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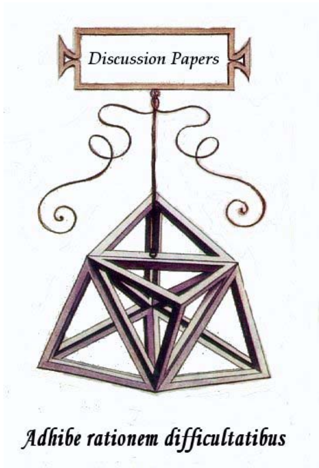

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Discussion Paper n. 36

2004

Discussion Paper n. 36, presentato: **settembre 2004**

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Si prega di citare così: Luciano Fanti - Luca Spataro, THE OPTIMAL FISCAL POLICY IN AN OLG MODEL WITH ENDOGENOUS FERTILITY, Discussion Papers del Dipartimento di Scienze Economiche – Università di Pisa, n. 36 (http://www-dse.ec.unipi.it/ricerca/discussion-papers.htm).

THE OPTIMAL FISCAL POLICY IN AN OLG MODEL WITH ENDOGENOUS FERTILITY

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Abstract

In this paper we show that, when endogenous fertility choices are accounted for, the traditional rule provided by Diamond (1965) should be amended in order to effectively implement the first best allocation of an OLG economy, even in the presence a non distortionary tax for financing national debt. With Cobb-Douglas preferences and fixed costs for rearing children, it turns out that the implementable equilibrium is in general suboptimal and dynamically inefficient (i.e. with overaccumulation of physical capital). The reason for this result is that, when fertility choices are taken as endogenous, a further policy instrument is necessary for controlling it: in this respect, by combining a lump sum tax upon the young adult with a subsidy for each child or with a lump sum trasfer to the old, the first best allocation can be implemented.

J.E.L. classification: D91, E62, H63, J13.

Keywords: Overlapping Generations, endogenous fertility, dynamic inefficiency, debt.

1 Introduction

The recognition that fertility choices depend, at least partially, on economic variables and the endogenization of such choices in economic models have, on the one hand, delivered a number of new results which have significantly modified some longly established propositions of economic theory and, on the other hand, have raised new issues, the analysis of which can improve our understanding of the functioning of modern economies.

Fruitful applications have been, among others, the analysis of the relationship between endogenous fertility choices and the optimality of Social Security systems (Zang and Nishimura (1992 and 1993), Cigno (1993), Rosati (1996) and Lagerlöf (1997)), of family (redistributive) taxation (Balestrino et al. (2002),

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Cigno et al. (2003), Balestrino (1997)). In particular, a key feature common to these contributions is the fact that children are important not only to their own parents, but also for society at large, via either positive externalities or redistributional issues due to comparative advantages to some families in raising children. In any of these situations there is room for corrective, Pareto improving public intevention interfering with parental fertility choices.

Other relevant fields of investigation of the role of endogenous fertility have been those focusing upon demographic transitions (Galor and Weil (1996)), on economic growth (Barro and Becker (1989), Barro and Sala-i-Martin (1999), Boldrin and Jones (2002)) and on poverty issues (Dasgupta (2000)). Although promising, these studies represent still pioneering contributions whose implication are far from being fully investigated.

By adopting an OLG framework, in this work we aim at providing a new insight into the effects of endogenous fertility on the shape of optimal fiscal policy designs relying on debt issuing. As fairly known, OLG economies can experience dynamic inefficiency (DI), that is overaccumulation of capital relative to the level which would generate the maximization of steady state consumption (see Samuelson (1958) and Diamond (1965))[1]. This occurrence, which invalidates the First Theorem of Welfare for perfectly competitive OLG economies, may well apply even to the simplest scenario in which only savings are endogenous and government is absent. However, the introduction of public debt, as pioneered by Diamond (1965), can correct such situation by crowding out the steady state level of savings and, thus, of capital.

Dispite the population growth rate is clearly crucial in determining dynamic inefficiency, so far it has been usually assumed as exogenous. A notable exception is represented by Wildasin (1990) in which, with altruistic individuals and endogenous fertility, the public debt neutrality turns out to fail. Although related, our work departs from the latter in that it aims at addressing the following questions: When the number of children is a choice variable 1) Under which conditions an optimal level of debt (that is correcting DI) does exist and what are the determinants of such optimal level? 2) What are the conditions under which the Pareto optimal policy can be effectively implemented by the policymaker? The analysis shows that the introduction of endogenous fertility changes significantly the results delivered so far by the literature on the optimality of debt management and of fiscal policies.

More precisely, we show that, when preferences, for simplicity, are of the Cobb-Douglas type and rearing costs per children are constant: 1) the existence of an optimal positive amount of public debt is favoured, among other things, by a sufficiently low preference for children; moreover, such level is higher the

[1] Such level of consumption and capital accumulation path are usually referred to as "Golden Rule" values. When the maximization problem concerns a "Social Welfare function", rather than steady state consumption, the solutions are called the "Modified Golden Rule" allocations. As for the most recent empirical evidence on such issue, see the contributions by Abel et al. (1989) and Anderson (1993): in fact, while the first work confirms that dynamic efficiency has been satisfied by the U.S. economy and other six developed countries, the second study casts doubt on such conclusion as for the U.S., Canada and Great Britain.

bigger the child rearing cost and the lower the preference for children; 2) when the number of children is endogenous, the first best solution is in general not implementable in a decentralized economy in presence of a single policy instrument (i.e. lump-sum tax on the young adult); as a consequence, Diamond's (1965) is no more necessary nor sufficient for correcting suboptimality (i.e. DI). In this case, in fact, only a "Pareto suboptimal" allocation is achievable, in which, however, the overaccumulation problem still persists. We explain this somehow puzzling outcome by the lack of an independent policy instrument: in fact, since the endogenous variables in this model are two (savings and number of children) two are also the instruments necessary to control them so as to implement the Pareto optimal allocation. In this respect we show that both a subsidy for each child or a lump-sum subsidy to the elderly can successfully serve the scope.

The paper is organized as follows: in the section 2 we lay out the basic framework and then we look at the effect of the introduction of debt upon the steady state level of capital. Next, after characterizing the allocation of resources, and in particular of capital, stemming from the Golden Rule, in section 3 we analyze the existence and the determinants of the optimal positive level of debt. Finally, in section 4 we show the failure of Diamond's rule and assess the conditions under which a Pareto optimal allocation can be effectively implemented by the policymaker in presence of endogenous fertility choices.

2 The model set up

2.1 Individuals

Following a standard way to endogeneize fertility in an OLG framework (e.g. Strulik (1999) and (2003)) life is separated into three periods: childhood, young adulthood, and old–age. During childhood, individuals do not make any decisions. Young adult individuals belonging to generation, say, $t-1$, have an utility function U, defined over c_{1t}, c_{2t+1}, n_t, that is consumption in the first and second period of adulthood and number of children, respectively: thus, in such a period of life agents, who receive a working income w_t, choose their optimal intertemporal allocations of consumption and fertility. By assuming for simplicity that every single young adult can have children, it is easy to see that the population at the steady state will be stationary or increasing if n is equal or bigger than 1 respectively (with $n-1$ being the long run growth rate of the economy as well). Moreover, rearing a child requires a fixed cost, e[2].

2.2 Firms

Firms own a constant returns to scale production technology $F(K_t, L_t)$ by which they transform physical capital (K_t) and labor (L_t), into the consumption good.

[2]This assumption departs from Strulik ((1999) and (2003)) who assumes the rearing cost as a fixed fraction of w.

We assume a perfect competitive market: thus, firms hire capital and labor by remunerating them according to their marginal productivity. Moreover, due to the homogeneity of degree one of F, it follows that $w_t = f(k_t) - f'_k(k_t)\,k_t$ and $r_t = f'_k(k_t)$ (in the case of absence of depreciation) or $r_t = f'_k(k_t) - 1$ (in the case of full depreciation), where low letters (apart from factor prices) indicate variables expressed in per worker terms and the subscript of the derivative function f' indicates the derivation variable. In Appendix A, however, we show that the steady state level of capital is independent of the assumptions on the production side and on the market structure.

2.3 Government

Now, following Diamond (1965), suppose that the government at each date t issues an amount B_t of national debt and levies lump sum taxes upon the young adults, according to the ordinary dynamic equation: $B_{t+1} = B_t(1 + r_t) - \tau_{1t}N_{t-1}$ (where τ_{1t} is the lump sum tax) which, by reckoning that $N_{t-1} = L_t$, in per worker terms is:

$$b_{t+1}n_t = b_t(1 + r_t) - \tau_{1t}; \tag{1}$$

Finally, by following Diamond (1965), we make the assumption that the debt in per worker terms be constant, so that the level of taxes, τ_{1t}, is equal to $b(1 + r_t - n_t)$.

2.4 Decentralized solution

The young adults solve the following maximization problem[3]:

$$\max U(c_{1t}, c_{2t+1}, n_t) = b_1 \log(c_{1t}) + b_2 \log(c_{2t+1}) + b_3 \log(n_t)$$

where

$$c_{1t} = w_t - \tau_{1t} - en_t - s_t$$

$$c_{2t+1} = (1 + r_{t+1})s_t.$$

Two alternative assumptions could be adopted as for the individuals' behavior with respect to the choice of the number of children: 1) they are ultra-perfect foresighted, so that they account for the effect of their choice of children on the level of taxes; 2) they are atomistic and, since each individual's has a negligible influence on the aggregate rate of growth of population, they take n_t as exogenous in the term $b(1 + r_t - n_t)$ while solving their maximization problem with respect to n_t. Notice that the difference between the two assumptions is crucial in determining the difference between the choice of a benevolent planner

[3]In the utility function of individuals only the number of children, but not their quality, matters. We leave the analysis of the implications of relaxing this hypothesis for future research.

and the decentralized solutions: that is, in case hypothesis 1) is assumed, the "social" optimal choice of savings and population is different (typically, suboptimal) from the decentralized optimal choice. For simplicity, in this paper we focus only on the second case, in which the steady state solutions are:

$$s^* = \frac{b_2 e(w - b(1+r))}{e\beta - bb_3} \tag{2}$$

$$n^* = \frac{b_3(w - b(1+r))}{e\beta - bb_3}, \tag{3}$$

where $\beta = b_1 + b_2 + b_3$; necessary and sufficient conditions for $s, n > 0$ are alternatively: i) $b > max\left[\frac{w}{1+r}, Q\right]$; ii) $b < min\left[\frac{w}{1+r}, Q\right]$, where $Q = \frac{eb_2}{b_3}$.

Finally, the capital market clearing condition for period t is: $s_t N_{t-1} = K_{t+1}$, where N_{t-1} is the size of the young adults of period t, born at the beginning of period $t-1$; now, by recalling that these individuals are also the workers of the economy (L_t) we get, in per worker terms:

$$s_t = (k_{t+1} + b)\, n_t. \tag{4}$$

From the solutions for s, n above, the market clearing condition boils down to the following long run per worker capital[4]:

$$k^* = \frac{eb_2}{b_3} - b. \tag{5}$$

It is interesting to note that at the equilibrium public debt fully "crowds out" the stock of capital, that is in a one-to-one correspondence.

Moreover, the condition for a positive long run capital stock is the following:

$$b < Q.$$

In particular, by assuming the usual CD production function in per worker terms $y = Bk^a$ and full depreciation of capital (that is $aBk^{a-1} = 1 + r$), necessary and sufficient conditions for $s, n > 0$ are alternatively: i) $b > max\left[(1-a)Q, Q\right]$; *ii)* $b < min\left[(1-a)Q, Q\right]$. Hence, the overall conditions for the positivity of the long run values of s, n, k simply boil down to the ii) above mentioned, which, recalling that $a < 1$, is $b < (1-a)\,Q$. In other words public debt must be sufficiently low, especially when rearing costs and the degree of patience are low and preference for children is high.

[4]Of course, also a zero equilibrium does exist. Although a zero equilibrium could be considered as a special case of "poverty trap", we focus only on the positive steady state.

3 The Golden Rule with endogenous fertility choices and the optimal level of debt

The Golden Rule provides the first best allocations, that is the level of consumption, fertility and capital which maximize a social welfare function[5]. Thus, suppose that the benevolent policymaker solves the following problem:

$$\max U(c_{1t+s}, c_{2t+s}, n) = b_1 \log(c_{1t+s}) + b_2 \log(c_{2t+s}) + b_3 \log(n_{t+s})$$

$$sub\ \ c_{1t+s} + \frac{c_{2t+s}}{n_{t+s-1}} + en_{t+s} = f\left(k_{t+s}\right) - k_{t+1+s}n_{t+s} + k_{t+s}\left(1-\delta\right), \forall s > 0. \quad (6)$$

where eq. (6) is the time $t+s$ resource constraint. The FOCs conditions are the following[6]:

$$c_{1t}:\ U_1^t - \lambda_t = 0 \quad (7)$$

$$c_{2t}:\ U_2^t - \frac{\lambda_t}{n_{t-1}} = 0 \quad (8)$$

$$n_t:\ U_3^t - \lambda_t\left(e + k_{t+1}\right) + \lambda_{t+1}\frac{c_{2t+1}}{n_t^2} = 0 \quad (9)$$

$$k_{t+1}: -\lambda_t n_t + \lambda_{t+1}\left(f'_{k_{t+1}} + 1 - \delta\right) = 0 \quad (10)$$

where λ_t is the Lagrangian multiplier associated to the time $t+s$ resource constraint. Then, by substituting for λ_{t+1} from equation (10) and dividing equations (9) and (8) by the (7) we get:

$$\frac{\lambda_{t+1}}{\lambda_t} = \frac{n_t}{\left(f'_{k_{t+1}} + 1 - \delta\right)} \quad (11)$$

$$\frac{U_2^t}{U_1^t} = \frac{1}{(n_{t-1})} \quad (12)$$

$$\frac{U_3^t}{U_1^t} = e + k_{t+1} - \frac{n_t}{\left(f'_{k_{t+1}} + 1 - \delta\right)}\frac{c_{2t+1}}{n_t^2} \quad (13)$$

Finally, supposing for simplicity that $\delta = 1$ and exploiting the resource constraint, it follows that, at the steady state:

[5]More precisely, when the maximization problem deals with a "Social Welfare function", rather than the steady state consumption, the solutions are referred to as the "Modified Golden Rule" allocations.

[6]For the sake of notational simplicity we omit the s indicator.

$$\frac{U_2}{U_1} = \frac{1}{n} \tag{14}$$

$$f'_k = n \tag{15}$$

$$\frac{U_3}{U_1} = e + k - \frac{c_2}{n^2}. \tag{16}$$

Note that eqs. (14) and (15) replicate the well known Golden Rule conditions; precisely, the first expression says that the social marginal rate of substitution between consumption in the two periods of life must equal the marginal rate of transformation (n),whereas the second one prescribes that, in order to achieve Pareto efficiency, the marginal productivity of capital must equal the gross growth rate of the population (and of the economy). As for eq. (16), it is a further condition directly stemming from the assumption of fertility choice endogeneity: it states that, along the optimal steady state path, the marginal rate of substitution between fertility and consumption must be equal to its social cost.

In our case, with Cobb-Douglas preferences and by exploiting eqs. (14) to (16) and eq. (6), we get:

$$k^{GR} = \frac{ae\gamma}{(b_2 + b_3) - a\gamma} \tag{17}$$

where $\gamma = b_1 + 2b_2 + b_3$. The positivity of the Golden Rule capital requires as necessary condition that $a < \frac{(b_2+b_3)}{\gamma}$. We can now give the following proposition:

Proposition 1 *The optimal level of debt is:*

$$b^{GR} = \frac{eb_2}{b_3} - \frac{ae\gamma}{b_2 + b_3 - a\gamma}. \tag{18}$$

Proof. The proof is straightforward by substracting eq. (5) from eq. (17) and by solving for b. ■

A positive level of debt is needed for correcting overaccumulation (i.e. DI); conversely, in the case of underaccumulation, the Golden Rule accumulation path would require a negative level of debt (in this case the government should buy bonds issued by private agents). We disregard here the case of negative debt and focus on that of dynamic inefficiency, aiming to investigate the properties of the corrective level of debt.

Thus, in order for the economy to be in the DI case, in the absence of debt, and thus to need a positive level of debt for correcting DI, the necessary and sufficient condition is $a < \frac{b_2}{\gamma}$[7]. In fact, it is easy to verify that, in case a

[7] Moreover, in order to ensure the positivity of s, n, k, another inequality must hold: $a > \frac{b_2}{\gamma} - \frac{b_3}{b_2\gamma}(\gamma - b_2)$. Simple realistic numerical examples show that only the upper bound for a is binding, since the lower bound $\frac{b_2}{\gamma} - \frac{b_3}{b_2\gamma}(\gamma - b_2)$ is likely to be negative as long as the preference for children is sufficiently positive.

happens to be equal to $\frac{b_2}{\gamma}$, the decentralized solution is Pareto optimal, since, in the absence of debt, $k^* = k^{GR}$.

As a consequence, we remark that the existence of a positive level of debt is favoured by a small capital income share (a) and a sufficiently high degree of patience (or thriftiness, b_2) and a sufficiently low preference for children (b_3). Such an existence, instead, does not depend on the cost of rearing children. We can conclude that, *for given technology and thriftiness, the higher the preference for children is, the more likely is the occurrence of DI.*

4 The welfare effects of debt variations

Following Diamond (1965), in this section we investigate the effects of debt variations on the steady state utility level.

At the steady state, the individual budget constraint has the form:

$$c_1 + \frac{c_2}{1+r} + en = w + b\left(\overline{n} - r - 1\right), \tag{19}$$

where $\overline{n}$ is the economy gross fertility rate, which individuals take as given. Then, by differentiating it with respect to b we get:

$$\frac{dc_1}{db} + \frac{dc_2}{db}\frac{1}{(1+r)} + e\frac{dn}{db} = \frac{c_2}{(1+r)^2}\frac{dr}{db} + \frac{dw}{db} + (\overline{n} - r) - b\frac{dr}{db} + b\frac{d\overline{n}}{db}. \tag{20}$$

Next, by assuming that the policymaker is benevolent and that all individuals have the same lifetime consumption/leisure pattern, by exploiting the Envelope Theorem the following equality holds:

$$\frac{dU}{db} = U_1\frac{dc_1}{db} + U_2\frac{dc_2}{db} + U_3\frac{dn}{db} = U_1\left[\frac{dc_1}{db} + \frac{dc_2}{db}\frac{1}{1+r} + e\frac{dn}{db}\right]; \tag{21}$$

finally, by reckoning that $c_2 = s\,(1+r)$, $\frac{s}{\overline{n}} = k + b$, $\frac{dw}{dr} = -k$, and exploiting equation (20) it follows that:

$$\frac{dU}{db} = U_1\left\{\underbrace{(\overline{n} - r - 1)\left[1 + \frac{k+b}{1+r}\frac{dr}{db}\right]}_{D} + \underbrace{b\frac{d\overline{n}}{db}}_{FS}\right\}. \tag{22}$$

Note that, the equation above amends Diamond's formula (that is, eq. 29, page 1142, part D of the equation), with an extra term $\left(b\frac{d\overline{n}}{db}\text{ or } FS \text{ factor}\right)$, which represents the novelty of the present work. In fact Diamond concludes that *"utility is decreased in the efficient case and increased in the inefficient* *case"*[8]. From inspection of equation (22) , instead, we get the following proposition, which generalizes Diamond's rule:

Proposition 2 *With lump sum taxation, increasing the level of debt is always Pareto worsening (improving) if and only if* $(\overline{n}-r-1)\left[1+\frac{k+b}{1+r}\frac{dr}{db}\right]+b\frac{d\overline{n}}{db}<(>)\,0$.

From the proposition above two corollaries descend:

Corollary 1: $\overline{n}-r-1>0$ *is neither a necessary nor a sufficient condition for debt issuing to be Pareto improving.*
Proof. When $r+1<\overline{n}$, $\frac{dU}{db}$ can be negative if $\frac{d\overline{n}}{db}<0$ and $(\overline{n}-r-1)\left[1+\frac{k+b}{1+r}\frac{dr}{db}\right]+b\frac{d\overline{n}}{db}<0$; on the other hand, if $r+1>\overline{n}$, it can be the case that $\frac{dU}{db}$ is positive, provided that $\frac{d\overline{n}}{db}>0$ and $(\overline{n}-r-1)\left[1+\frac{k+b}{1+r}\frac{dr}{db}\right]+b\frac{d\overline{n}}{db}>0$. ■

Note that the such result, again, is due to the fact that the sign of $\frac{d\overline{n}}{db}$ brings about an ambiguity on the overall effect on the steady state utility level[9]. As a consequence, the corollary above can revert Diamond's conclusions: in the "inefficient (efficient) case", utility can be decreased (increased) rather than increased (decreased) by an increase of the long run stock of public debt.

Moreover, it emerges that the long run allocation (or stock of capital and fertility rates) maximizing the steady state utility and implementable by a policymaker in a market economy via debt issuing and lump sum taxes levied upon the young adults, is generally different from the one prescribed by the Golden Rule. We refer to the former as to the "implementable optimal" (IO) level of capital $\left(k^{IO}\right)$, and, similarly, we call b^{IO} the amount of debt associated to that level of capital.

Corollary 2: *The Golden Rule allocation is not feasible, i.e. non implementable via a lump sum tax redistribution (or debt issuing) operated by the government in a market economy.*
Proof. The proof of this corollary consists in showing that the stock of debt solving $\overline{n}=(1+r)$ does not in general solve $\frac{d\overline{n}}{db}=0$. In fact, by substituting the decentralized equilibrium solutions for k, r and n into eq. (22), it follows that $r+1=\overline{n}$ if $b=\frac{e}{b_3(1-a)}\left(b_2(1-a)-a\beta\right)\equiv b^D$; however, when substituting such expression into $\frac{d\overline{n}}{db}$, one gets: $\frac{d\overline{n}}{db}=-\left(b_1+ab_2+b_3\right)^{-1}\beta^{-2}a^{-1}e^{-2}(a-1)^4\left(\left(\frac{ea\beta}{b_3(1-a)}\right)^a\right)b_2Bb_3^2$, which is negative; hence, we get that $\left.\frac{dU}{db}\right|_{b^D}<0$. ■

We are also able to investigate the properties of the IO capital level:

Proposition 3 *If a IO level of debt does exist, it is lower than the GR level.*

[8] We recall that an economy is referred to as "dynamic inefficient" when $r<n$. Note that in our model DI still implies overaccumulation, due to the fact that $\frac{\partial r}{\partial k}<0$, so that $r<n$ implies $k>k^{GR}$.

[9] Note that, under our assumptions, $\frac{\partial r}{\partial b}=Ba(1-a)\left(e\frac{b_2}{b_3}-b\right)^{a-2}>0$.

Proof. Although we do not have any explicit solution for $\frac{dU}{db} = 0$, after some calculus we get that eq. (22), after substituting the decentralized equilibrium expressions for all the variables, can be written as: $E\left[b_2\left(b_2b_3 + e^2a^2\beta\left(\beta + b_2\right)\right) + bb_3aeH + b^2b_3^2\left(\beta + b_2\right)\left(1 - a\right)\right]$,where $E = \frac{\left(\left(\frac{1}{b_3}\left(eb_2 - bb_3\right)\right)^a\right)eBb_3}{\left(e\beta - bb_3\right)^2\left(bb_3 - eb_2\right)^2}$, which is strictly positive if $k > 0$, and $H = 6b_1b_2 - 3b_1b_2/a + b_3^2 - 3b_2b_3/a - 4b_2^2/a + 2b_1b_3 + b_1^2 + 6b_2b_3 + 7b_2^2 - ab_1b_2 - ab_2b_3 - 2ab_2^2$. The expression in brackets is a second order polynomial in b, which has at most two possible positive roots and only the smaller one is a candidate (local) maximum for the U function, which is b^{IO}. Next, since we know that when $b = b^D$, $FS < 0$, so that $\frac{dU}{db} < 0$, we get that $b^{IO} < b^D$. Moreover, by comparing b^{GR} (i.e. eq. 18) and b^D it turns out that $b^D < b^{GR}$, so that $b^{IO} < b^{GR}$. ■

Figure 1 in Appendix B shows a numerical example for a set of preferences and technological parameters[10]; other comparative statistics are reported in Table 1 in the same Appendix, which confirm the findings above. Moreover, the properties of the GR level of capital are formally derived in Appendix A.

Finally, note that the condition $b^{IO} < b^D$ implies that $(\overline{n} - r - 1) > 0$, that is, $k^{IO} > k^D > k^{GR}$: in other words, at the IO allocation the economy is overaccumulating; however, the government is prevented from modifying this allocation, since it would do anything but worsen the welfare of the society as a whole.

4.1 An interpretation of the results

We now provide an explanation of the possible failure of Diamond's rule stemming from our model, which is substantially due to the extra term $b\frac{d\overline{n}}{db}$. First of all, it is worth noting that such term does not descend from any "static" distortion, in that in the present work taxes are supposed to be lump-sum. In fact the nature of such object is the fact that the instruments for correcting DI are numerically insufficient for handling the endogenous variables: more precisely, these variables are two: savings and number of children, while the former is exclusively the per worker level of debt (or, equivalently, the level of taxes). In other words, the steady state optimal allocations are described by two independent conditions (i.e. eqs. 14 and 16[11]) rather than by the traditional one with exogenous fertility (i.e. $f_k' = \overline{n}$), so that two independent instrument are necessary in order to implement the first best allocation.

We now show that, when introducing another independent policy instrument, the coincidence between implementable optimal and Golden Rule allocation is restored[12].

[10] Several numerical simulations have shown that, for realistic values of parameters, the $\frac{dU}{db}$ locus has only one root in the existence set of the problem.

[11] Note that condition 15 is guaranteed by the decentralized economy provided that $r+1 = n$ and that the tax is non distortionary. Hence, in our model it is always satisfied when the equality above holds, and, thus, when the capital stock is at the Golden Rule level (or at the IO level).

[12] Another possibile way to think of the FS element is to interpret it as an externality. In fact, the private advantage of rearing a child is different from (in general lower than) the social

4.1.1 A subsidy for children

Suppose that the government provides each family with a subsidy for each child equal to $e\tau_2$, so that the long run public budget constraint is $-\tau_1 + e\overline{n}\tau_2 = b(1 + r - \overline{n})$. Hence, the individual lifetime budget constraint is

$$c_1 + \frac{c_2}{1+r} + e(1-\tau_2)n = w - \tau_1. \tag{23}$$

Then, by differentiating it with respect to b we get:

$$\frac{dc_1}{db} + \frac{dc_2}{db}\frac{1}{(1+r)} + e(1-\tau_2)\frac{dn}{db} = \frac{c_2}{(1+r)^2}\frac{dr}{db} + \frac{dw}{db} - \frac{d\tau_1}{db} + en\frac{d\tau_2}{db}. \tag{24}$$

After some manipulation, similar to that worked out in the previous section, and recalling that, now, $\frac{U_3}{U_1} = e(1-\tau_2)$, one gets:

$$\frac{dU}{db} = U_1 \left\{ (\overline{n} - r - 1)\left[1 + \frac{k+b}{1+r}\frac{dr}{db}\right] + (b - e\tau_2)\frac{d\overline{n}}{db} \right\}, \tag{25}$$

which is zero if $\overline{n} = r + 1$ and $\tau_2 = \frac{b}{e}$[13].

4.1.2 A lump sum transfer to the old

Let us now suppose that the government choose a lump sum subsidy to the old as second instrument. The individual lifetime budget constraint becomes:

$$c_1 + \frac{c_2}{1+r} + en = w - \tau_1 + \frac{\tau_2}{1+r}. \tag{26}$$

After some manipulation, and recalling that, now, $-\tau_1 + \frac{\tau_2}{\overline{n}} = b(1 + r - \overline{n})$ and $c_2 = s(1+r) + \tau_2$, one obtains:

$$\frac{dU}{db} = U_1 \left\{ (\overline{n} - r - 1)\left[1 + \frac{1}{1+r}\left((k+b)\frac{dr}{db} + \frac{1}{\overline{n}}\frac{d\tau_2}{db}\right)\right] + \left(b - \frac{\tau_2}{\overline{n}^2}\right)\frac{d\overline{n}}{db} \right\}, \tag{27}$$

which is zero if $\overline{n} = r+1$ and $b\overline{n}^2 = \tau_2$. Note that τ_2 must be necessarily positive (i.e. a subsidy or pension) with positive debt and fertility rates[14]. Finally, in both examples Diamond's rule is necessary, although not sufficient for delivering the GR allocation.

one, since a higher fertility rate reduces the amount of taxes burdening the young adults. Hence, a Pigouvian taxation (subsidy), by making individuals internalizing such aggegrate effect, may restore the general equilibrium conditions for optimality. The second solution we propose (tranfer to the old) although in a less obviuos manner, relies on the same line of reasoning.

[13] If the subsidy is proportional to the number of children and not to the cost of rearing them, eq. (25) becomes: $\frac{dU}{db} = U_1 \left\{ (\overline{n} - r - 1)\left[1 + \frac{k+b}{1+r}\frac{dr}{db}\right] + (b - \tau_2)\frac{d\overline{n}}{db} \right\}$, that $b = \tau_2$ and $n = 1 + r$ are the solutions of the maximization problem.

[14] By the same reasoning, it is easy to show that a balanced PAYG pension system (i.e. with $b = 0$), in presence of endogenous fertility choices, would not be able to implement the first best allocation.

5 Conclusions

This paper extends Diamond's (1965) OLG framework by allowing for endogenous fertility choices. Our results can be summarized as follows: 1) the existence of dynamic inefficiency (and, thus, the necessity for issuing national debt so as to correct it) is favoured by a small capital income share, on the technological side, a sufficiently high degree of patience and a sufficiently low preference for children, on preferences grounds. Moreover, we argue that such level is higher the bigger the child rearing cost, the lower the capital share, the higher individuals' degree of patience and the lower the preference for children; 2) In general Diamond's (1965) rule turns out to be neither necessary nor sufficient for Pareto optimality to obtain in the presence of endogenous fertility, so that the optimal policy cannot be implemented in a decentralized economy unless another independent instrument is introduced further to lump sum taxes upon the young adults. In fact, the number of instruments must equal the number of endogenous variables to be controlled (namely, savings and fertility rates). In this respect, the introduction of both a subsidy for each child or a lump-sum transfer to the old turn out to be successful in implementing the first best allocation.

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A Implications of the model

A.1 The decentralized equilibrium level of capital

By inspection of eq. (5) two remarks are worth making:

Remark 1 *As expected, the long run per worker capital is inversely linked with the factors increasing the population growth and, thus, depends positively on the rearing cost (e) and the preferences for children (b_3) and positively linked with the factor increasing accumulation, that is with the degree of patience (b_2).*

Remark 2 *Rather interestingly, the long run per worker capital is independent of the technology: for whatever technology (CD, CES, Leontief and so on) the long run per worker capital is the same. But this also means that it is independent of all the usual assumptions on the side of firms, typical of the OLG framework: i.e. the constant returns to scale and competitive market.*

In other words, when fertility is endogenously chosen by individuals with CD preferences and constant rearing costs, the steady-state capital and the rate of growth of the economy depends only on the preferences and not on the technology.

A.2 The optimal level of debt

The following proposition contains some comparative-statics results relating the optimal debt level to preference and technological parameters and to the rearing cost:

Proposition 4 *The optimal level of debt depends a) negatively on the elasticity of physical capital; b) positively on the children rearing cost; c) positively on the degree of patience of individuals; d) negatively on the preference for children of individuals.*

Proof. Part a) of the proposition simply follows from the sign of $\frac{db^{GR}}{da} < 0$. As for part b), we note that both $\frac{\partial k^M}{\partial e} > 0$ and $\frac{\partial k^{GR}}{\partial e} > 0$. Since we are in the DI case, in absence of debt the economy is overaccumulating, so that $b^{GR} = k^M - k^{GR} > 0$; moreover, by reckoning that $b^{GR} = e\left(\frac{\partial k^M}{\partial e} - \frac{\partial k^{GR}}{\partial e}\right) > 0$, then $\frac{\partial k^M}{\partial e} > \frac{\partial k^{GR}}{\partial e}$, and, hence, $\frac{db^{GR}}{de} = \left(\frac{\partial k^M}{\partial e} - \frac{\partial k^{GR}}{\partial e}\right) > 0$. As for part c), the following derivative $\frac{db^{GR}}{db_2} = \frac{e}{b_3} + \frac{ae(b_1-b_3)}{(b_2+b_3-a\gamma)^2}$ shows that the effect of b_2 could be, at first sight, ambiguous; however, by exploiting condition $a < \frac{b_2}{\gamma}$, and writing $a = \frac{b_2}{\gamma} - \varepsilon$, where $0 < \varepsilon < b_2$ is an arbitrarily small constant, which ensure us to deal with the DI case, we get that: $\frac{db^{GR}}{db_2} = \frac{1}{\gamma(\varepsilon\gamma+b_3)^2 b_3}\left[\varepsilon^2(\gamma)^3 + \varepsilon\gamma b_3(\gamma + 2(b_2+b_3)) + b_3(b_1+b_3)(b_2+b_3)\right]e$, which is unambiguously positive. Finally, as for part d), by adopting the same strategy of part c), it can be shown that

$$\frac{db^{GR}}{db_3} = -\frac{\left[\varepsilon^2 b_2 (\gamma)^3 + \varepsilon b_3 \left(b_1 b_3 + 2b_1 b_2 + 3b_2 b_3 + 4b_2^2\right)(\gamma) + b_3^2 b_2 (b_2 + b_3)\right] e}{(b_3 + \varepsilon\gamma)^2 (\gamma) b_3^2} < 0. \blacksquare$$

As for the role of the elasticity of the physical capital a, its economic meaning is straightforward: since the Golden Rule level of capital depends positively on such parameter (whereas the steady state capital emerging from the market solution does not depend on it) the lower a is, the more likely is that the economy undergoes the risk of overaccumulation and, thus, the bigger is the debt the government has to issue in order to correct the DI of the market allocation. As far as the role played by the cost of rearing children e is concerned, it is firstly worth noting that the decentralized-economy level of capital is positively related with it. In particular, this relationship descends from eq. (2) and (3), in that $\frac{\partial s}{\partial e} > 0$ and $\frac{\partial n}{\partial e} < 0$ imply a lower level of per-worker capital when e decreases $\left(\frac{\partial k^M}{\partial e} > 0\right)$; however, also the Golden Rule capital increases if e gets bigger $\left(\frac{\partial k^{GR}}{\partial e} > 0\right)$; nevertheless, in the DI case an increase of the children rearing cost increases the market capital stock more than the capital stock optimally chosen by the benevolent policymaker.

Hence the following remark holds:

Remark 4: *In our economy the children rearing cost plays solely a "factor scale role", as for the DI issue: in fact, while being irrelevant for the occurrence of DI, higher levels of such cost increase the optimal level of debt.*

Finally, as expected, a higher thriftiness of the individuals increases overaccumulation, consequently requiring a higher level of debt.

In the light of our analysis we can say that a high (low) level of debt is more likely to be optimal for countries with a relatively low (high) share of capital, with high (low) costs for rearing children, with low (high) preference for children and with high (low) degree of patience.

B Figures and Tables

Figure 1: D, $\frac{dU}{db}$ and $\frac{dn}{db}$ loci. Parameters: $b_1 = 1$, $b_2 = 0.9$, $b_3 = 0.9$, $e = 2$, $B = 1$, $a = 0.1$.

Table 1: *Comparative Statistics*

e	b_2	b_3	a	b^{GR}	b^D	b^{IO}	$\frac{b^{IO}}{f}$	$\frac{e}{w}$
0.3	0.9	0.4	0.1	0.58	0.48	0.45	0.52	0.38
0.3	0.9	0.4	0.2	0.38	0.24	0.21	0.25	0.43
0.3	0.5	0.4	0.1	0.27	0.22	0.20	0.24	0.40
0.6	0.9	0.4	0.1	1.15	0.97	0.90	0.97	0.72
0.3	0.9	1	0.1	0.19	0.17	0.16	0.20	0.42

Other parameters imposed: $b_1 = 1$, $B = 1$.

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