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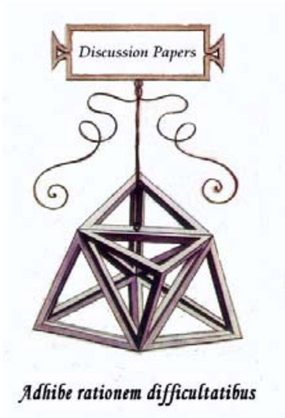

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Fiscal policy and tax collection lags: stability, cycles and chaos.

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Abstract

Fiscal policy and tax collection lags: stability, cycles and chaos

A simple dynamic IS-LM model with 'balanced budget' and money financing deficit can show, in the presence of realistic tax collection lags, regular and chaotic cycles depending on the "mix" of fiscal policies. We argue that also rigid and transparent expenditure limits which an increasing number of countries have recently adopted, could be ineffective in terms of stability if the role of collection lags is neglected. Moreover also the "dimension" of the public intervention in the economy matters for stability, especially as a determinant of chaotic behaviours.

Keywords: IS-LM models, fiscal policy, collection lags, Hopf bifurcation, chaos
J.E.L.Classification:E300,E620

INDEX

1.Introduction

A well established fact concerning income taxation in all countries is the existence of lags in tax collection.

There is broad empirical evidence in all countries for the existence of collection lags in major categories of government revenue. Furthermore, among the categories of revenues the evidence indicates a wide variation in collection lags (Choudry, 1991). The importance of the effects of collection lags has also been recently extensively analysed, mainly focusing on 1) the estimation of revenue-eroding effects of inflation within the standard model of inflationary finance (Choudry, 1991) where it is evidenced that the erosion of real fiscal revenue due to collection lags, although varied among countries, counterbalances the gains from the inflation tax, thereby severely restricting the use of the latter in generating government resources ; 2) analyzing the optimal rate of monetary expansion when there are collection lags in the tax system and the government resorts to inflationary finance to support public investment for enhancing growth (Choudry, 1992), demonstrating that the marginal cost of using inflationary finance is higher than it would be without collection lags; 3) investigating the optimal design of taxation instruments by using inflationary finance and commodity taxation when there are collection lags (Dixit, 1991), in which - challenging Tanzi’s (1978) argument that inflationary finance may further weaken the public finances in the case that both high inflation and collection lags erode the real value of fiscal revenue -

the presence of collection lags raises the excess burden of income taxes with a greater reliance on seignorage than when no lags exist; 4) reconsidering the optimality of inflation tax in the presence of collection lags as claimed by Dixit, showing that depending on the size of the expenditure ratio and the specification of the collection cost function, lags may increase, leave unaltered or reduce the desired rate of inflation (Mourmouras - Tijerina, 1994).

However so far, despite the vast literature on some effects of collection lags, the dynamical effects of such lags have not been sufficiently investigated and, as such lags can also be considered as partially controllable by policy-makers, the usefulness for stabilisation policy of this investigation seems evident. We start from a simple static IS-LM model, in which the existence of public deficit with money financing introduces intrinsically dynamical features, as already noted in the macroeconomic literature (e.g. Turnovski, 1977). This system obviously has a steady-state equilibrium when the ‘balanced budget’ is imposed. However also for this simplified economy so far the stability of the equilibrium has often been postulated or simply sketched rather than investigated with appropriate mathematical tools. We show, by using a simple model with a “textbook” specification both of the functional relationships and of the disequilibrium features, that the emergence of Hopf bifurcations and instability is also possible in such a simple model, but in any case it would require a very high level of public expenditure. In contrast with this case where the possible stabilization policy is straightforward and feasible – it would be sufficient to avoid “exaggerated” public expenditure as recent forms of fiscal rules adopted in many countries try to attain (Kopits, 2001) - the existence of collection lags can cause the emergence of either regularly oscillating or chaotic paths for ‘plausible’ values of the economic parameters. Cycles and chaos depend on the interactions between the various policy “control” variables, namely, in this ‘balanced budget’ frame, public expenditure, income taxation, «targeted» income and tax collection lags. Both persistent periodic dynamics and the phenomenon of the bi-stability of the equilibrium arise for plausible fiscal revenue lags when the income taxation is ‘low’, while when this latter is ‘high’ a completely chaotic dynamics is displayed. This suggests that the mechanism of the delay in income taxation may trigger short-run regular and chaotic cycles which can be entirely ascribed to the “mix” of fiscal policy choices, and in particular that the qualitative dynamical behaviour depends not only on expenditure levels and collection lags, but also on income taxation or in other words on the “dimension” of the “public intervention” in the economic system.

After this introduction, in section 2 we set out the general dynamic model; section 3 presents the model with standard behavioural functions; in section 4 revenue collection lags are introduced. For all models of these sections steady-state as well as dynamic analysis are performed. Section 5 shows the numerical simulations of the model. The final section 6 provides some concluding comments.

2- The general model

We begin by showing a very simple Keynesian macroeconomic dynamical model, which are based on an IS-LM frame with some extension (e.g. Schinasi 1981, 1982; Boldrin, 1984; Lorenz, 1993, 1994; Sasakura, 1994)[1]:

$$\begin{aligned} \dot{Y} &= a\left[I(Y,r) + G - tY - S(Y(1-t),r)\right] \\ \dot{r} &= b(M^d(Y.r) - M) \\ \dot{M} &= G - tY \end{aligned} \tag{1}$$

with a dot denoting the time derivative operator *d/dt.* The three state variables are income, *Y,* the interest rate, *r,* and the money stock, *M.* Parameters *a* and *b* are adjustment coefficients, *t* is the constant income tax rate, and *G* denotes government expenditure. The functions *I(.), S(.),* and $M^d(.)$ are the investment function, the savings function, and the liquidity preference function, respectively. The signs of the first-order derivatives are $I_Y > 0$, $I_r < 0$, $S_Y > 0$, S_r

[1] For example Schinasi (1981, 1982) and Sasakura (1994) extend the traditional IS-LM scheme with a government budget constraint in which both money and bond financing are alternatively used.

> 0, $M^d_Y > 0$, and $M^d_r < 0$ with an index denoting the partial derivative of a function with respect to the particular argument. Moreover we assume that $\Delta_Y = I_Y - S_Y < 0$ (and obviously also $\Delta_r = I_r - S_r < 0$). One or several of the functions could be assumed to be non-linear.

Basically, the model (1) is a short-run dynamic IS-LM-model. In (1), income (production) changes when an excess demand/supply prevails in the goods market. The interest rate increases (decreases) when there exists an excess supply (excess demand) of/for the money The change in the money supply is defined as the government's deficit/surplus which is entirely financed via increases/decreases in the money supply. Government expenditure, G, does not depend on endogenous variables but is assumed constant.

Equation system (1) is a three-dimensional continuous-time dynamical system. Depending on the specification of the functions $I(.)$, $S(.)$, and $M^d(.)$, the system possesses one or several fixed points[2].

The dynamic behaviour of the linearised system around an (assumed) equilibrium point, or in other words of the local behaviour of (1) in the neighbourhood of such an equilibrium, is known to be determined by the properties of the following Jacobian matrix J:

$$J(Y,r,M) = \begin{bmatrix} a\Delta_Y & a\Delta_r & 0 \\ 0 & b\psi & -b \\ -t & 0 & 0 \end{bmatrix} \qquad (2)$$

where

$$\psi = \frac{\partial M^d}{\partial r} < 0$$

The characteristic equation is

$$\lambda^3 + \lambda^2\alpha_1 + \lambda\alpha_2 + \alpha_3$$
$$\alpha_1 = -(a\Delta_Y + b\psi); \alpha_2 = ab\psi\Delta_Y; \alpha_3 = ab\Delta_r t \qquad (3)$$

[2] For the sake of simplicity the demand for money is assumed independent of income; focusing demand for money determination only upon the "speculative" motive, this can make easy closed-form solutions of the system. Thus the findings of this paper are always valid when the "speculative" model is the prevalent reason for money demand dynamics.

All coefficients are positive, such that the system can lose stability only because of a Hopf bifurcation, which occurs when[3]

$$\Delta_2 = \alpha_1\alpha_2 - \alpha_3 = -ab\left[b\psi^2\Delta_Y + a\psi(1-t)^2 - \Delta_r t\right] = 0 \quad (4)$$

By simple inspection of (4), we see that a Hopf bifurcation can emerge only if Δ_r is sufficiently high or in other words the following remark holds: persistent economic fluctuations can be generated by sufficiently high sensitivity of investments with respect to the rate of interest. Although the possibility of Hopf bifurcations has been shown by (4), the fact that the square brackets of such an equation can include in an unspecified form all economic parameters prevents a more detailed and economically meaningful interpretation of the bifurcational process. For instance from (4) nothing can be said about the role of fiscal policies. Therefore we turn to a specification of the model (1).

3- An illustrative textbook model

We specify the model (1) aiming 1) to replicate standard textbook formulations; 2) to simplify the behavioural relations as far as possible; 3) to have only minimal and well justifiable non-linearities. Although our illustrative model may be very "minimalist", we believe that the possible detection of complex behaviours in a "minimalist" model is the first step to understand dynamical models that admit other real-world complexities.

The behavioural relationships follow traditional Keynesian textbook formulations. The investment function shows an inverse (hyperbolic)

[3] The conditions for the existence of a Hopf bifurcation in three- and four-dimensional systems are extensively treated in Fanti-Manfredi (1998). By using the notion of "bifurcation boundary", Manfredi-Fanti (2001) show that in models such as those investigated in the present paper, in order to detect Hopf bifurcations only the satisfaction of the equation $\Delta_{n-1}=0$ is required (where Δ is the Routh-Hurwitz determinant and n is the dimension of the system). From now on, for the sake of brevity and for the economic rather than mathematical aim of this paper, we omit details of the proofs of the existence of Hopf bifurcations.

relationship with the rate of interest and a direct one with production.[4] Total saving depends only on income with a standard constant propensity to save. Demand for money follows the standard (hyperbolic) liquidity preference schedule[5]. The model (1) is reformulated thus:

$$\dot{Y} = a\left[i\frac{Y}{r} + G - tY - sY(1-t))\right]$$
$$\dot{r} = b(\mu r^{-\rho} - M)$$
$$\dot{M} = G - tY$$

(5)

The unique positive equilibrium E is

$$E = \left[Y^* = \frac{G}{t}; r^* = \frac{i}{s(1-t)}; M^{d*} = M^* = \mu\left(\frac{i}{s(1-t)}\right)^{-\rho}\right]$$

(6)

The stability analysis of this 3-D model can be summarised in the following remark: the economy is locally asymptotically stable (LAS) provided that public expenditure is not too high. The proof is simple. The Jacobian of (5) evaluated in (6) is

$$J(Y,r,M)=\begin{bmatrix} a\left[\frac{i}{r^*} - t - s(1-t)\right] & \frac{-aiY^*}{r^{*2}} & 0 \\ 0 & b\psi & -b \\ -t & 0 & 0 \end{bmatrix}$$

(7)

where

$$\psi = -\mu\rho r^{*-\rho-1}$$

[4] For instance Schinasi, Sasakura and Lorenz use a highly non-linear (sigmoid shape) investment function according to Kaldorian lines, while we assume a textbook Keynesian function to have a minimal non-linearity.

[5] This specification of the variable M^d is usual in textbooks; for instance, following Bergstrom (1967), $M^d = \mu Y^u r^{-\rho}$, and for the sake of simplicity we let $u=0$ (relaxation of the latter simplification would preserve the qualitative dynamical results of the paper but, though in some cases it could further enrich the taxonomy of the dynamical behaviours of the model, it would also reduce its analytical tractability).

The characteristic equation is

$$\lambda^3 + \lambda^2\alpha_1 + \lambda\alpha_2 + \alpha_3$$
$$\alpha_1 = at - b\psi; \quad \alpha_2 = -abt\psi; \quad \alpha_3 = \frac{abitY^*}{r^{*2}}$$

(8)

All coefficients are positive (as easily shown also for α_1 by substituting the equilibrium value for r*), so that the system can lose stability only because of an emergence of positive real parts of a pair of complex eigenvalues;, the following Routh-Hurwitz condition is known to ensure the local stability of system (5):

$$\Delta_2 = \alpha_1\alpha_2 - \alpha_3 = \frac{ab}{r^{*2}}\left[\psi(r^*)^2 t(b\psi - at) - iG\right] > 0$$

(9a)

Therefore Hopf bifurcation occurs at the following "bifurcation boundary":

$$\Delta_2 = \alpha_1\alpha_2 - \alpha_3 = 0 \quad \Rightarrow \left[a\rho\mu t^2 (r^*)^{-\rho+1} - b\rho^2\mu^2 t(r^*)^{-2\rho} - iG\right] = 0$$

(9b)

By simple inspection of (9), we see that a Hopf bifurcation[6] can emerge only if the expenditure *G* is sufficiently high. In other words a sufficient policy in order to preserve the stability is to control the level of expenditure.

For an illustrative example suppose $a=1$, $b=12$, $\mu=1$, $\rho=1.2$, $i=0.011$, $s=0.01$, and taking two polar cases of tax rate, namely a low $t=0.1$ and a high $t=0.5$, the system becomes unstable for respectively $G>98.1$ and $G>28.4$ which means $Y^*= 981$ and $Y^*=56.8$. We can assume that, unless policy-makers make abnormal expenditures, economic stability is always preserved. Things change considerably when there are tax collection lags.

Indeed, by assuming the existence of lags in the collection of fiscal revenue, the tax rate should be applied to the delayed income *Z* rather than to the current income *Y*.

[6] Some properties of this Hopf bifurcation will be discussed in Appendix 1.

4- The model (5) with tax collection lags

We explicitly assume that the time delay is of the typical form of exponential distributed type, as usual in economics. This assumption corresponds closely to the realistic case of many agents with different lags as regards income tax collection (Invernizzi-Medio, 1991). To highlight the meaning of the assumption of the exponential distributed form of the tax collection lags, we recall that the possible use of a fixed lag would imply a system having a single representative agent with an arbitrary fixed delay in the revenue collection (that is, the revenue collected at time '*t*' is the revenue of the taxation of income evaluated at time '*t-1*'). Such an assumption would be equivalent to assuming, removing the unrealistic hypothesis of a single representative agent, that the "degree of homogeneity" of the agents in the system tends to be infinite or, formally, that the weighting function of the distribution tends to a Dirac-delta function, the latter in turn meaning that the aggregate fixed delay is attained as a limit case of the continuous-time distributed lag when the order of the lag tends to infinity. The aggregate fixed delay would imply a crude form of dynamic aggregation with no economic underpinning; indeed, given the extensive empirical evidence in all countries on the existence of wide variations in collection lags concerning various categories of government revenue, it would be unrealistic to postulate the existence of a unique "period" of delay that would hold for the infinitely large number of agents. Therefore the case of an exponential distribution of the collection lags is more realistic than the case of fixed delay.[7]

With the assumption of tax collection lags[8], the model (2) becomes

[7] Moreover, for the sake of precision, we note that the assumption of a fixed delay in tax collection would imply a mixed delay-differential equations system, whose analytical complexity would prevent a clearcut economic interpretation of the results.

[8] As is well known, parameter d represents the inverse of the average time lag: for instance, on a yearly base d=0.5 would mean an average lag of about two years.

$$\dot{Y} = a\left[\frac{iY}{r} + G - tZ - s(Y - tZ)\right]$$
$$\dot{r} = b(M^d - M)$$
$$\dot{M} = G - tZ$$
$$\dot{Z} = d(Y - Z)$$

(10)

The system (10) clearly has the same equilibria with respect to variables *Y, r, M* of its unlagged counterpart represented by the model (5), with in addition *Z*=Y**.

We are interested to investigate the conditions under which the introduction of collection lags in the simple short-run IS-LM model with money financed deficit can be responsible for the appearance of cyclical and possibly chaotic behaviours which are induced by policy choices. For this we will be mainly concerned with the local properties of the positive equilibrium E, performing a standard local stability analysis. A global picture of the effects displayed by the existence of collection lags can be investigated only via numerical simulation, as will be seen in section 5.

The Jacobian of (10) is

$$J(Y,r,M,Z)=\begin{bmatrix} a\left[\frac{i}{r^*} - s\right] & \frac{-aiY^*}{r^{*2}} & 0 & -at(1-s) \\ 0 & b\psi & -b & 0 \\ 0 & 0 & 0 & -t \\ d & 0 & 0 & -d \end{bmatrix}$$

(11)

The characteristic equation is

$$\lambda^4 + \lambda^3\alpha_1 + \lambda^2\alpha_2 + \lambda\alpha_3 + \alpha_4$$

$$\alpha_1 = \frac{[r^*(d - b\psi) - a(i - r^*s)]}{r^*}; \quad \alpha_2 = \frac{a[b\psi(i - r^*s) - d(i - r^*(s(1-t)+t)] - bdr^*\psi}{r^*};$$

$$\alpha_3 = \frac{[adb\psi(i - r^*(t + s(1-t)))]}{r^*}; \quad \alpha_4 = \frac{abidtY^*}{r^{*2}}$$

(12)

By substituting the equilibrium values of Y* and r*, the above coefficients become

$$\alpha_1 = ast + d - b\psi; \quad \alpha_2 = \left[at(d - b\psi s) - b\psi d\right];$$
$$\alpha_3 = -ab\psi td; \quad \alpha_4 = \frac{abs^2 d(t-1)^2 G}{i}$$

(13)

Again it is easy to see the strict positivity of the coefficients, which rules out the existence of both positive and zero real roots, while in order to rule out complex eigenvalues with positive real parts it is required in addition that[9]

$$\Delta_1 = \alpha_1 > 0; \qquad \Delta_3 = \alpha_1\alpha_2\alpha_3 - a_3{}^2 - \alpha_1{}^2\alpha_4 > 0$$

We therefore only need to analyse the last condition, for which the positive equilibrium E will be locally asymptotically stable for those parameter sets in correspondence of which the following inequality holds:

$$\Delta_3 = \frac{-abd}{i}\begin{Bmatrix} b^2\psi^3 it(ast+d) - b\psi^2\left[a^2s^2t^3i + 2t^2aids - Gbs^2(t-1)^2 + id^2\right] + \\ \psi(ast+d)(t^2aid - 2Gbs^2(t-1)^2) + Gs^2(t-1)^2(ast+d^2) \end{Bmatrix} > 0$$

(14)

Therefore, given the invariant positive signs of the coefficients of the characteristic equation, we claim that losses of stability and corresponding emergence of Hopf bifurcations can only occur when the "bifurcation boundary" set *B(d,G)*, defined by solutions of the equation:

$$\begin{aligned} &B(d,G) = d^2\left[-it(b\psi^2 - \psi at) - Gs^2(t-1)^2\right] + \\ &d\left[-bit\psi^3 + 2abist^2 + \psi(2Gbs^2(t-1)^2 - a^2t^3) - 2Gats^3(t-1)^2\right] + \\ &s\left[-ai(bt)^2\psi^2 + \psi^2(a^2t^3bis - Gbs(t-1)^2) + 2G\psi abts^2(t-1)^2 - G(at)^2s^3(t-1)^2\right] = 0 \end{aligned}$$

(15)

is crossed.

We have expressed the bifurcation boundary set in terms of two parameters, *d* and *G*, which can be considered, for a given *t*, the policy parameters of interest in this paper. By choosing *d* as bifurcation parameter, the quadratic equation (15) defines two solutions for the *d* parameter: it is possible to see that at most only one solution, the larger one, can be positive and then acceptable, while the smaller one is always negative. Moreover with a simple use of the Descartes rule we can see that when *G* tends to zero no value of the average collection lag

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[9] Recall that Δ_i is the Routh-Hurwitz determinant of order *i*.

can destabilise (the system is always stable), while when *G* becomes very high the system is always unstable for any lag. The following holds: one positive 'critical' value of the collection lag always exists, provided that *G* assumes a sufficiently intermediate value.

By choosing *G* as bifurcation parameter, we note that as regards stability the role of *G* is even more clearcut than the role of *d*: as shown from

$$\frac{\partial \Delta_3}{\partial G} = \frac{-it\psi(ast+d)((b\psi)^2 - b\psi(ast+d) + adt)}{s^2\left[(b\psi)^2 - sb\psi(ast+d) + (ast+d)^2\right](t-1)^2} < 0 \tag{16}$$

public expenditure is always destabilising.

The main result of local stability analysis is summarised in the following proposition:

Proposition 1: the economy is locally asymptotically stable (LAS) when the expenditure is sufficiently small, that is for $G<G_H$. Moreover if, starting from a stable situation, the expenditure *G* is increased, the equilibrium point undergoes a Hopf bifurcation at the above value $G=G_H$, where

$$G_H = \frac{-it\psi(ast+d)\left[(b\psi)^2 - b\psi(ast+2d) + adt\right]}{s^2\left[(b\psi)^2 - sb\psi(ast+d) + (ast+d)^2\right](t-1)^2} > 0 \tag{17}$$

Furthermore, although the bifurcation value of *G* depends in a very complicated non-linear manner on all parameters of the model, we are able to establish the following two propositions:

Proposition 2: the larger the collection lag, the smaller the 'sustainable' public expenditure.

The proof easily emerges from the following derivative

$$\frac{\partial G_H}{\partial d} = \frac{-it\psi\left[(b\psi)^3 - (b\psi)^2(ast+d) + b\psi at(ast+2d) - (at)^2 s(ast+d)\right]}{s^2\left[(b\psi)^2 - sb\psi(ast+d) + (ast+d)^2\right](b\psi - (ast+d))(t-1)^2} < 0 \tag{18}$$

Proposition 3: the larger the income taxation, the smaller the 'sustainable' public expenditure.

The above proposition is easily proved by the following derivative:

$$\frac{\partial G_H}{\partial t}=\frac{-i\psi\left[(b\psi)^3(2ast+d(1+t))-(b\psi)^2(ast+d)(ast(3+t)+2d(1+t))\right]}{s^2\left[(b\psi)^2-sb\psi(ast+d)+(ast+d)^2\right](b\psi-(ast+d))(t-1)^2}+$$
$$\frac{b\psi a(a^3s^3t^3(1+t))+(at)^2ds(3s(1+t)+3-t)}{s^2\left[(b\psi)^2-sb\psi(ast+d)+(ast+d)^2\right](b\psi-(ast+d))(t-1)^2}+$$
$$\frac{d^2at(3s(1+t)+2)+d^3(1+t)-adt(ast+d)(ast(1+t)+2d)}{s^2\left[(b\psi)^2-sb\psi(ast+d)+(ast+d)^2\right](b\psi-(ast+d))(t-1)^2}<0 \quad (19)$$

Although we have analytically proved the existence of a Hopf bifurcation (see Prop. 1 and eq. 17), the economic interpretation of the stability condition (14) is very difficult. But with the aid of some numerical examples, even the economic interpretation of the emergence of cycles is possible.

As in this standard framework the policy dimension is very important, we focus on the latter. For policy purposes, we can assume that the policy-maker can control *G, t* and in a certain sense also *d*, while speeds of adjustment in the goods and money markets, interest elasticity of money demand and propensities to save and to invest are exogenously given. Moreover the policy-maker has a «targeted» income equilibrium, *Y**.[10] In this context, policy instruments are therefore interacting in a clearcut way with both the endogenous variables and the exogenous parameters, as neatly shown by equation (15) despite its complexity. For instance, given the target *Y**, policy instruments are *G, t* and possibly *d*, or, alternatively, given the income taxation system (represented by the level of *t*), policy instruments are *G* (therefore determining *Y**) and possibly *d*.

For the sake of presentation we distinguish two types of income taxation policy: 'low' taxation (*t=0.1*) and 'high' taxation (*t=0.5*). This distinction also corresponds to a small versus large "weight" of public sector in the output at the "equilibrium" or in other words a small G/Y* ratio (10%) versus a large G/Y* (50%).

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[10] We note that in this type of macromodel (without wage-price sector and wealth), from a static point of view, the fiscal multiplier would allow an 'ad libitum' increase in the short-run income equilibrium by increasing expenditure and/or reducing income taxation (while the rate of interest in equilibrium depends only on the propensities and is independent of income). But unfortunately income increases would generate supply-side problems, and money creation to finance momentary deficits would cause wealth effects, such that the policy maker will have a well limited «targeted» income equilibrium.

In this class of short-run models, given income taxation policy, the higher the public expenditure, the higher is income equilibrium or, in other words, the policy can choose the income equilibrium "scale". We will see that this absolute level of income equilibrium affects the dynamical behaviour or, in other words, there is an income scale effect on stability

Given the different "weight" of the public sector, we will investigate: 1) the income scale effect on stability (we recall that, for fixed t, the choice of the "targeted" income also means the choice of the corresponding level of G) for a given average collection lag; 2) the role of the average collection lag on stability given the "targeted" income. For this investigation, of interest is the relation between the two policy parameters *d,G* (or alternatively Y^*), given the third policy parameter, the tax rate *t*. This leads to the investigation of the shape of the bifurcation curve expressed by (17), by focusing upon the relation $G_H=G_H(d)$.

Let $a=1, b=12, \mu=1, \rho=1.2, i=0.011, s=0.01$. [11]

For each of the two cases of taxation, the shapes of the bifurcation curve in the policy parameter space (*d,G*) are represented for a large scale and for a small scale with respect to *G* (or alternatively Y^*) respectively in figs. 1a,b and 2a,b.

Inspection and comparison of such figures permit the following important policy remarks: 1) when there is 'high' taxation, the value of the «targeted» equilibrium income below which 'strong stability' exists is appreciably higher than when there is 'low' taxation: in the first case, stability can never be lost independently of collection lags when equilibrium income is $Y^*<0.5$, while in the second case only for $Y^*<0.12$; 2) when there is 'high' taxation, a high value of income equilibrium causes cycles and instability for very small collection lags as compared with those necessary to bifurcate when there is 'low' taxation: for instance, as depicted in figs. 2a,b, and in Table 1 of the next section, given the same Y*=200 a Hopf bifurcation occurs at $d=2$ in the 'low' taxation case and at $d=7.15$ in the 'high' taxation case.

[11] For sake of illustration, if our 'benchmark' period is one month, the above speeds of adjustment are, respectively in the goods market and in the money market, about one month and one workweek.

We sum up the main message of the above dynamical analysis as follows: in the dynamical context, in contrast with the static context, income taxation policy (ceteris *paribus* regarding the level of «targeted» income as well as the preservation of the balanced budget) matters for stability.

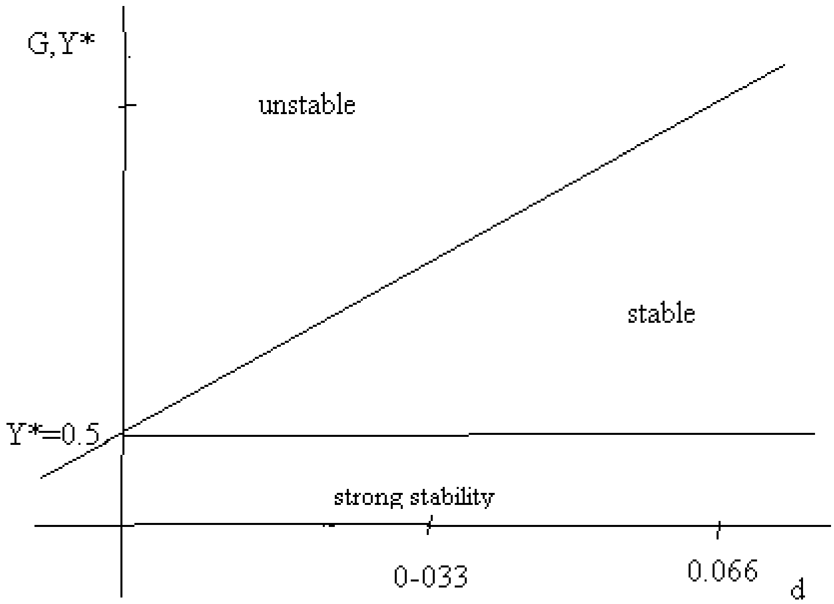

FIG. 1a

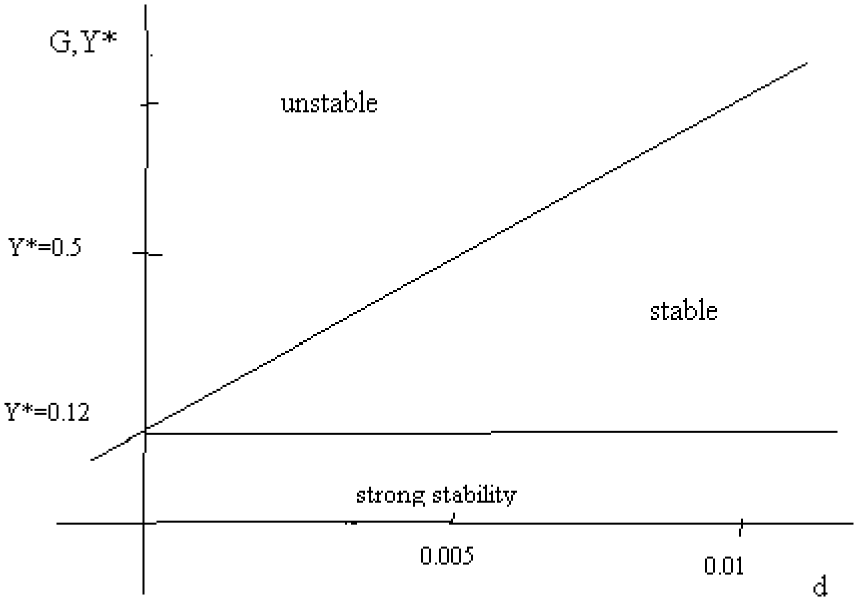

FIG. 1b

FIG. 1a,b- *Shapes of the bifurcation curve in the (G,d) domain with a) t=0.1 and b) t=0.5 for small scale of G (that also means small scale of Y*).*

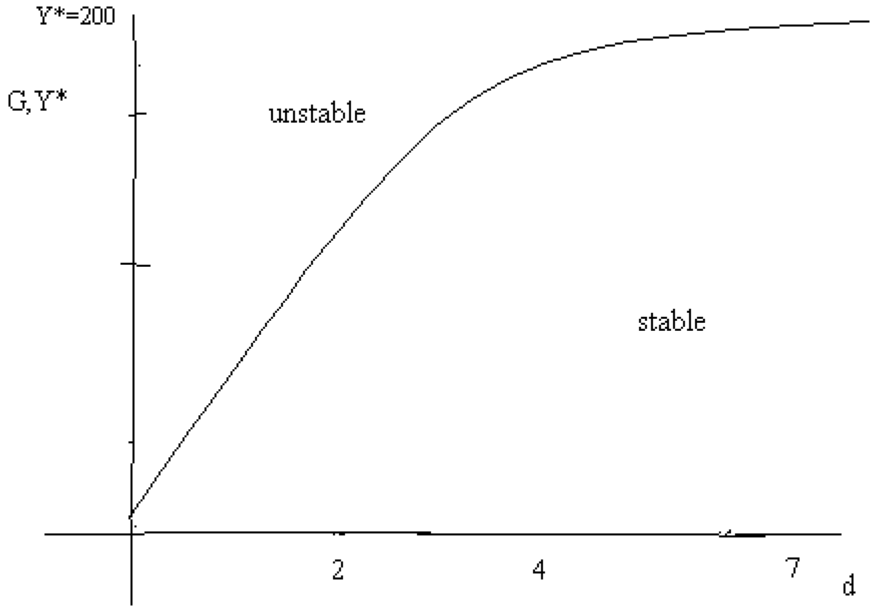

FIG. 2a

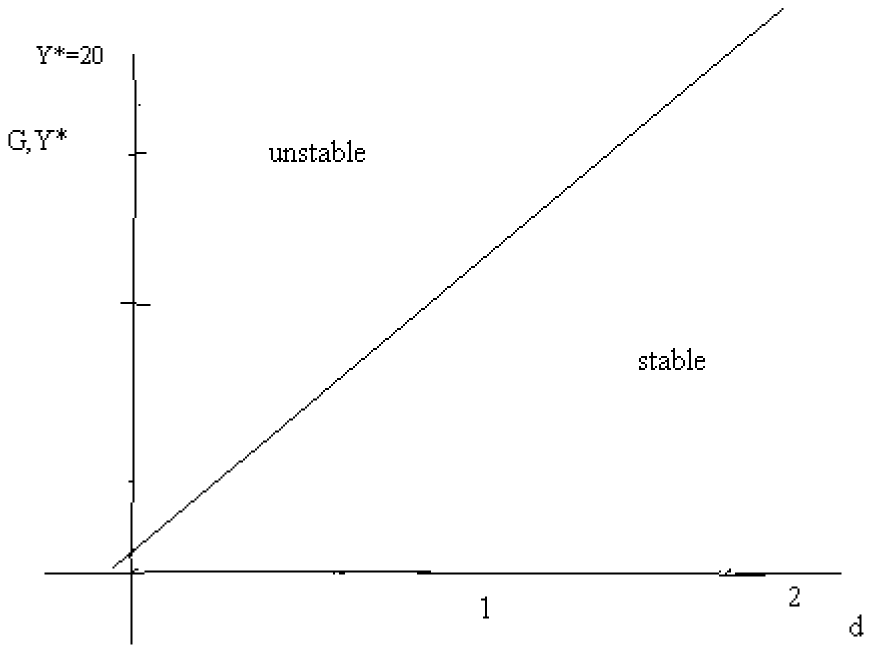

FIG. 2b

FIG. 2a,b- *Shapes of the bifurcation curve in the (G,d) domain with a) t=0.1 and b) t=0.5 for large scale of G (that also means large scale of Y*).*

Therefore the policy maker could be capable of controlling cycles in the economy by using its fiscal instruments in line with the explicit formulations of stability and Hopf bifurcation conditions. In order to illustrate the features of the cycles emerging from the Hopf bifurcation ascertained by using (15) or (17) as well as the global behaviour of the economy modelled in (10), we resort to numerical simulation in the next section.

5. Numerical results

The Hopf bifurcation theorem used in the previous section is a local result which only permits us to detect the existence of a (local) bifurcation of an equilibrium point in periodic orbits. Conversely, it says nothing about uniqueness and stability of the limit cycles emerging from the bifurcation. In particular, it says nothing on the question whether the bifurcation is supercritical or subcritical, i.e. whether or not the limit cycles which bifurcate from the stationary state are (at least) locally stable[12]. Moreover predictions of the theorem are local also in terms of parameter space: “the Hopf bifurcation theorem is local in character and only makes predictions for regions of parameter and phase space of unspecified size” (Medio, 1992, ch.2). This last fact makes it interesting to resort to numerical simulations to investigate matters such as i) the stability properties of the periodic orbits involved, ii) their uniqueness, iii) the size and shape of its (or their) basin of attractions, iv)

[12] This last question, which is of critical relevance from the point of view of a substantive economic analysis, is not tackled here analytically. As is well known, its analytical investigation needs a huge amount of cumbersome algebra (see for instance Farkas (1995) or Guckenheimer-Holmes(1983)), whose interpretation is generally economically meaningless.

the size and shape of the parameter region in which limit cycles exist, v) the global rather than local behaviour of the system.

We will show that in addition to the local persistent periodic behaviour discussed (and analytically proved) in the previous sections, other, more complex, dynamics of the system are possible when the global behaviour of the model is investigated.

As is well known, the existence of chaotic attractors can be proved analytically only in a few rather special cases [13]. The global behaviour of this macrodynamic system is then analyzed numerically [14]. The main finding of the numerical analysis will be that the dynamics of the system presents a *period-doubling* route to chaos.

Although an exhaustive understanding of the transition to chaos is still lacking, in order to investigate such a transition some numerical tools, for instance the numerical computation of the Poincarè-maps [15] and the so-called bifurcation diagrams, can be used.

By using the previous parameter set, we investigate the two cases of 'low' and 'high' taxation, for different levels of income equilibrium (or for different levels of public expenditure) by choosing d as bifurcation parameter [16]; the results are reported in the following table:

Table 1- Bifurcation value of the revenue collection average lag (d_H) for different income taxation and different income equilibrium levels.

Y^*	d_H with $t=0.1$	d_H with $t=0.5$
1	*0.05*	*0.0058*

[13] At present "complete (though approximate) information of the structure of orbits of continuous dynamical systems of dimension greater than two…can only be obtained by numerically integrating the equations of the systems…" (Medio, 1992, pp. 82).

[14] We recall the usual caveats with respect to the effects of computing approximation in continuous-time formulation, above all when detecting chaos, for which we refer to Lorenz (1993), Appendix 4, 276-281.

[15] The shape of some Poincarè-maps, for brevity not reported here, confirms the existence of the period-doubling route to chaos.

[16] Because changes in only one parameter are involved, the investigation regards only the codimension one route to chaos.

10	*0.099*	*0.106*
50	*0.50*	*0.64*
200	*2.39*	*7.14*

Below we report the numerical results for Y*=10. In the 'low' taxation case we observed the following phenomenon: when starting from a situation of stability parameter d decreases continuously, for values of d in the right neighbourhood of d_H a subcritical bifurcation appears generating an unstable limit cycle, for values of d in a left neighbourhood of d_H a so-called 'crater' bifurcation appears generating an unstable limit cycle surrounded by a stable limit cycle, and for further decrease of d only a stable limit cycle appears; finally for $d<0.08$ the system explodes. We note the case of 'crater' bifurcation for its economic relevance: in that case there is a co-existence of two equilibria, one a stable point, the other a stable oscillation.[17]

In the 'high' taxation case when the parameter decreases further beyond its bifurcation value, the stable limit cycle generated by Hopf bifurcation bifurcates in turn, generating a "subsequent" cycle with different period and amplitude, so that two periods and two amplitudes coexist: this is the first step of the gradual onset of chaotic solutions by the so-called *period-doubling* route, as d is further decreased.

Figure 3 displays the chaotic trajectories in the bidimensional phase space (Y,r), which are bounded in a geometrically very 'nice' attractor.

[17] 'Crater' bifurcation is rarely discussed in an economic context, despite its importance for economics, with the exception of Kind (1999) (who attributes such a definition to Lauwerier, 1986). In fact it allows for a better description of the idea of 'corridor stability' proposed by Leijonhufvud (1973) and Howitt (1978), explaining how large shocks do not lead to a totally unstable dynamics – as in the case of subcritical bifurcation - but result in self-sustained oscillations. Furthermore, this bifurcation scenario, in contrast with both supercritical and subcritical bifurcation scenarios, is capable of displaying phenomena like hysteresis loops and catastrophic transitions. Kind (1999) ascribes to this type of 'local' bifurcation the phenomenon of the co-existence of two equilibria (or in the words of Grasman, 1994, of 'bi-stability'). Such co-existence, to our knowledge, has been shown in economic models only in Semmler-Sieveking (1993), Grasman (1994) and Manfredi-Fanti (1999), but with an interpretative difference with respect to Kind: in fact the first authors showed co-existence by means of a 'global' stability analysis while the other authors attributed the phenomenon to the existence of a "relaxation" oscillation mechanism.

Figure 4 shows a tridimensional view of the same trajectories.

In the bifurcation diagram in figure 5, where the successive bifurcations of the solution of a system obtained by varying a chosen bifurcation parameter can be viewed, we can neatly argue for a *period-doubling* route to chaos. The chaotic regions are interspersed from at least three appreciable regions of regular multiple oscillations. The robustness of chaos with respect to variations in the tuning parameter is easily ascertained by observing the ample 'chaotic' range of the parameter values.

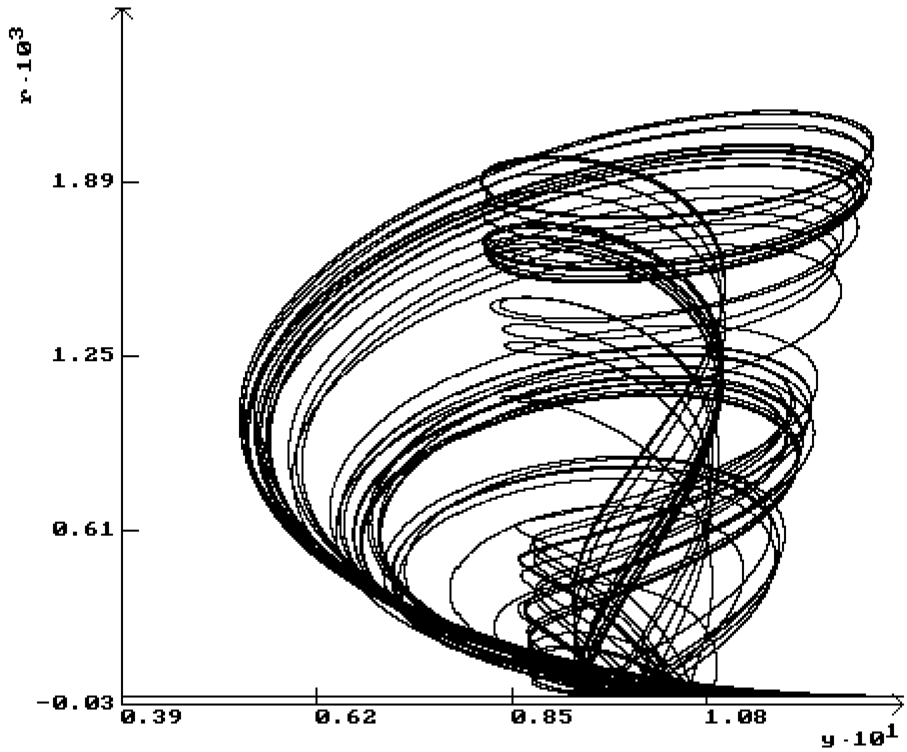

Fig. 3- Chaotic behaviour in the phase space Y,r.

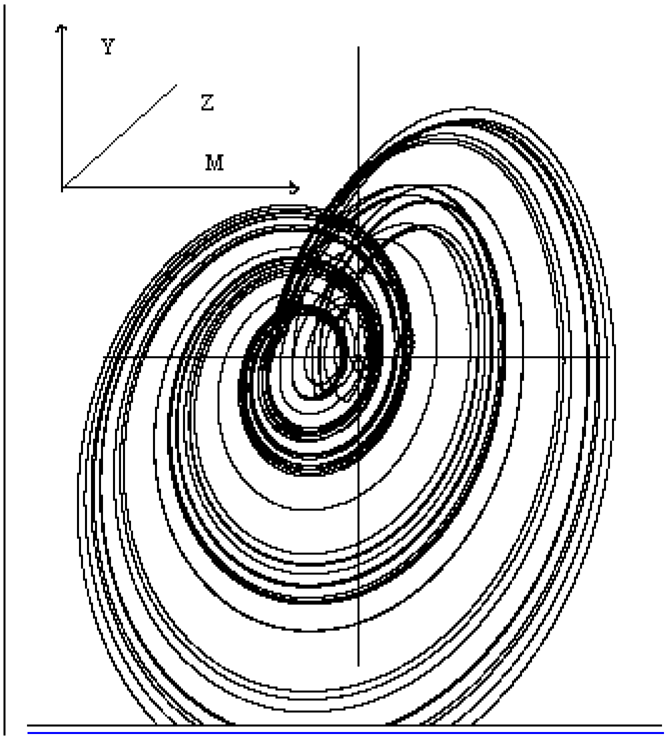

Fig. 4 – Tridimensional view, in the phase space (M,Z,Y), of the chaotic behaviour.

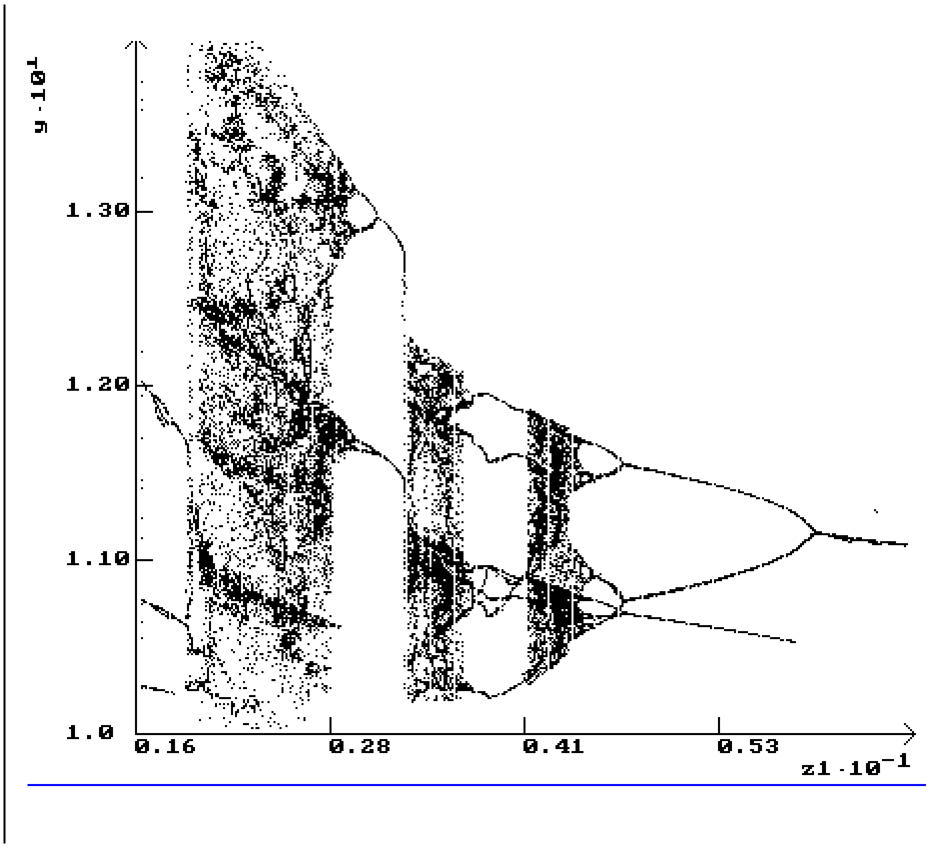

Fig 5 - Bifurcation diagram with respect to parameter d (in the interval 0.065-0.016) and the variable Y.

Interestingly, the result of the existence of cycles (and chaos) is very robust with respect to a wide range of parameter sets. For instance by investigating the effect of different interest elasticities of money demand, for which the econometric evidence is very controversial, ranging from small values close to 0.1 to values much greater than one [18], we can see (table 2) that the existence of bifurcations is independent of such an elasticity and that the smaller the elasticity, the higher is the likelihood of bifurcations.

Table 2- Bifurcation value of the revenue collection average lag (d_H) for different interest elasticity of money demand ρ (with Y=10 and t=0.5).*

ρ	d_H
0.2	0.21
0.5	0.15
0.8	0.11
1.2	0.106

The numerical simulations allow the following observations to be made with regard to the two different "weights" of the public sector in the economic system: 1) Hopf bifurcations occurred for a slightly smaller average collection lag in the case of high 'public weight' (high ratio G/Y*), but the difference is not substantial (for instance, with a ratio of 0.1 the critical inverse of the lag is 0.099 while with a ratio of 0.5 it is 0.106); 2) by contrast both "qualitative" and "quantitative" behaviours for further increases of the average collection lag beyond the Hopf bifurcation value are very different: i) from the qualitative point of view, while with the small ratio nothing more complicated than a twofold limit cycle (the inside one unstable, the outside one stable) occurs, with the large ratio a true chaotic behaviour appears; ii) from the quantitative point of view, while with the small ratio an average collection lag sufficiently

[18] For instance, for a synoptical view of the econometric evidence, see table A in the Appendix 1 in Goodhart-Crockett (1970), which reports the values of interest elasticity estimated by 36 econometric works.

greater than the Hopf bifurcation value causes the explosion of the system (in the example this occurs for $d<0.08$), in the large ratio case, also for very high collection lags the economic dynamics remains steadily confined in a strange attractor – in the above example until $d<0.016$. It is worth stressing that when, for instance, the benchmark period is one month, there is a very appreciable difference in the collection lags as regards the "persistence" of the system: with a small "public presence" the system explodes for collection lags higher than one year, while with a large "public presence" the system continues to fluctuate for collection lags below five-six years. This means that, given the taxation rules prevailing in most of the world, regular and mostly chaotic fluctuations could well be the rule rather than the exception.

Finally, given the strong policy dimension of this model, we should note that the existence of chaos has far-reaching implications for policy: for instance (Goodwin-Pacini, 1992) 1) inability to achieve perfect forecasting,[19] 2) impossibility of fine-tuning,[20] 3) persistence of temporary disturbances,[21] 4) policy intervention of a structural type[22].

6 - Conclusions

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[19] This is mainly due to a twofold reason: sudden qualitative breaks in the time path and Sensitive Dependence on Initial Conditions (SDIC).

[20] Because of the presence of SDIC and the consequent inaccuracy of measures "any attempt to fine-tune (is) likely to obtain undesired effects" (Goodwin-Pacini, p. 254).

[21] In contrast with the stochastic approach of the Real Business Cycles theory in which if a policy intervention has to have permanent effects it must be permanent, in a system with deterministic chaos, "since the system never forgets its past, actions of the 'hit and run' type must also take into account all possible strong distortionary effects in the future."(Goodwin-Pacini, p. 255).

[22] In fact "policy intervention acting only on the state of the system may succeed in affecting the actual time paths but not their qualitative features….structural intervention is required to promote changes in the value of the governing parameters if stabilization is required. This is in striking contrast with much of the recent policy debate centred around the feasibility and optimality of interventions affecting the level of the state variables (e.g. money stocks) leaving unaltered the structural relations of the economy." (Goodwin-Pacini, p.255).

We investigated a simple dynamic IS-LM model with 'balanced budget' at the equilibrium and money financing of the temporary public deficit, focusing upon the issue of stability and cycles, especially in the presence of realistic revenue collection lags.

Using the Hopf theorem we showed that cyclical behaviours can emerge as public expenditure rises and when collection lags are appreciable (while a strong interest elasticity of money demand favours economic stability). We also showed, by means of numerical simulations, a wide variety of dynamic behaviours; when there is 'low' income taxation, stable local limit cycles as well as "bi-stable" behaviours are shown, while when the income taxation becomes higher chaotic oscillations can also be generated. The chaotic behaviour is very robust with respect to changes in the parameters as well as in the system states, with corresponding far-reaching implications for fiscal policy.

An increasing number of countries have recently adopted various forms of fiscal rules, mainly i) balanced-budget requirements, ii) debt limits and iii) expenditure limits which, as claimed by Kopits (2001), could contribute to stability and growth. As regards the present model, while the first two requirements are satisfied by assumption, the third requirement appears to be crucial for stability. To this end, this paper suggests two conclusions, contributing to shed new light on the stability effects of fiscal rules: 1) independent of the balance of the budget or the absence of debt financing, the "dimension" of the public intervention in the economy matters for stability; 2) also the introduction of rigid and transparent expenditures limits could be ineffective in terms of stability if the role of collection lags is not considered as well. Therefore our results suggest that income tax collection lags may trigger short-run regular and chaotic cycles, which can be entirely ascribed to the "mix" of fiscal policies; this contributes to shed new light on the stability effects of fiscal rules.

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Appendix 1. A numerical simulation of the system (5).

The above system (5) is a specific as well as simplified form of the system (1),(2),(3) of Sasakura (1994, p.436). [23] Although the latter author proves the existence of closed orbits, the issue of their uniqueness or attractivity is neglected. He is aware of the preference from the point of view of the business cycle theory for an attracting closed orbit, so he simply assumes it. However he recognizes that "this assumption might be quite unsatisfactory from a practical point of view….one may work out an example for the present 3-D case by means of the approximation of nonlinear investment function by an appropriate cubic term and the utilization of computer simulation technique…indeed investigation in this direction would be also worth making.." (p.439). Since the present model can also be viewed as an example of Sasakura's model, it would be interesting to check the existence of closed orbits and their uniqueness and stability.

In his proposition (4) on p.437 Sasakura provides the "bifurcation" value for $b=b_H$ generating at least one closed orbit, but leaving open the question as to whether this occurs for $b<b_H$ (that is the bifurcation into a closed orbit is supercritical and such an orbit is attracting) or for $b>b_H$ (that is the bifurcation into a closed orbit is subcritical and such an orbit is repelling). We notice that the definition of the question above obviously means that the loss of stability occurs for increasing b, or in economic terms, too slow adjusting monetary markets may destabilise. In the present model the "bifurcation" value of the same bifurcation parameter used by Sasakura, is

$$b_H = \frac{iG - a\rho\mu t^2\left(\dfrac{i}{s(1-t)}\right)^{-\rho+1}}{\rho^2\mu^2 t\left(\dfrac{i}{s(1-t)}\right)^{-2\rho}}$$

[23] We are grateful to an anonymous referee for having pointed out the similarity between the two models and suggested the numerical simulation.

A positive critical value ($b_H > 0$) always exists, provided that the public expenditure is sufficiently high, as already noted in the main text. Numerical investigations have shown that the bifurcation always occurs for $b > b_H$ and therefore, unfortunately in contrast with the assumption of Sasakura ("therefore we assume the supercritical case where $b < b_H$ "p.439), the closed orbit is repelling[24]. However, we conjecture that this may be due to the present investment function which is a monotonic decreasing function of the interest rate, whereas Sasakura (in line with many other business cycle models) assumes a non-monotonic investment function (for which he suggests an approximation by means of an appropriate cubic term).

Figure 6 reports a phase plane representation of the system (5) restricted to the two main variables Y, r for the following parameter configuration: $a=1$, $i=0.011$, $s=0.04$, $\rho=1.2$, $\mu=1$, $G=80$, $t=0.1$. The equilibrium values are ($Y^*=800$, $r^*=0.305$, $M^{s*}=4.14$) and the Hopf bifurcation value is $b=b_H=0.3489$: the picture shows the unstable limit cycle and the corresponding "corridor stability" when $b=0.35$; the corridor provides the measure of the sustainable shocks on the equilibrium values for Y,r (i.e. the stability is preserved if the rate of interest does not vary more than about 5% with respect to the equilibrium rate).

[24] Repelling closed orbits always occur also (MW: even??) when positive values of income elasticity of demand for money ($u>0$) are introduced.

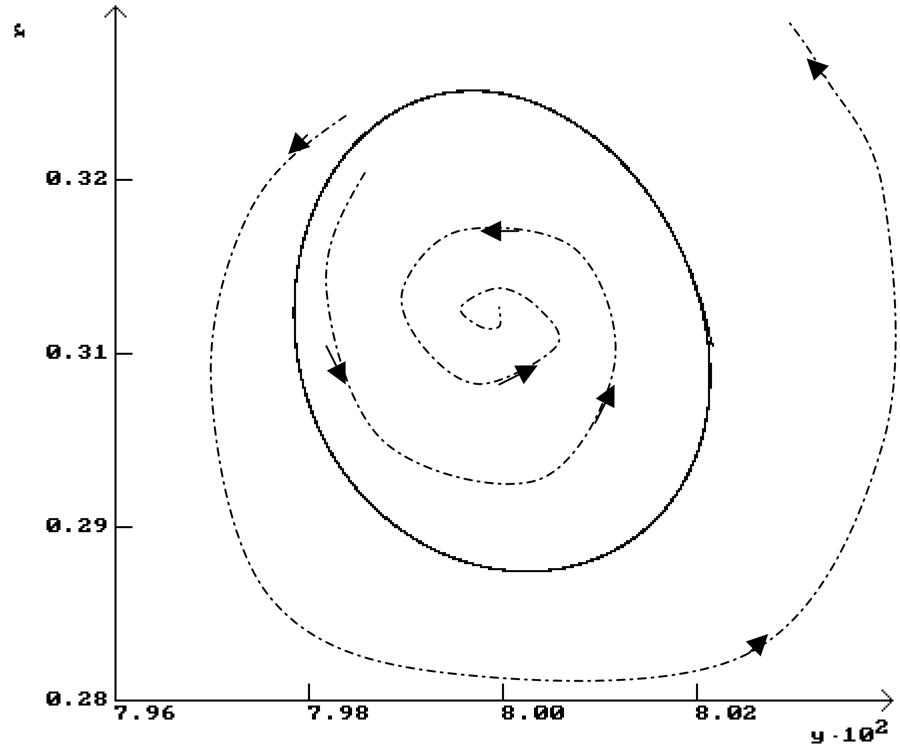

FIG. 6. Unstable limit cycle in the phase space *Y, r*.

Appendix 2. ***Lyapunov exponents and detection of chaos in the system (10).***

Lyapunov characteristic exponents (LCE) are widely known to be "the clearest measure to prove the existence of, and to quantify, chaos in a dynamic system or time series" (Brown, 1997, p. 53). In order to prove the existence of chaos we use the following definition: "A dissipative dynamical system is chaotic if the largest Lyapunov exponent is positive" (Lorenz, 1993, p.218) and estimate the LCEs under parameter values within the chaotic "zone" (as shown in the numerical example in the main text).

LCEs are related to various features of chaos and not only to its existence.

According to standard definitions an attractor is chaotic if 1) it is a closed invariant set indecomposable into smaller closed invariant sets; 2) it has a regular physical structure summarised by its fractal dimension (e.g. the

minimum number of degrees of freedom needed to have chaotic behaviour; of course the dimension of the chaotic attractor is smaller than the dimension of the phase space within which it is embedded); 3) it exhibits appreciable dependence on initial conditions (SDIC). The latter deals with the amplification of any arbitrarily small interval of initial values under the flow.
The Lyapunov exponent represents a quantity for measuring the rate of divergence of two initial values. The chaotic motion in the chaotic attractor remains bounded, implying that the 'flows' contracts the volume in some directions and stretches it in other directions; in the long term the stretching and folding of trajectories on a manifold usually (but not necessarily) determine a fractal geometry of the attractor. The Lyapunov exponent is closely related to the fractal dimension of the attractor according to the conjecture of Kaplan-Yorke: the general definition for an *n*-dimensional system of the fractal dimension via the Lyapunov exponent (or Lyapunov dimension) is $D_L = m + \frac{\sum_{i=1}^{m} LCE_i}{|LCE_{m+1}|}, \quad m < n$ where *m* satisfies the condition that

$$i)\ \sum_{i=1}^{m} LCE_i > 0; \quad ii)\ \sum_{i=1}^{m+1} LCE_i < 0 \quad .$$

In the present example, with the parameters in the main text and *d*=0.025 we have: *LCE1*=+0.68; *LCE2*=0; *LCE3*=- 0.0035; *LCE4*=- 117. Since the largest LCE is steadily positive (see Fig. 7), the existence of chaos is proven. Moreover the Lyapunov dimension is D_L=3.006 indicating a fractal dimension of the attractor.

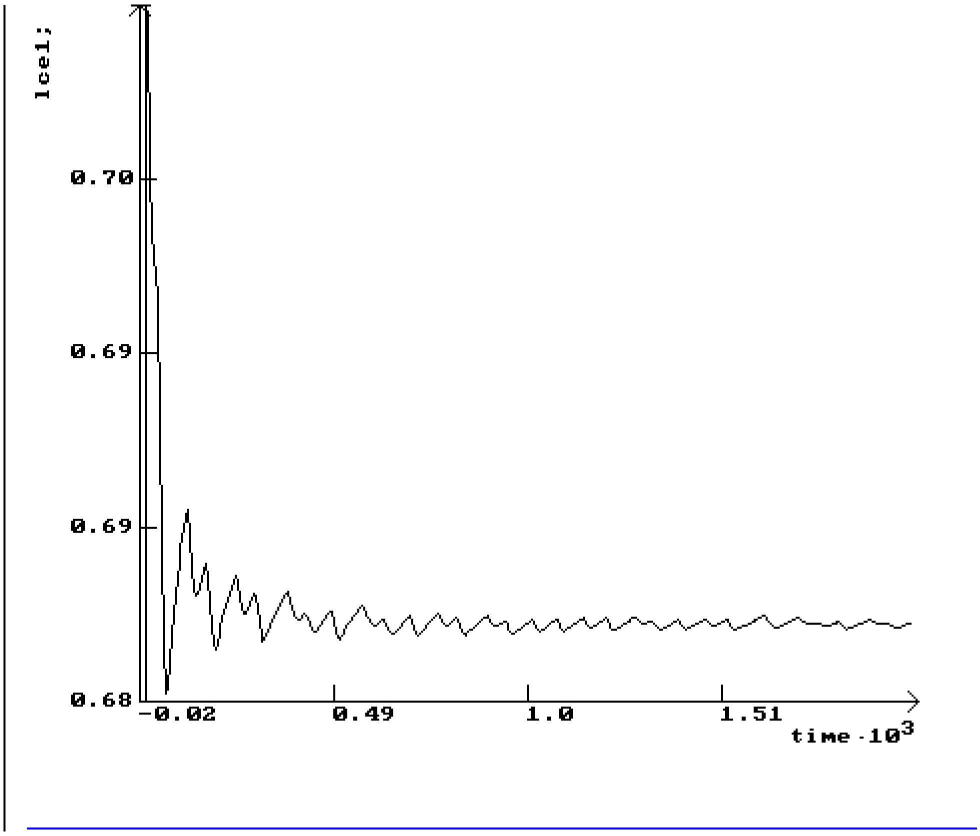

FIG. 7. The largest Lyapunov exponent

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