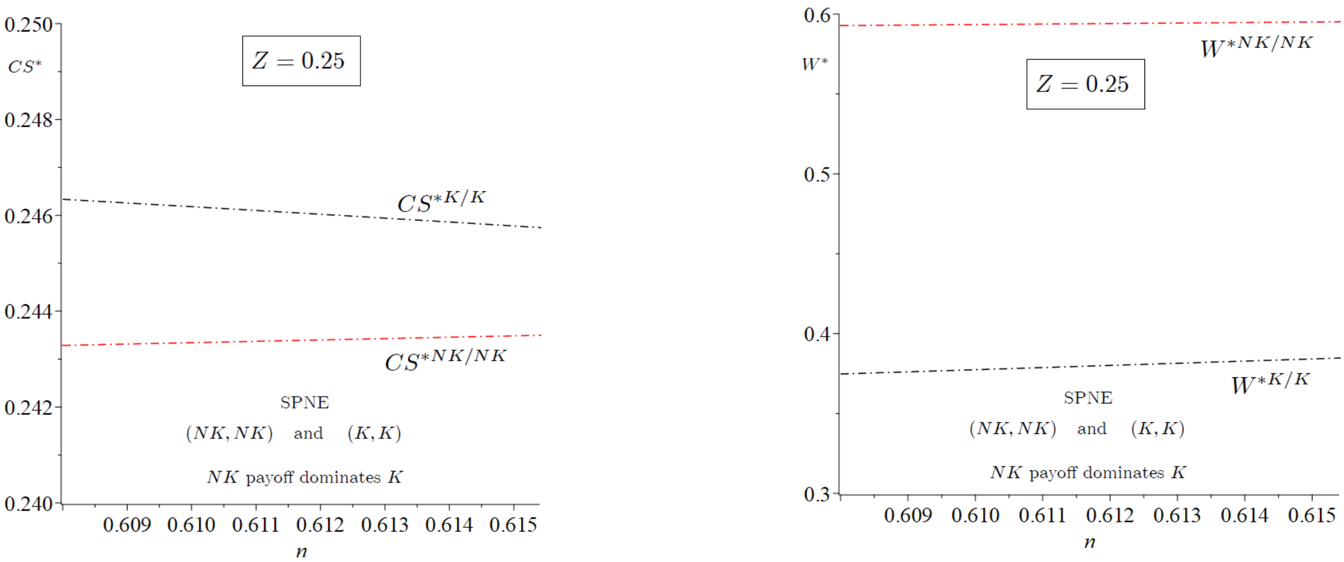


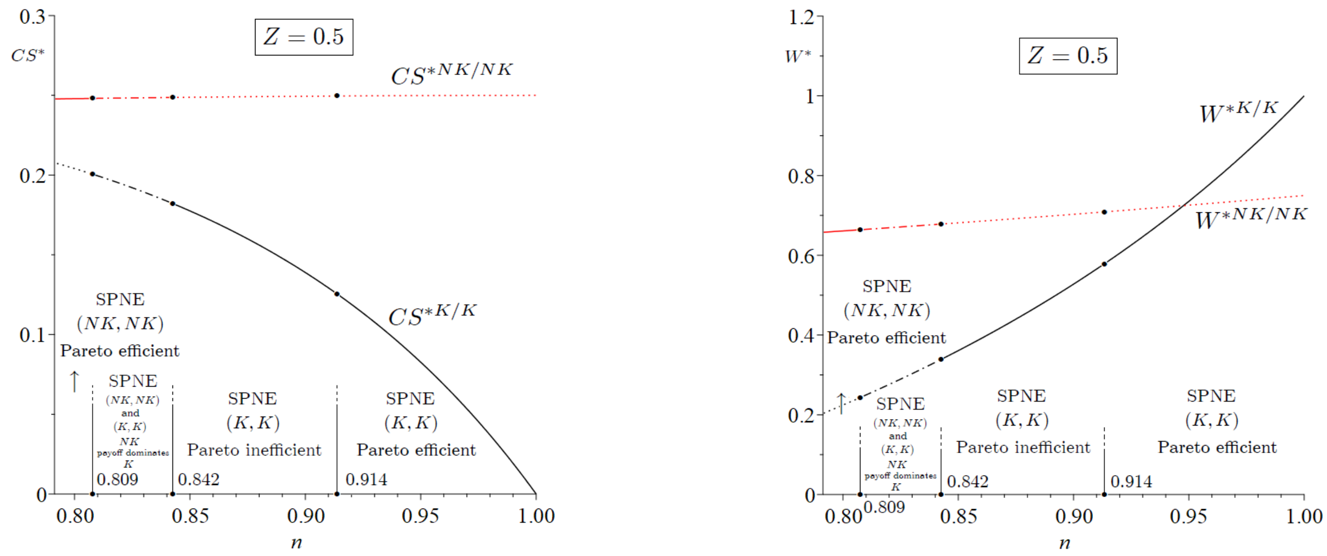
Figure 2. Equilibrium consumer surplus (Panel A) and social welfare (Panel B) when $Z = 0.1$. Panels C and D represent the corresponding enlarged view. The black (resp. red) curve refers to the case K/K (resp. NK/NK). The solid lines represent the levels prevailing at the SPNE. The dash-dotted lines represent the levels that can emerge at the SPNE when there is indeterminacy, i.e., a multiplicity of Nash equilibria in pure strategies (coordination game). The dotted lines are fictitious and are drawn only for comparison purposes with the prevailing equilibrium values.





(C) (D)

Figure 3. Equilibrium consumer surplus (Panel A) and social welfare (Panel B) when $Z = 0.25$. Panels C and D represent the corresponding enlarged view. The black (resp. red) curve refers to the case K/K (resp. NK/NK). The solid lines represent the levels prevailing at the SPNE. The dash-dotted lines represent the levels that can emerge at the SPNE when there is indeterminacy, i.e., a multiplicity of Nash equilibria in pure strategies (coordination game). The dotted lines are fictitious and are drawn only for comparison purposes with the prevailing equilibrium values.



(A) (B)

Figure 4. Equilibrium consumer surplus (Panel A) and social welfare (Panel B) when $Z = 0.5$. The black (resp. red) curve refers to the case K/K (resp. NK/NK). The solid lines represent the levels prevailing at the SPNE. The dash-dotted lines represent the levels that can emerge at the SPNE when there is indeterminacy, i.e., a multiplicity of Nash equilibria in pure strategies (coordination game). The dotted lines are fictitious and are drawn only for comparison purposes with the prevailing equilibrium values.

4. Exogenous product compatibility

Unlike Section 3, this section considers the case of exogenous product compatibility and then assumes that firms cannot choose the degree compatibility of their products to maximise profits because of technological reasons. Therefore, the non-cooperative CDG now develops along two (instead of) three stages. At the first stage, each firm chooses whether to let its product be (in)compatible given the state of the art of the technology. At the second stage, each firm chooses the output in the product market. The Nash equilibrium values ($*$) of quantity and profits in each sub-game are respectively given by Eqs. (5) and (6) under K/K, (14) and (15) under NK/NK, (20), (21) and (22), (23) under K/NK. For clarity, we report the equations below by stressing their dependency on n and k:

$$q_i^{*K/K}(n,k) = \frac{1}{3-n(1+k)}, \tag{37}$$

and

$$\Pi_i^{*K/K}(n,k) = \frac{1}{[3-n(1+k)]^2} - Z, \tag{38}$$

for the sub-game K/K;

$$q_i^{*NK/NK}(n,0) = \frac{1}{3-n}, \tag{39}$$

and

$$\Pi_i^{*NK/NK}(n,0) = \frac{1}{(3-n)^2}, \tag{40}$$

for the sub-game NK/NK;

$$q_i^{*K/NK}(n,k) = \frac{1-n(1-k)}{(3-n)(1-n)+nk}, \tag{41}$$

$$q_j^{*K/NK}(n,k) = \frac{1-n}{(3-n)(1-n)+nk}, \tag{42}$$

and

$$\Pi_i^{*K/NK}(n,k) = \frac{[1-n(1-k)]^2}{[(3-n)(1-n)+nk]^2} - Z, \tag{43}$$

$$\Pi_j^{*K/NK}(n,k) = \frac{(1-n)^2}{[(3-n)(1-n)+nk]^2}. \tag{44}$$

for the sub-game K/NK.

Before proceeding with the analysis of the SPNE, we investigate how the degree of compatibility and the intensity of the network externality affect the output and – knowing that the cost of compatibility is fixed and the competition in each sub-game occurs in strategic substitutes – the corresponding values of the profits. This is done by considering the case of unilateral deviation from the situations of universal compatibility and incompatibility. In other words, we study how the incentive to deviate from a Nash equilibrium depends on compatibility and the network effect. The results are summarised in the following lemmas.

Lemma 1. In the case of positive consumption externality, the output reduces (resp. increases) with the degree of compatibility in the case of unilateral deviation from K to NK (resp. from NK to K).

Proof. The proof proceeds by considering the sign of the following derivatives: $\frac{\partial q_i^{NK/K}}{\partial k} = \frac{-n(1-n)}{[(3-n)(1-n)+nk]^2} < 0$ for any $n > 0$ and $\frac{\partial q_i^{K/NK}}{\partial k} = \frac{n(1-n)(2-n)}{[(3-n)(1-n)+nk]^2} > 0$ for any $n > 0$. **Q.E.D.**

Lemma 1 is intuitive as – by considering the case of positive externality – switching towards NK reduces the product demand via a reduced size of the network of the firm. However, the network effect may reduce or increase the effects of the increasing degree of compatibility depending on whether the firm is cheating from the equilibrium with or without compatibility.

Lemma 2. In the case of positive consumption externality, the output-reducing (resp. increasing) effect of the increasing degree of compatibility in the case of unilateral deviation from K to NK [resp. NK to K] is mitigated (for sufficiently low values of k) [resp. for sufficiently high values of k] or magnified (for sufficiently high values of k) [resp. for sufficiently low values of k] by the intensity of the network effect.

Proof. The proof proceeds by considering the sign of the following derivatives: $\frac{\partial^2 q_i^{NK/K}}{\partial k \partial n} = \frac{-(3-2n)(1-n^2)+kn}{[(3-n)(1-n)+nk]^3} \gtreqless 0 \Leftrightarrow k \gtreqless \frac{(3-2n)(1-n^2)}{n} := \tilde{k}$ for any $n > 0$ and $\frac{\partial^2 q_i^{K/NK}}{\partial k \partial n} = \frac{(1-n)[2+(1-n)(4-n^2)]-kn(2-n^2)}{[(3-n)(1-n)+nk]^3} \gtreqless 0 \Leftrightarrow k \lesseqgtr \frac{(1-n)[2+(1-n)(4-n^2)]}{n(2-n^2)} := \bar{k}$ for any $n > 0$. **Q.E.D.**

We now investigate how the intensity of the network externality affects output and profits in both cases of unilateral deviation.

Lemma 3. The positive network effect monotonically increases (resp. can increase or reduce depending on whether k and n are sufficiently low or high) output and profits in the case of unilateral deviation from NK to K (resp. from K to NK).

Proof. The proof proceeds by considering the sign of the following derivatives: $\frac{\partial q_i^{NK/K}}{\partial n} = \frac{(1-n)^2-k}{[(3-n)(1-n)+nk]^2} \gtreqless 0 \Leftrightarrow k \lesseqgtr (1-n)^2 := \hat{k}$ for any $n > 0$ and $\frac{\partial q_i^{K/NK}}{\partial n} = \frac{(1-n)^2+k(2-n^2)}{[(3-n)(1-n)+nk]^2} > 0$ for any $n > 0$. **Q.E.D.**

Lemma 3 implies, when k and n are sufficiently high, that an increase in the network effect always reduces the convenience for one firm to choose to play NK. This is counterintuitive as an existing high level of both compatibility and network externality should make detrimental deviate from K.

Lemma 4. In both cases of deviation, the output-reducing (resp. increasing) effect of an increasing degree of network effect in the case of unilateral deviation from K to NK

[resp. NK to K] is mitigated (for sufficiently low values of k) [resp. for sufficiently high values of k] or magnified (for sufficiently high values of k) [resp. for sufficiently low values of k] by an increasing degree of compatibility.

Proof. The proof proceeds by considering the sign of the following derivatives: $\frac{\partial^2 q_i^{NK/K}}{\partial n \partial k} = \frac{-(3-2n)(1-n^2)+kn}{[(3-n)(1-n)+nk]^3} \gtrless 0 \Leftrightarrow k \gtrless \frac{(3-2n)(1-n^2)}{n} := \tilde{k}$ for any $n > 0$ and $\frac{\partial^2 q_i^{K/NK}}{\partial n \partial k} = \frac{(1-n)[2+(1-n)(4-n^2)]-kn(2-n^2)}{[(3-n)(1-n)+nk]^3} \gtrless 0 \Leftrightarrow k \lessgtr \frac{(1-n)[2+(1-n)(4-n^2)]}{n(2-n^2)} := \bar{k}$ for any $n > 0$. **Q.E.D.**

The complicated interactions of the effects of the levels of the compatibility and the network externality on the incentive to cheat from a symmetric Nash equilibrium will drive, together with the levels of the quasi-fixed cost, the occurrence of a very rich set of possible SPNE.

Therefore, at the first stage of the game each firm chooses the convenience to produce compatible (at an exogenous degree) or incompatible products. The main variables of the problem are summarised in Table 3 (payoff matrix) that summarises the equilibrium profit functions given by the expressions (38), (40), (43) and (44), which now depend on the extent of the network externality (n) and the degree of product compatibility (k).

Table 3. The compatibility decision game (payoff matrix) when k is exogenous.

Firm j $\rightarrow$ Firm i $\downarrow$	K	NK
K	$\Pi_i^{*K/K}(n,k)$, $\Pi_j^{*K/K}(n,k)$	$\Pi_i^{*K/NK}(n,k)$, $\Pi_j^{*K/NK}(n,k)$
NK	$\Pi_i^{*NK/K}(n,k)$, $\Pi_j^{*NK/K}(n,k)$	$\Pi_i^{*NK/NK}(n,k)$, $\Pi_j^{*NK/NK}(n,k)$

The technical restrictions that must be satisfied to have well-defined equilibria in pure strategies for every strategic profile (one for each player) are the same as those detailed so far in Section 3. However, for reasons of tractability we re-write these conditions for as a function of n and Z as follows: $k > k_{TH}^{K/K}(n,Z)$ and $k > k_{TH}^{K/NK}(n,Z)$, where $k_{TH}^{K/K}(n,Z) > k_{TH}^{K/NK}(n,Z)$. Therefore, the unique feasibility condition that must hold to guarantee meaningful Nash equilibria is $k > k_{TH}^{K/K}(n,Z)$. Then, to derive all the possible SPNE of this non-cooperative game, one must study the sign of the profit differentials for $i = \{1,2\}, i \neq j$, that is:

$$\Delta\Pi_A(n,k) := \Pi_i^{*K/NK}(n,k) - \Pi_i^{*NK/NK}(n,k), \tag{45}$$

$$\Delta\Pi_B(n,k) := \Pi_i^{*NK/K}(n,k) - \Pi_i^{*K/K}(n,k), \tag{46}$$

and

$$\Delta\Pi_C(n,k) := \Pi_i^{*NK/NK}(n,k) - \Pi_i^{*K/K}(n,k). \tag{47}$$

From (45), the sign of $\Delta\Pi_A(n,k)$ is positive (resp. negative) if $k > k_A(n,Z)$ (resp. $k < k_A(n,Z)$, where $k_A(n,Z)$ is the threshold value of the degree of product compatibility (as a function of the extent of the network externality and the fixed cost of compatibility) such that $\Delta\Pi_A(n,Z) = 0$.

From (46), the sign of $\Delta\Pi_B(n,k)$ is positive (resp. negative) if $k < k_B(n,Z)$ (resp. $k > k_B(n,Z)$, where $k_B(n,Z)$ is the threshold value of the degree of product compatibility (as a function of the extent of the network externality and the fixed cost of compatibility) such that $\Delta\Pi_B(n,Z) = 0$.

From (47), the sign of $\Delta\Pi_C(n,k)$ is positive (resp. negative) if $k < k_C(n,Z)$ (resp. $k > k_C(n,Z)$, where $k_C(n,Z)$ is the threshold value of the degree of product compatibility (as a function of the extent of the network externality and the fixed cost of compatibility) such that $\Delta\Pi_C(n,Z) = 0$.

The SPNE of the CDG when k is exogenous are illustrated in Figure 5, 6 and 7 that report the Nash equilibrium values emerging in the space (k,n) for $Z = 0.1$, $Z = 0.25$ and $Z = 0.5$, respectively. These figures fully replace the related proposition that we do not present as the outcomes emerging in this case are the same as those pinpointed in Section 3 (areas A, B, C, and D in Figures 5-7) with one relevant exception: when product are partially compatible it is possible also to observe an anti-coordination game in which the SPNE is consistent with the production of a compatible good by one firm only (area E in Figures 5-7). This result holds irrespective of the fixed cost of compatibility and is favoured by relatively low (but not too much low) values of Z (see area E of Figure 6 in comparison with area E of Figures 5 and 7). This outcome, based on a game-theoretic approach, can indeed explain several cases of compatibility in actual markets, the most popular and remarkable of which is Apple versus Microsoft in the computer market: Apple products are based on macOS but can also be used with Windows OS. Differently, Microsoft products are based on Windows OS and cannot be used with macOS. The complete set of SPNE is the following:

- area A: (K,K) is the unique Pareto efficient SPNE ($\Delta\Pi_A(n,k) > 0$, $\Delta\Pi_B(n,k) < 0$ and $\Delta\Pi_C(n,k) < 0$);
- area B: (K,K) is the unique Pareto inefficient SPNE ($\Delta\Pi_A(n,k) > 0$, $\Delta\Pi_B(n,k) < 0$ and $\Delta\Pi_C(n,k) > 0$);
- area C: (K,K) and (NK,NK) are two symmetric SPNE but NK payoff dominates K ($\Delta\Pi_A(n,k) < 0$, $\Delta\Pi_B(n,k) < 0$ and $\Delta\Pi_C(n,k) > 0$);
- area D: (NK,NK) is the unique Pareto efficient SPNE ($\Delta\Pi_A(n,k) < 0$, $\Delta\Pi_B(n,k) > 0$ and $\Delta\Pi_C(n,k) > 0$);
- area E: (NK,K) and (K,NK) are two asymmetric Pareto efficient SPNE ($\Delta\Pi_A(n,k) > 0$, $\Delta\Pi_B(n,k) > 0$ and $\Delta\Pi_C(n,k) > 0$).

The welfare outcomes related to the SPNE when k is exogenous resemble those found when k is endogenous. Therefore, we do not report a detailed analysis of the shape of the consumer surplus and the social welfare. We simply report that the social welfare values related to the asymmetric SPNE are intermediate compared to those related to (K, K) and (NK, NK) so that there are not relevant differences about the win-win results found in the previous section.

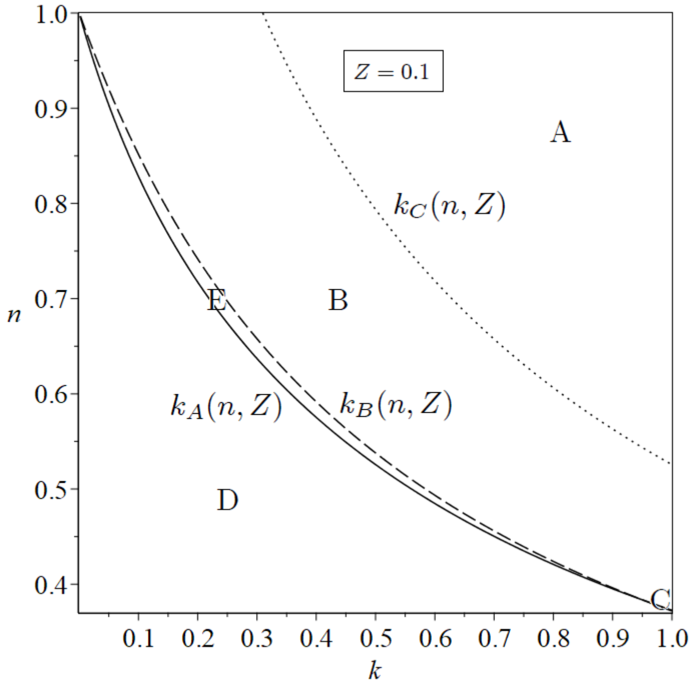


Figure 5. The compatibility decision game when k is exogenous: SPNE in the space (k, n) for $Z = 0.1$. The sand-coloured region represents unfeasibility. Area A: (K, K) is the unique Pareto efficient SPNE. Area B: (K, K) is the unique Pareto inefficient SPNE. Area C: (K, K) and (NK, NK) are two symmetric SPNE (NK payoff dominates K). Area D: (NK, NK) is the unique Pareto efficient SPNE. Area E: (NK, K) and (K, NK) are two asymmetric Pareto efficient SPNE.

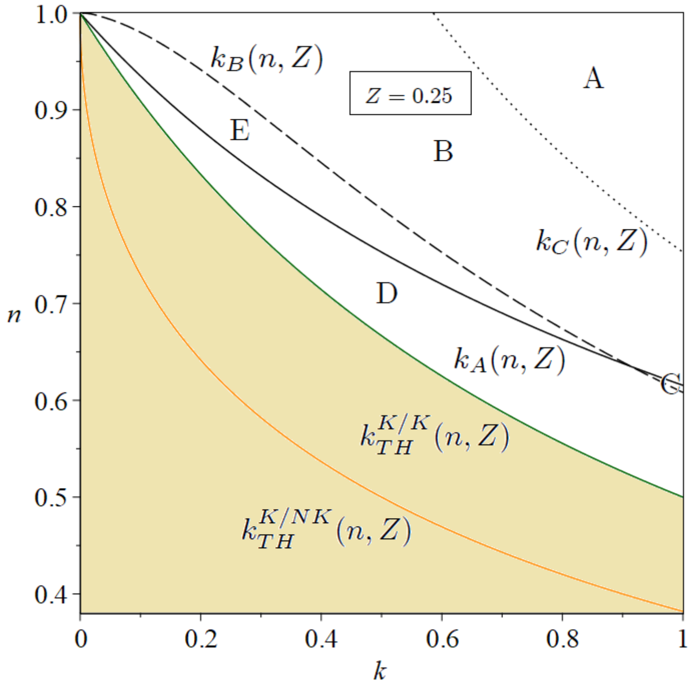


Figure 6. The compatibility decision game when k is exogenous: SPNE in the space (k, n) for $Z = 0.25$. The sand-coloured region represents unfeasibility. Area A: (K, K) is the unique Pareto efficient SPNE. Area B: (K, K) is the unique Pareto inefficient SPNE. Area C: (K, K) and (NK, NK) are two symmetric SPNE (NK payoff dominates K). Area D: (NK, NK) is the unique Pareto efficient SPNE. Area E: (NK, K) and (K, NK) are two asymmetric Pareto efficient SPNE.

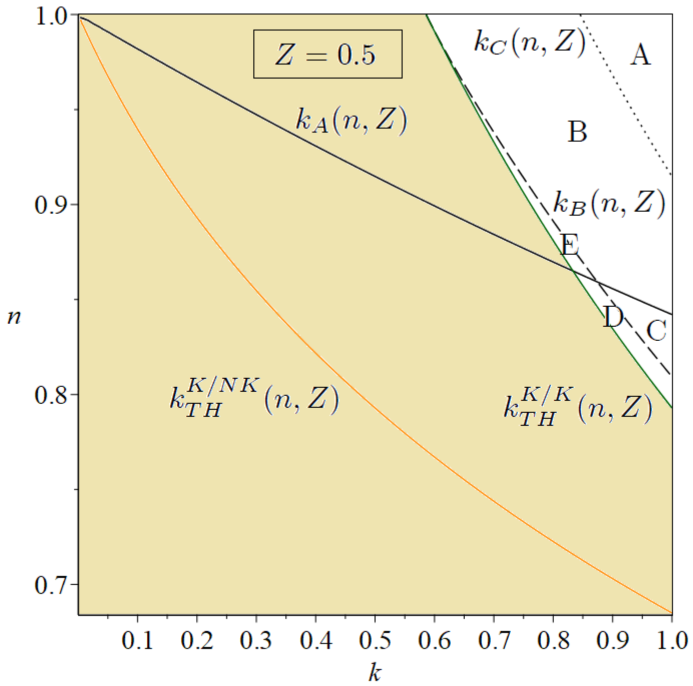


Figure 7. The compatibility decision game when k is exogenous: SPNE in the space (k, n) for $Z = 0.5$. The sand-coloured region represents unfeasibility. Area A: (K, K) is the unique Pareto efficient SPNE. Area B: (K, K) is the unique Pareto inefficient SPNE. Area C: (K, K) and (NK, NK) are two symmetric SPNE (NK payoff dominates K). Area D: (NK, NK) is the unique Pareto efficient SPNE. Area E: (NK, K) and (K, NK) are two asymmetric Pareto efficient SPNE.

5. Conclusions

This research has developed the non-cooperative compatibility decision game in a Cournot duopoly with network (consumption) externalities, along the lines of Katz and Shapiro (1985), which represents the closest contribution to the present work. The main innovation of this article is to consider the degree of product compatibility as a strategic variable in a strategic setting. Although some existing contributions already studied the problem of choosing the degree of compatibility to maximise profits (e.g., Economides, 1989; Kim and Choi 2015; Stadler et al., 2022), no one tackled – to the best of our knowledge – the issue of considering the individual strategic incentive to let products becomes (in)compatible with those of the rival in a non-cooperative game. To this purpose, the paper developed the compatibility decision game (CDG) played by two quantity-setting firms that should sustain fixed costs of compatibility. In this regard, letting products become compatible in a network market with a positive externality is not a trivial choice. Indeed, there exist benefits and costs of compatibility and this, in turn, implies the existence of a wide spectrum of endogenous Nash equilibrium outcomes, ranging from a situation in which self-interest and mutual benefit of producing compatible goods conflict to situations in which self-interest and mutual benefit of producing (in)compatible goods do not conflict. The article also provided an (endogenous) explanation for the existence of one-way compatibility and the outcome of the CGD can also be the anti-coordination scenario.

To the best of our knowledge, this article represents a first attempt to model a non-cooperative game in which product compatibility is considered a strategic device in a quantity-setting duopoly and has the aim of opening a debate on this issue. A possible future research agenda can include differences in product quality, horizontal product differentiation, managerial delegation and corporate social responsibility.

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Compliance with ethical standards

Disclosure of potential conflict of interest The authors declare that they have no conflict of interest.

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Declarations of interest None.

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