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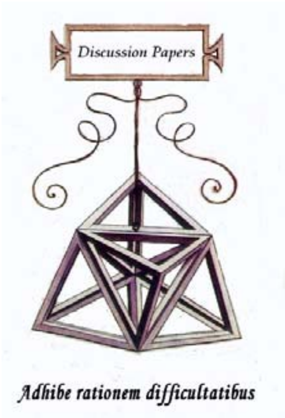

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The growth cycle and labour contract length

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Abstract

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This paper extends the growth cycle model à la Goodwin (1967) by introducing the risk-averse behaviour of the agents and a consequent positive correlation between wages and profitability. This extension is motivated by the impressive evidence on the joint role played by aggregate unemployment and lagged profitability in explaining wage determination. The effects of this extension in the growth cycle context are analysed, and the global and local dynamic effects of the union contract length are investigated. The following somewhat unexpected stabilisation policy rules have been argued: 1) in contrast with conventional wisdom the "local" stability criterion would suggest reducing the average length of labour contracts; 2) "global" considerations, however, could suggest a completely opposite policy, in the case in which the policy-makers prefer an almost globally stable fluctuation (resistant to strong shocks) to a small stable "corridor" (destroyed by very small shocks). Finally the deterministic business cycles triggered by the centralised contract length with risk-averse agents shown in this paper propose a role of uncertainty in determining economic fluctuations which somewhat differs from the manifestations of exogenous shocks to a fundamentally stable equilibrium proposed by Real Business Cycle theorists.

Classificazione JEL: E300, O4.
Keywords: growth cycle models, business cycles, labour contract length, Hopf bifurcation.

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I- Introduction

In recent years, corroborative evidence has emerged for the joint influence on wages of firm profitability and aggregate unemployment. This evidence has even been described as a statistical regularity or empirical “law” of economics (Blanchflower – Oswald, 1994). Indeed the dependence of wage dynamics on both unemployment and profitability components is extensively corroborated by various econometric works (Gregory et al., 1985; Rowlatt, 1986; Carruth-Oswald, 1987; Blanchflower et al., 1990, 1994, 1996) which reach the conclusion that wages are positively correlated with (lagged) levels of company profitability[1]. The gist of this view of wage determination can

[1] For example, from econometric estimation of aggregate time series data Carruth and Oswald (1987, p. 75) conclude that “we have found support from our wage equation for the idea that aggregate profit has a direct influence on real pay. (...) Britain has a rather small unemployment elasticity of real wages (just below -0.1) and a small profit elasticity of real wages (of approximately 0.05)”. Also, as regards the United States, both the pioneering contribution by Perry (1964) and the long-run econometric study by Dumenil and Levy (1993) acknowledge the statistical significance of the relationship between wages and the profit rate.

be actually summarised in the words of Blanchflower et al. (1990, p. 159): "British wage determination may be seen as a kind of rent-sharing in which workers appropriate a portion of profits and high external unemployment weakens workers' bargaining strength". On the other hand, this empirical evidence – namely the influence of the two components (unemployment and profitability) on wage dynamics – can be consistent with the theoretical literature on bargaining, on trade unions, on the economics of insider and outsider, on the efficiency wage and on the 'implicit' contract theory. We focus on the latter theory[2], borrowing from Blanchflower – Oswald (1990, 1994, 1996), to show that a link between the dynamics of wages and of profits emerges and that the original Phillips curve can be then augmented to take account of this new type of relationship.[3]

The aim of this paper is to develop a macroeconomic dynamic model by using Goodwin's model (1967) framework, embodying the issue of the relationship between wages and centralised labour contracts under uncertainty. In particular the existence, especially in Europe, of centralised "staggered" labour contracts[4], motivates the investigation of

[2] In recent decades characterized by economic shocks and rising unemployment, this theory has regained the interest of economists in uncertainty in imperfect labour markets, particularly in many industrialized countries with large unionized sectors, where labour contracts can provide efficient 'insurance' for firms and workers (e.g. Baily, 1974; Azariadis, 1975).

[3] This reconciles, as regards the dynamics of wage determination, the two lines of approach to the problem of wage rigidity: indeed so far modern theories concerning wage rigidity, such as the 'implicit contract' theory or the 'efficiency wage' theory, have focused on internal variables at firm level such as product demand and profits, whereas macroeconomics has followed the Phillips curve tradition, concentrating on external variables such as aggregate unemployment.

[4] For instance, as regards Italy, centralised contracts have a validity of about four years with respect to the normative aspects and about two years with

the macrodynamic effects of union contract length. Indeed, policy-makers and unions could be comforted if they knew whether and under what conditions customary union contract lengths are detrimental for economic stability; for instance, as regards Italy, the length of centralised contracts as well as the choice between centralised and decentralised contracts is a current political issue. As for the matter of contract length, the literature has mainly concentrated on static bargaining models limited to the labour market, also with uncertainty and risk-aversion (Danziger, 1988; Ragan, 1995) or, more rarely, extended in a static macroeconomic context (Holden, 1997)[5]. Dynamic features of such contracts have recently been investigated, but only in a dynamic strategic game context (Houba-von Lomwel, 2001). In all these cases many positive and negative aspects of the duration of labour contracts have been singled out but their dynamic role in a macrodynamic model has not yet been explored.

As regards the macroeconomic framework, Goodwin's model, although criticised for the lack of microfoundations, provides a convincing endogenous explanation of profit-unemployment fluctuations[6], typically arising as a consequence of profit-squeezes, which represent an important stylised fact of modern economies (Goldstein, 1999). As pointed out by Chiarella and Flaschel (1999), the cyclical profit-squeeze is such an ubiquitous phenomenon in realistic large-scale macroeconomic models that it is hard to discard this basic principle of

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respect to the economic aspects. In the U.S literature a union contract with three-year length is more frequently cited.

[5] For instance, Holden, via the Nash bargaining solution, determines the nominal wage in a small macroeconomic model with inflation, endogenous employment, decentralized wage bargaining and one-year contracts.

[6] The term profit-unemployment fluctuations is used to denote the synchronised oscillations in time series of the profit share and the unemployment rate in capitalist economies, as documented in Goldstein (1999).

Goodwin's model (which therefore represent a building block for "realistic" macro models). Moreover Harvie (2000) presents a direct econometric test of Goodwin's model, using postwar data for ten OECD countries and concludes that "the evidence presented here for the existence of Goodwin-type cycles is extremely encouraging, justifying both existing theoretical extensions of Goodwin's model and further empirical work in this area" (p. 349). Finally we note that Goodwin's model framework is a cornerstone of the recent revival of the work of economists in the field of economic fluctuations and cyclical growth, aiming at an endogenous explanation of economic fluctuations (e.g. Lorenz, 1993, 1994), contrasting, for instance, with the stochastically-driven fluctuations predicted by the Real Business Cycle theory.

This paper shows that, although the wage-profit dynamic correlation introduced by centralised optimal wage contracts would tend to dampen the fluctuations of the growth cycle model, since centralised contracts are plausibly both periodical and "staggered", therefore stable regular economic cycles can be generated. Such economic cycles crucially depend on the average length of labour contracts. Moreover such a length may play a different role depending on the global dynamic features of the economy.

The plan for the remainder of the paper is as follows: in section II we introduce the wage determination based on the 'implicit contract' hypothesis; in section III we set out the basic dynamic (I see you use the word "dynamic" here rather than "dynamic". Why don't you use "dynamic" throughout? It seems vastly preferable) model in Goodwin's framework, extended to the context of a labour market with bargaining and uncertainty; in section IV the steady state analysis of the model is performed; in section V the dynamic analysis is performed. Section VI extends the model presented in section III by considering length and staggering of labour contracts and analyses mostly the dynamic consequences of such an extension. Section VII shows the numerical simulations of the model. The final section VIII provides some concluding comments.

II. Wage dynamics and implicit contract theory

As described in the introduction, the empirical evidence reveals three stylised facts with a broad temporal and spatial robustness: a negative unemployment elasticity of wages; 2) a positive profit elasticity of wages; 3) a positive effect of lagged profits on current wages. Whatever theoretical interpretation underlies such statistical results, this paper investigates the macro-dynamic effects of the resulting more complex wage determination (which, in comparison with the traditional Phillips curve, involves a new variable such as total profits as well as time-delays). However, though only for illustrative purpose, we outline a theory in which a positive wage-profit dynamics correlation is to be expected.

As is well known, the crucial assumption of the implicit contract theory is that firms are risk-neutral and hence unaffected by profit variability while the workers are risk-averse and unable to access capital markets – that is unable to transfer via insurance, or simply by self-insurance via saving accumulation, the risk to capital markets - and hence interested to reduce wage variability. The lower risk aversion of firms is motivated by the fact that entrepreneurs may diversify the risk by diversifying their investment as well as by transferring part of the firm's risk to the capital market. However it is also plausible that firms are risk-averse rather risk-neutral, at least for two motives, the first concerning the shareholder owners of firms, the second concerning the firm's managers: 1) portfolio diversification ceases to be an efficient method to diversify the firm's risk when a positive correlation between profits of different firms appears; 2) even if the shareholders were risk-neutral, the existence of managers with private inside information who are not monitored, causing the problem of moral hazard, may ultimately make a firm become risk-averse. When firms are risk-averse there are two main implications: i) optimal wages vary when profitability varies and ii) wage variability increases when firms' risk aversion increases.

The model discussed in Blanchflower-Oswald (1994, ch.3) can be considered a generalization of an optimal labour contract model, with symmetric information, as in the 'implicit contract' literature, but in contrast with such a literature firms are risk-averse (instead of the original assumption of risk neutrality). Such a model assumes that firms and workers stipulate an implicit contract in which wages are determined taking account of the need to provide efficient 'insurance' against random demand shocks. The contract is the solution, in terms of a wage function defined on demand shocks, to the problem of the maximization of the firm's utility subject to the satisfaction of the predetermined minimum workers' utility. Following Blanchflower-Oswald (1994) we hypothesize that: i) demand shocks follow a probability density function $g(\mu)$; ii) the firm's and workers' utility depends upon respectively profits (P) and wages (w); iii) both the latter are represented by concave functions, respectively $v(P)$ and $u(w)$. Assuming for simplicity that the size of labour supply is normalized to unity, hence the probability of employment and unemployment are respectively L and (1-L), and assuming that an unemployment benefit (b) is exogenously given, then the workers' utility can be expressed as an average of the utility of the employment and unemployment states weighted for the respective probabilities to be in such states. Given the profit function $P=\mu f(L)-wL$, the problem is to find a wage function in terms of demand shocks as a solution to the following maximization

$$\max \int v(P) g(\mu) d\mu$$
$$\textit{subject to}$$
$$\int \left[u(w)L + u(b)(1-L)\right] g(\mu) d\mu = u^{\circ}$$

(1)

where $u°$ represents the market level of expected utility that the firms must satisfy; the first order conditions are[7]

$$v'(P)=\theta u'(w) \quad (1.1)$$

$$v'(P)[w-\mu f'(L)]=\theta[u(w)-u(b)] \quad (1.2)$$

where θ is the costate variable in the maximization problem (which is independent of μ). Eq. (1.1) defines implicitly wages as a function of profits, whose differentiation implies that

$$dw=v''(P)dP/[\theta u''(w)] \quad (1.3)$$

Remembering the relation between the second derivative of the utility function and risk behaviour, we can see that labour contracts imply a positive correlation between wage and profits as both parties are strictly risk-averse (or risk-lovers), while the relation is null (undefined) if firms are risk-neutral (the workers are risk-neutral). If we assume that the utility functions of both parties are of Constant relative risk aversion (Crra) type, we have, for instance in the case of the workers:

$$u(w)=\frac{w^{1-r}}{1-r}, r>0, r\neq 1 \quad (1.4)$$

where r is the inverse of the elasticity of the marginal utility. In a context of uncertainty, r is known to represent also the coefficient of relative risk aversion[8], which is defined (Diamond-Rotschild,1978) as

$$r= -u''(w)w/u'(w) \quad (1.5)$$

In a similar way, firms' relative risk aversion is

$$f= -v''(P)P/v'(P) \quad (1.6)$$

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[7] As usual, one dot represents the first derivative, two dots the second derivative.

[8] From now onward, for economy of space, the term 'risk aversion' will be used as synonymous with 'coefficient of relative risk aversion'.

From (1.1) we obtain $\theta = v'/u'$ and using equations (1.5-1.6) of the coefficients *r,f*
in the (1.3), we obtain

$$\frac{dw}{dP} = \frac{f}{r}\frac{w}{P} \tag{1.7}$$

and therefore the following dynamic wage-profit relation holds:

$$\frac{\dot{w}}{w} = \frac{f}{r}\frac{\dot{P}}{P} \tag{1.8}$$

In this way we have obtained a microfounded positive relation between the wage and profits dynamics, which is based on the assumption that in stipulating centralized labour contracts firms and unions choose to share the risk of demand fluctuations[9]. Interestingly, (1.7) shows that 1) elasticity of wages with respect to profits is equal to the ratio of the firms' relative risk aversion to the workers' relative risk aversion, 2) the predicted wage-profit correlation is with total profits rather than with profit-per-employee.[10]

[9] The extension of the static relation (equation 1.7) to the dynamic relation (equation 1.8) requires some qualifications. Indeed the former relation – according to which the insurance contract implies that wages and profits move together as a consequence of both being a function of a common exogenous shock - is "reduced form" and not structural in nature. Thus, extending this relationship *over time* (according to equation 1.8), although mathematically correct, is in general not warranted. In order to achieve consistency it could be argued that uncertainty only lies at the microeconomic level, but since the economy is not stationary, microeconomic shocks cannot be purely idiosyncratic as happens in a stationary economy and must be averaged on some sort of aggregate fundamental. This means that the distribution of shocks changes from time and agents can perfectly forecast this change. I am grateful to an anonymous referee for having made this qualification.

[10] This is preferable for an aggregate macroeconomic model, while it may be seen as a disadvantage for a partial micro model. However it must be noted that, unfortunately, the "labour contract" model is not also able to explain the role of the external unemployment rate resulting from the empirical evidence.

III. The basic model

As is well known, the growth cycle model of Goodwin is defined by the following system of two differential equations:

$$\begin{aligned} \frac{\dot{V}}{V} &= -\gamma + \rho U - \alpha \\ \frac{\dot{U}}{U} &= m - \alpha - n - mV \end{aligned} \qquad (2.1)$$

in two variables, U and V, where $U=U(t)$ is the employment rate level at time t (this level is defined as a ratio between the effective employment $L(t)$ and the supply of labour $N(t)$ (or alternatively the total population), and $V(t)$ is the distributive share of the wage earners, given by the ratio $V=wL/Q$, where w is the wage and Q is the production per unit of time. Moreover V can be expressed as: $V=w/a$, where a is the average productivity of work: $a=Q/L$. The model (2.1) is easily derived starting from the following "dynamic identity":

$$\frac{\dot{U}}{U} = \frac{\dot{L}}{L} - \frac{\dot{N}}{N}; \quad \frac{\dot{V}}{V} = \frac{\dot{w}}{w} - \frac{\dot{a}}{a} \qquad (2.2)$$

through six fundamental assumptions:

1) the wage dynamics is given by a Phillips relationship which is assumed to be simply linear:

$$\frac{\dot{w}}{w} = -\gamma + \rho U \qquad (2.3)$$

2) the wage earners don't save;

3) the profits are entirely reinvested;

4) the technology is 'Leontief-type';

5) the capital/production ratio, $K/Q=1/m$, is a constant;

6) the supply of labour and the labour productivity are growing at a constant rate (respectively $n>0$ and $\alpha>0$);

The model has only one non-zero equilibrium point (given plausible values of the parameters) with coordinates[11]:

$$U^* = \frac{\alpha + \gamma}{\rho}$$
$$V^* = \frac{m - \alpha - n}{m} \qquad (2.4)$$

As is well known the system (2.1) is a prey-predator model à la Lotka-Volterra, which shows conservative oscillatory behaviour, and which has, by definition of the two variables *U,V*, a dynamics bounded by the feasible set *T*=[0,1]x[0,1].

As regards the building of the present model, following the discussion of the previous sections, we assume, at variance with the original Goodwin model, that wage dynamics depends on two components[12]: 1) a component based on the unemployment in the labour market, as the usual Phillips relation, influencing mainly the workers' decentralized bargaining strength;[13] 2) a component bargained at the centralized level based on the 'implicit contract theory'; the latter component can imply a positive correlation of profits and wages or, in other words, a correlation between wage and profit dynamics[14].

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[11] It is easy to see that the condition for $U^*<1$ is $\rho>\gamma+\alpha$ and the condition for $0<V^*<1$ is $m>\alpha+n$.

[12] As a matter of fact, the only new hypothesis of our model is the introduction of a second component (profitability) in the wage equation à la Phillips.

[13] As regards this component Blanchflower-Oswald (1994) argue that " the foundation here is the notion that it is relative power that decides the wage outcome and that workers' power declines as local joblessness grows. When it is hard to get another job, the wage settlement tends to be at a lower level." (p. 96).

[14] In passing we note that the enrichment of the 'usual' Phillips relationship between wages and unemployment with a profit component could also be justified relating to Weitzman's approach (Weitzman,1985) on the grounds of which a profit-sharing agreement is assumed so that the growth rate of wages thus could include a bargained component indexed on the profit rate (see, in

The wage dynamics is described by the following[15]:

$$\frac{\dot{w}}{w}=-\gamma+\rho U+\frac{f}{r}\frac{\dot{P}}{P}=\frac{(-\gamma+\rho U+\frac{f}{r}m(1-V)+\alpha\frac{V}{1-V}\frac{f}{r})r(1-V)}{r(1-V)+fV}$$

(2.5)

The model (2.1), after substitution of (2.3) with (2.5), is the following:

$$\frac{\dot{V}}{V}=\frac{(-\gamma+\rho U+\frac{f}{r}m(1-V)-\alpha)r(1-V)}{r(1-V)+fV}$$

$$\frac{\dot{U}}{U}=m-\alpha-n-mV$$

(2.6)

IV. Steady-state analysis

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The model always shows the zero equilibrium (*V=0,U=0*) which obviously is not interesting; furthermore, according to the shape of the isoclines, the model can show two further non-zero equilibria localized on the axes *V* (*U=0* and respectively *V=1* or $V= 1-(\alpha+\gamma)r/(fm)$) and one equilibrium with positive values. The isoclines of the system, in addition to the isoclines (*V=0,U=0*), are

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the framework of Goodwin's model, Lordon, 1997 and Fanti-Manfredi, 1998). For the empirical evidence of the profit-sharing rule see for the Italian case, amongst others, Del Boca-Ichino (1992), and Del Boca-Cupaiolo (1997). Likewise the importance of the profit component could go back to Kaldor's (1959) hypothesis, arguing that the Phillips curve should have, as the independent variable, profits rather than unemployment.

[15] The rate of change of profit is derived with simple algebraic manipulations from the identity *P=(1-V)Q*.

$$U^* = \frac{r(\alpha+\gamma) - fm}{r\rho} + \frac{fmV}{r\rho}$$
$$V^* = \frac{m - \alpha - n}{m}$$

(3)

We are only interested in the positive equilibrium of the system[16], which is economically meaningful.

The positive equilibrium point of the model (2.7) is

$$U^* = \frac{r(\alpha+\gamma) - f(\alpha+n)}{r\rho} \qquad V^* = \frac{m - \alpha - n}{m} \tag{3.1}$$

In the steady-state analysis, it will be worth comparing the rate of employment in our model and in Goodwin's model:

$$U^* = U^{*G} - f(\alpha+n)/(r\rho)$$

(3.2)

where U^{*G} is the rate of employment of Goodwin's model (eq. 2.4). The equilibrium wage share is the same in both models.

The conditions for $0<U^*<1$ are:

1) $U^* >0$ if $r/f>(\alpha+n)/ (\alpha+\gamma)$ which is easily satisfied as soon as the workers' risk aversion is sufficiently high; it is evident that steady-state employment is lower when *i*) the firms are very risk-averse, *ii*) the workers are very strong irrespective of the unemployment situation, and *iii*) there is a high rate of growth of the labour supply. Note that the

[16] Nevertheless we can summarize the algebraic conditions for the existence of the second axes equilibrium; such an equilibrium exists: 1) in coexistence with the positive equilibrium, when $(\alpha+\gamma) < fm/r$; 2) as one and only equilibrium when $r(\alpha+\gamma)< f(\alpha+n)$.

Moreover it can be easily shown that the axes equilibrium, if it exists, is always bounded in the economically significant range of $V^*(0<V^*<1)$. By the way it can be noted that the undesirable outcome of a sole equilibrium with zero employment is favoured by a firm's risk-aversion higher than workers' risk-aversion, a high rate of growth of population and a strong autonomous workers' resistance (very low γ).

condition for the existence of a positive employment rate is also the condition for the existence of the positive equilibrium.

2) $U^{*}<1$ always holds given (3.2).

The main long-run results can be summarised as follows:

Remark 1: the long-run rate of employment is always lower than that of Goodwin's model; the extent of such a reduction is increasing (decreasing) with the firms' (workers') risk aversion.

Remark 2: the growth of labour supply (or of the population) reduces the rate of employment, in contrast with Goodwin's model.

Remark 3: in the present model the effect of labour productivity growth (LPG) on employment is ambiguous, whereas in Goodwin's model it always increases employment. Our model is capable of encompassing any relationship (positive, negative or null) between LPG and employment, depending on the relative degree of risk aversion of workers and firms. This may contribute to shed another light on the vexed question about the relation between labour productivity growth and employment. A well-known example arguing for the independence of the employment of the behaviour of productivity and capital accumulation is the influential work of Layard-Nickell-Jackman (1991) which, modelling wage bargaining and unemployment, reaches the powerful conclusion that equilibrium unemployment is completely unaffected by changes in exogenous labour productivity[17]. The consequences for policy-makers are clear: unemployment depends only on the labour market frictions and especially on the behaviour of the trade unions and on the link between wages and unemployment benefits, so that whatever policy aiming to influence employment should focus only on the labour market. Note that only in the special case of equal risk aversion for both firms and workers is the result popularized by Layard et al. (1991) obtained in the present model.

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[17] The reason for this is that, for example when LPG exogenously changes, bargainers will adjust wages downwards or upwards to keep the unemployment rate constant.

The economic intuition of the remarks above is as follows: in contrast with the original Goodwin model, the wage share also depends on profits and the latter are positively linked to the population and labour productivity growth via the steady-state relation V*(α,n), as the profits dynamics equation shows.[18] Since the wage share "preys" on employment, the novel results of the presence of a negative effect both of the population growth rate (*n*) and LPG rate (α) on equilibrium employment are due to the positive effect of *n* and α on the "predator" wage share (which in turn are due to the introduction of profits into the wage share dynamics).

V. Dynamic analysis

It is possible to show that the positive equilibrium is a locally stable focus (or node); its stability can be studied by looking at the following Jacobian matrix *J(V,U)*:

$$J=\begin{bmatrix} -\dfrac{fmV^*(1-V^*)}{fV^*+r(1-V^*)} & \dfrac{r\rho V^*(1-V^*)}{fV^*+r(1-V^*)} \\ -mU^* & 0 \end{bmatrix} \tag{4}$$

Simple inspection of this Jacobian shows that the trace is negative and the determinant is positive, so that the system is locally stable, as summarised in the following:

Proposition 1: compared with Goodwin's model, the positive equilibrium (3.1) is a stable focus (or node) instead of a centre; such an equilibrium is both locally asymptotically stable (Las) and globally asymptotically stable (Gas)[19].

[18] The profits dynamics equation is: $\frac{\dot{P}}{P}=-\frac{\dot{w}}{w}+m(1-V(\alpha,n))-\alpha$

[19] For an example of the methodology to demonstrate the Gas property in this type of model see Fanti-Manfredi (1995).

The negativity of the trace is a stability condition which can be interpreted in economic terms: an increase in the labour distributive share will decrease the bargaining pressure on wages, and consequently will reduce the growth rate of the same wage share, thereby favouring a convergent dynamics. Therefore the introduction of "implicit contract" motivations in the wage dynamics, in addition to the usual Phillips equation, tends to stabilise the fluctuations of the original Goodwin model. This result requires that labour contracts take account of current profitability or, in other words, assumes that they are continuously renewed. However, the latter assumption is not realistic, as we shall see in the next section.

VI- Periodic and "staggered" labour contracts: lagged profitability and dynamic consequences

The empirical evidence reveals that "the effect of profits takes time to work through into employees' remuneration" (Blanchflower et al*.,* 1996, p.234) and "the bulk of these wage rises would (..) come through by the third year after a burst of profits" (Blanchflower et al*.,* 1996, p. 232). One possible reason for the influence of lagged profitability on current wages is that labour contracts are periodic.

The importance of the periodicity of such contracts has been amply recognised in the economic literature: one of the most important recent macroeconomic theories is based on the existence of "staggered" labour contracts (e.g. Taylor, 1980).

Moreover many authors (e.g. Taylor, 1980) have emphasized that the effectiveness of macroeconomic policy depends on the length of time that wages are fixed or, in other words, on the length of wage contracts, mostly in the sense that when contract length increases, in comparison with the length of the reaction time of the policy makers, the role for stabilisation policy increases. Although the present formal framework is completely different from that of the "staggered contracts" literature, it is worth investigating the dynamic role played by the length of contracts. In contrast with such a literature, where the contracted wage is formed on the basis of what determines spot wages and the rule to obtain aggregate wages depends on the assumptions on the duration of contracts,[20] we simply assume that contracted wage depends on profitability and the rule of aggregation of wages – different by industry, region, firm, period of validity, and so on – is based on a Gamma distribution of (past) profitability.

Centralised wage bargaining is periodic (a labour contract is modified at long intervals)[21] and "staggered" because of numerous industry-wide bargaining practices with different time intervals[22], and especially in Europe the existence of unions as well as centralised labour contracts of different sectors with different periods of validity is evident. It should therefore be useful to explore the dynamic role of contract length from a policy viewpoint. Consequently, our next step will be the introduction of the lagged (rather than current) profit within our wage determination rule. Moreover we explicitly assume that the time delay be of the distributed type (as explicitly suggested in Invernizzi and Medio, 1991 or Medio,1992): such an approach seems to be certainly appropriate if the equations of the model do constitute aggregate representations of very large collections of firms and workers.

Hence the current profit should be substituted with a lagged profit, which implies substitution in (2.6) of the current wage share *V(t)* with

$$S(t)=\int_{-\infty}^{t}(V)(\tau)R(t-\tau)d\tau \qquad (5)$$

[20] We are indebted to an anonymous referee for having pointed out this distinction.

[21] For instance a frequently cited contract length in the "staggered contracts" literature, as regards the U.S., is the three-year union contract.

[22] For instance Taylor (1980, p. 2) argues that "wages are staggered, that is not all wage decisions in the economy are made at the same time".

where *R:[0,∞)→R*. The *R* function is also called “lag” function and its shape can be crucial for the stability of the system.

The density function *R* can have different forms: a very general form proposed in McDonald (1978) is the Gamma-type. The simplest forms (usual in economic dynamics) of the function R are the decreasing exponential distribution

$$R(\tau) = be^{-b\tau}, \quad b > 0 \qquad (5.1)$$

and the ‘humped’ distribution[23]:

$$R(\tau) = b^2 se^{-b\tau}, \quad b > 0$$

(5.2)

If system (2.7), with the substitution of (5), is an integro-differential system, that is it contains a continuous infinite lag, it is also equivalent to a differential ordinary equations system of higher order, provided that the density function R represents the solution of a linear differential equation with constant coefficients, as is the case of any polynomial multiplying an exponential function such as (5.1).

As regards the economic “rationale” underlying the choice of the time delay, we suggest (for instance, in order to better fit the ‘staggering’ contracts practice) that the the lagged profitability be represented through a humped distribution. In this event the humped distribution (eq. 5.2) could represent the (stationary over time) statistical distribution of the delay between consecutive bargaining rounds (see for instance Invernizzi and Medio, 1991).

Hence the system (2.7), taking account of the existence of periodic and staggered contracts, is transformed[24] into the following:

[23] The average value of the two distributions (exponential, humped) is respectively $\mu_E = \frac{1}{b}; \qquad \mu_H = \frac{2}{b}$

[24] For details about the transformation of an integro-differential system into an ODE system see McDonald (1978) and Fanti–Manfredi (1998).

$$\frac{\dot{Z}}{Z} = b(\frac{V}{Z} - 1)$$
$$\frac{\dot{S}}{S} = b(\frac{Z}{S} - 1)$$
$$\frac{\dot{V}}{V} = \frac{(-\gamma + \rho U + \frac{f}{r} m(1-S) - \alpha) r(1-S)}{r(1-S) + fS}$$
$$\frac{\dot{U}}{U} = m - \alpha - n - mV$$

(5.3)

The system (5.3) can represent a fair proxy in continuous time of the econometric evidence arguing that "changes in workers' remuneration follow earlier movements in profitability" [25] (Blanchflower et al., 1996, p.236)[26]. However, we note that the Gamma distribution of lagged profits is a statistical device which aggregates different agents' behaviours, whose possibly microfounded behavioural relationships are not investigated in this paper.

The positive equilibrium of this system is the same as that of system (2.7) *(U*, Z*=S*=V**).

[25] For instance Blanchflower et al., 1996, tables II and III, show that the effect of current unemployment and of the second and the third lag of profits - with a humped profile of the level of importance - are highly significant. By contrast, lagged unemployment is not significant.

[26] Note that in wage dynamics, while profitability is delayed, employment is still the current one. This is mainly due to the econometric evidence: for instance Blanchflower-Oswald–Sanfey (1996, p.232) claim that "*lagged unemployment* contributed nothing extra". However, in order to be consistent with the assumption of staggered contracts, the implicit assumption should hold that if such contracts, possibly fixed for many years, allow for future (un)employment to affect current wages, then (un)employment should also be perfectly forecastable. We thank an anonymous referee for this suggestion.

As is well-known, through a bifurcational approach the dynamic system is considered to belong to a family of systems which depend on a “control” parameter which will be responsible through its own changes of the morphogenesis of the system; when, following a change in the “control” parameter, in the case of a fourth-dimensional system, two eigenvalues of the linearized system around the equilibrium cross the imaginary axis at non-zero speed (and the other two eigenvalues are non-zero) a Hopf-bifurcation emerges which guarantees, under very general conditions, the existence of periodic solutions in the neighbourhood of the equilibrium point. Then we can identify the critical level of the “control” parameter which generates a limit cycle. Unfortunately, as in our case, this critical level could be represented by a complicated non-linear function of all the other parameters, so that it could be difficult to provide a thorough analysis of all the economic factors influencing the stability.

Since we are interested in the dynamic role of contract length, we choose the inverse of the average profit lag, b, as the "bifurcation" parameter. Then the dynamics of the system (5.3) can be summarized thus:

Proposition 2: if the average lag of profits considered in the stipulation of the wage contract is sufficiently low ($b>b_H$) – that is the period of validity of labour contracts is sufficiently short - the economy is locally stable (Las); when parameter b decreases, the equilibrium point ($U^*,Z^*=S^*=V^*$), which was stable, shows a Hopf’s bifurcation at $b=b_H$ and a local limit cycle can be generated, where

$$b_H = \frac{\sqrt{mV^*(1-V^*)\left[V^* f^2(m(1-V^*)+16(\alpha+n))+16fr(\alpha+\gamma V^*+n(1-V^*))+16r^2\gamma(1-V^*)(\alpha+\gamma)\right]}}{4\psi} > 0$$

where $\psi = fV^*+r(1-V^*)$

(5.4)

Proof:

The linearization of the system (5.3) around its equilibrium point gives us the following Jacobian:

$$J=\begin{bmatrix} -b & 0 & b & 0 \\ b & -b & 0 & 0 \\ 0 & \dfrac{-fmV^*(1-V^*)}{fV^*+r(1-V^*)} & 0 & \dfrac{r\rho V^*(1-V^*)}{fV^*+r(1-V^*)} \\ 0 & 0 & -mU^* & 0 \end{bmatrix}$$

(5.5)

We write the polynomial form of the characteristic equation in order to ascertain if the system is stable around its equilibrium point *U**, *Z*=V*=S**, controlling that all the roots of the polynomial have negative real parts. The characteristic equation is

$$k^4+2bk^3+\frac{b^2\psi+rmU^*V^*\rho}{\psi}k^2+\frac{b^2fmV^*+2brmU^*V^*\rho}{\psi}k+\frac{b^2rm\rho U^*V^*(1-V^*)}{\psi}=0$$

(5.6)

Assume the following polynomial coefficients

$$a_0=1;a_1=2b;a_2=\frac{b^2\psi+rmU^*V^*\rho}{\psi};a_3=\frac{b^2fmV^*+2brmU^*V^*\rho}{\psi};a_4=\frac{b^2rm\rho U^*V^*(1-V^*)}{\psi}$$

(5.7)

To demonstrate the existence[27] of Hopf's bifurcation, we apply one of the sets of necessary and sufficient conditions of Lienard-Chipart[28]: the polynomial roots have a negative real part if and only if the following inequalities are simultaneously satisfied: $a_4>0;a_2>0;\Delta_1>0;\Delta_3>0$, where Δ indicates a determinant of a specified order. As demonstrated in Fanti-Manfredi (1998), with reference to the dynamic system of fourth-order, such above conditions with $\Delta_3=0$ are necessary and sufficient to have two complex eigenvalues of the linearized system around the equilibrium crossing the imaginary axis and the other two non-zero eigenvalues. The crossing of the imaginary axes at non-zero speed is the other requisite for the existence of the bifurcation (for

[27] The demonstration strictly follows Fanti-Manfredi (1998).

[28] For a treatment of these conditions see Gandolfo (1996, ch.16).

instance to exclude that the eigenvalue function had an horizontal inflection point at the bifurcation value); such a requisite is satisfied here[29].

From a simple observation of the characteristic equation, we see that the first three inequalities are always satisfied. By contrast, the fourth

$$\Delta_3 = \frac{b^3 fmV^*(1-V^*)}{\psi}\left[2b^2\psi - bfmV^*(1-V^*) - 2rm\rho U^*V^*(1-V^*)\right] > 0 \qquad (5.8)$$

requires, to be satisfied, that the parameters satisfy certain relationships.

The fourth inequality is satisfied *iff*

$$F(b) = \left[2b^2\psi - bfmV^*(1-V^*) - 2rm\rho U^*V^*(1-V^*)\right] > 0 \qquad (5.9)$$

From inspection of (5.9) we can obtain the following remark:

Remark 4: since $F(b=0)<0$ and the coefficient of the quadratic term is positive (the function is increasing to the right towards infinity) the equation $F(b)=0$ has two distinct real roots (if they exist), which are of opposite sign. Obviously only the positive root (positive average lag) has an economic meaning.

The bifurcation value of the lag appears in a complicated relation with all the model parameters. We wish particularly to stress the interplay between the lag in the profit dynamics incorporated in the wage contract and the firms' and workers' risk aversion. Observation of the bifurcation function (5.4) leads to the following :

Remark 5: the following conditions on the parameters affect stability and cycles: an increase in productivity growth α and a reduction (a higher γ) in autonomous workers' resistance favour stability; on the contrary, the appearance of the limit cycle is favoured by a sufficiently high average lag of profits and a sufficiently low workers' risk aversion.

The latter relation is depicted in figure 1: the workers' risk aversion,

[29] We do not report here the calculations (available on request) of $dRe(k)/db \neq 0$ at $b=b_h$ where $Re(k)$ is the real part of the complex eigenvalues, for which a method in the case of the fourth-dimensional system is shown in Fanti-Manfredi (1998).

given the average profit lag, must be high enough to generate a limit cycle behaviour. However the bifurcation curve $b_H(r)$ appears fairly flat such that, for a low profit average lag, the system is always locally stable however high the workers' risk aversion. The same figure illustrates the preceding remark as regards the relation between the 'critical' lag and parameters α and γ.

Remark 6: the rise in the firm's risk aversion raises the "critical" lag; this also means that the firm's risk neutrality (often postulated in economics) would reduce the region of stability. Summarising the firm's risk-averse behaviour enlarges the region of stability, while the workers' risk-averse behaviour reduces it.

Remark 7: the role of the agents' risk aversion differs according to whether the focus is on the stationary state value or on economic stability: while on the one hand long-run employment is increased by the risk-averse behaviour of workers (and vice versa as regards firms), on the other such risk-averse behaviour tends to reduce the region of stability (and again vice versa as regards firms). Overall, the degree of the risk aversion of the different agents may imply in a certain sense a trade-off in social welfare (more employment versus more economic stability). In other words, high profit elasticity of real wages reduces the likelihood of fluctuations but unfortunately also increases the long-run unemployment rate.

Finally the main remark emerging from proposition 2 is the following:

Remark 8: the shorter the period of validity of the labour contracts incorporating the "insurance" motive, the more likely is the stability.

The intuition behind these results is straightforward and depends on the fact that actually labour contracts linking wage dynamics to current profitability are stabilising (as claimed in proposition 1): consequently we observe that, as regards remark 6, the effect of (stabilising) profit dynamics is larger (smaller) when the firms' risk aversion is larger (smaller) (and conversely as regards workers' risk aversion). With regard to remark 8 it becomes important that the lagged profitability embodied in the contracts are not “too” lagged, or, in other words, that labour contracts are not too long-lasting.

Resulting from remark 8 one stabilisation policy rule would seem to emerge: policy-makers and agents should work for the reduction of the period of validity of labour contracts. However this clear result – which could appear somewhat counterintuitive[30] - has only “local” validity: indeed global analysis via numerical simulation reveals richer dynamic outcomes as we shall see in the next section.

VII - A numerical simulation

The Hopf bifurcation theorem does not affirm the uniqueness or the stability of the resulting limit cycles. The question whether the bifurcation is supercritical or subcritical - that is whether or not the periodic orbit which bifurcates from the stationary state is itself stable - is not tackled here because it requires very cumbersome algebra whose interpretation is generally economic meaningless. Moreover it is worth remembering that “the Hopf bifurcation theorem is local in character and only makes predictions for regions of parameter and phase space of unspecified size” (Medio, 1992, ch.2).

Without going into detail about the qualitative analysis of uniqueness (Farkas, 1995, ch.7) or stability (Guckenheimer-Holmes,1983) problems, we use numerical simulations to investigate the properties of the limit cycles, in particular the “global” validity of the “local” analytical results of previous sections.

[30] Recall that, for instance, the message implied in the “staggered” contract literature (e.g. Taylor, 1980) is that the effectiveness of macroeconomic policy depends on the length of wage contracts in the sense that the role for stabilisation policy increases when wage contract length increases (in comparison with the length of the reaction time of the policy-makers).

We turn now to numerically investigate model (5.3). Assume the following parameter values:

$f=0.5;\ r=7;\ m=0.25;\ \alpha=n=0.01;\ \gamma=1;\ \rho=2$

Note that the choice of values of the firms' and workers' relative risk aversion, implying a profit elasticity of wages of about 0.075, is compatible with many econometric estimates (*e.g.* see note 1). In this case the steady state is given by U*=0.505; V*=S*= Z*=0.92

In line with the bifurcation function (5.4), the system is locally stable for $b \geq 0.35$; for values lower than, but close to, $b=b_H=0.35$ a supercritical Hopf bifurcation emerges.

Figure 3 shows, in the *U,V* phase diagram, that, with the following starting points $U°=0.52;\ V°=0.90;\ S°=.70,\ Z°=0.70$ and $b=0.30$, the trajectories approach a stable limit cycle and the movement is clockwise; the limit cycle shows an amplitude with oscillations between about *49%* and *52%* for the employment rate and between *88%* and *96%* for the wage share. It is worth noting that all the stable economic cycles obtained in our simulations are not only viable[31], but show a fairly realistic amplitude.

In particular we observe very interesting, and so far unobserved, global dynamic behaviours: 1) when *b* lies in the region of instability one stable local limit cycle always persists for a wide range of *b* values (that is the economy never explodes for plausible predetermined values of the variables) and – even more economically significant - the

[31] That is to say, the limit cycle stays within economically meaningful bounds when the initial conditions are within these limits; for instance, by definition of the variables U,V the economically feasible set is T= [0,1]x[0,1]. Goodwin's model has no type of "control" capable of ensuring that the natural dynamics of the model is always bounded in the admissible region, unless we introduce some rather artificial assumption concerning possible dynamics on its boundaries. Therefore the fact of obtaining a meaningful dynamics of employment and distributive shares for economically plausible values of the structural parameters may appear remarkable.

amplitude of the stable fluctuation is always included within a small acceptable range also when *b* tends to zero; 2) the external basin of attraction (from outside) of the stable limit cycle is very ample above all when *b* is small (so that in most of the feasible set of the phase space the system converges to the stable limit cycle, that is the economy appears almost globally stable), but tends to reduce when *b* increases approaching b_H; 3) when *b* goes beyond b_H, the economy becomes locally stable, but the basin of attraction of the equilibrium - so-called "corridor stability" according to the terminology of Leijonfvhud (1973) - unfortunately becomes smaller and continues to be small, albeit slightly enlarging, also when *b* further increases. In the following table, for some values of *b*, the corresponding predetermined values of the rate of employment at which the economy escapes from the basins of attraction either of the stable limit cycle or of the stable equilibrium point – that is the "critical exploding" values - are shown[32] (*ceteribus paribus* as regards the other state variables *V, S, Z*)[33]:

Table 1

B values (b_H=0.35)	*Type of local behaviour*	*"Exploding" U(t) values (in brackets the % change w.r.t U*=0.505)*
b=0.15	Stable limit cycle and unstable equilibrium point	U(t)=0.22 (-56.5%) U(t)= 0.59 (+36.6%)
b= 0.20	Stable limit cycle and	U(t)= 0.36 (- 28.7%) U(t)= 0.567 (+ 30.1%)

32 Obviously at any instant, the rate of employment may differ from the equilibrium value *U*=0.505* due for instance to an economic shock; the "critical" difference is also shown in percentage terms in brackets.

33 Obviously a similar table can be presented with respect to the other state variables *V,S,Z*.

	unstable equilibrium point	
b=0.37	Stable equilibrium point	U(t)= 0.496 (- 1.7%) U(t)= 0.513 (+ 1.5 %)
b=0.50	Stable equilibrium point	U(t)= 0.493 (- 4.3 %) U(t)= 0.515 (+ 2 %)
b= 1	Stable equilibrium point	U(t)= 0.4905 (- 3 %) U(t)= 0.518 (+ 2.6%)

The observation of the table 1 confirms the aforementioned considerations: when *b= 0.15* (average length of labour contract about 80 months) there is a stable small regular fluctuation which can be destroyed (thus causing the explosion of the economy) only if a strong implausible shock on employment occurs: e.g. a negative shock of *–56.6%* or a positive shock of *+36.6%.* By contrast, when *b=0.37* (average labour contract length about 32 months) the equilibrium point is stable but unfortunately a very small shock on employment can displace the economy from the corridor stability: e.g. a negative shock of *–1.7%* or a positive shock of *+1.5%,* which frequently occurs in the economic system, is sufficient to cause the economy to explode. Furthermore, while on the one hand the amplitude of the oscillations only increases in a really negligible manner[34], on the other hand the region of stability is enormously enlarged (see table 1), tending to

[34] For instance when *b=0.3* (average labour contract length about 40 months) the oscillations are between about *49%* and *52%* for the employment rate and between *88%* and *96%* for the wage share and when *b=0.15* the oscillations are between about *48.5%* and *52.5%* for the employment rate and between *87%* and *97%* for the wage share.

occupy the entire phase space when *b* decreases towards zero. The above remarks lead us to the following unexpected stabilisation policy rule: if the local stability criterion suggests reducing the average labour contract length (*b* increases)- as shown in the previous section- by taking account of global behaviour it could be worth considering the completely opposite policy: that is to increase the average length (*b* decreases), in that an almost globally stable fluctuation could be welfare-preferred (resistant to strong shocks) to a stable equilibrium point surrounded by a small stable "corridor" (destroyed by very small shocks)[35]. This point is clearly illustrated by figs. 2-3, (which are drawn with the same size of the phase space to facilitate comparison): for example (with the same parameters and initial conditions in the text above) when the average contract length is about four years (*b=0.25*) the equilibrium point is unstable (fig. 2), while when the length is reduced to about 32 months (*b=0.37)* the equilibrium is stable (fig.3). However, inspection of the dynamic behaviour in the "large" allows for richer interpretations of the stability in distinguishing between the two cases. In the case of the unstable equilibrium point the trajectories converge to a stable limit cycle, showing a very small amplitude of fluctuation. Such a limit cycle is stable both from inside and from outside and its external basin of attraction is sufficiently large to avoid a possible realistic shock on employment driving the economy out of such a basin. For instance, only sudden reductions of the employment rate greater than about 20% (U(t)<41%) or sudden increases greater than about 18% (U(t)>58.5%) with respect to the equilibrium employment rate can lead to global destabilisation. In the case of the stable equilibrium, unfortunately, the basin of attraction of the equilibrium (the "corridor" stability) is very narrow, as fig. 3 neatly shows: from table 1 we know that even small shock between 1% and

[35] Fanti–Manfredi (1997) argue that the policy of stabilisation be revisited in the non-linear dynamic macromodels, because of the new notions of orbital stability or instability and the relative complex management of policies.

2% can move the economy far from the corridor. A simple glance at the two figures shows that the size of "corridor" stability is far smaller than the portion of the phase space where the trajectories are always attracted from the stable limit cycle.

INSERT FIGS.2-3.

Therefore proposition 2 and the numerical simulation jointly taken show a very interesting result since, given the initial conditions and the likelihood of external shocks on such conditions, they allow us to single out the most appropriate policy as regards the choice of the labour contract length.

VIII. Conclusions

This paper analysed the influence of centralised "implicit" labour contracts on wage dynamics and provided answers to questions regarding the global and local dynamic effects of the union contract length, in the growth cycle context (Goodwin, 1967).

The first result is that such contracts would stabilize the economy, sterilizing the well-known oscillatory conservative behaviour of the original Goodwin model, but due to the existence of periodic and staggered contracts – which introduce lagged profitability in the optimal wage contract - a stable economic cycle may occur.

The investigation of the role of average contract length allows for the following somewhat unexpected stabilisation policy rules: 1) the local stability criterion would suggest – in contrast with conventional wisdom - reducing the average length of labour contracts; 2) taking account of the global behaviour a completely opposite policy could be suggested, to the extent that an almost globally stable fluctuation (resistant to strong shocks) could be preferred, for instance from a welfare viewpoint, to a stable equilibrium point surrounded by a small stable "corridor" (destroyed by very small shocks).

Moreover the role of risk-aversion parameters implied in the implicit wage contract both on long-run employment and on stability has been singled out, by evidencing a 'trade-off' between higher long-run employment and stronger economic stability: the firms' risk aversion both extends the region of stability and also reduces long-run employment, while the workers' risk-aversion produces the opposite results.

It is worth noting that the "implicit contracts" assumption is capable from a static viewpoint to reduce long-run employment and from a dynamic viewpoint allows for economic cycles as well as introducing possible sensitivity of the economic equilibrium to small external shocks.

To sum up, this paper has enriched the conventional growth cycle model, by introducing risk-averse behaviour of the agents and a positive correlation between wages and profitability - widely corroborated by econometric results - by propping it upon a microfounded optimal labour contract with firm's demand uncertainty, and has shown a very wide range of different dynamic results, depending on multiple factors (e.g. 'history' of the economy and/or external shocks on the economy). Finally the deterministic business cycles triggered by risk-averse behaviour of the agents shown in this paper propose a role of uncertainty in determining economic fluctuations somewhat different from the manifestations of exogenous shocks to a fundamentally stable equilibrium proposed by Real Business Cycle theorists (e.g. Kydland-Prescott, 1982).

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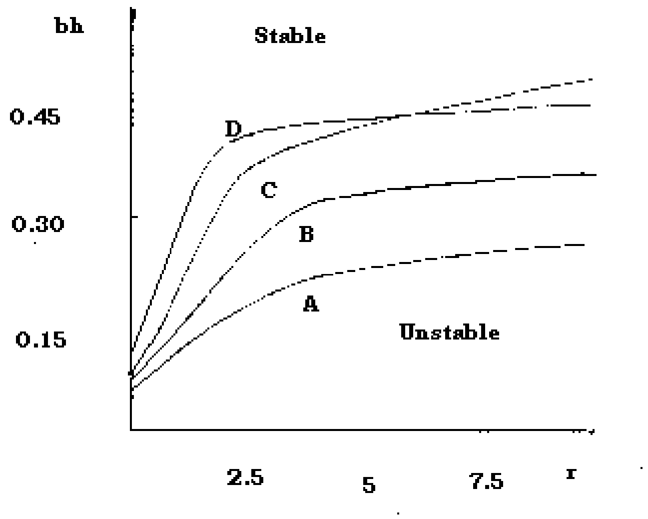

Figure 1. The bifurcation value b_h as a function of workers' relative risk aversion coefficient *r*, given different values of others parameters. Legend*: A: α=0.01;γ=.5; f=0.5; B: α=0.01; γ=1; f=0.5; C: α=0.01; γ=1; f=0.25; D: α=0.05; γ=1; f= 0.5.* For *A,B,C,D m=0.25, n=0.008.*

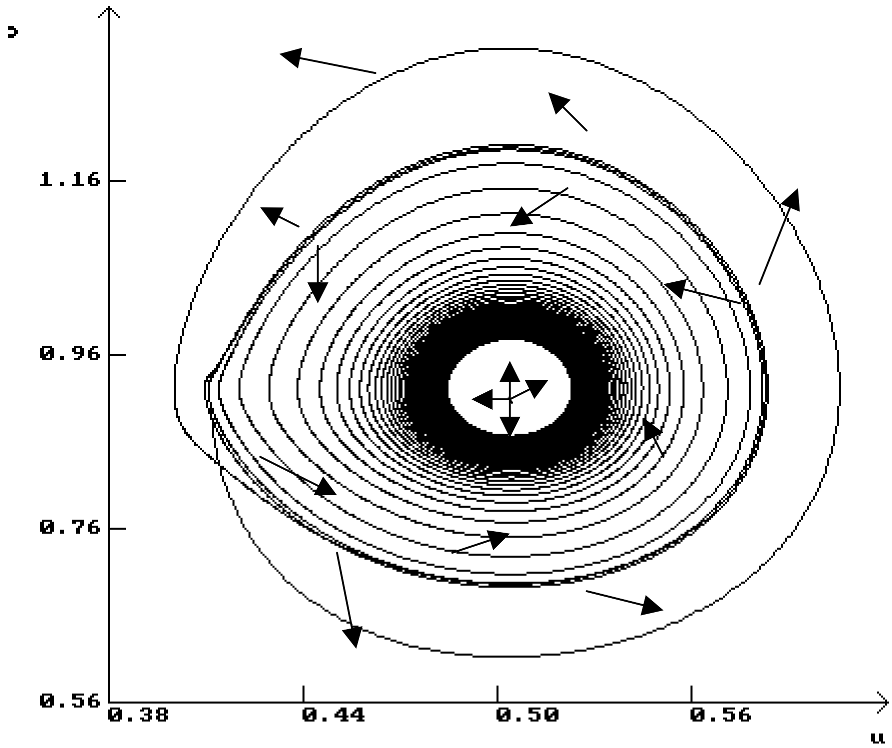

Figure 2. The phase portrait of the system 5.3 when *b= 0.25. Legend: parameter set in the main text, software DMC, fixed step size 0.01*

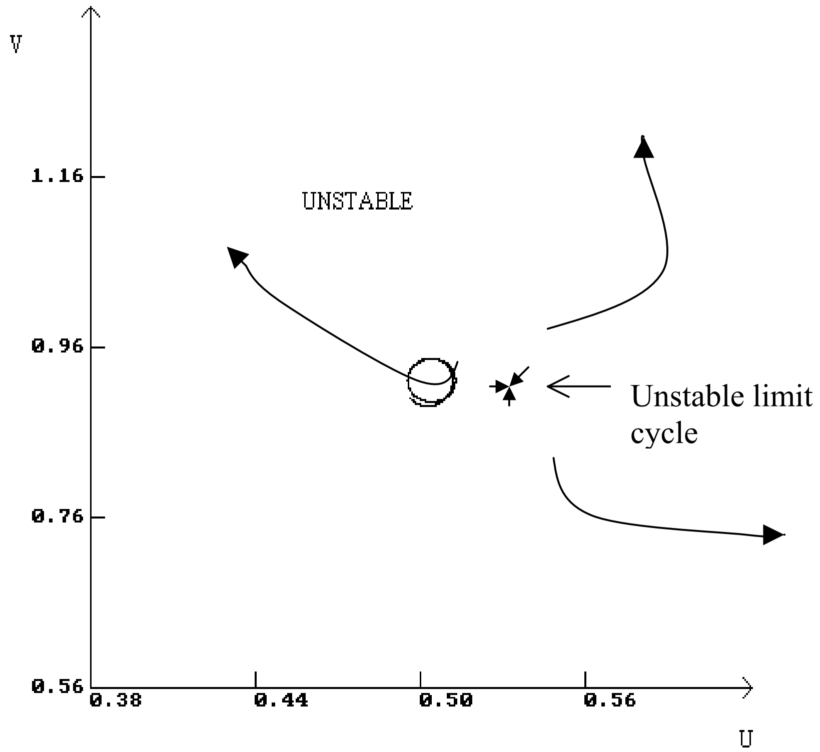

Figure 3. The phase portrait of the system 5.3 when *b= 0.25. Legend: as in fig. 2.*

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