Hierarchical Fleet Mix Problems with risk-aversion: a CVaR approach.

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Abstract

In this paper a two-stage multiperiod stochastic hierarchical workforce problem is studied from both a risk neutral and a risk averse approach. In the considered hierarchical models workforce units can be substituted by higher qualified ones; external workforce can also be hired to cover unfulfilled jobs and to avoid penalties. Demand for jobs is assumed to be stochastic. In the light of including agent risk aversion, the CVaR measure has been used following the lines of Rockafellar and Uryasev. The results of an extensive computational test are provided in order to validate the models.

Key words: Hierarchical fleet mix, Stochastic programming, CVaR.
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1 Introduction

The classical fleet size problem can be summarized as follows: how many vehicles should a firm have in its transport fleet to meet a fluctuating work load? And which should be the optimal fleet mix in the case different kinds of vehicles can be employed?

The study of this kind of problems is an important topic of management science because of their practical applications (see [2, 9, 10, 20]). As an example, emergency medical services involve decisions about the optimal fleet mix since ambulance services can be divided into categories according to the urgency of the requests, the need of a doctor on board, the equipment installed in the vehicle. Obviously, fleet mix problems are not related to vehicles only, since in general they involve units (that is to say single workforce elements such as workers, machines, trucks, ambulances, and so on) which have to fulfill jobs requests.

Such a kind of problems are inherently based on demand uncertainty. The decisions on the optimal fleet mix have to be taken before the jobs requests are revealed. From a firm point of view, a two stage process arises: first of all the fleet mix has to be chosen on the basis of a forecasted demand, then it can be adjusted taking into account of the actual requests. It is worth noticing that a good first stage decision is crucial for the firm from both an economical and a managerial point of view, and that different approaches for managing demand uncertainty may produce different first stage solutions.

Two different peculiarities can be considered. The first one is the possibility to hire external units to cover demand peaks; this leads to the presence of both internal and external units to be employed in order to satisfy the jobs requests, which yields the need to determine the optimal number of internal units to own and of external units to hire. A second critical market factor is the capability to fulfill the jobs in a requested amount of time. In particular, contractual constraints could fix penalties in the case the jobs are not fulfilled in time; this leads to the need to manage the peaks at a reasonable cost (for combined fleet size and vehicle routing problems see, for instance, Dullaert et al. [7]).

Furthermore, fleet mix problems are said to be *hierarchical* when the fleet is heterogeneous and the relative units can be grouped in hierarchical sets according to different capabilities or peculiarities (see for all [2, 10]); for example, an equipped ambulance with a medicine doctor on board can fulfill any request, an equipped ambulance with no doctor on board has a more limited use, an unequipped ambulance with no doctor on board can be used only to move not suffering patients. Hierarchical problems arise in scheduling of health care staff, job shop, maintenance crew and so on. In the literature the main research topic has been the shift scheduling of hierarchical workforce. In the paper by Narasimhan [14], for example, a single shift scheduling of hierarchical workforce has been considered in order to determine the best workforce composition which guarantees the full demand satisfaction and two days off for each employee every week. Burns and Koop [4] provide a modular approach for solving workforce scheduling problems characterized by multiple shifts and single category of employees. In all these models daily requests have to be completely fulfilled. On the contrary, in [5] Cambini and Riccardi propose a hierarchical fleet mix problem where demand violations are admitted and external units can be hired only to fulfill less qualified jobs.

It is worth noticing that demand for services is usually assumed in the literature to be deterministic, that is to say that it is fixed on the basis of a suitable demand estimation.

The aim of this paper is to present fleet mix models which consider in an unified framework all of the following crucial aspects: demand uncertainty, demand violation, hierarchical fleet structure, external units for all the hierarchical levels, risk aversion. The proposed models, thanks to the various considered fleet mix peculiarities, can be a useful strategic management tool aimed to determine an optimal long term strategy.

In particular, the service demand is assumed to be stochastic, depending on a random factor θ. This means that in a first stage, when demand is unknown, decisions on the optimal internal fleet have to be taken on the basis of a suitable estimation of future demand. In a second stage, the possibility of hiring external units for all the considered hierarchical levels can compensate the units shortage in case of demand peaks. Considering an heterogeneous fleet both in the case of internal and external units is a generalized version of the model proposed by Cambini and Riccardi [5]. Also quality of service constraints (QOS) are included: for each type of request k a minimum number of fulfilled jobs can be required. In other words, the firm may want to guarantee a certain service level in order to preserve an efficient corporate image. In this light, demand violations are admitted and a penalty cost function is included when the total demand is not completely fulfilled. The objective of these models is to minimize the expected total costs (including penalties) satisfying the quality of service constraints. Two different model formulations are proposed in this paper: a risk neutral approach, minimizing the expected total cost, and a risk aversion approach using a CVaR measure in the sense of Rockafellar and Uryasev [15, 16]. Both these formulations are two-stage multi-period stochastic programming models and their deterministic equivalent formulations are deduced with a scenarios approach. According to the features of the problem, the models belong to nonlinear integer programming and suitable equivalent linearized versions are provided in order to point out how they can be computationally managed.

In Section 2 a first model formulation, based on a risk neutral approach, is introduced. In this model a risk neutral agent aims to minimize the expected value of its cost function. The linearization of the model and its deterministic equivalent formulation are provided in Section 3. Neverthless, operators are usually risk averse in real applications, so that a more precise cost estimation, even if higher, is preferred to the expected cost estimation. For this reason, in Section 4 a Conditional Value-at-Risk (CVaR) approach is proposed and a suitable linearized version of the model is obtained too. All these models have been computationally tested by comparing them with two expected value formulations (EV and EV0) and the results are presented in Section 5. The test has been carried out by generating 24 different instances according to different QOS levels, different internal, external and penalty costs, high and low demand fluctuations.

2 Statement of the problem

The aim of the considered model is to determine an optimal hierarchical workforce in which a higher qualified unit can substitute for a lower qualified one but not vice versa. Both internal and external units can be hired to fulfill customers demand. All internal units are full working units, while external units can be employed for a single job. Internal and external units are classified into k levels from the less to the most qualified one. The service demand is assumed to be stochastic, depending on a random factor θ.

Also quality of service constraints are included: for each type of request k a minimum number of fulfilled jobs can be required, in order to guarantee a certain quality of service level and to preserve an efficient corporate image. The objective is to minimize total costs (including penalties) satisfying the quality of service constraints.

In order to provide a formal definition of the model, various notations have to be introduced. In particular, as regards to the internal units, the following parameters and variables will be used (recall that θ represents the stochastic rumor):

- $j = 1, \dots J$, is the number of different specialization levels of internal units;
- $k = 1, \dots J$, is the number of different jobs for each level;
- $n = 1, \dots, N$, is the number of working periods;

- $a^L = (a_j^L) \in \mathbb{Z}^J$, $a^L \geqq 0$, where a_j^L is the minimum number of internal units of level j;
- $a^U = (a_j^U) \in \mathbb{Z}^J$, $a^U \geqq a^L \geqq 0$, where a_j^U is the maximum number of internal units of level j;
- $x = (x_j) \in \mathbb{Z}^J$, $a^L \leqq x \leqq a^U$, where x_j is the variable representing the number of internal units of type j;
- $b = (b_j) \in \mathbb{Z}^J$, $b > 0$, where b_j is the number of jobs that a single internal unit of level j can fulfill in a working period.
- $Y(\theta) = [y_{jk}^n(\theta)] \in \mathbb{Z}^{N \times J \times J}$, where $y_{jk}^n(\theta)$ is the variable representing the number of internal units of level j, employed to fulfill stochastic requests of type k in period n;

In the light of the hierarchical structure of the model, level j units will be employed to fulfill requests of type $k \leq j$.

For the external units we need of the followings:

- $G = (g_j^n) \in \mathbb{Z}^{N \times J}$, $G \geqq 0$, where h_{nj}^L is the minimum number of external units of type j at the n-th working period;
- $H = (h_j^n) \in \mathbb{Z}^{N \times J}$, $H \leqq G$, where h_{nj}^U is the maximum number of external units of type j at the n-th working period;
- $Z(\theta) = [z_{jk}^n(\theta)] \in \mathbb{Z}^{N \times J \times J}$,$Z(\theta) \geqq 0$, where $z_{jk}^n(\theta)$ is the variable representing the number of external units of level j requested at the n-th working period to execute the job of type k depending on the random factor θ.

The stochastic requests are described by the following parameters:

- $M(\theta) = [m_k^n(\theta)] \in \mathbb{Z}^{N \times J}$, $M(\theta) \geqq 0$, where $m_k^n(\theta)$ is the maximum number of jobs of type k at the n-th working period depending on the random factor θ;
- $D(\theta) = [d_k^n(\theta)] \in \mathbb{Z}^{N \times J}$, $0 \leqq D(\theta) \leqq M(\theta)$, where $d_k^n(\theta)$ is the minimum number of jobs of type k to be fulfilled at the n-th working period depending on the random factor θ;

while the various costs are represented by:

- $c_x = (c_{x_j}) \in \mathbb{R}^J$, $c_x > 0$, where c_{x_j} is the cost of a single internal unit of type j in each period n;
- $C_z = (c_{z_j^n}) \in \mathbb{R}^{N \times J}$, $c_z > 0$, where $c_{z_j^n}$ is the cost of a single job done by an external unit of level j at the n-th working period;
- $C_w = (c_{w_k^n}) \in \mathbb{R}^{N \times J}$, $c_w \geqq 0$, where $c_{w_k^n}$ is the penalty cost of a single job of type k at the n-th working period which has not been fulfilled.

According to the hierarchical structure of the model, it is reasonable to suppose that higher qualified units have an higher marginal cost, that is $c_{x_J} \geq c_{x_{J-1}} \geq \cdots \geq c_{x_1}$ and $c_{z_J} \geq c_{z_{J-1}} \geq \cdots \geq c_{z_1}$.

By means of the introduced notations, the following formal definition of the problem is given.

Definition 2.1 Let P be the following stochastic problem:

(P)

$$\min_{x,Y,Z} \quad N\sum_{j=1}^{J} c_{x_j} x_j + \sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{j} c_{z_j^n} z_{jk}^n(\theta) + \sum_{n=1}^{N}\sum_{k=1}^{J} c_{w_k^n}\omega_k^n(\theta) \tag{1}$$

$$s.t. \quad a_j^L \le x_j \le a_j^U \qquad j = 1, \ldots, k \tag{2}$$

$$b_j x_j = \sum_{k=1}^{j} y_{jk}^n(\theta) \quad n = 1, \ldots, N, \; j = 1, \ldots, J \tag{3}$$

$$\sum_{j=k}^{J} \left(y_{jk}^n(\theta) + z_{jk}^n(\theta) \right) \ge d_k^n \quad n = 1, \ldots, N, \; k = 1, \ldots, J \tag{4}$$

$$g_j^n \le \sum_{k=1}^{j} z_{jk}^n(\theta) \le h_j^n \quad n = 1, \ldots, N, \; j = 1, \ldots, J \tag{5}$$

$$z_{jk}^n(\theta), \; y_{jk}^n(\theta) \ge 0 \quad n = 1, \ldots, N, \; j = 1, \ldots, J, \quad k = 1, \ldots, J \tag{6}$$

where $\forall \; n = 1, \ldots, N, \; k = 1, \ldots, J$:

$$R_k^n(\theta) \;=\; m_k^n(\theta) - \sum_{j=k}^{J} \left(y_{jk}^n(\theta) + z_{jk}^n(\theta) \right) \tag{7}$$

$$\omega_k^n(\theta) \;=\; \max\{0, \; R_k^n(\theta)\} \tag{8}$$

The objective function of problem P is the sum of three cost factors: the cost of internal units $N\sum_{j=1}^{J} c_{x_j} x_j$, employed for the whole considered period, the expected cost for external units $\sum_{n=1}^{N}\sum_{j=1}^{J}\sum_{k=1}^{j} c_{z_k^n} z_{jk}^n(\theta)$, proportional to the number of temporary units employed each working period n for each kind of requests j, and the expected penalty cost $\sum_{n=1}^{N}\sum_{k=1}^{J} c_{w_k^n}\omega_k^n(\theta)$. The penalty cost function is the cost related to the unfulfilled requests. When the requests are not easily predictable, for example in the case of maintenance units, some requests can not be fulfilled in the contractual time window. This happens, for instance, when it is too expensive to employ an additional internal unit to cover demand peaks. Internal units, in facts, are hired for the whole period under consideration so that there is a trade off between increasing units costs and paying a penalty. In this model we suppose that the penalty cost is proportional to the number of unfulfilled requests $\omega_k^n(\theta)$ (see constraints (8)). Notice that variables $R_k^n(\theta)$ in constraints (7) is unrestricted in sign and represent the excess of demand or supply in maintenance services for each working period n and each kind of request j, depending on its positive or negative value.

As regards to the feasible region, constraints (3) determine the distribution of internal units, for each working period n, among the different jobs they are able to execute. These constraints express the hierarchical structure of the workforce: for each level of qualification j the total number of internal employees are allocated to the different jobs of kind $k \leq j$. Constraints (4) represent the quality of service constraints: as higher is parameter d_k^n as more restrictive is the firm policy on demand satisfaction. The case $d_k^n = m_k^n$ reduces to the equilibrium condition in which demand has to be fully satisfied. Box constraints (2) and (5) include the case of lower and upper bound on labour supply.

3 Discretization of the problem

The problem described in the previous section is an integer multi-period two-stage stochastic model with fixed recourse. In facts, vector variable x represents the first stage decision variables, taking into account that internal units have to be determined at the beginning of the time period, while external units $z_{jk}^n(\theta)$ and internal units allocation between different jobs $y_{jk}^n(\theta)$ are determined after the realizations of the random events θ. If we consider the case in which an analytical representation of the density function is not available, but we have S demand scenarios, sampled from the density $p(\theta)$, the deterministic equivalent formulation of problem P can be rewritten in the following expanded model:

$(\tilde{P})$

$$\min_{x, Y^s, Z^s} \tilde{f}(x, Y^s, Z^s) \tag{9}$$

$$s.t. \quad a_j^L \leq x_j \leq a_j^U \qquad j = 1, \ldots, k \tag{10}$$

$$b_j x_j = \sum_{k=1}^{j} y_{jk}^{ns} \quad n = 1, \ldots, N, \; j = 1, \ldots, J \; s = 1, \ldots, S \tag{11}$$

$$\sum_{j=k}^{J} \left(y_{jk}^{ns} + z_{jk}^{ns} \right) \geq d_k^{ns} \quad n = 1, \ldots, N, \; k = 1, \ldots, J \; s = 1, \ldots, S \tag{12}$$

$$g_j^n \leq \sum_{k=1}^{j} z_{jk}^n(\theta) \leq h_j^n \quad n = 1, \ldots, N, \; j = 1, \ldots, J \tag{13}$$

$$R_k^{ns} = m_k^{ns} - \sum_{j=k}^{J} \left(y_{jk}^{ns} + z_{jk}^{ns} \right) \quad n = 1, \ldots, N, \; k = 1, \ldots, J \; s = 1, \ldots, S \tag{14}$$

$$\omega_k^{ns} = \max\{0, \; R_k^{ns}\} \quad n = 1, \ldots, N, \; k = 1, \ldots J, \; s = 1, \ldots, S \tag{15}$$

$$z_{jk}^{ns}, \; y_{jk}^{ns} \geq 0 \quad n = 1, \ldots, N, \; j = 1, \ldots, J, \quad k = 1, \ldots, J \; s = 1, \ldots, S \tag{16}$$

where

$$\tilde{f}(x, Y^s, Z^s) = N \sum_{j=1}^{J} c_{x_j} x_j + \sum_{n=1}^{N} \sum_{j=1}^{J} \sum_{k=1}^{j} c_{z_j^n} \sum_{s=1}^{S} p^s z_{jk}^{ns} + \sum_{n=1}^{N} \sum_{k=1}^{J} c_{w_k^n} \sum_{s=1}^{S} p^s \omega_k^{ns} \tag{17}$$

and $Y^s = (y_{jk}^{ns}), Z^s = (z_{jk}^{ns})$.

In addition constraints (15) are nonlinear, so that well known solution algorithms for linear programming can not be directly used without an equivalent linear formulation. In this light, since variables ω_k^{ns} are minimized in the objective function, constraints (15) can be linearized by substituting them with the followings:

$$\omega_k^{ns} \geq R_k^{ns},\ \omega_k^{ns} \geq 0, \quad n = 1, \ldots, N,\ k = 1, \ldots, J\ s = 1, \ldots, S \tag{18}$$

In this light, the linearized version $\tilde{P}$ of problem P is suitable for computational purpose and, in particular, it will be used for obtaining first stage optimal solutions. Clearly, the discretized first stage problem is a large dimension linear problem which needs of suitable softwares (like AMPL+CPLEX) in order to be computationally solved.

4 Risk-Aversion: a CVaR approach

In the previous sections we referred to a risk neutral agent who aims to minimize the expected value of its cost function. In real applications, usually, operators are risk averse, so that they prefer a more precise cost estimation, even if higher, than the expected cost estimation. In order to include risk aversion in our optimization model we consider a Conditional Value-at-Risk (CVaR) approach in the sense of Rockafellar and Uryasev [15, 16]. CVaR represents the expected value of loss, conditional on being on the bad tail of the distribution. In other words it is the expected loss exceeding the corresponding Value-at-Risk (VaR). This risk measure overcomes the limitations of VaR, since it is a coherent risk measure (among others, it verifies the subadditivity property) and it is a convex function. Following the lines of Rockafellar and Uryasev [15], the CVaR optimization problem can be modeled as follows.

Definition 4.1 Let P_{CVaR} be the following stochastic problem:

$$P_{CVaR} : \left\{ \begin{array}{c} \min f_{CVaR}\left(\xi, x, Y(\theta), Z(\theta)\right) \\ (x, Y(\theta), Z(\theta))\ \text{ verify } (2) - (6),\ \xi \in \Re \end{array} \right.$$

where

$$f_{CVaR}\left(\xi, x, Y(\theta), Z(\theta)\right) = \xi + \frac{1}{(1-\alpha)} \int \max\{0,\ f(x, Y(\theta), Z(\theta)) - \xi\} p(\theta) d\theta$$

and $f(x, Y(\theta), Z(\theta))$ is defined as in Problem P, $p(\theta)$ is the probability density function of θ.

If we consider the case in which an analytical representation of the density function is not available, but we have S demand scenarios, sampled from the density $p(\theta)$, we can approximately calculate function $f_{CVaR}\left(\xi, x, Y(\theta), Z(\theta)\right)$ as follows:

$$\tilde{f}_{CVaR}\left(\xi, x, Y^s, Z^s, v^s\right) = \xi + \frac{1}{(1-\alpha)} \sum_{s=1}^{S} v^s p(s) \tag{19}$$

where

$$v^s = \max\{0,\ \tilde{f}(x, Y^s, Z^s) - \xi\} \quad s = 1, \ldots, S \tag{20}$$

and $\tilde{f}(x, Y^s, Z^s)$ is defined in (17). Clearly, constraints (2)-(6) can be discretized by means of constraints (10)-(16).

In order to concretely solve the discretized version of problem P_{CVaR} with the use of standard linear programming tools, constraints (15) and (20) need to be linearized. Since variables v^s are minimized in the objective function, constraints (20) can be linearized by substituting them with the followings:

$$v^s \geq \tilde{f}(x, Y^s, Z^s) - \xi,\ v^s \geq 0, \quad s = 1, \ldots, S \tag{21}$$

On the other hand, constraints (15) can be linearized by introducing the binary variables $\delta_k^{ns} \in \{0, 1\}$, $n = 1, \ldots, N$, $k = 1, \ldots, J$, $s = 1 \ldots S$ and a value $\mathbf{Big} >> 0$. The resulting linear discretized problem is:

$$\tilde{P}_{CVaR} : \begin{cases} \min \tilde{f}_{CVaR}(\xi, x, Y^s, Z^s, v^s) \\ (\xi, x, Y^s, Z^s, v^s, \delta^s) \in \tilde{D}_{CVaR} \end{cases}$$

where:

$$\tilde{D}_{CVaR} = \left\{ \begin{array}{c} \text{for all } n = 1, \ldots, N,\ j = 1, \ldots, J,\ k = 1, \ldots, J,\ s = 1 \ldots S : \\ \\ v^s + \xi \geq \tilde{f}(x, Y^s, Z^s) \\ \\ -\mathbf{Big} \cdot \delta_k^{ns} \leq R_k^{ns} \leq \mathbf{Big}(1 - \delta_k^{ns}) \\ \\ \omega_k^{ns} \leq R_k^{ns} + \mathbf{Big} \cdot \delta_k^{ns} \\ \\ \omega_k^{ns} \leq \mathbf{Big}(1 - \delta_k^{ns}) \\ \\ v^s \geqq 0,\ \delta_k^{ns} \in \{0, 1\} \\ \\ \text{constraints (10)-(14), (16), (18) of } \tilde{P} \text{ hold} \end{array} \right\}$$

5 Computational results

Models $\tilde{P}$ and $\tilde{P}_{CVaR}$ have been implemented in order to test the goodness of the stochastic approach with respect to the Expected Value solution (EV). Different demand distributions have been considered in both first and second stages: uniform, normal and beta distribution. The body of the computational test, and in particular the data generation and the solution procedures for the second stage problems, has been implemented in MatLab 2010a. The first stage decision variables are obtained by solving the linearized version of problem P with AMPL+CPLEX v.12.

In particular, the first step optimal solution has been tested generating 5000 out of sample scenarios. Different problem instances have been considered varying cost parameters, quality of service levels, in sample scenarios. For the sake of convenience three hierarchical levels are assumed. The parameters of the problem a^L,a^U, b, g, h, c_x, C_z and C_w have been generated by using the "rand()" MatLab function. Specifically speaking, the cost parameters have been generated according to a hierarchical cost structure, that is to say that the components of c_x and c_w are in increasing order, while the components of the parameter b are in decreasing order. Stochastic matrix parameter M has been generated according to three different distributions: uniform, normal and beta. The distribution generation has been carried out using the Halton Sequences MatLab Code and the components in the various rows of the matrix parameter M are in decreasing order with respect to the column. The matrix parameter D has been generated from M taking into account different levels of quality of service. For the sake of clearness, the results of the computational test are described and discussed in two different subsections. In the first one, the results of a single instance test are given and described in details in order to clarify their meanings. In the second subsection the overall results of the extensive computational test are given and the validity of the proposed models is pointed out.

5.1 A single instance computational results

In order to clarify the meanings of the various computational results, in this subsection a single instance test is described and discussed in details. In particular, the case $k = 3$, $n = 20$ is considered and the model consistency is investigated with respect to two deterministic case (expected value case and adjusted expected value case). Some statistic measures have been calculated: average errors, absolute average errors, errors standard deviation, Expected Value of Perfect Information (EVPI), Value of Stochastic Solution (VSS) etc. As regards to the generated values, for all j, k, n, we have $c_{x_j} \in [20, 100]$, $c_{w_k^n} \in [10, 100]$, $c_{z_j^n} \in [50, 120]$, $a_j^L \in [0, 10]$, $a_j^U \in [1000, 3000]$, $g_j^n = 0$, $h_j^n \in [1000, 2000]$ and $b_j \in [1, 4]$. Quality of service levels have been fixed equal to $[0.7, 0.8, 0.9]$. This means that for the lower service demand the firm wants to fulfill at least the 70% of the total requests for each time period, for the second level of requests the 80% of requests has to be guaranteed and for the most qualified demand for services the 90% of requests has to be covered. Demand for service stochastic parameter M has been simulated starting from a mean, for each level, respectively equal to $\mu = [1000, 500, 300]$ and a standard deviation equal to $\sigma = [500, 250, 150]$ respectively. Different numbers of in sample scenarios have been considered, in the case shown below we referred to 500 in sample scenarios. A sort of peak effect has been considered varying mean values in some critical days (for example peaks can be reached on Monday and Friday). For this reasons the Two-Stage stochastic model (TS) and CVaR model have been compared with two different deterministic models: pure expected model (EV0) and an adjusted expected model (EV) taking into account the peak effect. Some statistic measures have been calculated: VSS, average errors, absolute average errors, errors standard deviation, VaR etc. The main results are presented in the following Tables 1 and 2 and in Figures 1-4.

Statistics/Models	EV0	EV	TS	CVaR − 95%
FValStage1	2′079′960	2′522′124	3′044′600	3′298′400
Mean FValStage2	3′337′700	3′115′200	3′045′000	3′099′400
MinFVal Stage2	2′286′252	2′528′096	2′602′911	2′863′080
MaxFVal Stage2	4′921′256	4′239′431	4′101′478	3′879′064
Standard Error	362′350	238′930	210′240	136′970
VaR − 95%	3′972′045	3′504′502	3′426′968	3′372′445
EVPI	-	-	30′748	-
VSS − EV0	-	-	293′380	-
VSS − EV	-	-	37′705	-

Table 1: Models optimal values comparison: normal distribution

Table 1 collects the results related to the considered instance in the case of normal distrubution. The same instance has been solved with EV0, EV, Two-Stage stochastic model and CVaR model finding the first stage optimal solution, then 5000 out of sample scenarios have been generated and the second stage optimal solutions are calculated. It can be easily observed that the optimal first stage value for problems EV0 and EV significantly differs from the second stage one while in the stochastic models the two solutions are almost the same. Also the Value at Risk comparison at level $\alpha = 0.05$ shows that the stochastic model is more stable around the first stage optimal solution and the expected total cost is lower. CVaR model with $\alpha = 0.95$ ensures the lowest VaR value. In order to clarify this behaviour, in Figure 1, 2, 3 and 4 the distribution of second stage optimal solutions for the four models are presented in the case of normal demand distribution. It can be easily verified that the EV0 distribution is more flat and variable, moreover the right tail is heavy. This means that the risk of higher cost is greater in EV0 and EV models than in the stochastic one. The CVaR distribution shows a different behaviour. The distribution is very tight with high peakedness and the right tail is flat with low volatility.

Similar results are obtained in the case of uniform and beta distributions.

In Table 2 the results concerning errors measures are presented. Model EV and EV0 can not be efficiently used to determine first stage optimal solutions since the errors in cost prevision are respectively higher than 20% and 60% as average values. The two stage stochastic model (TS) can forecast quite well the exact second stage solution, while the CVaR model slightly overestimates the second stage optimal solution. This is the price of risk aversion in the sense that a more prudential perspective is able to face demand fluctuations by reducing forecast errors. Notice also that both stochastic models (TS and CVaR) significantly reduce the maximum errors oscillations with respect to the deterministic ones, in particular CVaR model seems to be the better response for a risk averse agent who wants to minimize the probability to incur in higher cost levels than the ones predicted in the first stage solution.

Errors/Models	EV0	EV	TS	CVaR – 95%
Min Errors	0.09918	0.002367	−0.14508	−0.13198
Max Errors	1.366	0.6809	0.34713	0.15283
Mean Errors	0.60469	0.21979	0.00012	−0.0603
Standard Error	0.1742	0.09474	0.06905	0.03574
Min AbsErrors	0.0992	0.0024	0	0
Max AbsErrors	1.366	0.6809	0.34713	0.15283
Mean AbsErrors	0.60469	0.21979	0.05478	0.0527
Standard AbsError	0.17421	0.0947	0.04203	0.03304

Table 2: Model EV, EV0, Two-Stage, CVaR: second stage errors measures

5.2 Results of the extensive computational tests

The extensive computational test includes 24 different instances generated varying internal unit costs, penalty costs, quality of service constraints and demand variability. For each instance, three different demand distributions have been used (Uniform, Normal and Beta). Internal units costs and penalty costs have been tested low and high, three different quality of service levels have been considered (low, medium, high) and scenarios have been generated with high and low volatility for each distribution and with (Stag 1) or without (Stag 0) seasonality effects. Different values of in sample scenarios have been considered from 200 to 500 scenarios. In the CVaR model different α values have been also tested. For each instance 5000 out of sample scenarios have been generated and the optimal first stage solution has been compared with the out of sample results. Let us describe more in detail the data used in the simulation tests.

Quality of Service Levels (QoS):

- low (L): $[0.3; 0.4; 0.6]$
- medium (M): $[0.5; 0.6; 0.8]$
- high (H): $[0.7; 0.8; 0.9]$

Internal units daily costs: random values with hierarchical structure in the following intervals

- low: $[20; 100]$
- high: $[120; 240]$

Penalty costs: random values in the following intervals (min and max values for each level)

- low: $[10\ 30; 30\ 50; 50\ 100]$
- high: $[100\ 150; 150\ 200; 200\ 250]$

All these possible combinations have been investigated (see Table 3). For each instance 10 different test problems have been generated in order to avoid outliers values and the means of the outcomes are given in Tables 4, 5 and 6 as the results of the computational test.

Instances	**InternalCosts**	**ExternalCosts**	**PenaltyCosts**
Prob 1	High	Low	Low
Prob 2	Low	High	Low
Prob 3	Low	High	High
Prob 4	High	Low	High

Table 3: Instances description

The obtained outcomes seem to confirm the goodness of the stochastic models. In particular both the stochastic models show a better behaviour with respect to the deterministic ones in correspondence to high penalty costs and high quality of service levels. This can be easily explained taking into account that the shortage of internal units to cover the peaks (first stage optimal solutions) leads to employ more external units at higher costs in order to avoid penalties (see for instance the case of $P3$ where the cost structure produces high errors in the case of deterministic models). The high accuracy of the first stage solution in the Two-Stage (TS) model is stable also in presence of different second stage demand realizations. On one hand, the CVaR models (both for the two different α values) overestimate the total cost of the second stage, on the other hand the maximum error (that is a total cost higher than the estimated one in the fist stage) is always the lowest. This means that a risk averse agent can predict with more accuracy the effective cost he will face in the second stage and it will occur in an higher total cost in very few situations. Moreover, in case of high demand volatility (seasonality effects, demand peaks denoted as Stag 1) and with asymmetric distributions the CVaR model performs better than the TS one. As more the firm is risk averse as bigger the value α has to be taken in order to reduce cost variability. In the case of low penalties, low external costs, low quality of service constraints and low demand variability the error interval is tighter and it seems reasonable to use a deterministic model saving computational effort. The cases of high volatility tend to enlarge the second stage interval of variation for the optimal value in each considered instance except for the CVaR models. Finally, the use of a high number of in sample scenarios reduces second stage mean errors, specially in the case of high volatility, but has a small effect on the optimal solution when exceeding 500 scenarios. As a consequence, there is no incentive in furthermore increasing in sample scenarios taking into account the corresponding rise in computational times.

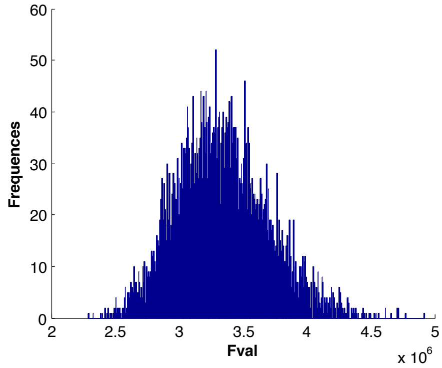

Figure 1: FVAL EV0 model out of sample distribution

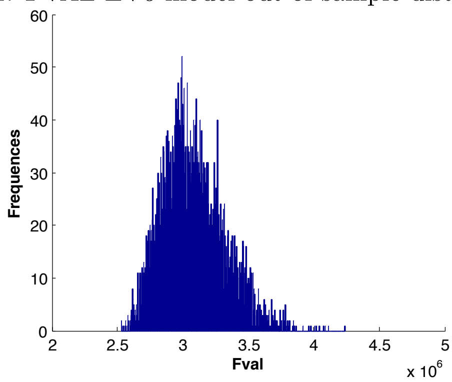

Figure 2: FVAL EV model out of sample distribution

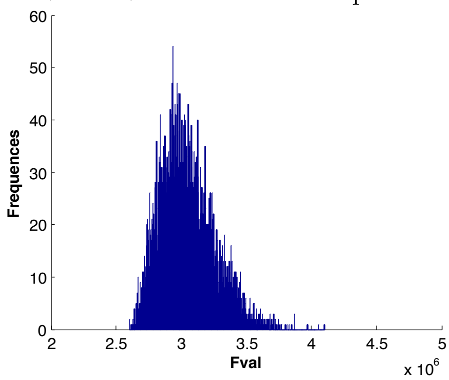

Figure 3: FVAL Two-Stage stochastic model out of sample distribution

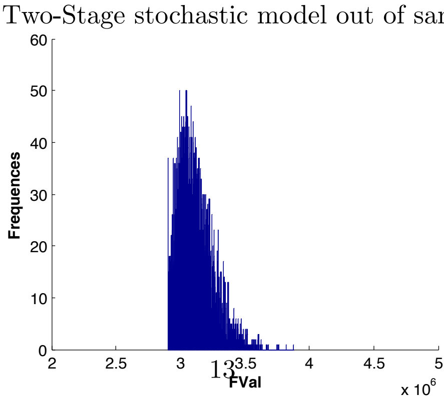

Figure 4: FVAL CVaR model out of sample distribution

P QOS	Model	Stag 0 Min Err	Stag 0 Max Err	Stag 0 Mean Err	Stag 0 Std Err	Stag 0 MAbs Err	Stag 0 StdAbs Err	Stag 1 Min Err	Stag 1 Max Err	Stag 1 Mean Err	Stag 1 Std Err	Stag 1 MAbs Err	Stag 1 StdAbs Err
P1 L	**TS**	-0.4957	0.5093	-0.0005	0.1434	0.1150	0.0857	-0.4510	0.4766	-0.0006	0.1344	0.1077	0.0804
	EV	-0.4560	0.5612	0.0425	0.1455	0.1215	0.0920	-0.4009	0.5491	0.0563	0.1380	0.1205	0.0912
	EV0	-0.3943	0.5849	0.0825	0.1406	0.1329	0.0986	-0.3312	0.5803	0.1051	0.1334	0.1403	0.1012
	CVaR (0.95)	-0.5719	0.2064	-0.1901	0.1113	0.1942	0.1038	-0.5083	0.2034	-0.1647	0.1035	0.1700	0.0945
	CVaR (0.975)	-0.5757	0.1857	-0.2023	0.1089	0.2051	0.1036	-0.5102	0.1781	-0.1784	0.1001	0.1817	0.0939
P1 M	**TS**	-0.5085	0.5500	-0.0006	0.1549	0.1243	0.0923	-0.4284	0.4942	-0.0006	0.1337	0.1072	0.0800
	EV	-0.4354	0.6703	0.0885	0.1625	0.1480	0.1123	-0.3617	0.6746	0.1193	0.1507	0.1555	0.1149
	EV0	-0.3527	0.7330	0.1515	0.1598	0.1818	0.1278	-0.2683	0.7466	0.1938	0.1481	0.2088	0.1301
	CVaR (0.95)	-0.5679	0.2358	-0.1890	0.1177	0.1950	0.1074	-0.4469	0.2009	-0.1493	0.0941	0.1546	0.0852
	CVaR (0.975)	-0.5803	0.1934	-0.2158	0.1132	0.2185	0.1079	-0.4477	0.1735	-0.1628	0.0903	0.1660	0.0845
P1 H	**TS**	-0.5212	0.5797	-0.0006	0.1636	0.1317	0.0970	-0.4157	0.5038	-0.0006	0.1340	0.1074	0.0801
	EV	-0.4311	0.7094	0.0971	0.1704	0.1575	0.1190	-0.3249	0.7679	0.1686	0.1601	0.1908	0.1339
	EV0	-0.3146	0.8745	0.2207	0.1788	0.2395	0.1556	-0.2059	0.9054	0.2858	0.1635	0.2920	0.1544
	CVaR (0.95)	-0.5716	0.2644	-0.1870	0.1246	0.1952	0.1114	-0.4038	0.2094	-0.1332	0.0889	0.1398	0.0784
	CVaR (0.975)	-0.5839	0.2207	-0.2142	0.1199	0.2184	0.1124	-0.4014	0.1911	-0.1409	0.0859	0.1467	0.0772
P2 L	**TS**	-0.3399	0.5126	-0.0006	0.1299	0.1045	0.0771	-0.3076	0.4491	-0.0001	0.1112	0.0891	0.0664
	EV	-0.2085	1.0485	0.3422	0.1903	0.3453	0.1847	-0.0927	1.0404	0.3679	0.1657	0.3702	0.1623
	EV0	-0.0196	1.4654	0.6186	0.2221	0.6200	0.2201	0.0196	1.4698	0.6384	0.2106	0.6389	0.2097
	CVaR (0.95)	-0.3726	0.2136	-0.1579	0.0907	0.1621	0.0829	-0.2796	0.1607	-0.1105	0.0644	0.1137	0.0585
	CVaR (0.975)	-0.3805	0.1828	-0.1756	0.0872	0.1778	0.0828	-0.2807	0.1349	-0.1219	0.0606	0.1235	0.0573
P2 M	**TS**	-0.3309	0.5694	-0.0005	0.1380	0.1112	0.0818	-0.2855	0.4519	-0.0004	0.1072	0.0860	0.0641
	EV	-0.2235	1.2538	0.3637	0.2199	0.3683	0.2121	-0.0791	1.1876	0.4234	0.1828	0.4248	0.1806
	EV0	-0.0108	1.7815	0.7061	0.2669	0.7067	0.2658	0.0330	1.7192	0.7329	0.2439	0.7330	0.2437
	CVaR (0.95)	-0.3540	0.2289	-0.1592	0.0918	0.1639	0.0833	-0.2312	0.1542	-0.0965	0.0566	0.0997	0.0509
	CVaR (0.975)	-0.3648	0.1914	-0.1799	0.0877	0.1821	0.0831	-0.2329	0.1253	-0.1087	0.0526	0.1101	0.0496
P2 H	**TS**	-0.3256	0.6582	-0.0006	0.1512	0.1218	0.0896	-0.2511	0.4417	-0.0003	0.1013	0.0812	0.0605
	EV	-0.2358	1.3688	0.3448	0.2451	0.3549	0.2304	-0.0485	1.4898	0.5361	0.2222	0.5367	0.2213
	EV0	-0.0023	2.3624	0.8778	0.3595	0.8780	0.3590	0.0489	2.3274	0.9584	0.3283	0.9584	0.3283
	CVaR (0.95)	-0.3228	0.2380	-0.1535	0.0878	0.1581	0.0791	-0.1900	0.1406	-0.0846	0.0491	0.0874	0.0438
	CVaR (0.975)	-0.3287	0.1936	-0.1715	0.0815	0.1736	0.0771	-0.1883	0.1129	-0.0936	0.0445	0.0948	0.0418
P3 L	**TS**	-0.2918	0.6964	-0.0005	0.1544	0.1244	0.0914	-0.2163	0.4185	-0.0007	0.0931	0.0746	0.0557
	EV	-0.1514	1.5168	0.4315	0.2569	0.4337	0.2531	-0.0221	2.2036	0.8128	0.3216	0.8129	0.3212
	EV0	0.0533	3.2832	1.3423	0.4885	1.3423	0.4885	0.1005	3.4555	1.4652	0.4821	1.4652	0.4821
	CVaR (0.95)	-0.2649	0.2322	-0.1288	0.0753	0.1337	0.0665	-0.1655	0.1359	-0.0759	0.0440	0.0786	0.0389
	CVaR (0.975)	-0.2724	0.1938	-0.1445	0.0704	0.1469	0.0655	-0.1627	0.1053	-0.0832	0.0391	0.0844	0.0366
P3 M	**TS**	-0.2919	0.6962	-0.0005	0.1544	0.1245	0.0914	-0.2163	0.4185	-0.0007	0.0931	0.0746	0.0557
	EV	-0.1514	1.509	0.4307	0.2562	0.4329	0.2524	-0.0221	2.2036	0.8128	0.3216	0.8129	0.3212
	EV0	0.0533	3.2814	1.342	0.4882	1.342	0.4882	0.1005	3.4547	1.4649	0.4819	1.4649	0.4819
	CVaR (0.95)	-0.2669	0.2289	-0.1313	0.0751	0.1357	0.0668	-0.1653	0.1361	-0.0758	0.0440	0.0785	0.0389
	CVaR (0.975)	-0.2711	0.1962	-0.1427	0.0706	0.1458	0.0650	-0.1619	0.1065	-0.0823	0.0391	0.0835	0.0365
P3 H	**TS**	-0.2907	0.6972	-0.0004	0.1541	0.1242	0.0912	-0.2163	0.4185	-0.0007	0.0931	0.0746	0.0557
	EV	-0.1514	1.5188	0.4320	0.2573	0.4342	0.2534	-0.0221	2.2077	0.8139	0.3223	0.8141	0.3219
	EV0	0.0533	3.3255	1.352	0.4950	1.352	0.4950	0.1005	3.4847	1.4731	0.4862	1.4731	0.4862
	CVaR (0.95)	-0.2618	0.2371	-0.1250	0.0755	0.1313	0.0656	-0.1655	0.1359	-0.0760	0.0440	0.0787	0.0389
	CVaR (0.975)	-0.2690	0.1990	-0.1406	0.0706	0.1442	0.0645	-0.1596	0.1100	-0.0795	0.0393	0.0810	0.0363
P4 L	**TS**	-0.3942	0.5449	-0.0005	0.1401	0.1125	0.0835	-0.3539	0.4963	-0.0005	0.1239	0.0993	0.0741
	EV	-0.2700	1.0105	0.2851	0.1915	0.3009	0.1748	-0.1011	0.8978	0.2981	0.1449	0.3038	0.1383
	EV0	-0.0736	1.2532	0.4718	0.1981	0.4782	0.1907	-0.0415	1.2931	0.5160	0.1927	0.5205	0.1871
	CVaR (0.95)	-0.4218	0.2448	-0.1608	0.1010	0.1673	0.0901	-0.3217	0.1949	-0.1143	0.0760	0.1204	0.0664
	CVaR (0.975)	-0.4264	0.2146	-0.1778	0.0973	0.1817	0.0902	-0.3225	0.1629	-0.1284	0.0713	0.1317	0.0655
P4 M	**TS**	-0.3942	0.5306	-0.0005	0.1389	0.1116	0.0827	-0.3574	0.4961	-0.0005	0.1249	0.1001	0.0747
	EV	-0.2700	0.9835	0.2808	0.1887	0.2970	0.1718	-0.1016	0.8953	0.2972	0.1445	0.3030	0.1378
	EV0	-0.0736	1.2278	0.4682	0.1957	0.4748	0.1881	-0.0421	1.2885	0.5147	0.1920	0.5192	0.1864
	CVaR (0.95)	-0.4239	0.2441	-0.1578	0.1021	0.1647	0.0904	-0.3193	0.1990	-0.1110	0.0762	0.1181	0.0658
	CVaR (0.975)	-0.4267	0.2247	-0.1685	0.0996	0.1745	0.0900	-0.3243	0.1551	-0.1329	0.0704	0.1353	0.0657
P4 H	**TS**	-0.3947	0.5325	-0.0005	0.1395	0.1120	0.0831	-0.3553	0.4945	-0.0005	0.1245	0.0998	0.0745
	EV	-0.2706	0.9872	0.2822	0.1897	0.2986	0.1727	-0.1016	0.8992	0.2983	0.1451	0.3041	0.1384
	EV0	-0.0736	1.2365	0.4709	0.1972	0.4773	0.1896	-0.0418	1.2983	0.5178	0.1934	0.5222	0.1879
	CVaR (0.95)	-0.4218	0.2488	-0.1557	0.1025	0.1633	0.0901	-0.3193	0.1902	-0.1159	0.0749	0.1211	0.0662
	CVaR (0.975)	-0.4244	0.2198	-0.1717	0.0985	0.1763	0.0903	-0.3194	0.1596	-0.1289	0.0704	0.1317	0.0651

Table 4: Uniform distribution - comparison between 24 different instances

P QOS	Model	Stag 0 Min Err	Stag 0 Max Err	Stag 0 Mean Err	Stag 0 Std Err	Stag 0 MAbs Err	Stag 0 StdAbs Err	Stag 1 Min Err	Stag 1 Max Err	Stag 1 Mean Err	Stag 1 Std Err	Stag 1 MAbs Err	Stag 1 StdAbs Err
	TS	-0.2343	0.2496	-0.0001	0.0622	0.0496	0.0376	-0.2087	0.2158	-0.0001	0.0598	0.0477	0.0359
	EV	-0.2235	0.2603	0.0108	0.0622	0.0503	0.0386	-0.1918	0.2297	0.0130	0.0596	0.0488	0.0374
P1	**EV0**	-0.1794	0.2957	0.0528	0.0611	0.0672	0.0470	-0.1551	0.2741	0.0560	0.0606	0.0688	0.0479
L	**CVaR (0.95)**	-0.3024	0.1307	-0.0923	0.0557	0.0947	0.0514	-0.2681	0.1125	-0.0811	0.0537	0.0844	0.0483
	CVaR (0.975)	-0.3111	0.1144	-0.1045	0.0548	0.1058	0.0522	-0.2758	0.0973	-0.0926	0.0527	0.0945	0.0493
	TS	-0.2491	0.3074	-0.0001	0.0706	0.0563	0.0425	-0.2140	0.2372	-0.0001	0.0625	0.0499	0.0376
	EV	-0.2282	0.3416	0.0266	0.0724	0.0609	0.0476	-0.1889	0.2827	0.0333	0.0659	0.0584	0.0454
P1	**EV0**	-0.1595	0.4014	0.0898	0.0706	0.0969	0.0617	-0.1339	0.3401	0.0935	0.0655	0.0989	0.0587
M	**CVaR (0.95)**	-0.3183	0.1691	-0.1014	0.0615	0.1045	0.0561	-0.2664	0.1215	-0.0826	0.0536	0.0860	0.0480
	CVaR (0.975)	-0.3363	0.1377	-0.1254	0.0598	0.1264	0.0575	-0.2748	0.1024	-0.0961	0.0521	0.0978	0.0489
	TS	-0.2702	0.3347	-0.0001	0.0779	0.0623	0.0467	-0.2186	0.2551	-0.0001	0.0651	0.0520	0.0392
	EV	-0.2485	0.3651	0.0252	0.0789	0.0659	0.0509	-0.1907	0.3242	0.0473	0.0716	0.0682	0.0526
P1	**EV0**	-0.1396	0.5059	0.1413	0.0830	0.1442	0.0785	-0.1063	0.4250	0.1412	0.0729	0.1430	0.0700
H	**CVaR (0.95)**	-0.3452	0.1788	-0.1138	0.0673	0.1168	0.0619	-0.2612	0.1262	-0.0846	0.0530	0.0878	0.0475
	CVaR (0.975)	-0.3601	0.1514	-0.1343	0.0657	0.1357	0.0629	-0.2690	0.1069	-0.0980	0.0514	0.0996	0.0482
	TS	-0.2458	0.3533	-0.0001	0.0773	0.0618	0.0464	-0.1884	0.2793	-0.0001	0.0647	0.0516	0.0389
	EV	-0.1823	0.5119	0.1143	0.0893	0.1217	0.0792	-0.0831	0.4598	0.1308	0.0758	0.1352	0.0707
P2	**EV0**	0.0163	0.8203	0.3505	0.1035	0.3508	0.1031	0.0378	0.7949	0.3684	0.1041	0.3686	0.1038
L	**CVaR (0.95)**	-0.3090	0.1855	-0.1108	0.0641	0.1138	0.0587	-0.2127	0.1365	-0.0802	0.0480	0.0833	0.0424
	CVaR (0.975)	-0.3231	0.1581	-0.1303	0.0625	0.1316	0.0597	-0.2203	0.1126	-0.0950	0.0456	0.0963	0.0426
	TS	-0.2545	0.3989	-0.0000	0.0852	0.0681	0.0512	-0.1761	0.3020	-0.0001	0.0660	0.0527	0.0399
	EV	-0.2017	0.5727	0.1122	0.1005	0.1242	0.0861	-0.0711	0.5063	0.1435	0.0802	0.1468	0.0763
P2	**EV0**	0.0211	0.9715	0.3945	0.1222	0.3947	0.1220	0.0460	0.8797	0.3993	0.1144	0.3993	0.1143
M	**CVaR (0.95)**	-0.3098	0.2039	-0.1185	0.0680	0.1219	0.0618	-0.1876	0.1415	-0.0789	0.0456	0.0818	0.0401
	CVaR (0.975)	-0.3231	0.1677	-0.1412	0.0652	0.1425	0.0623	-0.1944	0.1146	-0.0936	0.0427	0.0949	0.0399
	TS	-0.2598	0.4654	-0.0000	0.0952	0.0759	0.0575	-0.1597	0.3227	0.0000	0.0676	0.0538	0.0410
	EV	-0.2167	0.6182	0.0890	0.1084	0.1112	0.0862	-0.0479	0.5900	0.1711	0.0888	0.1729	0.0868
P2	**EV0**	0.0294	1.2662	0.4890	0.1598	0.4891	0.1597	0.0613	1.095	0.4761	0.1420	0.4761	0.1420
H	**CVaR (0.95)**	-0.2928	0.2162	-0.1208	0.0679	0.1244	0.0610	-0.1621	0.1479	-0.0759	0.0432	0.0790	0.0372
	CVaR (0.975)	-0.3018	0.1744	-0.1423	0.0634	0.1437	0.0602	-0.1710	0.1160	-0.0929	0.0399	0.0941	0.0371
	TS	-0.2423	0.5315	-0.0001	0.1011	0.0804	0.0612	-0.1450	0.3583	0.0007	0.0704	0.0556	0.0432
	EV	-0.1469	0.7277	0.1427	0.1124	0.1489	0.1042	-0.0211	0.8765	0.2724	0.1248	0.2728	0.1241
P3	**EV0**	0.0814	1.8398	0.7404	0.2306	0.7404	0.2306	0.1183	1.7239	0.7492	0.2180	0.7492	0.2180
L	**CVaR (0.95)**	-0.2574	0.2294	-0.1122	0.0615	0.1160	0.0539	-0.1489	0.1499	-0.0740	0.0407	0.0772	0.0343
	CVaR (0.975)	-0.2619	0.1866	-0.1290	0.0566	0.1307	0.0526	-0.1577	0.1169	-0.0897	0.0372	0.0910	0.0340
	TS	-0.2423	0.5315	-0.0001	0.1011	0.0804	0.0612	-0.1450	0.3583	0.0007	0.0704	0.0556	0.0432
	EV	-0.1469	0.7277	0.1427	0.1124	0.1489	0.1042	-0.0211	0.8765	0.2724	0.1248	0.2728	0.1241
P3	**EV0**	0.0814	1.8398	0.7404	0.2306	0.7404	0.2306	0.1183	1.7239	0.7492	0.2180	0.7492	0.2180
M	**CVaR (0.95)**	-0.2581	0.2285	-0.1129	0.0615	0.1166	0.0540	-0.1460	0.1537	-0.0709	0.0408	0.0747	0.0338
	CVaR (0.975)	-0.2623	0.1860	-0.1295	0.0566	0.1311	0.0526	-0.1577	0.1169	-0.0897	0.0372	0.0910	0.0340
	TS	-0.2422	0.5318	-0.0000	0.1011	0.0804	0.0613	-0.1450	0.3583	0.0007	0.0704	0.0556	0.0432
	EV	-0.1469	0.7286	0.1427	0.1124	0.1489	0.1043	-0.0211	0.8765	0.2725	0.1248	0.2728	0.1242
P3	**EV0**	0.0814	1.8573	0.7420	0.2322	0.7420	0.2322	0.1183	1.7251	0.7494	0.2182	0.7494	0.2182
H	**CVaR (0.95)**	-0.2537	0.2349	-0.1079	0.0617	0.1128	0.0532	-0.1489	0.1499	-0.0740	0.0407	0.0772	0.0343
	CVaR (0.975)	-0.2613	0.1870	-0.1286	0.0566	0.1303	0.0525	-0.1577	0.1169	-0.0897	0.0372	0.0910	0.0340
	TS	-0.2593	0.3551	-0.0001	0.0795	0.0636	0.0477	-0.2044	0.2926	-0.0001	0.0684	0.0546	0.0412
	EV	-0.2083	0.4754	0.0860	0.0890	0.1028	0.0724	-0.0699	0.3631	0.0914	0.0599	0.0974	0.0542
P4	**EV0**	-0.0196	0.6813	0.2586	0.0922	0.2607	0.0894	0.0007	0.6442	0.2621	0.0883	0.2639	0.0860
L	**CVaR (0.95)**	-0.3285	0.1901	-0.1156	0.0676	0.1188	0.0618	-0.2303	0.1415	-0.0849	0.0511	0.0881	0.0454
	CVaR (0.975)	-0.3463	0.1594	-0.1383	0.0659	0.1396	0.0632	-0.2378	0.1159	-0.1005	0.0487	0.1019	0.0456
	TS	-0.2591	0.3550	-0.0001	0.0794	0.0635	0.0477	-0.2043	0.2926	-0.0001	0.0684	0.0546	0.0411
	EV	-0.2083	0.4763	0.0862	0.0890	0.1029	0.0724	-0.0699	0.3631	0.0914	0.0599	0.0974	0.0542
P4	**EV0**	-0.0196	0.6821	0.2586	0.0922	0.2607	0.0895	0.0007	0.6442	0.2621	0.0883	0.2639	0.0859
M	**CVaR (0.95)**	-0.3290	0.1905	-0.1157	0.0675	0.1189	0.0618	-0.2303	0.1415	-0.0849	0.0511	0.0881	0.0454
	CVaR (0.975)	-0.3479	0.1604	-0.1385	0.0661	0.1398	0.0633	-0.2381	0.1154	-0.1008	0.0486	0.1023	0.0456
	TS	-0.2589	0.3582	-0.0001	0.0797	0.0638	0.0479	-0.2040	0.2928	-0.0001	0.0684	0.0546	0.0412
	EV	-0.2090	0.4806	0.0867	0.0896	0.1037	0.0730	-0.0699	0.3632	0.0914	0.0599	0.0974	0.0542
P4	**EV0**	-0.0196	0.6876	0.2597	0.0928	0.2617	0.0901	0.0007	0.6457	0.2625	0.0885	0.2643	0.0862
H	**CVaR (0.95)**	-0.3290	0.1927	-0.1161	0.0677	0.1192	0.0620	-0.2296	0.1411	-0.0850	0.0509	0.0881	0.0453
	CVaR (0.975)	-0.3476	0.1614	-0.1394	0.0662	0.1406	0.0634	-0.2279	0.1310	-0.0882	0.0494	0.0930	0.0433

Table 5: Normal distribution - comparison between 24 different instances

P QOS	Model	Stag 0 Min Err	Stag 0 Max Err	Stag 0 Mean Err	Stag 0 Std Err	Stag 0 MAbs Err	Stag 0 StdAbs Err	Stag 1 Min Err	Stag 1 Max Err	Stag 1 Mean Err	Stag 1 Std Err	Stag 1 MAbs Err	Stag 1 StdAbs Err
	TS	-0.4451	0.5632	-0.0003	0.1317	0.1052	0.0792	-0.4043	0.4806	-0.0003	0.1275	0.1020	0.0765
	EV	-0.4183	0.5913	0.0244	0.1319	0.1067	0.0824	-0.3613	0.5129	0.0318	0.1262	0.1048	0.0809
P1	**EV0**	-0.3661	0.6304	0.0665	0.1298	0.1170	0.0907	-0.3022	0.5630	0.0835	0.1257	0.1224	0.0930
L	**CVaR (0.95)**	-0.5390	0.2713	-0.1826	0.1058	0.1873	0.0974	-0.4886	0.2274	-0.1632	0.1034	0.1694	0.0930
	CVaR (0.975)	-0.5495	0.2370	-0.2037	0.1027	0.2062	0.0976	-0.4993	0.1859	-0.1889	0.0990	0.1918	0.0933
	TS	-0.4564	0.6740	-0.0003	0.1438	0.1148	0.0867	-0.4071	0.5415	-0.0004	0.1342	0.1073	0.0806
	EV	-0.4139	0.7526	0.0530	0.1483	0.1234	0.0986	-0.3443	0.6353	0.0733	0.1397	0.1254	0.0987
P1	**EV0**	-0.3330	0.8011	0.1113	0.1434	0.1457	0.1119	-0.2538	0.6959	0.1400	0.1352	0.1615	0.1134
M	**CVaR (0.95)**	-0.5444	0.3385	-0.1914	0.1119	0.1971	0.1015	-0.4612	0.2492	-0.1607	0.1009	0.1671	0.0899
	CVaR (0.975)	-0.5618	0.2819	-0.2246	0.1069	0.2270	0.1017	-0.4682	0.2107	-0.1815	0.0965	0.1849	0.0898
	TS	-0.4702	0.7379	-0.0004	0.1557	0.1243	0.0937	-0.4031	0.5839	-0.0004	0.1391	0.1112	0.0836
	EV	-0.4026	0.8769	0.0880	0.1649	0.1457	0.1177	-0.3394	0.7633	0.1126	0.1562	0.1525	0.1191
P1	**EV0**	-0.3008	0.9623	0.1662	0.1623	0.1889	0.1382	-0.2082	0.8652	0.2109	0.1512	0.2218	0.1381
H	**CVaR (0.95)**	-0.5476	0.3892	-0.1906	0.1202	0.1989	0.1058	-0.4322	0.2877	-0.1493	0.1008	0.1582	0.0866
	CVaR (0.975)	-0.5649	0.3320	-0.2234	0.1151	0.2273	0.1073	-0.4299	0.2631	-0.1587	0.0970	0.1659	0.0854
	TS	-0.3528	0.7195	-0.0004	0.1421	0.1133	0.0857	-0.3190	0.5830	-0.0003	0.1281	0.1023	0.0770
	EV	-0.2372	1.229	0.2899	0.1932	0.2961	0.1839	-0.1087	1.1075	0.3199	0.1731	0.3241	0.1676
P2	**EV0**	-0.0280	1.6919	0.5632	0.2272	0.5648	0.2250	0.0007	1.5453	0.5779	0.2178	0.5787	0.2166
L	**CVaR (0.95)**	-0.3995	0.3477	-0.1785	0.0991	0.1839	0.0887	-0.3244	0.2577	-0.1389	0.0822	0.1444	0.0721
	CVaR (0.975)	-0.4182	0.2951	-0.2077	0.0948	0.2101	0.0893	-0.3372	0.2035	-0.1671	0.0761	0.1691	0.0715
	TS	-0.3493	0.8121	-0.0004	0.1536	0.1224	0.0927	-0.3029	0.6187	-0.0002	0.1303	0.1040	0.0784
	EV	-0.2475	1.4124	0.2989	0.2169	0.3073	0.2049	-0.0930	1.2844	0.3700	0.1935	0.3727	0.1900
P2	**EV0**	-0.0206	2.0209	0.6353	0.2690	0.6361	0.2678	0.0117	1.8348	0.6618	0.2549	0.6620	0.2545
M	**CVaR (0.95)**	-0.3869	0.3737	-0.1824	0.1015	0.1885	0.0896	-0.2882	0.2598	-0.1357	0.0771	0.1407	0.0675
	CVaR (0.975)	-0.4000	0.3045	-0.2137	0.0943	0.2162	0.0885	-0.2994	0.2085	-0.1602	0.0712	0.1622	0.0665
	TS	-0.3473	0.9452	-0.0004	0.1701	0.1353	0.1030	-0.2799	0.6551	-0.0001	0.1326	0.1056	0.0801
	EV	-0.2524	1.5743	0.2792	0.2389	0.2942	0.2203	-0.0604	1.6086	0.4617	0.2339	0.4630	0.2322
P2	**EV0**	-0.0133	2.6108	0.7587	0.3432	0.7591	0.3426	0.0228	2.4115	0.8316	0.3305	0.8316	0.3305
H	**CVaR (0.95)**	-0.3669	0.3809	-0.1846	0.0996	0.1907	0.0872	-0.2538	0.2613	-0.1279	0.0717	0.1328	0.0619
	CVaR (0.975)	-0.3738	0.3108	-0.2108	0.0915	0.2135	0.0849	-0.2681	0.2038	-0.1537	0.0654	0.1555	0.0609
	TS	-0.3072	1.0153	-0.0003	0.1731	0.1379	0.1046	-0.2540	0.7063	-0.0001	0.1333	0.1058	0.0810
	EV	-0.1803	1.8196	0.3764	0.2586	0.3801	0.2530	-0.0361	2.2513	0.6672	0.3207	0.6677	0.3199
P3	**EV0**	0.0392	3.6308	1.1516	0.4709	1.1516	0.4709	0.0707	3.4174	1.2214	0.4667	1.2214	0.4667
L	**CVaR (0.95)**	-0.3141	0.3928	-0.1660	0.0907	0.1730	0.0767	-0.2212	0.2525	-0.1119	0.0647	0.1176	0.0540
	CVaR (0.975)	-0.3122	0.3100	-0.1840	0.0800	0.1871	0.0726	-0.2293	0.1835	-0.1356	0.0562	0.1373	0.0518
	TS	-0.3071	1.0155	-0.0003	0.1730	0.1379	0.1045	-0.2540	0.7063	-0.0001	0.1333	0.1058	0.0810
	EV	-0.1803	1.8213	0.3765	0.2586	0.3801	0.2531	-0.0361	2.2513	0.6672	0.3207	0.6677	0.3199
P3	**EV0**	0.0392	3.6355	1.1519	0.4713	1.1519	0.4713	0.0707	3.417	1.2214	0.4667	1.2214	0.4667
M	**CVaR (0.95)**	-0.3138	0.3937	-0.1654	0.0907	0.1727	0.0766	-0.2215	0.2526	-0.1118	0.0648	0.1179	0.0539
	CVaR (0.975)	-0.3094	0.3153	-0.1806	0.0803	0.1840	0.0723	-0.2283	0.1852	-0.1342	0.0563	0.1361	0.0517
	TS	-0.3066	1.0192	-0.0003	0.1732	0.1379	0.1048	-0.2539	0.7063	-0.0001	0.1333	0.1058	0.0810
	EV	-0.1803	1.8341	0.3777	0.2600	0.3814	0.2545	-0.0360	2.2617	0.6686	0.3220	0.6690	0.3212
P3	**EV0**	0.0392	3.6799	1.157	0.4762	1.157	0.4762	0.0707	3.4479	1.2259	0.4703	1.2259	0.4703
H	**CVaR (0.95)**	-0.3107	0.3960	-0.1628	0.0906	0.1705	0.0759	-0.2149	0.2634	-0.1041	0.0653	0.1120	0.0527
	CVaR (0.975)	-0.3078	0.3168	-0.1790	0.0802	0.1827	0.0720	-0.2274	0.1867	-0.1331	0.0564	0.1351	0.0516
	TS	-0.4044	0.7463	-0.0003	0.1500	0.1196	0.0906	-0.3636	0.6149	-0.0004	0.1394	0.1113	0.0838
	EV	-0.2626	1.1678	0.2418	0.1868	0.2627	0.1688	-0.1172	0.9318	0.2432	0.1495	0.2520	0.1401
P4	**EV0**	-0.0827	1.3715	0.3961	0.1909	0.4061	0.1814	-0.0537	1.2833	0.4267	0.1887	0.4337	0.1813
L	**CVaR (0.95)**	-0.4553	0.3879	-0.1781	0.1102	0.1862	0.0960	-0.3748	0.2754	-0.1524	0.0925	0.1590	0.0810
	CVaR (0.975)	-0.4707	0.3404	-0.2051	0.1061	0.2095	0.0973	-0.3825	0.2309	-0.1739	0.0872	0.1774	0.0803
	TS	-0.4043	0.7380	-0.0003	0.1495	0.1192	0.0902	-0.3637	0.6129	-0.0004	0.1392	0.1112	0.0837
	EV	-0.2626	1.1592	0.2416	0.1863	0.2624	0.1683	-0.1172	0.9301	0.2430	0.1493	0.2518	0.1399
P4	**EV0**	-0.0827	1.3669	0.3961	0.1905	0.4060	0.1811	-0.0537	1.2822	0.4267	0.1885	0.4336	0.1811
M	**CVaR (0.95)**	-0.4585	0.3828	-0.1802	0.1101	0.1879	0.0965	-0.3699	0.2892	-0.1439	0.0938	0.1529	0.0798
	CVaR (0.975)	-0.4703	0.3439	-0.2022	0.1066	0.2069	0.0975	-0.3863	0.2269	-0.1782	0.0870	0.1812	0.0808
	TS	-0.4044	0.7421	-0.0004	0.1500	0.1197	0.0905	-0.3625	0.6152	-0.0004	0.1393	0.1113	0.0837
	EV	-0.2629	1.167	0.2429	0.1874	0.2638	0.1693	-0.1172	0.9357	0.2440	0.1500	0.2527	0.1406
P4	**EV0**	-0.0827	1.3768	0.3979	0.1918	0.4077	0.1824	-0.0535	1.2921	0.4288	0.1898	0.4356	0.1825
H	**CVaR (0.95)**	-0.4535	0.3962	-0.1740	0.1111	0.1829	0.0958	-0.3650	0.2925	-0.1412	0.0934	0.1497	0.0794
	CVaR (0.975)	-0.4671	0.3529	-0.1986	0.1074	0.2038	0.0975	-0.3704	0.2533	-0.1599	0.0884	0.1646	0.0795

Table 6: Beta distribution - comparison between 24 different instances

6 Conclusion

A large class of two-stage multi-period stochastic hierarchical fleet mix problems, with various concrete applications, has been introduced and fully studied from a theoretical and a computational point of view. A risk aversion perspective has been also proposed by adopting a CVaR optimization model in the sense of Rockafellar and Uryasev [16]. The computational test proves the goodness of the stochastic models compared with two alternative deterministic versions. The risk aversion approach is able to reduce cost increase facing demand variability. Further developments could be in the direction of a more sophisticated scenarios representation (including demand correlation or scenario reduction techniques) and of a multi-stage stochastic model.

References

[1] Azmat, C.S., Hürliman, T. and M. Widmer (2004), "Mixed integer programming to schedule a single-shift workforce under annualized hours", *Annals of Operations Research*, vol.128, pp.199-215.

[2] Billionet A. (1999), "Integer programming to schedule a hierarchical workforce with variable demands", *European Journal of Operational Research*, vol.114, pp.105-114.

[3] Birge, J.R. and F. Louveaux (1997), "Introduction to stochastic programming", Heidelberg: Springer-Verlag.

[4] Burns and R.N. and G.J. Koop (1987), "A modular approach to optimal multiple-shift manpower scheduling", *Operations Research*, vol.35, pp.100-110.

[5] Cambini R. and R. Riccardi (2009), "Theoretical and algorithmic results for a class of hierarchical fleet mix problems", *European Journal of Operational Research*, vol.198, pp.741-747.

[6] Cambini R., Riccardi R. and Ü. Yüceer (2007), "An approach to discrete convexity and its use in an optimal fleet mix problem", *Generalized Convexity and Related Topics*, I.V. Konnov, D.T. Luc, A.M. Rubinov (eds.), Lecture Notes in Economics and Mathematical Systems, vol. 583, ISBN 3-540-37006-4, Springer, Berlin, pp.133-148.

[7] Dullaert, W., Janssens, G.K., Sörensen, K. and B. Vernimmen (2002), "New heuristics for the fleet size and mix vehicle routing problem with time windows", *The Journal of the Operational Research Society*, vol.53, n.11, pp.1232-1238.

[8] Etezadi, T. and J.E. Beasley . (1983), "Vehicle fleet composition", *The Journal of the Operational Research Society*, vol.34, pp.87-91.

[9] Gheysen F., Golden B. and A. Assad (1984), "The Fleet Size and Mix Vehicle Routing Problem", *Computers&Operations Research*, vol.11, pp.49-66.

[10] Hung R. (1994), "Single-shift off-day scheduling of a hierarchical workforce with variable demands", *European Journal of Operational Research*, vol.78, n.1, pp.49-57.

[11] Konno H., Thach P.T. and H. Tuy (1997), *Optimization on low rank nonconvex structures*, Nonconvex Optimization and Its Applications, vol.15, Kluwer Academic Publishers, Dordrecht.

[12] Liu, F.H. and S.Y. Shen (1999), "The fleet size and mix vehicle routing problem with time windows", *The Journal of the Operational Research Society*, vol.50, pp.721-732.

[13] Mole, R.H. (1975), "Dynamic optimization of vehicle fleet size", *Operational Research Quarterly*, vol.26, n.1, pp.25-34.

[14] Narasimhan R. (1996), "An algorithm for single shift scheduling of hierarchical workforce", *European Journal of Operational Research*, vol.96, pp.113-121.

[15] Rockafellar, R.T. and S. Uryasev (2000) "Optimization of conditional value-at-risk", *Journal of Risk*, vol.2, pp.2141.

[16] Rockafellar, R.T. and S. Uryasev (2002) "Conditional value-at-risk for general loss distributions", *Journal of Banking&Finance*, vol.26, pp.14431471.

[17] Sayarshad, H.R. and R. Tavakkoli-Moghaddam (2010), "Solving a multi periodic stochastic model of the railcar fleet sizing by two-stage optimization formulation". *Applied Mathematical Modelling*, vol.34, pp.11641174.

[18] Seçkiner S.U., Gökçen, H. and M. Kurt (2007), "An integer programming model for hierarchical workforce scheduling problem", *European Journal of Operational Research*, vol.183, pp.694-699.

[19] Tarantilis, C.D., Kiranoudis, C.T. and V.S. Vassiliadis (2003), "A list based threshold accepting metaheuristic for the heterogeneous fixed fleet vehicle routing problem", *The Journal of the OperationalResearch Society*, vol.54, pp.65-71.

[20] Wyatt J.K. (1961), "Optimal Fleet Size". *Operational Research Quarterly*, vol.12, pp.187-188.