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Economic growth, poverty trap and intergenerational transfers

Discussion Paper n. 39

2004
Economic growth, poverty trap and intergenerational transfers

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Abstract

We adopt the traditional competitive OLG model a là Samuelson (1958)-Diamond (1965) with two-period-living individuals, fixed fertility rates and labor supply, where the government can pursue retributive policies between generations by levying levy lump sum taxes/subsidies.

By using standard logarithmic preferences and a CES technology with low factor substitution, we show that the taxation of the old can be used: 1) to escape from a poverty trap; 2) to increase the per-capita income in the positive high steady state. Conversely, the taxation of the young worsens the stationary per capita income and may in fact lead to the explosion of the economy.

Our results may apply to the policy analysis concerning developing countries in that they show that the introduction of a PAYG social security scheme as a means of redistributing among generations may be detrimental for economic growth and for the poverty trap problem. This argument may also apply to rich countries ascaped from poverty traps. Conversely, the introduction of such instruments as public education or subsidies to children may be positive as for both economic growth and the solution of the poverty trap problem.

1. Introduction

There is a renewed interest about the relation between economic growth and income distribution. In particular, the literature has mainly focused on the issue of the functional distribution of income between classes, social groups and so on, and of the individual distribution. However, less attention has been devoted to the relation between growth and intergenerational distribution. In addition, the recent literature on economic growth has highlighted the role of multiple equilibria and poverty traps in explaining the different long run performance of rich and poor countries (Galor (1996) and Azariadis (1996)).

In this paper we put an emphasis on the role of distribution of income among generations. In particular, we assess the possibility of redistributing resources among generations as a means for escaping from a poverty trap in a traditional competitive OLG model a là Samuelson (1958)-Diamond (1965).

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The work is organized as follows: in section 2 we present the model and characterize the general equilibrium of the decentralized economy in presence of redistributive public policies; in section 3 we discuss the occurrence of poverty traps and the qualitative effects of such policies in determining the long run outcomes of the economy. In section 4, we provide an analytical discussion of two alternative policies and suggest some policy implication of the results. Finally, conclusive remarks follow.

2. The model setup

The economy is populated by two-period living individuals, who maximize a utility function $U$, defined over $c_1$ and $c_2$, that is consumption in the first and second period of life; hence, in their youth the $N_t$ agents born in period $t$ must choose how much to save out of their income, which is comprised of the wages received from their fixed labor supply (and normalized to 1). The population $P_t$ grows at a constant growth rate, $n$, so that $N_t = N_{t-1}(1+n)$.

Moreover, in this closed-production economy perfect competition is assumed. As a consequence, firms, owning an CRS technology, can hire capital and labor by remunerating them at their marginal productivity.

Finally, the government can levy lump sum taxes/subsidies on the young ($\tau_1$) and/or on the old ($\tau_2$) for redistributive purposes.

2.1 Householders

Individuals, at time $t$, choose their optimal intertemporal allocations of consumption/savings. By assuming for simplicity a logarithmic utility they solve the following problem:

$$\text{Max } U_t = \log c_{1t} + \beta \log c_{2t+1}$$

subject to $c_{1t} + \frac{c_{2t+1}}{1+r_{t+1}} = w_t - \tau_1 - \tau_2 (1+r_{t+1})$.

First order conditions deliver the following equations:

$$c_{1t} = \frac{1}{\lambda}$$

$$c_{2t+1} = 1 + r_{t+1} \frac{\beta}{\lambda} \left( w_t - \tau_1 - \frac{\tau_2}{1+r_{t+1}} \right)$$

where $\lambda$ is the Lagrangean multiplier, so that:

$$c_{1t} = \frac{1}{1+\beta} \left( w_t - \tau_1 - \frac{\tau_2}{1+r_{t+1}} \right) \quad [1']$$

$$c_{2t+1} = (1+r_{t+1}) \frac{\beta}{1+\beta} \left( w_t - \tau_1 - \frac{\tau_2}{1+r_{t+1}} \right) \quad [2]$$
\[ s_t \equiv w_t - \tau_t - c_{1t} = \frac{\beta}{1 + \beta} (w_t - \tau_t) + \frac{\tau_2}{(1 + r_{t+1})(1 + \beta)}, \quad c_{1t}, c_{2t+1}, s_t > 0. \]  

**2.2 The Production sector**

We assume a perfect competitive market with identical firms: thus, firms hire capital and labor by remunerating them according to their marginal productivity. Moreover, due to the homogeneity of degree one of the aggregate production function \( F(K, L) \) and defining \( k \equiv K \) and \( f(k) \equiv \frac{F(K, L)}{L} = F(k, 1) \), it follows that:

\[ F'_k \equiv f'_k = r_t \quad \text{and} \quad F'_L \equiv f - f'_k k_t = w_t, \]  

where \( r_t \) and \( w_t \) are the interest rate and the wage in period \( t \), and the subscript of the derivative function \( F' \) and \( f' \) indicates the derivation variable.

In the numerical examples of the subsequent sections we use a CES production function of the following type:

\[ F(K, L) = A\left[\alpha K^{-\rho} + (1 - \alpha)L^{-\rho}\right]^{\frac{1}{\rho}}, \quad \rho > -1, \rho \neq 0, A > 0, \alpha \in (0, 1) \]  

which, in per worker terms, has the form:

\[ f(k) = A\left[\alpha k^{-\rho} + (1 - \alpha)\right]^{\frac{1}{\rho}}, \]  

with

\[ f'_k = \alpha A^{-\rho} \left(\frac{f}{k}\right)^{(1+\rho)} \quad \text{and} \quad f - f'_k k_t = (1 - \alpha)A^{-\rho} f^{(1+\rho)}. \]  

**2.3 Public sector and the good market clearing condition**

The government levies lump sum taxes or subsidies on both young and old individuals. The public sector displays the following budget constraint

\[ \tau_t N_t + \tau_{t+1} N_t - r_t D_t = D_{t+1} - D_t \]  

where \( D \) stands for debt. For the sake of simplicity we assume that the policy keeps the budget balanced in each period, so that, in per worker terms, we get:

\[ \tau_{t+1} = 0. \]  

Finally, the equilibrium implies the satisfaction of individuals’ and firms’ optimality equations and the good market clearing condition, whereby aggregate savings \( (S_t) \) must equal the capital of the subsequent period:

\[ S_t = K_{t+1} \]  

In per worker terms, and substituting \( s_t \) for eq. [3] and eq. [8] one gets:
$$k_{t+1} = \frac{1}{1 + n} \left[ \frac{1}{1+\beta} w_t - \frac{\tau_1}{1+\beta} \frac{\beta(1+r_{z+1}) + (1+n)}{1+r_{z+1}} \right]$$ \[10\]

or

$$k_{t+1} = \frac{s_t(w_t,r_{z+1},\tau_2)}{1 + n} = \phi(k_t,k_{z+1}) = \frac{1}{1 + n} \left[ \frac{\beta}{1+\beta} w_t + \frac{\tau_2}{1+\beta} \frac{\beta(1+r_{z+1}) + (1+n)}{1+r_{z+1}} \right].$$ \[10'\]

3 Poverty traps

In this section we analyze the possibility of poverty trap occurrences in an OLG economy without and with public policies.

3.1 Poverty traps without intergenerational transfers

In absence of public policy (i.e. \(\tau_1,\tau_2=0\)), an OLG economy with Cobb-Douglas preferences and CES technology has been extensively discussed by De La Croix-Michel (2001, ch. 1.6). We limit us in this paper to consider the case of \(\rho>0\), which appears of some empirical relevance (Rowthorn, 1999a, 1999b). In such a case the equation resuming the entire dynamics of the model is

$$k_{t+1} = \frac{1}{1 + n} \left[ \frac{1}{1+\beta} w_t \right].$$ \[11\]

The existence and the number of the steady states emerge from the analysis of the following implicit equation:

$$k - \frac{1}{1 + n} \left[ \frac{\beta}{1+\beta} w(k) \right] = 0,$$ \[12\]

which provides the following possible outcomes:

- w1) one equilibrium: the trivial one (\(k=0\));
- w2) two equilibria: \(k=0\) and a stable one (tangent) (this is obviously a very special case);
- w3) three equilibria: \(k=0\), one (unstable) and one stable.

As for the stability issue, we recall that: 1) when the zero equilibrium is the only equilibrium, it is globally stable; 2) if there are two positive equilibria, the higher one and the zero equilibrium are locally stable.

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1 Rowthorn 1999a reports the results for the elasticity of substitution \(\sigma=1/(1+\rho)\) of 33 econometric studies, according to which the overall median of the summary values (median of the medians) is equals to 0.58 (and only in 7 cases the elasticity is greater than 0.8). Rowthorn 1999b) estimates in indirect way the elasticity of substitution (based on some empirical values of the labour demand elasticity, profit share and price elasticity), generating the following examples for the parameter \(\rho\).

<table>
<thead>
<tr>
<th>Italy</th>
<th>UK</th>
<th>France</th>
<th>Germany</th>
<th>Sweden</th>
<th>Japan</th>
<th>USA</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\rho)</td>
<td>13.3</td>
<td>3.95</td>
<td>15.6</td>
<td>1.63</td>
<td>24</td>
<td>5.25</td>
</tr>
</tbody>
</table>
This dynamic analysis allows for the definitions of the poverty trap\(^2\): 1) when only the trivial equilibrium exists the poverty trap is said to be inescapable (for whatever initial capital stock the zero point is a catching one); 2) when a positive \(k\) value (namely \(k^+\)) exists corresponding to the lowest positive steady state, then the poverty trap can be escaped provided that the initial capital stock is high enough \((k(0)> k^+)\).

The Figure 1 illustrates cases w1)-w3); by passing we remark that the increases in the technological index \(A\) (that is positive technological shocks) can only lead to escape from the inescapable poverty trap situation (i.e. zero equilibrium as a global attractive point, see Fanti-Spataro, 2004), although the zero equilibrium remains as a local attractive point.

### 3.2 Poverty traps and intergenerational transfers

In the presence of intergenerational transfers (i.e. \(\tau_1, \tau_2 \neq 0\)), the dynamics of the model is represented by the equation \([10']\).

Note that, the analysis of the existence and of the number of the equilibria can carried out analytically by studying the implicit equation.

\[(1+n)k-s(w,r,\tau_2)=0\]  \(\Rightarrow\)  \([13']\)

\[k - \frac{1}{1+n} \left[ \frac{\beta w(k)}{1+\beta} + \frac{\tau_2}{1+\beta} \frac{(1+r(k))+(1+n)}{(1+r(k))(1+n)} \right] = 0\]

The compared view between the dynamic equations \([12]\) and \([11]\) and the two implicit equations \([13]\) and \([10']\) makes clear the effects of public policies both on the stability issues and on the (existence and number) steady states.

We neglect, for brevity, to derive some (necessary and/or sufficient) conditions for the existence of a poverty trap situation, and resort to some simulations and their graphical representation.

This simulative analysis shows that six cases may exist; according to whether \(\tau_2\) is greater or less than zero:

**A) \(\tau_2 > 0\)**

\(a1\) three positive equilibria: the lowest and the highest are stable, the middle one is unstable;
\(a2\) two positive equilibria: the lowest, which is tangent (this is obviously a very special case) and the highest stable;
\(a3\) only one positive equilibrium, which is also globally stable.

Possible configurations of cases a1)-a3) are illustrated in Figure 2a.

**B) \(\tau_2 < 0\)**

\(a4\) no equilibrium;
\(a5\) only one positive equilibrium (tangent, this is obviously a very special case);
\(a6\) two positive equilibria: the lowest, which is unstable, and the highest which is stable.

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\(^2\) For an extensive discussion of multiple equilibria see Galor (1996) and Azariadis (1996).
Cases a4)-a6) are presented in Figure 2b.

As to the existence of poverty traps we see that:

1) when no equilibrium exists (case a4) the economy is not existing (for whatever initial capital stock the economy “explodes”); 2) when a finite $k$ value (namely $k^+$) exists corresponding to the lowest (case a6) or middle (case a1) positive steady state, therefore the poverty trap emerges, but the latter can be escaped provided that the initial capital stock is high enough ($k(0) > k^+$); 3) when only one positive equilibrium exists (case a3), it is globally stable and the poverty trap is disappeared independently of the initial capital stock.

Some comments are worth making. The comparison of cases w1-w3) and a1)-a6) shows that, in presence of a public policy, the following new features emerge:

1) the zero equilibrium disappears;
2) the economy may “explode”;
3) the poverty trap may disappear independently of the initial capital stock.

In particular, the points 2) and 3) highlight the relevance of the introduction of the public policy: on the one side, policy should be cautious in order to avoid the explosion of the economy; on the other side an opportunely implemented policy may definitely eliminate the poverty trap problem.

For illustrative purposes we show, in the Figure 3, the role played by the value of the tax (subsidy) on the old in determining the possible long run dynamics of the economy. In this Figure the combinations of $\tau_2$ and the long run equilibrium capital stock are reported.

In particular, the level of the lump sum tax on the old originates three long run equilibrium regions, according to the following parametric “windows” of $\tau_2$:

- $\tau_2 < \tau_2^*$: non existence (explosion) of the economy
- $\tau_2^* \leq \tau_2 \leq \tau_2^{**}$: poverty trap situations (the escape from which depends only on the initial (or on the shocks on) capital stock
- $\tau_2 > \tau_2^{**}$: no poverty trap; only one “rich” equilibrium exists.

4 Effects of Public policies on the economic growth and on the poverty trap.

We now focus on the analysis of the effects that public policies may play on the poverty traps.

First of all, let us analyze the effect of a change in the level of $\tau_2$ on the existence of a steady state value of the capital intensity: it is evident from eq. [10'] that a sufficiently negative $\tau_2$ value may render the function $\phi$ negative for whatever value of $k$, and therefore no economy may exist (see case a4).

Secondly, we consider the cases where multiple (non special) equilibria exist (one, two or three, that is, respectively, cases a3, a6 and a1). In this case we are able to derive analytically the effects of small changes in $\tau_2$ on each equilibrium. In fact, by implicitly differentiating eq. [13'], exploiting eqs. [4] and recalling that the factor price frontier implies $\frac{dw}{dr} = -k$, one obtains:

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3 The two special tangent equilibria cases (a2 and b2) are not considered for brevity.
\[
\frac{dk}{d\tau_2} = \frac{s_{\tau_2}}{(1+n) - f_k s_r + f_k k s_w}
\]  \[14\]

which, in our case, becomes:

\[
\frac{dk}{d\tau_2} = \frac{1}{1+\beta} \left[ \frac{\beta}{1+n} + \frac{1}{1+r} \right] \frac{1}{(1+n) + \frac{f_k'\tau_2}{(1+\beta)(1+r)} - f_k' k \frac{\beta}{1+\beta}}
\]  \[14']

In addition the stability analysis can be easily performed. In fact, by differentiating eq. [10'] it can be shown that:

\[
\frac{dk_{i+1}}{dk_i} = -\frac{f_k' k s_w}{(1+n) - f_k s_r}
\]  \[15\]

which, under our assumptions on the utility function, becomes:

\[
\frac{dk_{i+1}}{dk_i} = -\frac{f_k' k \frac{\beta}{1+\beta}}{(1+n) + \frac{f_k' \tau_2}{(1+\beta)(1+r_{i+1})}}
\]  \[15'

Clearly, when \(\tau_2\) is negative (i.e. a subsidy), since \(f_k''\) is negative, the locus is always positive around the equilibrium; on the other hand, when \(\tau_2\) is positive, the denominator of eq. [15] must be imposed to be positive.

Finally, the following inequalities will hold:

\[
0 < \frac{dk_{i+1}}{dk_i} < (>)
\]

according to whether the equilibrium under analysis is stable or unstable, respectively.

Hence, taking account of the comparative static equations [14] and the stability equations [15] the following proposition holds:

**Proposition 1:** If the equilibrium is stable, then an increase of the tax on the old increases the steady state capital intensity. If the equilibrium is unstable, then such increase reduces the steady state capital intensity.

**Proof.** Since the numerator of eq. [14'] is strictly positive, the sign of \(dk/d\tau_2\) is univocally driven by the sign of the denominator; now, by reckoning that the denominator is the difference between the denominator and numerator of eq. [15'], under stability \(0 < \frac{dk_{i+1}}{dk_i} < 1\), such a difference is positive so that \(dk/d\tau_2\) is positive; on the other hand, when the equilibrium is unstable \(dk_{i+1}/dk_i > 1\), the denominator of eq. [14'] is negative, so that \(dk/d\tau_2\) is negative as well.
The content of Proposition 1 is shown by Figure 4.

On policy grounds, an interesting interpretation of our results is to think about the intergenerational transfers as a PAYG social security scheme. In the light our analysis, the following remark descends:

**Remark 1:** the introduction (or an increase) of a PAYG Social Security system reduces the “good” and increases the “bad” equilibrium. Moreover, a sufficiently high increase of the PAYG system may also lead to the “explosion” of the economy.

Both parts of Remark 1 are illustrated in Figure 5. The economic explanation of this result can be posed as follows: in an economy stuck in a poverty trap, taxing the young so as to finance the PAYG system will crowd out private investments and, consequently, will reduce the rhythm of accumulation of resources. In case such tax is too high, the accumulation path will be even insufficient to replace the capital consumed by the old generations, so that the capital stock will progressively decumulate, going to zero in a finite time. Note that this possibility can also apply to rich economies: imagine that a country is situated in a “rich” equilibrium (after escaping from the poor equilibrium during its historical transition). The introduction of an unfunded pension system has always a twofold effect: 1) reducing the level of income and 2) enlarging the region of instability, rendering the economy more vulnerable to negative temporary shocks, which can, for a sufficiently high level of redistribution, become disruptive for the economy. These arguments can be easily reckoned by observing Figure 2b.

Finally, if we interpret the positive $\tau_2$ as a tax financing a subsidy to the young in the form of a public allowance for educational fees or a subsidy to fertility, the following remark descends:

**Remark 2:** The introduction of subsidy to education or to fertility is in general positive for the economy since it may improve, help escaping or even eliminate the poverty trap equilibrium.

The content of this last Remark is depicted in Figure 6. In particular, the case of the escape from the “poor” attractor, toward the rich equilibrium via a transfer to the young at a date $t_0$ is depicted. The economic rationale behind our results can be explained as follows: the intergenerational transfer reduces the disposable income of the “first old generations” through taxation and redistributes the revenues to the young through a negative tax. By doing this the government boosts private savings and, by favoring the formation of new capital, lets the economy escape from the poverty trap (in case the latter exists) and, in any case, allows the convergence to a high steady state level of capital.

**Conclusions**

In this paper we have adopted the traditional competitive OLG model a la Samuelson (1958)-Diamond (1965) with two-period-living individuals, logarithmic preferences, CES

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4 In case a subsidy per child (g) is provided, the public budget constraint would become: $\tau_1+\tau_2/(1+n)-gn=0$, while the individual one would be: $c_1t+c_2t+1=w_t-\tau_1-\tau_2/(1+r)+gn$. Substituting for $\tau_1-gn$ from the first into the second equation obtains the same analytical setting and results as those presented in the text.

5 The case of elimination of the poverty trap and the increase of the “poor” equilibrium are reported in Figure 2a and 4 respectively.

6 This explanation may resemble to the argument developed by De La Croix-Michel (2002), pages 226-228, according to which nationalizing part of the capital stock detained by the first old generation and subsequently redistributing the dividends to the young may help to escape from the poverty trap situation.
technology with low factor substitution and exogenous fertility rates, where the government can pursue retributive policies between generations by levying levy lump sum taxes/subsidies.

We show that the taxation of the old can be used: 1) to escape from a poverty trap; 2) to increase the per-capita income in the positive high steady state.

Conversely, the taxation of the young worsens the stationary per capita income and may in fact lead to the explosion of the economy.

Hence, we unveil that the introduction of a redistributive policy enriches the range of possible long run dynamic outcomes and in particular, that, on the one side, such a policy should be cautious in order to avoid the explosion of the economy; on the other side, if opportunely implemented, such a policy could definitely eliminate the poverty trap problem.

Our results may apply to the policy analysis concerning developing countries in that they show that the introduction of a PAYG social security scheme as a means of redistributing among generations may be detrimental for economic growth and for the poverty trap problem. This argument may also apply to rich countries escaped from poverty traps. Conversely, the introduction of such instruments as public education or subsidies to children may be positive as for both economic growth and the solution of the poverty trap problem.

These results have been obtained under simplified assumptions: only lump sum taxation, no government spending and no debt, labor supply and fertility exogenously given. The relaxation of any of such hypotheses are interesting avenues for future research.

References

APPENDIX: FIGURES

Figure 1: Multiple equilibria and poverty trap: \( \beta=0.6, \alpha=0.3, n=0.4, \rho=2, \tau_2=0.8 \).

Figure 2a: Multiple equilibria with \( \tau_2>0 \): \( \beta=0.6, \alpha=0.3, n=0.4, \rho=2, A=6 \).
Figure 2b: Multiple equilibria with $\tau_2 < 0$: $\beta = 0.6, \alpha = 0.3, n = 0.4, \rho = 2, A = 7$.

Figure 3: Long run dynamics in presence of a redistributive policy: equilibrium combinations of $k$ and $\tau_2$. $\beta = 0.6, \alpha = 0.3, n = 0.4, \rho = 2, A = 5$. 
Figure 4: The effects of an increase of $\tau_2$ on the stable and unstable equilibria.

Figure 5: Effects of a PAYG tax increase on the stable and unstable equilibria and the case of explosion.
Figure 6: Escaping from the poverty trap trajectory via a tax increase on the old (a subsidy increase to the young) at time $t_0$. 
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