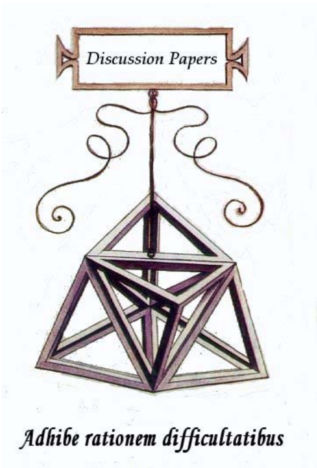
*Discussion Papers*
Collana di
E-papers del Dipartimento di Scienze Economiche – Università di Pisa




Davide Fiaschi

# Fiscal policy and welfare in an endogenous growth model with heterogeneous endowments

*Discussion Paper* n. 18

2003

*Discussion Paper* n. 18, presentato: **ottobre 2003**

**Indirizzi dell'Autore**: Davide Fiaschi
Dipartimento di scienze economiche, via Ridolfi 10, 56100 PISA
fax: (39 +) 050 598040
e-mail : dfiaschi@ec.unipi.it



Si prega di citare così: Davide Fiaschi, "Fiscal policy and welfare in an endogenous growth model with heterogeneous endowments", Discussion Papers del Dipartimento di Scienze Economiche – Università di Pisa, n. 18 (http://www-dse.ec.unipi.it/ricerca/discussion-papers.htm).

*Discussion Paper*
n. 18

Davide Fiaschi

# Fiscal policy and welfare in an endogenous growth model with heterogeneous endowments

**Abstract**

This paper analyzes an endogenous growth model where agents have different factor endowments and government finances public expenditure by imposing two flat-tax rates, one on capital income and one on labor income. The main finding is that, in the absence of lump-sum redistributions, heterogeneity of endowments is crucial to determine the optimal fiscal policy; in particular, taxing capital income is always optimal.

**Classificazione JEL:** H21; E13; D31; D3; H23.
**Keywords:** Heterogeneous agents, Efficiency, Equity; Majority voting.

*Contents*

## I. Introduction

This paper analyses optimal taxation in a model of endogenous growth where agents have different endowments and government finances public investment by means of two flat-tax rates, one on labor income and the other on capital income.

It is a common finding in literature that optimal fiscal policy involves zero capital tax. Judd (1985) shows that the optimal fiscal policy in the standard neoclassical growth model should not tax capital whenever other financing sources are available (see also Chamley (1986)) and Lucas (1990) extends this conclusion to endogenous growth models with human capital (see also Jones, Manuelli and Rossi (1997)). Finally Judd (1999) finds the same result in a growth model with public expenditure.[1]

However all these results are based on a representative agent hypothesis[2]; when we consider agents with heterogeneous endowments, this has implications in terms of income distribution. The fiscal regime where the tax rate on capital income is zero maximizes the growth rate but damages agents who have a low endowment of capital; then, even for a social planner indifferent to equity taxing income capital would be optimal.[3] The result holds provided that more efficient redistributive instruments are not available.[4]

Welfare analysis is here based on the Lorenz dominance concept.[5] Two paradigmatic fiscal policies are determined: the former maximizing a Rawlsian welfare function (the social planner cares only for the worst-off agent) and the latter the simple sum of all the in-

[1] Many contributions highlight the time inconsistency of this fiscal policy (see for example Alesina and Rodrik (1994)). However, as we will show, the optimal fiscal policy in our model is time-consistent because it also maximizes the output of every period (see Benhabib, Rustichini and Velasco (2001)).

[2] Judd (1985) claims that his result also holds for an economy where individuals have different factor endowments.

[3] A social planner is considered to be indifferent to equity if she/he ranks alternative fiscal policies on the basis of the simple sum of individual utilities. This may be considered as a measure of efficiency in an economy where agents have heterogeneous endowments.

[4] Stiglitz (1987) stresses the importance for welfare economics to study a world without discriminatory lump-sum taxes, given their difficult implementation.

[5] See Shorrocks and Foster (1987). This is the main difference with respect to Correia (1999).

dividual utilities (a possible measure of efficiency). These two polar cases are the bounds of the interval to which all socially optimal fiscal policies belong; intuitively the higher the social planner's inequality aversion, the closer will be the welfare-maximizing fiscal policy to that which maximizes the Rawlsian welfare function.

The main result is that for any uneven distribution of initial factor endowments, the fiscal policy which maximizes growth does not belong to the set of socially optimal fiscal policies. In other words, if initial endowments are heterogeneous, taxing capital income is a *necessary* condition to maximize welfare. The distortion in the accumulation of capital for poor agents is more than offset by the increase in redistribution, at least for the fiscal policy where tax on capital income is zero.

The key aspect of the model is to consider explicitely a growing economy: Judd (1999), pag. 5, shows that a constant positive tax rate on capital income is equivalent to an explosive commodity tax rate; the latter implies a explosive distortion and therefore it suggests not to tax capital income. However, to balance Government budget, the lower is the tax on capital income the higher is the tax on wage income. In turn, higher tax on wage income implies a decrease in the initial consumption of all agents, whose effect is also explosive in a growing economy.[6] Moreover, the poor agents are the more affected by this decrease in the initial consumption, since they are relatively more endowed of labor. Therefore some agents, typically the poorest, get the maximum utility by a positive tax on capital income, which implies a lower tax on wage income. This also explains because differently from Judd (1985), pag. 72, in our model workers prefer a positive tax on capital income in the long-run.

The paper extends the literature on optimal taxation of capital income. Judd (1985) . From an analytical point of view the model is close to Alesina and Rodrik (1994), but they consider taxation only on capital and do not focus their attention on the welfare effects

[6]The difference in the consumption paths corresponding to different levels of tax on wage income is growing at the same rate of the growth rate of economy.

of different fiscal policies; the same consideration holds for Bertola (1993). Judd (1999) analyzes the welfare property of the taxation of capital income in a model where public expenditure is productive and optimally decided, but all agents have equal endowments, which leads to the standard result of zero optimal taxation on capital income.[7] Judd (1985) find the same result in a heterogeneous agents economy, but as stated above, he considers an economy not growing in the long-run and this is crucial for the result.

It is worth to remark that the positive tax on capital income is not the result of limiting our analysis to log-utility as in Lansing (1999); in fact, Appendix E shows that the same finding is found when agents have an intertemporal elasticity of substitution of consumption different from 1.

Finally our result is a contribute to the literature on time-consistent fiscal policy

We refer to Shorrocks and Foster (1987) for the (generalized) Lorenz dominance concept; however, while there are many applications of this concept in static welfare analysis, in a dynamic framework, to our knowledge Karcher, Moyes and Trannoy (1995) is the only contribution. Finally Correia (1999) is very close to our approach; however, she uses a different criterion (differential dominance instead of generalized Lorenz dominance) to rank alternative fiscal policies. In this respect, our results are more directly related to the standard welfare analysis.

Appendix D extends the model to an economy with leisure.

The paper is organized as follow. In Section II. the basic model is presented and it is determined the fiscal policy which maximizes the growth rate; Section III. introduces the welfare analysis and compares socially optimal equilibrium and maximizing growth fiscal policies. Finally, Section IV. shows how taxing capital income is always optimal if agents have heterogeneous endowments. Conclusions and references close the paper. Calculations and extensions

[7] Judd (1999), p. 16, claims that Jones, Manuelli and Rossi (1997)'s result of positive tax on capital income in steady state is due to the exogenous rule adopted by government to decide public expenditure.

are gathered in the Appendix.

## *II. The model*

Assume that the aggregate production function is:[8]

$$Y = AK^{\alpha}G^{1-\alpha}L^{1-\alpha}, \text{ with } 0 < \alpha < 1, \tag{1}$$

where $Y$ is aggregate production, $K$ is the capital, $L$ is the labor, $A$ is a scale parameter and $G$ is a factor provided by the government, which produces a positive externality on all the other productive factors. $G$ could be considered as productive services supplied by the government to every firm.[9]

Assume $G$ is financed with a balanced budget, such that

$$G = \tau r K + \gamma \hat{w} L, \tag{2}$$

where $\tau$ and $\gamma$ are two different flat taxes, respectively, on capital income and on labor income. Private factors receive their marginal productivity; this implies that:[10]

$$r = \alpha A^{\frac{1}{\alpha}} \left[\alpha\tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} = r(\tau,\gamma) \text{ and} \tag{3}$$

$$\hat{w} = (1-\alpha) A^{\frac{1}{\alpha}} \left[\alpha\tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} K = w(\tau,\gamma) K. \tag{4}$$

Notice that $\dfrac{\partial r}{\partial \tau} > 0$, $\dfrac{\partial r}{\partial \gamma} > 0$, $\dfrac{\partial w}{\partial \gamma} > 0$ and $\dfrac{\partial w}{\partial \gamma} > 0$, i.e. there is a positive externality of public investment on factors' income. By substituting (2), (3) and (4) in (1), it follows that:

$$Y = A^{\frac{1}{\alpha}} \left[\alpha\tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} K. \tag{5}$$

Equation (5) makes it clear that this is an $AK$ model, that is a model where the marginal return on the cumulative factor remains constant, instead of decreasing, as the factor accumulates. Moreover, assume that factors cannot be subsidized and therefore $\tau$ and $\gamma$ belong to the range $[0,1]$.

---

[8] The time index will always be omitted if this does not create confusion.

[9] $Y$ can be seen as the aggregation of $N$ production functions $y_i = AG^{1-\alpha}k_i^{1-\alpha}l_i^{1-\alpha}$.

[10] For the sake of simplicity the total quantity of labor $L$ is normalized to 1.

There are $N$ consumers with different initial factor endowments. Each consumer maximizes his intertemporal utility, taking as given the aggregate variable paths.

Let $l_i$ be the labor endowment of agent $i$ and $k_i$ her/his capital endowment; therefore, her/his income will be expressed by

$$y_i = k_i \left[(1-\tau)r(\tau,\gamma) + (1-\gamma)\, w(\tau,\gamma)\sigma_i\right], \tag{6}$$

where

$$\sigma_i = \frac{Kl_i}{k_i}, \qquad \sigma_i \in [0, +\infty) \tag{7}$$

is the relative factor endowment of the *i-th* agent.

The instantaneous utility function has a logarithmic form, such that every agent maximizes[11]

$$U_i = \int_0^\infty e^{-\rho t} \log(c_i)\, dt, \tag{8}$$

where $\rho$ is the intertemporal discount rate.

Given the time paths both of the tax rates and of the capital aggregate stock,[12] the maximization of $U_i$, subject to:

$$\dot{k}_i = k_i \left[(1-\tau)r(\tau,\gamma) + (1-\gamma)\, w(\tau,\gamma)\sigma_i\right] - c_i \tag{9}$$

yields the following optimal consumption path:

$$\frac{\dot{c}_i}{c_i} = (1-\tau)r(\tau,\gamma) - \rho. \tag{10}$$

Suppose that fiscal policy is constant over time (like Alesina and Rodrik (1994)).[13] Then it is possible to demonstrate that equation

[11] See Appendix E for an extension of analysis to the case of CES utility function $U_i = \frac{c_i^{1-\mu}-1}{1-\mu}$, where $\frac{1}{\mu}$ is the intertemporal elasticity of substitution of consumption.

[12] Interactions among individuals actually occur only through the decisions on $\tau$ and $\gamma$ because $r$ and $w$ are independent of capital stock (see (3) and (4)).

[13] This is not source of limitation of the analysis; Appendix A contains a proof that the optimal fiscal policy is actually constant over time. However, the more general procedure of solution appears very tedious and not very intuitive from an economic point of view.

(10) is also the growth rate of $K$ and $k_i$ along the individual optimal path,[14] such that

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = \frac{\dot{K}}{K} = (1-\tau)r(\tau,\gamma) - \rho = \eta\left(\tau,\gamma\right). \tag{11}$$

Therefore $\eta\left(\tau,\gamma\right)$ represents the steady state growth rate as a function of the sequence of tax rates on capital income and labor income; moreover (7) and (11) highlight the fact that the relative factor endowment $\sigma$ of every agent does not change over time.

By (9), (10) and (11) the instantaneous level of consumption along the optimal path can be calculated, that is:

$$c_i = \left[(1-\gamma)\, w(\tau,\gamma)\sigma_i + \rho\right] k_i. \tag{12}$$

The linear relationship between $c_i$ and $k_i$ causes aggregate saving to be independent of endowment distributions and therefore fiscal policy affects the dynamics of the economy just by changing the price of factors; this property directly derives from the assumption of homothetic preferences.[15]

The optimal fiscal policy for the *i-th* agent solves the following problem:

$$\max_{\{\tau,\gamma\}_{t=0}^{\infty}} U_i = \int_0^{\infty} e^{-\rho t} \log\left(c_i\right) dt \tag{P1}$$

$$s.t. \begin{cases} c_i = \left[(1-\gamma)\, w(\tau,\gamma)\sigma_i + \rho\right] k_i \\ \dfrac{\dot{k}_i}{k_i} = \dfrac{\dot{K}}{K} = \eta\left(\tau,\gamma\right) \\ \tau,\gamma \in [0,1]\,. \end{cases}$$

We stress that the two tax rates are not assumed to be constant; the current value Hamiltonian for problem (P1) is given by[16]

$$H = \ln\left\{k_i\left[(1-\gamma)\, w\left(\tau,\gamma\right)\sigma_i + \rho\right]\right\} + \lambda k_i\left[(1-\tau)\, r\left(\tau,\gamma\right) - \rho\right].$$

[14]This is a standard result of AK growth models, where economy jumps instantaneously to its steady state (see Barro and Sala-I-Martin (1999)).

[15]For a similar argument see Correia (1999).

[16]We ignore the constraints on $\tau$ and $\gamma$ in the formulation of the Hamiltonian, but we will take them into account in the discussion of the results.

The following are necessary and sufficient conditions for the optimum:[17]

$$\frac{\partial H}{\partial \tau} = 0; \tag{13}$$

$$\frac{\partial H}{\partial \gamma} = 0; \tag{14}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial k_i}; \tag{15}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda k_i = 0. \tag{16}$$

From (13), (14) and (15), it follows that:[18]

$$\tau = \frac{(1-\alpha)(1-\gamma)}{\alpha}. \tag{17}$$

Equation (17) shows that there is a linear relation between the individual optimal tax rates; this will be fundamental in determining the politico-economic equilibrium because it allows a generalization of the median-voter theorem to be applied.

Equation (17) represents a static optimal condition; indeed, it can be verified that to maximize the net total output with respect to $\tau$ and $\gamma$, that is

$$Y - G = A^{\frac{1}{\alpha}} K \left\{ \left[\alpha\tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} - \left[\alpha\tau + (1-\alpha)\gamma\right]^{\frac{1}{\alpha}} \right\}, \tag{18}$$

tax rates have to satisfy equation (17). From the latter result we can conclude that every fiscal policy, and therefore also the optimal fiscal policy of *i-th* agent, satisfying equation (17) is not subject to the time inconsistency problem (see Benhabib, Rustichini and Velasco (2001)).

The optimal tax rate on capital income for agent $i$ follows by

[17] Under the usual hypothesis of concavity of the hamiltonian function with respect to state and control variables.

[18] See Appendix B.

substituting (17) in (13):[19]

$$\hat{\tau}_i = \max\left[0, \frac{\rho\left(\sigma_i - 1\right)}{\bar{r}\sigma_i}\right], \tag{19}$$

where $\bar{r} = \alpha A^{\frac{1}{\alpha}} \left(1-\alpha\right)^{\frac{1-\alpha}{\alpha}}$ is the value of $r$ when (17) holds. We are implicitly assuming that $\bar{r} > \rho$, which represents a necessary condition to have a positive steady state growth rate.

Therefore the optimal tax on labor income will be:

$$\hat{\gamma}_i = \min\left[1, 1 - \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\rho\left(\sigma_i - 1\right)}{\bar{r}\sigma_i}\right)\right]. \tag{20}$$

The optimal tax rate on capital income for the *i-th* individual $\hat{\tau}_i$ will be zero if her/his relative factor endowments is less than or equal to 1 and positive for values greater than 1. Moreover, $\hat{\tau}_i$ is constant over time, such that the optimal fiscal policy implies constant tax rates. The individual optimal tax rate on a factor is inversely proportional to her/his relative endowment of that factor; for example if her/his relative capital endowment falls (i.e. $\sigma_i$ increases), then the optimal tax rate on capital income increases (see (19)). Moreover, provided that equation (17) holds, $w$ and $r$ are independent of $\tau$ and $\gamma$.

To examine this point in greater depth, assume constant tax rates (this property holds for each agent's optimal fiscal policy, see (19)). Solving the integral in (P1) yields the following expression for individual utility:

$$U\left(\sigma_i, \tau, \gamma\right) = \frac{1}{\rho}\left\{\log\left\{k_i^0\left[\left(1-\gamma\right) w\left(\tau,\gamma\right)\sigma_i + \rho\right]\right\} + \frac{\left(1-\tau\right) r\left(\tau,\gamma\right) - \rho}{\rho}\right\}. \tag{21}$$

The first term of (21) is consumption at time 0 (denote this as level effect), while the second term is proportional to steady

[19]From (65), (11) and (71) it follows that

$$\frac{\rho}{\tau\bar{r}\sigma_i + \rho} = \frac{1}{\sigma_i}.$$

Solving for $\tau$ we obtain equation (19).

state growth rate $\eta$ (denote this as growth effect). The individual optimal fiscal policy is determined by the trade-off between these two effects; maximizing the growth rate, on the contrary, amounts to considering only the second effect. If equation (17) holds, the growth effect and the level effect are, respectively, negatively and positively related to $\tau$. The level effect positively depends on $\sigma$. This explains the relationship between individual preferences on fiscal policy and relative factor endowments given by (19).

## II.A. Maximum growth

To understand how fiscal policy affects the steady state growth rate expressed by (11), consider equation (12), that shows how the ratio between $c_i$ and $k_i$ varies along the balanced growth path as a function of the tax rates. Notice that the higher the tax rate on capital income, the higher is the instantaneous level of consumption for a given $k_i$; this, in turn, implies a lower investment rate and therefore a lower growth rate (see Bertola (1993)). The following Proposition states the fiscal policy which maximizes the growth rate:

**Proposition 1** *The fiscal policy which maximizes the growth rate* $\eta(\tau,\gamma)$ *is given by* $(\tau^*,\gamma^*) = (0,1)$.

**Proof.** Consider the derivative of equation (11) with respect to $\tau$ and $\gamma$. Since $\dfrac{\partial \eta(\tau,\gamma)}{\partial \tau} > 0 \Leftrightarrow \tau < (1-\alpha)(1-\gamma)$, $\dfrac{\partial \eta(\tau,\gamma)}{\partial \gamma} > 0 \quad \forall (\tau,\gamma)$ and $\tau,\gamma \in [0,1]$, then the fiscal policy which maximizes growth is given by $(\tau^*,\gamma^*) = (0,1)$. ■

The intuition is straightforward: the growth rate of output depends on the accumulation of capital and, because the saving rate positively depends on its return, taxation of capital income reduces the incentives to accumulate. This does not hold for labor because it has an inelastic supply. Allowing for leisure, the tax rate on labor income which maximizes the steady state growth rate $\gamma^*$ will be equal to $\pi < 1$, where $\pi$ is inversely proportional to the elasticity of substitution between leisure and working time; more details on this case are reported in Appendix D.

Finally, we note that it is the accumulation of capital that generates growth, while the level of $G$ affects only the production level; this rules out the possibility that, in order to maximize the growth rate, capital income has to be taxed.

For simplicity and because this is not a source of limitation of the analysis, in the following assume that every agent has the same labor endowment, such that a difference in $\sigma$ reflects different capital endowments, that is:

$$l_i = l = \frac{1}{N} \; \forall i \in \{1, ..., N\}. \tag{22}$$

This assumption makes $\sigma$ inversely related to the level of individual income, so that all results can be also interpreted in terms of income.

Moreover, to avoid trivial results in the following assume, if not otherwise specified, that $\sigma$ is uneven distributed, that is:

$$\exists i, j \in \{1, ..., N\} \text{ with } i \neq j \text{ such that } \sigma_i \neq \sigma_j. \tag{23}$$

## *III. Normative analysis*

The aim of this section is to rank consumption paths in welfare terms. In this contest the distribution of capital will be crucial to determine the optimal fiscal policy. Heuristically if every agent has the same capital endowment, then $\sigma = 1$ and the fiscal policy $\tau = 0$ and $\gamma = 1$, which maximizes steady state growth rate, will maximize welfare as well. Thus in this representative agent economy there would be not any trade-off between equity and efficiency (this is the Judd (1999)'s finding).[20]

However this result does not hold any more if agents have heterogeneous endowments; in particular, on the condition that *lump-sum* redistributions are not available, we will demonstrate that there is no more a perfect correspondence between fiscal policies which

[20]Notice that if assumption (22) does not hold, $\sigma_i = 1 \forall i$ does not imply all agents' endowments are equal.

maximize welfare and steady state growth rate.[21]

In the economy there are many Pareto optima; indeed the optimal fiscal policy of every agent is a Pareto optimal allocation, but these are not the only ones. In particular, since the individual utility function is concave,[22] the set of Pareto optima is an interval whose bounds are for every tax rate, respectively, the optimal fiscal policy of the agents with the highest and the smallest value of $\sigma$. Thus the fiscal policy which maximizes the growth rate is not the optimum, but only one among all the possible Pareto optima.

The following analysis is focused on all combinations of $(\tau, \gamma)$, constant over time, belonging to the line segment (17).[23] Given these assumptions, the indirect utility of agent $i$ becomes:

$$\bar{U}(\sigma_i, \tau) = \frac{1}{\rho}\left\{\log\left\{k_i^0\left[\tau\bar{r}\sigma_i + \rho\right]\right\} + \frac{(1-\tau)\,\bar{r} - \rho}{\rho}\right\}.$$

Since every fiscal policy can be expressed as a function of only tax rate of capital income $\tau$, for the sake of simplicity in the following we will refer only to this variable to indicate fiscal policy, when this is not source of confusion.

To rank in welfare terms the elements of the set of Pareto optima we need to postulate a social welfare function. Assume that the welfare function is additive-separable, that is:

$$W = \sum_{j=1}^{N} \phi\left(\bar{U}(\sigma_j, \tau)\right).$$

The social planner's inequality aversion is expressed by the form of $\phi(\cdot)$; for example, if she/he were indifferent to inequality, $\phi(\cdot)$

[21] We do not discuss the case of redistribution of initial endowment (i.e. confiscation of capital) leading to a uniform distribution of factors among agents, due to of both its possible intertemporal inconsistency (see Alesina and Rodrik (1994)) and its difficult implementation (see Lucas (1990)).

[22] This can be checked by (21), using equation (17).

[23] This is not a source of limitation for the analysis because relationship (17) between the two optimal tax rates does not depend on agents' endowments. Moreover, we have shown that also individual optimal fiscal policy provides for constant tax rates. Finally, it will be shown that relationship (17) is always satisfied when the social planner maximizes the simple sum of individual utilities.

would be linear in its arguments. In an economy with heterogeneous endowments the latter case is a natural candidate for representing an efficient index of fiscal policy.

Finally, we define the optimal fiscal policy:

**Definition 2** ***(Optimal fiscal policy)*** *Let* $\left(\tau^W, \gamma^W\right)$ *be the optimal fiscal policy, where*

$$\tau^W = \arg\max_{\tau \in [0,1]} \left[ \sum_{j=1}^{N} \phi\left(\bar{U}\left(\sigma_j, \tau\right)\right) \right]; \tag{24}$$

$$\gamma^W = 1 - \left(\frac{\alpha}{1-\alpha}\right) \tau^W. \tag{25}$$

## III.A. Generalized Lorenz dominance and ranking of fiscal policies

A very useful concept in welfare analysis is the (generalized) Lorenz dominance; the latter allows to rank alternative fiscal policies in welfare terms. In particular, Shorrocks and Foster (1987) state the following Theorem on the relationship between additive separable welfare function and (generalized) Lorenz dominance[24]:

**Theorem 3** *Assume that* $\phi'(\cdot) \geq 0$ *and* $\phi''(\cdot) \leq 0$ *and let* $\bar{U}^{\tau^q} = \left[\bar{U}\left(\sigma_1, \tau^q\right), \ ..., \bar{U}\left(\sigma_N, \tau^q\right)\right]$ *be a utility vector of length N, whose elements are ranked in an increasing order, i.e.* $\bar{U}\left(\sigma_1, \tau^q\right) \leq \bar{U}\left(\sigma_2, \tau^q\right) \leq ... \leq \bar{U}\left(\sigma_N, \tau^q\right)$*, then*

$$GL^{\tau^A}(p) \geq GL^{\tau^B}(p), \forall p \in [0,1] \Leftrightarrow \sum_{j=1}^{N} \phi\left(\bar{U}\left(\sigma_j, \tau^A\right)\right) \geq \sum_{j=1}^{N} \phi\left(\bar{U}\left(\sigma_j, \tau^B\right)\right),$$

*where* $GL^{\tau^q}(p) = \frac{\sum_{j=1}^{S} \bar{U}(\sigma_j, \tau^q)}{N}$*,* $q = A, B$*,* $p = \dfrac{S}{N}$ *and* $S = 1, ..., N$*.*

[24] In general Shorrocks and Foster's Theorem refers to a ranking of alternative (income) vectors. Since balanced consumption path is a matrix (the set of consumption paths of all agents) and not a vector, an aggregation is necessary. We use an aggregate index for the consumption path of each agent, her/his intertemporal utility $U$. Other approaches are possible; see for example Karcher, Moyes and Trannoy (1995).

The $GL^{\tau}(p)$ is the (generalized) Lorenz curve and $\bar{U}^{\tau^A}$ is said to dominate $\bar{U}^{\tau^B}$ according to the Lorenz dominance principle; refer to Shorrocks and Foster (1987) for the proof. If $GL^{\tau^A}(p)$ were not always above $GL^{\tau^B}(p)$, then a univocal conclusion could not be reached without imposing other constraints on the form of $\phi(\cdot)$ (see Dardanoni and Lambert (1988)).

The definition of efficient fiscal policy in heterogeneous agent economies is controversial; according to our approach we measure the efficiency of a fiscal policy by the simple sum of individual utilities. In the following we give the definition of efficiency improving fiscal policy:[25]

**Definition 4** *(**Efficiency improving fiscal policy**) Let $\tau^A$ and $\tau^B$ be two fiscal policies; then $\tau^A$ is more efficient than $\tau^B$ if $GL^{\tau^A}(1) > GL^{\tau^B}(1)$.*

Finally, as regards equity the following definition states the fiscal policies that present an efficiency-equity trade-off:

**Definition 5** *(**Efficiency-equity trade-off fiscal policy**) Let $\tau^A$ and $\tau^B$ be two fiscal policies; then $\tau^A$ is more equitable than $\tau^B$ and $\tau^B$ is more efficient than $\tau^A$ if $\exists \hat{p} \in (0,1)$ such that $\forall p \in [0,\hat{p}]$ $GL^{\tau^A}(p) \geq GL^{\tau^B}(p)$ and $\forall p \in (\hat{p},1]$ $GL^{\tau^A}(p) \leq GL^{\tau^B}(p)$.*

The intuition is straightforward: if the change from fiscal policy $\tau^B$ to $\tau^A$ causes an upward movement in the part of the Lorenz curve nearer to the origin and a downward movement in the other part, then fiscal policy $\tau^A$ is related to a lower inequality but a lower efficiency than $\tau^B$. Figure 1 shows this case.

FIGURE 1 HERE

In Figure 1 $\tau^B$ is an efficiency improving policy with respect to $\tau^A$ (the sum of individual utilities increases), but it also implies a

[25]Our definition of efficient improving fiscal policy is different from that used by Correia (1999); in fact we refer to the sum of individual utilities, while Correia (1999) considers the utility related to average endowment. In the latter case the most efficient fiscal policy is always $\tau = 0$, because average endowment corresponds to $\sigma = 1$.

greater inequality (the poor agent has a lower (relative) utility). Thus the change from fiscal policy $\tau^B$ to $\tau^A$ implies an efficiency-equity trade-off.

In our framework the generalized Lorenz curve is defined as (see Theorem 3):

$$GL^{\tau}(p) = \frac{\sum_{j=1}^{S} \bar{U}(\sigma_j, \tau)}{N} = \\ = \frac{\sum_{j=1}^{S} \left[\ln(\tau\bar{r}\sigma_j + \rho) k_j^0\right] + S\left[\frac{(1-\tau)\bar{r}-\rho}{\rho}\right]}{\rho N}$$

for $S = 1, ..., N$; the derivative of $GL^{\tau}(p)$ with respect to $\tau$ shows how fiscal policy affects the Lorenz curve:

$$\frac{\partial GL^{\tau}(p)}{\partial \tau} = \left(\frac{1}{\rho N}\right)\left[\sum_{j=1}^{S} \frac{\bar{r}\sigma_j}{\tau\bar{r}\sigma_j + \rho} - \frac{S\bar{r}}{\rho}\right]. \tag{26}$$

Theorem 3 can be applied only if $\frac{\partial GL^{\tau}(p)}{\partial \tau}$ does not change its sign for $S = 1, ..., N$.

From equation (26) it follows that:

$$\text{if } \frac{\sum_{j=1}^{S} \sigma_j}{S} = \mu_{\sigma}^{S} > 1 \Rightarrow \left.\frac{\partial GL^{\tau}(p)}{\partial \tau}\right|_{\tau=0} > 0 \quad S = 1, ..., N, \tag{27}$$

where $\mu_{\sigma}^{S}$ is the mean of distribution of $\sigma$ of the first $S$ agents. The condition expressed by (27) is intuitive: since agents with a $\sigma$ no greater than 1 prefer $\tau = 0$ (see (19)), for every $S = 1, ..., N$ the Lorenz curve shifts up or down as $\tau$ becomes greater than 0 depending on the mean of the $\sigma$ of the first $S$ agents being, respectively, greater or smaller than 1. Therefore from Definition 4 if $\left.\frac{\partial GL^{\tau}(1)}{\partial \tau}\right|_{\tau=0}$ were greater than 0 then $\tau = 0$ could not be the most efficient fiscal policy.

### III.B. Optimal fiscal policy

This section characterizes the set of all possible optimal fiscal policies.

Firstly, we defines two extreme fiscal policies, one maximizing a Rawlsian welfare function, corresponding to a social planner interested only in the worst-off agent's utility, and one maximizing the simple sum of individual utilities, corresponding to a social planner that aims only at efficiency. Secondly, we show how within this range there are all candidate optimal fiscal policies; the effectively optimal one will depend on the social planner's inequality aversion.

According to Rawls' principle the welfare of an economy is represented by the utility of the worst-off; this means that social welfare is given by the value of the Lorenz curve corresponding to abscissa $\frac{1}{N}$. The following Proposition states the optimal fiscal policy of a Rawlsian social planner:

**Proposition 6** *Let* $\left(\tau^R, \gamma^R\right)$ *be the optimal fiscal policy when the social planner has Rawlsian preferences. Then* $\tau^R$ *maximizes* $GL^\tau\left(\frac{1}{N}\right)$*, that is:*

$$\tau^R = \underset{\tau\in[0,1]}{\arg\max}\left[GL^\tau\left(\frac{1}{N}\right)\right] = \max\left[0, \frac{\rho\left(\sigma_1-1\right)}{\bar{r}\sigma_1}\right] \tag{28}$$

*and*

$$\gamma^R = \min\left[1, 1-\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{\rho\left(\sigma_1-1\right)}{\bar{r}\sigma_1}\right)\right]. \tag{29}$$

**Proof.** To prove (28) it is sufficient to note that $\bar{U}\left(\sigma_1,\tau\right) \leq \bar{U}\left(\sigma_2,\tau\right) \leq ... \leq \bar{U}\left(\sigma_N,\tau\right)$, while $\gamma^R$ is derived from (17). ■

On the contrary, if efficiency is the only goal of the social planner, social welfare is given by the value of the Lorenz curves corresponding to abscissa 1. The following Proposition states the optimal fiscal policy in this case:

**Proposition 7** *Let* $\left(\tau^E, \gamma^E\right)$ *be the optimal fiscal policy when efficiency is the only goal of social planner. Then* $\tau^E$ *maximizes*

*$GL^{\tau}(1)$, that is:*

$$\left.\frac{\partial GL^{\tau}(1)}{\partial \tau}\right|_{\tau=\tau^{E}} = 0 \tag{30}$$

*and*

$$\gamma^{E} = 1 - \left(\frac{\alpha}{1-\alpha}\right)\tau^{E}. \tag{31}$$

**Proof.** The proof of (30) is given by the definition of efficiency improving fiscal policy (see Definition 4), while $\gamma^{E}$ is derived from (17). ■

Proposition 7 yields the following condition:

$$\left.\frac{\partial GL^{\tau}(1)}{\partial \tau}\right|_{\tau=\tau^{E}} = 0 \Leftrightarrow \sum_{j=1}^{N} \frac{\sigma_j}{\tau^{E}\bar{r}\sigma_j + \rho} = \frac{N}{\rho} \tag{32}$$

From condition (32) it follows that $\tau^{R} \geq \tau^{E}$.[26] This suggests that $\tau^{W}$ has to belong to the interval $\left[\tau^{E}, \tau^{R}\right]$ and the higher is the social planner's inequality aversion the closer is $\tau^{W}$ to $\tau^{R}$. With respect to this point, note that since $\sigma_1 \geq \sigma_1 \geq ... \geq \sigma_N$ then:

$$\frac{\sum_{j=1}^{S} \frac{\sigma_j}{\tau\bar{r}\sigma_j + \rho}}{S} \geq \frac{\sum_{j=1}^{S+1} \frac{\sigma_j}{\tau\bar{r}\sigma_j + \rho}}{S+1} \text{ for } S = 1, ..., N-1 \text{ and } \forall \tau \in [0,1]. \tag{33}$$

Given that $\tau^{E}$ has to satisfy the following equality:

$$\sum_{j=1}^{N} \frac{\sigma_j}{\tau^{E}\bar{r}\sigma_j + \rho} - \frac{N}{\rho} = 0,$$

it follows that:

$$\left.\frac{\partial GL^{\tau}(p)}{\partial \tau}\right|_{\tau=\tau^{E}} = \sum_{j=1}^{S} \frac{\sigma_j}{\tau^{E}\bar{r}\sigma_j + \rho} - \frac{N}{\rho} \geq 0 \text{ for } S = 1, ..., N-1.$$

---

[26] Note that $\frac{\sigma_1}{\tau\bar{r}\sigma_1+\rho} \geq \frac{\sigma_j}{\tau\bar{r}\sigma_j+\rho}$ $\forall j = 2, ..., N$ and $\forall \tau \in [0,1]$, hence $\sum_{j=1}^{N} \frac{\sigma_j}{\tau^{R}\bar{r}\sigma_j+\rho} \leq \frac{N}{\rho}$ and therefore $\tau^{R} \geq \tau^{E}$.

This means that the Lorenz curve corresponding to $\tau^E$ is everywhere increasing in $\tau^E$, except for $p = 1$; in other words an increase in $\tau^E$ causes a decrease in income inequality (but also a decrease in average utility).

Let $\tau_S$ be the level of tax rate that, if increased, would leave $GL^{\tau}(p)$ unchanged (remember that $p = \frac{S}{N}$), that is:

$$\left.\frac{\partial GL^{\tau}(p)}{\partial \tau}\right|_{\tau=\tau_S} = 0.$$

From (33) it follows that vector $\{\tau_S\}_{S=1}^{N}$ is ranked in decreasing order, i.e. $\tau_S \geq \tau_{S+1}$, such that $\tau_1 = \tau^R$ and $\tau_N = \tau^E$ (see Propositions 6 and 7).

We see that an increase in $\tau$ causes an upward movement of the part of $GL^{\tau_S}$ on the left with respect to the point of abscissa $p = \frac{S}{N}$ and a downward movement of the part on the right (here the application of Definition 5 of efficiency-equity trade-off fiscal policy to $\tau_S$ is straightforward). Figure 2 shows two Lorenz curves, one corresponding to $\tau_S$ and another one corresponding to $\tau > \tau_S$; they have to cross only in a single point.

FIGURE 2 HERE

Each $\tau < \tau^E = \tau_N$ never maximizes social welfare because such an increase causes an upward movement of the whole $GL^{\tau}(p)$ curve and therefore a welfare gain (see Theorem 3).

For every $\tau > \tau_N$ an increase in tax rate causes only the left part of the $GL^{\tau}(p)$ curve to shift up, while the right part shifts down; this means that the poorest agents increase their welfare, while all the others experience reduced status; moreover, for every $\tau > \tau_N$ an increase in $\tau$ leads to a fall in the average social welfare. Since the higher is $\tau$ the smaller is the number of agents preferring a rise in $\tau$, then if $\tau$ were increased over $\tau_1$ also the worst-off agent would worsen her/his status; this implies that $\tau_1 = \tau^R$.

This confirms the previous intuition that $\tau^W \in \left[\tau^E, \tau^R\right]$ and that the level of tax rate maximizing social welfare will positively depend on the social planner's inequality aversion.

The following Proposition summarizes the results:

**Proposition 8** *Let* $W=\sum_{j=1}^{N}\phi\left(\bar{U}\left(\sigma_j,\tau\right)\right)$ *be the social welfare function, where* $\phi'\geq 0$, $\phi''\leq 0$, $\bar{U}\left(\sigma_i,\tau\right)$ *is the intertemporal utility of agent* $i$ *and* $\bar{U}\left(\sigma_i,\tau\right)\leq\bar{U}\left(\sigma_{i+1},\tau\right)$ *for* $i=1,...,N-1$. *Then the tax rate on capital income maximizing welfare function* $\tau^W$ *has to belong to the interval* $\left[\tau^E,\tau^R\right]$, *where* $\tau^R=\max\left[0,\frac{\rho(\sigma_1-1)}{\bar{r}\sigma_1}\right]$ *and* $\tau^E$ *implicitly solves* $\sum_{j=1}^{N}\frac{\sigma_j}{\tau^E\bar{r}\sigma_j+\rho}=\frac{N}{\rho}$.
*Moreover, according to Definitions 4 and 5,* $\tau^E$ *is the most efficient fiscal policy, while all the other fiscal policies belonging to the interval* $\left(\tau^E,\tau^R\right)$ *present an efficiency-equity trade-off.*

**Proof.** From equation (33) it follows that $\tau_S\geq\tau_{S+1}$, and since $\tau^E$ maximizes the simple sum of individual utilities, then in every economy where $\tau<\tau^E$ average individual utility is lower and inequality is higher than in the economy where $\tau=\tau^E$; this means that no $\tau<\tau^E$ can maximize social welfare $W$. Moreover in every economy where $\tau>\tau^R$ average individual utility is lower and the utility of the poorest agent is less than in the economy where $\tau=\tau^R$; therefore $\tau^W$ has to belong to the interval $\left[\tau^E,\tau^R\right]$. This proves the first statement. The proof of the second statement directly follows from Definitions 4 and 5 and from equation (33). ■

Moreover:

**Remark 9** *The efficient fiscal policy* $\tau^E$ *dominates according to the Lorenz dominance principle every fiscal policy* $\tau<\tau^E$.

**Proof.** From equation (33) and since $\tau^E$ maximizes the sum of individual utilities, a decrease in $\tau^E$ causes a downward movement of the whole Lorenz curve, which holds for every $\tau<\tau^E$. This completes the proof. ■

The next section analyzes thoroughly the case of efficient fiscal policy. This case is particularly interesting because $\tau^E$ is both the lower bound of the set of possible optimal fiscal policies and the most efficient fiscal policy (see Remark 9).

## *IV. Efficient fiscal policy*

In this section we show that taxing capital income is always optimal if agents have heterogeneous endowments. According to Definition 4 the most efficient fiscal policy maximizes the simple sum of individual utilities, that is:

$$\left(\tau^E, \gamma^E\right) = \arg \max_{\tau,\gamma \in [0,1]^2} W =$$

$$= \frac{\ln \prod_{j=1}^{N} k_j^0 \left[(1-\gamma) w(\tau,\gamma) \sigma_i + \rho\right] + N \dfrac{(1-\tau) r(\tau,\gamma) - \rho}{\rho}}{\rho}.$$

By the first order conditions it follows that:

$$\tau = \frac{(1-\alpha)(1-\gamma)}{\alpha}.$$

This result supports the previous choice to focus only on the fiscal policies belonging to the straight line given by (17); substituting (17) in the first order conditions of (IV.) and solving for $\gamma$ yields:[27]

$$\frac{\sum_{i=1}^{N} \dfrac{\sigma_i}{\tau^E \bar{r} \sigma_i + \rho}}{N} = \frac{1}{\rho}. \tag{34}$$

Setting $\tau^E = 0$ in (34) yields a sufficient condition to have $\tau^E = 0$, that is:[28]

$$\mu_\sigma = \frac{\sum_{j=1}^{N} \sigma_j}{N} = 1. \tag{35}$$

It can be demonstrated that $\mu_\sigma = 1$ implies $\sigma_i = 1 \; \forall i$;[29] therefore, provided that condition (35) holds, since every agent has the same relative endowment $\sigma = 1$ and prefers $\tau = 0$, it follows that $\tau^E = \tau^W = 0$. This case corresponds to the representative agent economy and the result coincides with Judd's (1999) finding.

[27] The logarithmic form of utility function prevents fiscal policy maximizing growth from maximizing social welfare if at least one agent has no quantity of capital.

[28] Condition (35) could also be derived from (27).

[29] See Appendix C.

The mean of the $\sigma$ distribution proves a crucial parameter; intuitively the greater the mean of $\sigma$, the higher is the number of agents preferring high tax rate on capital income. Moreover, $\sigma_i = 1\ \forall i$ is also the allocation of endowments minimizing $\mu_\sigma$, which suggests that if $\mu_\sigma > 1$ then $\tau^E > 0$ (that is (35) is not only a sufficient, but also necessary condition to have $\tau^E = 0$). The following Proposition confirms this intuition:

**Proposition 10** *The most efficient fiscal policy (or the optimal fiscal policy when the social planner is indifferent to inequality) involves a positive tax rate on capital income, but the case of even distribution of individual endowments (i.e. the representative agent economy), that is:*

$$\tau^E = \begin{cases} 0 & \textit{if } \sigma_i = 1\ \forall i \\ \psi > 0 & \textit{if } \exists i \textit{ such that } \sigma_i \neq 1 \end{cases}$$

**Proof.** See Appendix C. ■

The intuition is straightforward: given an unequal distribution of resources, the poor agents (i.e. laborers) have such a low level of consumption that they benefit from trading off lower growth with higher initial levels of consumption. In fact, a positive taxation on capital income means a greater share of total income is allocated to labor, which implies a higher consumption in the initial periods for all agents, but a lower growth rate. The richest agents have a loss from this consumption reallocation, but the concave form of individual utility function always makes the latter lower than the gain of the poor agents.[30] Appendix E extends this result to an economy where agents have a CES utility function.

In the optimal taxation literature there exists a standard result (for a representative agent economy), that states that the optimal

[30] Here the difference with the definition of efficiency proposed by Correia (1999) is crucial: she considers the utility of the agent with average endowment as a measure of efficiency of fiscal policy, but this implies that concavity of the individual utility function does not matter. In other words, the distribution of endowments would not affect the efficiency of fiscal policy. Adopting the definition of efficiency proposed by Correia (1999), in this economy, independent of endowment distribution, the most efficient fiscal policy would be always to tax zero capital income.

tax rate on capital income is zero (e.g. Judd (1999)); Proposition 10 states that for a heterogeneous agents' economy exactly the opposite holds.

### IV.A. Two agent economy

This section analyzes a simple example, a two agent economy (e.g. a capitalist and a worker like in Judd (1985)). Denote as the iso-tax rate curve the set of all the pairs $(\sigma_1, \sigma_2)$ that imply the same level of $\tau^E$; from (34) this can be expressed as:

$$\sigma_2 = -\rho \left[ \frac{\sigma_1 \left( 2\tau^E \bar{r} - \rho \right) + 2\rho}{2\tau^E \bar{r} \sigma_1 \left( \tau^E \bar{r} - \rho \right) + \rho \left( 2\tau^E \bar{r} - \rho \right)} \right].$$

If $\tau^E = 0$ the slope of iso-tax rate curve is always equal to $-1$ and therefore the iso-tax rate curve is a straight line, while if $\tau^E > 0$ the iso-tax rate curve is concave. Since all the feasible pairs $(\sigma_1, \sigma_2)$ have to belong to the curve $\sigma_2 = \frac{\sigma_1}{2\sigma_1 - 1}$,[31] it is easy to verify that $\mu_\sigma = 1 \Leftrightarrow \sigma_1 = \sigma_2 = 1$. Moreover, for any $\sigma_1, \sigma_2 \neq 1$ we have $\tau^E > 0$ and only if $\sigma_1 = \sigma_2 = 1$, then $\tau^E = 0$ (these results are just an application of Proposition 10).

Finally, from (34) with a little algebra it follows that[32]

$$\tau^E = \left( \frac{\rho}{\bar{r}} \right) \left[ \sqrt{5 - \frac{4}{\mu_\sigma}} - 1 \right],$$

which highlights the positive relationship between $\tau^E$ and the mean of $\sigma$. It is easy to show that $\mu_\sigma$ increases as distribution becomes more unequal.[33]

Figure 3 shows in $(\sigma_1, \sigma_2)$ space the resource constraint $\sigma_2 = \frac{\sigma_1}{2 \cdot \sigma_1 - 1}$ and two iso-tax rate curves.

[31] To see this, consider that the resource constraint implies $\frac{1}{\sigma_1} + \frac{1}{\sigma_2} = 2$, hence $\sigma_2 = \frac{\sigma_1}{2\sigma_1 - 1}$.

[32] Mean and variance of distribution of $\sigma$ have a positive relationship if $\sigma_1 > 1$; in particular $var_\sigma = \mu_\sigma \left[ \frac{(\sigma_1 - 1)^2}{2\sigma_1 - 1} \right]$.

[33] By definiton of mean $\mu_\sigma = \left( \frac{1}{2} \right) \left[ \sigma_1 + \frac{\sigma_1}{2\sigma_1 - 1} \right] = \frac{\sigma_1^2}{2\sigma_1 - 1}$, then $\frac{d\mu_\sigma}{d\sigma_1} = \frac{2(\sigma_1 - 1)}{(2\sigma_1 - 1)^2} > 0$ if $\sigma_1 > 1$ (by resource constraint $\frac{d\sigma_2}{d\sigma_1} < 0$).

FIGURE 3 HERE

Notice that the space between the straight line defined by $\mu_\sigma = 1$ (the iso-tax curve closer to the origin) and the origin is the locus where $\tau^E = 0$; outside this space for every feasible pair $(\sigma_1, \sigma_2)$ $\tau^E$ will be positive and increasing as long as we go away from the origin.

If economy is populated only by a capitalist (i.e. $\sigma_C = \frac{1}{2}$) and by a worker (i.e. $\sigma_W \to \infty$) like in Judd (1985), then $\mu_\sigma \to \infty$ and $\tau^E = \left(\frac{\rho}{\bar{r}}\right)\left[\sqrt{5} - 1\right] > 0$.

## *V. Conclusions*

The main finding of the paper is that in order to maximize welfare taxing capital income is a necessary condition, provide that agents have an uneven distribution of initial factor endowments and lump-sum redistributions are not available. Therefore the representative agent hypothesis is decisive to evaluate the welfare properties of fiscal policy. Moreover, in the politico-economic equilibrium social welfare can be maximized. In fact, even if fiscal policy of the politico-economic equilibrium does not maximize growth, this could be socially optimal if the social planner is sufficiently averse to inequality.

**Acknowledgements**

I am very grateful to my Ph.D. supervisor Vincenzo Denicolò, whose suggestions and encouragements have been crucial for this work. Carlo Casarosa, Valeria De Bonis, Fausto Gozzi, Kenneth Judd, Pier Mario Pacini, Piero Manfredi and seminar participants at Pisa and Bologna are kindly acknowledged for their comments. The usual disclaimers apply.

**Author's affiliation**

Dipartimento di Scienze Economiche
University of Pisa

# Appendix

## A Optimal fiscal policy for agent $i$

Given the paths of flat tax rates, agent $i$'s solves the following problem:

$$U_i = \int_0^\infty e^{-\rho t} \log(c_i)\, dt, \tag{P1}$$
$$s.t. \begin{cases} \dot{k}_i = (1-\tau) r(\tau,\gamma) k_i + (1-\gamma)\, w(\tau,\gamma) K l_i - c_i; \\ k_i^0 = \bar{k}_i. \end{cases}$$

From the FOCs of Problem P1 optimal consumption path is given by:

$$\frac{\dot{c}_i}{c_i} = (1-\tau) r(\tau,\gamma) - \rho = \eta(\tau,\gamma), \tag{36}$$

while trasversality condition is given by:

$$\lim_{t\to\infty} \frac{k_i}{c_i} \exp(-\rho t) = 0 \tag{37}$$

The optimal fiscal policy for agent $i$ solves the following problem (arguments of functions are not reported):

$$\max_{\{\tau,\gamma\}_{t=0}^{\infty}} U_i = \int_0^\infty e^{-\rho t} \log(c_i)\, dt \tag{P2}$$
$$s.t. \begin{cases} \dot{k}_i = (1-\tau) r k_i + (1-\gamma)\, w K l_i - c_i; \\ \dot{c}_i = c_i \left[(1-\tau) r - \rho\right]; \\ \dot{K} = (1-\tau) r K + (1-\gamma)\, w K - C; \\ \dot{C} = C \left[(1-\tau) r - \rho\right]; \\ \gamma, \tau \in [0,1] \\ k_i^0 = \bar{k}_i; \\ \lim_{t\to\infty} \frac{k_i}{c_i} \exp(-\rho t) = 0. \end{cases}$$

Hamiltonian function for this problem is given by:[34]

$$H = \log(c_i) + \lambda_k \left[(\eta+\rho) k_i + (1-\gamma) wKl_i - c_i\right] + \lambda_c c_i \eta + \\ + \mu_K \left[(\eta+\rho) K + (1-\gamma) wK - C\right] + \mu_C C\eta. \tag{38}$$

We make the standard assumptions that $H$ is concave in control and state variables and that constraint qualifications are satisfied (see Seierstad and Sydsaeter (1985), pag. 381, Theorem 9 and Note 11) as well as the nonnegative constraints on $c_j$ for $j = 1, ..., N$.

Then necessary and sufficient conditions for the optimum are the following (see Seierstad and Sydsaeter (1985), pag. 385 and the following):

$$(\hat{\tau}_i, \hat{\gamma}_i) = \arg \max_{(\tau,\gamma)\in[0,1]^2} H; \tag{39}$$

$$\dot{\lambda}_k = \lambda_k \rho - \frac{\partial H}{\partial k_i}; \tag{40}$$

$$\dot{\lambda}_c = \lambda_c \rho - \frac{\partial H}{\partial c_i}; \tag{41}$$

$$\dot{\mu}_K = \mu_K \rho - \frac{\partial H}{\partial K}; \tag{42}$$

$$\dot{\mu}_C = \mu_C \rho - \frac{\partial H}{\partial C}; \tag{43}$$

$$\lim_{t\to+\infty} \lambda_k k_i e^{-\rho t} = 0; \tag{44}$$

$$\lim_{t\to+\infty} \lambda_c c_i e^{-\rho t} = 0; \tag{45}$$

$$\lim_{t\to+\infty} \mu_K K e^{-\rho t} = 0; \tag{46}$$

$$\lim_{t\to+\infty} \mu_C C e^{-\rho t} = 0; \tag{47}$$

$$k_i^0 = \bar{k}_i; \tag{48}$$

$$\lim_{t\to\infty} \frac{k_i}{c_i} \exp(-\rho t) = 0. \tag{49}$$

[34] An equivalent way to tackle our problem is to introduce a Lagrangian function (see Chiang (1992), pag. 279). On the contrary we use the approach suggested in Seierstad and Sydsaeter (1985) (Section 6.5 and 6.6).

From (39)-(43) we have:

$$\left.\frac{\partial H}{\partial \tau}\right|_{\tau=\hat{\tau}_i,\gamma=\hat{\gamma}_i}=\begin{cases} 0 \text{ if } 0<\hat{\tau}_i<1; \\ \geq 0 \text{ if } \hat{\tau}_i=1; \\ \leq 0 \text{ if } \hat{\tau}_i=0; \end{cases} \tag{50}$$

$$\left.\frac{\partial H}{\partial \gamma}\right|_{\tau=\hat{\tau}_i,\gamma=\hat{\gamma}_i}=\begin{cases} 0 \text{ if } 0<\hat{\gamma}_i<1; \\ \geq 0 \text{ if } \hat{\gamma}_i=1; \\ \leq 0 \text{ if } \hat{\gamma}_i=0; \end{cases} \tag{51}$$

$$\frac{\dot{\lambda}_k}{\lambda_k}=\rho-(1-\tau)\,r; \tag{52}$$

$$\frac{\dot{\lambda}_c}{\lambda_c}=2\rho-\frac{1}{c_i\lambda_c}+\frac{\lambda_k}{\lambda_c}-(1-\tau)\,r; \tag{53}$$

$$\frac{\dot{\mu}_K}{\mu_K}=\rho-r\frac{\lambda_k}{\mu_K}(1-\gamma)\left(\frac{1-\alpha}{\alpha}\right)l_i-r\left[(1-\tau)+(1-\gamma)\left(\frac{1-\alpha}{\alpha}\right)\right]; \tag{54}$$

$$\frac{\dot{\mu}_C}{\mu_C}=2\rho+\frac{\mu_K}{\mu_C}-(1-\tau)\,r, \tag{55}$$

where $\sigma_i=\frac{Kl_i}{k_i}$ and

$$\frac{\partial H}{\partial \tau}=\lambda_k\left[\eta_\tau k_i+(1-\gamma)\,w_\tau K l_i\right]+\lambda_c c_i\eta_\tau+\mu_K K\left[\eta_\tau+(1-\gamma)\,w_\tau\right]+\mu_C C\eta_\tau;$$

$$\frac{\partial H}{\partial \gamma}=\lambda_k\left[\eta_\gamma k_i+(1-\gamma)\,w_\gamma K l_i-wKl_i\right]+\lambda_c c_i\eta_\gamma+\mu_K K\left[\eta_\gamma+(1-\gamma)\,w_\gamma-w\right]+\mu_C C\eta_\gamma.$$

## AA. Candidate solution

A way to solve Problem P2 is to verify if a candidate solution satisfies necessary and sufficient conditions. In particular, we guess the solution (19)-(20) satisfies these conditions. This implies that:

$$\hat{\tau}_i = \max\left[0, \frac{\rho(\sigma_i - 1)}{\bar{r}\sigma_i}\right]; \tag{56}$$

$$\hat{\gamma}_i = 1 - \frac{\alpha\hat{\tau}_i}{1-\alpha}; \tag{57}$$

$$\hat{c}_i = \rho\hat{k}_i \max[1, \sigma_i]; \tag{58}$$

$$\hat{C} = \rho\hat{K}; \tag{59}$$

$$\hat{c}_i = \frac{1}{\lambda_k + \rho\lambda_c}; \tag{60}$$

$$\mu_K = -\lambda_k l_i; \tag{61}$$

$$\mu_C = \frac{\lambda_k l_i}{\rho} \tag{62}$$

is the solution of Problem P2, where $\bar{r} = \alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}$.

## AB. Check of candidate solution

Equations (56)-(62) causes variables $k_i$, $c_i$, $K$ and $C$ to grow at rate $\bar{\eta} = (1-\hat{\tau}_i)\bar{r}$ (see constraints Problem P1), while $\lambda_k$, $\lambda_c$, $\mu_K$, $\mu_C$ and $\mu_K$ to grow at rate $-\bar{\eta}$ (see equations (40)-(43)). This implies that trasversality conditions (44)-(47) and (49) are satisfied.

To complete the check we verify that (50) and (51) (i.e. 39) are satisfied.

From (3), (4), (57), (61) and (62) we have:

$$\left.\frac{\partial H}{\partial \tau}\right|_{\tau=\hat{\tau}_i,\gamma=\hat{\gamma}_i} = -\hat{\tau}_i\bar{r}k_i\left(\lambda_k + \lambda_c\frac{\hat{c}_i}{\hat{k}_i}\right); \tag{63}$$

$$\left.\frac{\partial H}{\partial \gamma}\right|_{\tau=\hat{\tau}_i,\gamma=\hat{\gamma}_i} = \frac{(1-\hat{\tau}_i)(1-\alpha)\bar{r}}{\alpha}\left(\lambda_k + \lambda_c\frac{\hat{c}_i}{\hat{k}_i}\right). \tag{64}$$

Firstly suppose that $\sigma_i > 1$. Equations (56)-(58) imply that:

$$\begin{aligned}\hat{\tau}_i &= \frac{\rho(\sigma_i - 1)}{\bar{r}\sigma_i} > 0;\\ \hat{\gamma}_i &= 1 - \frac{\alpha\hat{\tau}_i}{1-\alpha} < 1;\\ \hat{c}_i &= \rho\hat{k}_i\sigma_i\end{aligned}$$

and therefore from (63)-(64) and (50)-(51):

$$\begin{aligned}\hat{\tau}_i\bar{r}k_i\left(\lambda_k + \lambda_c\rho\sigma_i\right) &= 0;\\ \frac{(1-\hat{\tau}_i)(1-\alpha)\bar{r}}{\alpha}\left(\lambda_k + \lambda_c\rho\sigma_i\right) &= 0;\end{aligned}$$

By setting $\lambda_k^0 = \frac{1}{\rho\bar{k}_i(\sigma_i - 1)}$ and $\lambda_c^0 = -\frac{1}{\rho_i^2\bar{k}_i\sigma_i(\sigma_i-1)}$ (see equation (60)) both conditions are satisfied, i.e. $\lambda_k + \lambda_c\rho\sigma_i = 0$.

Finally suppose that $\sigma_i \leq 1$. Equations (56)-(58) imply that:

$$\begin{aligned}\hat{\tau}_i &= 0;\\ \hat{\gamma}_i &= 1;\\ \hat{c}_i &= \rho\hat{k}_i\end{aligned}$$

and therefore from (63)-(64) and (50)-(51):

$$\begin{aligned}-\hat{\tau}_i\bar{r}k_i\left(\lambda_k + \lambda_c\frac{\hat{c}_i}{\hat{k}_i}\right) &\leq 0;\\ \frac{(1-\alpha)\bar{r}}{\alpha\hat{c}_i} &\geq 0.\end{aligned}$$

Again both conditions are satisfied. This completes the check.

## B Derivation of equation (17)

The following are the first order conditions of problem (P1)

$$\frac{\partial H}{\partial \tau} = \frac{(1-\gamma)\, w_\tau \sigma_i}{(1-\gamma)\, w\sigma_i + \rho} + \lambda k_i \left[(1-\tau)\, r_\tau - r\right]; \tag{65}$$

$$\frac{\partial H}{\partial \gamma} = \frac{\sigma_i \left[(1-\gamma)\, w_\gamma - w\right]}{(1-\gamma)\, w\sigma_i + \rho} + \lambda k_i \left[(1-\tau)\, r_\gamma\right]; \tag{66}$$

$$\frac{\dot{\lambda}}{\lambda} = \rho - \frac{1}{\lambda k_i} - \left[(1-\tau)\, r - \rho\right], \tag{67}$$

where $r_\tau = \frac{\partial r}{\partial \tau}$, $r_\gamma = \frac{\partial r}{\partial \gamma}$, $w_\tau = \frac{\partial w}{\partial \tau}$ and $w_\gamma = \frac{\partial w}{\partial \gamma}$.

A solution to (67) is

$$\frac{1}{k_i \lambda} = \rho; \tag{68}$$

Equations (68), (65) and (66) yield:

$$\frac{\rho}{(1-\gamma)\, w\sigma_i + \rho} = \frac{r - (1-\tau)\, r_\tau}{(1-\gamma)\, w_\tau \sigma_i}; \tag{69}$$

$$\frac{\rho}{(1-\gamma)\, w\sigma_i + \rho} = -\frac{(1-\tau)\, r_\gamma}{\sigma_i \left[(1-\gamma)\, w_\gamma - w\right]}. \tag{70}$$

Substituting equation (69) in equation (70), given that $w = \frac{(1-\alpha)}{\alpha} r$, leads to

$$\frac{r - (1-\gamma)\, r_\gamma}{(1-\tau)\, r_\gamma} = \frac{r_\tau (1-\gamma)}{r - (1-\tau)\, r_\tau},$$

from which

$$\frac{(1-\gamma)(1-\alpha)^2 - \alpha^2 \tau - \alpha (1-\alpha)\gamma}{-(1-\tau)(1-\alpha)^2} = \frac{(1-\gamma)(1-\alpha)}{\alpha\tau + (1-\alpha)\gamma - (1-\alpha)(1-\tau)},$$

and finally

$$\tau = \frac{(1-\alpha)(1-\gamma)}{\alpha}, \tag{71}$$

which represents equation (17). Finally, notice that (68) satisfies the transversality condition (16).

## C Proof of Proposition 10

The proof consists in two steps: the first is to prove that if factor endowments are even distributed, i.e. $\sigma_i = 1 \; \forall i$, then $\tau^E = 0$, while the second step proves that if factor endowments are unevenly distributed then $\tau^E > 0$.

The first step is proved by verifying that 1) $\tau^E = 0$ is a solution of condition (34) if $\mu_\sigma = 1$ and that 2) $\mu_\sigma = 1 \Leftrightarrow \sigma_i = 1 \; \forall i$. The first point follows directly from (35), while the second requires a lengthier demonstration. First of all, note that the resource constraint implies $\frac{k_1}{lK} + ... + \frac{k_N}{lK} = \frac{1}{l}$ or $\frac{1}{\sigma_1} + ... + \frac{1}{\sigma_N} = N$[35], which yields $\mu_\sigma = \prod_{j=1}^N \sigma_j$. Now let's suppose for argument' sake that $\mu_\sigma = \prod_{j=1}^N \sigma_j = 1$, but that $\sigma_q, \sigma_z \neq 1$, where $q, z \leq\in \{1, ..., N\}$ and $q \neq z$. Since $\mu_\sigma = \prod_{j=1}^N \sigma_j = 1$ then $\frac{\sigma i + \sigma_z}{2} = 1$, from which $\sigma_q = 2 - \sigma_z$ and $\sigma_q \sigma_z = 1$; but $(2 - \sigma_z)\,\sigma_z = 1$ implies $(\sigma_z - 1)^2 = 0$ which is verified only for $\sigma_z = 1$; this contradicts the assumption $\sigma_z \neq 1$. Therefore $\mu_\sigma = 1 \Leftrightarrow \sigma_i = 1 \; \forall i$.

The second step is proved by induction. In particular, we verify that an increase in inequality of endowment distribution due to a reallocation of capital between two agents implies an increase in $\tau^E$. In fact, starting from an even distribution of resources, it is intuitive to conclude that any other feasible distribution can be generated by a series of reallocations between two agents that increases inequality; therefore, provided that an increase in inequality of endowment distribution implies an increase in $\tau^E$, all uneven distributions will be characterized by a $\tau^E > 0$.

Suppose that $\sigma_q > \sigma_z$, where $q, z \in \{1, ..., N\}$ and to redistribute some quantity of capital from agent $q$ to agent $z$; this causes an increase in $\sigma_q$ and a fall in $\sigma_z$, that is an increase in inequality. Moreover let $\bar{k} = k_q + k_z$ be the total amount of capital of two agents, from which $\frac{1}{\sigma_q} + \frac{1}{\sigma_z} = \frac{1}{\bar{\sigma}}$, where $\bar{\sigma} = \frac{Kl}{\bar{k}}$ is a constant. It easily follows that $\sigma_z = \frac{\bar{\sigma}\sigma_q}{\sigma_q - \bar{\sigma}}$ and $\frac{d\sigma_z}{d\sigma_q} = -\frac{\bar{\sigma}^2}{(\sigma_q - \bar{\sigma})^2}$.

[35] Indeed if $L = 1$ and $l_i = l \; \forall i$, then $l = \frac{1}{N}$.

Now calculate the total differential of (34), that is:

$$d\tau^E \left( \sum\nolimits_{j=1}^{N} \frac{\bar{r}\sigma_j^2}{\left(\tau^E \bar{r}\sigma_j + \rho\right)^2} \right) = \sum\nolimits_{j=1}^{N} \left[ \frac{\rho}{\left(\tau^E \bar{r}\sigma_j + \rho\right)^2} \right] d\sigma_j$$

and, provided that $\frac{1}{\sigma_q} + \frac{1}{\sigma_z} = \frac{1}{\bar{\sigma}}$ and $d\sigma_i = 0 \ \forall i, \ i \neq q, z$, the above equation becomes

$$d\tau^E \left( \sum\nolimits_{j=1}^{N} \frac{\bar{r}\sigma_j^2}{\left(\tau^E \bar{r}\sigma_j + \rho\right)^2} \right) =$$

$$= \rho \left[ \frac{1}{\left(\tau^E \bar{r}\sigma_q + \rho\right)^2} - \frac{\bar{\sigma}^2}{\left(\tau^E \bar{r} \left(\frac{\bar{\sigma}\sigma_q}{\sigma_q - \bar{\sigma}}\right) + \rho\right)^2 \left(\sigma_q - \bar{\sigma}\right)^2} \right] d\sigma_q,$$

hence:

$$d\tau^E \left( \sum\nolimits_{j=1}^{N} \frac{\bar{r}\sigma_j^2}{\left(\tau^E \bar{r}\sigma_j + \rho\right)^2} \right) = \rho \left[ \frac{\left(2\tau^E \bar{r}\sigma_q + \rho\right)\left(\sigma_q - 2\bar{\sigma}\right)\sigma_q \rho \bar{r}}{\left(\tau^E \bar{r}\sigma_q + \rho\right)^2 \left(\tau^E \bar{r}\bar{\sigma}\sigma_q + \rho\left(\sigma_q - \bar{\sigma}\right)\right)^2} \right] d\sigma_q.$$

It follows that:

$$d\tau^E > 0 \Leftrightarrow \sigma_q - 2\bar{\sigma} > 0,$$

that is

$$d\tau^E > 0 \Leftrightarrow \sigma_q > \sigma_z.$$

This completes the proof.

## D Leisure in utility function

The aim of this Appendix is to show how the fiscal policy which maximizes the growth rate changes if labor has an elastic supply. For simplicity the log-linear utility case will be analyzed, that is:

$$V_i = (1 - \phi) \log c_i + \phi \log p_i,$$

where $p_i$ is leisure and $\phi \in [0, 1]$ measures the elasticity of substitution between leisure and working time. By normalizing to $\dfrac{1}{N}$ the total amount of time disposable per period to each agent it follows that:

$$p_i = \frac{1}{N} - l_i.$$

As in the case of inelastic labor supply, wage and interest rate are defined by:

$$r = \alpha A^{\frac{1}{\alpha}} \left[\alpha\tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} L^{\frac{1-\alpha}{\alpha}} = r(\tau,\gamma); \tag{72}$$

$$\hat{w} = (1-\alpha) A^{\frac{1}{\alpha}} \left[\alpha\tau + (1-\alpha)\gamma\right]^{\frac{1-\alpha}{\alpha}} L^{\frac{1-2\alpha}{\alpha}} K = w(\tau,\gamma) K, \tag{73}$$

where $L = \sum_{i=1}^{N} l_i$.

Agent $i$ solves

$$\max_{\{c_i,l_i\}_{t=0}^{\infty}} U_i = \int_0^{\infty} e^{-\rho t} \left[(1-\phi)\log c_i + \phi \log\left(\frac{1}{N} - l_i\right)\right] dt \tag{74}$$

$$\dot{k}_i = (1-\tau) r(\tau,\gamma) k_i + (1-\gamma)\, w(\tau,\gamma) K l_i - c_i. \tag{75}$$

The first order conditions (necessary and sufficient for the optimum) are

$$\frac{1-\phi}{c_i} = \lambda; \tag{76}$$

$$\frac{\phi}{\frac{1}{N} - l_i} = \lambda (1-\gamma)\, w(\tau,\gamma) K; \tag{77}$$

$$\dot{\lambda} = \lambda \left[\rho - (1-\tau)\, r(\tau,\gamma)\right]; \tag{78}$$

$$\lim_{t\to\infty} e^{-\rho t} \lambda k_i = 0. \tag{79}$$

It is possible to show (see (76), (78) and (79)) that the steady state growth rate is

$$\eta(\tau,\gamma) = \frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = (1-\tau)\, r(\tau,\gamma) - \rho \tag{80}$$

and therefore from (75) the level of consumption is:

$$c_i = (1-\gamma)\, w(\tau,\gamma) l_i K + \rho k_i.$$

The latter can be used to calculate the optimal level of working time:

$$l_i = \frac{1-\phi}{N} - \frac{\phi\rho k_i}{(1-\gamma)\, w(\tau,\gamma) K}.$$

By aggregating it yields the aggregate supply curve of labor

$$L = (1-\phi) - \frac{\phi\rho}{(1-\gamma)\, w(\tau,\gamma)}. \tag{81}$$

Note that $w(\tau,\gamma)$ is a nonlinear function of $L$ (see (73)) and this, in general, does not allow an analytical solution. Since the intention of this Appendix is only to provide an example of the effects of an elastic supply of labor, for the sake of simplicity in order to obtain an analytical solution set $\alpha = \frac{1}{2}$.

The steady state growth rate will be given by ( see (72), (80) and (81))

$$\eta(\tau,\gamma) = \frac{1}{2}(1-\tau)\left[\left(\tfrac{\gamma+\tau}{2}\right)(1-\phi) - \tfrac{2\phi\rho}{(1-\gamma)}\right] - \rho.$$

The derivative of $\eta$ with respect to $\gamma$ is equal to

$$\frac{\partial\eta(\tau,\gamma)}{\partial\gamma} = \frac{1}{2}(1-\tau)\left[\frac{1-\phi}{2} - \frac{2\phi\rho}{(1-\gamma)^2}\right],$$

from which:

$$\gamma^* = \min\left[0, 1 - 2\sqrt{\frac{\phi\rho}{(1-\phi)}}\right].$$

Thus, if $\phi > 0$ then $\gamma^* < 1$ and $\dfrac{\partial\gamma^*}{\partial\phi} \leq 0$, strictly if $\phi < \frac{1}{1+4\rho}$ (see Section II.A.); it is worth noting that this result contrasts with Lucas (1990). However many empirical works have found $\phi$ very low, so that setting $\phi = 0$ does not seem a very strong assumption.

The derivative of $\eta$ with respect to $\tau$ is equal to

$$\frac{\partial\eta(\tau,\gamma)}{\partial\tau} = \frac{1}{2}\left[\frac{2\phi\rho}{(1-\gamma)^2} + \frac{(1-\phi)(1-\gamma)}{2} - \tau(1-\phi)\right],$$

from which:

$$\tau^* = \min\left[\frac{2\phi\rho}{\sqrt{\phi\rho(1-\phi)}}, 1\right].$$

Therefore, $\tau^* = 0$ iff $\phi = 0$ (the case of inelastic labor supply) and $\dfrac{\partial \tau^*}{\partial \phi} \geq 0$, strictly if $\phi < \frac{1}{1+4\rho}$; finally, if $\phi > \frac{1}{1+4\rho}$ then $(\tau^*, \gamma^*) = (1, 0)$ and the steady state growth rate will be negative.

## *E CES utility function*

This Appendix extends the analysis to the case where the utility is:

$$\frac{(c_i)^{1-\mu} - 1}{1-\mu},$$

that is, the elasticity of substitution of consumption is constant and equal to $\frac{1}{\mu}$. We stress that a CES utility function is a necessary condition to have a steady state.

The *i-th* agent solves

$$\max_{\{c_i\}_{t=0}^{\infty}} U_i = \int_0^{\infty} e^{-\rho t} \frac{(c_i)^{1-\mu} - 1}{1-\mu} dt \tag{P1.A}$$

$$s.t.\ \dot{k}_i = k_i \left[(1-\tau) r(\tau, \gamma) + (1-\gamma)\, w(\tau, \gamma) \sigma_i\right] - c_i.$$

Given the time paths both of the taxes and of the capital aggregate stock, the solution of problem (P1.A) yields the following optimal consumption path:

$$\frac{\dot{c}_i}{c_i} = \frac{(1-\tau) r(\tau, \gamma) - \rho}{\mu}. \tag{82}$$

It is possible to demonstrate that (82) is also the growth rate of $K$ and $k_i$, such that

$$\frac{\dot{c}_i}{c_i} = \frac{\dot{k}_i}{k_i} = \frac{\dot{K}}{K} = \frac{(1-\tau) r(\tau, \gamma) - \rho}{\mu} = \eta\,(\tau, \gamma)\,. \tag{83}$$

Therefore $\eta\,(\tau, \gamma)$ is the steady state growth rate; finally, the instantaneous level of consumption along the optimal path is given by:

$$c_i = \frac{\left[(\mu - 1)\,(1-\tau)\, r(\tau, \gamma) + \mu\,(1-\gamma)\, w(\tau, \gamma) \sigma_i + \rho\right] k_i}{\mu}.$$

The optimal fiscal policy for the *i-th* agent solves:

$$\max_{\{\tau,\gamma\}_{t=0}^{\infty}} U_i = \int_0^{\infty} e^{-\rho t} \frac{(c_i)^{1-\mu} - 1}{1-\mu}\, dt$$

$$s.t. \begin{cases} c_i = \dfrac{\left[(\mu-1)(1-\tau)\, r(\tau,\gamma) + \mu (1-\gamma)\, w(\tau,\gamma)\sigma_i + \rho\right] k_i}{\mu} \\ \dfrac{\dot{k}_i}{k_i} = \dfrac{\dot{K}}{K} = \eta(\tau,\gamma). \end{cases} \tag{P2.B}$$

Assume that $\frac{\rho}{\bar{r}} + (\mu - 1) > 0$ in order to have for every feasible $\tau$ a definite integral in (P2.B). The current value Hamiltonian of problem (P2.B) is given by

$$H = \frac{\left\{\left[(\mu-1)(1-\tau)\, r(\tau,\gamma) + \mu(1-\gamma)\, w(\tau,\gamma)\sigma_i + \rho\right] k_i\right\}^{1-\mu}}{\mu^{1-\mu}(1-\mu)} + \\ + \lambda k_i \left[\frac{(1-\tau)\, r(\tau,\gamma) - \rho}{\mu}\right] - \frac{1}{\mu^{1-\mu}(1-\mu)}. \tag{84}$$

The following are the necessary and sufficient conditions for the optimum:

$$\frac{\partial H}{\partial \tau} = 0, \tag{85}$$

$$\frac{\partial H}{\partial \gamma} = 0, \tag{86}$$

$$\dot{\lambda} = \rho\lambda - \frac{\partial H}{\partial k_i} \text{ and} \tag{87}$$

$$\lim_{t\to\infty} e^{-\rho t}\lambda k_i = 0. \tag{88}$$

From equations (85), (86) and (87), it follows that:[36]

$$\tau = \frac{(1-\alpha)(1-\gamma)}{\alpha}. \tag{89}$$

[36] The calculations are available on request.

Hence, substituting (89) in (85):

$$\hat{\tau}_i = \max\left[0, \frac{(\sigma_i - 1)\left[\frac{\rho}{\bar{r}} + (\mu - 1)\right]}{\mu(\sigma_i - 1) + 1}\right], \tag{90}$$

where $\bar{r} = \alpha A^{\frac{1}{\alpha}}(1-\alpha)^{\frac{1-\alpha}{\alpha}}$ is the value of $r$ when (89) holds.

On the assumption that tax rates are constant, solving the integral in (P2.B) yields the following expression for individual utility:

$$U_i(\sigma_i, \tau, \gamma) = \frac{[(\mu - 1)(1-\tau) r(\tau,\gamma) + \mu(1-\gamma) w(\tau,\gamma)\sigma_i + \rho]^{1-\mu}}{\mu^{1-\mu}(1-\mu)(k_0^i)^{\mu-1}[\rho - (1-\mu)\eta(\tau,\gamma)]} + \tag{91}$$
$$- \frac{1}{\rho(1-\mu)}.$$

It is analytically convenient to focus attention only on fiscal policies which can be a Pareto optimum; substituting (89) in (91) and by using the relationship $w(\tau,\gamma) = \left(\frac{1-\alpha}{\alpha}\right) r(\tau,\gamma)$, it follows that:

$$\bar{U}(\sigma_i, \tau) = \frac{\mu^{\mu}\left(k_0^i\right)^{1-\mu}\left[\mu\tau\sigma_i - (1-\mu)(1-\tau) + \frac{\rho}{\bar{r}}\right]^{1-\mu}}{(1-\mu)\bar{r}^{\mu}\left[\frac{\rho}{\bar{r}} - (1-\mu)(1-\tau)\right]} - \frac{1}{\rho(1-\mu)}.$$

Notice that $\bar{U}$ is concave with respect to $\tau$ if:[37]

$$\mu\sigma_i - \mu + 1 > 0;$$

since this condition has to hold for every $i$, assume that

$$\mu < \frac{1}{1 - \breve{\sigma}}, \tag{92}$$

where $\breve{\sigma} = \min\{\sigma_i\}_{i=1}^{N} \leq 1$.

[37]In fact:

$$\frac{\partial^2 U}{\partial \tau^2} = \bar{r}(Kl_i)^{1-\mu}\left[1 - \frac{\bar{r}^{-\mu-1}\sigma_i^{\mu-1}\mu^{\mu+1}(\mu\sigma_i - \mu + 1)}{\left(\mu\tau\sigma_i + (1-\tau)(\mu-1) + \frac{\rho}{\bar{r}}\right)^{\mu}\left(-(1-\tau)(\mu-1) - \frac{\rho}{\bar{r}}\right)^2}\right],$$

from which a necessary condition to have $\frac{\partial^2 U}{\partial \tau^2} > 0$ is $(\mu\sigma_i - \mu + 1) > 0$.

## EA. Maximum growth

The following Propositions state the fiscal policies that maximize the growth rate and that of politico-economic equilibrium:

**Proposition 11** *The fiscal policy that maximizes the growth rate* $\eta(\tau,\gamma)$ *is given by* $(\tau^*,\gamma^*) = (0,1)$.

**Proof.** Consider the derivative of equation (11) with respect to $\tau$ and $\gamma$. Since $\dfrac{\partial\eta(\tau,\gamma)}{\partial\tau} > 0 \Leftrightarrow \tau < (1-\alpha)(1-\gamma)$ and $\dfrac{\partial\eta(\tau,\gamma)}{\partial\gamma} > 0 \quad \forall(\tau,\gamma)$, then the fiscal policy which maximizes growth is given by $(\tau^*,\gamma^*) = (0,1)$. ■

In the following assume that every agent has the same labor endowment, such that a difference in $\sigma$ reflects different capital endowments, that is:

$$l_i = l = \frac{1}{N} \; \forall i \in \{1,...,N\}. \tag{93}$$

Moreover, to avoid trivial results assume, unless otherwise specified, that $\sigma$ is unevenly distributed, that is:

$$\exists i,j \in \{1,...,N\} \text{ with } i \neq j \text{ such that } \sigma_i \neq \sigma_j.$$

## EB. Normative analysis

As stated above we focus our attention only on fiscal policies satisfying relationship (89). Substituting for $\gamma$ in (91) yields the set of all individual utilities associated to $\tau$:

$$\left[\, \bar{U}(\sigma_1,\tau), \;\; ... \;\; ,\bar{U}(\sigma_N,\tau) \,\right],$$

whose elements are ranked in increasing order, i.e. $\bar{U}(\sigma_1,\tau) \leq \bar{U}(\sigma_2,\tau) \leq ... \leq \bar{U}(\sigma_N,\tau)$.

Assume that the welfare function is additive-separable, that is:

$$W = \sum_{j=1}^{N} \phi\left(\bar{U}(\sigma_j,\tau)\right); \tag{94}$$

Let $\tau^W$ be the tax rate on capital income which maximizes the welfare function, that is

$$\tau^W = \arg\max_{\tau} \left[ \sum_{j=1}^{N} \phi \left( \bar{U} \left( \sigma_j, \tau \right) \right) \right]. \tag{95}$$

Let the (generalized) Lorenz curve be:

$$GL^{\tau}(p) = \frac{\sum_{j=1}^{S} \bar{U}(\sigma_i, \tau)}{N} = $$
$$= \frac{1}{N} \sum_{j=1}^{S} \frac{\mu^{\mu} \left(k_0^i\right)^{1-\mu} \left[\mu\tau\sigma_i - (1-\mu)(1-\tau) + \frac{\rho}{\bar{r}}\right]^{1-\mu}}{(1-\mu)\bar{r}^{\mu} \left[\frac{\rho}{\bar{r}} - (1-\mu)(1-\tau)\right]} - \frac{S}{N\rho(1-\mu)}$$

for $S = 1, ..., N$ and where $p = \frac{S}{N}$ and $\bar{r} = \alpha A^{\frac{1}{\alpha}} (1-\alpha)^{\frac{1-\alpha}{\alpha}}$.

The first derivative of $GL(p, \tau)$ with respect to $\tau$ shows how fiscal policy affects the Lorenz curve:

$$\frac{\partial GL(p,\tau)}{\partial \tau} = \sum_{j=1}^{S} \frac{(Kl)^{1-\mu} \mu^{\mu+1} \sigma_j^{\mu-1} \left[(\sigma_j - 1)\left(\frac{\rho}{\bar{r}} - 1 + \mu(1-\tau)\right) - \tau\right]}{N\bar{r}^{\mu} \left[\frac{\rho}{\bar{r}} + \mu\tau\sigma_j - (1-\mu)(1-\tau)\right]^{\mu} \left[(1-\tau)(1-\mu) - \frac{\rho}{\bar{r}}\right]}. \tag{96}$$

The following Proposition states the optimal fiscal policy for a Rawlsian social planner:

**Proposition 12** *Let $(\tau^R, \gamma^R)$ be the optimal fiscal policy when the social planner has Rawlsian preferences. Then $\tau^R$ maximizes $GL\left(\frac{1}{N}, \tau\right)$, that is:*

$$\tau^R = \max \left[ 0, \frac{(\sigma_1 - 1)\left[\frac{\rho}{\bar{r}} + (\mu - 1)\right]}{\mu(\sigma_1 - 1) + 1} \right] \tag{97}$$

*and*

$$\gamma^R = \min \left[ 1, 1 - \frac{\alpha(\sigma_m - 1)\left[\frac{\rho}{\bar{r}} + (\mu-1)\right]}{(1-\alpha)\left[\mu(\sigma_m - 1) + 1\right]} \right]. \tag{98}$$

**Proof.** To prove (97) it is sufficient to note that $\bar{U}(\sigma_1, \tau) \leq \bar{U}(\sigma_2, \tau) \leq ... \leq \bar{U}(\sigma_N, \tau)$, while $\gamma^R$ is derived from (89). ∎

The following Proposition states the optimal fiscal policy when welfare is the simple sum of individual utilities, that is efficiency is the only goal of the social planner:

**Proposition 13** *Let $\left(\tau^E, \gamma^E\right)$ be the optimal fiscal policy when welfare is the simple sum of individual utilities. Then $\tau^E$ maximizes $GL^{\tau}(1)$, that is:*

$$\left.\frac{\partial GL^{\tau}(1)}{\partial \tau}\right|_{\tau=\tau^E} = 0 \tag{99}$$

*and*

$$\gamma^E = 1 - \left(\frac{\alpha}{1-\alpha}\right)\tau^E. \tag{100}$$

**Proof.** The proof of (99) is given by the same definition of the generalized Lorenz curve, while $\gamma^E$ is derived from (89). ■

Since $\tau^E$ solves the condition (99), i.e.

$$\left.\frac{\partial GL^{\tau}(1)}{\partial \tau}\right|_{\tau=\tau^E} = 0 \tag{101}$$

$$\Leftrightarrow \sum_{j=1}^{N} \frac{\sigma_j^{\mu-1}\left[(\sigma_j - 1)\left(\frac{\rho}{\bar{r}} - 1 + \mu\left(1-\tau^E\right)\right) - \tau^E\right]}{\left[\frac{\rho}{\bar{r}} + \mu\tau^E\sigma_j - (1-\mu)\left(1-\tau^E\right)\right]^{\mu}} = 0,$$

it follows that $\tau^R \geq \tau^E$. Indeed, note that:[38]

$$\frac{\sigma_1^{\mu-1}\left[(\sigma_1 - 1)\left(\frac{\rho}{\bar{r}} - 1 + \mu(1-\tau)\right) - \tau\right]}{\left[\frac{\rho}{\bar{r}} + \mu\tau\sigma_1 - (1-\mu)(1-\tau)\right]^{\mu}} \geq$$

$$\frac{\sigma_j^{\mu-1}\left[(\sigma_j - 1)\left(\frac{\rho}{\bar{r}} - 1 + \mu(1-\tau)\right) - \tau\right]}{\left[\frac{\rho}{\bar{r}} + \mu\tau\sigma_j - (1-\mu)(1-\tau)\right]^{\mu}}$$

$\forall j = 2, ..., N$ and $\forall \tau \in [0, 1]$, from which

$$\sum_{j=1}^{N} \frac{\sigma_j^{\mu-1}\left[(\sigma_j - 1)\left(\frac{\rho}{\bar{r}} - 1 + \mu\left(1-\tau^R\right)\right) - \tau^R\right]}{\left[\frac{\rho}{\bar{r}} + \mu\tau^R\sigma_j - (1-\mu)\left(1-\tau^R\right)\right]^{\mu}} \leq 0,$$

[38] Since $(\mu\sigma_i - \mu + 1) > 0\ \forall i$ (see (92)), then

$$\frac{\partial^2 U}{\partial \tau \partial \sigma_j} = \frac{\sigma_j^{\mu-2}(Kl)^{1-\mu}\,\bar{r}^{-\mu}\mu^{\mu+1}(\mu\sigma_j - \mu + 1)}{\left(\mu\tau\sigma_j + (1-\tau)(\mu-1) + \frac{\rho}{\bar{r}}\right)^{1+\mu}} > 0.$$

that yields $\tau^R \geq \tau^E$. Heuristically the set of all possible $\tau^W$ belongs to the interval $\left[\tau^E, \tau^R\right]$, and the higher the social planner's inequality aversion, the closer is $\tau^W$ to $\tau^R$.

To examine this point in greater depth, note that since $\sigma_1 \geq \sigma_1 \geq ... \geq \sigma_N$ then:

$$\frac{\sum_{j=1}^{S} \frac{\sigma_j^{\mu-1}\left[(\sigma_j-1)\left(\frac{\rho}{\bar{r}}-1+\mu(1-\tau)\right)-\tau\right]}{\left[\frac{\rho}{\bar{r}}+\mu\tau\sigma_j-(1-\mu)(1-\tau)\right]^{\mu}}}{S} \geq \frac{\sum_{j=1}^{S+1} \frac{\sigma_j^{\mu-1}\left[(\sigma_j-1)\left(\frac{\rho}{\bar{r}}-1+\mu(1-\tau)\right)-\tau\right]}{\left[\frac{\rho}{\bar{r}}+\mu\tau\sigma_j-(1-\mu)(1-\tau)\right]^{\mu}}}{S+1} \tag{102}$$

for $S \in [1, N-1]$ and $\forall \tau \in [0,1]$.

Given that $\tau^E$ has to satisfy the following equality:

$$\sum_{j=1}^{N} \frac{\sigma_j^{\mu-1}\left[(\sigma_j-1)\left(\frac{\rho}{\bar{r}}-1+\mu\left(1-\tau^E\right)\right)-\tau^E\right]}{\left[\frac{\rho}{\bar{r}}+\mu\tau^E\sigma_j-(1-\mu)\left(1-\tau^E\right)\right]^{\mu}} = 0,$$

it follows that:

$$\left.\frac{\partial GL^{\tau}(p)}{\partial \tau}\right|_{\tau=\tau^E} = \sum_{j=1}^{S} \frac{\sigma_j^{\mu-1}\left[(\sigma_j-1)\left(\frac{\rho}{\bar{r}}-1+\mu\left(1-\tau^E\right)\right)-\tau^E\right]}{\left[\frac{\rho}{\bar{r}}+\mu\tau^E\sigma_j-(1-\mu)\left(1-\tau^E\right)\right]^{\mu}} \geq 0$$

for $S = 1, ..., N-1$.

This means that the Lorenz curve corresponding to $\tau^E$ is everywhere increasing in $\tau^E$, except for $p = 1$; in other words, an increase in $\tau^E$ causes a decrease in inequality, but also a decrease in average utility. Let $\tau_S$ be the level of tax rate that if increased would leave $GL^{\tau}(p)$ unchanged, that is:

$$\left.\frac{\partial GL^{\tau}(p)}{\partial \tau}\right|_{\tau=\tau_S} = 0.$$

From (102) it follows that vector $\{\tau_S\}_{S=1}^{N}$ is ranked in decreasing order, i.e. $\tau_S \geq \tau_{S+1}$, such that $\tau_1 = \tau^R$ and $\tau_N = \tau^E$ (see Propositions 12 and 13).

**Proposition 14** *Let* $W = \sum_{j=1}^{N} \phi\left(\bar{U}(\sigma_j, \tau)\right)$ *be the social welfare function, where* $\phi' \geq 0$, $\phi'' \leq 0$, $\bar{U}(\sigma_i, \tau)$ *the intertemporal utility* *of agent i and* $\bar{U}(\sigma_i, \tau) \leq \bar{U}(\sigma_{i+1}, \tau)$ *for* $i = 1, ..., N-1$. *Then the tax rate on capital income maximizing welfare function* $\tau^W$ *has to belong to the interval* $\left[\tau^E, \tau^R\right]$, *where* $\tau^R = \max\left[0, \frac{(\sigma_1-1)\left[\frac{\rho}{\bar{r}}+(\mu-1)\right]}{\mu(\sigma_1-1)+1}\right]$ *and* $\tau^E$ *implicitly solves* $\sum_{j=1}^{N} \frac{\sigma_j^{\mu-1}\left[(\sigma_j-1)\left(\frac{\rho}{\bar{r}}-1+\mu\left(1-\tau^E\right)\right)-\tau^E\right]}{\left[\frac{\rho}{\bar{r}}+\mu\tau^E\sigma_j-(1-\mu)(1-\tau^E)\right]^{\mu}} = 0.$ *Moreover, according to Definitions 4 and 5,* $\tau^E$ *is the most efficient fiscal policy, while all fiscal policies belonging to the interval* $\left(\tau^E, \tau^R\right)$ *present an efficiency-equity trade-off.*

**Proof.** From (102) it follows that $\tau_S \geq \tau_{S+1}$, and since $\tau^E$ maximizes the simple sum of individual utility, then in every economy where $\tau < \tau^E$ average individual utility is lower and inequality is higher than in the economy where $\tau = \tau^E$; this means that any $\tau < \tau^E$ cannot maximize social welfare. Moreover, in every economy where $\tau > \tau^R$ average individual utility is lower and the poorest agent is worse off than in the economy where $\tau = \tau^R$; therefore the tax rate on capital income maximizing welfare function $\tau^W$ has to belong to the range $\left[\tau^E, \tau^R\right]$. This proves the first statement. The proof of the second statement directly follows from Definitions 4 and 5 and equation (96). ■

*EB.i. Efficient fiscal policy*

According to Definition 4 the most efficient fiscal policy maximizes the simple sum of individual utilities, that is:

$$\left(\tau^E, \tau^E\right) = \arg\max_{\tau,\gamma} W =$$

$$= \frac{1}{N}\sum_{j=1}^{S} \frac{\mu^{\mu}\left(k_0^i\right)^{1-\mu}\left[\mu\tau\sigma_i - (1-\mu)(1-\tau) + \frac{\rho}{\bar{r}}\right]^{1-\mu}}{(1-\mu)\,\bar{r}^{\mu}\left[\frac{\rho}{\bar{r}} - (1-\mu)(1-\tau)\right]} - \frac{S}{N\rho(1-\mu)}$$

By the first order conditions it follows that:

$$\tau = \frac{(1-\alpha)(1-\gamma)}{\alpha}$$

and

$$\sum_{j=1}^{N} \frac{\sigma_j^{\mu-1} \left[ (\sigma_j - 1) \left( \frac{\rho}{\bar{r}} - 1 + \mu \left( 1 - \tau^E \right) \right) - \tau^E \right]}{\left[ \frac{\rho}{\bar{r}} + \mu \tau^E \sigma_j - (1-\mu)\left(1 - \tau^E\right) \right]^{\mu}} = 0. \tag{103}$$

Setting $\tau^E = 0$ in (34) yields a sufficient condition to obtain $\tau^E = 0$, that is

$$\sum_{j=1}^{N} \sigma_j^{\mu-1} (\sigma_j - 1) = 0, \tag{104}$$

that is $\sigma_i = 1 \; \forall i$ (or $\mu_\sigma = 1$).[39]

The following Proposition characterizes the most efficient fiscal policy:

**Proposition 15** *The most efficient fiscal policy (or the optimal fiscal policy when the social planner is indifferent to inequality) involves a positive tax rate on capital income, but the case of even distribution of individual endowments (i.e. the representative agent economy); that is:*

$$\tau^E = \begin{cases} 0 & \text{if } \sigma_i = 1 \; \forall i \\ \psi > 0 & \text{if } \exists i \text{ such that } \sigma_i \neq 1 \end{cases}$$

**Proof.** This proof follows closely that of Appendix C. The proof consists of two steps: the first is to prove that if factor endowments are evenly distributed, i.e. $\sigma_i = 1 \; \forall i$, then $\tau^E = 0$, while the second that, if factor endowments are unevenly distributed, then $\tau^E > 0$. The first step is proved by verifying that 1) $\tau^E = 0$ is a solution of condition (103) if $\mu_\sigma = 1$ and that 2) $\mu_\sigma = 1 \Leftrightarrow \sigma_i = 1 \; \forall i$. The first point follows directly from (104), while for the second refer to Appendix C. The second step is proved by verifying that an increase in inequality of endowments' distribution due to a reallocation of capital between two agents implies an increase in $\tau^E$. Suppose that $\sigma_q > \sigma_z$, where $q, z \in [1, N]$ and to redistribute some quantity of capital from agent $q$ to agent $z$; this causes an increase in $\sigma_q$ and a

[39] See Appendix C.

fall in $\sigma_z$, that is an increase in inequality. Moreover, let $\bar{k} = k_q + k_z$ be the total amount of capital of two agents, from which $\frac{1}{\sigma_q} + \frac{1}{\sigma_z} = \frac{1}{\bar{\sigma}}$, where $\bar{\sigma} = \frac{Kl}{\bar{k}}$ is a constant. It easily follows that $\sigma_z = \frac{\bar{\sigma}\sigma_q}{\sigma_q - \bar{\sigma}}$ and $\frac{d\sigma_z}{d\sigma_q} = -\frac{\bar{\sigma}^2}{(\sigma_q - \bar{\sigma})^2}$. Given the total differential of (94), that is:

$$d\tau^E \left( \sum_{j=1}^{N} \frac{\partial^2 W}{\partial \tau^2} \right) = -\sum_{j=1}^{N} \frac{\partial^2 W}{\partial \tau \partial \sigma_j} d\sigma_j,$$

where $\frac{\partial^2 W}{\partial \tau^2} \leq 0$,[40] provided that $\frac{1}{\sigma_q} + \frac{1}{\sigma_z} = \frac{1}{\bar{\sigma}}$, $d\sigma_q > 0$, $d\sigma_z = -\frac{\bar{\sigma}^2 d\sigma_q}{(\sigma_q - \bar{\sigma})^2}$ and $d\sigma_i = 0 \ \forall i$, $i \neq q, z$, the above equation becomes

$$d\tau^E \left( -\sum_{j=1}^{N} \frac{\partial^2 W}{\partial \tau^2} \right) = \frac{\mu^{\mu+1} (Kl)^{1-\mu} \bar{r}}{\sigma_q^2} \left\{ \frac{\sigma_q^\mu (\mu\sigma_q - \mu + 1)}{[\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_q]^{(1+\mu)}} + \right.$$
$$\left. - \frac{\sigma_z^\mu (\mu\sigma_z - \mu + 1)}{[\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_z]^{(1+\mu)}} \right\} d\sigma_q,$$

hence:

$$d\tau^E > 0 \Leftrightarrow$$
$$\frac{\sigma_q^\mu (\mu\sigma_q - \mu + 1)}{[\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_q]^{(1+\mu)}} +$$
$$- \frac{\sigma_z^\mu (\mu\sigma_z - \mu + 1)}{[\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_z]^{(1+\mu)}} > 0.$$

The last inequality holds if:[41]

$$\frac{\sigma_q^\mu (\mu\sigma_q - \mu + 1) [\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_z]^{(1+\mu)}}{\sigma_z^\mu (\mu\sigma_z - \mu + 1) [\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_q]^{(1+\mu)}} > 1,$$

that is:

$$\left( \frac{\sigma_q [\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_z]}{\sigma_z [\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_q]} \right)^\mu *$$
$$* \left( \frac{(\mu\sigma_q - \mu + 1) [\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_z]}{(\mu\sigma_z - \mu + 1) [\bar{r}(1-\tau)(\mu-1) + \rho + \mu\tau\bar{r}\sigma_q]} \right) > 1.$$

It is easy to check that both members are greater than 1, such that $d\tau^E > 0$. ■

[40] This is the second order condition to have a maximum.

[41] For condition (92) both $\mu\sigma_q - \mu + 1$ and $\mu\sigma_z - \mu + 1$ are nonegative.

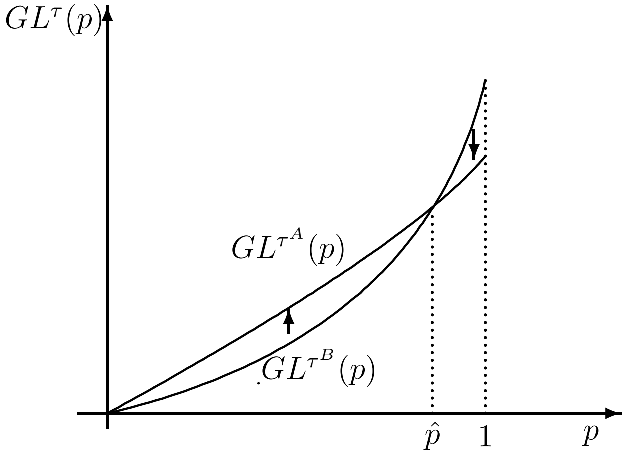



Figure 1: efficiency-equity trade-off

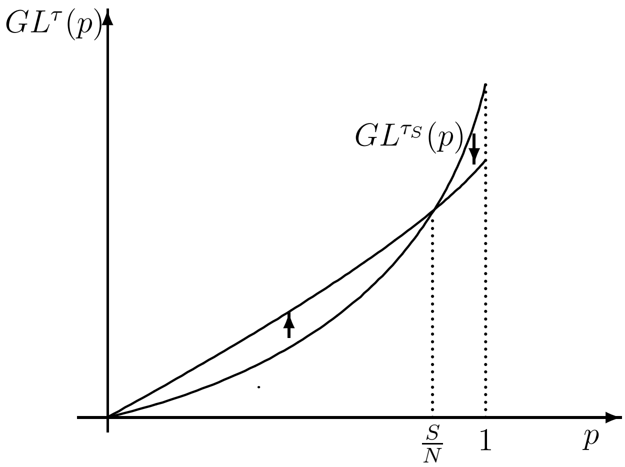



Figure 2: effect of an increase in $\tau$

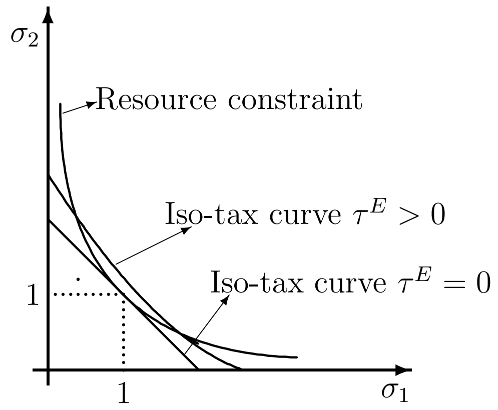



Figure 3: two agent economy

### *References*

Alesina, A. and Rodrik, D.: 1994, Distributive politics and economic growth, *Quarterly Journal of Economics* **109**(2), 493–490.

Barro, R. and Sala-I-Martin, X.: 1999, *Economic Growth*, MIT Press, London, England.

Benhabib, J., Rustichini, A. and Velasco, A.: 2001, Public spending and optimal taxes without commitment, *Review of Economic Design* **6**, 371–396.

Bertola, G.: 1993, Factor shares and savings in endogenous growth, *American Economic Review* **81**, 1184–1198.

Chamley, C. P.: 1986, Optimal taxation of capital income in general equilibrium with infinite lives, *Econometrica* **54**(3), 607–622.

Chiang, A.: 1992, *Elements of Dynamics Optimization*, McGraw Hill, New York.

Correia, I. H.: 1999, On the efficiency and equity trade-off, *Journal of Monetary Economics* **44**, 581–603.

Dardanoni, V. and Lambert, P.: 1988, Welfare ranking of income distribution: A role for the variance and some insights for tax reforms, *Social Choice and Welfare* **5**(1), 1–17.

Jones, L., Manuelli, R. and Rossi, P.: 1997, On the optimal taxation of capital income, *Journal of Economic Theory* **73**, 93–117.

Judd, K.: 1985, Redistributive taxation in a simple perfect foresight model, *Journal of Public Economics* **28**, 59–83.

Judd, K. L.: 1999, Optimal taxation and spending in general competitive growth models, *Journal of Public Economics* **71**, 1–26.

Karcher, T., Moyes, P. and Trannoy, A.: 1995, The stochastic dominance ordering of income distributions over time: The disconted sum of the expected utilities of incomes, *in* W. B. et Al. (ed.), *Social Choice, Welfare, and Ethics: Proceedings of the Eighth International Symposium in Economic Theory and Econometrics*, Cambridge University Press, New York and Melbourne, pp. 375–408.

Lansing, K.: 1999, Optimal redistributive capital taxation in a neoclassical growth model, *Journal of Public Economics* **73**, 423–453.

Lucas, R. E.: 1990, Supply-side economics: an analytical review, *Oxford Economic Papers* **42**, 293–316.

Seierstad, A. and Sydsaeter, K.: 1985, *Optimal Control Theory with Economic Applications*, North-Holland, Amsterdam.

Shorrocks, A. F. and Foster, J. E.: 1987, Transfer sensitive inequality measures, *Review of Economic Studies* **3**(179), 485–497.

Stiglitz, J.: 1987, Pareto efficient and optimal taxatation and the new welfare economics, *in* A. Auerbach and M. Feldstein (eds), *Handbook of Public Economics Vol. II*, Vol. II, North-Holland.

19. Luciano Fanti, Notes on Keynesian models of recession and depression, (ottobre 2003)
20. Luciano Fanti, Technological Diffusion and Cyclical Growth, (ottobre 2003)
21. Luciano Fanti - Piero Manfredi, Neo-classical labour market dynamics, chaos and the Phillips Curve, (ottobre 2003)
22.