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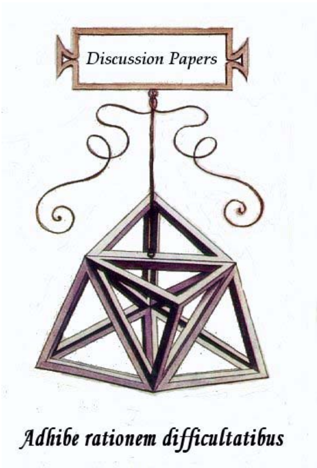

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Endogenous labor supply and Diamond's (1965) model: a reconsideration of the debt role

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Abstract

In this paper we show that, when elastic labor supply is considered via Cobb Douglas preferences, dynamic inefficiency of OLG economies, while being still a necessary condition, is no longer sufficient for an internal public debt increase to generate a Pareto improvement. This is due to the fact that the equilibrium interest rate can move in the "wrong" direction due to the reaction of labor supply. Consequently, raising the level of debt when the economy is experiencing dynamic inefficiency could even be welfare-worsening, in contrast with Diamond (1965).

Keywords: Overlapping Generations, endogenous labor supply, dynamic inefficiency, debt
Classificazione JEL: D91, E62, H63, J22

Contents

I. Introduction

As well known, OLG economies may happen to be "dynamically inefficient" whenever agents are accumulating too much capital with respect to the level which would maximize steady state consumption[1]. Diamond (1965) provides a general rule concerning the possibility of correcting such dynamic inefficiency (DI) of production economies via debt issuing. As far as the internal debt case is concerned, he finds that DI is in fact a necessary and sufficient condition for increasing the steady state level of welfare. However, his findings are obtained under the hypothesis of fixed labor supply. Despite the relevance of Diamond's result in the economic literature, the more general case of endogenous labor supply has not been sufficiently investigated. The aim of this work is to contribute to fill this gap.

So far, first attempts to investigate the role of labor supply in correcting dynamic inefficiency have been: 1) Hu (1979), who however deals with Social Security instead of debt: his main result is that Diamond's rule can fail to generate a Pareto improvement in the economy due to the distortion brought about by the payroll tax on wages[2]; 2) Spataro (2003), who explicitly complements Diamond's analysis by introducing endogenous labor supply in the presence of internal public debt. He points out that, contrary to Hu, the violation of the Diamond's rule can occur even in the presence of non distortionary taxation (i.e. lump sum taxes), due to the effect that debt issuing has on the labor supply and, thus, on the equilibrium interest rate.

In this paper we further analyze the issue of correcting suboptimality via debt issuing in an OLG framework, by assuming well behaved preferences (i.e. Cobb Douglas preferences).

The main results of the work can be summarized as follows: first, we provide a general discussion of the relationship between the equilibrium interest rate and the level of capital. In fact, such relationship turns out to be crucial for the possibility of correcting DI: we show that its sign can be negative in the DI case so as to potentially invalidate Diamond's rule (some numerical examples show that this can be the case for realistic values of the parameters). Second, we deliver the conditions of debt optimality in the presence of elastic labor supply. In the light of this analysis, we finally find that: 1) Diamond's rule is to be considered still valid, in the presence of elastic labor supply, when the economy is dynamically efficient: in fact, in such a case, the effect of raising debt is to unambiguously lower the steady state level of welfare; 2) on the other hand, in the case of DI, the ambiguity brought about by the change of the interest rate mentioned above leads to the possibility that debt issuing cannot correct DI in the presence of endogenous labor supply.

[1] Such optimal level of capital is usually referred to as the Golden Rule level.

[2] Other works dealing with the optimality of unfunded Social Security schemes in OLG economies, with endogenous labor supply are Homburg (1989) and Breyer and Straub (1993).

The work proceeds as follows: in section two we lay out the model and in section three we characterize the general equilibrium for the decentralized economy and analyze the long run relationship between debt issuing and the interest rate, illustrated also by some numerical examples. In the section four we obtain the general conditions for debt optimality. Final remarks will end the work.

II. The model set-up

Our economy is closed and populated by two-period lived individuals who, in order to maximize their lifetime utility, decide how long to work and how much to save in the first period so as to afford consumption in their second period, when they are retired. The economy grows at an exogenous rate n, which is entirely due to the population dynamics, so that $N_t = N_{t-1}(1+n)$, with N_t the size of cohort born in period t. The individuals belonging to generation t have an utility function U, defined over c_{1t}, c_{2t+1}, a_t, that is consumption in the first and second period and leisure, respectively. Firms own a constant returns to scale production technology $F(K_t, L_t)$ by which they transform physical capital (K_t) and labor (L_t), with $L_t = N_t(1-a_t)$, into the consumption good[3].

Furthermore, the government issues debt B_t and levies lump sum taxes upon the young, according to the ordinary dynamic equation: $B_{t+1} = B_t(1+r_t) - \tau_{1t}N_t$ (where τ_{1t} is the lump sum tax) which, in per young terms, is: $b_{t+1}(1+n) = b(1+r_t) - \tau_{1t}$.

III. The decentralized equilibrium

The solution for a decentralized economy implies firms' and individuals' maximization of profits and utility respectively, the market clearing condition and the satisfaction of the government budget constraint. We assume a perfect competitive market: thus, firms hire capital and labor by remunerating them according to their marginal productivity. Moreover, due to the homogeneity of degree one of F, it follows that $w_t(1-a_t) = f(k_t, a_t) - f'_k(k_t, a_t)k_t$ and $r_t = f'_k(k_t, a_t)$, where low letters (apart from factor prices) indicate variables expressed in per young terms and the subscript of the derivative function f' indicates the derivation variable.

As for individual maximization, considering individuals of generation t, they face the following lifetime budget constraint:

$$c_{1t} + \frac{c_{2t+1}}{(1+r_{t+1})} = w_t(1-a_t) - \tau_{1t}. \tag{1}$$

Now, the debt accumulation equation in per young terms can be written as follows:

[3] The usual Inada conditions on both the utility and the production function are assumed in order to insure interiority of the solutions.

$$b_{t+1}(1+n) = b_t(1+r_t) - \tau_{1t}; \tag{2}$$

then, the individual maximization problem is:

$$U_t = \log c_{1,t} + \lambda \log a_t + \beta \log c_{2,t+1} \tag{3}$$

$$sub \;\; c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = w_t(1-a_t) - \tau_{1t}$$

where $\lambda > 0$ and $\beta \in (0,1)$. The FOCs conditions are the following:

$$\beta \frac{c_{1,t}}{c_{2,t+1}} = \frac{1}{(1+r_{t+1})} \tag{4}$$

$$\lambda \frac{c_{1,t}}{a_{1,t}} = w_t. \tag{5}$$

As a consequence, the demand functions are the following[4]:

$$c_{1,t} = \frac{w_t - \tau_{1t}}{H\beta} \tag{6}$$

$$c_{2,t+1} = (1+r_{t+1})\left(\frac{w_t - \tau_{1t}}{H}\right) \tag{7}$$

$$a_t = \frac{\lambda}{w_t}\left(\frac{w_t - \tau_{1t}}{H\beta}\right) \tag{8}$$

and

$$s_{1,t} = w_t\left[(1-a_t) - \frac{1}{H\beta}\right] - \tau_{1t}\left(1 - \frac{1}{H\beta}\right), \tag{9}$$

where $H = \left(\frac{1+\beta+\lambda}{\beta}\right)$, and $w_t > \tau_{1t}$. By following Diamond, we make the assumption that the debt in per young terms be constant, so that the level of taxes, τ_{1t}, is equal to $b(r_t - n)$.

Finally, the market clearing condition completes the equilibrium conditions: $s_t N_t = K_{t+1} + B_{t+1}$, that is:

$$s_t = (k_{t+1} + b)(1+n). \tag{10}$$

As far as the stability of the equilibrium is concerned, we can start from the general relationship

[4]For simplicity we ignore the non-negativity constraints and the constraint $a_t < 1$, by assuming parameters which ensure that the constraints are satisfied (as it has been verified in the numerical examples that follow).

$$r_{t+1} = f'_k \left(\frac{s_t \left(w_t, a_t, b, r_t \right)}{(1+n)} - b, a_{t+1} \right); \qquad (11)$$

by differentiating it with respect to r_t, and recalling that $\frac{\partial w_t}{\partial r_t} = \frac{k_t}{(1-a_t)}$, assume that

$$0 < \frac{dr_{t+1}}{dr_t} = -\frac{f''_k \left[k_t s_{w_t} - s_{r_t} + s_{a_t} \left(-\frac{\partial a_t}{\partial r_t} + \frac{k_t}{(1-a_t)} \frac{\partial a_t}{\partial w_t} \right) \right]}{(1+n) \left[1 + \frac{\partial f'_k}{\partial a_{t+1}} \left(-\frac{\partial a_{t+1}}{\partial r_{t+1}} + \frac{k_{t+1}}{(1-a_{t+1})} \frac{\partial a_{t+1}}{\partial w_{t+1}} \right) \right]} \leq 1. \qquad (12)$$

Eq. (12), under our assumption of Cobb Douglas preferences, turns out to be:

$$0 < \frac{dr_{t+1}}{dr_t} = -\frac{f''_k (k_t + b)}{(1+n) H \left[1 + \frac{\partial f'_k}{\partial a_{t+1}} \frac{b\lambda}{H\beta w_{t+1}} \left(1 - \frac{k_{t+1}}{(1-a_{t+1})} \frac{(n - r_{t+1})}{H\beta w_{t+1}} \right) \right]} \leq 1. \qquad (13)$$

Note that, with Cobb Douglas preferences, the "substitution effect" exactly compensates the "income effect" generated by an increase of the interest rate, so that $\left(-\frac{\partial a_t}{\partial r_t} + \frac{k_t}{(1-a_t)} \frac{\partial a_t}{\partial w_t} \right) = 0$.

At the steady state (in which $r_{t+1} = r_t$), differentiating eq. (11) with respect to debt we have the following general result:

$$\frac{dr}{db} = \frac{f''_k \left[\frac{s_b + s_a \frac{\partial a}{\partial b}}{(1+n)} - 1 \right] + \frac{\partial f'_k}{\partial a} \frac{\partial a}{\partial b}}{(1+n) \left[1 + \frac{\partial f'_k}{\partial a} \left(-\frac{\partial a}{\partial r} + \frac{k}{(1-a)} \frac{\partial a}{\partial w} \right) \right] + f''_k \left[k s_w - s_r + s_a \left(-\frac{\partial a}{\partial r} + \frac{k}{(1-a)} \frac{\partial a}{\partial w} \right) \right]}. \qquad (14)$$

Now, since with Cobb Douglas preferences

$$\frac{da}{db} = (n - r) \frac{\lambda}{wH\beta} \Rightarrow \frac{da}{db} \gtreqless 0 \Leftrightarrow n \gtreqless r,$$

it emerges that under our preferences assumption the sign of the variation of leisure depends crucially on the relationship between n and r and on the sign of λ. In the sequel of the work we maintain the usual hypothesis that λ be strictly positive. The economic intuition behind the relationship between leisure and public debt is clear: public debt increases (reduces) the amount of leisure chosen by individuals according to whether it produces a positive (negative) wealth effect (i.e. a subsidy or tax respectively[5]). Consequently, under our assumptions, eq. (14) becomes:

[5] It is worth recalling that, in presence of DI ($n > r$), the assumption of constant per young debt, implies that the new debt issued subsidizes incomes more than it is necessary to burden them for interest payments on the outstanding debt stock.

$$\frac{dr}{db} = -\frac{f_k'' \left[1 - \frac{(n-r)}{H(1+n)}\right] - \frac{\partial f_k'}{\partial a} \frac{\lambda (n-r)}{H\beta w}}{1 + \frac{f_k''(k+b)}{(1+n)H} + \frac{\partial f_k'}{\partial a} \frac{b\lambda}{H\beta w} \left(1 - \frac{k}{(1-a)} \frac{(n-r)}{H\beta w}\right)}. \tag{15}$$

¿From the stability assumption the denominator of eq. (15) is positive; moreover, by recalling that, as usual in economics, $H > 1$[6], it follows that $\left[1 - \frac{(n-r)}{H(1+n)}\right]$ is always positive, so that the only source of ambiguity is given by the second term at the numerator, that is the effect of the labor supply on the marginal productivity of capital.

We can summarize our findings in the following proposition:

Proposition 1 *When the utility function has a Cobb Douglas form and under stability of the equilibrium, if $n < r$, $\frac{dr}{db}$ is always positive; conversely, when $n > r$, $\frac{dr}{db}$ may be negative if $\frac{\partial f_k'}{\partial a} \frac{\lambda(n-r)}{H\beta w} > f_k'' \left[1 - \frac{(n-r)}{H(1+n)}\right]$.*

Proof. When stability is satisfied and $n < r$, since both $\frac{\partial f_k'}{\partial a}$ and f_k'' are negative, $\frac{dr}{db}$ is positive. By the same reasoning, when $n > r$ the numerator of eq. (15) can be positive (and thus $\frac{dr}{db}$ negative) if inequality ii) of Proposition 1 holds. ■

Equation (15) is a generalization (including endogenous labor supply) of Diamond's analysis of the change in the equilibrium interest rate arising from the change of the quantity of debt (see Diamond (1965), eq. 27, page 1142); we recall that in Diamond's paper such a change is always positive. The rationale of the possibility of the reversion of Diamond's result shown above is simple: when debt produces a positive wealth effect, it lowers labor supply (which in Diamond's analysis was fixed) and the corresponding reduction in the labor input in production has a negative effect on the marginal product of the capital. If such negative effect is strong enough, then the increase of the quantity of debt may reduce the equilibrium interest rate.

Only for illustrative purposes, we show some numerical examples[7] whereby an increase of public debt brings about a reduction of the long run interest rate, according to the result of the Proposition 1. We choose a CES production function of the following type:

$$F = A \left[\alpha K^{-\rho} + (1-\alpha) L^{-\rho}\right]^{-\frac{1}{\rho}},$$

which, in per young terms, has the form:

$$f = A \left[\alpha k^{-\rho} + (1-\alpha)(1-a_t)^{-\rho}\right]^{-\frac{1}{\rho}}.$$

[6]It is worth noting that $H > 1$ if $\lambda > -1$. Of course this condition is always verified if work is "painful" (that is $\lambda > 0$).

[7]Of course the present model does not admit closed form solutions, so that it must be investigated only by numerical simulations. For this purpose we have used the software Maple 5.1.

By performing some numerical simulations, with the following parameter sets:

A	α	ρ	n	b	β	λ
5	0.1	4.63	0.4	0.01	0.5	0.7
20	0.1	6.8	0.77	0.2	0.5	2
20	0.1	1.9	0.77	0.7	0.5	3

we get, respectively, the following steady state values[8] of k, r, a and in particular negative values of $\frac{dr}{db}$:

k	r	a	$\frac{dr}{db}$
0.75	0.29	0.31	-0.01
0.58	0.2	0.57	-0.02
0.55	0.51	0.67	-0.03

Therefore, as shown above, debt issuing in a dynamically inefficient economy (with realistic parametric configurations), may reduce the equilibrium interest rate.

IV. *The welfare effects of debt variations*

In this section we obtain the expression for the effects of debt variations on the steady state utility level.

At the steady state, the individual budget constraint has the form:

$$c_1 + \frac{c_2}{1+r} = w(1-a) + b(n-r). \tag{16}$$

Then, by differentiating it with respect to b we get:

$$\frac{dc_1}{db} + \frac{dc_2}{db}\frac{1}{(1+r)} + w\frac{da}{db} = \frac{c_2}{(1+r)^2}\frac{dr}{db} + \frac{dw}{db}(1-a) + (n-r) - b\frac{dr}{db}. \tag{17}$$

Next, by assuming that the policymaker is benevolent and that all individuals have the same lifetime consumption/leisure pattern, by exploiting the Envelope Theorem the following equality holds:

$$\frac{dU}{db} = U_1\frac{dc_1}{db} + U_2\frac{dc_2}{db} + U_3\frac{da}{db} = U_1\left[\frac{dc_1}{db} + \frac{dc_2}{db}\frac{1}{1+r} + w\frac{da}{db}\right]; \tag{18}$$

[8]Of course all the economic constraints as well as the stability hold in these examples.

finally, by reckoning that $c_2 = s(1+r)$, $\frac{s}{(1+n)} = k+b$, $\frac{dw}{dr} = -\frac{k}{1-a}$, and exploiting equation (17) it follows that:

$$\frac{dU}{db} = U_1 (n-r) \left[1 + \frac{k+b}{1+r} \frac{dr}{db} \right]. \tag{19}$$

Note that, the equation above replicates Diamond's formula (that is, eq. 29, page 1142). Diamond concludes that *"utility is decreased in the efficient case and increased in the inefficient case"*[9].

¿From inspection of equation (19) we get the following:

Proposition 2 *Under Cobb Douglas preferences, and with lump sum taxation, increasing the level of debt is always Pareto worsening if $r > n$.*

Proof. Under stability assumption, if $r > n$, then $\frac{dr}{db} > 0$, and, since $U_1 > 0$, it follows that $\frac{dU}{db} < 0$. ■

This proposition is in line with Diamond's results. On the other hand, we get also the following:

Proposition 3 *Under Cobb Douglas preferences, if $n > r$, then increasing the level of debt can be Pareto worsening.*

Proof. When $n > r$, the sign of $\frac{dU}{db}$ is univocally determined by the expression in brackets; on the one hand, if $\frac{dr}{db} > 0$, then $\frac{dU}{db} > 0$. On the other hand, if $\frac{dr}{db} < 0$, then $\frac{dU}{db} > 0$ if $1 + \frac{k+b}{1+r}\frac{dr}{db} > 0$, which does not necessarily hold. ■

¿From the proposition above it emerges that dynamic inefficiency is still necessary but not a sufficient condition for the debt increase to produce a Pareto improvement, since the sign of $\frac{dr}{db}$ introduces an ambiguity on the overall effect on utility. Note that this proposition can revert Diamond's conclusions: in the "inefficient case", utility can be decreased rather than increased by an increase of the long run stock of public debt[10].

V. Conclusions

In this work we reconsider the analysis of Diamond (1965), on the possibility of correcting the dynamic inefficiency (DI) of an OLG economy via debt issuing, in the presence of endogenous labor supply and well behaved preferences.

[9] We recall that an economy is referred to as "dynamic inefficient" when $r < n$.

[10] Notice that whether the reversion of the sign of $\frac{dr}{db}$ (analytically proven and numerically illustrated in the previous section) is strong enough as to generate a reduction of the steady state utility in realistic economies, is left for future simulative research.

First of all, we demonstrate that, contrary to the previous literature, allowing for elastic labor supply causes an ambiguity concerning the sign of the change in the equilibrium interest rate generated by a debt variation. This means that, in principle, a raise in the level of debt might reduce the interest rate: that is, move it in either the bad or the good direction for the welfare, according to whether the economy undergoes dynamic inefficiency or efficiency respectively. When the utility function is a Cobb Douglas, we show that such ambiguity can occur only in the DI case. As a consequence, dynamic efficiency is still (as in Diamond) a sufficient condition for generating a Pareto worsening of the long run welfare. On the other hand, raising debt when the economy is overaccumulating, in contrast with the Diamond's belief, may not be always Pareto improving.

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