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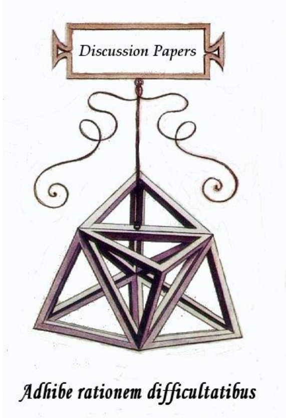

Luca Gori, Piero Manfredi, Simone Marsiglio and Mauro Sodini

A parsimonious model of optimal social distancing and vaccination during an outbreak

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Keywords Infectious diseases; COVID-19 transmission dynamics and interventions; Social distancing; Vaccination

JEL Classification C61; C62; J1, J22; O41; O47

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Luca Gori*• Piero Manfredi†• Simone Marsiglio‡• Mauro Sodini§

April 7, 2023

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1 Introduction

The recent COVID-19 pandemic with its dramatic impact worldwide has highlighted the lack of resiliency of Western societies to infectious diseases threats (WHO, 2022) due to the difficult handling of the trade-off between the direct protection of health from the consequences of infections, on the one hand, and the protection of the economy and the society as a whole from the nasty measures necessary to contain epidemic spread, on the other hand. This trade-off has provided a dramatic impulse to the study of the interplay between infection transmission and the economy that remained somewhat neglected before the pandemic (Gori et al., 2020 and references therein).

As a consequence of this momentum, virtually any area of economic research and policy have tackled the interplay between a serious epidemic and the macro-economy with an endless list of contributions (e.g., Alvarez et al., 2021; Acemoglu et al., 2021; Eichenbaum et al., 2021; Glover et al., 2021; Gori et al., 2021a, 2021b; La Torre et al., 2021; Davin et al., 2022) of which we can obviously only cite a few.

A major area of research amongst this new wave of economic epidemiology has regarded the optimal control of serious epidemic outbreaks by also accounting for the nasty implications of the undertaken measures for the economy and the society. This has led for the first time to consider optimal control problems of a social planner trying to globally minimise not only the direct effect of the disease but also the potentially devastating indirect effects induced by the pandemic control measures at all societal level (WHO, 2022).

In this direction, the road map has been indicated by a few pioneering works, amongst which we recall Alvarez et al. (2021) and Acemoglu et al. (2021), who were first in considering the potential effect of long-lasting lockdown measures on the economy. The present article follows and contributes to this vein by investigating the optimal control of a Susceptible-Infective-Removed-Susceptible (SIRS) epidemic of an infection, which is directly transmitted from person to person by social contacts and imparts only temporary immunity. In particular, we consider the situation in which two control stages (say, Stage 1 versus Stage 2) are identified by the availability of effective control tools, as has been the case for COVID-19 (WHO, 2022). The realm of COVID-19 has indeed shown that social distancing has been the major control option in the first stage (due to the lack of a vaccine and to the secondary role of testing and tracing), whereas a combination of social distancing and vaccination by a vaccine imparting short-lasting immunity has been the control option available during the second stage. Consistently, we study a *two-control optimal policy* targeting social distancing and vaccination in a simple economic set up in which the social planner minimises an objective function weighting epidemiological and economic costs.

More in detail, we consider two main finite-horizon control problems. The former one is the generic problem of epidemic control in the presence of two control instruments, in which the second instrument, i.e., vaccination by a fully effective vaccine, appears only in the second stage of the epidemic. Here we assume vaccine efficacy to be maximal simply because we are interested in the optimal vaccination schedule. This control problem is further distinguished into two main sub cases depending on the planner's degree of rationality. More precisely, we considered an optimal control problem that we termed *two-stage disjoint optimal control*, where the social planner optimally controls the epidemic since the beginning of Stage 1 by tuning the strength of social distancing only and then re-optimises at the beginning of Stage 2 (i.e., at the arrival time of the vaccine) with respect to both control instruments, as opposed to a *joint optimal control over the two stages* of the epidemic, where the social planner attempts at optimally controlling the epidemic over the entire horizon since the beginning of the alert. The latter one is given by the optimal control by a single instrument (social distancing) combined with a fully exogenous vaccination schedule reflecting *vaccine supply rationing* or scarcity (that we fitted to observed Italian data), regardless of the scales (international, national, local, etc.) at which such vaccine shortages might appear, thereby preventing the optimal use of vaccination. In particular, in all the problems analysed we consider the following temporal sequence of actions: (a) an early phase of free epidemic growth due to the lack of any planned or spontaneous control action, (b) a social distancing phase, (c) a vaccination phase, where the two controls coexist.

Our main results across the various sub-cases considered show the fine dependence of optimal trajectories on the weight attributed to direct cost of the epidemic compare to its indirect (i.e., economical) costs and on the characteristics (efficacy of *take* and duration of vaccine immunity) of the vaccine.

In the fully optimal case (*the joint problem*), it happens that the social distancing is enacted early and massively and can be substantially decreased even before the vaccine arrival and completely lifted once the vaccine is available without compromising full epidemic control. The *disjoint problem* results are clearly sub-optimal compare to the joint problem. On one hand, for high values of the weight of the epidemiological cost, social distancing must be persistently maintained at high levels (to also account for immunity losses) during Stage 1, but the availability of an effective vaccine whose distribution can be optimised allows to rapidly achieve high levels of epidemic control. On the other hand, for low values of the weight of the epidemiological cost, social distancing is obviously kept at moderate levels during Stage 1, but the arrival of an effective vaccine at Stage 2 allows to achieve analogous levels of epidemic control by joining the natural immunity acquired during Stage 1 with the optimal vaccine-induced immunity during Stage 2.

When the vaccine coverage is an exogenously given function, social distancing can never be fully relaxed for high values of the weight of the epidemiological cost: relaxation is possible in an initial phase of Stage 2 and can be substantial at high levels of vaccine efficacy but needs to be always strengthen in a later phase regardless of vaccine efficacy.

Finally, when the temporal profile of vaccination coverage is given (due to organisational and logistic constraints), and therefore social distancing is the only available control over the entire horizon, we show the conditions allowing the relaxation of social distancing and the subsequent need for relapsing it due to the progressive loss of immunity amongst vaccinated individuals. Here, our results are fairly general and hold for both the case of linear vaccine rationing and of a logistic increasing vaccine coverage.

The rest of the article proceeds as follows. Section 2 presents the epidemiological and economic modelling frameworks. Section 3 formulates the control problems, reporting the first-order conditions and the main theoretical results. Section 4 reports the main simulative results and the ensuing substantive implications. Concluding remarks follow in Section 5.

2 The model

2.1 The epidemiological model: SIRS with vaccination

The model describes the active population only, whose size $N(t)$ is divided into three classes: susceptible, infected and recovered individuals, denoted respectively as $S(t)$, $I(t)$ and $R(t)$, where $N(t) = S(t) + I(t) + R(t)$ and $t \in \mathbb{R}_+$ is the time index. The population is assumed to mix homogeneously with a time-varying per capita transmission rate, $\beta(t)$, representing the number of secondary infections caused an infective individual per unit of time in a wholly susceptible population. The overall infection incidence, therefore, is $\beta(t)I(t)s\,(t)$, where $s(t) = S\,(t)\,/N\,(t)$ is the susceptible fraction. The time dependency in the transmission rate mirrors the effects of public interventions aimed to mitigate transmission (e.g., by social distancing) and the ensuing spontaneous behavioural responses by individuals. Infected individuals are assumed to recover and acquire immunity at a constant rate $\gamma > 0$. Recovered individual acquire temporary natural immunity after which they return to full susceptibility at a constant immunity waning rate $\rho > 0$. The model also considers vaccination according to a time-varying function $V\,(t)$ representing the absolute number of susceptible individuals who are effectively immunised against the infection per unit of time. For the sake of simplicity, we assume that vaccine-related immunity vanishes at the same rate ρ as natural immunity. The ensuing model has the following SIRS structure:

$$\begin{cases} \dot{S}(t) = -\beta(t)\frac{I(t)S(t)}{N(t)} - V(t) + \rho R(t) \\ \dot{I}(t) = \beta(t)\frac{I(t)S(t)}{N(t)} - \gamma I(t) \\ \dot{R}(t) = \gamma I(t) + V(t) - \rho R(t) \end{cases}, \tag{1}$$

where the dots denote time derivatives. We remark that the epidemiological set up (1) represents a *pure transmission-infection-control model* and, therefore, it disregards: (i) realistic complications such as vital dynamics and infection-related mortality (for instance, infection related mortality from COVID-19 was dramatic amongst the elderlies but rather negligible in the active population), implying that the population is constant over time: $N\,(t) = N$; (ii) a latency phase and other possibly realistic feature shared by COVID-19, such as asymptomatic transmission; (iii) other possible intervention options such as testing, tracing and isolating. By this, we do not mean that these interventions are not relevant, rather that their role is expected to be secondary compared to the two major instruments considered in this article. As for the two main control actions (social distancing and vaccination), they are embedded through the time-dependent functions $\beta(t)$ and $V(t)$ that will be suitably modelled later on.

By considering the per capita quantities, i.e., $s(t) = \frac{S(t)}{N}$, $i(t) = \frac{I(t)}{N}$, $r(t) = \frac{R(t)}{N}$ and $v(t) = \frac{V(t)}{N}$, with $s(t) + i\,(t) + r\,(t) = 1$, one gets the final epidemiological system:

$$\begin{cases} \dot{s}(t) = -\beta(t)i(t)s(t) - v(t) + \rho r(t) \\ \dot{i}(t) = \beta(t)i(t)s(t) - \gamma i(t) \\ \dot{r}(t) = \gamma i(t) + v\,(t) - \rho r(t) \end{cases}. \tag{2}$$

2.2 The economy

Healthy (that is, susceptible and recovered) individuals ($N - I(t)$) actively work to produce consumption goods ($Q(t)$) using a linear technology employing only labour. For parsimony and to focus efficiently on a two-control (first-best) optimal policy in the short term, we disregard capital accumulation[1] and assume that the social contacts determining infection transmission are the same contacts (the so-called work contacts) that allow productive activities (Eichenbaum et al., 2021).[2]

The consumption goods are produced, in the short term, by a linear (constant-return-to-scale) production function given by:

$$Q(t) = AL(t). \tag{3}$$

where $A > 0$ is a scaling parameter and $L(t) = N - I(t) = S(t) + R(t)$ is the available labour force.

As $s(t) + r(t) = 1 - i(t)$, one can get the production function in per capita terms, that is:

$$q(t) = A(1 - i(t)). \tag{4}$$

The share $0 \leq u(t) \leq 1$ of workers that are forced to unemployment following the closure of firms represents the strength of public social distancing, which determines net production as $Y(t) = (1 - u(t))Q(t)$ that can be written in per capita terms as follows:

$$y(t) = (1 - u(t))q(t), \tag{5}$$

where $y(t) = \frac{Y(t)}{N(t)}$. Therefore, $Q(t)$ represents the potential output at time t and $Y(t)$ the actual output at the same time given the constraints on production through social distancing measures for infection containment.

The social planner can also levy (capital and labour) income taxes at the rate $0 \leq \tau(t) \leq 1$ and use the revenues so collected to finance and organise the R&D-related activity, production, logistics and administrations of vaccines. Although in the actual world these tasks occur at different moments in time, we assume they coincide in the logical timing of the events detailed in the present work.

The disposable income (net of taxation) is entirely distributed and consumed, so that aggregate consumption is $C(t) = (1 - \tau(t))Y(t)$, which can be written in per capita terms as follows:

$$c(t) = (1 - \tau(t))y(t). \tag{6}$$

The tax revenue $T(t) = \tau(t)Y(t)$ is entirely used to finance vaccination spending $X(t)$ at a balanced budget. This (public) expenditure works exactly out as a subsidy to the pharmaceutical sector directly devoted to the production of vaccines (and subsequent logistics). Therefore, for any t, the government satisfies the budget constraint:

$$X(t) = T(t). \tag{7}$$

Eq. (7) can also be written in per capita terms as $x(t) = \tau(t)(1 - u(t))q(t)$, where $x(t) = \frac{X(t)}{N(t)}$.

[1] See Gori et al. (2021) for an analysis of second-best optimal policies targeted on social distancing and testing, tracing and isolation in an epi-econ long-term Solow-like growth model with capital accumulation.

[2] The introduction of a fraction of social contacts at work resulting in a transmission rate different from the one emerging by overall social contacts does not change the qualitative results of the model.

The supply of vaccines made available by the government, $V^S(t)$, is a function of the public expenditure, $X(t)$, that is:

$$V^S(t) = Z(X(t)). \tag{8}$$

By assuming that Z is linear technology transforming the public spending in available vaccines to be administrated to susceptible individuals we get $V^S(t) = BX(t)$, where $B^{-1} > 0$ represents the public expenditure per single vaccine dose (or pairs of doses). From (8), the per capita supply of vaccines is given by:

$$v^S(t) = Bx(t). \tag{9}$$

Remark 1 *Eq. (9) aims to capture the two main public activities undertaken to develop a COVID-19 vaccine: the large-scale public initiative aimed to the fast mapping of the virus genome and the massive transfer of public resources to the pharmaceutical industry for supporting the COVID-19 vaccine R&D costs.*

Equation (9) represents the number of vaccines available to the planner when the tax revenue is given by $T(t)$. In this work, we will mainly consider two cases: (1) the planner can optimally choose the desired number of vaccines and then design the tax trajectory to finance vaccination. Under this assumption, we have that $v(t) = v^S(t)$, where $v(t)$ is the number of actually administered vaccines; (2) the number of available vaccine is exogenously given (and then rationed) by some logistic constraints. Under this assumption, the planner will set the tax rate to purchase the exogenously given number of vaccines. The rationing model (fixed capacity) represents the case in which the actual vaccine administrations remain below the desired supply, i.e., $v(t) < v^S(t)$.

Some clarifications regarding the timing of the events and the model structure are now useful. From a chronological point of view, we will consider the evolution and control of the epidemic as sub-divided into three stages: (1) an initial phase (Stage 0) characterised by free growth of the epidemic due to the lack of control by the authorities. This delay in intervention is basically due lack of knowledge characterising new infections in the very early epidemic stages. At Stage 0, the epidemiological dynamics are completely characterised by system (2), in the absence of any control, that is $v(t) = 0$ and $\beta(t) = \beta$. We will assume that the length of this initial phase is 30 days; (2) a subsequent phase (Stage 1), characterised by the intervention of the social planner through social distancing only. We will assume that the length of Stage 1 is almost one year; (3) a final phase (Stage 2), where the availability of a vaccine (regardless its efficacy) allows the social planner to intervene through both social distancing and vaccination, possibly aiming at relaxing social distancing. The last two phases can be related to each other in various ways, either in relation to the predictive capacity of the vaccine discovery, or to the foresight and aims of the social planner, or – alternatively – in relation to the production/distribution/administration capacity of the vaccine. We will assume that the length of Stage 2 is almost one year. In the remainder of the article, we will describe Stage 1 and Stage 2 separately, and then analyse how different hypotheses on the behaviour of the planner can generate different outcomes.

3 The control problem and theoretical results

Owing to the staged structure of interventions, the control problem will be designed as follows.

Stage 1. The epidemic containment at this stage can be realised only through social distancing. We assume that the planner at the beginning of Stage 1 expects that the vaccine will be available at the expected time $t = T_V^e$. Therefore, the aim of the public authority is to compare benefits and costs of the intervention by specifically considering the direct epidemiological costs of the infection and the indirect economic costs due the undertaken control measures, which, in turn, result in a decrease in aggregate production and thus in aggregate consumption. Eventually, the aim of the social planner will be that of minimising the following functional:

$$F_1(s(t), i(t), u(t)) = \int_{T_{SD}}^{T_V^e} \frac{1}{2} \left[ki^2(t) + (1-k)u^2(t)q^2(t)\right] e^{-\theta(t-T_{SD})} dt, \tag{10}$$

where $q(t)$ is defined in (4), by taking into account the dynamics of the epidemic, that is:

$$\begin{aligned} \dot{s}(t) &= -(1-u(t))\beta i(t)s(t) + \rho(1-s(t)-i(t)) \\ \dot{i}(t) &= (1-u(t))\beta i(t)s(t) - \gamma i(t) \end{aligned}, \tag{11}$$

and the initial conditions $s(T_{SD}), i(T_{SD}) \in (0,1)$, where $u(t) \in [0,1]$ is the strength of social distancing and $t = T_{SD}$ is the date the government initiates control actions through social distancing measures. We assume that the social planner also fixes at this stage an objective about the number of infective in the population at time $t = T_V^e$, i.e., $i(T_V^e)$.

The first addendum in brackets in Eq. (10), weighted by k, represents the epidemiological costs, i.e., a summary of the cost of hospitalisations/deaths resulting from the infection and thus both related to $i(t)$, for the sake of simplicity, in view of the parsimonious structure of our model). The second term in brackets in Eq. (10), weighted by $1-k$, represents the economic cost following the production loss due to social distancing. Therefore, $0 \leq k \leq 1$ weights the direct epidemiological costs in functional (10) and incorporates the health costs of hospitalisation and those attributed to the losses of human life used, e.g., in see Acemoglu et al. (2021) and Alvarez et al. (2021).

The expression in $F_1(s(t), i(t), u(t))$ should be minimised by the planner by choosing $u(t)$ subject to the relevant dynamic constraints.

Stage 2. By assuming that vaccines become actually available at time $t = T_V$, the government can act since T_V onwards through two controls, that is social distancing $u(t) \in [0,1]$ and the tax rate $\tau(t) \in [0,1]$. Then, the total cost of implementing a two-control policy is given by:

$$F_2(s(t), i(t), u(t), \tau(t)) = \int_{T_V}^{T} \frac{1}{2} \left\{ki^2(t) + (1-k)[u^2(t)q^2(t) + x^2(t)]\right\} e^{-\theta(t-T_V)} dt + \phi_V i_T e^{-\theta(T-T_V)}, \tag{12}$$

where the economic costs are augmented by the public expenditure $x(t) = \tau(t)(1-u(t))q(t)$ to finance the production/administration of vaccines. In this case, the epidemic dynamics are described by the equations:

$$\begin{aligned}\dot{s}(t) &= -(1-u(t))\beta i(t)s(t) - v(t) + \rho[1-s(t)-i(t)] \\ \dot{i}(t) &= (1-u(t))\beta i(t)s(t) - \gamma i(t)\end{aligned}, \tag{13}$$

given $s(T_V), i(T_V) \in (0,1)$, $v(t) = Bx(t)$ and $v(t) = v^S(t)$. The expression in $F_2(s(t), i(t), u(t), \tau(t))$ should be minimised by the planner by choosing $u(t)$ and $\tau(t)$. The term $\phi_V i_T e^{-\theta(T-T_V)}$ accounts for the number of infective individuals at the end of the epidemic control plan, where $\phi_V \geq 0$ is its relative weight. We recall that though the tax rate is the control variable nothing prevents to refer to optimal vaccination as the (optimal) tax rate is eventually used to collect resources to finance $x(t)$.

Remark 2 *[Reproduction numbers]. In the proposed model, the basic reproduction number – representing the number of secondary infection in a fully susceptible population in the absence of intervention (that is, in a free epidemic) – is given by $\mathcal{R}_0 = \beta/\gamma$. The corresponding effective reproduction number is equal to $\mathcal{R}_e = \mathcal{R}_0 \cdot s(t)$. In the presence of social distancing, infection reproduction is summarised by the current (effective) reproduction number $\mathcal{R}_t = \mathcal{R}_0 \cdot (1-u(t)) \cdot s(t)$. The last expression remains valid under vaccination since successful immunisation against infection simply acts by reducing the susceptible fraction.*

3.1 Formulation of the control problem

Let us now consider the control programme. Different formulations are possible depending on the degree of rationality and foresight ability of the social planner, ranging from a naïve behaviour to a rational one (first best).

The naïve control at Stage 1 and the optimal control at Stage 2. Let us first consider the case in which the social planner does not have adequate information to optimally choose $u(t)$ in the first period and then it cannot define an optimal plan at Stage 1. In this case, following Gori et al. (2021), we assume that the planner adopts a fixed rule consisting of keeping the effective reproduction number $\mathcal{R}_t \cong 1$ during Stage 1 and only at Stage 2 it optimally enacts two instruments, i.e., social distancing and vaccination, to define a first best solution. The problem thus consists of minimising at Stage 2 the expression in (12) with respect to $u(t)$ and $\tau(t)$ subject to the dynamics in (13). The existence of solutions to the optimal control problem is guaranteed because the requirements of classical existence theorems (see Fleming and Rishel, 1975) are satisfied.

Substituting the expressions of $x(t)$ and $v(t)$ from (7) and the per capita formulation referred to (9) into (12), the corresponding Hamiltonian is:

$$\begin{aligned}\mathcal{H}(s(t), i(t), u(t), \tau(t), \lambda_1(t), \lambda_2(t)) = k\frac{i(t)^2}{2} + (1-k)\frac{[A(1-i(t))(u(t)+\tau(t)(1-u(t))]^2}{2} + \\ + \lambda_1(t)\left[(1-u(t))((AB\tau(t)-s(t))i(t) - AB\tau(t))\beta + \rho(1-i(t)-s(t))\right] + \\ + \lambda_2(t)\left[\beta s(t)i(t)(1-u(t)) - \gamma i(t)\right]\end{aligned} \tag{14}$$

from which the optimality conditions read as follows:

$$
\begin{aligned}
\tau(t)^* &= \min\left(1, \max\left(0, \frac{((1-k)A^2(1-i(t))^2 + \beta i(t)s(t)(\lambda_1(t)-\lambda_2(t)))}{((1-i(t))A((1-i(t))(1-k)A - B\beta\lambda_1(t)))}\right)\right)\\
u(t)^* &= \min\left(1, \max\left(0, \frac{\beta i(t)s(t)(\lambda_2-\lambda_1)}{(1-k)A^2(1-it)^2}\right)\right) \text{ if } \tau(t)=0 \text{ and } u(t)^*=0 \text{ if } \tau(t)^*>0\\
\dot{s}(t) &= -(1-u(t))\beta i(t)s(t) - v(t) + \rho[1-s(t)-i(t)]\\
\dot{i}(t) &= (1-u(t)^*)\beta i(t)s(t) - \gamma i(t)\\
\dot{\lambda}_1(t) &= \lambda_1(t)\left[\theta + \beta(1-u(t)^*)i(t) + \rho\right] - \lambda_2(t)(1-u(t))\beta i(t) \qquad (15)\\
\dot{\lambda}_2(t) &= \lambda_2(t)\left[\theta - ((1-u(t)^*)\beta s(t) - \gamma)\right] + \lambda_1(t)((1-u(t)^*)(s(t) - AB\tau(t)^*)\beta + \rho) - ki(t) +\\
&\quad + (1-k)(1-i(t))\left[A(u(t)^* + \tau(t)^*(1-u(t)^*)\right]^2\\
s(T_V) &= s_{T_V^N},\ i(T_V) = i_{T_V^N}\\
\lambda_1(T) &= 0,\ \lambda_2(T) = \phi_V e^{-\theta(T-T_V)},
\end{aligned}
$$

where $i_{T_V^N}$ and $s_{T_V^N}$ are the actual value of infective and susceptible observed at the time of the arrival of the vaccine under the naïve policy at the end of Stage 1. From (15) it is clear that the conditions that guarantee at the optimum the existence of a positive income tax rate to finance the vaccination campaign imply setting the extent of social distancing measures to zero.

We pinpoint here that we will not present simulation results for this scenario for economy of space and avoid increasing the number of figures in the article, but they are available upon request.

The two-stage disjoint optimal control. Consider now the case in which the goal of the social planner is to optimally control the epidemic at Stage 1, with the aim of leaving a given level of infection at the beginning of Stage 2, in which the vaccine is available. The problem of the social planner then consists in minimising F_1 given the dynamic constraints in (11). We consider a fixed target on the function $i(T_V^e)$ denoting infections considered optimal at the expected date of vaccine arrival (T_V^e). The associated Hamiltonian is:

$$
\begin{aligned}
\mathcal{H}(s(t), i(t), u(t), \lambda_1(t), \lambda_2(t)) = k\frac{i(t)^2}{2} + (1-k)\frac{\left[A(1-i(t))(u(t)\right]^2}{2} + \qquad (16)\\
+ \lambda_1(t)\left[-\beta s(t)i(t)(1-u(t)) + \rho(1-i(t)-s(t))\right] +\\
+ \lambda_2(t)\left[\beta s(t)i(t)(1-u(t)) - \gamma i(t)\right]
\end{aligned}
$$

and the optimality conditions read as follows:

$$
\begin{aligned}
u(t)^* &= \min\left(1, \max\left(0, \frac{\beta i(t)s(t)(\lambda_2-\lambda_1)}{(1-k)A^2(1-it)^2}\right)\right)\\
\dot{s}(t) &= -\beta s(t)i(t)(1-u(t)^*) + \rho(1-i(t)-s(t))\\
\dot{i}(t) &= (1-u(t)^*)\beta i(t)s(t) - \gamma i(t)\\
\dot{\lambda}_1(t) &= \lambda_1(t)\left[\theta + \beta(1-u(t)^*)i(t) + \rho\right] - \lambda_2(t)(1-u(t))\beta i(t)\\
\dot{\lambda}_2(t) &= \lambda_2(t)\left[\theta - ((1-u(t)^*)\beta s(t) - \gamma)\right] + \lambda_1(t)((1-u(t)^*)\beta s(t) + \rho) - ki(t) +\\
&\quad + (1-k)(1-i(t))\left[Au(t)^*\right]^2\\
s(T_{SD}) &= s_{T_{SD}},\ i(T_{SD}) = i_{T_{SD}}\\
\lambda_1(T_V^e) &= 0,\ i(T_V^e) = i_{T_V^e}.
\end{aligned}
\tag{17}
$$

where $i_{T_V^e}$ represents the percentage of infective individuals considered optimal by the social planner when the vaccine is expected to be available. As for Stage 2, the same conditions stated in (15) hold, where however $s(T_V) = s_{T_V^N}$ and $i(T_V) = i_{T_V^N}$ should be replaced with $s(T_V) = s_{T_V^{OPT}}$ and $i(T_V) = i_{T_V^{OPT}}$, where $s_{T_V^{OPT}}$ and $i_{T_V^{OPT}}$ are the values of the susceptible and infective defined by the optimal control at Stage 1 at the actual time of vaccine arrival.

The joint optimal control. Under the assumption in which the social planner attempts at (jointly) optimally controlling the epidemic over both Stage 1 and Stage 2 already at the end of Stage 0 (i.e., when it becomes aware of the epidemic seriousness), the optimisation problem consists in minimising

$$
F_1(s(t), i(t), u(t)) + F_2(s(t), i(t), u(t), \tau(t)), \tag{18}
$$

subject to the dynamics of $s(t)$ and $i(t)$ described by a piecewise dynamic system defined in (11) and (13), given the continuity conditions on $s(t)$ and $i(t)$ when the vaccine arrives. We do not report the optimality conditions for this case as they are similar to those computed previously. We pinpoint, however, that the latter optimisation problem resembles those in Acemoglu et al. (2021) and Alvarez et al. (2021). Unlike them, however, we build on a vaccination plan beginning at the time of the arrival of the vaccine.

Vaccine rationing during Stage 2. As typically observed during the COVID-19 pandemic, we consider the case of a non-optimal vaccination schedule $\dot{v}(t) = g(t) > 0$, where function $g(t)$ is exogenously given due to the concurrency of constraints on the organisational and logistic side. In what follows we analyse the stylised case of linear rationing in which the number of vaccines that can be administered per day is constant ($\dot{v}(t) = g > 0$), generating a linearly increasing cumulative coverage (in the simulation section, we will also consider the case of a generic data-based daily vaccine coverage function, $g(t)$).

The optimal programme for the rationing problem reads as follows:

$$
\min_{u(t)\in[0,1]} \int_{T_V}^{T} \frac{1}{2}\left\{ki^2(t) + (1-k)[u^2(t)q^2(t) + \left(\frac{g(t)}{B}\right)^2]\right\} e^{-\theta(t-T_V)}dt + \phi_V i_T e^{-\theta(T-T_V)}, \tag{19}
$$

subject to

$$\begin{aligned}
\dot{s}(t) &= -(1-u(t))\beta i(t)s(t) - E_V \cdot g(t) + \rho[1-s(t)-i(t)] \\
\dot{i}(t) &= (1-u(t))\beta i(t)s(t) - \gamma i(t)
\end{aligned},$$

and the initial conditions $s(T_V), i(T_V) \in (0,1)$, with $v(T_V) = 0$. The parameter $0 \leq E_V \leq 1$ represents the actual efficacy of the vaccine against the infection, which can be interpreted as the probability that a randomly selected person for vaccine administration is successfully immunised against the infection. The optimality conditions for this problem are the following:

$$\begin{aligned}
u(t)^* &= \min\left(1, \max\left(0, \frac{\beta i(t)s(t)(\lambda_2-\lambda_1)}{(1-k)A^2(1-it)^2}\right)\right) \\
\dot{s}(t) &= -\beta s(t)i(t)(1-u(t)^*) - E_V \cdot g(t) + \rho(1-i(t)-s(t)) \\
\dot{i}(t) &= (1-u(t)^*)\beta i(t)s(t) - \gamma i(t) \\
\dot{\lambda}_1(t) &= \lambda_1(t)\left[\theta + \beta(1-u(t)^*)i(t) + \rho\right] - \lambda_2(t)(1-u(t))\beta i(t) \\
\dot{\lambda}_2(t) &= \lambda_2(t)\left[\theta - ((1-u(t)^*)\beta s(t) - \gamma)\right] + \lambda_1(t)((1-u(t)^*)\beta s(t) + \rho) - ki(t) + \\
&\quad + (1-k)(1-i(t))\left[Au(t)^*\right]^2 \\
s(T_V) &= s_{T_V},\ i(T_V) = i_{T_V} \\
\lambda_1(T) &= 0,\ \lambda_2(T) = \phi_V e^{-\theta(T-T_V)}.
\end{aligned} \tag{20}$$

4 Results: simulations

Consistently with our theoretical objectives, we now report the main simulation results for a sample of sub-cases. In particular, we will illustrate our main results for the generic abstract problem of epidemic control in the presence of two control instruments, in which the second instrument (vaccination by an effective vaccine) appears only in the second stage of the epidemic. As discussed in the theoretical part, this analysis will distinguish two main sub-cases, namely the two-stage disjoint optimal control problem and the full joint optimal control problem. Additionally, we will report a number of results on the problem in which the vaccination schedule (i.e., the number $V(t)$ of susceptible successfully immunised against the infection per unit of time) is given, reflecting *vaccine supply rationing* at various scales (international, national, local, etc.), thereby yielding an optimal control problem in which social distancing is the unique available instrument.

The values of the parameters used in the simulations are all drawn from the COVID-19 literature (Table 1). In particular, we adopted a value of the basic reproduction number of $\mathcal{R}_0 = 3.0$ as consistent with the values estimated from the original strains responsible for the first and the second pandemic waves in Italy. The waning rate of natural and vaccine-related immunity (ρ) is set to $1/135$, reflecting the short average duration of protection (less than five months) offered by the vaccine against infection. The efficacy of the vaccine against the infection (E_V) is drawn from the entire range of COVID-19 efficacies against the original strains and the various COVID-19 variants of concern (VoC).

We preliminary recall that in the full absence of any control the infection shows an initial epidemic phase but, unlike pure SIR models, the infection does not go extinct due to acquired immunity, instead it steadily oscillates around an endemic state whose appearance and stability are induced by the waning of natural immunity (Figure 1).

Parameter	Baseline value or range	Source
A	0.3	Arbitrary (economic scaling parameter)
θ	0.03/365	Literature (subjective discount rate)
$\mathcal{R}_0$	3.0	Literature
γ	$1/7$ (day^{-1})	Literature
β	$3/7$ (day^{-1})	Computed from the relation $\mathcal{R}_0 = \beta/\gamma$
ρ	$1/135$ (day^{-1})	Literature
E_V	$0.2 - 0.8$	Literature.

Table 1. Values of the main epidemiological and economic parameters used in the numerical simulations.

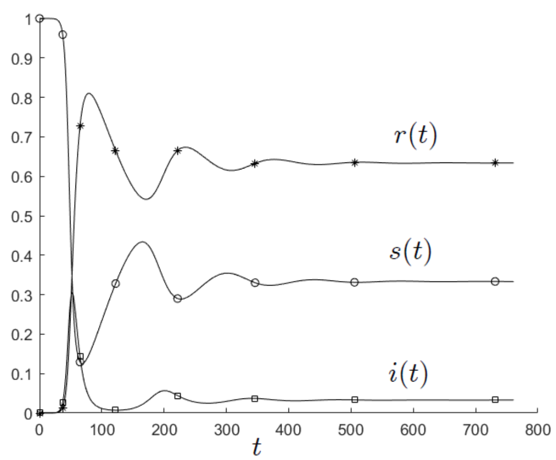

Figure 1. The free epidemic case showing (for $\mathcal{R}_0 > 1$) the temporal trend of state variables (s, i, r) resulting in damped oscillations about the endemic state.

In what follows, we report the main simulation results for the various cases considered in the theoretical analysis. We recall that in all simulations presented the social distancing is activated (during Stage 1) after an initial stage (Stage 0) of free epidemic growth lasting one month initialised from an initial infective case in a fully susceptible population.

4.1 The two-stage disjoint optimal control problem with realised expectations on vaccine arrival

The simulation results related to this case refer to the *two-stage disjoint optimal control problem* discussed so far by assuming that the expectations of the planner about the arrival date of the vaccine are fully realised. In this scenario, the social planner aims to control (since the beginning of Stage 1) the epidemic only by means of the social distancing instrument, taking into account their effects in terms of both epidemiological and economic costs. In fact, faced with great uncertainty about the arrival of the vaccine during Stage 1, the social planner's objective is to control the epidemic over a relatively short time horizon until the expected arrival date of the vaccine with a given level of infection. This follows the spirit of Acemoglu et al. (2021) and Alvarez et al. (2021).

We then assume that the vaccine will become available at the beginning of Stage 2 and then the social planner will seek to optimally tune (by re-optimising) the social distancing strength and the vaccination plan by inheriting the exit conditions from Stage 1. This approach is not trivial as, for example, it reflects the intervention undertaken in Italy (and in other European countries), in which a change of the Italian central government temporally occurred at the start of the vaccination campaign, implying that the decisions about the control over Stage 1 and Stage 2 of the epidemic were completely disjoint.

In this regard, Figures 2 and 3 report the outcomes of the optimal control problems for different values ($k = k^{High} = 0.97$ and $k = k^{Low} = 0.2$) of the relative weight (k) of the direct versus indirect component of the costs to be minimised by the planner.

For $k = k^{High}$ (Figure 2), the social distancing intensity in Stage 1 fastly reaches a maximum, then relaxes in view of the achieved high degree of control and finally relapses to contrast the effects of the immunity loss, and of the control of the potential increase of epidemic activity at the end of Stage 1 horizon. Once the vaccine becomes available, which occurs at a rather low level of population immunity (that is, for a susceptible fraction rather large) due to the intense control activity during the pre-vaccine period, the vaccination component of the optimal programme starts with a massive initial campaign (thanks to the free accessibility to the vaccine) which is subsequently relaxed, but it allows to bring epidemic activity to highly controlled levels for the entire control horizon, and consequently to fully relax the social distancing instrument. Notably, the social distancing component of the optimal programme can be completely relaxed (i.e., decreased to zero) because the administered vaccination allows the full control of the epidemic over the horizon considered. Remarkably, the (postulated) full effectiveness of the vaccine allows to optimally joining the two components of the programme and achieving a very high level of epidemic control by a total cumulative vaccination coverage slightly above 60%.

For $k = k^{Low}$ (Figure 3), the low weight attributed to the epidemiological cost implies that the social distancing is enacted at a much lower intensity. Therefore, the natural waves due to the epidemic are not prevented and social distancing is activated just to mitigate them, consequently yielding a much larger societal impact of the epidemic compare to k^{High}. Therefore, prior to arrival of the vaccine, three epidemic waves are observed with a time spacing much closer to the duration of the immunity. Nonetheless, the high degree of natural immunity allowed by the mild social distancing implemented prior to vaccine arrival allows – once the vaccine becomes available – to rapidly bring epidemic activity at fully controlled levels with a very low vaccination effort compare to k^{High}. Notably, the building of susceptibles rapidly restarts, but the epidemic remains well controlled for the entire residual control horizon. Last, note that also the optimal tax rate oscillates to follow epidemiological trends.

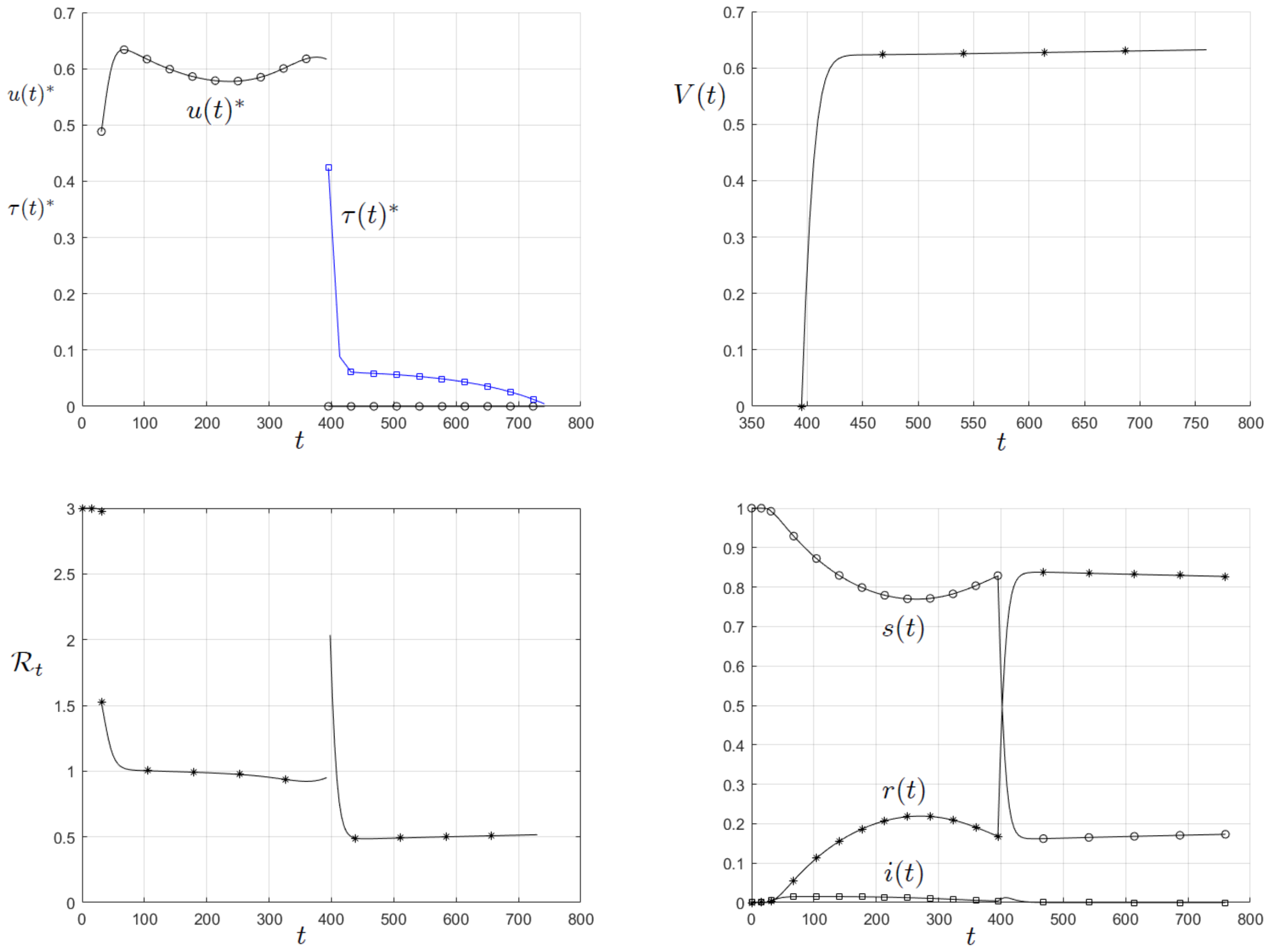

Figure 2. The two-stage disjoint optimal control problem under realised expectations for k^{High}. Temporal trends of (i) optimal social distancing and the tax rate to finance vaccination (top left panel), (ii) cumulative vaccine function (top right panel), (iii) $\mathcal{R}_t$ (bottom left panel), (iv) epidemiological state variables (bottom right panel).

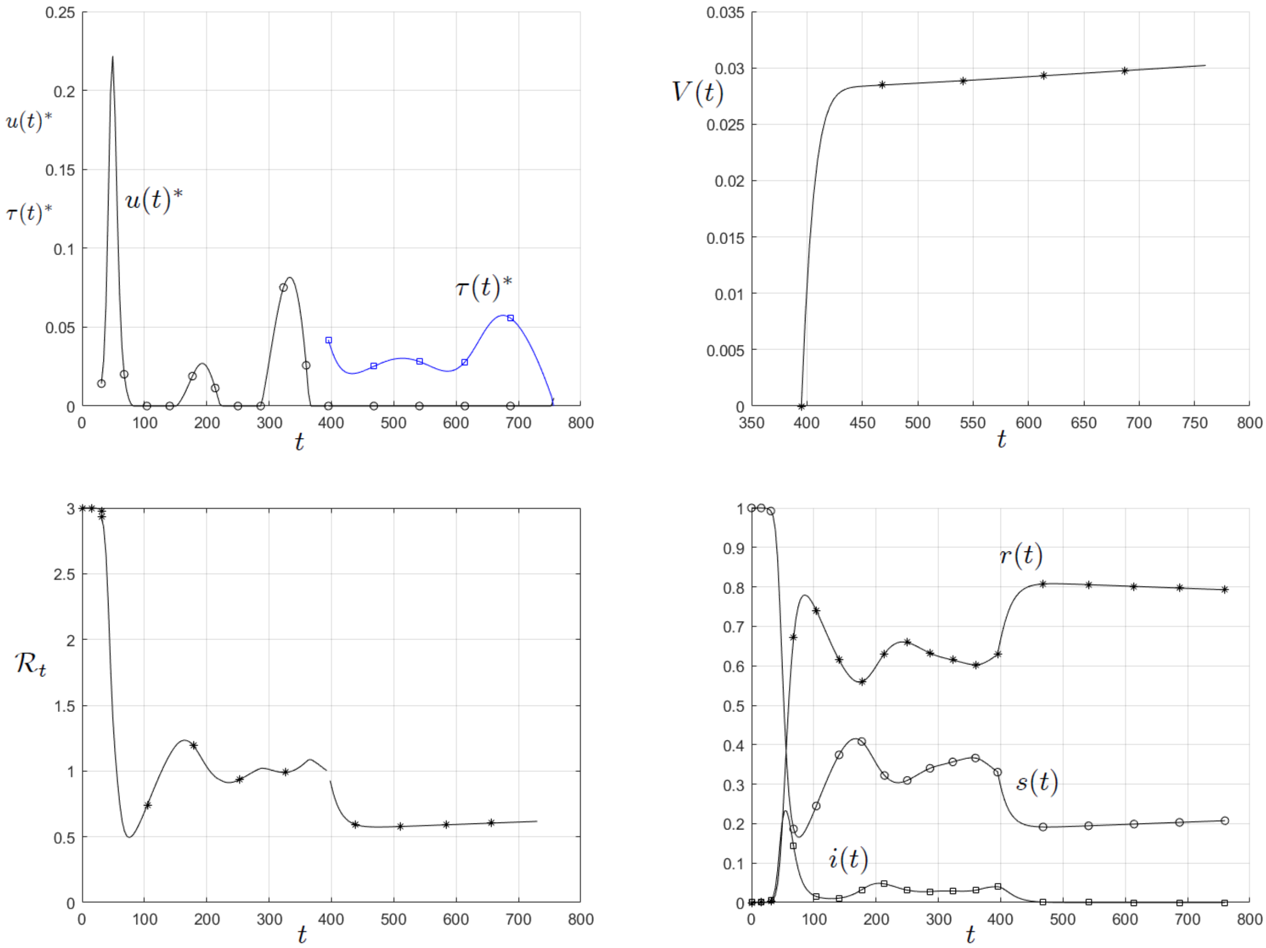

Figure 3. The two-stage disjoint optimal control problem under realised expectations for k^{Low}. Temporal trends of (i) optimal social distancing and the tax rate to finance vaccination (top left panel), (ii) cumulative vaccine function (top right panel), (iii) $\mathcal{R}_t$ (bottom left panel), (iv) epidemiological state variables (bottom right panel).

4.2 The two-stage disjoint optimal control problem with non-realised expectations

The simulation results now refer to the *two-stage disjoint optimal control problem* by assuming that the actual arrival of the vaccine occurs prior to the expected arrival date ($k = k^{High}$). In Figure 4 we assume that the time horizon of the social planner is still two years as in the previous experiment, but the arrival of the vaccine that was expected by the end of year 2 actually occurs at the end of year 1. At Stage 2, the planner will then re-optimise by leaving the previous plan (chosen at Stage 1) and choosing a new optimal mix of social distancing and vaccination. This discrepancy between expected and actual arrival date of the vaccine yields (compare to the case of realised expectations) an interesting effect by which a lower social distancing is rapidly crowded out by a more intense and concentrated vaccination effort.

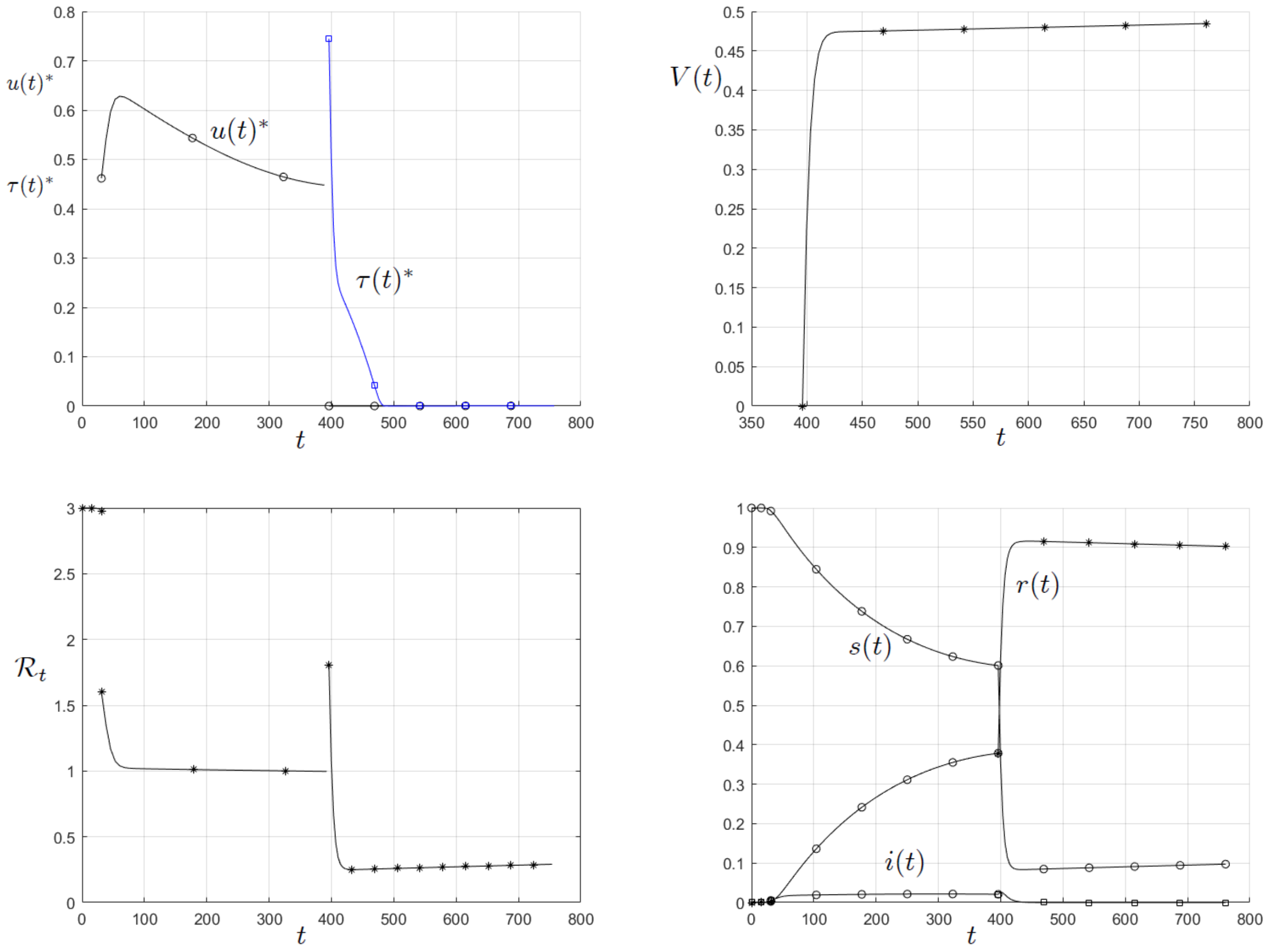

Figure 4. The two-stage disjoint optimal control problem in the case of anticipated arrival of the vaccine compared to expectations for k^{High}. Temporal trends of (i) optimal social distancing and the tax rate to finance vaccination (top left panel), (ii) cumulative vaccine function (top right panel), (iii) $\mathcal{R}_t$ (bottom left panel), (iv) epidemiological state variables (bottom right panel).

4.3 The joint optimal control problem over the two stages of the epidemic

As for this case, we only report simulations referred to the case in which the expectations of the planner about the arrival date of the vaccine are fully realised. The planner builds an optimal plan since the beginning of Stage 1 ($t = T_{SD}$) by relying only social distancing instrument at Stage 1 and switching to a mix of social distancing and vaccination at Stage 2, instead of re-optimising at the beginning of Stage 2 as in the disjoint problem. Sticking on the case of $k = k^{High}$, the main qualitative news (Figure 5, top left panel), compared to the disjoint problem reported in Figure 2 stems in the shape of both the optimal controls: the optimal social distancing can be substantially relaxed prior to the arrival of the vaccine, while the optimal vaccination does not set to its maximum value at the programme start, but it follows a plateau for a quite long period of time before declining to zero.

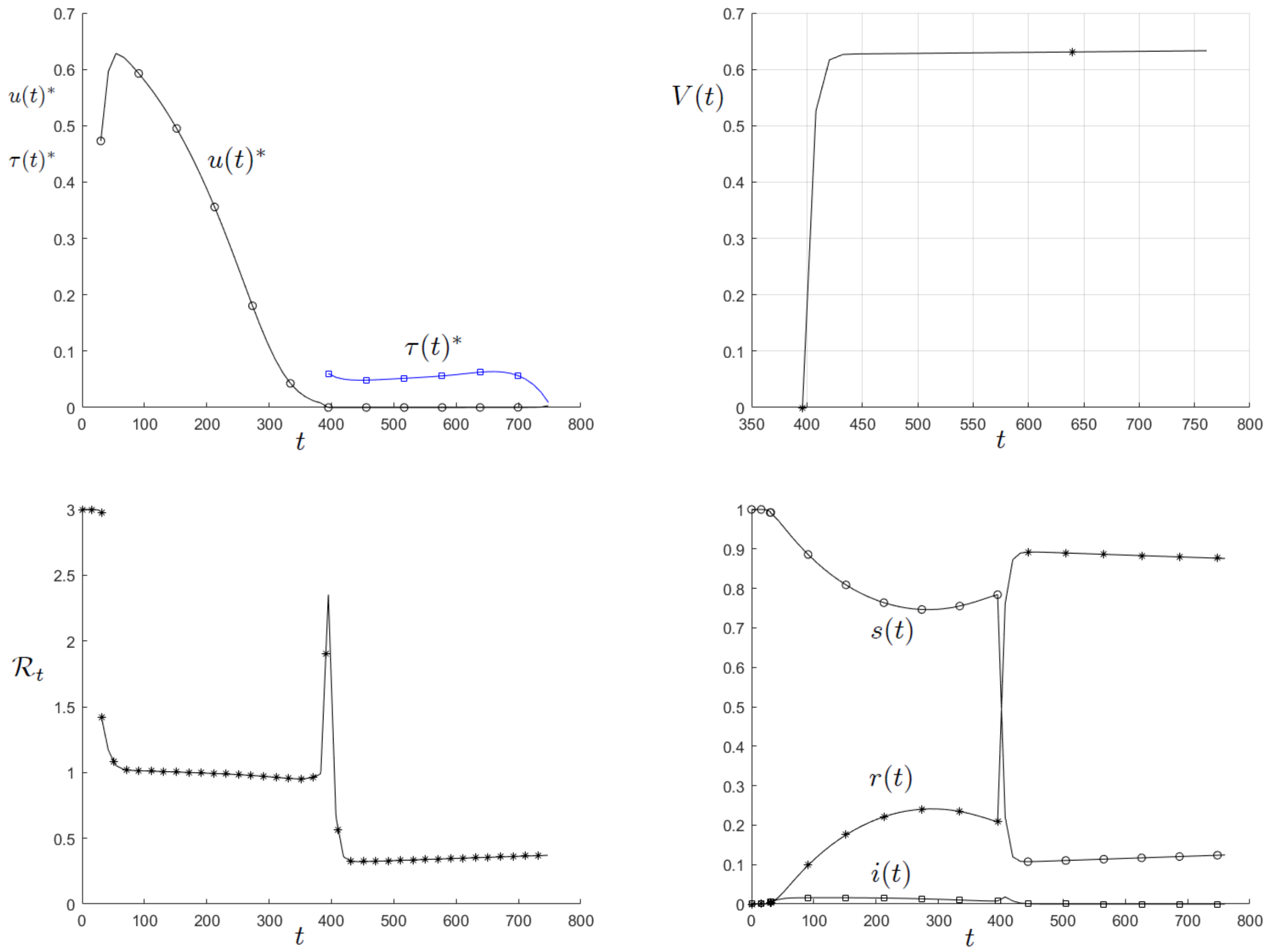

Figure 5. The joint optimal control problem over the two stages of the epidemic under realised expectations for k^{High}. Temporal trends of (i) optimal social distancing and the tax rate to finance vaccination (top left panel), (ii) cumulative vaccine function (top right panel), (iii) $\mathcal{R}_t$ (bottom left panel), (iv) epidemiological state variables (bottom right panel).

4.4 Optimal social distancing under an observed vaccination schedule

The simulations related to this case refer to vaccine rationing – as was developed in the theoretical part – under a generic data-based vaccine coverage function $\tilde{V}(t)$, reflecting a range of phenomena such as vaccine scarcity, logistic bottlenecks and any other non-optimal allocation. Therefore, the objective of this section is to investigate the optimal lifting of social distancing owing to the given temporal profile of vaccination. We will specifically refer to COVID-19 as a case study and use actual data from the Italian COVID-19 vaccination campaign to parametrise the $V(t)$ curve under a grid of values of the efficacy of the vaccine against the infection. From a theoretical perspective, the planner has an initial aim to contain the epidemic in Stage 1 through social distancing as was the case in the disjoint optimal control problem analysed so far. To relate $V(t)$ to actual data, we define it as:

$$V(t) = E_V \cdot N_V(t), \tag{21}$$

where $N_V(t)$ represents the actual number of vaccine administrations per day and $0 \leq E_V \leq 1$ represents the actual efficacy of the vaccine against the infection, which is the probability that a randomly selected person for vaccine administration is successfully immunised. In our experiments, we will let E_V vary (Table 1) in a grid reflecting the different efficacies of the vaccine against the different VoC of COVID-19. In particular, we consider the following values: $E_V = 0.8$ (roughly corresponding to the original strains), $E_V = 0.5 - 0.8$ (delta/alpha VoC) and $E_V = 0.2 - 0.3$ (omicron VoC). As for $N_V(t)$, we will measure it by daily data of second doses of COVID-19 vaccine administrations because the available vaccines achieve their declared efficacy only after the completion of a two-dose cycle. We delayed the vaccination signal by two weeks to capture the time of full *take* of the vaccine. In particular, we smoothed available cumulative vaccination data by a logistic curve showing an excellent fit (Figure 6). The smoothed curve ($\tilde{V}(t)$) was used as an input for the optimal control programmes.

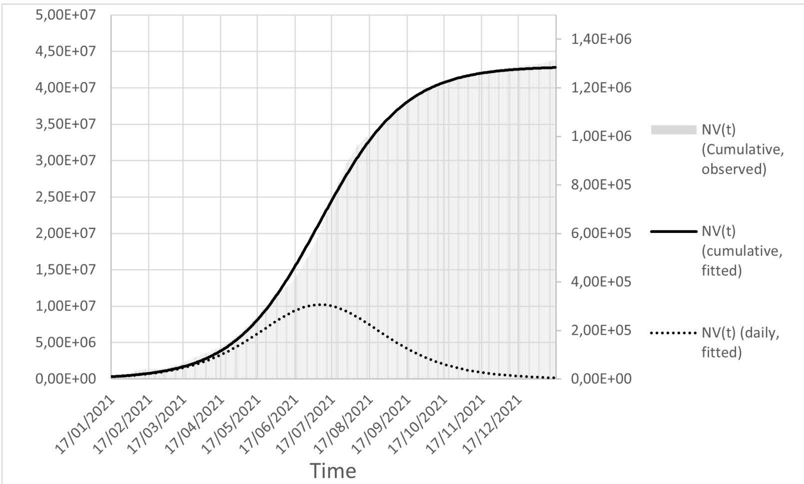

Figure 6. Aggregate (all eligible age groups) cumulative temporal profile of second dose administrations of COVID-19 vaccines in Italy since the onset of the Italian vaccination campaign on December 27th, 2020, and the corresponding logistic smoothing curve. The eventual cumulative aggregate (in the eligible population) second dose coverage was 77.6%.

The corresponding outcomes for the case $k = 0.97$ are reported in Figure 7. The optimal social distancing (top left panel) keeps the already found U-shaped trend during Stage 1, whereas its shape during Stage 2 strongly reflects the vaccine efficacy and duration. In particular, the optimal trajectory $u(t)^*$ in Stage 2 rapidly climbs (disregarding the initial discontinuity) to values nearby those of Stage 1, then it declines to a minimum in a time span of about 5 months, but it needs to relapse thereafter due to the waning of vaccine protection, as expected. The extent of the decline is dramatically dependent on E_V: for $E_V = 0.8$ (black) most social distancing can be relaxed, whereas for $E_V = 0.2$ (blue), $u(t)^*$ can never decline beyond 0.5. The (non-optimal) ensuing tax rate (top right panel) reveals a non-trivial dynamics due to an indirect equilibrium economic effect. Indeed, the number of administered vaccine is the same in all scenarios, but the different intensity of social distancing poses different costs to the economy, which requires to finely tune the tax rate (the lower the efficacy of the vaccine against the infection, the higher the tax rate). The corresponding trajectories of the epidemiological variables (bottom left panel) clearly confirm the need for the relapse of the optimal social distancing due to the decline in (primarily) vaccine-related immunity caused by the short duration of vaccine protection. The trajectories of $\mathcal{R}_t$ essentially overlap and (regardless the level of vaccine efficacy) set on a temporal trend that plateaus slightly below the unit threshold.

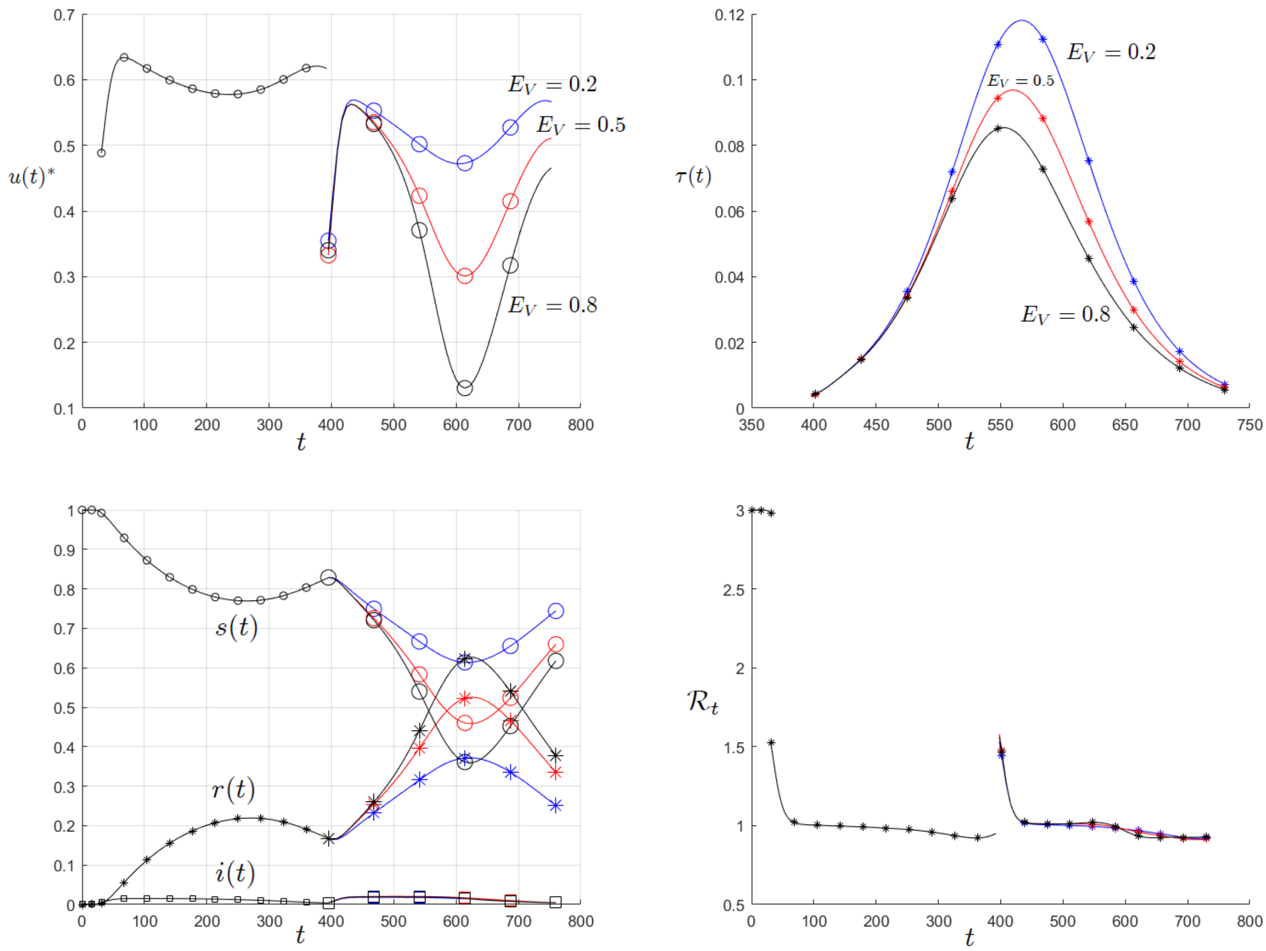

Figure 7. The one-control case with the logistic vaccination schedule reported in Figure 6 for k^{High}. Colours at Stage 2: black curve $E_V = 0.8$; red curve $E_V = 0.5$; blue curve $E_V = 0.2$. Temporal trends of (i) optimal social distancing (top left panel), (ii) (non-optimal) tax rate to finance vaccination (top right panel), (iii) epidemiological state variables (bottom left panel). (iv) $\mathcal{R}_t$ (bottom right panel).

4.5 Optimal social distancing under linear vaccine rationing

We now report a few summary results on the case of linear vaccine rationing. Still relying on the Italian case, we simply assume that the cumulative second dose coverage eventually achieved (see the previous sub-section) was distributed linearly with a constant number of vaccine administrations per day ($g = 117.000/55.000.000$). Our findings for $k = 0.97$ are summarised in Figure 8, showing that the outcome in this case is strongly overlapped with the one presented in Figure 7, with the exception (besides some negligible loss of smoothing in the control solution) of the non-optimal temporal trend of the tax rate. Unlike the logistic case, in which the tax rate followed somewhat straightforwardly the daily number of effective immunisations, in this case the constant vaccination is more sensitive to epidemiological trends.

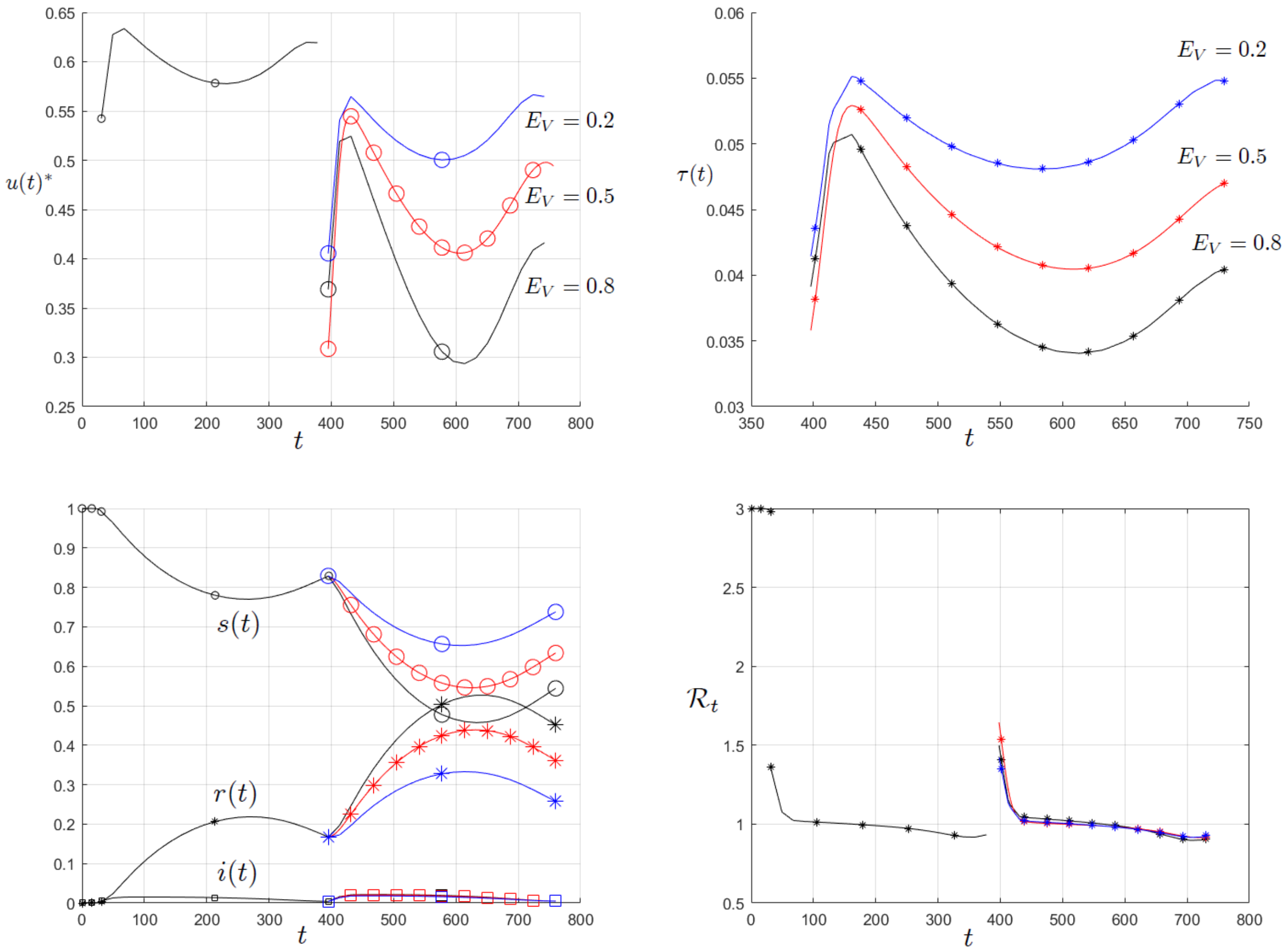

Figure 8. The one-control case with linear vaccine rationing for k^{High}. Colours at Stage 2: black curve $E_V = 0.8$; red curve $E_V = 0.5$; blue curve $E_V = 0.2$. Temporal trends of (i) optimal social distancing (top left panel), (ii) (non-optimal) tax rate to finance vaccination (top right panel), (iii) epidemiological state variables (bottom left panel). (iv) $\mathcal{R}_t$ (bottom right panel).

5 Conclusions

This article aimed to tackle the issue of designing a two-control optimal policy against an outbreak of a communicable disease conferring temporary immunity. Our analysis aimed to augment the post-COVID-19 literature on economic epidemiology triggered by the early developments by Acemoglu et al. (2021) and Alvarez et al. (2021). With this aim, we proposed a parsimonious economic-epidemiological framework mixing a Susceptible-Infective-Removed-Susceptible (SIRS) model for a socially transmitted infection with a (short-term) economic set up, in which the social planner minimises an objective function by choosing the strength of social distancing and the tax rate to finance the vaccination campaign. However, owing to the realm of COVID-19, the social distancing is the only available control tool in a first stage of the epidemic because an effective vaccine only arrives in a second stage. The use of two control instruments depends on the degree of rationality and foresight ability of the planner as well as on the availability of effective vaccines against the infection. In addition, still owing to the realm of COVID-19, we also considered the more realistic case where the amount of vaccine stockpile is fully exogenously given due to a number of constraints (e.g., due to scarcity, rationing, logistic bottlenecks and so on).

Being highly parsimonious, our framework resorted to a number of simplifying hypotheses. On the epidemiological side, we disregarded realistic epidemiological factors used, for example, to describe the COVID-19 epidemic, such as, e.g., a latency phase after infection, asymptomatic states of infection, infection-related mortality, age structure, etc. Moreover, we disregarded further control options such as testing and tracing (used, e.g., in both Acemoglu et al., 2021 and Alvarez et al., 2021). Another limitation lies in the postulated equality between the rates of waning of both natural and vaccine-related immunity. This issue is easily solved by adding an additional compartment distinguishing natural versus vaccine-related removed individuals. Additionally, we kept the economic side as simple as possible to concentrate on the short-term analysis of a planner that aims to trade-off economic and epidemiological costs having taxation as the only economic tool, without accounting for capital accumulation. All previous points are easily removable – at least in principle – and will be the object of our future research agenda. However, the parsimonious framework posed here included a great deal of the fundamental qualitative features of the problem.

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Conflict of Interest The authors declare that they have no conflict of interest.

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