***Discussion Papers***
Collana di
E-papers del Dipartimento di Scienze Economiche – Università di Pisa

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# Endogenous fertility and cycles in Solow's growth model.

*Discussion Paper* n. 8
2003

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*Discussion Paper* n. 8, presentato: settembre 2003

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Si prega di citare così:
Luciano Fanti and Piero Manfredi (2003), "Endogenous fertility and cycles in Solow's growth model", Discussion Papers del Dipartimento di Scienze Economiche – Università di Pisa, n. 8 (http://www-dse.ec.unipi.it/ricerca/discussion-papers.htm).

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# Endogenous fertility and cycles in Solow's growth model.

**Abstract**

It is shown here that the Solow (1956) neo-classical growth paradigm not only explains the "first" stylised fact of economic growth, namely the existence of a globally stable state of balanced growth, but, once endowed with a demographically founded formulation of the labour supply, is also capable to endogenously explain a second main stylised fact of growth, i.e. the generation of globally stable oscillations around the path of balanced growth.

**J.E.L** Classification codes: E3,J0

**Keywords**: Solow's balanced growth model, endogenous population, neoclassical growth-cycle model.

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## *1. Introduction*

The centrality of the neo-classical growth model of Solow (1956) for economic theory is witnessed by the current persistency of new contributions stimulated by his work (for instance Bajo-Rubio, 2000). The aim of the present paper is to investigate the effects of endogenous labour supply within Solow's model.

This endogenisation is based on a coupling of an age structure argument (absent in the original Solow) plus a Malthusian relation between fertility and income, which was well recognised by Solow himself. A realistic formulation of labour supply must take into account past demographic behaviours in that the new entries into the labour force at time $t$ are the outcome of fertility behaviours of past generations. This requires the introduction of the age structure prevailing in the population. This fact was well known to the "classical" economists: "the supply of labourers in the market can neither be speedily increased when wage rise, nor speedily diminished when they fall. When wages rise a period of eighteen or twenty years must elapse before the stimulus which the rise, given the principle of population, can be felt in the market" (McCulloch 1854, p. 34). The previous McCulloch's sentence contains, according to the current literature in population economics, a terse definition of Malthusian cycles. Malthusian cycles are a firm theoretical point in population economics. They are the consequence : "...of the lags between the response of fertility to current labour market conditions and the time when the resulting births actually enter the labour force" (Lee 1997, pp. 1097). The existence of such cycles was lucidly predicted, at least theoretically, by Malthus in several loci in his famous Essay (Malthus 1992).

Long run population oscillations (and the economy) are an important, though sometimes neglected (Barro and Sala i Martin 1995), stylised fact. They are well documented in the recent empirical study by Lee & Anderson (2002), see in particular fig.1,2,3 reporting UK historical time series of the death and birth rates, and of the wage rate. They state: "Both series, especially the birth rate, show broad swings over hundreds of years, and short term fluctuations are substantial, especially prior to 1750" (ibidem pp 201). Another fundamental example is represented by so called post-transitional fertility fluctuations, such as the baby-booms observed in most the Western World, sometimes explained by the Easterlin's effect (Easterlin 1961). We finally feel that if Lowest Low Fertility, as recently observed in the Western World (Kohler et al. 2002), should come to an end - and this is considered more or

less a necessary event (Bongaarts 1999) - this would almost necessarily give rise to a new fertility wave.

In this paper the age structure mechanism, which governs Malthusian cycles, is embedded in our Solow-type model by resorting, in a manner very close to the classical economists thought, to time-lags.[1] . We show that persistent oscillations may occur in the Solow's model when the rate of change of the labour supply is correctly assumed to depend (even in the simplest manner) on past demographic behaviours. Thus we argue that the neoclassical growth paradigm is capable not only to explain the stylised fact of balanced growth, but, once endowed with a correctly demographically founded formulation of the labour supply, also to endogenously explain a second main stylised fact of economic growth, namely the appearance of oscillations around a stable path of balanced growth.

In order to position the present paper in the literature on the economic growth, we note that our model is a Solovian model of balanced growth with endogenous population. In this sense it may be considered an "endogenous" growth model in that it endogenously explains the rate of total income growth [2] ; obviously it is a "descriptive" model rather than a "fully optimized" one as those surveyed in Tamura (2000). Further our model : 1) is a growth cycle model; 2) it endogeneises the age structure of the population in a macroeconomic setting. The two latter features are not shared by most avalaible growth models, whether they belong to the Solovian tradition or to the "new growth theory".

The present paper is organised as follows. In section two a basic Solow-type model with endogenous population depending on current fertility is introduced, and its properties are studied. In section three we introduce a more general model embedding a time-lag in the reaction of the rate of change of the labour supply to past income, and its properties are investigated by means of local stability analysis plus Hopf bifurcation. Section four completes the analysis of the model by investigating its global properties via numerical simulations and discussing the economic meaning of its main results. The "core" results are summarised in the conclusions.

## *2. A basic Solow's model with endogenous population*

[1]As largely recognised, time-delays represent a simple and clever strategy to embed age structure within complex models (see for instance McDonald (1978,1989)). This strategy is of course an approximation of the true underlying age structure model, based on the assumption of "stability" of the age composition, i.e. the constancy of the ratios between different cohorts of the underlying population.

[2]We note that conversely our model does not allow a positive rate of growth of the per-capita income growth, which is a main task of the recent "new growth theory".

The original Solow growth model (Solow 1956) is defined by the ordinary differential equation (ODE):

$$\dot{k} = sf(k) - n_s k \qquad (1)$$

where $k=K/L$ denotes the capital-labour ratio, $f(k)$ the production per unit of labour, $s$ the saving rate ($0<s<1$), $n_S>0$ the rate of change of the supply of labour ($\dot{L}/L$) which was initially assumed exogenous by Solow. As in the Solow 1956 paper, we have assumed $\delta=0$ where $\delta$ is the rate of capital depreciation. When a Cobb-Douglas production function is chosen (ibidem, example 2, p.76), the model collapses in the ODE:

$$\dot{k} = sk^{\alpha} - n_s k \qquad (2)$$

where $0<\alpha<1$. Contrary to most subsequent developments, where the supply of labour was treated as exogenously determined, Solow (1956) also tried, still in his 1956 paper, to endogenise it. He wrote (*ibidem*, p. 90) the rate of change of the supply of labour as a function of the current level of percapita income: $n_s = n_s(f(k))$, which becomes in the Cobb-Douglas case

$$n_S=n_S(k^{\alpha}) \qquad (3)$$

He also gave some qualitative analysis of the effects of a general non monotonic $n_S$ function.[3]

As in this paper we are essentially interested in the effects of forces of "fundamental" nature, in what follows we will assume, for simplicity, that the map $n_S$ be linear and increasing, i.e. we consider $n_S=nk^{\alpha}$ where $n>0$ is a constant parameter [4], tuning the reaction of the rate of change of supply of labour to changes in per-capita income. We therefore have the model:

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[3]For sake of precision, we notice that he considered a qualitatively complicated form of the fertility function, generating three equilibria, and also predicted, in a pioneering way, the possibility of an endogenous self-sustained growth allowed by the endogeneity of labour supply. Such idea will be better formalised only many years later, for instance by Becker, Murphy and Tamura (1990).

[4]According to a purely demographic interpretation of labour supply, the positive relationship between labour supply and wage adopted here may be interpreted as a Malthusian relation between fertility and the wage, rather than a "well behaved" labour supply function. We did not explicitly considered income-related mortality but the extension is straightforward.

Referring to the original ideas of Malthus, the rate of change of labor supply could also be specified as $\dot{k} = sk^{\alpha a} - nk^{1+\alpha}$, where $y^0$ is subsistence income and $n>0$. In such a specification, labor supply growth, when the per capita income is below the subsistence level, would be negative, which is a realistic fact. But also this specification (as well as any non linear saturating

$$\dot{k} = sk^{\alpha a} - nk^{1+\alpha} \tag{4}$$

It is easy to see that system (4) always admits, as in the basic Solow's model, the zero equilibrium ($E_0$) and a unique positive equilibrium ($E_1$) which is globally asymptotically stable (the previous conclusions are preserved if we take $n(.)$ as a general increasing function, saturating or not). In particular the capital-labour ratio at $E_1$ is $k=k_1=s/n$.

It is worth noting that the long term growth rate of the population, and thus of total income ($Y$) in model (4), is:

$$\frac{\dot{Y}}{Y} = \frac{\dot{y}}{y} + n_s = nk_1^{\alpha} = n^{1-\alpha}s^{\alpha} \tag{5}$$

Thus the long term growth rate of output depends on the joint action of population (as summarised through the Malthusian parameter $n$) and saving. The interest for this result lies in the relevance of its message, and in the simplicity with which it is obtained: contrary to Solow's original result, where, paradoxically, saving determines the accumulation but does not play any role for long term growth of total income, saving matters for growth: indeed only through saving, e.g. new investments, population change may promote economic growth.

Clearly, the assumption that the rate of change of the supply of labour is a function of the current income, as in (4), might perhaps reflect an underlying participation effect, but is hardly defendable as determined by population change. We argue, however, that this simple model is useful to cast the investigation of the consequences of fully endogenous population dynamics, as will be done in the next section by resorting to time-lags.

## *3. Age structure and labour supply: the demographic delay*

To take into account of the overall demographic mechanism of age structure in economic models would lead to a larger dimension and thus more analytical complexity. Since the mathematical literature (Mac Donald 1978) has shown that time-delays often represent simple approximations of age structure mechanisms, in this paper we follow this route. The intuitive idea here is that the current rate of change of the supply of labour is related to past fertility, and thus to past levels of the wage, following a prescribed pattern of delay. There are two main alternatives: fixed delays and distributed delays. The former is better suited when there is no variability in the process of transmission of the past into the future: for instance when we assume that all individuals are

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specifications) leads to the same conclusions shown with the more simple specification adopted here.

recruited in the labour force at the same fixed age. Conversely when recruitment may occur at different ages, i.e. with different delays (for instance because the time needed to complete formal education is heterogeneous within the population) distributed delays appear more suitable[5] . The introduction of a distributed delay in the population term in (4) leads to the following integro-differential (IDE) equation:

$$\dot{k} = sk^{\alpha} - \left( \int_{-\infty}^{t} n\left(k^{\alpha}(\tau)\right) G(t-\tau) d\tau \right) k \qquad (6)$$

where the term $n\left(k^{\alpha}(\tau)\right)$ captures past (rather than current), income-related fertility, and $G(t-\tau)$ is the corresponding delaying kernel. As well known (MacDonald 1978), by assuming that the $G$ kernel belongs to the so-called Erlangian family[6], IDEs as (6) may be reduced to higher order systems of ordinary differential equations.

## 3.1 The model: equilibrium and dynamical analysis.

In economics the most frequent type of delaying kernel is based on the notion of exponentially fading memory (obtained for $r$=1 in the Erlangian family). However an exponentially fading memory, which implies to give maximal weight to the "more recent" past, cannot be considered a reasonable representation of the demographic mechanism. Rather the new entries in the labour supply at each moment $t$ of time are due to individuals born in a rather narrow interval of time in the past. We argue that "humped" memories, rather than exponential, give a satisfactory representation of the demographic process (i.e. past fertility plus delayed transition into the labour force). We therefore choosen as the delaying kernel the second member of the Erlangian family ($r$=2):

$$G(u) = \beta^2 u e^{-\beta u} \qquad u > 0 \ ; \ \beta > 0 \qquad (7)$$

because it is the simplest type of "humped" Erlangian density (having a mean delay $T=2/\beta$). This implies that the maximal weight is given to some appropriate time interval in the past, which heuristically well corresponds to

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[5]See Invernizzi and Medio (1991) for an economically oriented discussion of the role played by distributed delays.

[6]A density function $f(x)$ is erlangian with parameters $(r,\beta)$ when it has the form:

$$f(x;r,\beta) = \frac{\beta^r}{(r-1)!} x^{r-1} e^{-\beta x} \qquad x > 0 \ ; \ r = 1,2,\ldots \ ; \ \beta > 0$$

the idea of demographic delay.[7] Under (7) the IDE (6) may be reduced to an ODE system by the linear chain trick. This is obtained by introducing a set of $r$ suitable auxiliary variables, one of which is the integral term in (6), and the other arise by differentiation of the integral term. In our case we obtain, after the further change of variable $k^{\alpha}=Z$, the 3-dimensional ODE system:

$$\begin{aligned} \dot{Z} &= \alpha Z\left( sZ^{\frac{\alpha-1}{\alpha}} - nX \right) \\ \dot{X} &= \beta(R - X) \\ \dot{R} &= \beta(Z - R) \end{aligned} \qquad (8)$$

In a nutshell the procedure implies $r$ additional linear equations, where $r$ is the order of the (Erlangian) delaying kernel. In particular every additional differential equation represents an adaptive mechanism[8] . In our model, where $r$=2, the procedure thus leads to two additional linear equations both representing a (continuous time) adaptive mechanism. As in Solow's model (we recall that the equilibria of the underlying unlagged model are preserved, so that equation (5) holds with its interesting economic interpretation) we have the zero equilibrium $E_0$ and the positive equilibrium $E_1=(Z_1,X_1,R_1)$ where:

$$Z_1 = X_1 = R_1 = \left(\frac{s}{n}\right)^{\alpha} \qquad (9)$$

The local stability analysis of the positive equilibrium $E_1$ gives the third order characteristic polynomial: $P(X) = X^3 + a_1X^2 + a_2X + a_3$ with coefficients:

$$a_1 = 2\beta + (1-\alpha)nZ_1 \quad , \quad a_2 = 2\beta(1-\alpha)nZ_1 + \beta^2 \quad , \quad a_3 = \beta^2 nZ_1$$

which are all strictly positive. Therefore, by the Routh-Hurwitz stability test, $E_1$ will be locally stable provided the condition $\Delta_2=a_1a_2-a_3>0$ holds. This leads to the stability condition:

$$f(\beta) = 2\beta^2 + nZ_1(4-5\alpha)\beta + 2\left((1-\alpha)nZ_1\right)^2 > 0 \qquad (10)$$

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[7]The use of a "more humped" distribution in the Erlangian family (as erlangian densities of higher order, i.e. $r$=3,4,5...), which could perhaps more realistically capture the phenomenon of the delayed entry into the labour force, simply confirms the more important results of this paper (e.g. the onset of persistent oscillations as shown in next section). We did not investigate analytically the properties of higher order systems with higher order kernels because they become more and more analytically cumbersome without offering any extra "qualitative" insight from the economic point of view.

[8]For more technical details on the linear chain trick see Mac Donald (1978), Fanti and Manfredi (1998).

The discussion of (10) shows that: i) for $\alpha<\alpha_2=0.88$ no loss of stability is possible; ii) for $\alpha>\alpha_2$, $f(\beta)$ always has two strictly positive real roots, $\beta_1$, $\beta_2$ (with $\beta_1<\beta_2$), implying that losses of stability may occur. In particular both $\beta_1$, $\beta_2$ values represent Hopf bifurcation values of the $\beta$ parameter.[9] They are given by:

$$\beta_{1,2}=\frac{n^{1-\alpha}}{4}s^{\alpha}\left((5\alpha-4)\pm\sqrt{(9\alpha-8)\alpha}\right) \qquad (11)$$

The latter formula provides a nice relation between the demographic delay and the other economic parameters.

We may summarise our main findings by the following:

**Proposition 1**. When the profit share $\alpha$ is below a prescribed threshold value ($\alpha<\alpha_2$) the system (8) replies the behaviour of the original Solow's model, with convergence to the unique and globally stable equilibrium $E_1$. When $\alpha>\alpha_2$ the system (8) continues to converge to the globally stable equilibrium $E_1$ only when $\beta$ is sufficiently large of sufficiently small, i.e. for $\beta > \beta_2$, or $\beta<\beta_1$. In the whole window $\beta_1<\beta<\beta_2$ the $E_1$ equilibrium is locally unstable. At the points $\beta=\beta_1$. $\beta=\beta_2$, Hopf bifurcations occur.

In words: an endogenous labour supply according to the demographic mechanism adopted here is able to destabilise the equilibrium of the Solow's model and to generate persistent oscillations[10] via Hopf bifurcations.

## *4. Simulative evidence and working of the system*

The fact to know that a Hopf bifurcation exists nothing says about the stability properties of the involved periodic orbits, e.g. does not say whether the bifurcation is supercritical or subcritical, i.e. whether the periodic orbit is locally stable or unstable. Unfortunately the investigation of stability of periodic orbits emerged via Hopf bifurcation at dimensions greater than the

[9]The formal proof requires to show that: i) purely imaginay eigenvalues exist for the linearised system at $\beta=\beta_1$, or $\beta=\beta_2$, due to a "continuous" movement of a pair of complex eigenvalues; ii)the crossing of the imaginary axis by the involved complex pair occurs with nonzero speed. The proof, omitted here for brevity, is available on request.

[10]Of course this is not the only avenue through which to obtain population induced persistent oscillations in a neoclassical growth model: for instance, this could be obtained by a two-period overlapping generation model – rather than the Solovian one developed in our paper – including a model of adaptive expectation on fertility determination.

second is a hard task (Marsden and McCracken 1976). Moreover the predictions of the Hopf theorem are "local": they nothing say about global behaviours.

We therefore resorted to numerical simulation to clarify the stability nature of the Hopf bifurcations occurred at the points $\beta=\beta_1$, or $\beta=\beta_2$, and more generally to investigate the global properties of our model.

Let us now summarise our main dynamical findings and illustrate the working of the model. There is a region, defined by $\alpha<\alpha_2=0.88$, in which the traditional behaviour of Solow's model is confirmed: the economy converges to a long term (globally stable) steady state. Conversely, for very large $\alpha$ ($\alpha>\alpha_2$) the economy may be destabilised by the action of the demographic delay. More in detail, as long as $\beta$ is very large (in relative terms), i.e. for $\beta>\beta_2$ (we recall that this means "very" small mean delays), the $E_1$ equilibrium preserves its stability. But as $\beta$ is decreased (this happens for increasing mean delays) stability may be lost. This happens for $\beta=\beta_2$ where a first Hopf bifurcation occurs and $E_1$ exchanges its stability with a stable limit cycle. Finally, by further decreasing $\beta$, a further bifurcation occurs at $\beta=\beta_1$ where the local stability of the $E_1$ equilibrium is restored. Therefore for very large mean delays the Solow's behaviour is recovered once again. Table 2 reports a synoptic view of the process of phase transition of the positive equilibrium when $\alpha>\alpha_2$.[11]

*Table 1. Windows of the delay parameter $\beta$ and the corresponding behaviour of the model as summarised by the nature of its unique positive equilibrium $E_1$.*

| $\beta$ | $(0,\beta_1)$ | $[\beta_1,\beta_2)$ | $[\beta_2,+\infty)$ |
|---|---|---|---|
| $E_1$ | stable (node or focus) | unstable (surrounded by a stable limit cycle) | stable (node or focus) |

It is worth to remark the following further facts: i) both the points $\beta=\beta_1$, $\beta=\beta_2$, generate supercritical bifurcations (i.e. locally stable oscillations); ii) the whole window $\beta_1<\beta<\beta_2$ is a region of stable oscillations, iii) all the

[11]We notice that, although persistent oscillations seem to require a rather large value of the elasticity of capital $\alpha$ (even higher than those estimated by using a broad definition of capital stock including the human capital), this is just a feature of the low-order kernel ($r$=2) considered here for purposes of analytical simplicity. Indeed by resorting to higher order distributions of the delaying kernel (e.g. erlangian densities of order $r$=3,4,5...), which perhaps are even more realistic, we obtained, but only via simulation, more realistic "critical" values of the elasticity of capital.

properties of the model seem to hold *globally*: when the $E_1$ equilibrium is either locally stable, or surrounded by locally stable limit cycles, the system always seems globally stable.

Thus our results are consistent with two important stylized facts: 1) the existence of global stability of the economy, thus confirming Solow's result; 2) the presence of fluctuating behaviours around the state of balanced growth of the economy, which greatly enriches Solow's result.

Figs. 1a and 1b report a two-dimensional view of the emerging cycles respectively at the largest bifurcation point $\beta_2=0.064$, and at the smaller one $\beta_1=0.0056$ (other parameter values: $\alpha=0.92$, $s=0.3$, $n=0.01$).

*Fig. 1. a) A stable limit cycle appeared at $\beta=\beta_2=0.064$; b) a stable limit cycle appeared at $\beta=\beta_1=0.0056$*

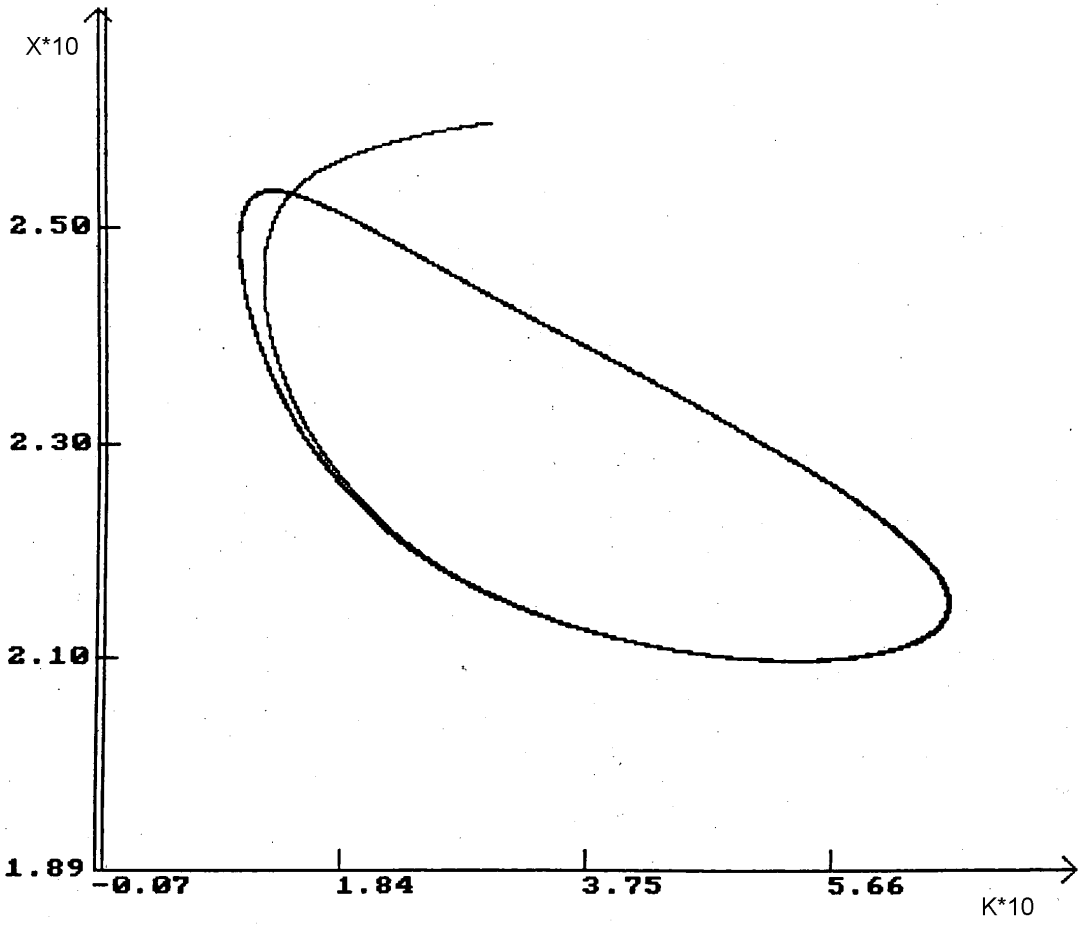


Fig. 1a

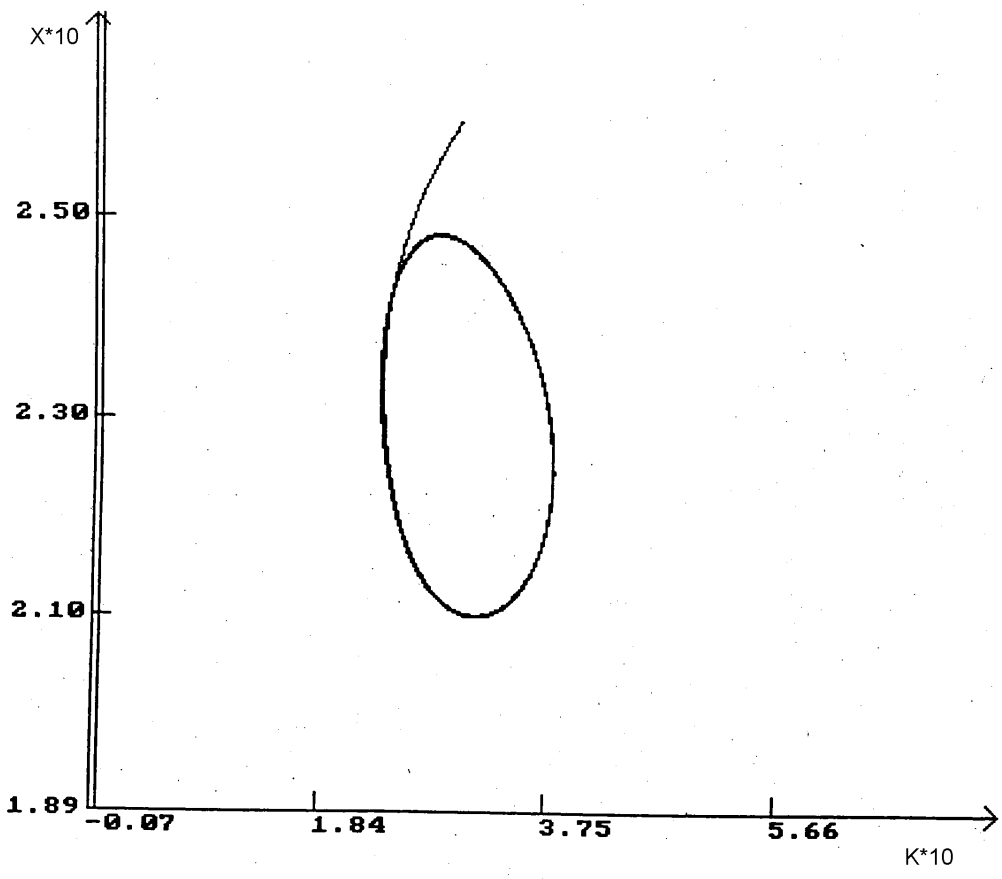


Fig. 1b

The economic interpretation of our findings is straightforward: when returns are "not too decreasing" (i.e. $\alpha > \alpha_2$), increasing the age of entrance in the labour force may destabilise the equilibrium of the Solow's model, and trigger persistent oscillations. These latter appear to be bounded, so that the global stability, in broader terms, is preserved. These fluctuations appear to be the outcome of the demographic memory operating through the age structure delay. The fact that periodic behaviours may persist also for very long mean delays seems to suggest the existence of possible "supergenerational" echoes, deriving from patterns ascribed to birth cohorts different from the last one.

## *5. Conclusions*

In this paper we have shown that the Solow's model, once extended to take account for: i) the existence of a delay in the process of recruitment in the labour force, due to the age structure process, ii) the existence of a Malthusian relation between past fertility and wage, is capable of generating stably fluctuating growth paths. This fact shows that the neoclassical growth paradigm not only explains the stylised fact of balanced growth, but, once endowed with a correctly demographically founded formulation of the labour supply, becomes capable to endogenously explain the other main stylised fact of economic growth, namely the generation of globally stable oscillations around a path of balanced growth. An interesting consequence of the presence of endogenous population is that, contrary to Solow's original result, where, as well known, saving determines the accumulation but does not play any role for long term growth, saving matters for (long term) growth. The main message is thus that it is only through saving, e.g. new investments, that population growth may eventually promote economic growth.

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