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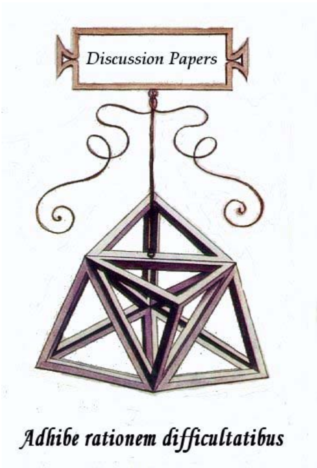

Gaetano Alfredo Minerva

Location and Horizontal Differentiation under Duopoly with Marshallian Externalities

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Indirizzi dell'Autore:
Dipartimento di scienze economiche, via Ridolfi 10, 56100 PISA
fax: (39 +) 050 598040

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Location and Horizontal Differentiation under Duopoly with Marshallian Externalities

Abstract

A classical differentiated duopoly model is cast in a two regions' framework. An entrant firm, which locates in the other region from the incumbent, comes in and competes with the rival, under markets' segmentation. In the first stage of the game, it has to choose product differentiation. The resulting equilibrium is compared under two different settings: competition in prices (Bertrand) and quantities (Cournot). It is shown that, under both modes of competition, the entrant maximizes product differentiation, producing a completely different good.

Afterwards, the Cournot model is extended by assuming that, when firms are located together, they benefit from Marshallian localization economies. First, the minimum cost reduction inducing agglomeration is computed. Second, the implications of a linear spillover function (linking product differentiation to marginal cost reduction) against a quadratic specification, with respect to location and product differentiation, are investigated.

Classificazione JEL: D43; F12; L13.
Keywords: Intra-industry trade; product differentiation; location; Marshallian externalities.

Contents

I. Introduction

This paper wants to add some insights to the existing literature on role played by product differentiation in the formation of clusters of firms. We focus on a duopolistic framework, where firms can differentiate their products as much as they want, because differentiation is costless, and consumers *ceteris paribus* love variety. First we establish that maximum differentiation and dispersion is the profit maximizing configuration. Then we show how this result is affected by the presence of a spillover leading to marginal cost reduction, and being dependent on product differentiation.

The framework of analysis hinges upon that of Belleflamme, Picard, and Thisse (2000), BPT hereafter, though the problem we want to investigate is somewhat different. In particular, from that model we take the idea of casting the Dixit (1979) duopolistic framework in a two regions context. We set up a game where the choice of product differentiation is endogenous. There is an incumbent firm and an entrant firm. The entrant, in the first stage of the game, chooses the product differentiation parameter, i.e. decides how much to differentiate its production from the one of the incumbent. Then it has to deal with the problem of selecting the region where to organize production (second stage), and finally both firms compete in prices or quantities (third stage).

We will assume that firms *use spatial separation to segment their markets*, i.e. they set a price specific to each region in which the product is sold. This is the same assumption of Brander and Krugman (1983), that focus on reciprocal dumping in a two regions economy where firms compete à la Cournot, and sell a homogeneous product. Our model features the emergence of dumping and trade in equilibrium under both modes of competition, independently of the level of transport costs, given that then entrant firm will maximize differentiation in equilibrium so to allow trade also under price competition.

Product selection in this paper is achieved by setting the parameter of the demand function for the differentiated varieties linking own price (or quantity) to rival's quantity (or price), so that it is closely related to cross elasticity of substitution. This is different from non-address approaches, as the usual version of Dixit and Stiglitz (1977), where product differentiation is defined in terms of the number of active firms, in the framework of a CES utility function defined over all the potential range of differentiated products.

As to address models à la Hotelling, where product differentiation consists in different locations in the product space (think to d'Aspremont, Gabszewicz, and Thisse (1979) quadratic transport costs framework), our paper retains the strategic aspect of product differentiation (the incentive to differentiate in order to relax price competition), while it lacks the demand effect (the incentive to locate at the market centre in order to capture as many customers as possible) given its non-spatial nature. This explains why firms will target maximum differentiation in the absence of any other incentive to lessen product differentiation. Moreover, as suggested by Harrington (1995), the parameter of product differentiation that we employ, more than being linked to a measure of distance on the segment, is the counterpart of transport costs in Hotelling. The higher transport costs, the poorer substitutes the varieties are, because the higher is the loss incurred by customers in buying a product different from the preferred one.

The two aspects of location and product differentiation have been addressed together in the literature in models à la Hotelling. Schmitt (1995) analyzes a model where two countries trade differentiated products subject to a barrier to trade t, representing either transport costs or a tariff. Preferences in each country are modelled as a segment of unit length, and each firm can adjust its specialization, along the segment, in addition to its location (region A or B). In his paper, if demand density is the same across regions, the case we actually consider in this work, then, depending on transport costs, the Nash equilibrium will be either imitation ($x_1^* = x_2^*$) or maximum differentiation ($x_1^* = 0$ and $x_2^* = 1$). Imitation is the profit maximizing choice when the barrier to trade between regions is sufficiently high as well as *the degree of substitution between products* (as captured by low transport costs on the Hotelling segment), and corresponds to a no trade pattern between the two regions. On the contrary, when the degree of substitution is relatively poor with respect to the barrier to trade between regions, firms maximally differentiate their products in equilibrium, and intra-industry trade consequently is established. Our results differ from Schmitt (1995) to the extent that we work out just the convenience of *increasing* the degree of substitution between products.

After having established that producing independent products is the profit maximizing choice, the second part of the paper shows how this result could be modified by assuming that the entrant benefits from marginal cost reduction if it locates in the same region of the incumbent. To our knowledge, this is the first attempt in the literature to build a model where profit maximizing firms compute costs and benefits of decreasing differentiation in order to exploit a cost reduction, due to localization economies, whose intensity depends on product differentiation itself.

The analysis shows how the equilibrium level of product differentiation depends on the autonomous marginal cost (i.e. the cost incurred when the externality is null, which occurs when products are independent). When cost decreases linearly in product differentiation, the choice will be either to maximize differentiation (when autonomous cost is low) or to produce a homogenous product in order to fully exploit the externality (when cost is high). When the spillover is quadratic, maximizing differentiation is still the profit maximizing choice for low levels of marginal autonomous cost, while it decreases smoothly as cost increases.

The organization of the paper is as follows. Section II. introduces the model. Section III. solves for equilibrium quantities in the Bertrand and Cournot games. Section IV. computes the profit maximizing product differentiation without spillover, while Section V. shows the effect of introducing them in the context of Cournot competition. Concluding remarks end the paper.

II. The model

Our economy is made up of two regions A and B. One firm, producing a certain good, is already established either in A or in B. Another firm wishes to entry in the market, selling an horizontally differentiated variety of the good. Thus, the actions the entrant must take are the following: *a)* to choose the level of product differentiation of its own production with respect to the good produced by the incumbent; *b)* to choose a region where to locate. As a last step, both firms non-cooperatively compete in prices or quantities for the home and foreign market.

Let us start from the consumption side of the model. Each region shows the same demand conditions, which are taken as given, that is firms' location decisions do not affect them. Since we deal with entry by a new firm in the market, we distinguish between preferences *before* entry takes place and *after* it does. When only one variety and the numeraire are available, preferences are of the following form:

$$U(q_0, q_1) = \alpha q_1 - \frac{\beta}{2} q_1^2 + q_0 \tag{1}$$

where marginal utility from good 1 is non-negative as long as $q_1 \leq \alpha/\beta$. This shows that our preference relation displays satiation in the consumption of the good, thus making it well-suited for partial equilibrium modelling.

Entry by the other firm modifies preferences into the following quadratic structure:

$$U(q_0, q_1, q_2, \delta) = \alpha(q_1 + q_2) - (\beta/2)(q_1^2 + q_2^2) - \delta q_1 q_2 + q_0 \tag{2}$$

The condition for the non-negativity of marginal utility from good 1 is:

$$q_1 \leq \frac{\alpha}{\beta} - \frac{\delta}{\beta} q_2$$

saying that as consumption of good 2 increases, the upper bound in the consumption of good 1 decreases.

Utility function (2) depends on the product differentiation variable δ. We assume that there is an incumbent in the market producing some good, and an entrant coming in and deciding, through the choice of δ, how much to differentiate its product either because of the physical characteristics of the products (e.g. through different design) or because of firms' ability to manipulate in some sense sheer consumers' perception of varieties (i.e. through advertising). In each region the representative consumer shows a symmetric preference relation over the two horizontally differentiated varieties of the monopolistic sector, separable and linear in the homogeneous good q_0, the numeraire good. α, β, and δ are all assumed to be positive[1], and in order to get a concave utility function we need $\delta^2 < \beta^2$. The convexity of the indifference curves with respect to the differentiated products q_1 and q_2 implies that consumers love variety[2]. Given that the quantity of the homogeneous good that can be consumed is the residual, from the initial fixed endowment $\bar{y}$, of what is spent on the two varieties, we get that consumer's problem is to maximize

$$\underset{q_1,q_2}{\text{Max}}\ U(q_1,q_2) = \alpha(q_1+q_2) - (\beta/2)(q_1^2+q_2^2) - \delta q_1 q_2 + \bar{y} - (q_1 p_1 + q_2 p_2) \tag{3}$$

The necessary (and sufficient) first order conditions imply that the demand for one of the two differentiated varieties $i = \{1,2\}$ is:

$$p_i = \alpha - \beta q_i - \delta q_j \tag{4}$$

with j the other variety.

After few manipulations, it can be expressed in terms of prices as:

$$q_i = \frac{\alpha}{\beta+\delta} - \frac{\beta}{(\beta+\delta)(\beta-\delta)} p_i + \frac{\delta}{(\beta+\delta)(\beta-\delta)} p_j \tag{5}$$

When $\delta = 0$ the two products belong to *independent sectors*, whose cross price elasticity of substitution is zero. When $\delta \to \beta$, the

[1] Note that the case $\delta < 0$ still retains an interesting economic interpretation in terms of complementary products, but in what follows we will deal just with substitutes, i.e. $\delta > 0$.

[2] The intensity of the love for variety depends on the magnitude of δ. For fixed prices and quantities, utility decreases as we increase δ.

two products are *perfect substitutes*, since quantities in the demand function (5) react hugely even to a slight difference in prices.

As far as the production side of the model is concerned, there are no fixed costs to start up production. Each firm faces just a constant marginal cost equal to c, in order to produce one unit of output.

Hence, profits accruing to firm i when located in region K may be represented as:

$$\Pi_{iK} = (p_{iK} - c)\left[\frac{\alpha}{\beta+\delta} - \frac{\beta}{(\beta+\delta)(\beta-\delta)}p_{iK} + \frac{\delta}{(\beta+\delta)(\beta-\delta)}p_{jK}\right] + \tag{6}$$

$$+(p_{iL} - c - t)[\frac{\alpha}{\beta+\delta} - \frac{\beta}{(\beta+\delta)(\beta-\delta)}p_{iL} + \frac{\delta}{(\beta+\delta)(\beta-\delta)}p_{jL}]$$

when firms maximize with respect to prices (Bertrand case), where the variables p_{iK}, p_{jK}, p_{iL}, p_{jL} are the prices charged by firm i or j in region K (home region for firm i) or L (foreign region for firm i) respectively. A similar expression holds if firms set quantities (Cournot case):

$$\Pi_{iK} = (\alpha - \beta q_{iK} - \delta q_{jK} - c)q_{iK} + (\alpha - \beta q_{iL} - \delta q_{jL} - c - t)q_{iL} \tag{7}$$

Two spatial configurations are possible, *agglomeration* of both firms in one region (e.g. in K) or *dispersion* (e.g. firm i located in K and j in L).

Profits can be normalized. *Multiplying* by the constant β in (6) simply amounts to scale equilibrium quantities, so that we can replace in the normalized profit function 1 to β, $\omega \equiv \delta/\beta$ to δ. The range of variation of the product differentiation parameter is $\omega \in [0, 1]$, with $\omega = 1$ the case of homogeneous varieties. The resulting demand corresponds to:

$$q_i = \frac{\alpha}{1+\omega} - \frac{1}{1-\omega^2}p_i + \frac{\omega}{1-\omega^2}p_j$$

For the profit function (7), expressed in terms of quantities, *dividing* by β we are allowed to substitute $\xi \equiv \alpha/\beta$ to α, 1 to β,

$\omega \equiv \delta/\beta =$ to δ, $C \equiv c/\beta$ to c and $T \equiv t/\beta$ to t. We get that inverse demand is equal to:

$$p_i = \xi - q_i - \omega q_j$$

Normalization makes it easier to derive analytical solutions, while to go back to the original variables it suffices to multiply by the appropriate scale factor.

III. The solution to the model

With a backward induction argument, we first compute equilibrium in the third stage of the game, where two equilibria are possible. One is a duopoly equilibrium with differentiated varieties, when firms non-cooperatively compete in prices or quantities. The other is monopoly pricing, occurring when equilibrium product differentiation is so low that export would be possible just accepting losses in the foreign market. In this case firms do not send their production abroad and strategic interaction ceases.

Then we go back to the second stage of the game, when the entrant chooses one region, locating either in the same place of the incumbent (agglomeration) or in the other region (dispersion).

Finally, product differentiation is chosen. As said, for low values of product differentiation intra-industry trade is not profitable, hence the entrant locating in the region not occupied by the incumbent can charge monopoly price.

Given the symmetric structure of the model, the entrant's equilibrium choice of the profit maximizing level d and of a location will be stable in the sense that the incumbent will not find convenient to modify it in any way. Profits accruing to both firms are equal, and so maximizing its own profits, a firm maximizes also its rival's.

As to existence and uniqueness, note that profit functions are concave with respect to their own control variable when firms share both markets. Hence, solving the first-order conditions for profit maximization yields the equilibrium prices. The monopoly case under autarchy is a standard one, and again existence is ensured.

III.A. Non-cooperative equilibrium

In the last stage of the game, if product differentiation is high enough and trade occurs, we have competition either in prices or in quantities: firms maximize their objective functions (profits) with respect to the strategic variable. The reaction curves are then solved simultaneously to derive the equilibrium configuration.

III.A.i. Equilibrium à la Bertrand

Let us start from *agglomeration*. Equilibrium prices are:

$$\begin{aligned} p_{iK} = p_{jK} &= \frac{\alpha(1-\omega)+c}{2-\omega} \equiv p_K^h \\ p_{iL} = p_{jL} &= \frac{\alpha(1-\omega)+c+t}{2-\omega} \equiv p_K^f \end{aligned} \tag{8}$$

Here p_K^h stands for home equilibrium price (i.e. prevailing in region K) when the two firms locate in region K, and p_K^f is the foreign equilibrium price (i.e. prevailing in region L) when the two firms are located in region K. Prices are decreasing in ω, i.e. are decreasing as varieties become more homogeneous. Equilibrium quantities are obtained substituting prices in the demand function (5):

$$\begin{aligned} q_{iK} = q_{jK} &= \frac{\alpha-c}{(2-\omega)(1+\omega)} \equiv q_K^h \\ q_{iL} = q_{jL} &= \frac{\alpha-c-t}{(2-\omega)(1+\omega)} \equiv q_K^f \end{aligned}$$

In the case of *dispersion*, assuming i is located in K and j is located in L, equilibrium quantities are:

$$p_{iK} = p_{jL} = \frac{2(\alpha + c) - \omega(\alpha - c - t) - \alpha\omega^2}{4 - \omega^2} \equiv p_S^h \tag{9}$$
$$p_{iL} = p_{jK} = \frac{2(\alpha + c + t) - \omega(\alpha - c) - \alpha\omega^2}{4 - \omega^2} \equiv p_S^f$$
$$q_{iK} = q_{jL} = \frac{(\alpha - c)(2 - \omega^2) - \omega(\alpha - c - t)}{(4 - \omega^2)(1 - \omega^2)} \equiv q_S^h$$
$$q_{iL} = q_{jK} = \frac{(\alpha - c - t)(2 - \omega^2) - \omega(\alpha - c)}{(4 - \omega^2)(1 - \omega^2)} \equiv q_S^f$$

Again prices are decreasing in ω. The following relations hold:

$$p_K^f - t < p_K^h < p_K^f \tag{10}$$
$$p_S^f - t < p_S^h < p_S^f \tag{11}$$

Expressions (10) and (11) show that the burden of transport costs is divided between consumers and producers, since, both under agglomeration and dispersion, the price the formers will pay for the imported variety is higher than the domestic one, while, net of transport costs, producers will get from the exported variety less of what they are able to obtain from the local market. In other terms we have 'reciprocal dumping', i.e. firms receive a lower mark-up from the foreign market.

Equilibrium prices are such that market segmentation turns out to be a *sustainable policy* in equilibrium. Making the realistic assumption that consumers would incur the same transport costs of producers in importing a variety from abroad, the resulting price equilibrium is robust against such arbitrage attempts given that:

$$p_K^h + t > p_K^f \tag{12}$$
$$p_S^h + t > p_S^f \tag{13}$$

We need to specify some non-negativity constraints, in order to ensure that quantities and mark-ups be positive in the duopoly setting. Calculations are reported in Appendix A. The condition ensuring positive equilibrium quantities under dispersion is[3]:

$$t < \frac{(\alpha - c)(2 - \omega^2 - \omega)}{2 - \omega^2} \equiv \tau_S(\omega) \tag{14}$$

$$\alpha - c > 0 \tag{15}$$

In (14), $\tau_S(\omega)$ is the threshold level of transport costs below which trade occurs under dispersion[4]. In particular, for values of t smaller than the threshold, we get positive equilibrium foreign mark-ups and quantities under dispersion.

The threshold level is a function of the product differentiation parameter ω. It can be easily proven that $\partial\tau_S(\omega)/\partial\omega < 0$: the maximum level of transport costs compatible with trade under dispersion is an increasing function of product differentiation. Put in the other way, $\bar{\omega}$, i.e. the minimum level of differentiation that must be introduced to get trade, is increasing in t. This means that when products are sufficiently homogeneous and transport costs are sufficiently high, consumers will buy only the local product: their love for variety (in this case rather weak, since the two products are not too much differentiated) is outweighed by the fact that foreign good is expensive due to the high incidence of transport costs.

In the extreme $\omega = 0$ the threshold level τ_S takes the value $\tau_S(0) \equiv \tau_K = \alpha - c$. The threshold τ_K specifies the value inhibiting trade among regions whatever product differentiation is.

If condition (14) is not checked, then it might be the case that foreign mark-ups and export still be positive under agglomeration, provided that transport costs satisfy:

$$t < \alpha - c = \tau_K$$

We summarize the above findings in Figure 1.

Equilibrium profits in the third stage of price competition are then distinguished whether agglomeration or dispersion occurs. If

[3]In turn, this will guarantee positive equilibrium quantities under agglomeration, see the Appendix.

[4]In Belleflamme, Picard, and Thisse (2000) the trade condition $\tau_S(\omega)$ is called t_{trade}.

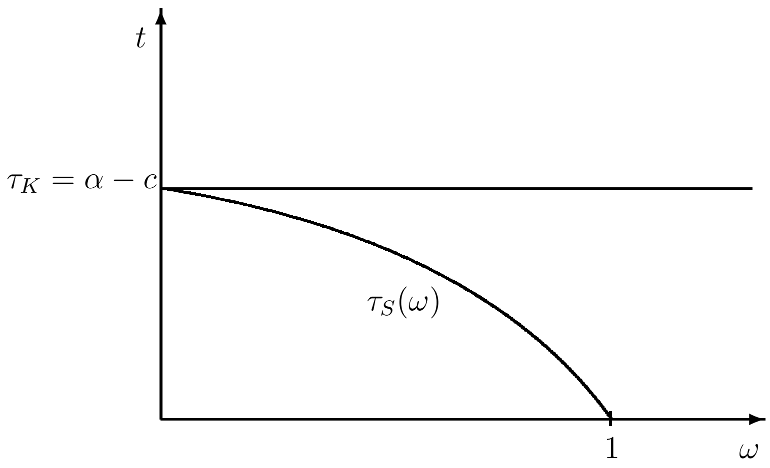

Figure 1: The trade condition under Bertrand competition

firms locate together, they will get:

$$\Pi_K = \frac{(1-\omega)[2(\alpha-c)^2 - 2t(\alpha-c) + t^2]}{(2-\omega)^2(1+\omega)} \tag{16}$$

If firms locate separately in space, they will get:

$$\Pi_S = \frac{2(\alpha-c)^2(1-\omega)^2(2+\omega)^2 + (4-3\omega^2+\omega^4)t^2 - 2(\alpha-c)(1-\omega)^2(2+\omega)^2 t}{(4-\omega^2)^2(1-\omega^2)} \tag{17}$$

The entrant firm in the second stage of the game is supposed to choose agglomeration whenever $\Pi_K > \Pi_S$. If the opposite holds it will choose dispersion. Actually, it turns out that

$$\Pi_K(t,\omega) < \Pi_S(t,\omega) \quad \forall\ (\omega,t) \neq 0 \tag{18}$$

for all ranges of parameters where dispersion is feasible (i.e. $t \leq \tau_S(\omega)$), and so firms will always choose to locate in different regions, given the higher level of profits they attain. When $\omega = 0$, profits under agglomeration or dispersion are the same because firms produce varieties whose cross elasticity of substitution is zero, and no interaction between them occurs: they belong to separate markets.

For $\tau_S(\omega) < t \leq \tau_K$, such that agglomeration is feasible while dispersion is not, we have that maximization of dispersion profits

(16) involves an increase in the level of product differentiation (lower ω), and so we go back to a parameters' region where dispersion is feasible again. So we can conclude that dispersion is always the profit maximizing configuration in this model.

III.A.ii. Equilibrium à la Cournot

We write the expression for equilibrium variables under agglomeration:

$$\begin{aligned} q_{iK} = q_{jK} &= \frac{\xi - C}{2+\omega} \equiv q_K^h \\ q_{iL} = q_{jL} &= \frac{\xi - C - T}{2+\omega} \equiv q_K^f \\ p_{iK} = p_{jK} &= \frac{\xi + C(1+\omega)}{2+\omega} \equiv p_K^h \\ p_{iL} = p_{jL} &= \frac{\xi + (1+\omega)(C+T)}{2+\omega} \equiv p_K^f \end{aligned} \tag{19}$$

and profits are:

$$\Pi_K = \frac{2(\xi - C)^2 - 2T(\xi - C) + T^2}{(2+\omega)^2}$$

Equilibrium prices and quantities under dispersion turn out to be:

$$\begin{aligned} q_{iK} = q_{jL} &= \frac{2(\xi - C) - \omega(\xi - C - T)}{4 - \omega^2} \equiv q_S^h \\ q_{iL} = q_{jK} &= \frac{2(\xi - C - T) - \omega(\xi - C)}{4 - \omega^2} \equiv q_S^f \\ p_{iK} = p_{jL} &= \frac{2(\xi + C) - \omega(\xi - C - T) - \omega^2 C}{4 - \omega^2} \equiv p_S^h \\ p_{iL} = p_{jK} &= \frac{2(\xi + C + T) - \omega(\xi - C) - \omega^2(C+T)}{4 - \omega^2} \equiv p_S^f \end{aligned} \tag{20}$$

Profits under dispersion become:

$$\Pi_S = \frac{2(\xi - C)^2(2-\omega)^2 + (4+\omega^2)T^2 - 2(\xi - C)(2-\omega)^2 T}{(2-\omega)^2(2+\omega)^2} \tag{21}$$

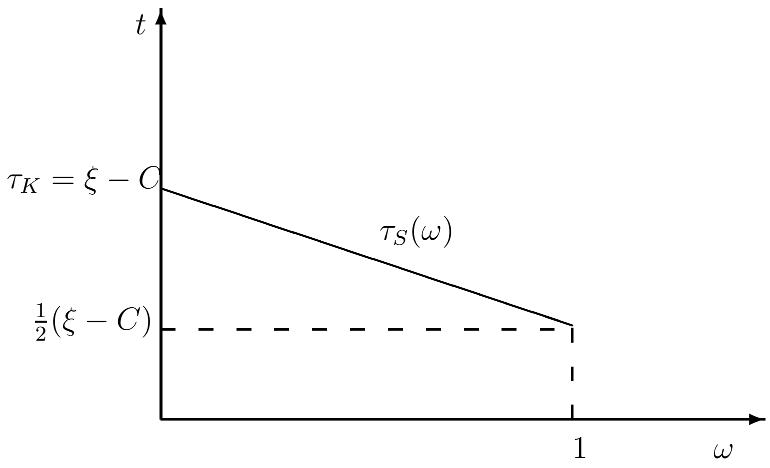

Figure 2: The trade condition under Cournot competition

In analogy to the Bertrand case, we impose the non-negativity of quantities and mark-ups. Home quantity and mark-up are always positive. The conditions for foreign variables are:

$$q_S^f > 0 \Rightarrow T < \frac{(2-\omega)(\xi - C)}{2} \equiv \tau_S(\omega) \tag{22}$$
$$p_S^f - C - T > 0 \Rightarrow T < \tau_S(\omega)$$

We identify a trade condition $\tau_S(\omega)$: there is trade only when transport costs are low enough. It can be interpreted equivalently as the minimum level of product differentiation $\bar{\omega}(T) \equiv \tau_S^{-1}(T)$ compatible with intra-industry trade, for a given level of transport costs. In Fig. 2 we plot the trade condition.

It is interesting to note that there is a range of transport costs for which intra-industry trade occurs also when products are perfectly homogeneous. This is a well known result since Brander and Krugman (1983), due to oligopolistic interaction between firms playing à la Cournot, and contrasts with competition in prices, where intra-industry trade in homogeneous products is impossible for a positive level of transport costs.

Similarly to the Bertrand case, it can be proven that dispersion is always more profitable than agglomeration.

III.B. Monopoly equilibrium

When firms sell just in their domestic market (autarchy), there is no longer strategic interaction and the model reduces to monopoly. This occurs under two circumstances: *a)* transport costs are greater or equal to τ_K; *b)* the equilibrium level of product differentiation is greater than $\bar{\omega}(t)$ or $\bar{\omega}(T)$.

Knowing that, under monopoly, Bertrand and Cournot equilibrium coincides, we compute just one of them. A subscript U is added for variables under monopoly. In the non-normalized version of the model, the demand in each region for the available variety is equal to:

$$p_U = \alpha - \beta q_U$$

and profits are

$$\Pi_U = (\alpha - \beta q_U - c)q_U$$

Maximization with respect to q_U yields the following equilibrium quantities:

$$\begin{aligned} p_U &= \frac{\alpha + c}{2} \\ q_U &= \frac{\alpha - c}{2\beta} \\ \Pi_U &= \frac{(\alpha - c)^2}{4\beta} \end{aligned} \tag{23}$$

IV. Profit maximizing product differentiation

Setting ω has an implication in terms of intra-industry trade patterns, and in terms of market structure, as for values of equilibrium product differentiation greater than $\bar{\omega}(t)$ we get an autarchic monopoly configuration. The entrant knows that if product differentiation is low enough, then there is no more strategic interaction. It then becomes crucial to compare monopoly profits with those under the non-cooperative duopoly equilibrium. If the entrant firm attains a higher profit in the monopoly setting, then equilibrium product differentiation will be $\omega > \bar{\omega}(t)$. Otherwise the equilibrium value will be that maximizing duopoly profits.

IV.A. The Bertrand game

The level $\omega^*(t)$ maximizing profits has to verify the following conditions:

$$\omega^*(t) = \arg\max_{\omega} \Pi_S(\omega, t) \quad \text{if} \quad \Pi_S(\omega, t) \geq \Pi_U \tag{24}$$

$$\omega^*(t) > \bar{\omega}(t) \qquad \text{otherwise} \tag{25}$$

where $\bar{\omega}(t)$ is the threshold level of product differentiation compatible with intra-industry trade.

When transport costs are null the level of product differentiation maximizing profits is $\omega^*(0) = 0$, corresponding to products belonging to independent sectors. This can be easily checked setting $t = 0$ in (17), and noting that the first order derivative is negative. When $t = 0$ the two regional markets collapse into a single one, given that both areas are accessible at the same (zero) transport cost. The problem reduces to determine, in a duopolistic framework with firms competing in prices, profit maximizing product differentiation.

We now look for the level of ω which maximizes profits when transport costs are positive, under the constraint $\omega < \bar{\omega}(t)$. After that we will compare duopoly profits with those under autarchy, because by backward induction the entrant firm has to decide whether to adopt a product differentiation level allowing trade, or whether to differentiate its product so slightly that monopoly arises.

We first compute the partial derivative of dispersion profits with respect to ω, in order to get necessary first order conditions. When $\omega = 0$ we get that

$$\left.\frac{\partial \Pi_S(\omega, t)}{\partial \omega}\right|_{\omega=0} = -\frac{(\alpha - c)t + (\alpha - c)^2}{2} < 0$$

and so it is a local maximum given that ω must be greater or equal than zero.

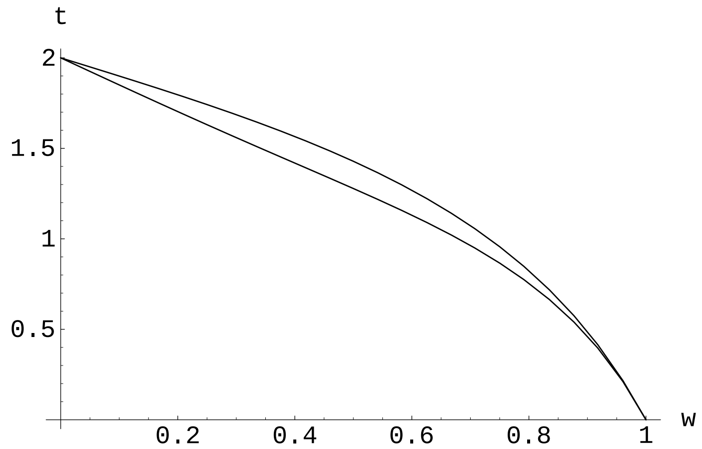

Figure 3: $t_2(\omega)$ (the lower curve) and $\tau_S(\omega)$ when $\alpha = 3$, $c = 1$.

Extensive calculations are reported in Appendix B for the case $\omega > 0$. The f.o.c. is equal to:

$$\frac{\partial \Pi_S(\omega, t)}{\partial \omega} = -\frac{\Phi(\omega, t)}{(4-\omega^2)^3(1-\omega^2)^2} = 0 \qquad (26)$$

where $\Phi(\omega, t)$ is a parabola in t with downward concavity for $\omega \neq 0$. From now on, for analytic tractability, we will explicit the f.o.c. in terms of the level of transport costs, instead of the original product differentiation parameter. We call the two solutions of equation (26) $t_1(\omega)$ and $t_2(\omega)$ respectively. For a positive ω, $t_1(\omega) < 0$ while $t_2(\omega) > 0$. We immediately derive that dispersion profits Π_S are increasing in ω (decreasing in the degree of product differentiation) for $t > t_2(\omega)$. On the contrary, when $t < t_2(\omega)$, dispersion profits are decreasing in ω (increasing in the degree of product differentiation). Moreover we have that $t_2(\omega) < \tau_S(\omega)$ for all ω. In Fig. 3 we plot $t_2(\omega)$ and $\tau_S(\omega)$ in the plane (ω, t) for the parameters' values $\alpha = 3$, $c = 1$. In this way we can conclude that $t_2(\omega)$ is the locus of minimum profit, and candidates to be global maxima are then the vertical axis (where products are made independent, $\omega = 0$), and the trade condition $\tau_S(\omega)$. We derive the following Proposition.

Proposition 1 *Under Bertrand competition, for every level of transport costs $t < \alpha - c$, dispersion profits are always maximized at*

$\omega = 0$, so that the entrant firm will always choose to maximally differentiate its product from the incumbent.

Proof. We begin by computing profits along the trade condition that are:

$$\Pi_S(\omega, \tau_S(\omega)) = \frac{(\alpha - c)^2(1-\omega^2)}{(2-\omega^2)^2}$$

Profits under monopoly, after having carried out the normalization by multiplying for β are:

$$\Pi_U = \frac{(\alpha - c)^2}{4}$$

and we note that, for $\omega \neq 0$, $\Pi_U > \Pi_S(\omega, \tau_S(\omega))$ (a discontinuity occurs in the profit function at $\bar{\omega}(t)$). The entrant would prefer to set product differentiation so low that the autarchic equilibrium arises (i.e. $\omega > \bar{\omega}(t)$) more than differentiating at the level $\omega = \bar{\omega}(t)$ and face the duopoly (which is degenerate in some sense, since exports are nil). But by setting $\omega = 0$, he can do ever better, because when it produces an independent product from the incumbent he will earn monopolist's profits in the home market (producing at the marginal cost c) and monopolist's profits in the foreign market (facing a total marginal cost of $c + t$, cost of production plus transport costs). Hence $\omega = 0$ maximizes profits. ■

IV.B. The Cournot game

We maximize dispersion duopoly profits under Cournot competition with respect to ω. We recall that the non-cooperative equilibrium concept applies only when the trade condition is checked, i.e. $t \leq \tau_S(\omega)$, otherwise we have monopoly. In other words, the level $\omega^*(t)$ maximizing profits has to verify the following conditions:

$$\omega^*(t) = \arg\max_{\omega} \Pi_S(\omega, t) \quad \text{if} \quad \Pi_S(\omega, t) \geq \Pi_T \tag{27}$$

$$\omega^*(t) > \bar{\omega}(t) \qquad \text{otherwise} \tag{28}$$

where $\bar{\omega}(T)$ is the threshold level of product differentiation compatible with intra-industry trade.

Setting $T = 0$ in (21) it can be readily checked that for zero transport costs profits are maximized when varieties are made independent. This result was already obtained under price competition. Let us now consider the more general case of positive transport costs. For extensive calculations report to Appendix C.

Maximization of dispersion profits (21) yields the following first order condition:

$$\frac{\partial \Pi_S(\omega, T)}{\partial \omega} = \frac{2\Phi(\omega, T)}{(2-\omega)^3(2+\omega)^3}$$

where $\Phi(\omega, T)$ is a parabola with upward concavity if $\omega \neq 0$.

Let us consider what happens when $\omega = 0$. In this case $\Phi(\omega, T)$ reduces to

$$\Phi(0, T) = 16(\xi - C)[T - (\xi - C)] < 0$$

and hence

$$\left.\frac{\partial \Pi_S(\omega, T)}{\partial \omega}\right|_{\omega=0} < 0$$

We now turn to the case of a positive ω. Since the parabola has a positive root $t_2(\omega)$, and a negative root $t_1(\omega)$ we can conclude that the derivative of dispersion profits will be decreasing in ω for every $T < t_2(\omega)$ and increasing in ω when $T > t_2(\omega)$. In other terms $t_2(d)$ is the locus of minima. We plot both $t_2(d)$ and $\tau_S(d)$ in Fig. 4, the term $(\xi - C)$ being just a scale factor. The two candidates to be global maxima, for a given t, are then the vertical axis ($\omega = 0$) and the trade condition line $\tau_S(\omega)$.

We derive the following Proposition.

Proposition 2 *Under Cournot competition, for every level of transport costs $T < \xi - C$, dispersion profits are always maximized at $\omega = 0$, that is the entrant firm will always choose to maximally differentiate its product from the incumbent.*

Proof. Profits computed along the trade condition amount to

$$\Pi_S(\omega, \tau_S(\omega)) = \frac{1}{4}(\xi - C)^2$$

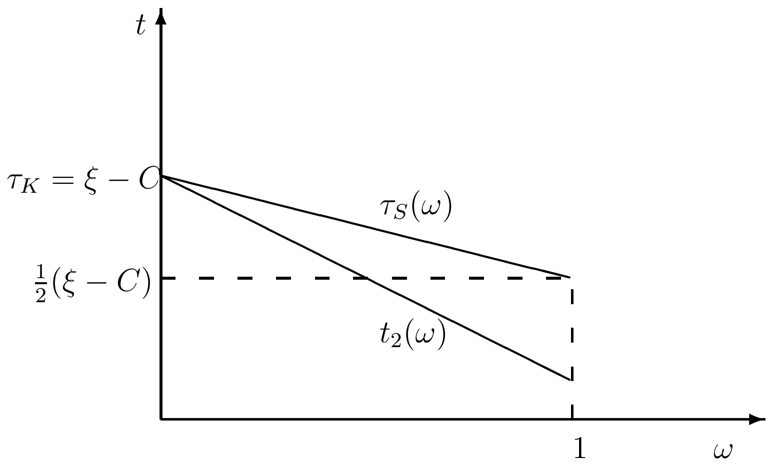

Figure 4: The trade condition and the $t_2(d)$ curve

that, after having multiplied by the scale factor, exactly equal monopoly profits. When the quantity sold abroad goes to zero, the model reduces to monopoly. Comparing monopoly profits with those earned by the firm when $\omega = 0$, it is straightforward to verify that the latter will be greater, since they are equivalent to the sum of profits made by a monopolist firm in two distinct markets, one where it produces at marginal cost C, the other where it produces at marginal cost $(C + T)$.

If transport costs are low enough $(T < 1/2(\xi - C))$ it is impossible to get autarchy (monopoly) by decreasing differentiation, and also intra-industry trade in homogeneous products becomes feasible. For a given T, comparing profits when $\omega = 0$ with those when $\omega = 1$, it is easy to show that they will be higher in the former case, because each firm acts as a monopolist (even though it faces different marginal costs according to the region where it sells the product), as opposed to the latter case, where they are duopolists producing the same homogeneous good. ■

The conclusion in this proposition should be compared with what is found in section IV.A.. A main relevant difference is worth mentioning. In the model expressed in terms of competition in prices, when the product differentiation parameter tends to the upper bound compatible with intra-industry trade under dispersion $\bar{\omega}(t)$, we do not retrieve monopoly profits. The reason is the following. Under Cournot, along the trade condition the residual demand the firm faces (in its own local market) is exactly that of a monopolist, because by definition on τ_S the quantity sold by the rival firm is zero. This is not the case under Bertrand, since the residual demand function on the trade condition is not equivalent to that of monopoly, because there still exists a positive price differential between the foreign and home variety.

The fact that there are strong incentives for product differentiation in the first stage of the game seems plausible, though somehow flawed by the partial equilibrium approach of the model, that ignores competition from other sectors, which is sensible to judge stronger the more the product is different from the one produced by the incumbent.

V. Location and product differentiation under Marshallian externalities

The main message from the discussion above is that firms maximize product differentiation in the absence of any incentive fostering agglomeration and some product homogeneity. To figure out the existence of regional clusters, as in Belleflamme, Picard, and Thisse (2000), we suppose that there are Marshallian externalities at work, leading to a reduction in marginal cost whenever firms are located together. We focus on competition in quantities. With a slight abuse of notation, we use lower case variables c and t instead of the normalized ones C and T. The difference between agglomeration and dispersion profits is:

$$\Pi_K(c_K(\omega), \omega, t) - \Pi_S(c_S, \omega, t) \tag{29}$$

where the arguments in the profit function are marginal cost, product differentiation, and transport costs respectively.

V.A. Level of the spillover inducing agglomeration

As a first step, we compute the magnitude of the spillover necessary to induce agglomeration whatever product differentiation is.

We make the simplifying assumption that it costs zero to produce one unit of the good under agglomeration ($c_K = 0$) and c under dispersion ($c_S = c$). This entails no loss of generality with respect to saying that firms produce at marginal cost a when located together, and $(a + c)$ when dispersed, but the former case is more tractable analytically. In this manner c measures productivity gains of agglomeration.

The spillover c° inducing agglomeration, if it exists, is that level of marginal cost under dispersion so that:

$$\begin{aligned} &\forall c < c^\circ(t), \quad \exists \quad \omega' \in (0,1) \quad \text{such that} \quad \forall \omega < \omega', \quad \Pi_K - \Pi_S > 0 \\ &\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\quad \forall \omega > \omega', \quad \Pi_K - \Pi_S < 0 \\ &\forall c > c^\circ(t), \quad \forall \omega : \qquad \Pi_K - \Pi_S > 0. \end{aligned}$$

If the spillover is high enough, then it is convenient to locate together for every level of product differentiation. If the spillover is not too high, then product differentiation will determine location.

We compute the value of product differentiation so that the difference in (29) is positive. It turns out that it must be $\omega(c) < \omega'(c)$ where

$$\omega'(c) \equiv \frac{2c^2 + 2ct - t^2 - 4c\xi + \sqrt{-4c^2t^2 - 4ct^3 + t^4 + 8ct^2\xi}}{c^2 + ct - 2c\xi}$$

and $c \neq 0$. Clearly, if $\omega' \geq 1$, dispersion is never possible. Given that ω' is monotonically increasing in c[5], the spillover c° inducing agglomeration for every level of product differentiation solves the equation:

$$\omega'(c^\circ) = 1.$$

[5] The derivative of $\omega(c)$ with respect to c is $\partial\omega(c)/\partial c = \omega(c)f(c)$, with $f(c)$ a positive function.

The admissible solutions are:

$$c^\circ{}_1(t) = \xi - \frac{1}{2}\left(t + \sqrt{(t-2\xi)^2 - 8t^2}\right)$$
$$c^\circ{}_2(t) = \xi - \frac{1}{2}\left(t - \sqrt{(t-2\xi)^2 - 8t^2}\right)$$

But $c^\circ{}_2$ does not satisfy the constraint τ_K, making feasible the export under agglomeration, since it turns out that:

$$c^\circ{}_2(t) > \xi - t.$$

We conclude that the level of the spillover necessary to induce agglomeration is $c^\circ{}_1(t)$. Since it is a function of transport costs, when they are sufficiently low, agglomeration is more likely, because the condition $c > c^\circ$ is verified more easily. This result was already established in BPT.

V.B. Specifying a spillover function

We now depart from BPT in assuming that localization economies are not constant, since they come to depend on product differentiation, i.e. on the distance in the product space. Marginal cost under agglomeration c_K depends on product differentiation according to two specific functional forms. One is a linear function of the type

$$c_K(\omega) = (1-\omega)c$$

the other is quadratic and reads

$$c_K(\omega) = [1 - 4(\omega - \omega^2)]c$$

with $0 \leq c_K(\omega) \leq c$ under both cost structures. We plot the two functions in Fig. 5.

V.B.i. Linear spillover

With a linear spillover, marginal cost c_K is zero when products are perfectly homogeneous, and it stays at the same level c of dispersion in the case of independent products. Agglomeration economies

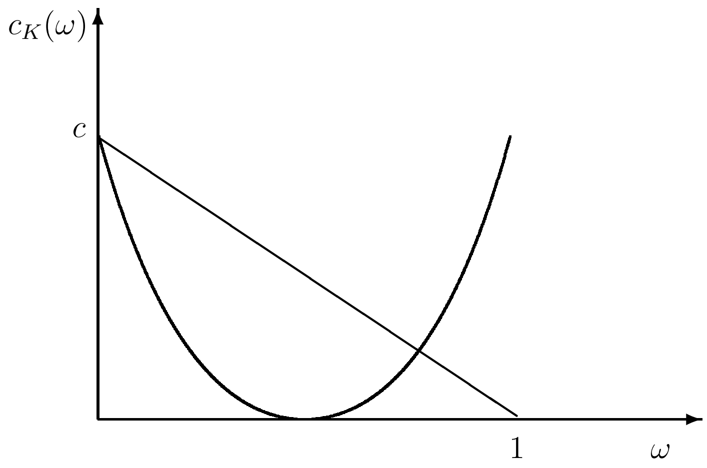

Figure 5: Marginal cost $c_K(\omega)$ under linear and quadratic spillover functions.

are linearly decreasing in product differentiation, and the rate of cost reduction is constant and equal to $-c$. First, we establish whether agglomeration or dispersion occurs *for a fixed level of product differentiation.* Then we look at the profit maximizing level of product differentiation, in so making implicitly the assumption that the entrant *sets product differentiation* in the first stage of the game.

After algebraic computations, being skipped here, the condition ensuring that (29) is greater than zero is $t < t^{\circ}(c,\omega)$ where:

$$t^{\circ}(c,\omega) \equiv \frac{1}{4}\left[-c(2-\omega)^2 + (2-\omega)\sqrt{c(-12c+16\xi+4c\omega+c\omega^2)}\right]$$

If $t > t^{\circ}(c,\omega)$ then the difference (29) is negative, and dispersion turns to be more profitable than agglomeration.

Since t° is a decreasing function of product differentiation[6], this demonstrates that there exists an intermediate range of transport costs where agglomeration is more profitable than dispersion *depending on product differentiation.* In particular, this is true when $t^{\circ}(c,0) < t < t^{\circ}(c,1)$: by increasing (decreasing) ω we make dispersion (agglomeration) the most profitable choice. In terms of Fig. 6, when we come to cross the $t^{\circ}(c,\omega)$ curve, the equilibrium configu-

[6] At least for all the parameters configurations we tried with.

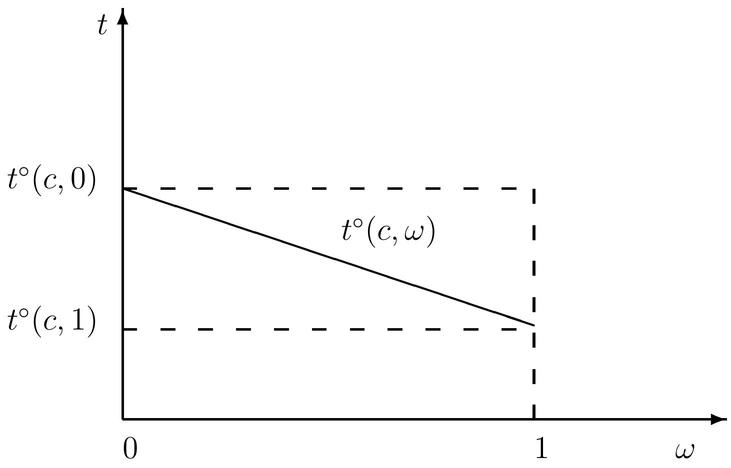

Figure 6: The intermediate range of t where location depends on product differentiation.

ration switches. The lower and upper bounds take the values:

$$t^{\circ}(c,0) = -c + \sqrt{4\xi c - 3c^2}$$
$$t^{\circ}(c,1) = \frac{1}{4}\left(-c + \sqrt{16\xi c - 7c^2}\right)$$

Our results imply, in addition, that an entrant firm producing a homogeneous product, under a linear spillover, will locate in the same region of the incumbent only if transport costs are sufficiently low, that is $t < t^{\circ}(c, 1)$.

We now want to determine the *profit maximizing level of product differentiation* an entrant would set.

Product differentiation has two contrasting effects on agglomeration profits. One is negative, since by increasing product differentiation the reduction in marginal cost is smaller. The other is positive, and is related to the benefits of weaker competition. A very important point is to note that, when $\omega = 0$, $\Pi_K = \Pi_S$: when varieties are indeed independent products, it does not really matter to be located together or separately, interaction among products being absent. In the first part of the paper we established that, without spillover, firms maximize product differentiation. Now we are left to check whether decreasing differentiation, with localization economies reducing marginal cost, increases agglomeration profits. If this is not the case, then the profit maximizing choice will be still to maximize differentiation.

In order to study the effect of ω on agglomeration profits Π_K, we decompose them in the home component Π_K^h (profits made in the home market), and in the foreign component Π_K^f (profits made in the foreign market). We get the following:

$$\frac{d\Pi_K}{d\omega} = \left(\frac{\partial \Pi_K^h}{\partial \omega} + \frac{\partial \Pi_K^f}{\partial \omega} \right) d\omega$$

where

$$\frac{\partial \Pi_K^h}{\partial \omega} + \frac{\partial \Pi_K^f}{\partial \omega} = \frac{2\left[(3c-\xi)(\xi - c + \omega c) + (3c + t - \xi)(\xi - c - t + \omega c)\right]}{(2+\omega)^3} \tag{30}$$

We prove the following result concerning optimum product differentiation ω^* in the linear case.

Proposition 3 *Under Cournot competition and a linear spillover function, the optimum level of product differentiation* ω^* *is the following.*

i) If $c \geq \xi/3$*, then* $\omega^* = 1$*, that is firms will produce a homogeneous product.*

ii) If $c \leq \xi/3 - t/6$*, then* $\omega^* = 0$*, that is firms will produce independent products.*

iii) There exists a point c°*, where* $\xi/3 - t/6 < c^\circ < \xi/3$*, such that* $\omega^* = 1$ *for* $c > c^\circ$*, and* $\omega^* = 0$ *for* $c < c^\circ$*. At* $c = c^\circ$ *the derivative of agglomeration profits with respect to* ω *is zero, that is profits stay constant irrespectively of product differentiation.*

Proof. Proof of *i)* descends from an inspection of (30). If $c > \xi/3$ the derivative is positive, for every ω.

To prove *ii)* we need to rearrange the derivative of profits in the following form.

$$\frac{\partial \Pi_K}{\partial \omega} = \frac{2\left\{[\xi - (1-\omega)c][(3c-\xi) + (3c+t-\xi)] - (3c+t-\xi)t\right\}}{(2+\omega)^3}$$

If $c \leq (\xi - t)/3 - t/6$, the term $[(3c - \xi) + (3c + t - \xi)]$ is negative, and the derivative as well.

To prove case *iii)*, we apply the Weierstrass theorem. We know that $g(c) \equiv \partial\Pi_K/\partial\omega$ is negative at $c = \xi/3 - t/6$ and positive at $c = \xi/3$. Moreover $g(c)$ is a continuous function. In order to apply the existence theorem we need just to prove that $g(c)$ is monotonically increasing:

$$\frac{\partial g(c)}{\partial c} = (4 - \omega)(2\xi - t) - 12c(1 - \omega)$$

being positive when

$$c < \frac{4 - \omega}{2(1 - \omega)} \left(\frac{\xi}{3} - \frac{t}{6}\right)$$

As a last step we note that

$$\frac{4 - \omega}{2(1 - \omega)} \left(\frac{\xi}{3} - \frac{t}{6}\right) > \frac{\xi}{3}$$

Since the derivative is zero whatever is ω, this means that profits are constant at c°. ■

The economic interpretation of this proposition suggests that, if the cost reduction attainable with agglomeration is significant, then the competition effect is overwhelmed by the cost saving one, and the entrant goes for minimum differentiation. The reverse holds if c is low. We proved the existence of a case where demand and cost conditions are such that the two effects exactly offset each other, so that ω does not affect profits, and the equilibrium level of product differentiation is indeterminate. A peculiarity of the profit maximizing level of product differentiation in the linear case is its "all or nothing" appearance: the two levels of ω^* are either 0 or 1.

V.B.ii. Quadratic spillover

With a quadratic spillover, the reduction in marginal cost is the highest when varieties show an intermediate level of product differentiation. Hence firms take advantage from cost reductions while not suffering excessively from competition.

Again, we have to study the condition making agglomeration profits higher than dispersion ones for a *fixed level of product differentiation.* The difference (29) is greater than zero if $t < t^{\circ}(c, \omega)$, where

$$t^{\circ}(c, \omega) \equiv -c(2-\omega)^2(1-\omega) + (2-\omega)\sqrt{c(1-\omega)(4\xi - 3c\omega^2 - c\omega^3)}$$

and less than zero when transport costs are greater than this threshold. Given that $t^{\circ}(c, \omega)$ is decreasing in product differentiation[7], and $t^{\circ}(c, 1) = 0$, there does not exist a level of transport low enough so to make agglomeration possible for every level of product differentiation. Actually, at least when products are homogeneous, it is convenient to separate in space, since, under the quadratic spillover function, producing a homogeneous good has no gain in terms of cost reduction. We remain only with the condition leading to dispersion whatever product differentiation is, $t > t^{\circ}(c, 0)$, where

$$t^{\circ}(c, 0) = 4(-c + \sqrt{\xi c})$$

We now compute *profit maximizing differentiation.* Given the symmetry of the quadratic spillover function around $\omega = 1/2$, a given cost reduction corresponds to two distinct levels of product differentiation. It is straightforward to verify that the most profitable level of product differentiation to be selected is the lower one, since it corresponds to a higher product differentiation. Hence, we restrict the analysis to the interval $\omega \leq 1/2$, where the marginal cost function is decreasing in ω.

We are able to derive the following Proposition.

Proposition 4 *Under Cournot competition and a quadratic spillover function, the optimum level of product differentiation ω^* is the following.*

i) If $c \leq c^{\circ}$, where

$$c^{\circ} \equiv \frac{5(2\xi - t) - \sqrt{(8\xi - 7t)(8\xi - t)}}{18}$$

[7]As in the case of the linear spillover function, we checked this through numerical examples.

then $\omega^* = 0$*, that is firms will produce independent products.*

ii) If $c > c^\circ$*, then* $0 < \omega^* < \omega_U < 1/2$*, which amounts to saying that firms will introduce some product homogeneity to benefit from the spillover. Moreover* $\omega^*(c)$ *is increasing in* c *until it reaches the upper bound* ω_U*, i.e. the minimum differentiation compatible with intra-industry trade.*

Proof. The profit maximizing level of product differentiation is found by solving the first order condition:

$$\frac{\partial \Pi_K}{\partial \omega} = 0 \tag{31}$$

This is true when

$$(32\omega^4 + 96\omega^3 - 192\omega^2 + 104\omega - 18)c^2 + \\ (-10t + 20\xi + 20\omega t - 40\omega\xi)c - t^2 + 2\xi t - 2\xi^2 = 0 \tag{32}$$

First of all we solve in c the inequality

$$\left.\frac{\partial \Pi_K}{\partial \omega}\right|_{\omega=0} \leq 0$$

The condition above ensures that $\omega^* = 0$ maximizes profits. There are two roots solving (32), c_1 and c_2, with $c_1 < c_2$, but the biggest one does not satisfy the constraint $c_2 < \xi - t$. Hence we get that

$$c \leq \frac{5(2\xi - t) - \sqrt{(8\xi - 7t)(8\xi - t)}}{18} \equiv c^\circ$$

Obviously, if $c > c^\circ$, optimum differentiation will be greater than zero. Proving that $\omega^* < 1/2$, simply requires to evaluate $\partial \Pi_K / \partial \omega$ at $\omega = 1/2$, and check that it is negative whatever is c.

To prove that $\omega^*(c)$ is increasing in c, it suffices to show that

$$\frac{\partial \Pi_K(\omega^*)}{\partial \omega \partial c} > 0 \tag{33}$$

Remember that, by increasing c and ω, the trade condition becomes tighter, and ω_U hits exactly the trade condition, where the minimum differentiation compatible with intra-industry trade is reached.

After some computations, it can be shown that

$$\frac{\partial \Pi_K}{\partial \omega \partial c} > 0 \tag{34}$$

when

$$c < \frac{5(2\xi - t)}{18 - 68\omega + 56\omega^2 + 16\omega^3} \equiv h(\omega)$$

and $\omega < 1/2$. It is easy to see that $c_1 < h(\omega)$, and this concludes the proof. ■

When marginal cost is a quadratic function of product differentiation, the optimum level of ω^* is smoothly increasing in c. The explanation underlying the difference in behaviour between the linear and quadratic spillover lies in the shape of the rate of marginal cost reduction: under the quadratic spillover the rate of reduction is $4c(2\omega - 1)$, which is not constant.

For similar reasons, the derivative of profits with respect to product differentiation becomes negative, whatever is c, at $\omega = 1/2$. Actually, the rate of marginal cost reduction shrinks as ω is tending to 1/2 from the left, and reaches zero in the limit. So, in the limit, the only effect corresponding to an increase in ω is the competition one, decreasing profits.

VI. Concluding remarks

This paper has shown first how product differentiation results from the interaction of two firms competing either in prices or in quantities in the last stage game, in a framework of two regions of equal size, with consumers showing a symmetric preference relation over the differentiated varieties: maximum differentiation is chosen both under Bertrand and Cournot settings.

With respect to this part, we shall not ignore the side effects that, by taking δ (or ω) as control variable, are introduced into the model. Changing δ, as was already outlined by Dixit (1979), shifts the demand function upward. Another critique that can be moved is that, by changing δ we are modifying *ex-ante* consumers' love for

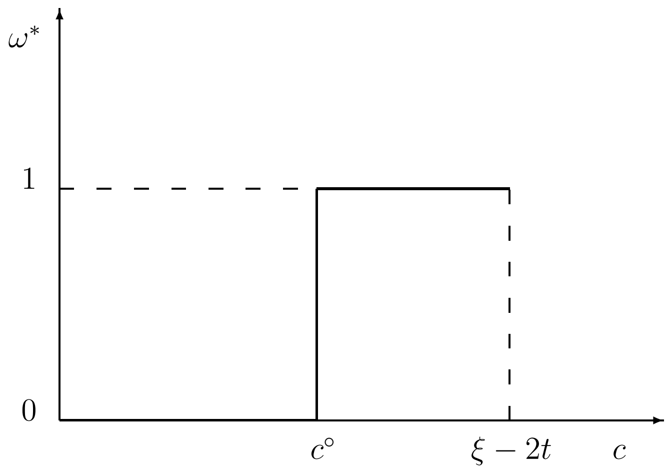

Figure 7: Equilibrium product differentiation as a function of c with linear spillover.

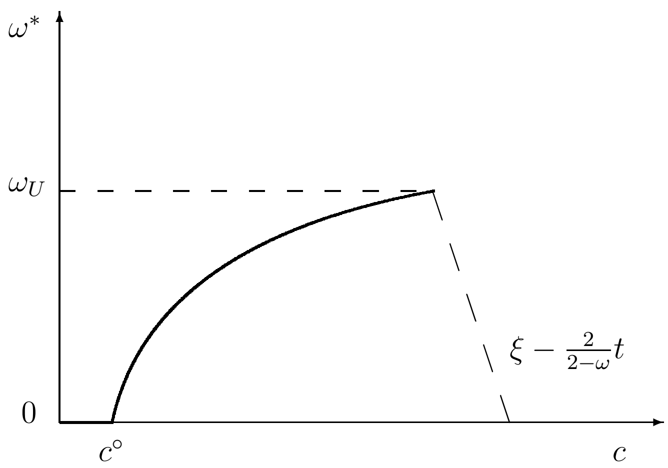

Figure 8: Equilibrium product differentiation as a function of c with quadratic spillover.

variety. A more satisfactory treatment of the issue would probably involve a utility function having two distinct parameters, one representing an exogenous *ex-ante* preference, the other representing the endogenous *realized* variety in production.

Then we went on showing, in a simple model making the phenomenon particularly clean, how Marshallian externalities lead to agglomeration and partial imitation in a model featuring otherwise maximum differentiation.

We have shown how location depends on transport cost, keeping exogenously given product differentiation, and then equilibrium product differentiation was derived as a function of the autonomous part of marginal cost c. The robustness of the results to different specification of the externality was tested. To this end, we considered two forms, a linear one and a quadratic one. Under the linear spillover, a kind of "all or nothing" differentiation arises, because either a homogeneous product is produced or an independent one.

Under the quadratic spillover, a smooth relation between autonomous marginal cost and product differentiation emerges.

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Author's affiliation and address

Dipartimento di Scienze Economiche, Università di Pisa, Via Ridolfi 10, I - 56124 Pisa

Appendix

A Derivation of non-negativity constraints

The non negativity constraints for quantities and mark-ups are in the dispersion case:

$$\begin{aligned} q_S^f > 0 &\Rightarrow t < \frac{(\alpha - c)(2 - \omega^2 - \omega)}{(2 - \omega^2)} \equiv \tau_S(\omega) \qquad (35)\\ p_S^f - c - t > 0 &\Rightarrow t < \tau_S(\omega)\\ q_S^h > 0 &\Rightarrow \alpha - c > 0\\ p_S^h - c > 0 &\Rightarrow \alpha - c > 0 \end{aligned}$$

In agglomeration, the following constraints must be met simultaneously:

$$\begin{aligned} q_K^f > 0 &\Rightarrow t < \alpha - c \equiv \tau_K \qquad (36)\\ p_K^f - c - t > 0 &\Rightarrow t < \tau_K\\ q_K^h > 0 &\Rightarrow \alpha - c > 0\\ p_K^h - c > 0 &\Rightarrow \alpha - c > 0 \end{aligned}$$

The condition ensuring the non negativity of dispersion equilibrium is more restrictive than the one ensuring a similar feature for agglomeration.

B Maximization of dispersion profits under price competition

The partial derivative of dispersion profits with respect to the product differentiation parameter ω is equal to

$$\frac{\partial \Pi_S}{\partial \omega} = -\frac{\Phi(\omega, t)}{(4 - \omega^2)^3 (1 - \omega^2)^2}$$

where

$$\begin{aligned} \Phi(\omega, t) = &- 2(\alpha - c)(1 - \omega)^2 (2 + \omega^3)(1 - \omega + \omega^2) t +\\ &- (12 - 7\omega^2 + 2\omega^4 - \omega^6)\omega t^2 + 2(\alpha - c)^2 (1 - \omega)^2 (2 + \omega)^3 (1 - \omega + \omega^2) \end{aligned}$$

$\Phi(\omega, t)$ is a parabola with downward concavity if $\omega \neq 0$, taking t as the independent variable, and $\omega \in [0, 1]$ as a parameter. We call $t_1(\omega)$ and $t_2(\omega)$ the two intercepts with the horizontal axis. They are solutions to the equation $\Phi(\cdot) = 0$:

$$t_{1,2}(\omega) = -\frac{f(\omega) \pm g(\omega)}{h(\omega)}$$

with

$$f(d) = (\alpha - c)(1 - \omega)^2(2 + \omega)^3(1 - \omega + \omega^2)$$
$$g(d) = (\alpha - c)(4 - \omega^2)(1 - \omega^2)\sqrt{(4 - \omega^2)(1 - \omega + \omega^2)(1 + \omega + \omega^2)}$$
$$h(d) = 2\omega(12 - 7\omega^2 + 2\omega^4 - \omega^6)$$

It is readily seen that $t_1(\omega) < 0$. But also $t_2(\omega) > 0$, because after some algebraic computations it can be shown that $f(\omega) < g(\omega)$.

Assigning some value to the positive multiplicative constant $(\alpha - c)$, the function $t_2(\omega)$ may be plotted, as we do in Fig. 3. We prefer to provide directly the graphical representation of the function, more than embarking in a cumbersome analytical study. Another useful relation is that:

$$t_2(\omega) < \tau_S(\omega), \quad \forall \omega.$$

To show this, it is simply a matter of computation.

C Maximization of dispersion profits under quantity competition

The derivative of dispersion profits with respect to the product differentiation parameter is:

$$\frac{\partial \Pi_S(\omega, T)}{\partial \omega} = \frac{2\Phi(\omega, T)}{(2 - \omega)^3(2 + \omega)^3}$$

If $\omega \neq 0$, then $\Phi(\omega, T)$ is a parabola with upward concavity, taking t as the independent variable and ω as a parameter. Its exact expression is:

$$\Phi(\omega, T) = (12\omega + \omega^3)T^2 + 2(\xi - C)(2 - \omega)^3 T - 2(\xi - C)^2(2 - \omega)^3$$

The roots of the equation $\Phi(\omega, T) = 0$ are:

$$t_{1,2}(\omega) = \frac{-(\xi - C)(2-\omega)^3 \mp \sqrt{(\xi - C)^2(2-\omega)^3(2+\omega)^3}}{12\omega + \omega^3}$$

An immediate result is that $t_1 < 0$. After some manipulations it can be also proved that $t_2 > 0$, and $t_2(\omega) < \tau_S(\omega)$ for every $\omega \in (0, 1]$.

References

Belleflamme P., Picard P., Thisse J.-F. (2000) An Economic Theory of Regional Clusters. *Journal of Urban Economics,* **48**, 158-184.

Brander J., Krugman P. (1983) A 'Reciprocal Dumping' Model of International Trade. *Journal of International Economics*, **15**, 313-321.

d'Aspremont C., Gabszewicz J., Thisse J.-F. (1979) On Hotelling's 'Stability in Competition', *Econometrica*, **47**, 1145-1150.

Dixit, A. K. (1979) A model of duopoly suggesting a theory of entry barriers. *Bell Journal of Economics*, **10**, 20-32.

Dixit A. K., Stiglitz J. E. (1977) Monopolistic Competition and Optimum Product Diversity, *American Economic Review*, **67**, 297-308.

Harrington, J. E. (1995) Experimentation and Learning in a Differentiated-Products Duopoly. *Journal of Economic Theory*, **66**, 275-288.

Lambertini L., Rossini G. (1998) Product homogeneity as a prisoner's dilemma in a duopoly with R&D. *Economics Letters*, **58**, 297-301.

Schmitt, N. (1995) Product Imitation, Product Differentiation and International Trade. *International Economic Review*, **36**, 583-608.

Singh N., Vives X. (1984) Price and quantity competition in a differentiated duopoly. *Rand Journal of Economics*, **15**, 546-554.

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