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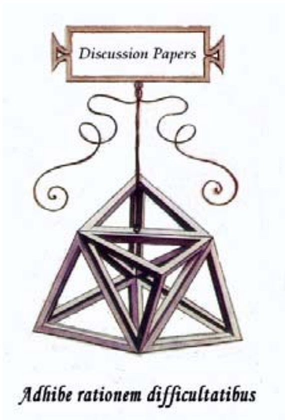

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TECHNOLOGICAL DIFFUSION AND CYCLICAL GROWTH

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Abstract

TECHNOLOGICAL DIFFUSION AND CYCLICAL GROWTH

The models of technology diffusion originally proposed by Metcalfe (1981), Batten (1987) and Amable (1992) are modified so as to allows for price expectations of adopters and suppliers of an innovation. We show many interesting and somewhat unexpected results, which were not noticed in the preceding models: *i*) productive technologies with higher returns or small fixed costs, *ii*) large market dimension (e.g. as a consequence of economic growth), iii) high speed of adoption, and iv) "cautious" investors in production of innovation, tend to prevent a balanced development of an innovation. Moreover *1*) a co-existence of multiple equilibria, depending on initial conditions of new technology diffusion, *2*) cyclical evolution of the new technique as a rule rather than an exception, are shown. Finally, some implications for policy interventions as well as firms' marketing policies emerge.

Classificazione JEL: O30
Keywords: technological diffusion, Hopf bifurcation, cyclical growth

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INTRODUCTION

In the recent years some models of growth through diffusion of innovations[1], which separately take account both of demand side and of supply side, have been developed by Metcalfe (1981), Batten (1987) and Amable (1992) (M-B-A afterwards). These models have some interesting main features: 1) they represent an improvement with respect to the traditional "epidemic" models in which the diffusion of an innovation, which is not instantaneous because of some frictions, is analysed only on the side of adopters of such an innovation (Mansfield, 1961); 2) on demand side the rate of growth of adopters of an innovation increases when price reductions occur; 3) on supply side, profits decrease when price decreases and therefore also the incentive to invest in capacity of the new technique is reduced; moreover profits are reducing when demand raises because of increasing[2] production costs;

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[1] An innovation can be a new technique or, alternatively, a new product. In this paper we refer to an innovation as a new technique.

[2] An extension of this type of models concerns the introduction of the possibility of decreasing costs, due for instance to improvements of the productivity after the initial innovation (i.e. as regards supply side, via

this can emphasise the typical Schumpeterian role of profit: "it is at the same time the child and the victim of the development" (Schumpeter, 1976, p. 154); 4) the development of the new technique, starting from an initial position of disequilibrium between supply and demand at the time of the introduction of the new technique, tends to a stable long period equilibrium (a level of saturation of demand), 5) they assume that "market mechanisms provide the necessary adjustments for supply to equal demand so that prices are the only adjustment variable" (Amable, 1992, p.151). In this paper we retain the same model of the preceding literature, but in contrast with it we assume that the decisions of adopters and suppliers are based on an expected price rather than on a current price, because of, for instance, informational delays. This assumption is realistic as well as coherent with the intrinsic dynamical nature of the process of diffusion of an innovation.[3]

Now it is largely recognised in the economic literature that a complex interdependence between demand and supply can be very sensible to the mechanisms of formation of the expectations about the price evolution. What the theory of innovation diffusion lacks so far is a simple analytical investigation of the effects of a distinction between expected and current price[4]. In this paper the novelty aspect is reflected in the assumption that

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learning by doing; also for what concerns demand side improvements in the utility of adopters when the time elapses can occur, i.e. via the so called learning by using). This can obviously modify the qualitative dynamics of an innovation diffusion (Amable, 1992).

[3] The possibility that price does not instantaneously adjust in each point of time to equal demand and supply growths was already explicit even in the pioneering work of Metcalfe (1981), who observed: "…a function for price to balance the growth of demand with the growth of productive capacity, not necessarily at each point in time but certainly on a secular basis" (p.402). Amable explicitly pointed out to have neglected the possibility of not instantaneous price adjustment: "[…] prices are the only adjustment variable. Thus, no disequilibrium is assumed here" (p.151).

[4] For instance Stoneman (1983) admits to have developed his model of diffusion (demand as well as supply-based) assuming completely myopic price expectations, because "despite the desirability of introducing expectations more explicitly, this proved impractical"(p.128).

price considered by the potential adopters as well as expected price used by suppliers of the new technique to predict their own profitability are delayed with respect to the current one. For instance, by simply assuming the realistic presence of some informational delay, agents at each point of time may perceive or expect a different price with respect to the current one; furthermore different agents can have different "frictional" informational delay. The presence in the market of numerous agents, which, although homogeneous with respect to preferences and technologies, can be heterogeneous with respect to the informational delay, may require a continuous distributed lag function to represent the overall expected market price. This latter assumption implies that expected price adjusts to the current price in an "adaptive" manner; this seems to us at all coherent with the "philosophy" of the preceding models. In fact the consideration of a population of agents with a possible heterogeneous behaviour rather than of the representative agent is crucial, as Metcalfe (1994) argues: "First and foremost the premise of representative behaviour ceases to be the foundation of the analysis [….]Rather, a population perspective adopted in which the entire distribution of behaviours shapes the pattern of change" (p.128-129).

Although all the assumptions of the preceding models are retained, except the assumption of the equality between expected (or perceived with a delay) and current prices, the results are, interestingly, very different. The contrast with the results of the preceding models involves the existence of: i) possible multiple equilibria, ii) stable long run fluctuations of production, iii) a role for (possibly random) shocks in order to determine the qualitative long term "fate" of the new technique (stability, fluctuation, extinction), iv) multiple and somewhat unexpected roles of the economic forces (e.g. speed of imitation, rashness or cautiousness of investors, cost technology) determining the qualitative dynamics of an innovation.

Section 2 develops the model of innovation diffusion. In Section 3 both the equilibrium and the stability analysis are developed. A numerical simulation illustrating the working and the results of the model is performed in section 4. Section 5 presents some concluding remarks.

2. THE MODEL.

Now we turn to the development of the model, which is based upon M-B-A's models.

The first basic assumption concerns the cost function of the new technique. The evolution of the production cost represents the potential of the new technique and depends on the installed capacity *x:*

$$c = c_0 + c_1 x \quad , \qquad c_0 , c_1 > 0$$

(1)

This means that technology exhibits decreasing returns to scale (as in M-B-A's model).[5]

The unit profit function π is equal to the difference between the price (p) and the production cost (c) of the new technique: $\pi = p - c$

(2)

Aggregate demand for the new technique is assumed to be the result of a process of imitation, which, coherently with the 'epidemic' model of diffusion, generates a logistic pattern:

$$\dot{D} = bD(y - D) \qquad b > 0$$

(3)

where D is the level of diffusion for the new technique at each instant of time, b is an "imitation" (or in the words of Metcalfe, "adoption") coefficient and y the level of saturation (that is the maximum attainable level) of demand for the new technique. This level of saturation is depending on the price, p, in accordance with the simple linear

[5] Amable (1992) do not limit themselves to a decreasing returns to scale technology, but also consider the case of increasing returns to scale, that is $c_1 < 0$.

downward-sloping curve; that is the law of demand of the new technique is

$$y = a_0 - a_1 p \qquad a_0 > 0, a_1 > 0$$

(4)

On supply side, investment in capacity of the new technique depends on profit: firms invest in the production of the new technique on the basis of profits they expect to make on it. Expected profits are assumed to be a simple linear function of current profit. Investment behaviour, loosely speaking, follows a "schumpeterian" scheme. The decision of investment is not based on profit maximization principle, which would be with difficulty implemented because of an assumed 'ignorance' about the main features both of the new technique (i.e. the unit cost function) and of demand function. On the contrary, firms' behaviour is assumed to depend on a mix of history-dependent routines and routine-breaking novelties (Dosi, 1991), and in particular the decisions are based on current profit, π, as a proxy of the evolution of future profits. So the rate of growth of capacity of the new technique is

$$\frac{\dot{x}}{x} = s\pi \qquad s > 0$$

(5)

where *s* is a parameter "reflecting the dependence of expected profits on current profits as well as the dependence of investment on expected profits" (Amable, p. 151).[6]

Such a rate of growth, by substituting (2) in (5), becomes

$$\frac{\dot{x}}{x} = s(p - c_0 - c_1 x)$$

(6)

Following Batten and Amable, we assume that the mechanisms of the market ensure the equilibrium between supply and demand through the

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[6] This parameter also measures whether investors are "rash" or "cautious" (according to the famous terminology by Joan Robinson).

price flexibility, so that the equilibrium condition $x=D$ implies that $\dot{x}=\dot{D}$. Price equalising supply and demand is

$$p = F(x) = \frac{x(c_1 s - b) + (c_0 s + a_0 b)}{s + a_1 b} \tag{7}$$

By inserting (7) in (6) we obtain the final expression for the evolution of capacity utilisation:

$$\frac{\dot{x}}{x} = A(B - x); \qquad A = \frac{sb(1 + c_1 a_1)}{s + a_1 b}; \quad B = \frac{a_0 - c_0 a_1}{(1 + c_1 a_1)} \tag{8}$$

However, in contrast with M-B-A's models, we assume that the relevant price for adopters and suppliers is an expected price based on past prices (or simply that the current price is known by both agents[7] with a time delay). This appears reasonable in the light of the realism and of the present imperfect information setting.[8]

The assumption of the lag implies the substitution of current price (p) in (7) with an expression of the following type:

$$p = \int_{-\infty}^{t} F(x) R(t - \tau) d\tau \tag{8}$$

where the function R is the so-called "weighting function" of the lag. Equation (8) implies that equation (6) shows the following integro-differential structure:

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[7] The analysis of other realistic cases, such as suppliers and adopters perceive a different price or only suppliers' decisions are based on an expected price, is not, for simplicity, presented here, but in any case notice that the main findings of this paper would hold for such cases as well.

[8] For instance, as regards the case of fixed investment to produce an innovation, Hillinger (1996) presents the following reasons to explain lags involved from changes in demand conditions and the operation of changes in capacity: 1) considerable lags before information becomes available to decision makers, because performance reports are typically on a quarterly or annual basis, time beyond the reporting period is necessary to be prepared and submitted to the top management, the information from many firms is assembled into national statistics, which are typically highly unreliable when they are first reported with about a year's delay and further revised over several years; 2) the problem of extracting a signal from noisy data imply delays, in that firms must decide when a change has persisted long enough to be considered permanent rather than transitory, and subsequently if a new factory is needed, it must be planned and built, and the work force expanded and trained. To sum up from the initial planning to routine operation of a new factory, several years will pass.

$$\frac{\dot{x}}{x} = s\left[\int_{-\infty}^{t} F(x)R(t-\tau)d\tau - c_0 - c_1 x\right] \qquad (9)$$

The weighting function of the lag can obviously assume different specifications. The more usual specification is the Gamma distribution with two parameters (n, γ); in the case of a Gamma distribution with parameter (1,γ) we have the usual decreasing exponential shape, while in the case of a Gamma distribution with parameters (2, γ) we have the simplest case of a "one-humped" distribution of the lag. As regards the Gamma distribution we remark that it can well represent a market consisting of an indefinitely large number of agents who respond to a certain signal with given discrete lags. Such lags can be different for different agents and distributed in a random manner over all the population (e.g. Invernizzi-Medio,1991). The parameter of the shape of the distribution, n, provides a measure of the degree of homogeneity of the market under consideration. We argue that probably the simple exponential lag[9] (n=1) as well as the opposed case of a fixed delay assumption ($n=\infty$) can be considered a rather special and crude formalisation of market reaction mechanisms. A more satisfactory modelling of the market response requires the assumption of the simplest "one-humped" Gamma distribution of parameter (2, γ); this amounts to assume a degree of non-homogeneity between the agents slightly less than that implied by the exponential shape assumption, but obviously always far from the 'unrealistic' perfect homogeneity implied by the fixed delay assumption.[10] Therefore the assumed function R is

$$R(z) = \gamma^2 z e^{-\gamma z}, \qquad \gamma > 0 \qquad (10)$$

[9] It can be also defined as exponential lag of order one.

[10] It is worth to notice that, by assuming for R the usual exponential shape $R(z) = \gamma e^{-\gamma z}, \quad \gamma > 0$, we obtain similar qualitative results as regards the role of the economic parameters on the stability, but limit cycles cannot occur.

The average price lag is equal to 2/γ. Then, by using the «linear chain trick»[11], the integro-differential equation (9) can be transformed in the following system of three ordinary differential equation:

$$\dot{x} = sx(p - c_0 - c_1 x)$$
$$\dot{p} = \gamma(p^e - p)$$
$$\dot{p}^e = \gamma\left[\frac{x(c_1 s - b) + (c_0 s + a_0 b)}{s + a_1 b} - p^e\right]$$

(S.1)

where the variables p, p^e are so defined:

$$p(t) = \int_{-\infty}^{t} F(x(\tau))\gamma^2 (t-\tau)e^{-\gamma(t-\tau)} d\tau \qquad (11a)$$

$$p^e(t) = \int_{-\infty}^{t} F(x(\tau))\gamma e^{-\gamma(t-\tau)} d\tau \qquad (11b)$$

3. STATIC AND DYNAMIC ANALYSIS.

The system (S.1) has the following two equilibria:

E_0: $\quad x_0 = 0; p_0 = p_0{}^e = \dfrac{a_0 b + c_0 s}{s + a_1 b}$

E_1: $\quad x^* = \dfrac{a_0 - c_0 a_1}{1 + a_1 c_1}; p^* = p^{e*} = \dfrac{a_0 c_1 + c_0}{1 + a_1 c_1}$

(13)

As to the axis equilibrium E_0, we note that it is economically not interesting (it will be further investigated in Appendix 1).

A simple inspection shows that a unique simple condition ensures the existence of the positive equilibrium:

$$c_0 a_1 - a_0 < 0 \qquad (14)$$

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[11] For details about the reduction method known as «linear chain trick» see McDonald (1978) and Fanti-Manfredi (1998,Appendix A.2).

Condition (14) is the same of M-B-A's models and holds as far as the assumption of increasing costs is maintained. Such a condition says that: an innovation will converge to its own long run level of saturation provided that 1) both the fixed cost and the sensitivity of demand to price are sufficiently low; 2) "potential " demand is high.

Another consideration is that the positive equilibrium value is independent of the coefficient of imitation of adopters of the new technique (*b*) as well as of the coefficient of investment in capacity of the new technique (*s*) (and, as expected, of the price delay parameter γ). [12] However, the speed of imitation and of extension in capacity of the new technique, as well as the average delay of the equilibrating market price, affect the qualitative dynamics of an innovation.

The usual jacobian of (S.1) is

$$J(x,p,p^e)_{|E_1} = \begin{bmatrix} s(c_0 - p^*) & sx^* & 0 \\ 0 & -\gamma & \gamma \\ \dfrac{c_1 s - b}{s + a_1 b} & 0 & -\gamma \end{bmatrix}$$

(15)

The characteristic equation is

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3 \qquad (16)$$

where

$$\begin{aligned} a_0 &= 1 \\ a_1 &= 2\gamma + s(p^* - c_0) \\ a_2 &= \gamma(\gamma + 2s(p^* - c_0) \\ a_3 &= \frac{\gamma^2 bs(p^* - c_0)(c_1 a_1 + 1)}{c_1(s + a_1 b)} \end{aligned}$$

(17)

We study the local stability, by invoking the well known Routh-Hurwitz theorem. As condition (14) for the existence of the positive equilibrium must hold, therefore $(p^* - c_0) = c_1(a_0 - c_0 c_1) > 0$ must hold as well.

[12] By passing we note that Batten (1987, p.78), because of an algebraic mistake in the evaluation of the equilibrium, erroneously states that the equilibrium of the new product only depends on the "rate of imitation".

Consequently, the first three Routh-Hurwitz conditions, that is $a_1, a_2, a_3 > 0$, are always satisfied.

It is worth to remark that in M-B-A's model the sole condition (14) is able to ensure not only the existence of the equilibrium but also its local (and global) stability. On the contrary, in our model such a condition, as regards the stability, is only capable to ensure the absence of positive real eigenvalues.

Moreover since

$$a_1 a_2 - a_3 = \frac{\gamma}{c_1(s + a_1 b)}[c_1[2a_1 b(s^2(c_0 - p^*)^2 + 2\gamma s(p^* - c_0) + \gamma^2) + + s(2s^2(c_0 - p^*)^2 + 5\gamma s(p^* - c_0) + 2\gamma^2)] - \gamma b s(p^* - c_0)] \tag{18}$$

we see that, defining $H = p^* - c_0$,

$$a_1 a_2 - a_3 > 0 \Rightarrow 2c_1(s + a_1 b)(s^2(-H)^2 + \gamma^2) + \gamma s H\left[b(4a_1 c_1 - 1) + 5c_1 s\right] > 0 \tag{19}$$

The condition (19) is only satisfied by specific parameter values.

Therefore from simple inspection of the above Routh-Hurwitz conditions, we can note that: i) the model (S.1) can be locally either stable or unstable; ii) if it is unstable, the instability can only be of the oscillatory-type.

In particular simple inspection shows that $a_1 c_1 > 0.25$ is a sufficient condition for the stability. This means that both 1) returns to scale sufficiently decreasing, and 2) a high sensitivity of demand to price can ensure a stable balanced growth of an innovation. A more complete analysis of the effect of the different economic parameters on the possibility either of a cyclical development or of an elimination of the new technique is developed by choosing the parameter *s* as bifurcation parameter[13] and reformulating equation (19) to explicit the possible existence of Hopf bifurcations:

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[13] A careful analysis of various stability conditions focusing on the role of each parameter could fill many pages. For economy of space we omit the formal analysis of such cases.

$$s^3 B_0 + s^2 B_1 + sB_2 + B_3 = 0$$
$$B_0 = 2c_1 H^2; \quad B_1 = 2bc_1 a_1 H^2 + 5c_1 \gamma H; \qquad (20)$$
$$B_2 = \gamma\left[bH(4_1 a_1 c_1 - 1) + 2c_1 \gamma\right]; \quad B_3 = 2a_1 c_1 b\gamma^2$$

It is easy to show, by using the Descartes' rule of signs, that two Hopf bifurcation values of s (if they are real) always exist when $B_2<0$.

Let's define s_{H1} and s_{H2} the two positive values of the parameter s generating a Hopf bifurcation; then the following holds:

$$B_2 < 0 \Rightarrow \ni (s_{H1}, s_{H2}) \in (0, \infty)$$

In order to obtain $B_2<0$, the following inequality must hold:

$$B_2 < 0 \Rightarrow c_1 < c_1{}^* = \frac{b\xi - 2\gamma}{2a_1(2b\xi + \gamma)}$$
$$\xi = a_0 - c_0 a_1$$
(21)

A first economic meaning of the above conditions is that a double Hopf bifurcation driven by a continuous change in the s parameter always exists when the returns to scale are not too decreasing; moreover as $c_1{}^*$ must be positive and provided that condition (14) (of existence of the positive equilibrium) must hold (that is $\xi>0$), a necessary condition for the existence of the (double) Hopf-bifurcation is

$$\xi > \frac{2\gamma}{b} \qquad (22)$$

In the table 1 the sensitivity of the threshold $c_1{}^*$ to changes in the involved five economic parameters is shown. The above stability conditions and the table 1 show that the stable trajectory is prevented and cyclical growth as well as also a possible extinction of the new technique can emerge when the following economic features exist: 1) a low unit fixed cost of production of the new technique; 2) a large extension of the "potential" market; 3) a high average price lag; 4) a high coefficient of imitation. As regards demand sensitivity to price of the new technique, it plays un ambiguous role[14].

Table 1 – Sensitivity of the threshold value c_1* with respect to the other parameters.

	a_0	a_1	c_0	b	γ
Sign of $dc_1*/(dx_i)$	+	?	-	+	-

We can sum up our main dynamical result in the following proposition:
Proposition 1: if the conditions (14) and (21) hold, when the parameter *s* increases from zero along with its entire positive domain the system (S.1) a first time switches from a stable behaviour to an unstable one and a second time re-switches from an unstable behaviour to a stable one and undergoes, at the parametric values corresponding to each switch of stability, a Hopf bifurcation, with the appearance of local limit cycles (at least one) surrounding the positive equilibrium (a sketched proof in Appendix).

The main remarks emerging from the stability analysis are the following:

1) the stability is influenced not only by demand and supply conditions (namely preferences and technologies), as in M-B-A's models, but, contrary to these latter models, also all the other parameters play a crucial role on an innovation dynamics, and some parameters, such as the "propensity" to invest in the new technique, may even play a multiple role;

2) the existence as well as the amplitude of a region of instability can be favoured by both low fixed costs and a high "potential" demand for the new technique;

3) the economic features required for a high level of saturation of the new technique in all the models - that is both low fixed costs and a high "potential" demand for the new technique - which in M-A-B's models

[14] Although low elasticity needs to satisfy the necessary condition (22), it is not always so as regards condition (21), and therefore the effect of demand price elasticity on the stability is depending on a highly complicated relation between all the system's parameters.

also guarantee the global stability of such a long term level, in our model may be harmful for the stability: indeed, in order to ensure the stability, a very low, however obviously positive, long term saturation level can be necessary;

4) in many cases, the larger is the "imitative" speed (*b*) the more likely is the destabilisation;

5) by reducing the speed of production of an innovation (*s*) the system may be destabilised.

In order *i*) to discover the properties of the limit cycles evidenced in Proposition 1, and *ii*) to investigate the sensitivity of the dynamics with respect to the various economic parameters, we resort, in the next section, to the numerical simulations.

4. NUMERICAL SIMULATIONS

It is known by Hopf theorem that the involved limit cycles are not necessarily stable. In fact while the Hopf theorem proves the existence of closed orbits, it gives no information on their *number* and their *stabilit*y. Regarding *stabilit*y, the cycles may either be attracting (stable) or repelling (unstable). Therefore, two cases of Hopf bifurcation are distinguished. In the subcritical case, repelling closed orbits emerge while the fixed point is still stable. Attracting cycles occur in the supercritical case, where they surround the unstable fixed point. Both phenomena are open to an economic interpretation, as pointed out by Benhabib and Miyao (1981). Stable closed orbits emerging in the supercritical case can be viewed as stylized business or growth cycles. On the other hand, the subcritical case corresponds to the concept of corridor stability as developed by Leijonhufvud (1973) and Howitt (1978). Such a concept implies that the system converges to its dynamic equilibrium for small perturbations, but shows no such tendency for larger shocks. The unstable orbits of the subcritical case enclose a region of stability in which all orbits inside that region converge to the fixed point. Unfortunately, as it is known, the complete characterisation of the stability properties of the periodic orbits emerging from Hopf bifurcations at dimensions higher than the second, is a quite hard task[15]. Usually this characterisation should involve the determination of that specific threshold in the parameter space which separates the *supercritical region* (in which the periodic orbits are locally stable) from the *subcritical region* (in which the involved periodic orbits are locally unstable).

We did not investigate analytically the stability properties of the periodic orbits appeared in the model, but rather resorted to numerical simulation. Several interesting dynamical facts arise, which were not noticed in the previous contributions. We have chosen as crucial parameters of the numerical investigation b and s, representing, broadly speaking, the intensity, respectively, of imitative and innovative forces, and the following set of economic parameters involved: b=13.3, a_0=1.5, a_1,=0.5, c_0,= 1, c_1 = 0.088, γ=0.1, for which the system has the equilibrium x^*=0.97, p^*=1.07. In what follows we summarise the main facts from the simulative evidence:

i) for every bifurcation curve $\Phi = \Phi(b,s,a_0,a_1,,c_0,c_1,\gamma)$ lying in the (s,b) plane there exists a s value, let us call it s_d, $0.125<s_d<\propto$, such that the point ($b_d,s_d=\Phi(s_d)$) cuts the bifurcation curve in two parts: points belonging to Φ at the left of (b_d,s_d) will give rise to supercritical bifurcations, while points to the right of (b_d,s_d) give rise to subcritical bifurcations. More precisely: if the crossing of the bifurcation curve $b=\Phi(s)$ takes place in the region $0.125<s<s_d$, then the corresponding periodic orbits are locally stable, and conversely if the crossing takes place in the

[15] Using the nonlinear parts of an equation system, a stability coefficient (as formulated for example by Guckenheimer-Holmes (1983)) may be calculated in order to determine the stability properties of the closed orbits.

region $s_d < s < \infty$, where the corresponding periodic orbits are locally unstable;

ii) there exist a parametric configuration in which multiple limit cycles coexist (namely a stable limit cycles with an unstable one); in particular there is a co-existence of a stable equilibrium and a stable limit cycle. This latter situation is called "bi-stability". We have been able to identify numerically the parametric window in which Hopf bifurcation brings about the bi-stability. We argue that, following the recent paper of Kind (1999), the emergence of bi-stability is due to a so-called "crater" bifurcation[16]. This may be verified by simulating the system with a bifurcation parameter close to but below its critical value in the region to the right of (b_d, s_d) giving rise to subcritical bifurcations. In this case, though Hopf bifurcation is subcritical, in addition to the appearance of an unstable limit cycle also an external stable limit cycle emerges. We remark the case of "crater" bifurcation for its economic relevance: in fact in such a case there is a co-existence of two equilibria, the one a stable point, the other a stable oscillation. As Kind (p.152) observes: "large shocks do not lead to a totally unstable dynamic (as in the case of a subcritical bifurcation) but result in permanent oscillations. In addition, phenomena like hysteresis loops and catastrophic transitions may all be described by this bifurcation scenario. The loss of stability in the case of a crater bifurcation thus resembles more closely a scenario of catastrophe than the smooth change of stability associated with a supercritical bifurcation". Notice that, if the bifurcation parameter re-crosses the

[16] The 'crater' bifurcation is rarely discussed in an economic context, despite of its importance for economics, with the exception of Kind (1999) (who attributes such a definition to Lauwerier, 1986). The co-existence of two equilibria (this co-existence is defined "bi-stability" by Grasman, 1994), at our knowledge, has been shown in economic models only in Semmler-Sieveking (1993), Grasman (1994), Manfredi-Fanti (1999) and Kind (1999), but with interpretative differences: in fact while Kind ascribes the phenomenon to a 'local' bifurcation of "crater" type, Semmler-Sieveking have shown the co-existence by means of a 'global' stability analysis while the other authors have attributed the phenomenon to the existence of a "relaxation" oscillation mechanism.

bifurcation boundary directed towards the region of stability, the system does not return to its previous equilibrium but rests in steady oscillation, that is shows a hysteresis loop.

The facts mentioned in *i*) and *ii*) are represented in fig. 1, where the supercritical, bi-stable and subcritical bands are qualitatively depicted[17].

The dynamical transformations of the system (S.1) by varying the crucial parameters *s,* for a given *b,* are presented in table 2 and fig. 1. In particular, as it was expected, we observe: the orbits converge *i)* to the positive equilibrium for $0>s>s_{1H}$, *ii)* to the stable limit cycle, emerged at s_{1H} = 0.132, for an intermediate window of the *s* parameter, $s_{1H} >s>s_{2H}$; *iii)* to the stable limit cycle or to the stable equilibrium point, depending on the initial conditions of price and quantity, because of a "crater" bifurcation, when $s_{2H} >s>s_s$.

Fig. 2 report numerical simulations of the system (S.1) for the above parametric set and *s*=5, displaying the shape of two limit cycles appearing through a Hopf bifurcation of the conjectured "crater" type.

Moreover, the simulations evidenced the following points: 1) the stability is only "local", in that the basin of attraction of the equilibrium point is only a subset of the phase space; in particular when the equilibrium is stable for high values of *s* (namely $s>s_s$=5.07) the basin of attraction is smaller than when the equilibrium is stable for low values of *s* (namely $s<s_{1H}=0.132$); 2) concerning the amplitude of the range of *s* generating stable fluctuations, the simulative evidence seems to indicate that the supercritical band tends to be quite large, by including almost the entire interval lying between the two "critical" values. This is important, because it means that the paths actually either converge to the equilibrium point or converge to the stable cycle, but in any case does not "explode" (that is an innovation cycle can be the rule rather than the exception). The evidence of this is reported in fig. 1, which also reports

[17] The figure displays the bifurcation function (19), reformulated explicating *b* in function of *s* (given the numerical values of the others parameters as in main text).

the amplitude of the subcritical and the bi-stable bands. Table 2 reports also the windows of the *s* parameter in which subcritical (approximately s_s=*5.07<s<5.05*), bi-stable (*5.05<s<4.87*) and stable cyclic (*4.87<s<0.132*) behaviours occur.

It is very important for the successful growth of an innovation to know the location of the basins of attraction of the long run equilibrium, of the external stable limit cycle and of the exploding region. It is worth to recall that the switch between the two possible stable long period configurations – that is convergence to the level of saturation rather than to the external limit cycle – or even the explosion of the system depends on the initial conditions of the variables; it is possible make predictions on these switches of regimes: for instance, as shown in fig. 2, with the above parametric configuration and an initial quantity close to the equilibrium value, an initial price about *p(0)*=1.24 permits to approach the stable equilibrium *p**=1.07, while a small change of the initial price, e.g. *p(0)*=1.241, is sufficient in order that the trajectories approach in a fluctuating way the external persistent limit cycle; finally, in the case of an initial price greater than 5 (*p(0)* >5) the trajectories explode.

Table 2 – Qualitative dynamical results when *s* continuously changes (other parameters as in main text)

Critical values of *s*	$0<s<s_{1H}=$ 0.134	$s_{1H}>s<s_{2H}$ = *4.87*	$s_{2H}>s<s_b$=*5.05*	$s>s_b$
"Global" qualitative dynamics	LAS (and large local basin of attraction)	Unique stable limit cycle	Bi-stability	LAS (and small local basin of attraction)

Legend: LAS= local asymptotic stability (of the equilibrium point).

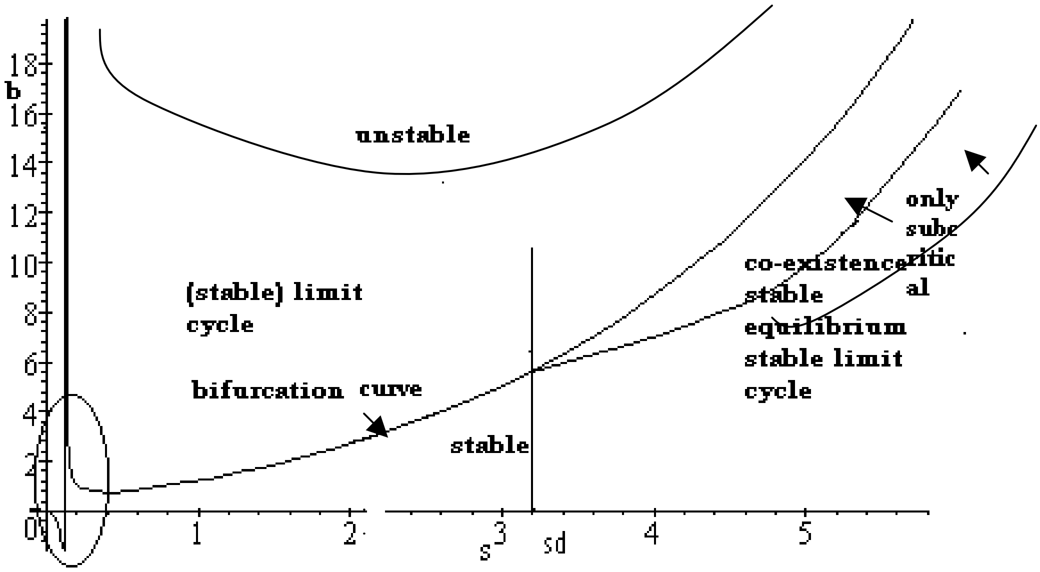


Fig. 1a- Bifurcation curve b_H in the plane s-b and location of the stable and unstable regions with the corresponding bands concerning the unique stable limit cycle and the bi-stable behaviour (depicted for the case illustrated in the main text).

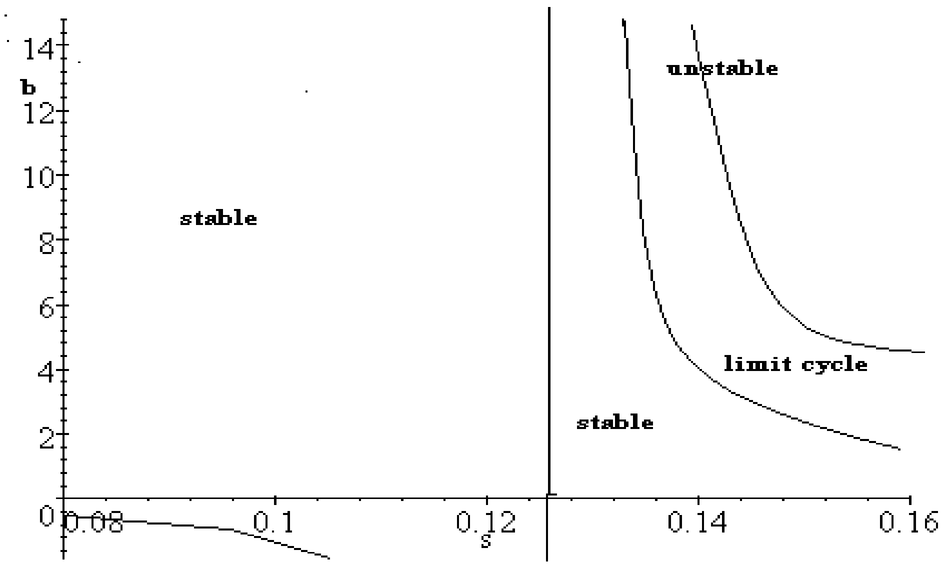
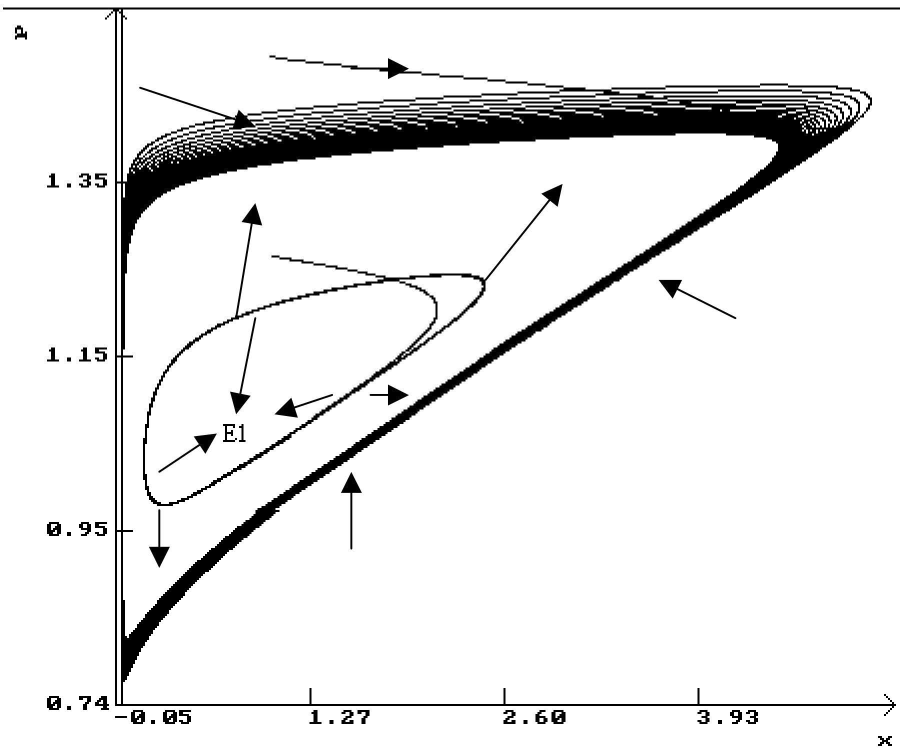


Fig.1b- Enlargement of the elliptic zone of the fig. 1a



.Fig. 2 *The bi-stable case: a stable limit cycle surrounds an unstable one enclosing the equilibrium of the system (parameter set as in main text).*

Therefore, from the above considerations, a neat implication for "marketing" policies emerges: in order to ensure the success of an innovation, the following somewhat unexpected conditions could be necessary: 1) a sufficiently low diffusion between adopters of the new technique, 2) a sufficiently small long term market of the new technique, 3) a very exact "guess" regarding both 3.1) the long term dimension of the market of the new technique at the time to offer the initial quantity and 3.2) the initial expected price which should be sufficiently close to long term price (that is, the opportune initial conditions necessary to ensure the balanced stable growth of an innovation).

5. CONCLUSIONS

In this paper we investigated the aspects of the dynamics of the development of an innovation in the models à la Metcalfe (1981), Batten (1987), Amable (1992), by arguing that the decisions of adopters and suppliers are based on an expected price rather than on a current price, because of, for instance, informational delays.

Therefore the contrast with the previous literature is sensible and involves the qualitative dynamics of an innovation as well as the economic interpretation of the role of various economic conditions. First of all, since the above analysis has shown that in many cases: *i*) the larger is the "imitative" speed (*b*) the more likely is the destabilisation; *ii*) by reducing the speed of production of an innovation (*s*) the system may be destabilised, therefore our results can entirely revert an usual interpretative "dogma" of the diffusion processes[18] claiming that, for instance in the words of Iwai (1984, p. 187): "it is the dynamic interaction between the continuous and equilibrating force of imitation and the discontinuous and disequilibrating force of innovation which governs the evolution of Entropian industry's state of technology". Moreover, the result *ii*) also implies, somewhat counter-intuitively, that too "cautious" investors in capacity of the new technique may prevent a stable development of such an innovation.

Second, in the M-B-A's models supply elasticity, c_1, and the parameters a_0 and c_0, representing shifts respectively of demand and cost curves, only contribute to determine the level of production at the equilibrium point but they cannot influence its stability; on the contrary, our model shows that: 1) too small decreasing costs, and 2) a too high a_0 (a too

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[18] As it is known, on supply side the diffusion of an innovation is driven by the intensity of the growth in investment in the new technique, which is represented by the parameter *s*, while on demand side the diffusion of an innovation is driven by the intensity of the imitation, which is represented by the parameter *b*.

small c_0), can trigger either instability with an explosion of the system or multiple long run attractors or stable fluctuations of the new technique.

As regards point 1) above, interestingly, it may imply that the more "efficient" is the new technique (in the sense that it shows higher returns to scale) the more likely is the deviation from the equilibrium path. Moreover this may also suggest a counterintuitive policy prescription: in fact, ceteris paribus, policies aiming at removing for instance the rigidity of labour supply or of the fixed supply of some other factor can prevent the successful development of a new technique. In addition another somewhat counterintuitive consequence could be suggested: imagining the Schumpeterian view of the twofold profit role as a pair of scissors with two blades (respectively "victim" and "child"), our results predict that the more relaxed is one blade (namely that of "victim") of the scissors, the more likely the long run equilibrium may be prevented; this latter outcome contrast with the intuitively expected attainment of a higher saturation level of the new technique as a consequence of the relaxation of the "squeezing" role of profit.

As regards point 2) above, it may imply some counterintuitive remarks: *i*) the overall economic growth and the total extension of the market, to the extent that they may shift upwards demand curve[19], can prevent the successful development of a new technique; *ii*) a production technology of an innovation which were too "variable costs" intensive – for instance innovations requiring a low initial stock of fixed capital[20]– can prevent an innovation to approach its long run equilibrium.

Finally we argue that: 1) while the preceding models predict a stable level of saturation for an innovation, the cyclical growth for an innovation can be the rule rather than an exception;[21] 2) consequently, an

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[19] For instance, economic growth is expected steadily to shift upwards demand curve.

[20] Such a situation could be considered by some economists in many cases desirable, since supply of an innovation could appear then very flexible in order to follow suitably the unknown demand of adopters.

[21] This regular cycle can be viewed as a first approximation to a majorly irregular, chaotic, "fate" of innovations, as it could occur with a

endogenous deterministic explanation to the observed market fluctuations is provided; 3) since these models represent, above all by emphasising the role of profit as child and victim of the progress, a very classical Schumpeterian view, our results concerning both cyclical growth and the transitory nature of the stable equilibrium could be appreciated in the spirit of the "Schumpeterian" view.

REFERENCES

Amable B., (1992), Competition in the presence of increasing return to scale, *Journal of Evolutionary Economics,* 1, 146-158.

Batten D., (1987), The balanced path of economic development: a fable for growth merchants, in Batten D., Casti J., Johansson B. (eds.), *Economic Evolution and Economic Adjustment*, Springer, Berlin Heidelberg New York.

Benhabib J. – Miyao T., (1981), Some new results on the dynamics of the generalized Tobin's model, *Journal of Economic Dynamics and Control*, 4.

Dosi G., (1991), Some thoughts on the promises, challenges and dangers of an 'evolutionary perspective" in economics, *Journal of Evolutionary Economics*,1, 5-7.

Fanti L.- Manfredi P., (1998), A Goodwin Growth Cycle Model with Profit-Sharing, *Economic Notes,* 3, 371-402.

Grasman J., (1994), Co-existence of a limit cycle and an equilibrium in Kaldor's business cycle model, *Journal of Economic Behaviour and Organization*, 24, 369-377.

Guckenheimer J. - Holmes P., (1983), *Nonlinear Oscillations, Dynamical Systems and Bifurcation of Vector Fields,* Springer-Verlag, New York, Tokio, Berlin.

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less agents' heterogeneity (as regards price delay), or, in other words, with a distribution of lags tending towards the Dirac distribution (i.e. a fixed delay).

Hillinger C., (1996), Dynamic Disequilibrium Economics: History, Conceptual Foundations, Possible Futures, in *Dynamic disequilibrium modeling: Theory and applications*: Proceedings of the Ninth International Symposium in Economic Theory and Econometrics, International Symposia in Economic Theory and Econometrics series. Cambridge; New York and Melbourne: Cambridge University Press, 27-65.

Howitt P., (1978), The limits to stability of a full-employment equilibrium, *Swedish Journal of Economics* 80, 265–282.

Kind C., (1999), Remarks on the economic interpretation of Hopf bifurcations, *Economics Letters*, 62, 147-154.

Invernizzi S.- Medio A., (1991), On Lags and Chaos in Economic Dynamic Models, *Journal of Mathematical Economics*, 20, 521-550.

Iwai K., (1984), Schumpeterian dynamics: an evolutionary model of innovation and imitation, *Journal of Economic Behaviour and Organization*, 5, 159-190.

Lauwerier H., (1986), Two-Dimensional iterative maps. In: Holden, A.V. (Ed.), *Chaos*, Manchester, 58–95.

Leijonhufvud A., (1973), On Keynesian Economics and the Economics of Keynes, *Swedish Journal of Economics, 75,* 27-48.

Lorenz H.W., (1993), *Nonlinear Economic Dynamics and Chaotic Motion*, Springer Verlag, New York, Tokio, Berlin.

Manfredi P.– Fanti L., (1999), Gestation Lags and Efficiency Wage Mechanisms in a Goodwin-type Growth Cycle Model, *Studi Economici*, n. 68, 95-126.

Mansfield E., (1961), Technical Change and the Rate of Imitation, *Econometrica*, vol. 29, 4, 741-766.

Marsden J., McCracken M., (1976), *The Hopf bifurcation and its applications*, Springer Verlag, New-York Tokio Berlin

McDonald N., (1978), *Time Lags in Biological Systems*, Lecture Notes Biomath. 29, Springer Verlag, New York, Tokio, Berlin

Metcalfe S., (1981), Impulse and Diffusion in the Study of Technical Change, *Futures*, vol. 13, ottobre, 347-358.

Metcalfe S., (1994), Competition, evolution and the capital market, *Metroeconomica* 45:2. 127-154

Schumpeter J.A., (1976), *Capitalism, socialism and democracy*, Allen and Unwin, London. (first edition 1943).

Semmler W. - Sieveking M., (1993), Nonlinear liquidity-growth dynamics with corridor-stability, *Journal of Economic Behavior and Organization*, 22, 189–208.

Stoneman P., (1983), *The Economic Analysis of Technological Change*, Oxford University Press.

APPENDIX 1

Although the E_0 equilibrium is not economically interesting by itself, the knowledge of its stability properties can be important in order to understand the global dynamical behaviour of the system.

The usual jacobian of the system (S.1) evaluated in E_0 is

$$J(x,p,p^e)_{|E_0} = \begin{bmatrix} s(p^* - c_0) & 0 & 0 \\ 0 & -\gamma & \gamma \\ 0 & 0 & -\gamma \end{bmatrix}$$

(A.1)

The characteristic equation is

$$a_0\lambda^3 + a_1\lambda^2 + a_2\lambda + a_3$$

(A,2)

where

$$a_0 = 1$$

$$a_1 = \frac{-bs\Psi + 2\gamma(s + a_1 b)}{(s + a_1 b)}$$

$$a_2 = \frac{-2bs\Psi + b\gamma a_1(1+s)}{(s + a_1 b)}$$

$$a_3 = -\frac{\gamma^2 bs\Psi}{(s + a_1 b)}$$

$$\Psi = a_0 - c_0 a_1$$

(A.3)

We remark two distinct situations: 1) when the positive equilibrium exists, the E_0 equilibrium is always unstable, because, by using the existence condition of the positive equilibrium (eq. 14) and by invoking the usual Routh-Hurwitz stability conditions, it is easily to see that $a_3 < 0$; 2) when the positive equilibrium does not exist, that is (14) does not hold, the E_0 equilibrium is always stable because Routh-Hurwitz conditions always hold.

APPENDIX 2

In a nutshell, proposition 1 claims that if the parameter s continuously increases starting from zero, provided a parametric configuration satisfying $B_2<0$, the system shows a first Hopf bifurcation at $s=s_{1H}>0$ and a second bifurcation a $s=s_{2H}>s_{1H}$.

The proof of this proposition is based on the mechanism of Hopf bifurcation[22]. In heuristic terms the Hopf bifurcation theorem states the existence of closed orbits in a neighbourhood of the equilibrium for some bifurcation parameter value; and this occurs as, being such a parameter increasing, the following conditions are satisfied: 1) complex eigenvalues exist or emerge; 2) the real parts of such eigenvalues are zero at the bifurcation value of the parameter; 3) all the other real eigenvalues are different from zero when the parameter is at its bifurcation value; 4) the real parts of the eigenvalues become positive when the bifurcation parameter value goes beyond the bifurcation value. As regards the case of three-dimensional system, it is well-known that (Lorenz,1993) the real parts of the complex eigenvalues are zero and the third real eigenvalue is negative when $a_1,a_2,a_3 >0$ and $a_1a_2-a_3=0$, and therefore the simple application of the Routh-Hurwitz conditions for the local stability

[22] A rigorous treatment of the Hopf theorem is, for example, in Marsden-McCracken (1976).

allows to demonstrate the existence of a Hopf bifurcation[23]. Indeed: *i*) if at the bifurcation point the Routh Hurwicz conditions $a_1,a_2,a_3 >0$ are satisfied then both the loss of stability can only occur when a pair of complex eigenvalues exists (that is the discriminant of the characteristic equation is positive) (satisfying point 1) and all the other real eigenvalues are different from zero (satisfying point 3); *ii*) therefore, provided that the derivative of the real parts of the complex eigenvalues with respect to the bifurcation parameter is different from zero (satisfying point 4)[24] (so that there is an effective crossing of the imaginary axes as the bifurcation parameter is increasing), the bifurcation appears as $a_1a_2-a_3=0,$ where the real parts of the complex eigenvalues are zero (satisfying point 2). Then, in the present model, being all the other conditions satisfied, when

$$B_2 = \gamma\left[bH(4_1 a_1 c_1 - 1) + 2c_1\gamma\right] < 0$$

there are always two positive (obviously if they are real) values of *s,* satisfying

$$a_1 a_2 - a_3 = s^3 B_0 + s^2 B_1 + sB_2 + B_3 = 0$$

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[23] As regards the demonstration of the necessary and sufficient conditions for the existence of Hopf bifurcations in systems of third, fourth (and higher) order see Fanti-Manfredi (1998).

[24] For what concerns the procedure to test the crossing of the imaginary axis with non zero speed see again Fanti-Manfredi (1998). Obviously we have verified that such a condition is satisfied here, but, for brevity, the calculations (disposable on request) are not shown here.

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