Indeterminacy, bifurcations and chaos in an overlaping generations model with negative environmental externalities

Angelo Antoci*
Dipartimento di Economia Impresa e Regolamentazione, Università di Sassari, via Torre Tonda 34, 07100 Sassari, Italy; e-mail: antoci@uniss.it

Mauro Sodini
Dipartimento di Statistica e Matematica Applicata all'Economia, Università di Pisa, via Cosimo Ridolfi 10, 56124 Pisa, Italy; e-mail: m.sodini@ec.unipi.it

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Abstract

We analyze an overlapping generations model where agent's welfare depends on three goods: leisure, environmental quality and consumption of a private good.
We assume that the production process of the private good depletes the natural resource and that the consumption of the private good alleviates the damages due to environmental deterioration. In such context, we show that individuals' reactions to environmental deterioration may lead to complex dynamics, in particular to the rise of periodic orbits and chaos.

Keywords: Defensive environmental expenditures, Overlapping generations models, Indeterminacy, Undesirable economic growth

1 Introduction

The literature on overlapping generations models with environmental goods is centered on the study of the context in which economic agents belonging to the same generation maximize their welfare by taking into account the negative impact of economic activity on environmental dynamics. Consequently, they allocate their resources between consumption, saving and environmental defensive expenditures that improve environmental quality by reducing the negative

*Corresponding author

effects of production processes. See, for example, the seminal paper of John and Pecchenino (1994); on the same research line we can find Jhon et al. (1995), Zhang (1999), Seegmuller and Verchère (2005) and many others. In these works, the authors obtain a long run positive correlation between wellbeing and economic growth; that is, the increase of the production of consumption goods is always a desirable outcome.

The main difference between the above cited literature and our model is that we obtain a negative correlation between economic growth and individuals' wellbeing. This result is due to the fact that we analyze an economy where individuals defend themselves from environmental degradation by increasing their consumption of produced goods. Since the production process of these goods has a negative impact on the environment, these self-protection choices generate further environmental degradation. In such context, a self-enforcing mechanism may be observed according to which environmental degradation leads to an increase of consumption of private goods, which in turn generates further environmental degradation and so on. This self-enforcing economic growth process is driven by the continuous increase of individuals' needs for private consumption generated by the progressive reduction of the free consumption of environmental goods.

The idea that environmental deterioration may lead individuals to become more dependent on consumption of private goods rather than on consumption of free access environmental resources is shared by several works on the subject of environmental defensive expenditures (see e.g. Hueting 1980; Shibata and Winrich 1983; Leipert and Simonis 1988; Leipert 1989; Shogren and Crocker 1993; Antoci and Bartolini 1999, 2004; Bartolini and Bonatti 2002, 2003; López 2003; Antoci et al. 2005, 2007; Escofet and Bravo-Peña 2007; Antoci et al. 2008). According to this literature, several produced goods can be used to alleviate the damages due to environmental degradation. For example, mineral water may substitute spring water or tap water. Medicines may mitigate the effects of respiratory diseases caused by air pollution. Individuals may react to the deterioration of the seaside near home by going to a less deteriorated seaside area by car or by boat, they may build a swimming pool in their gardens, they may purchase houses in exclusive areas at the seaside or buy holiday-packages in tropical paradises. Individuals may defend themselves from external sources of noise by installing sound-proofing devices, and so on. However, the general insight provided by said literature is that individual reactions to environmental deterioration can be diverse and are likely to deeply influence consumption patterns, increasing the consumption of private and expensive goods as opposed to free access environmental resources. Urban life offers a paradigmatic example of this substitution mechanism. Cities are often characterized by the scarcity of free access environmental resources and, at the same time, they are able to supply a considerable variety of private and expensive consumption opportunities (see e.g. Hueting 1980; Antoci and Bartolini 1999, 2004; Bartolini and Bonatti 2002, 2003; Antoci et al. 2008). The scarcity of areas where individuals can meet away from the dangers of city traffic brings on additional expenses for childcare (baby-sitters, playgrounds, etc.), as well as for the leisure of adults.

One reason for the constant increase in the consumption of "home entertainment" in the industrialized countries can indeed be found in the substitution mechanism as previously defined.

The context where self-protection choices generate a further deterioration of environmental goods has been examined by Shogren and Croker (1991), who demonstrate that if self-protection choices of economic agents generate environmental damage on other economic agents, and if economic agents do not coordinate themselves, then self-protection choices are enforced beyond the socially optimal level[1]. However, Shogren and Croker analyze a static model and do not develop their model in order to further examine the consequences that an "excess" of self-protecting choices may determine on economic growth dynamics. In our model we expand Shogren and Croker's work in this direction. In particular, our study points out that self-protection consumption choices may create a large set of parameters for which the dynamics of the economy is indeterminate; that is, given the initial value of the state variable, the economy may follow a continuum of economic growth orbits. Furthermore, periodic orbits and chaotic behavior may emerge. These are quite new results because the widespread view in literature on overlapping generations models is that the rise of periodic orbits and of indeterminacy is due to distortions in the production side of the economy: non perfect competition, positive externalities and so on (see e.g. Reichlin 1986; Grandmont et al. 1998; Cazzavillan et al. 1998; Cazzavillan 2001), while in our model these dynamic regimes can be observed assuming a very simple production technology (i.e. a Cobb-Douglas one) and perfect competition among firms. From this point of view, our paper is similar to the work of Seegmuller and Verchère (2005), which obtain analogous results but, as said above, in a different context.

The paper is structured as follows. Section 2 introduces the model. Section 3 analyzes local stability of fixed points. Section 4 shows some results obtained by numerical simulations. Section 5 concludes.

2 The overlapping generations model

Let us consider a perfectly competitive economy populated by a continuum of identical infinitely-lived firms, which produce a consumption good, and by a constant size population of identical individuals; so, we can consider the choices of a *representative firm* and of a *representative individual*. Time is discrete, $t = 1, 2, 3, ..., \infty$; each individual lives for two periods. In each period t, two generations coexist in the economy: the "young" and the "old". As usual in overlapping generations models, for the sake of analytical simplicity, we assume that individuals works only when they are young and consumes only when they are old (see e.g. Duranton 2001; Seegmuller and Verchère 2005)[2].

[1] The distinction between self-protecting devices that "transfer" negative externalities on to other economic agents and devices that instead "filter" them has been introduced by Bird (1987).

[2] See De la Croix and Michel (2002) for an introduction to overlaping generations models.

The representative individual is "young" in period t and "old" in $t+1$. In t, he is endowed with $L^* > 1$ units of time (L^* is a fixed parameter of the model) and supplies the labour input L_t ($0 \le L_t \le L^*$) to the representative firm, which employes it to produce a consumption good. Labour effort L_t is remunerated at the wage rate W_t and the sum received $W_t \cdot L_t$ is invested through the purchase of productive capital K_{t+1} (i.e. $K_{t+1} = W_t \cdot L_t$) which the individual will rent to the representative firm, when old, earning the interest factor R_{t+1}. The sum obtained, $W_t \cdot L_t \cdot R_{t+1}$, allows him to buy and consume the quantity $C_{t+1} = W_t \cdot L_t \cdot R_{t+1}$ of the good produced by the firm ($W_t \cdot L_t$ and $W_t \cdot L_t \cdot R_{t+1}$ are expressed as unities of the consumption good).

For simplicity, we assume that the life-time welfare of the representative individual is measured by the following logarithmic function

$$U(L_t, C_{t+1}, E_{t+1}) = \ln(L^* - L_t) + \frac{1}{1+\theta} \cdot \ln(P \cdot C_{t+1} + E_{t+1}) \tag{1}$$

where E_t represents the value of a given environmental quality index at time t; P, γ and θ are positive parameters; $\frac{1}{1+\theta}$ is the discount factor[3]. Notice that

$$\frac{\partial^2 U}{\partial E_{t+1} \partial C_{t+1}} = -\frac{\gamma \cdot P}{(P \cdot C_{t+1} + E_{t+1})^2} < 0$$

So the increase of U due to an increase of C_{t+1} in negatively correlated with the value of E_{t+1}. More precisely, C_{t+1} and E_{t+1} are substitutes with marginal rate of substitution equal to

$$\frac{\frac{\partial U}{\partial C_{t+1}}}{\frac{\partial U}{\partial E_{t+1}}} = P$$

The representative firm produces the private good using a very simple Cobb-Douglas technology

$$\begin{aligned} Y &= A \cdot F(K_t, L_t) = A \cdot L_t^{1-\alpha} \cdot K_t^{\alpha} = \\ &= A \cdot L_t \cdot k_t^{\alpha} = A \cdot L_t \cdot f(k_t) \end{aligned}$$

where K_t is physical capital, A is a positive parameter representing technological progress and $k_t := K_t / L_t$. The economy is assumed perfectly competitive and so, in each period t, the representative firm maximizes the profit function

$$A \cdot F(K_t, L_t) - W_t \cdot L_t - R_t \cdot K_t \tag{2}$$

taking the wage rate W_t and the interest factor R_t as exogenously given. As usual, this assumption gives rise to the following first order conditions

[3] It is easy to check that by assuming the alternative utility funcion

$$U(L_t, C_{t+1}, E_{t+1}) = \ln(L^* - L_t) + \ln(E_t) + \frac{1}{1+\theta} \cdot \ln(P \cdot C_{t+1} + E_{t+1}),$$

the same dynamics are obtained.

$$\begin{aligned} W_t &= A \cdot (1-\alpha) \cdot k_t^{\alpha} && (3) \\ R_t &= A \cdot \alpha \cdot k_t^{\alpha-1} && (4) \end{aligned}$$

The maximization problem of the representative individual is

$$\max U(L_t, C_{t+1}, E_{t+1})$$

subject to

$$\begin{aligned} C_{t+1} &= R_{t+1} \cdot W_t \cdot L_t && (5) \\ L_t &\in [0, L^*] && (6) \end{aligned}$$

According to (5), the representative individual, by working L_t, obtains the remuneration $W_t \cdot L_t$ which can be invested at the interest rate R_{t+1} obtaining $R_{t+1} \cdot W_t \cdot L_t$ when old; $R_{t+1} \cdot W_t \cdot L_t$ is entirely used to consume the good produced by the representative firm. In our perfectly competitive economy, W_t and R_{t+1} are considered as exogenously given by the representative individual. Furthermore, we assume that the representative individual, at time t, is able to perfectly foresee the value of E_{t+1}. However, E_{t+1} is considered as exogenously given; that is, the representative individual considers as negligible the impact of his choices on the environmental quality.

Under these assumptions, the first order condition for an interior solution (that is, with $0 < L_t < L^*$) of the representative individual's choice problem is

$$-\frac{1+\theta}{L^* - L_t} + \frac{\alpha \cdot (1-\alpha) \cdot P \cdot A^2 \cdot k_t^{\alpha} \cdot k_{t+1}^{\alpha-1}}{\alpha \cdot (1-\alpha) \cdot P \cdot A^2 \cdot k_t^{\alpha} \cdot k_{t+1}^{\alpha-1} \cdot L_t + E_{t+1}} = 0 \qquad (7)$$

where W_t and R_t have been replaced by the values given in (3) and (4), respectively.

To complete the model, we assume the dynamics of E_{t+1} defined by the equation

$$E_{t+1} = \overline{E} - \eta \cdot \left[F(\overline{K}_t, \overline{L}_t)\right]^{\beta} = \overline{E} - \eta \cdot (\overline{L}_t \overline{k}_t^{\alpha})^{\beta} \qquad (8)$$

where $\overline{E}$ is a positive parameter representing the value assumed by the environmental quality index without the negative impact due to economic activity; β and η are positive parameters measuring the negative impact of the production activity on environmental quality (the index E_{t+1} can assume negative values); $\overline{K}_t$, $\overline{L}_t$ and $\overline{k}_t$ represent the average values of K_t, L_t and k_t, respectively. The representative individual takes as given average values when maximizing his utility function; however, being individuals identical, ex post we have $\overline{K}_t = K_t$, $\overline{L}_t = L_t$ and $\overline{k}_t = k_t$. This assumption implies that the representative agent considers the negative impact of his choices on the environment as negligible; that is, with no effect on $\overline{K}_t$, $\overline{L}_t$ and $\overline{k}_t$. Therefore, in this model, the choices of the representative individual are not optimal and generate negative externalities. However, the orbits followed by the economy are Nash equilibria, in that no single individual has interest to modify his choices if also the others don't revise theirs.

Since it holds

$$K_{t+1} = k_{t+1} \cdot L_{t+1} = W_t \cdot L_t$$

that is, labour remuneration $W_t \cdot L_t$ of representative agent in period t is entirely saved and becomes the capital K_{t+1} used by the representative firm in period $t+1$, we obtain the following dynamical system

$$-\frac{1+\theta}{L^* - L_t} + \frac{\alpha \cdot (1-\alpha) \cdot P \cdot A^2 \cdot k_t^\alpha \cdot k_{t+1}^{\alpha-1}}{\alpha \cdot (1-\alpha) \cdot P \cdot A^2 \cdot k_t^\alpha \cdot k_{t+1}^{\alpha-1} \cdot L_t + (\overline{E} - \eta \cdot (k_t^\alpha L_t)^\beta)^\gamma} = 0 \quad (9)$$

$$k_{t+1} \cdot L_{t+1} = A \cdot (1-\alpha) \cdot k_t^\alpha \cdot L_t \quad (10)$$

3 Stability of the normalized fixed point

The system (9)-(10) defines k_{t+1} and L_{t+1} as functions of k_t and L_t. In this section, we study the stability of fixed points of such discrete dynamical system. Since our model contains a large number of parameters, to make clear the study we use the geometrical-graphical method developed by Grandmont and DeVilder (1999) that allows us to characterize the stability properties of this dynamical system. We impose some conditions on parameters under which a fixed point with coordinates

$$k = L = E = 1$$

exists. This allows us to analyze the effects on stability due to changes in parameters' values being sure that the fixed point doesn't disappear.

Requiring that $k = L = E = 1$, from (10) we obtain

$$A = \frac{1}{1-\alpha} \quad (11)$$

Furthermore, from (8), we obtain

$$\overline{E} = 1 + \eta \quad (12)$$

and, from (7), we obtain

$$-\frac{1+\theta}{L^* - 1} + \frac{\alpha P}{\alpha P + 1 - \alpha} = 0$$

which is satisfied if $L^* > 2 + \theta$ and

$$P = \frac{(1-\alpha)(1+\theta)}{\alpha(L^* - \theta - 2))} \tag{13}$$

Finally, the point with $k = L = E = 1$ must be a Nash equilibrium; that is, it must be a solution of the maximization problem of the representative individual. This amounts to require that $L = 1$, and not $L = 0$, represents the best reply of the representative agent if $\overline{k} = \overline{L} = 1$ and $E = 1$. Therefore, it must hold

$$U|_{L=k=E=1} = \ln(L^* - 1) + \frac{1}{1+\theta} \ln\left(\frac{1+\theta}{L^* - \theta - 2} + 1\right) > \ln(L^*) = U|_{L=0,\ k=E=1}$$

Under the conditions (11)-(13), the dynamical system (9)-(10) can be written as follows

$$\begin{aligned} k_{t+1} &= \left[\frac{k_t^\alpha \cdot (L^* - L_t \cdot (2+\theta))}{(L^* - \theta - 2) \cdot \left(1 + \eta - \eta \left(L_t k_t^\alpha\right)^\beta\right)}\right]^{\frac{1}{1-\alpha}} \\ L_{t+1} &= L_t \cdot k_t^\alpha \cdot \left(\frac{(L^* - \theta - 2) \cdot \left(1 + \eta - \eta \left(L_t k_t^\alpha\right)^\beta\right)}{k_t^\alpha (L^* - L_t \cdot (2+\theta))}\right)^{\frac{1}{1-\alpha}} \end{aligned} \tag{14}$$

Notice that $L_{t+1} \cdot k_{t+1} = L_t \cdot k_t^\alpha$.

The Jacobian matrix, evaluated at the normalized fixed point, is

$$J = \begin{pmatrix} \frac{\alpha(1+\beta\eta)}{1-\alpha} & \frac{\beta\eta}{1-\alpha} - \frac{2+\theta}{(1-\alpha)(L^*-\theta-2))} \\ -\frac{\alpha(\alpha+\beta\eta)}{1-\alpha} & \frac{L^*}{(1-\alpha)(L^*-\theta-2)} - \frac{\alpha+\beta\eta}{1-\alpha} \end{pmatrix}$$

with

$$Det(J) = \frac{\alpha \cdot L^*}{(1-\alpha) \cdot (L^* - \theta - 2)} \tag{15}$$

$$Tr(J) = \frac{L^*}{(1-\alpha) \cdot (L^* - \theta - 2)} - \beta\eta \tag{16}$$

Figure 1 indicates, for each subset of the plane $(Tr(J),\ Det(J))$, the corresponding stability regime. Let us consider, in the plane $(Tr(J),\ Det(J))$, the half-line $\Delta \equiv (Tr(J)|_{\beta=0}, Det(J)|_{\beta=0})$ parametrized by $\frac{L^*}{(L^*-\theta-2)} \in (1, +\infty)$ having positive slope lower than 1. For $\frac{L^*}{(L^*-\theta-2)} \to 1$, Δ approaches the segment AC at the point $(\frac{1}{1-\alpha}, \frac{\alpha}{1-\alpha})$, in the positive orthant of the plane $(Tr(J)$, $Det(J))$. Starting from any point (Tr^*, Det^*) belonging to Δ, and increasing the parameter β, we move along the horizontal line Ω, where $Det(J) = Det^*$, towards the left. So, for β high enough, $Tr(J)$ becomes negative.

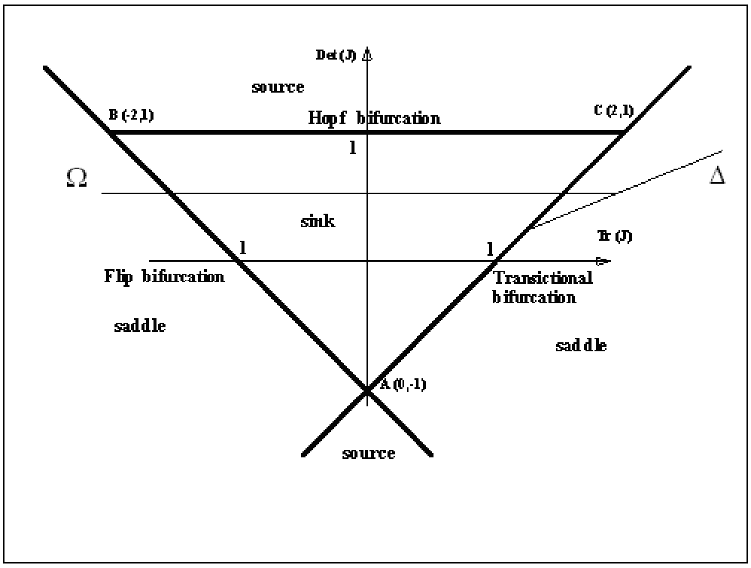

Figure 1.

Notice that, when Ω intersects AC, that is $Det(J) - Tr(j) + 1 = 0$, then a *transcritical bifurcation* occurs. It is easy to check that the bifurcation value of β is

$$\beta = \beta_{tr} \equiv \frac{2+\theta}{\eta \cdot (L^* - \theta - 2)}$$

When Ω intersects AB, that is $Det(J)+Tr(J)+1 = 0$, then a *flip bifurcation* occurs and the bifurcation value of β is given by

$$\beta = \beta_{fl} \equiv \frac{2L^* + (2+\theta)(\alpha - 1)}{\eta \cdot (1-\alpha) \cdot (L^* - 2 - \theta)}$$

Furthermore, note that when $\frac{1-\alpha}{\alpha} > 1$, the fixed point is a sink for $\beta \in (\beta_{tr}, \beta_{fl})$ and a saddle for $\beta \in (0, \beta_{tr}) \cup (\beta_{fl}, +\infty)$; while, when $\frac{1-\alpha}{\alpha} < 1$, the fixed point is a source for $\beta \in (\beta_{tr}, \beta_{fl})$ and a saddle for $\beta \in (0, \beta_{tr}) \cup (\beta_{fl}, +\infty)$.

Finally, a Hopf bifurcation occurs when Ω intersects the segment BC, that is, when $Det(J) = 1$ and $Tr(J) \in (-2, 2)$.

In our model, productive capital k_t represents a state variable; the economy starts from a given initial value of k_t, k_0, and then k_t evolves according to equations (14). Differently from k_t, the variable L_t is a "jumping" variable in that it is the representative individual's labor input, which is chosen taking into account of the average labour input in the economy, the expected environmental quality and the accumulated productive capital. So, when the normalized fixed point is a saddle under dynamics (14), if k_0 is near enough to 1, then there exists an unique initial value of L_t, L_0, such that the orbit passing through (k_0, L_0) approaches the fixed point. When the fixed point is a sink, given the initial value k_0, then there exists a continuum of initial values L_0 such that the orbit passing through (k_0, L_0) approaches the fixed point; consequently, the orbit the economy will follow is "indeterminate" in that it depends on the choice of the initial value L_0. The above results show how indeterminacy depends on the negative impact on environment generated by the production process of the consumption good. In particular, we have seen that indeterminacy occurs only when the relevance of labour input L_t (with respect to productive capital k_t) in the production function is high enough (i.e. if $\frac{1-\alpha}{\alpha} > 1$) and if the environmental impact is not "too low" or "too high", that is if $\beta \in (\beta_{tr}, \beta_{fl})$.

In Figure 2 the results of a numerical exercise are represented according to which a negative correlation exists between the level of production (economic activity) and individuals' wellbeing (measured by the utility function evaluated at an attracting fixed point). Parameters are chosen as follows: $\overline{E} = 3$, $L^* = 5$, $\beta = 2$, $\theta = 0.1$, $P = \frac{1}{\alpha(1-\alpha)A^2}$. The exercise shows that if η (the parameter measuring the negative impact of production on the environmental good) increases (in the interval $(0.1, 0.4)$), the production and consumption levels increase too; however, the increase in consumption of the produced good is not associated with an increase of wellbeing. This result implies that environmental degradation can be an engine of undesirable economic growth.

Figure 3 shows the evolution of the aggregated production y along a growth orbit starting near a repulsive fixed point where $L = 0.173$ and approaching the normalized fixed point (where $L = 1$), which is a sink; Figure 4 represents the evolution of wellbeing along such orbit. Parameters are chosen as follows: $\overline{E} = 7, L^* = 7$, $\alpha = 0.21$, $\beta = 2.46$, $\eta = 0.41$, $\theta = 0.3$, $P = 1.04$. Observe that individuals' wellbeing would be higher at the fixed point where the labour input is lower; however, such fixed point is repulsive under our dynamics.

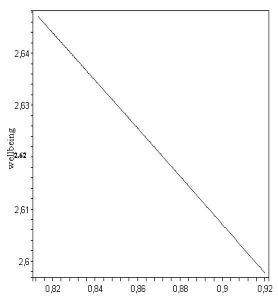

Figure 2.

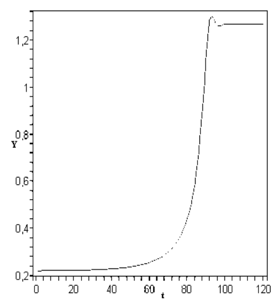

Figure 3.

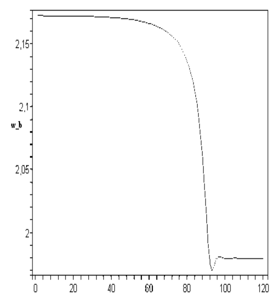

Figure 4.

These results are conform to analogous results obtained in the literature on undesirable economic growth by Bartolini and Bonatti (2002, 2003) and Antoci et al. (2005, 2007) in continuous time models. According to such results, environmental degradation may be an engine of economic growth in that it leads individuals to work more to have the possibility to increase their consumption of private goods. However, the increase in consumption doesn't compensate the reduction of wellbeing due to environmental degradation. Individuals' wellbeing would be higher by reducing labour input and consumption; however, no individual is incentivated to work and consume less if also the others don't do the same. Therefore, in this context, the increase of the production level is a consequence of a coordination failure.

4 Bifurcations and chaos

The dynamical system (14) may exhibit complex dynamics as the following numerical simulations show. We set $\alpha = 0.1$, $\eta = 0.41$, $L^* = 7$, $\theta = 0.2$. and use β as bifurcation parameter; when β increases, the normalized fixed point $(1, 1)$ loses its stability becoming a saddle and a period 2_cycle appears via a *supercritical flip bifurcation* (*period doubling bifurcation*). Subsequent increases of β lead to further flip bifurcations according to which cycles of periods $4, 8, ..., 2^n$ arise until the rise of a strange attractor (*period-doubling route to chaos*).

Figure 5 shows a period 2_cycle obtained (ceteris paribus) for $\beta = 6.79$ and Figure 6 represents a period 4_cycle obtained for $\beta = 8$.

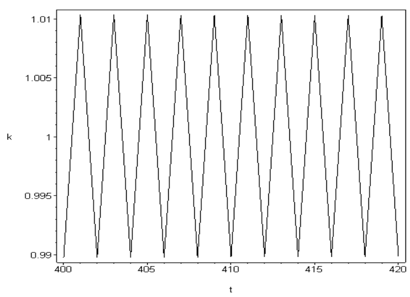

Figure 5.

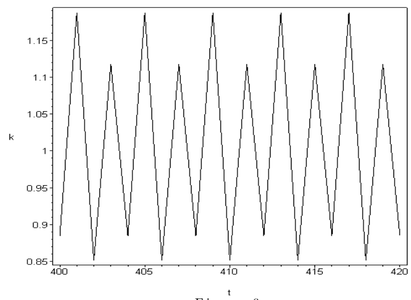

Figure 6.

Finally, by assuming $\beta = 8.22$ and $\beta = 9.13$, the chaotic attractors showed in Figures 7 and 8 arise, respectively. Figure 9 shows an enlargement of the region in the rectangle of Figure 8. Figure 10 shows the evolution of Lyapunov exponents obtained varying the value of β.

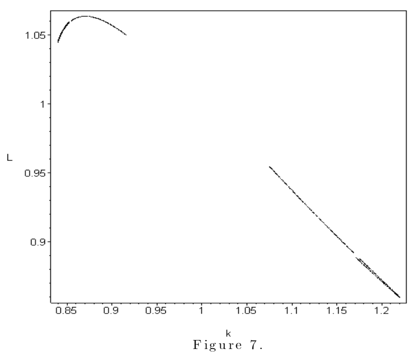

Figure 7.

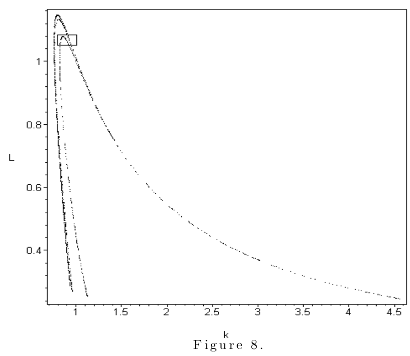

Figure 8.

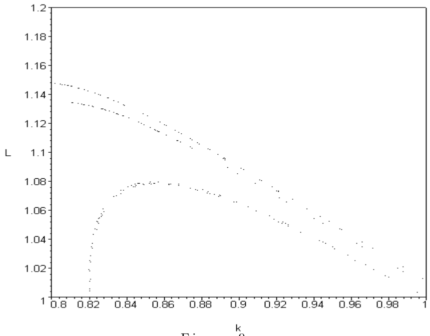

Figure 9.

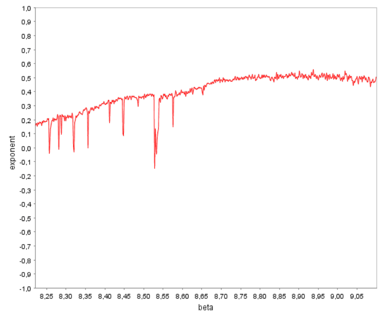

Figure 10.

Figures 11 and 12 represent the bifurcation diagrams, with respect to the variables L and k, obtained assuming $\alpha = 0.13$, $\eta = 0.341$, $L^* = 9$, $\theta = 0.012$, showing a sequence of supercritical flip bifurcations followed by chaotic behavior.

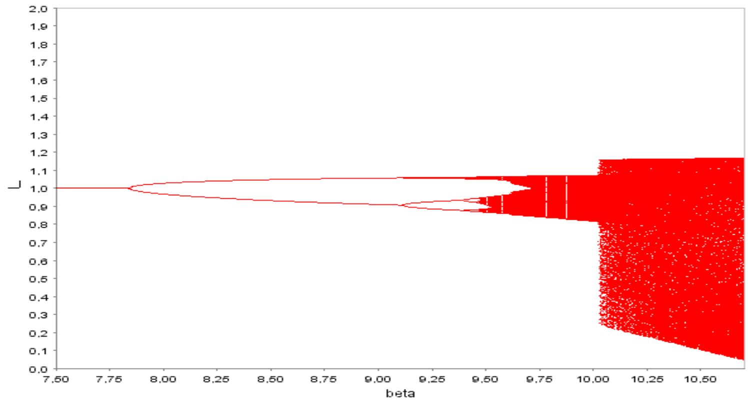

Figure 11.

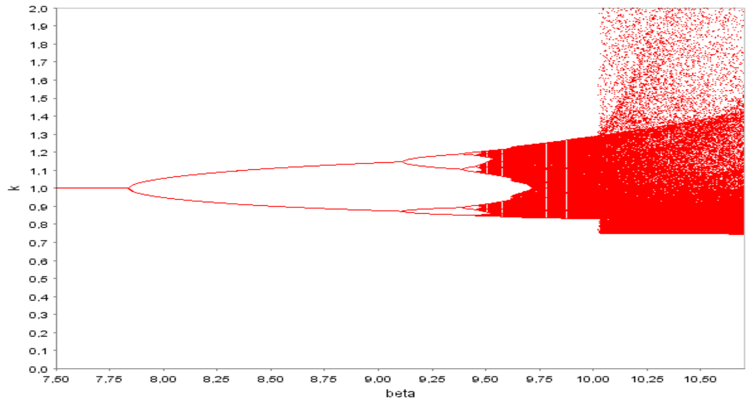

Figure 12.

In this model, complex dynamics can also occur via Hopf bifurcations. In fact, letting $\alpha = 0.41$, $\eta = 0.41$, $L^* = 7$, $\theta = 0.2$ and varying β in the interval

[1, 2.5] we first obtain an attracting limit cycle (Figures 13) which breaks in several attracting islands giving rise to complex dynamics (Figures 14-15).

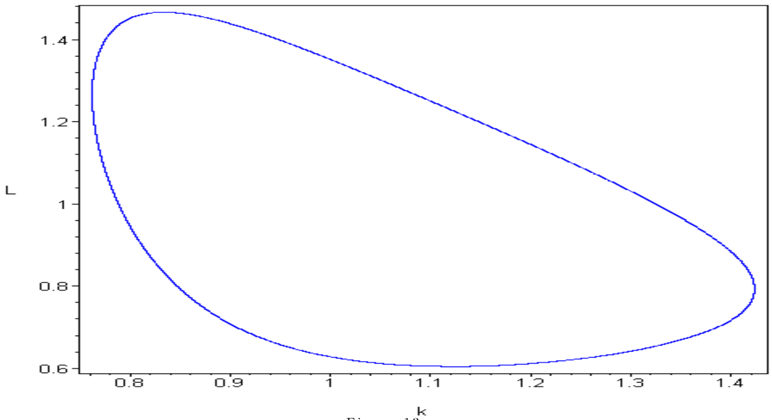

Figure 13.

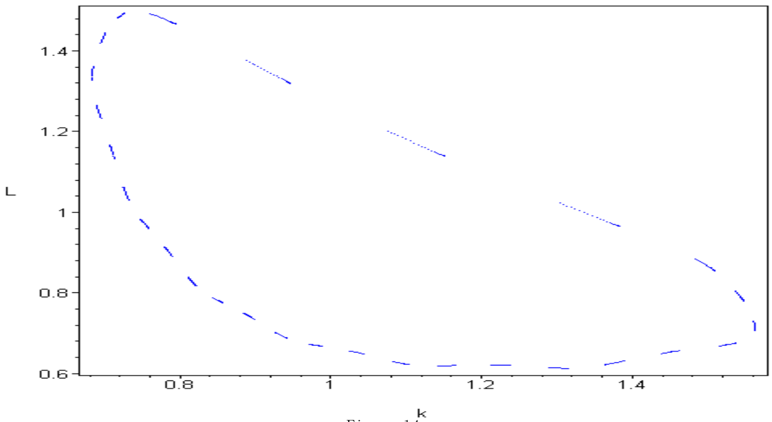

Figure 14.

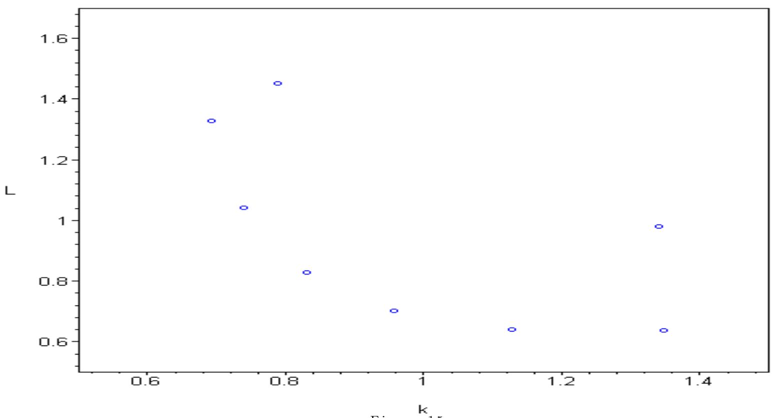

Figure 15.

Figure 16 shows the bifurcation diagram, with respect to the variable L, related to these cases.

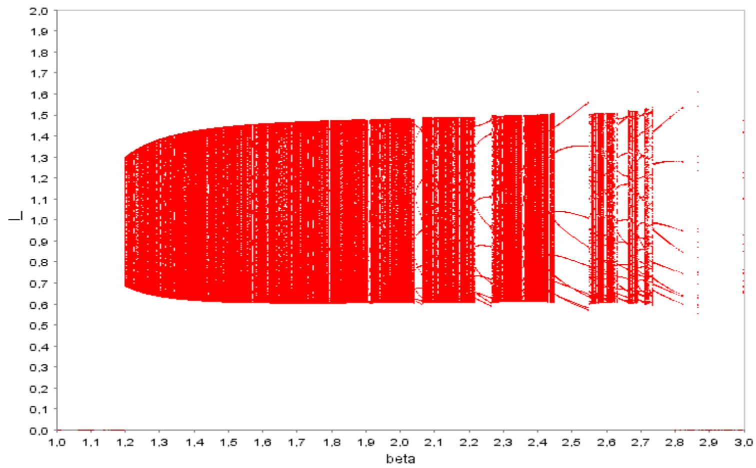

Figure 16.

5 Conclusions

Our work has highlighted a mechanism according to which environmental degradation may lead to complex dynamic behavior in an overlapping generation model described by a two-dimensional discrete dynamical system. We have shown that, ceteris paribus, an increase in the environmental impact of economic activity may lead to chaotic behavior. Differently from the mainstream literature concerning overlapping generation models, indeterminacy and chaotic dynamics don't occur in a context in which there are positive externalities in the production process but in a context where there are negative externalities generated by the production process.

Furthermore, we have shown that an increase of environmental degradation may rise economic activity via individuals' self-protection choices, which generate an increase of labour input and of consumption level. However, this rise in economic activity is undesirable in that a negative correlation may be observed between individuals' wellbeing and economic activity level.

The general prediction of the model is that the higher the environmental impact of the production process in the economy, the higher the economy's consumption level will be. An exogenous increase of η may generate an increase of the aggregate product and consumption in the economy; economic growth is fueled by the increase of "work motivation" of the economic agents, as a consequence of the gradual deterioration of the environmental resource, which induces agents to alter their consumption patterns, concentrating more and more on the consumption of private and expensive goods, rather than on the consumption of free access environmental goods.

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