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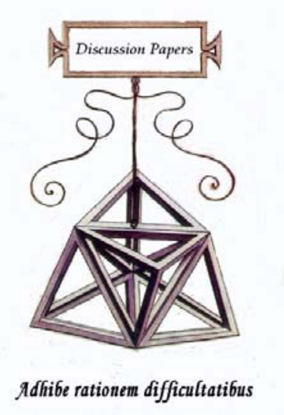

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Technological Innovation, Financial Fragility and Complex Dynamics

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Discussion Paper
n. 31

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Technological Innovation, Financial Fragility and Complex Dynamics

Abstract
Technological Innovation, Financial Fragility and Complex Dynamics

In this paper we suggest a scaling approach to business cycles. We develop a heterogeneous interacting agents (HIAs) model that replicates well known industrial dynamics stylized facts, as the power law distribution of firms' size and the Laplace distribution of firms' growth rates. In particular, the power law is a persistent but not time invariant feature of firms distribution. In order to account for the shifting behavior of the firms size power law distribution along business cycles, we propose a simple economic mechanism based on the interplay among R&D investment, technological innovation, wage dynamics and financial factors. Agent-based simulations show that power law shifts are a consequence of changes in firms' capital accumulation behavior due to technological progress and a wage – firm size relationship. We also find that the model simulation replicates important growth type stylized facts and a dynamic relationship between workers' wages and firms' profits.

JEL classification: C63, E32, O32
Keywords: business cycle, power-law distribution, agent-based model

Contents

1. Introduction

In this paper we develop a heterogeneous interacting agents (HIAs) model suitable to replicate well known industrial dynamics empirical regularities, as the power law distribution of firms' size (Okuyama *et al*., 1999; Ramsden *et al*., 2000; Axtell, 2001; Gaffeo *et al*., 2003) and the Laplace distribution of firms' growth rates (Stanley *et al*., 1996; Bottazzi and Secchi, 2003).

In general, in order to account for the scaling type stylized facts provided by the application of statistical physics concepts and tools to empirical evidence (see Mantegna and Stanley, 2000) we believe that economists have to adopt a methodological approach based on heterogeneous interacting agents (HIA), rejecting the *reductionist approach* centered on the *representative agent* hypothesis. In particular, two works have stressed the limits of this approach: Kirman (1992), from a theoretical point of view, and Stoker (1993), from an empirical perspective. In sum, the practice of combining heterogeneity and interactions is at odds with mainstream economics which reduces the analysis of the aggregate to that of a single representative agent and which is unable, by construction, to explain non-normal distributions, scaling behavior, self-similarity, self-organizing criticality (Bak, 1996) or the occurrence of large aggregate fluctuations as a consequence of small idiosyncratic shocks.

Starting from Delli Gatti *et al.* (2004a) and Gallegati *et al.* (2003), we have developed an agent-based model by extending the initial framework, in which a large number of firms (the financial fragility of which is proxied by their equity ratio) interact with a banking sector giving rise to complex dynamics, through the introduction of a labor-saving technological progress and a wage – firm size relationship.

In this model, discussing a scaling approach to business fluctuations, we are particularly interested in the analysis of the evolution and shifts of the distribution of firms' size. In fact, despite some work on this topic has been pursued in the last decade in physics, econophysics literature has sporadically dealt with such an issue. Scarce attention has been paid so far to establishing a link between power law shifts and the business cycle theory, mainly because mainstream economics lacks adequate conceptual and analytical tools to accomplish such an endeavor.

We provide a tentative interpretation of the shifting behavior of the power law distribution along business cycles based on the interplay among R&D investments, technological progress, wage dynamics, firms' productivity, and financial factors. In particular, we focus on the shifts of the production function towards the origin as an indicator of ongoing technological development, that is, a sequence of periodic arrival of innovations that leads to a permanent improvement in the production function (Schumpeter, 1939). Therefore, firms' productivity is proxied by the capital-labor ratio increasing over time due to labor-saving technological innovation. In addition, our agent-based model reproduces some of the growth type stylized facts provided by Kaldor (1961) and a Goodwin-like growth cycle (1967).

In general, our analysis suggests that there are significant changes in firms' distribution during different phases of the business cycle and that the power law scaling behavior, emerging as an invariant feature of size distribution of firms, is at the basis of the understanding of business fluctuations.

The model is presented in section 2. The model simulation and the discussion of the results are in section 3. Section 4 concludes.

2. The Model

The structure of the model can be divided into two parts: the real side and the financial side of the economy. The real (supply) side of the model – good market – is characterized by the behavior of firms that sell all output at a stochastic price and invest resources in the R&D activity with the aim to obtain innovations. The financial side – credit market – is constituted by a banking sector that, in presence of asymmetric information, allocates the supply of credit among firms on the basis of the collaterals they can provide.

2.1. The good market

Firms produce a homogeneous good by means of the following production function

[1] $$Y_{it} = \phi K_{it} \qquad (i=1, \ldots, F \text{ and } t = 1, \ldots, T)$$

where K_{it} is the stock of capital of the i-th firm, ϕ is the productivity of capital, constant and uniform across firms, F is a *large* number of firms and T is the length of the period of time considered.

In order to produce the output, firms need a given amount of labor N_{it} depending on its capital-labour ratio

[2] $$\lambda_{it} = \frac{K_{it}}{N_{it}}$$

Consequently, each firm has a requirement labor function $N_{it} = K_{it} / \lambda_{it}$.[1] There are no constraints on the labor market, that is, firms can hire (and fire) all workers they need at the wage:

[3] $$w_{it} = \rho\left(\delta K_{it}^{\varepsilon}\right) + (1-\rho) w_{it-1}$$

where $0 < \rho < 1$, $0 < \delta < 1$, and $0 < \varepsilon < 1$. We simply assume that there is a wage-size relation[2] combined with an adaptive term in the setting of the wage paid to workers.[3]

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[1] Alternatively, we can see the production function in the following way : $Y_{it} = \phi \lambda_{it} N_{it}$

[2] Many empirical studies have found the existence of a strong positive relationship between employer's size and wages, emphasizing different aspects of wage formation – labor quality, efficiency wages, etc. – and institutional factors – working conditions, the role of unions, etc. Brown and Medoff (1989), in their seminal paper, find a positive and significant employer's size – wages effect using US data. A possible theoretical explanation is related with the role of unions: larger firms are subject to a higher union influence about wage determination with respect to smaller firms. We simply assume this wage – firm size relationship instead of reproducing it by means of our agent-based model. This is only a first step toward a more complete model (see the following footnote).

Due to limited knowledge of market conditions, firms sell their output at an uncertain (relative) price u_{it} , where u_{it} is a an i.i.d. idiosyncratic shock.
The balance sheet of the firm is:

$$K_{it} = A_{it} + L_{it} \tag{4}$$

where A_{it} is the equity base and L_{it} is the demand for credit.
The firm's profit is equal to:

$$\pi_{it} = u_{ti} Y_{ti} - r_{ti} K_{it} - w_{it} N_{it} = u_{it} Y_{it} - \left(r_{it} + \frac{w_{it}}{\lambda_{it}} \right) K_{it} \tag{5}$$

Firms invest a portion of retained profits in R&D activity with the aim to obtain innovations in the upcoming periods:

$$RD_{it} = \begin{cases} \sigma \pi_{it-1} & if \quad \pi_{it-1} > 0 \\ 0 & otherwise \end{cases} \tag{6}$$

where 0 < σ < 1 .[4]
Consequently, profits after R&D expenditure are

$$\pi'_{it-1} = \pi_{it-1} - RD_{it} = (1-\sigma)\pi_{it-1} \tag{7}$$

Firms' technological level z_{it} enhances due to a Poisson distributed process depending on the R&D investments and to the possibility to imitate other firms.
Accordingly, the evolution of technology due to the internal innovation activity done by firms is given by

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[3] A further improvement of the model is a matching mechanism between firms and workers to jointly determine wage and employment levels, in a way that allow us to explain the wage-size effect as one of the emergent properties of the model.

[4] This implies that the initial equity base of the firm is equal to $A_{it} = A_{it-1} + (\pi_{it-1} - RD_{it})$.

[8] $$z'_{it} = \begin{cases} z^P_{it} & if \quad \pi_{it-1} > 0 \\ 0 & if \quad \pi_{it-1} \leq 0 \end{cases}$$

where z^P_{it} is the number of innovations at time t for the firm i, that is, the realization of a Poisson process with mean $\mu_{it} = RD_{it-1} / K_{it-1}$.[5]

The imitation process is simply characterized by a mean-interaction term, that is, by a term proportional to the average technological level of firms. Then, the level of technology due to imitation is equal to

[9] $$z''_{it} = \nu \bar{z}_{t-1}$$

where $\bar{z}_{t-1}$ is the average technological level in the past period and $0 < \nu < 1$. The technological level of the *i*-th firm at period *t* is equal to

[10] $$z_{it} = z_{it-1} + z'_{it} + z''_{it}$$

Finally, the capital-labor ratio is a function of the technological level :

[11] $$\lambda_{it} = \gamma z_{it-1}$$

where $0 \leq \gamma \leq 1$.

Each firm maximizes an objective function:

[12] $$\Gamma_{it} = E(\pi_{it}) - E(BC_{it})$$

where $E(\pi_{it})$ is the expected profit and $E(BC_{it})$ is the expected bankruptcy cost. We assume a quadratic functional form for the bankruptcy cost:

[13] $$BC_{it} = cY^2_{it}$$

[5] Accordingly to this relation, the effect of R&D investments on innovations is scaled by a factor K_{it-1}.

A firm goes bankrupt if the net worth becomes negative. Then, the bankruptcy condition is [6]

[14] $$A_{it+1} = A_{it} + \pi_{it} < 0$$

Substituting [5] in [14], we obtain

[15] $$u_{it} < \frac{(r_{it} + w_{it} / \lambda_{it})K_{it} - A_{it}}{Y_{it}}$$

where

[16] $$\bar{u}_{it} \equiv \frac{(r_{it} + w_{it} / \lambda_{it})K_{it} - A_{it}}{Y_{it}}$$

is the critical value for the relative price of a firm, below that bankruptcy occurs. For the sake of simplicity, we assume that u_{it} is a uniformly distributed variable with support (0,2); consequently, the probability of bankruptcy is

[17] $$BP_{it} = \frac{(r_{it} + w_{it} / \lambda_{it})K_{it} - A_{it}}{2Y_{it}}$$

and the expected bankruptcy cost is

$$E(BC_{it}) = \frac{c}{2}\phi K_{it}\left[\left(r_{it} + \frac{w_{it}}{\lambda_{it}}\right)K_{it} - A_{it}\right]$$

After that, the firm's objective function becomes:

[6] Remember that the initial equity base is $A_{it} = A_{it-1} + \pi'_{it-1}$ because a fraction of retained profits obtained in period *t*-1 is invested in R&D activity (see footnote 3).

$$[18] \qquad \Gamma_{it} = \phi K_{it} - r_{it} K_{it} - \frac{w_{it}}{\lambda_{it}} K_{it} - \frac{c}{2} \phi K_{it} \left[\left(r_{it} + \frac{w_{it}}{\lambda_{it}} \right) K_{it} - A_{it} \right]$$

From the maximization of the objective function, we obtain the optimal capital stock

$$[19] \qquad K_{it}^{*} = \frac{\phi - r_{it} - w_{it} / \lambda_{it}}{c\phi \left(r_{it} + w_{it} / \lambda_{it} \right)} + \frac{A_{it}}{2\left(r_{it} + w_{it} / \lambda_{it} \right)}$$

Investment is equal to

$$[20] \qquad I_{it} = K_{it}^{*} - K_{it-1}$$

The demand for credit is:

$$[21] \qquad L_{it}^{d} = L_{it-1} - \pi_{it-1} + I_{it} = K_{it} - A_{it}$$

Finally, substituting [19] in [21] we have the following relation for firms' loans:

$$[22] \qquad L_{it}^{d} = \frac{\phi - r_{it} - w_{it} / \lambda_{it}}{c\phi \left(r_{it} + w_{it} / \lambda_{it} \right)} + \frac{\left(1 - 2\left(r_{it} + w_{it} / \lambda_{it} \right) \right) A_{it}}{2\left(r_{it} + w_{it} / \lambda_{it} \right)}$$

2.2. The credit market

The banking sector is modeled as in Gallegati *et al.* (2003). Then, there is a bank[7] that allocates the total supply of credit among firms in function of the relative size of firms:

[7] We can interpret the only bank in the model as a vertical integrated banking sector.

[23] $$L_{it}^{s} = L_t \left(\frac{K_{it-1}}{K_{t-1}} \right)$$

where L_t is the total supply of credit at time t, $K_{it\text{-}1}$ is the capital of the i-th firm, and $K_{t\text{-}1}$ is the aggregate stock of capital in the precedent period of time. This rule of credit allocation is a way to face with asymmetric information in the credit market: the bank does not know the *true* financial conditions of the heterogeneous borrowers and uses collaterals, proxied by the capital stock of the firm relative to the aggregate stock of capital, to determine the individual supply of credit.

The supply of credit is vertical (it is independent of the interest rate) at the level

[24] $$L_t = \frac{E_{t-1}}{\alpha}$$

where a is a coefficient of risk (e.g., a prudential rule set up by a regulatory institution) that the bank has to respect and $E_{t\text{-}1}$ is the equity base of the bank in the previous period of time.

The balance sheet of the bank is

[25] $$L_t = E_t - D_t$$

where D_t are deposits.

The bank's equity base is equal to

[26] $$E_t = E_{t-1} + \Pi_t - \sum_i B_{it-1}$$

where Π_t is the bank's profit and B_{it} is the *bad debt* of a bankrupted firm, that is

[27] $$B_{it} = \begin{cases} -A_{it} & if \quad A_{it} < 0 \\ 0 & if \quad A_{it} \geq 0 \end{cases}$$

The bank's profit is

$$\Pi_t = \sum_i r_{it} L_{it} - (1-\omega)\bar{r}_t D_{t-1} - \bar{r}_t E_{t-1} \qquad [28]$$

where $\bar{r}_t$ is the average interest rate and ω is the mark-up for the bank.
Finally, the individual rate of interest is endogenously determined when $L_{it}^d = L_{it}^s$, that is, when [22] is equal to [23]:

$$r_{it}^* = \frac{2 + cA_{it}}{2c\left[L_t\left(\frac{K_{it-1}}{K_{t-1}}\right) + \frac{1}{c\phi} + A_{it}\right]} - \frac{w_{it}}{\lambda_{it}} \qquad [29]$$

3. *Simulating the model*

We simulate an artificial economy in which operate F = 1000 firms and a banking sector under the assumption that if a firm goes bankrupt it is replaced by a new firm (with initial conditions), so that F is fixed. [8]
As we can see in Fig. 1, the aggregate output fluctuates showing phases of smooth growth and periods of large variability; [9] in addition, sudden drifts and different slopes appear time to time.[10]

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[8] The parameters' values and the initial conditions are illustrated in Appendix A.

[9] All figures presented in the following are relative to simulations from period 1501 to period 3000. The first part of the simulation is considered as a transition phase.

[10] For a discussion of other important features of the model (without considering technological progress) see Delli Gatti *et al.* (2004a,b) and Gallegati *et al.* (2003).

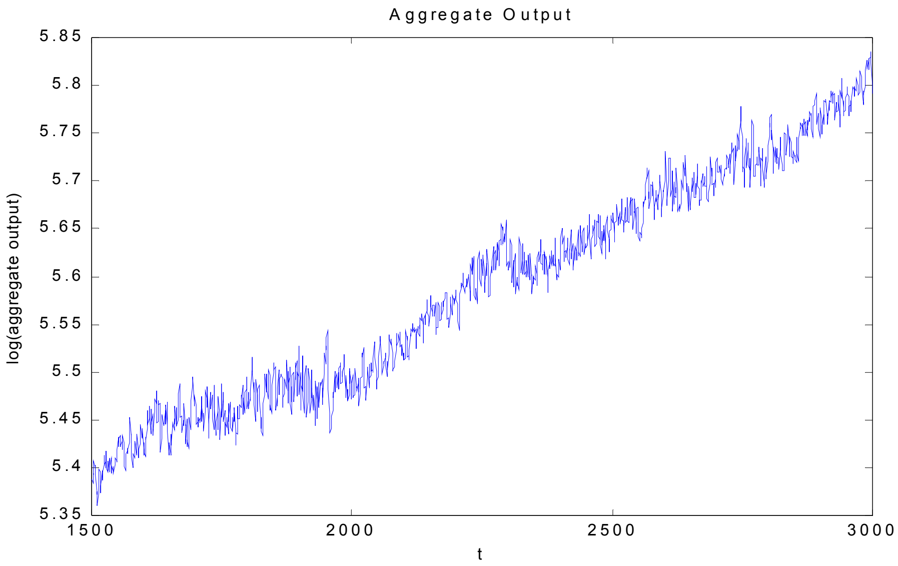

Fig. 1 – Aggregate Output

The growth process is due to the growth of firms' size and to productivity enhancements. Note that we model a supply driven economic system in which all output produced by firms is demanded at a stochastic price. Then, the growth of firms due to investment choices and financial factors has no (quantity) constraints from the demand side, even if the volatility of prices has important consequences on firms' dynamics.

In particular, it is important to note that a domino effect through a balance sheet contagion may develop because of firms' bankruptcies. In fact, when a firm goes bankruptcy it leaves the market and it does not pay back the debt to the bank. Consequently, the bank has a *bad debt* and the total supply of credit reduces producing an increase of the interest faced by surviving firms. Since debt commitments rise, firms' insolvencies increase even more, thus self-reinforcing this vicious circle.

In addition, firms' growth is due to technological progress, that is, labor-saving innovations due to R&D investments. In fact, when a firm obtains an innovation it can produce the same output with a smaller amount of the labor input. Then, it can accumulate more capital and grow faster with respect to firms that have not innovated. In particular, we focus on the shifts of the production function towards the origin as an indicator of ongoing

technological development (that is, labour-saving innovations that allow to produce the same output using less input), instead of analysing the shifts along the production function due to factor substitution (Schumpeter, 1939).[11]
The capital-labor ratio grows along time due to a diffused technological progress that saves labor inputs in the production process. We can see the time evolution of this ratio in Fig. 2. The model simulation reproduces some important growth type stylized facts (Kaldor, 1961):

- the capital-labor ratio (fig. 2) and the output-labor ratio increase over time (due to labor-saving technological progress);
- the capital-output ratio is constant; [12]
- the investment-output ratio is roughly constant;
- the rate of return on capital is roughly constant;
- the real wage increases over time;
- the relative share of capital and the relative share of labor are roughly constant.[13]

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[11] Here we are analyzing only the effect of a diffused innovation process on the labor quantity used in the production process. A further improvement of the model relates with the possibility to model the monopolistic power of a firm that can diminish the price of the output as a consequence of an innovation, that is the competitive advantage of an innovation for a firm that can sell its output at a price lower than that of the competitors'.

[12] By construction, see eq. [1].

[13] This result holds only for a particular setting of the model's parameters. As we will see in the following, the relative share of labor is equal to the ratio between the real wage and the labor productivity; this ratio oscillates as a consequence of the wage-profit dynamics. It is possible to calibrate the model in order to have a stationary wage-productivity ratio. If this is not the case, one of the two classes (workers or capitalists) dominates the other one in the long run, that is the relative share of capital (labor) increases over time while the relative share of labor (capital) diminishes.

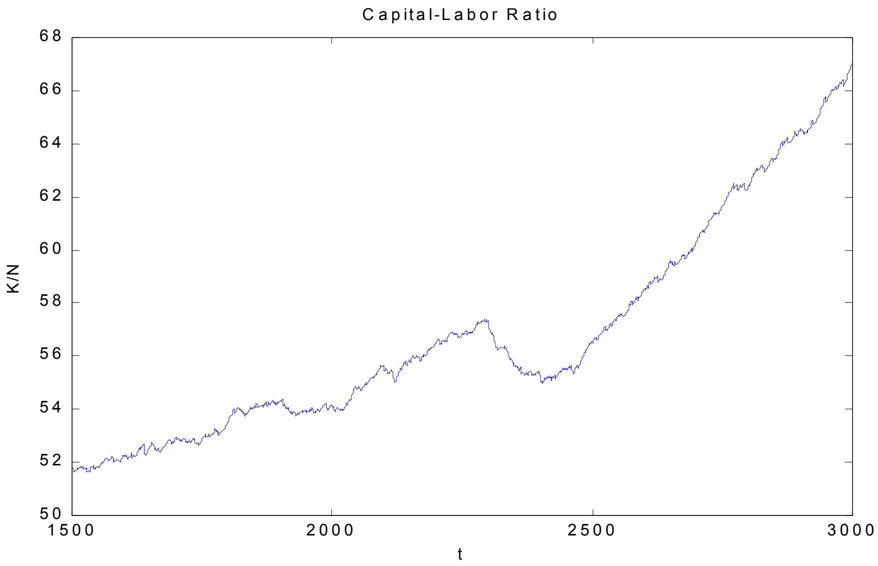

Fig. 2 – The time evolution of the capital-labor ratio of the economy

Firms are characterized by an asymmetric distribution of capital-labor ratios (Fig. 3). Since productivity improvements are due to an incremental innovation process and to an imitation term, older firms that have had positive profits for many periods of time are more likely to have higher capital-labor ratios with respect to young firms with no R&D experience and a limited time to imitate others. Firm size distribution (FSD) is right skew and it is distributed according to a power law (Fig. 4 and 5). Then, the model simulation well replicates a stylized fact that the empirical literature on industrial dynamics has recently highlighted (Okuyama *et al.*, 1999; Ramsden *et al.*, 2000; Axtell, 2001; Gaffeo *et al.*, 2003).

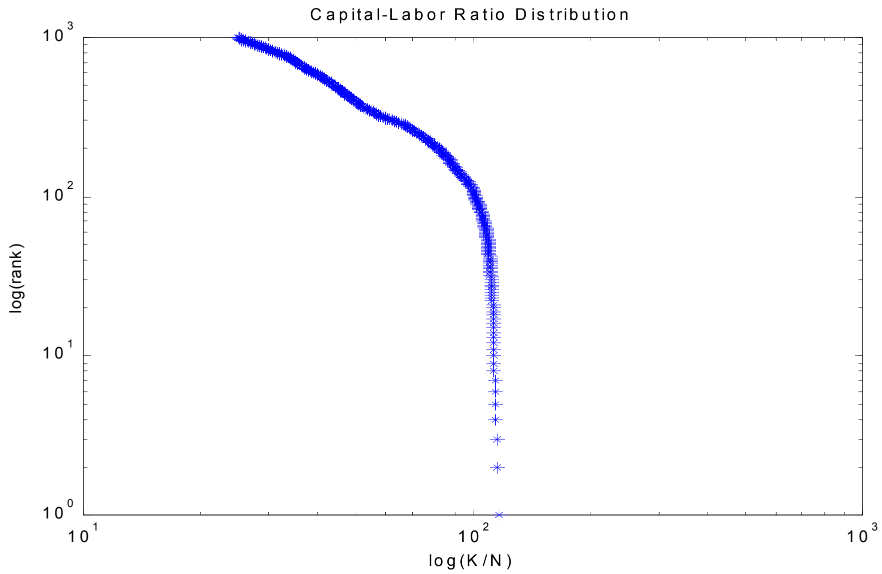

Fig. 3 – The distribution of the capital-labor ratio across firms

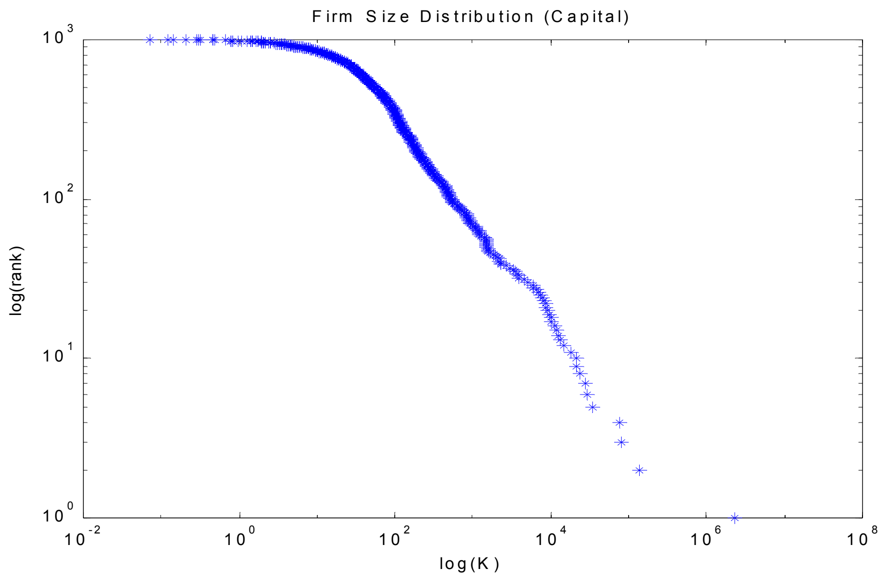

Fig. 4 – Power law distribution of firms' size (proxied by capital)

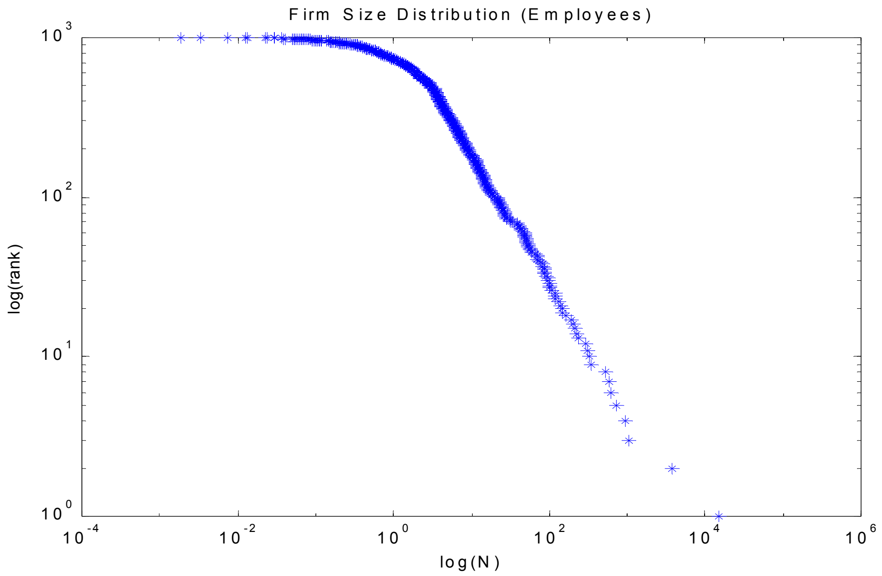

Fig. 5 – Power law distribution of firms' size (proxied by number of employees)

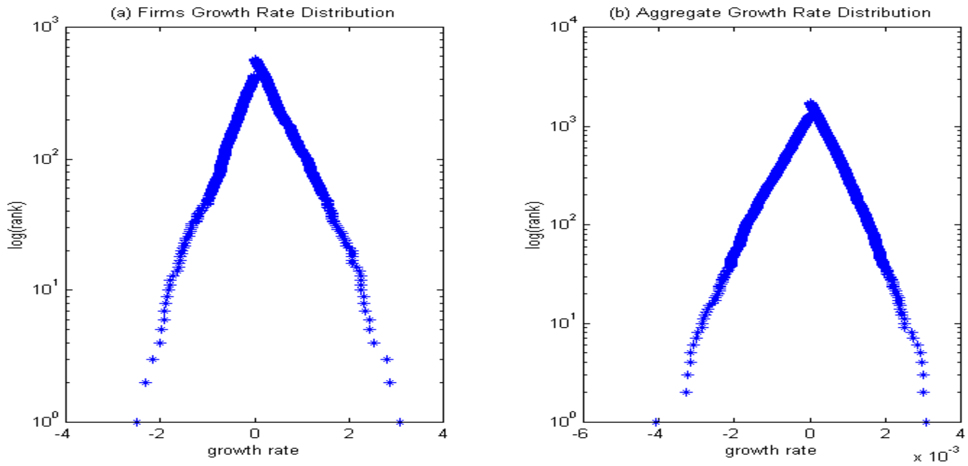

Fig. 6 – Laplace distribution of (a) firms' growth rates, and (b) aggregate growth rates

Moreover, Gaffeo *et al*. (2003) find that there are significant shifts of the FSD during different phases of the business cycle. In other terms, power law is a persistent but not time invariant feature of the FSD. In the following, we will show that our agent-based model is able to replicate also the shifting behavior of the FSD.

Another important stylized fact (Stanley *et al*., 1996; Bottazzi and Secchi, 2003) is about the firms' growth rates that follow a Laplace distribution (Fig. 6a). Aggregate output growth rates are also tent-shaped (Fig. 6b). Interestingly enough, simulations show that behavior of greatest units (the industrial sector) reproduces the behavior of smaller units (firms) (Lee *et al*., 1998). [14]

In order to obtain an explanation of the shifting behavior of the FSD, we will propose a simple economic mechanism based on the interplay among R&D investment, technological innovation, firms' productivity, and wage dynamics.

In this model we assume a firm size – wage relationship as a simply way to determine the wage that firms pay to workers (jointly with an adaptive term).[15] Since there is a capital growth in the economy, the average wage level increases in time.

Let's analyze the joint behavior of wage levels and productivity dynamics, given that there are different implications for firms, depending on the size and the capital-labor ratio. In particular, we examine the behavior of the ratio between the average wage paid to workers and the labor productivity. Fig. 7 shows that the wage-productivity ratio fluctuates and presents many cycles of different length. Clearly, an increment (decrement) in the ratio can be due to an increase (decrease) in the average level of wage or to a decline (increase) in firms' productivity.

The typical shape of a business cycle that we analyze has the following structure. Firms accumulate capital due to technological progress that allows the production of the same output using less quantities of labor as input (e.g., labor productivity increases). The growth of firms' size implied by labor-saving innovations and financial factors, generates an increase of wages, due to the wage-firm size relationship and a shift towards north-east of the firms' size power law distribution in the double logarithmic space (from the

[14] For a general comparison between simulation and empirical data relative to scaling, industrial, financial, and business cycle stylized facts, see Delli Gatti *et al.* (2004c).

[15] See equation [3].

distribution A to the distribution B in Fig. 8). Since the wage-productivity ratio increases in this phase of the cycle, wage levels grow faster than firms' productivity (firms' productivity increases along all the cycle, as we can see in fig. 9). This process continues until the wage level reaches the peak of the cycle, after that the capital size of firms starts to diminish. [16] Consequently, wages decrease and the power law moves towards south-west (from B to C in Fig. 8). [17]

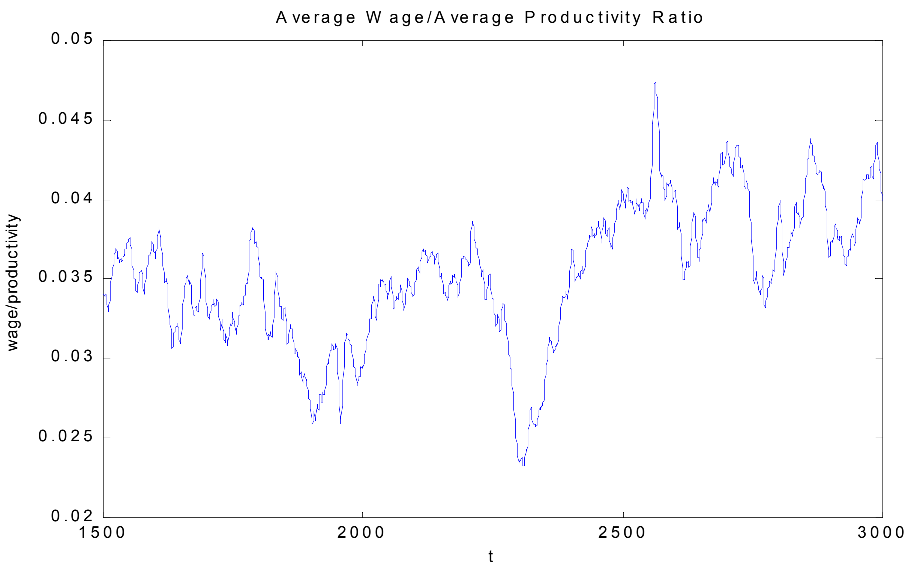

Fig. 7 – Joint evolution of average wage and productivity

The fluctuating behavior of the wage-productivity ratio suggests that in the model there is also a Goodwin-like growth cycle at work, that is a cyclical relationship between workers' wage and firms' profit. In fact, we can see the wage-productivity ratio as equivalent to the relative share of labor (see Fig. 7).[18]

[16] In the following we provide a Goodwinian interpretation of this fact.

[17] In addition, shifts of the firms' size power law can also affect the slope of the distribution. See Delli Gatti *et al*. (2004b) on different slopes of power law in expansions and recessions.

[18] The relative share of labor is equal to wN / Y, where wN is the wage bill (w is the wage and N the number of employees) and Y is the output. We can consider it in the following way: $w / (Y / N)$, where Y/N is the labor productivity.

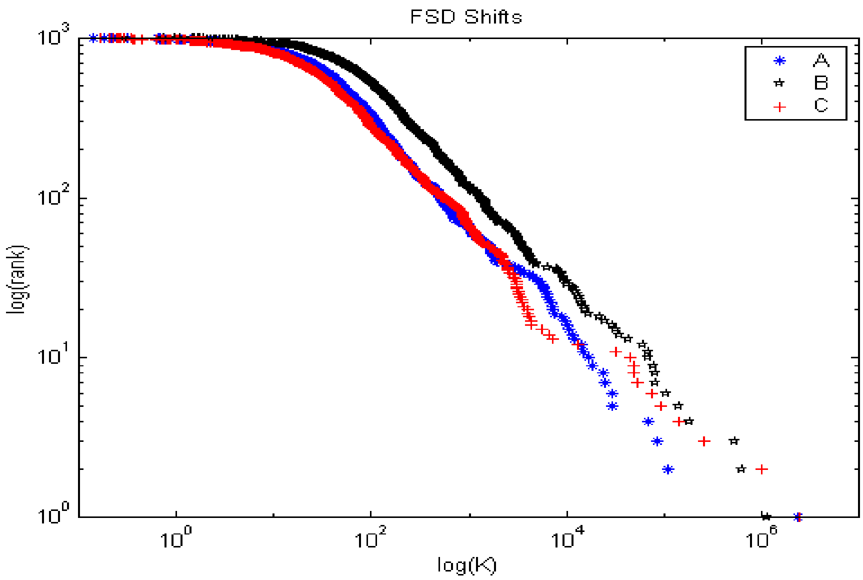

Fig. 8 – Power law shifts along the business cycle. The distributions are relative to periods: 1903 (A); 2210 (B); 2305 (C). In Fig. 7 these three periods correspond to the begin (A), the peak (B) and the end (C) of a business cycle.

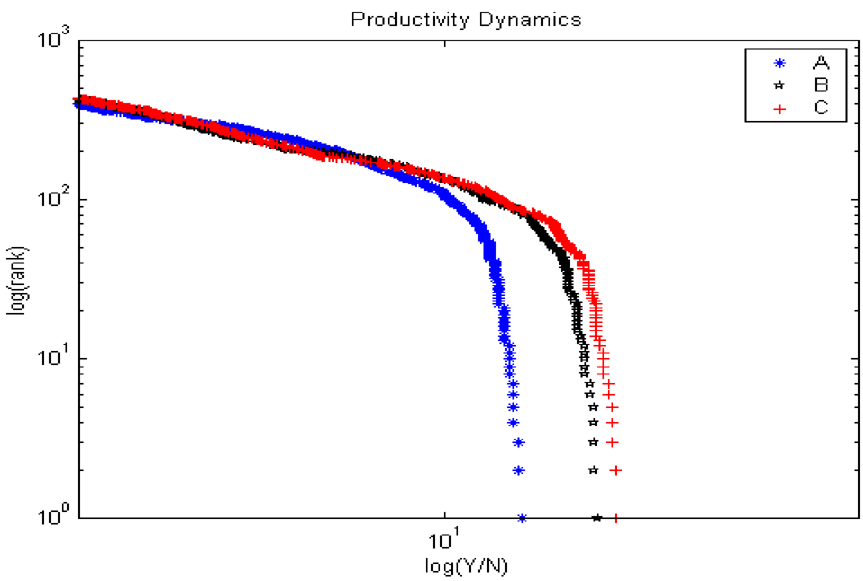

Fig. 9 – Firms' productivity shifts.

What happens in the model is that firms accumulate capital (due to technological and financial factors) and, because of a wage – firm size relationship, growing firms pay higher wages. In other terms, firms' capital accumulation increases the labor demand and, at the same time, the wage levels.[19] Consequently, the relative share of labor increases, while firms' profits and investments diminish. This imply a lower capital accumulation that generates a decrease of wages and thus a decline of the relative share of labor, producing the condition for the capital accumulation to re-start.

4. *Conclusion*

In this paper we propose a heterogeneous interacting agents (HIAs) model in which a large number of financially fragile firms interact with a banking sector giving rise to complex dynamics. In particular, we focus on the role of technological progress and its effects on firms' dynamics. We propose a simple economic mechanism based on the interplay among R&D investments, technological progress, wage dynamics, firms' productivity, and financial factors, providing a tentative explanation of the shifting behavior of the firms' size distribution (FSD) along business cycles. Assuming a wage – firms size relationship and considering that firms obtain productivity enhancements by means of labor-saving innovations, we find that FSD shifts are linked to the co-movement of wages and labor productivity. We find that the model simulation also replicate important growth type stylized facts (Kaldor, 1961) and a dynamic relationship between workers' wages and firms' profits (Goodwin, 1967).

[19] In other terms, if we consider that larger firms are subject to a higher union power with respect to the smaller ones, the wage – firm size relationship that we assume in the model implies that the bargaining power of workers improves when firms' size increase.

Appendix A: Parameters setting and initial conditions.

The parameters values used in the numerical simulations of the model are showed in the following. Note that we have verified the robustness of the model simulation by means of Monte Carlo experiments. Simulations showed very similar qualitative results under alternative configurations of the parameters setting and different initial conditions. In particular, even though the quantitative outcomes can vary in different simulations, the scaling properties of the model are present in very different scenarios, suggesting that this characteristic is an intrinsic feature of a model showing complex dynamics.

(i) Firms' specific parameters:
$\phi = 0.1$ (productivity of capital); $c = 1$ (bankruptcy function parameter); $\sigma = 0.5$ (weight in the wage equation); $\delta = 0.01$ (wage equation parameter); $\varepsilon = 0.5$ (wage equation parameter); $\nu = 0.001$ (imitation coefficient); $\gamma = 1$ (technological level vs. capital-labor ratio); $\sigma = 0.075$ (percentage of retained profit invested in R&D).

(ii) Firms' initial conditions:
$A_{i0} = 20$ (equity base); $L_{i0} = 80$ (loan); $B_{i0} = 0$ (bad debt); $\lambda_{i0} = 25$ (capital-labor ratio); $w_{i0} = 0.1$ (individual wage).

(iii) Bank's specific parameters:
$\alpha = 0.25$ (risk coefficient); $\omega = 0.01$ (mark-up).

(iv) Bank's initial conditions:
$L_0 = \Sigma L_{i0} = 8000$ (total supply of credit); $E_0 = \alpha L_0 = 2000$ (equity base); $D_0 = L_0 - E_0 = 6000$ (deposits); $\Pi_0 = 0$ (profit).

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