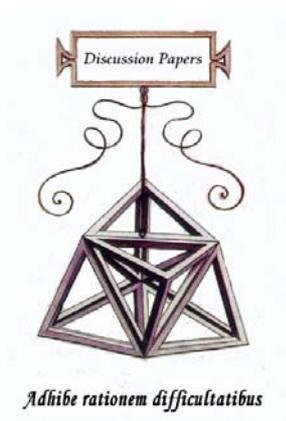




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***Notes on Keynesian models of
recession and depression***

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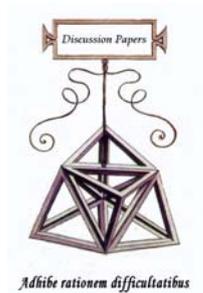
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Luciano Fanti

Notes on Keynesian models of recession and depression

Abstract

Notes on Keynesian models of recession and depression.

In this paper we have developed a “minimalist” Keynesian model (simplified version of Tobin’s model, 1975) aiming at demonstrating the existence of endogenous cycles.

We have shown that the Tobin’s interpretation of the forces governing the stability can be misleading, in that 1) the speculative effect on demand is not necessary for instability, 2) this latter depends on the relative strength of the speeds of quantity adjustment and of price expectations to experience, given the “propensity to spend”, in contrast with the condition claimed by Tobin, requiring that the “speculative” effects are prevailing on the “price” effects on aggregate demand. Moreover, we have shown that 1) on the one side a stable business cycle can emerge, despite of the almost linear assumptions; 2) on the other side the Tobin’s belief that the economy might be stable for small deviations from its equilibrium but unstable for large shocks, has been confirmed in consequence of a local subcritical bifurcation in some parametric cases; 3) both price flexibility and price effects (i.e. Keynes and Pigou effects) does not play any role in restoring full employment equilibrium, in contrast with a common macroeconomic belief; 4) the business cycle not only is endogenous, but, as a matter of fact, it is the result of a traditional

Walras-Keynes-Phillips macroeconomic model even with linear behavioural functions.

Classificazione JEL: E120, E320

Keywords: Keynesian macrodynamics, endogenous business cycle.

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Introduction.

The investigation for a deterministic explanation of the business economic cycle, in contrast with the stochastic view of the new classical macroeconomics (i.e. Real Business Cycle theory), has renewed the interest for Keynesian macroeconomic models. However, in general, this latter type of models has been characterised by two, in a somewhat sense limiting, features: on the one side they are based on too restrictive Keynesian features as money wage and/or price rigidity, liquidity trap, IS-LM frame, etc., and on the other side have postulated somewhat ‘ad-hoc’ strong nonlinearities in the structural relations. For instance, most of the recent typical dynamic Keynesian models are based on an IS-LM frame with some extension; for example Schinasi (1981, 1982) and Sasakura (1994) extend the traditional IS-LM scheme with a government budget constraint in which both money and bond financing are alternatively used, Benassy (1984) adds an explicit aggregate production function and a wage determination sector, Lorenz (1994) adds the accumulation of capital and the public sector, Kiefer (1996) adds an expectational Phillips curve, adaptive expectations and the dependence – following Benassy - of the investment function on expected output rather than on current output. All these authors have introduced strong nonlinearities which could initiate the critique of a so-called ad-hoc procedure: Schinasi, Sasakura and Lorenz use a highly non-linear (sigmoid shape) investment function according to Kaldorian lines, Benassy and Kiefer incorporate a ceiling on supply in the Phillips curve as well

as an assumption of increasing partial derivative of output with respect to its expectation.

Apart from the IS-LM frame, Fanti (2001) has shown the existence of short run (also possibly chaotic) cycles in a model of the sole goods market implementing the view of the Keynes' General Theory based on the marshallian microfoundation of the firm's behaviour.

A cornerstone of the renaissance of Keynesian models of business cycle has been the article of Tobin (1975). Tobin does not aim to build explicitly a model of business cycle, as in the tradition of the famous foregoing models of Goodwin, Hicks, Samuelson, but rather develops a simple model aiming at showing that "even with stable monetary and fiscal policy, combined with price and wage flexibility, the adjustment mechanisms of the economy may be too weak to eliminate persistent unemployment." (p. 202), in contrast with the view, starting from the Pigou's critique to Keynes, that the private market economy can and will, without government's interventions, restore the equilibrium at full employment level with reduced or zero inflation.

Tobin reinterprets the debate on Keynesian view pointing out that "the real issue is not the existence of a long-run static equilibrium with unemployment, but the possibility of protracted unemployment which the natural adjustments of a market economy remedy very slow if at all"(p. 195). The Tobin's dynamic analysis has successfully demonstrated the possibility of instability implicit in the adjustment of a pure market economy, when, in line with a Keynesian view, "price-level effects are weak relative to speculative effects [on aggregate effective demand]" (Tobin, p.200).

However, unfortunately, his interpretation of such a result has two major flaws: 1) the economic forces governing the instability suggested by Tobin – i.e. price effects versus "speculative" effects on aggregate demand – can be neither necessary nor sufficient; 2) Tobin's analysis is unable, in contrast with the Tobin's conclusions, to represent an economy, "stable for small deviations from its equilibrium but unstable for large shocks" (p. 201) as well as an economy evolving according to a continuous succession of booms and depressions (a true business cycle). The present paper aims to show, starting from a very "minimalist" model, which is in turn a further simplification of Tobin's model, the possibility both of instability of the private market economy and, most of all, of a persistent stable true business cycle. It is worth to note that the persistent cycle shown in this paper relies only on the basic interaction between quantities and prices which could be the shared "core" of any macroeconomic model instead of on the assumption of somewhat "ad-hoc" complicated structural relations; on the contrary, in this paper only linear relations, as in a standard elementary textbook, are assumed.

The format of the paper is as follows. Section 2 discusses the main features of the Tobin's model and section 3 develops the present

model. Section 4 contains the local stability analysis of the present model. Section 5 contains a brief discussion of the stability features of Tobin's model compared with those of the present model. Section 6 presents a numerical simulation, while section 7 offers some concluding comments.

2- Review of the features of Tobin's model.

Tobin's model has two major features. First it develops two different dynamic versions of a simple macroeconomic model, implementing keynesian views and evaluating explicitly the dynamic stability implications of Walrasian versus Marshallian assumptions about quantity adjustment. As to the choice of a type of adjustment process, the consideration of Friedman (1971, p. 18) on the method used by Keynes is stimulating: according to Friedman, Keynes was "Marshallian" in method but "where he deviated from Marshall, and it was a momentous deviation, was in reversing the roles assigned to price and quantity. He assumed that, at least for changes in aggregate demand, quantity was the variable that adjusted rapidly, while price was the variable that adjusted slowly, at least in a downward direction". By quoting this sentence of Friedman, Tobin (p. 196) agrees with it and says that "one way to appreciate the point is to look explicitly at the dynamic implications of Walrasian vs. Marshallian assumptions about quantity adjustment". Second, for both dynamical models – defined respectively as Walras-Keynes-Phillips (*WKP*) and Marshallian - he found the same necessary condition of stability¹, for which the economic interpretation is clearcut: departure from full employment equilibrium could not be remedied by market forces when, in line with a Keynesian view, price-level effects are weak relative to speculative effects.

Tobin's dynamic analysis, though formally corrected, is only partial and this implies two problems worthwhile to comment: 1) even if the stability condition stressed by Tobin (the eq. (3.4) at p. 199) were not met the economy could only show exploding instability, but it could never oscillate between recessions and recoveries governed by parametric changes due to policy interventions; 2) again more seriously, the unique force governing the possibility to trigger the business cycle and preventing the possibility to eliminate persistent unemployment, seems to be the effect of the expected rate of change of prices or in other (more Keynesian) words the "speculative" effect. Therefore it would appear that without such a

¹ For this reason in this paper we limit us to focus only upon the *WKP* model.

speculative force lapses from full employment will be automatically remedied by market forces.

Instead, we show that a simple WKP model, again more simplified than Tobin's one - in which such a speculative effect is not considered - can be a true business cycle model.

3 – A “minimalist” model

Critical to Tobin's way of thinking about the instability forces is the role of the speculative effects on effective demand and production and in particular the relative strength, on the one side, of “Keynes” and “Pigou” effects and on the other side of the speculative effects.

Our starting point is a “minimalist” model which in order to focus the basic mechanism generating cycle and instability ignores the features of Tobin's model relative to the speculative effects on demand and production.

The purpose of our “minimalist” model is to capture the two elements we see as essentials to generate a Keynesian cycle, that is the relative speeds of adjustment of quantities and expectations (instead of price and speculative effects on demand as claimed by Tobin).

Moreover we note that, although Keynesian macroeconomists can be much more open to the assumption of non linearities as nobody really knows the nonlinear functional form of the behaviour of the economy on the aggregate level, they also can be more exposed to the critique of a so-called “ad hoc” procedure (e.g. Lorenz, 1994). In order to avoid the critique of “ad-hocery” as regards such nonlinear assumptions, we postulate linear behavioural functions, by showing however that the highly simplified as far as possible *WKP* model endogenously has the seeds of recession and expansion.

Let's define Y as aggregate real output, Y^* as its value of equilibrium, i.e. at the “natural rate” of unemployment, D as aggregate real effective demand, which is the sum of consumption C , private investment I and government purchases g :

$$D = C + I + g \quad (1)$$

In short run disequilibrium, D cannot equalise both current production Y and equilibrium production, Y^* .

Following Tobin, we detail the components of the aggregate real demand as

$$D = C(Y, Y^*, T, R, x \frac{M}{p}, W) + I(Y, Y^*, K, R) + g \quad (2)$$

where the behavioural relations are assumed of standard type. In this frame there are only three endogenous variables, from which

demand depends on: p , the price level, x , its expected rate of change, and Y , the level of output and real income (while are exogenously given the government purchases g , taxes T , the nominal stock of outside money M , the stock of capital K).

The private wealth W is

$$W = \frac{M}{p} + qK; \quad (3)$$

where q is the ratio of market valuation of capital equity to replacement cost. Tobin makes q depending on the relative strength of the real interest rate R and the marginal efficiency of capital E (“an increase in the real interest rate relative to the marginal efficiency of capital makes q to fall” p. 197)

In turn the marginal efficiency of capital depends positively on Y and Y^* and negatively on K , so that the coefficient q can be expressed as

$$q = \frac{E(Y, Y^*, K)}{R} \quad (4)$$

The real interest rate R depends inversely on both M/p and x , and positively on Y and W :

$$R = R\left(\frac{M}{p}, x, Y, W\right) \quad (5)$$

Now, in order to build a very simple example of the Tobin’s model, we specify eq. (1)-(5) postulating the following textbook linear relations:

$$R = \rho_1 Y + \rho_2 W - \rho_3 \frac{M}{p} - \rho_4 x \quad (6)$$

$$C = a_1 Y - a_2 R + a_3 W; \quad (7)$$

$$I = i_1 Y - i_2 K - i_3 R \quad (8)$$

$$E = \varepsilon_1 Y - \varepsilon_2 K \quad (9)$$

Taking account for eq.(4), (9) and under the following further simplification, needed to have a tractable and economically interpretable system, $\varepsilon_2 = \rho_4 = 0$ and $\rho_2 = \rho_3$, after substituting in the eq. (6) we obtain the following unique positive level of the real rate of interest:

$$R = \frac{\rho_1 Y + \sqrt{(\rho_1 Y)^2 + (4\varepsilon_1 \rho_2 KY)}}{2} \quad (10)$$

Thanks to the previous simplifications the real rate is independent of price and expected inflation².

From the goods market equilibrium, we obtain the level of price in equilibrium (R^* means that it is evaluated at the income equilibrium point Y^*):

$$p^* = \frac{a_3 MR^*}{R^{*2} (a_2 + i_3) - R^* ((a_1 + i_1 - 1)Y^* + g - i_2 K) - a_1 a_3 KY^*} \quad (11)$$

Therefore, to sum up, we have further simplified the macroeconomic structure of the Tobin's model, eliminating some effects (i.e. the effect of equilibrium income Y^* both on the aggregate demand and on the marginal efficiency of capital, the effect of expected inflation on the real rate of interest, the effect of the level of the capital stock on the marginal efficiency of capital³) and postulating linear functional forms.

The dynamic version of the present model, in line with the *WKP* Tobin's model, is the following:

$$\begin{aligned} \dot{Y} &= s_1(D - Y) \\ \frac{\dot{p}}{p} &= s_2(Y - Y^*) + x \\ \dot{x} &= s_3\left(\frac{\dot{p}}{p} - x\right) \end{aligned} \quad (12)$$

where $D = (a_1 + i_1)Y - (a_2 + i_2)R - i_2 K + a_3 \left(\frac{M}{p} + \frac{\varepsilon_1 YK}{R} \right) + g$

The system (12) is a linear version of Tobin's system (p.198); the sole difference of (12) with respect to Tobin's model is that here D is independent of x , that is the "speculative" effect, which was crucial for Tobin's conclusions, is absent.

We briefly recall the economic interpretation suggested by Tobin for what concerns the above equations: the first one, saying that

² This assumption, however unrealistic, allows us for showing that the possibility of instability and cycles of the model is not necessarily due, as Tobin believed, to this "speculative" effect. Of course, it is worth to note that our dynamical results would be reinforced in the case in which the "speculative" effects was maintained.

³ The ignorance of the effect of K on the marginal efficiency of capital would seem to favour the destabilising forces, as argued by Tobin: "The failure of automatic market processes to restore full employment would be reinforced if large and prolonged recession caused investors to gear their estimates of the marginal efficiency of capital more to current than to equilibrium demand and profitability" (p. 201). However, since K is solely an exogenous constant in this frame, our simplification is not relevant.

production Y adjusts in response to discrepancies of D and Y , represents the Keynesian view that in the very short run money wages and prices are set and output moves in response to variations of demand; the second one is a natural-rate version of the Phillips curve⁴, finally the third one represents the well-known mechanism of adaptive expectations⁵.

By substituting the second in the third equation, this latter becomes

$$\dot{x} = s_3 s_2 (Y - Y^*) \quad (12.1)$$

The differential equation system represented by (12) and (12.1) has the following equilibrium: $p=p^*$ (where p^* is again represented by eq. 11), $Y=Y^*$, and $\frac{\dot{p}}{p} = x^* = 0$.

4- Local stability analysis of the model (12).

Let us now consider the local stability of the system (12). The linearised equations of the *WKP* model (12) are:

$$\begin{bmatrix} \dot{Y} \\ \dot{p} \\ \dot{x} \end{bmatrix} = \begin{bmatrix} s_1(D_Y - 1) & -\frac{s_1 a_3 M}{p^{*2}} & 0 \\ s_2 p^* & 0 & p^* \\ s_3 s_2 & 0 & 0 \end{bmatrix} \begin{bmatrix} Y - Y^* \\ p - p^* \\ x - x^* \end{bmatrix} \quad (13)$$

The characteristic equation of the above jacobian is

$$b_0 \lambda^3 + b_1 \lambda^2 + b_2 \lambda + b_3 \quad (14)$$

where

⁴ However, curiously, Tobin pointed out that assuming such an equation “I don not mean necessarily associate myself with the natural-rate hypothesis in all its power and glory” (p. 198).

⁵ As known, it could be irrational for agents to form expectations according to the adaptive rule. As Kiefer (1996, p.39) observes “this assumption is viewed with suspicion by many economists, for whom rational behavior is axiomatic [...] Nevertheless, adaptive expectations are often implicit in the continuing wide adherence to Keynesian doctrine”. Moreover we use adaptive rules in philological adherence to Tobin’s model.

$$\begin{aligned}
b_0 &= 1 \\
b_1 &= p^* s_1 (1 - D_Y) \\
b_2 &= a_3 s_1 s_2 p^* M \\
b_3 &= a_3 s_1 s_2 s_3 M \\
D_Y &= \frac{4\varepsilon_1 \rho_2 K^2 Y^* [a_1 a_3 - \varepsilon_1 \rho_2 (a_2 + i_2)]}{(H_2 + \rho_1 Y^*)^2 H_2} + a_1 + i_1 - \rho_1 (a_2 + i_3) \\
H_2 &= \sqrt{Y^* (\rho_1^2 Y^* + 4\varepsilon_1 \rho_2 K)};
\end{aligned} \tag{15}$$

A necessary and sufficient condition (Routh-Hurwitz) for local asymptotic stability requires

$$\begin{aligned}
b_2, b_3 > 0 \quad b_1 > 0 &\Rightarrow D_Y < 1 \\
b_1 b_2 - b_3 = a_3 M s_1 s_2 (s_1 (1 - D_Y) - s_3) > 0 &\Rightarrow s_1 (1 - D_Y) - s_3 > 0
\end{aligned} \tag{16}$$

We assume that “the marginal propensity to spend D_Y is taken to lie between 0 and 1 on usual Keynesian grounds” (Tobin, p.197).

Therefore the first three inequalities in condition (16) are satisfied, while the fourth one depends on specific parameter values. Therefore from simple inspection of condition (16) we can note that: i) the economic system can be locally either stable or unstable; ii) if it is unstable, the instability is of the oscillatory-type.

We are able to interpret the cyclical dynamics of the economic variables in the following manner. The signs of the elements of the jacobian are as follows:

$$J(Y, p, x) = \begin{vmatrix} - & - & 0 \\ + & 0 & + \\ + & 0 & 0 \end{vmatrix}$$

If initially all the variables are growing, the first variable that changes direction must be income (owing to the growth both of p and x); when Y falls, it will be followed later by x , which in turn will lower p . But when p is reduced to the point that the well-known price effects on demand become sufficiently strong, therefore income begins to rise followed by price expectations x and later by current prices⁶. For appropriate speeds of adjustment, we can expect a persistent oscillation to emerge, so that we formulate the following proposition:

Proposition 1: when, starting from a situation in which the system is locally stable, parameter s_I decreases, the system shows a Hopf bifurcation at $s_I = s_{IH} > 0$ (a short proof is in Appendix).

We have chosen the parameter s_I as the bifurcation parameter; we note that such a parameter has no effect on the equilibrium values,

⁶ This interpretation is confirmed by inspecting the profile of the time paths of the variables in the numerical example in figure 2 in section 6.

but only affects the local stability of such an equilibrium. The bifurcation value expressed in the proposition 1 is

$$s_{1H} = \frac{s_3}{1 - D_Y} > 0 \quad (17)$$

Simple inspection of condition (17) allows us to analyse the stability effects of changes in the various parameters.

An implication of proposition 1 is that changes in each of the factors in condition (17) can cause changes in the qualitative behaviour of the system; but it is also possible that changes in these parameters may cancel one another out, so that their endogeneisation would be required in order to reach definitive findings. On the other hand, if all but one of the parameters are fixed the qualitative behaviour can be accurately depicted according to a comparative dynamics exercise, as has been previously done with respect to the parameter s_I .

We can interpret the economic meaning of the stability condition (16) and of the bifurcation curve (17) through the following remarks: 1) the speed of quantity adjustment works for stability, viceversa for what concerns the speed of response of price expectations to experience; 2) the region of stability is contracted by a high propensity to spend; 3) the qualitative dynamic behaviour of this economy is independent of i) the speed of price adjustment (therefore full price flexibility does not play any role in restoring full employment equilibrium); ii) the price effects (i.e. Keynes and Pigou effects do not matter for stability and therefore are not capable to restore full employment equilibrium as thought by a common macroeconomic belief); iii) monetary policy (i.e. money supply as instrument is ineffective for stability).

5- Some remarks on the stability features of Tobin's model.

In this section the stability features of Tobin's model are reviewed, comparing them with those of the present model, and the interpretations provided by Tobin are critically discussed.

The stability conditions of original Tobin's model can be so resumed

$$\begin{aligned} b_3 > 0, \quad b_1 > 0 &\Rightarrow D_Y < 1 \\ b_2 > 0 &\Rightarrow (p * D_p + s_3 D_x) < 0 \end{aligned} \quad (16.1)$$

(16.2)

$$b_1 b_2 - b_3 = \{p * D_p [s_1(1 - D_Y) - s_3] - s_3 D_x s_1(1 - D_Y)\} > 0 \quad (16.3)$$

where D_p and D_x represent respectively the price effect and the speculative effect on aggregate demand.

It is easy to see that the conditions of Tobin's model collapse to our conditions (16) when the speculative effect on demand is absent ($D_x=0$).

In the paper of Tobin only the necessary condition (16.2) (corresponding to the equation 3.4 at p. 199) is explicitly showed and discussed⁷. However the condition (16.2) can say nothing as regards the possibility of economic cycles, but only as to a "disruptive explosion" of the economy. In fact in the case of a violation of (16.2), the equilibrium would show a saddle-type instability – in particular two positive roots of the characteristic of the jacobian – which, given the absence of so-called "jump" variables in the Keynesian model, simply would mean an economically disruptive instability⁸. Let us better clarify the economic interpretation of this point.

In effect Tobin does not argue for the existence of cyclical properties but for the possibility of both global instability and local stability, implicitly postulating a type of the so-called, in terms of current dynamic systems theory, "corridor" stability⁹. The situation argued by Tobin is that "the system might be stable for small deviations from its equilibrium but unstable for large shocks" (p. 201)¹⁰. However this situation can be originated only as a consequence of the emergence of an unstable local limit cycle around the full employment equilibrium or, in technical words, of a sub-critical Hopf bifurcation. However for this latter occurs as a consequence of a parametric change, condition (16.2) must be always met, while it is crucial that the expression (16.3) is tending to zero (as known in a neighbourhood of the bifurcation value of the bifurcation parameter for which (16.3) is positive, a sub-critical Hopf bifurcation could emerge). Therefore the economically unstable situation depicted by

⁷ In the words of Tobin "as would be expected, a strong negative price-level effect on aggregate demand, a weak price-expectation effect, and a slow response of price expectations to experience are conducive to stability"(p.199-200).

⁸ It is worth to note that, abandoning the Keynesian way of thinking and then assuming perfect foresight, therefore only this "unstable" case with prevailing "speculative" effects should become the sole meaningful stable case, while, on the contrary, all the cases of stable equilibrium of the Keynesian model (the present one as well as Tobin's one) would become cases of "indeterminate" equilibrium points.

⁹ The economic meaning of this situation has been emphasised by Leijonhfvud (1973) and Benhabib-Myiao (1981).

¹⁰ Moreover Tobin notes that Solow has pointed out to him that the possible global instability can be only a conjecture, requiring further investigation.

Tobin could never occur as condition (16.2) is violated. On the contrary, it is easy to see that, in order to show that the market economy can not always restore short run full-employment equilibrium, the crucial condition is not (16.2) (excluding the uninteresting case in which the economy necessarily explodes), but the expression (16.3): in particular, in order to have oscillatory behaviours mimicking more realistically the occurrence of recession and depression, both the weakness of the propensity to spend and the fastness of the quantity adjustment on the goods market - instead of the weakness of “price-level effects relative to speculative effects on aggregate effective demand” claimed by Tobin - are crucial for stability.

6 - A numerical simulation.

The proposition of the previous section only makes up the existence part of Hopf theorem, whereas the question is still opened to know whether the bifurcation is supercritical or subcritical, that is whether the periodic orbit which bifurcates from the equilibrium is stable or not. As it is well known, in the supercritical case the equilibrium point is an unstable focus which is surrounded from a stable orbit, whereas in the subcritical case the orbit surrounds a stable focus and is repulsive. Nevertheless also in this latter case, in which self sustained fluctuations can never be observed, can exist useful interpretation for economic analysis: that is the repulsive orbit can define the basin of attraction of the stable equilibrium that it surrounds (Leijonhufvud, 1973; Benhabib-Miyao, 1981). This latter case indeed would correspond to the possibility that the economy might be stable for small deviations from its equilibrium but unstable for large shocks, as argued by Tobin, who unfortunately attributes this possibility to the violation of a stability condition which rather would provoke a bifurcation of the saddle-node type. Therefore the investigation of the stability of the limit cycle means to investigate on the possibility of a business cycle versus the situation argued by Tobin.

Unfortunately the stability requirements for the limit cycle emerging from Hopf bifurcation are very complicated¹¹; instead of qualitative analysis we employ numerical simulations in order to investigate the stability properties of the limit cycles.

We illustrate the working of the model by concrete examples in which we will show that both the situation – a true business cycle and the “corridor” stability – are possible, depending on the

¹¹ Moreover, as Gandolfo (ch.25,1996) remarks: “Apart from mathematical difficulties, there is little hope to extricate an economic meaning from these stability conditions, especially insofar as third-order mixed partial derivatives have not clear economic interpretation”.

parameter values. We concentrate only on the dynamical effects of the price adjustment parameter, *ceteris paribus* for what concerns all the other parameters. The parameter set is the following¹²: $\rho_1=0.02$, $\rho_2=0.02$, $\varepsilon_1=0.1$, $a_1=0.3698$, $a_2=0$, $a_3=0.1$, $i_1=0.085$, $M=6$, $K=2$, $Y^*=1$, $s_2=24$, $s_3=2$. The corresponding equilibrium values are $Y^*=1$, $p^*=2.19$, $x^*=0$, the rate of interest R^* is 7.4% e the “propensity to spend” is 0.871. The simulation shows that increasing the sluggishness of prices (that is reducing s_1) the phase portrait of the system undergoes the following transformations: convergence to a stable node \rightarrow convergence to a stable focus \rightarrow convergence to a stable limit cycle \rightarrow possible “explosions” of the orbits. More in detail, the equilibrium point is a stable node or focus as long as¹³ $s_1 > s_{1H}=4$; at $s_1 = s_{1H}=4$ (corresponding for example to a quarterly average price adjustment) a stable limit cycle appears and for further reductions of such a parameter the cycles show an increasing amplitude and subsequently trajectories become explosive. Fig. 1 illustrates the business cycle in the phase space Y, p ; the counter-clockwise direction of the oscillations corresponds with empirical observations. Fig. 2 shows time series of income, price and expected inflation arising from the limit cycle, where both amplitude and timing of turning point are shown as well. As for the succession of the turning point of the three variables, fig. 2 confirms the results already inferred in the discussion of the cyclical dynamics in section four above. Since the model (12) is almost linear the occurrence of a true business cycle could be somewhat surprising: this means that cycles originate endogenously in the working of basic macroeconomic relations and not in some special non-linear relation. However with a different parameter set - for example when the parameter set is the following¹⁴: $\rho_1=0.1$, $\rho_2=0.3$, $\varepsilon_1=0.1$, $a_1=0.3132$, $a_3=0.1$, $i_1=0.16$, $M=2$, $K=2$, $Y^*=1$, $s_2=24$, $s_3=2$ - Hopf bifurcation is subcritical, and the emerging unstable limit cycle defines the basin of attraction of the stable equilibrium. In this latter case, the simulation shows that increasing the sluggishness of prices (that is reducing s_1) the phase portrait of the system undergoes the following transformations: convergence to a stable node \rightarrow convergence to a stable focus (as long as $s_1 > s_{1H}=4$) \rightarrow emergence of

¹² For simplicity, we let $a_2=i_2=i_3=g=0$. A positive value for a_2 and i_3 simply reinforces the negative effect of the interest rate on aggregate demand, already represented via the variable q , while i_2 and g represent only constant values which qualitatively do not affect the dynamics.

¹³ Let's define s_{1H} as the Hopf bifurcation value.

¹⁴ For simplicity, we let $a_2=i_2=i_3=g=0$. A positive value for a_2 and i_3 simply reinforces the negative effect of the interest rate on aggregate demand, already represented via the variable q , while i_2 and g represent only constant values which qualitatively do not affect the dynamics.

an unstable limit cycle at $s_I=s_{IH}=4$ (where either the convergence to a locally stable focus or possible “exploding” oscillations depend on the “history” of the economy); for further reductions of such a parameter the trajectories become explosive. Figure 1 reports a phase-plane representation of the model (12) restricted to the two variables p and Y : the trajectories starting within the “corridor” defined from the unstable limit cycle converge to the equilibrium point, while the trajectories starting from outside the corridor are exploding. To sum up the numerical simulation, on the one hand shows the existence of a business cycle, on the other hand positively answers to the point stressed by Tobin: in fact the economy might be stable for small deviations from its equilibrium but unstable for large shocks in consequence of a local subcritical bifurcation, but, in contrast with Tobin, this occurs when the speed of quantity adjustment is sufficiently low in comparison with the speed of response of price expectations to experience, for a given “propensity to spend”. And this is at all different from the condition claimed by Tobin, requiring that the “speculative” effects are prevailing on the “price” effects on aggregate demand.

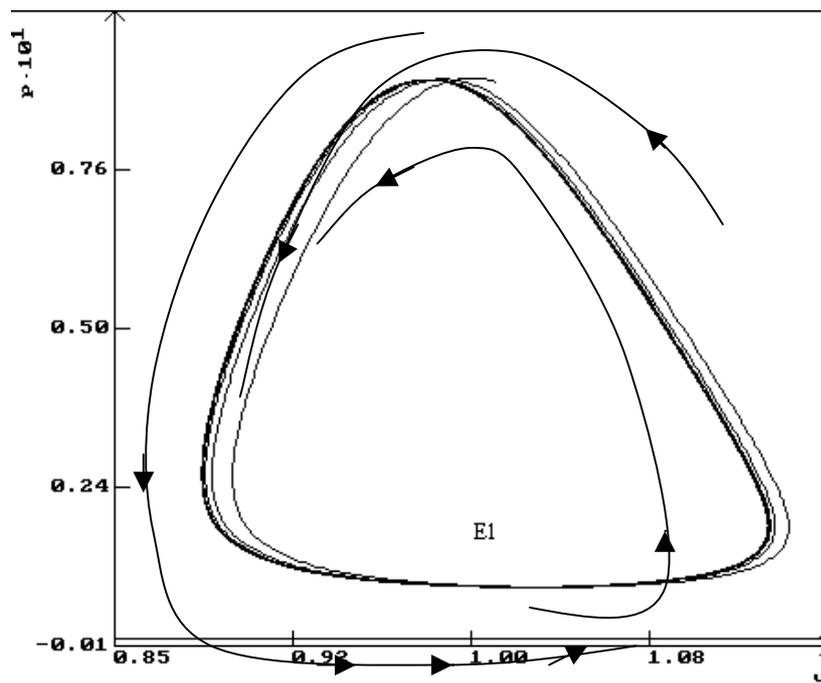


FIG. 1 - Limit cycle in the phase space $Y-p$, I.C. ($Y^0=1.01$, $p^0=8.95$, $x^0=0.01$) (Runge-Kutta 4th order, step 0.0075)

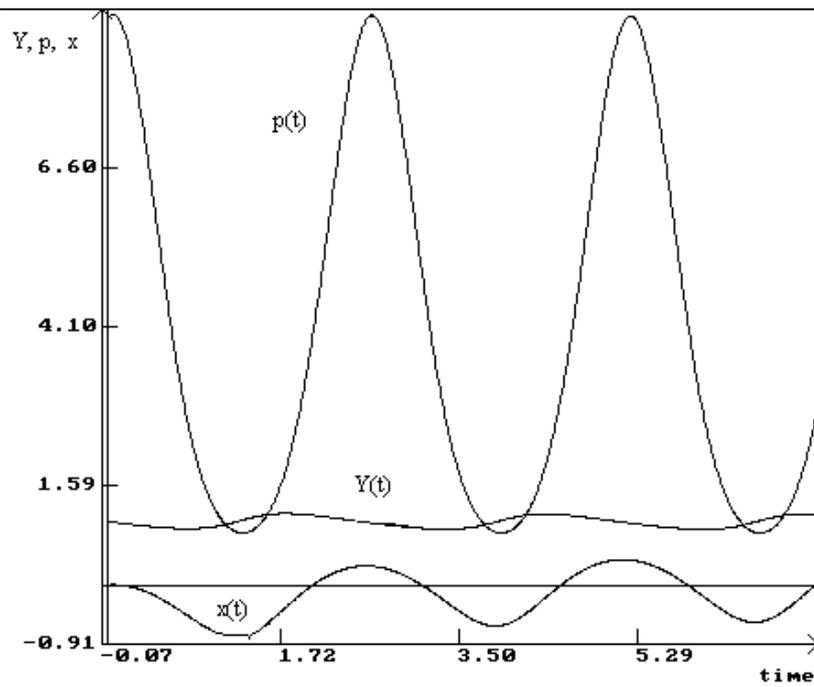


FIG 2 - Time paths of the variables Y , p and x (iterates 10000-10500, parameter set as in fig.1)

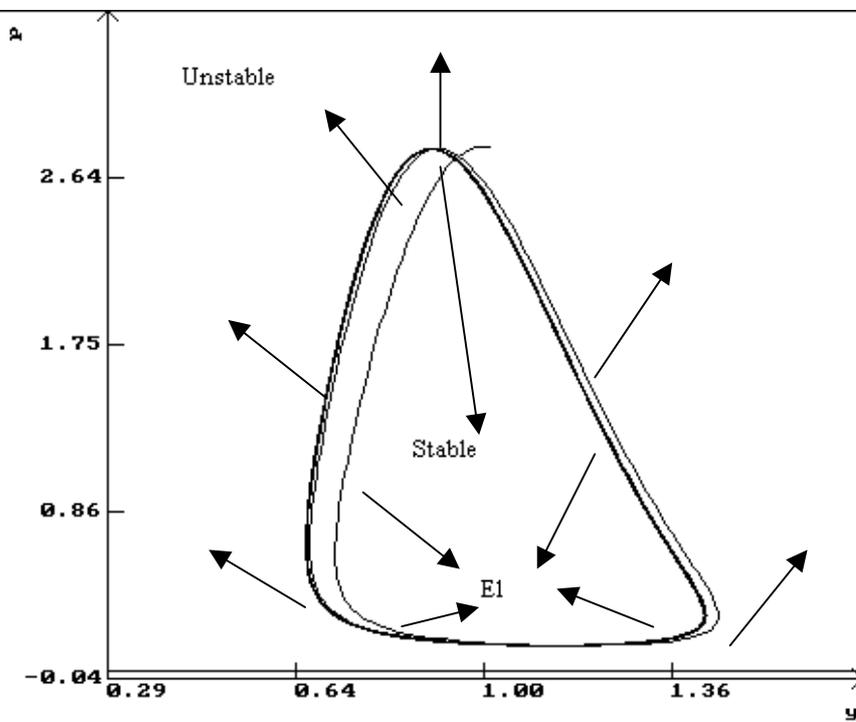


FIG. 3 - Unstable limit cycle in the phase space $Y-p$, I.C. ($Y^{\circ}=1.01$, $p^{\circ}=2.8$, $x^{\circ}=0.01$) (Runge-Kutta 4th order, step 0.0075)

7- Conclusions

In this paper we have developed a “minimalist” Keynesian model – where all the behavioural functions are assumed to be linear - aiming at demonstrating the existence of endogenous cycles and evidencing the economic forces responsible for them. Our model is a simplified version of Tobin’s model (1975), which can be considered a cornerstone of the renewed interest for the Keynesian macrodynamics.

Tobin’s article had the merit both to reinterpret the debate on Keynesian view pointing out that “the real issue is not the existence of a long-run static equilibrium with unemployment, but the possibility of protracted unemployment which the natural adjustments of a market economy remedy very slow if at all” (p. 195) and to demonstrate the possibility of instability implicit in the adjustment of a pure market economy. By interpreting his own results, Tobin suggests that: 1) there are two economic forces governing the instability process, i.e. “prices” effects versus “speculative” effects on aggregate demand and only when the “speculative” effects are prevailing the market economy can be destabilised, and 2) the economy “might be stable for small deviations from its equilibrium but unstable for large shocks”, postulating implicitly the existence of a “corridor” stability.

However both the suggestions are not well founded. In particular the Tobin’s interpretation of the forces governing the stability can be misleading, in that the speculative effect on demand is not necessary for instability: in fact we have shown that in absence of any speculative effect on demand and production, the relative strength of the speeds of quantity adjustment and of response of price expectations to experience, given the “propensity to spend”, is sufficient to explain the entire dynamical behaviour of the economy. Moreover, we have shown that a stable¹⁵ business cycle can emerge. This result is very noticeable taking account for the linearity assumptions of all the macro behavioural functions.

Nevertheless, the Tobin’s belief that the economy might be stable for small deviations from its equilibrium but unstable for large shocks in consequence of a local subcritical bifurcation, has been confirmed in some parametric cases, but it is worth to note that this occurs when the speed of quantity adjustment is sufficiently low in comparison with the speed of response of price expectations to

¹⁵ We recall that the stability of the cycle has been ascertained only via numerical simulation.

experience, for a given “propensity to spend”. Therefore our result is in contrast with the condition claimed by Tobin, requiring that the “speculative” effects are prevailing on the “price” effects on aggregate demand.

Finally notice that the cycle emerges independently of: 1) the speed of price adjustment, implying that therefore full price flexibility does not play any role in restoring full employment equilibrium; 2) also the price effects (i.e. Keynes and Pigou effects) do not matter for stability and therefore are not capable to restore full employment equilibrium, in contrast with a common macroeconomic belief ; 3) monetary policy (i.e. money supply as instrument) is ineffective for stability.

This paper belongs to the literature of the dynamical Keynesian macroeconomics and in particular of that aiming at an endogenous deterministic explanation of the economic cycles, in contrast with the exogenous stochastic explanation proposed by the new classical macroeconomics. The main contributions this paper makes to that literature are the following: 1) the main dynamical interpretations of the pioneering model of Tobin (1975) are critically discussed, by shedding new light on the true forces capable to generate the economic cycle; 2) the economic cycle emerges through the standard macroeconomic interaction between quantities and prices and does not depend, as in most part of that literature, on the assumption of somewhat “ad-hoc” non linear structural relations. To sum up, the essential message is: the business cycle not only is endogenous, but, as a matter of fact, it is the result of a traditional Walras-Keynes-Phillips macroeconomic model even with linear behavioural functions.

APPENDIX

The proof of the proposition 1 is based on the mechanism of the Hopf bifurcation¹⁶. In heuristic terms the Hopf bifurcation theorem states the existence of closed orbits in a neighbourhood of the equilibrium for some bifurcation parameter value; and this occurs insofar as, when such a parameter is increasing, the following conditions are satisfied: 1) complex eigenvalues exist or emerge; 2) the real parts of such eigenvalues are zero at the bifurcation value of the parameter; 3) all other real eigenvalues are different from zero when the parameter is at its bifurcation value; 4) the real parts of the eigenvalues become positive when the bifurcation parameter value goes beyond the bifurcation value. With respect to the case of three-

¹⁶A rigorous treatment of the Hopf theorem is, for example, in Guckenheimer-Holmes (1984).

dimensional system¹⁷, it is well-known that (e.g. Lorenz, 1994) the real parts of the complex eigenvalues are zero and the third real eigenvalue is negative when $b_1, b_2, b_3 > 0$ and $b_1 b_2 - b_3 = 0$. It then follows that through simple application of the Routh-Hurwitz conditions for local stability it is possible to demonstrate the existence of the Hopf bifurcation, since in the point where the above Routh-Hurwitz conditions are satisfied a pair of complex eigenvalues exists, that is to say, the discriminant of the characteristic equation is positive (satisfying point 1)); obviously the derivative of the real parts of the complex eigenvalues with respect to the bifurcation parameter must likewise be different from zero (satisfying point 4)¹⁸ (so that there is an effective crossing of the imaginary axes as the bifurcation parameter is increasing). Then, all other being conditions satisfied, the bifurcation appears as $b_1 b_2 - b_3 = 0$, that is in the present model as

$$b_1 b_2 - b_3 = 0 \Rightarrow \frac{s_3}{s_1} = 1 - D_Y$$

or by using s_1 as bifurcation parameter, as

$$s_{1H} = \frac{s_3}{1 - D_Y} > 0 \quad (17).$$

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¹⁷ As regards the demonstration of the necessary and sufficient conditions for the existence of the Hopf bifurcation in third, fourth (and higher) order systems, see Fanti-Manfredi (1998).

¹⁸ As regards the procedure for testing the crossing of the imaginary axis with non zero speed, see again Fanti-Manfredi (1998). Such a condition is satisfied in this case as well, but the calculations are not shown here for sake of brevity.

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