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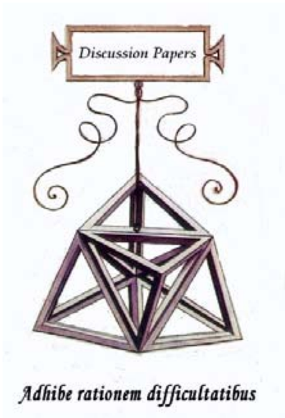

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Intertemporal Preferences, Distributive Shares, and Local Dynamics*

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Abstract

This paper provides a simplified version of the perfectly flexible wages OLG model proposed by Hahn and Solow (1995). Using a Cobb-Douglas specification for the utility and the production functions, we demonstrate that the local stability of the steady-state equilibrium depends only on intertemporal preferences and distributive shares. Furthermore, we show that local stability might be related to consumption smoothing considerations.

JEL Classification: D11, D24

Keywords: Overlapping Generations, Market Clearing, and Local Stability

1 Introduction

One way to deal with intertemporal issues is to study, simultaneously and explicitly, the entire time span or time horizon that is relevant for the question at hand[1]. Whenever the time horizon is infinite there are two useful schemes to describe an economy: models in which agents live forever and overlapping generations (OLG) models.

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[1]This is the perspective proposed by Irving Fisher.

If taken too seriously, an infinite-lifetime model leads to look at the economy as if it were guided by a Ramsey optimiser. When there are no discontinuities, this means that the economy, starting from historically given conditions, will follow a unique perfect-foresight path to the appropriate steady-state[2]. This normative model, helpful to working out what an omniscient planner should do, is not meant to capture undesired macroeconomic behaviours, because it simply assumes that they do not occur.

In an OLG model new generations of people appear before all of the people from the earlier period die off. There are many well-founded reasons to describe an infinite-horizon economy using an OLG model. First, it forces the researcher to model agents which think about the future because they may be engaged in transactions concerning goods produced by individuals who are not yet alive. Second, keeping a simple algebra, it preserves the possibility to conclude that the economy may display a (relatively) wide range of macroeconomic patterns. Third, even if it is more controversial, it provides the option to include fiat money as a valuable asset[3].

Following a popular contribution by Hahn and Solow (1995), we build a two-periods perfect foresight OLG model in which we allow wages and prices to be perfectly flexible. By "perfect flexibility" we mean that at every instant wages and prices are at values that equate demand and supply in all the markets. Our immediate goal is to show that this model can display a wide range of local dynamic patterns in which stability is not the likeliest outcome.

Resorting to a version of the Clower constraint, which capture the role of money as a medium of exchange, we model an OLG economy where young households can allocate their saving in two different assets: money and bonds. Once intertemporal preferences and distributive shares are given, the analysis of a steady-state equilibrium in which the return earned by bonds is higher than the return on money shows that there are specific bounds for the eligible demand for cash balances. Furthermore, we provide an explanation of local stability in terms of consumption smoothing.

The paper is organised as follow. Section 2 illustrates the model. Section 3 concludes.

[2]More interesting possibilities are analysed - *inter alia* - by Benhabib and Farmer (1994).

[3]With perfect foresight or rational expectations, this is not possible in a finite economy.

2 The Model

We consider a two-periods OLG model with price (and wage) flexibility and perfect foresight. In a given period t, young households born at the very beginning of t, and old households born at the very beginning of $t-1$, live together. Only young households are allowed to work: they provide a fixed amount of labour services l_t (normalised to unity), which is always fully employed by firms. For sake of simplicity, old households do not leave bequests.

A remarkable feature of this model is that young households can allocate their savings to either or both of two assets, that is, money balances and bonds issued by the firms. The former give no nominal return, but they may earn a real return which depends on the change in the nominal prices of consumption goods between the two periods of households' life. The latter grants a proportional share of the firms profits which is equal to the difference between the nominal output and the wage bill. Assuming perfect foresight, there is no difference in the two assets riskiness.

Such an economy will be in one of two possible phases[4]. In one phase the real return of investment on bonds is higher than the real return on money balances. In this case, because of the lack of uncertainty, young households would prefer to invest all of their saved resources on bonds, vanishing the demand for money. In order to avoid this possibility, we follow the contribution by Hahn and Solow (1995) and we impose a partial *cash-in-advance* constraint known as Clower constraint. According to the Clower constraint, a young household planning to spend a certain nominal amount in its old period, has to demand an amount of money that is at least equal to a fixed fraction of its planned expenditure[5]. Whenever the real return on bonds is higher than the real return on money balances young households are *liquidity constrained*, that is, they will not wish to hold money more than the Clower constraint obliges them to hold.

In the other phase, the real return on the two assets is the same, so young households are indifferent between them. We may refer to this situations as *portfolio indifference.*

The productive sector of the model is quite traditional. Firms behave competitively, taking prices and wages as given. A typical firm produces a perishable homogeneous good employing the labour supplied by young households and the capital raised issuing the bonds. The capital entering

[4]Assuming that capital is an indispensable factor, we do not consider the situation in which the real return on money is higher than the real return on bonds.

[5]Clower (1975) imposed the rule that "only money buys goods". Hahn (1982) proposed a weaker axiom, that is, "money buys goods *more cheaply* than do other assets".

in the production function is of the circulating type, therefore, it disappears completely after one period of use.

The model is closed deriving the (unique) steady-state characterising a situation of continuos market-clearing in which agents are liquidity constrained, and discussing its local stability conditions.

2.1 The Households Side

Taking the current real wage w_t as given, the typical household choose presents (c_t) and future (c_{t+1}) consumption in real terms maximising its utility function (u) under the relevant budget constraints:

$$\max_{c_t, c_{t+1}} u = c_t^{\delta} c_{t+1}^{(1-\delta)} \qquad 0 < \delta < 1 \tag{1}$$

$$s.to$$

$$c_t + s_t = w_t l_t \tag{2}$$

$$c_{t+1} = R_t \left[s_t - m_t \right] + \frac{1}{x_t} m_t \tag{3}$$

where s_t are total savings, R_t is the real gross return on resources lent to the productive sector, m_t is the demand for real cash balances, and $x_t \equiv \frac{p_{t+1}}{p_t}$ is the inverse of the real return on money[6]. As stated above, young household can allocate savings to money and/or bonds. However, the nominal demand for money balances has to satisfy the Clower constraint:

$$p_t m_t \geqslant \frac{1}{\xi} p_{t+1} c_{t+1} \tag{4}$$

where $\xi \geqslant 1$[7]. Whenever the Clower constraint holds as equality, equation (3) reduces to:

$$c_{t+1} = \frac{\xi}{\xi + x_t R_t - 1} R_t s_t \equiv \theta_t R_t s_t \tag{5}$$

As a consequence, we may refer to $\theta_t R_t$ as the "effective" rate of return on savings. Obviously, when money and bonds grant the same real return θ_t is equal to one. On the other hand, when equities are more profitable then

[6]Money earns a positive real return only in the case of deflation.

[7]In details, $\xi = 1$ means "cash in advance", while $\xi \to \infty$ means that the Clower contraint can be ignored.

money balances, the real return on the former has to be reduced by a factor that is a increasing function of ξ[8].

Deriving s_t from equation (5), and substituting in equation (2), we derive an intertemporal budget constraint:

$$c_t + \frac{c_{t+1}}{\theta_t R_t} = w_t l_t \tag{6}$$

Imposing full employment, that is $l_t = 1$, the maximization of (1) subject to (6) leads to the following solutions:

$$c_t^* = \delta w_t \qquad c_{t+1}^* = \theta_t R_t (1-\delta) w_t \tag{7}$$

Necessarily, it also follows that $s_t^* = (1-\delta)w_t$. For a graphical exposition see figure 1.

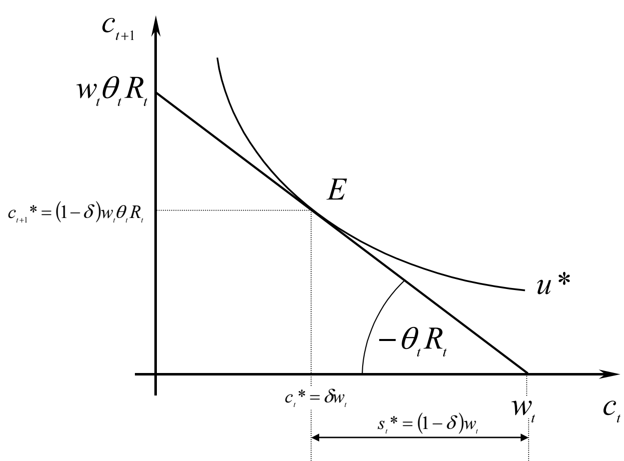

Figure 1: The households consumption choices

Note that in the simple Cobb-Douglas case, the marginal propensities to consume and to save are not affected by the real returns on savings[9]. Furthermore, notice that when young households are liquidity constrained, that is when $x_t R_t > 1$, a lower demand for money - *i.e.* a higher ξ - leads to a higher consumption in the old age.

[8]Notice that $v_t \equiv x_t R_t$ represents the ratio between the real return on equities and the real return on money.

[9]This is no longer true with a C.E.S. utility function.

2.2 The Productive Sector

Current production (y_t) is obtained through the utilisation of circulating capital (k_t) and labour supplied by young households (h_t). The production function is a traditional constant-return to scale Cobb-Douglas:

$$y_t = k_t^{\beta} h_t^{1-\beta} \qquad\qquad 0 < \beta < 1 \tag{8}$$

Normalising the total labour force to unity, the labour market-clearing hypothesis implies that $h_t = 1$ for all t.

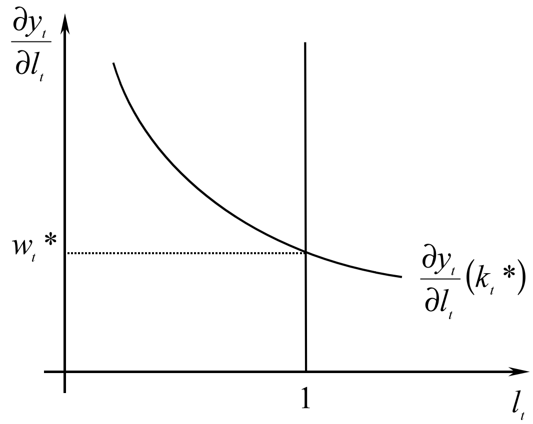

Figure 2: The labour market

In a competitive environment, the remunerations of each factor are given by the respective marginal productivities. As a consequence, the quantity signal given by the labour market-clearing hypothesis (see figure 2) allows to derive the following expressions:

$$w_t = \frac{\partial y_t}{\partial h_t} = (1-\beta)k_t^{\beta} \qquad\qquad R_t = \frac{\partial y_t}{\partial k_t} = \beta k_t^{\beta-1} \tag{9}$$

Considering the expression for R_t, we can easily derive the employed capital as a function of the real rate of return at time t:

$$k_t^* = \left(\frac{R_t}{\beta}\right)^{\frac{1}{\beta-1}} \tag{10}$$

From the previous equation, it follows immediately that:

$$w_t^* = (1-\beta)\left(\frac{R_t}{\beta}\right)^{\frac{\beta}{\beta-1}} = (1-\beta)y_t^* \tag{11}$$

Knowing the real wage, we can finally determine the households consumption choices as a function of the real gross return on bonds:

$$c_t^* = \delta(1-\beta)\left(\frac{R_t}{\beta}\right)^{\frac{\beta}{\beta-1}} \qquad c_{t+1}^* = \theta_t R_t(1-\delta)(1-\beta)\left(\frac{R_t}{\beta}\right)^{\frac{\beta}{\beta-1}}$$

Before going on, it may be worth to note that our model entails a small inconsistency. Since wages and profits are shares of the same output, one may ask the reason why young households have to wait for the old age to obtain the earnings deriving from capital investments. See equation (3). The reason is trivial. Production takes time, all the time underlying the single period considered in our analysis. At the very beginning of that, when young households plan their consumption, firms have to decide how much capital to employ[10]. In a market-clearing equilibrium, given the real interest rate, the investment level maximising profits coincides with the amount of resources lent to the productive sector.

In order to allow households to subscribe the debt issued by the firms, it is necessary to provide them the required resources. In other words, firms must to pay the overall wage bill before starting the production. The simplest way to overwhelm this inconsistency is to assume that in the background there is a bank which loan to the productive sector - without charging any interest - the corresponding amount of resources. Therefore, at very beginning of a given period, firms use the capital raised issuing the bonds, and hire the labour supplied by young households. At the end of the period, when the output is available, firms pays the resulting profits to the households which subscribed bonds[11], and repay their loans to the bank. At the beginning of the next period this financial cycle starts again and again.

2.3 The Walras's Law

Now we consider the aggregate situation at a given period t. At this time two generations coexist: young households born at the very beginning of t and old households born at the very beginning of $t-1$ (see figure 3). The former consume a share δ of the current real wage w_t. The latter consume

[10] We already know that the employed labour is fixed.

[11] The households that at the biginning of the period subcribed bonds are crossing the line of the old age.

the "effective" return on the saving they planned which, on turn, is equal to the share $(1-\delta)$ of the real wage prevailing in the previous period w_{t-1}.

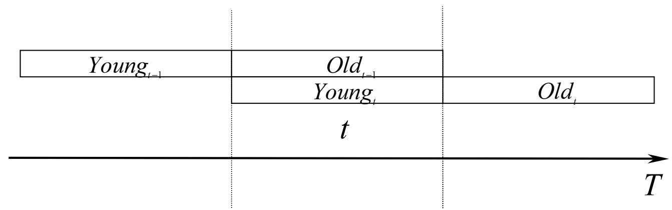

Figure 3: Generations overlap

Exploiting the results derived above, young households consumption $\left(c_t^Y\right)$ may be expressed as:

$$c_t^Y = \delta w_t = \delta(1-\beta)\left(\frac{R_t}{\beta}\right)^{\frac{\beta}{\beta-1}} \tag{12}$$

On the other hand, old households consumption $\left(c_t^O\right)$ is:

$$c_t^O = \theta_{t-1}R_{t-1}s_{t-1} = (1-\delta)(1-\beta)\theta_{t-1}R_{t-1}\left(\frac{R_{t-1}}{\beta}\right)^{\frac{\beta}{\beta-1}} \tag{13}$$

Deriving the expression for capital demand at time t from equation (10), we may write the real excess demand for goods as:

$$\chi_G \equiv c_t^Y + c_t^O + k_t^d - y_t \tag{14}$$

Using equations (10), (11), (12) and (13), we can derive the real excess demand for goods per unit of output:

$$\widehat{\chi}_G \equiv (1-\delta)(1-\beta)\theta_{t-1}R_{t-1}\left(\frac{R_{t-1}}{R_t}\right)^{\frac{\beta}{\beta-1}} + \delta(1-\beta) + \frac{\beta}{R_t} - 1 \tag{15}$$

where $\widehat{\chi}_G \equiv \frac{\chi_G}{y_t}$.

Let us turn to the real excess demand for debt. In that market, equation (10) defines the supply in real terms. To derive the demand we have to go back to the households side of the model. At the very beginning of time t, the demand for real debt comes from young households deciding to save a share of their current income. Assuming that the Clower constraint holds as equality, we can easily derive the optimal real cash balances as a fraction of planned savings:

$$m_t^* = \frac{\theta_t R_t x_t}{\xi} s_t^* \tag{16}$$

Now we can write the expression for the real demand of debt:

$$k_t^s = s_t^* - m_t^* = \frac{\xi - 1}{\xi + R_t x_t - 1} s_t^* \equiv \Psi_t s_t^*$$

Therefore, the real excess of demand for real debt per unit of output is the following:

$$\widehat{\chi}_B \equiv \frac{\chi_B}{y_t} = \frac{k_t^s - k_t^d}{y_t} = (1-\delta)(1-\beta)\Psi_t - \frac{\beta}{R_t} \tag{17}$$

Finally, let us turn to the real excess of demand for cash balances. In this model, the supply of money at time t comes from the cash balances demanded in the previous period by the old households augmented by its real return. As a consequence:

$$\chi_M \equiv m_t - m_{t-1} \frac{1}{x_{t-1}} \tag{18}$$

Assuming that the nominal stock of money is constant and equal to M, the model implies a regular circular flow of cash. In this particular case, the equality $\chi_M = 0$ is always verified.

The Walras's law states that:

$$\chi_G + \chi_B + \chi_M \equiv 0 \tag{19}$$

The equality holds necessarily. It could be easy verified following the traditional way in which the Walras's law is usually derived, that is, adding the budget constraints that at a given time period are binding for old and young households[12].

2.4 Steady State Equilibria

The first step toward studying the local dynamics of the model is the analysis of configurations of variables that are capable of reproducing themselves

[12]The Walras's law derived above does not contain a term for the excess demand for labour because we assumed from the beginning that the corresponding market always clears.

through time. A perfect foresight equilibrium such that, for all t, $R_t = R^*$ and $x_t = x^* = 1$ is said to be a steady-state equilibrium.

In our construction, there are two possible types of such an equilibrium: a *portfolio indifference steady-state* (PIS) and a *liquidity constrained steady-state* (LCS). The steady-state equations are the following:

$$\widehat{\chi}_G^* \equiv (1-\delta)(1-\beta)\,\theta^* R^* + \delta(1-\beta) + \frac{\beta}{R^*} - 1 = 0 \tag{20}$$

$$\widehat{\chi}_B^* \equiv (1-\delta)(1-\beta)\,\Psi^* - \frac{\beta}{R^*} = 0 \tag{21}$$

$$\chi_M^* \equiv m_t - m_{t-1} = 0 \tag{22}$$

In studying steady-state equilibria we will concentrate on the market for goods and the market for debt. With a constant nominal stock of money, equation (22) is automatically resolved.

Consider the case of a PIS, that is, the case in which $x^* = R^* = 1$:

$$\widehat{\chi}_G\,(\mathrm{PIS}) = (1-\delta)(1-\beta) + \delta(1-\beta) + \beta - 1 = 0 \tag{23}$$

$$\widehat{\chi}_B\,(\mathrm{PIS}) = (1-\delta)(1-\beta)\,\frac{\xi-1}{\xi} - \beta = 0 \tag{24}$$

The first equality is always verified. The second is verified whenever

$$\frac{\xi-1}{\xi} = \frac{\beta}{(1-\delta)(1-\beta)} \tag{25}$$

Equation (25) suggests that a PIS is a quite particular case[13]. The very Hahn and Solow (1995) state that the portfolio indifference story is almost unbelievable when we deal with perfect prices and wages flexibility. In the light of these observations, we will concentrate our attention only to the case in which households are liquidity constrained.

A situation of LCS implies that

$$x^* = 1 \quad \text{and} \quad R_t = R^* > 1$$

In term of real excesses of demand, this leads to

[13] As we shall see, solving equation (25) for ξ provides the upper bound for this parameter whenever δ and β impose a lower bound different from zero to the steady-state demand for money.

$$\widehat{\chi}_G\left(\text{LCS}\right) = \xi\frac{\left(1-\delta\right)\left(1-\beta\right)}{\xi+R^*-1}R^* + \delta\left(1-\beta\right) + \frac{\beta}{R^*} - 1 = 0 \tag{26}$$

$$\widehat{\chi}_B\left(\text{LCS}\right) = \frac{\left(1-\delta\right)\left(1-\beta\right)\left(\xi-1\right)}{\xi+R^*-1} - \frac{\beta}{R^*} = 0 \tag{27}$$

Equality (26) holds in two cases which may be distinct, that is

$$R^* = 1 \quad \text{and} \quad R^* = \beta\frac{\xi-1}{\left(1-\beta\right)\left[\delta+\xi\left(1-\delta\right)\right]-1}$$

Equality (27) holds only in the second case[14]. To rule out the non parametric solution $R* = 1$ from $\widehat{\chi}_G\left(\text{LCS}\right) = 0$, it is sufficient to observe that whenever agents are liquidity constrained (see the appendix) old households consumption as a fraction of current income is given by

$$\frac{\xi}{\xi-1}\beta\left(\frac{R_{t-1}}{R_t}\right)^{\frac{\beta}{\beta-1}} \tag{28}$$

Thanks to the previous expression, equations (26) and (27) now share the same parametric solution.

At this stage of analysis, we have to give a closer look at the unique expression for the steady-state level of the real gross return on bonds:

$$R^* = \frac{\beta\left(\xi-1\right)}{\left(1-\beta\right)\left[\delta+\xi\left(1-\delta\right)\right]-1} \equiv \frac{\beta\left(\xi-1\right)}{A} \tag{29}$$

In the plan (ξ, R), equation (29) represents a parametric hyperbola family with the following properties:

- $R^* = 0 \Leftrightarrow \xi = 1$
- $R^* = 1 \Leftrightarrow \xi = \frac{(1-\beta)(\delta-1)}{2\beta-1+\delta(1-\beta)} \equiv \xi_{\max}$
- if $\xi = 0$, then $R^* = \frac{\beta}{1-\delta(1-\beta)} < 1$
- $\lim_{\xi\to+\infty} R^* = \frac{\beta}{(1-\beta)(1-\delta)}$ *horizontal asymptote*
- $\xi_{\min} \equiv \frac{1-\delta(1-\beta)}{(1-\beta)(1-\delta)} > 1$ *vertical asymptote*
- $\frac{\partial R^*}{\partial \delta} > 0$, $\frac{\partial R^*}{\partial \beta} > 0$, and $\frac{\partial R^*}{\partial \xi} < 0$

[14] $\widehat{\chi}_B\left(\text{LCS}\right) = 0$ does not necessarily show the solution $R^* = 1$ because it is suited from the the beginning for the LCS case.

Since in a LCS R^* has to be higher than one, the listed points allow to derive some interesting conclusions. First, a situation of cash-in-advance ($\xi = 1$) is not consistent with a LCS. Second, the vertical asymptote ($\xi_{\min}$) constitutes the lower bound for the eligible values of ξ. Third, depending on the values of δ and β, there are situations in which there exist an upper bound ($\xi_{\max}$) for the eligible values of ξ, and situations in which such values are unbounded[15]. Simple algebra allows to state that the eligible values of ξ are not bounded from above whenever

$$\beta \geqslant \frac{1-\delta}{2-\delta}$$

The previous inequality is very important because it allows to distinguish situations in which there is an open limited interval for the admissible steady-state demand for money (bounded cases), from situations in which such a demand might be vanished (unbounded cases)[16]. For a graphical exposition see figures (4) and (5).

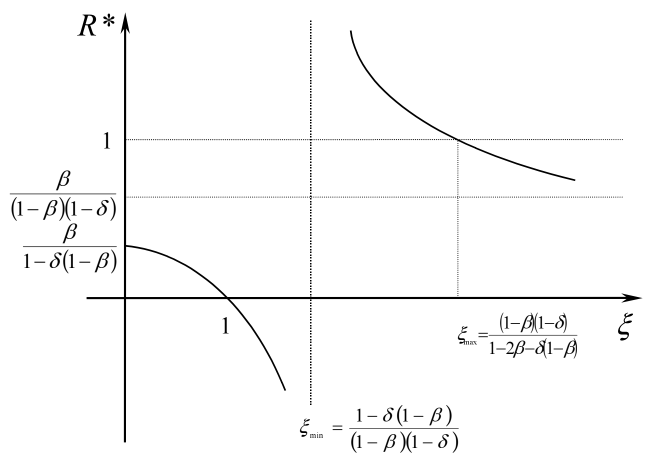

Figure 4: The bounded case

[15]The former are the situations in which the horizontal asymptote for R^* is lower than one. The latter are the cases in which such an asymptote is higher or equal to one.

[16]The algebraic analysis of $R*$ allows also to clarify the nature of the portfolio indifference steady-state. In details, a PIS in the asymptotic outcome toward which the economy tends when demand for money falls to its minimum admissible level.

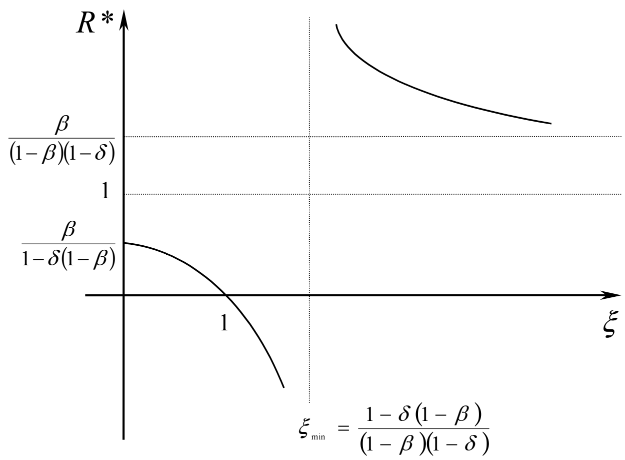

Figure 5: The unbounded case

A noteworthy attribute of figures (4) and (5) is the (positive) unexpected relationship between R^* and the demand for money. However, keeping in mind the model transaction technology, it is straightforward to verify that it is due to the positive relationship between the consumption in the old age and R^*. In fact,

$$\frac{\partial\left(\frac{c^{O*}}{y^*}\right)}{\partial R^*} = \frac{\xi\,(1-\delta)\,(1-\beta)\,(\xi-1)}{(\xi+R^*-1)^2} > 0$$

Summing up, the fundamental conclusion that should be drawn from the study of the hyperbola family representing the steady-state gross rate of return on bonds is that, once δ and β are given, it is also given the eligible interval for ξ. For short[17],

[17] If we consider the extreme bounds for the parameter δ, the inequality defining the eligible interval for ξ is easily explained in economic terms. When δ equals to 0, a LCS in which the demand for money can be vanished is possible when β is higher or equal to $\frac{1}{2}$, that is, when capital share is higher or equal to labour share. In this case, households do not care to consume in young age. Therefore, they will be willing to transfer all their labour income to old age. In such a transfer, households will avoid money only if bonds provide a return that in relative terms is higher than labour income. On the other hand, when δ is equal to 1, a LCS in which the Clower constraint can be ignored is possible for all the eligible values of β. In this case, households do not care to consume in old age. Therefore, whatever is the relative return on capital investment, they will avoid to transfer resources.

$$\forall\,(\delta,\beta)\in\{(0,1)\times(0,1)\}\,,$$

$$\xi\in(\xi_{\min},\xi_{\max})\quad\text{whenever }\beta<\frac{1-\delta}{2-\delta}$$

$$\xi\in(\xi_{\min},+\infty)\quad\text{whenever }\beta\geqslant\frac{1-\delta}{2-\delta}$$

2.5 Local Dynamics

In order to study the local dynamics around a generic LCS, it is necessary to make some assumptions about the consumption prices dynamics. Imposing $\chi_m^*=0$, it is straightforward to derive that

$$x_{t-1}=\frac{m_{t-1}}{m_t}$$

Therefore, if we assume that real money balances are constant over time we have:

$$x_{t-1}=x_t=1\qquad\forall t \tag{30}$$

In other words, this means to assume that the nominal amount of money and the consumption prices are following a dynamics which is consistent with the quantity theory of money.

The previous arguments lead to the following simplified steady-state equations:

$$\widehat{\chi}_G^*=\frac{\xi}{\xi-1}\beta+\delta\,(1-\beta)+\frac{\beta}{R^*}-1=0 \tag{31}$$

$$\widehat{\chi}_B^*=(1-\delta)\,(1-\beta)\,\frac{\xi-1}{\xi+R^*-1}-\frac{\beta}{R^*}=0 \tag{32}$$

The linearization around a generic LCS is given by the following expression[18]:

$$dR_t=\frac{\xi\beta^2B\,(\xi-1)^2}{B\,(\xi-1)^2\,[\xi\beta^2+A\,(\beta-1)]+(B-\beta)}dR_{t-1} \tag{33}$$

where $B\equiv(\xi-1)\,(1-\delta)\,(1-\beta)$.

The differential equation (33) states that the local dynamic properties of our system depend on the values of the parameters δ, β, and ξ. However, we

[18] The complete derivation is given in appendix.

have already shown that the eligible values of ξ depends exclusively on δ and β. Therefore, we conclude that the local dynamic properties of the system are conditioned solely by intertemporal preferences for consumption (δ) and the capital share (β). In other words, given an eligible value of ξ

$$dR_t \equiv \varphi(\beta, \delta)\, dR_{t-1}$$

In spite of its apparent simplicity, it is well know from the elementary theory on first-order differential equations that our model might display a quite large variety of local dynamic patterns, each of them depending on the actual value of $\varphi(\beta, \delta)$. Those local patterns are resumed in table 1.

$\varphi(\delta, \beta)$	Steady-State	Local Dynamics
$(-\infty, -1)$	*unstable*	*oscillations*
-1	*"flip" unstable*	*oscillations*
$(-1, 0)$	*stable*	*oscillations*
$(0, 1)$	*stable*	*monotone*
1	*indeterminate*	*indeterminate*
$(1, +\infty)$	*unstable*	*monotone*

Table 1: Local dynamic patterns

The occurrence of a "flip" instability, that is, the possibility of a closed two-periods cycle for admissible parameters values, is the case stressed by Hahn and Solow (1995).

2.6 Numerical Simulations

The expression for $\varphi(\beta, \delta)$ is not so manageable to be studied in an analytical way. Therefore, in order to establish how the local dynamic behaviour of the model is conditioned by δ and β, we proceeded performing some numerical simulations.

Before showing the simulation procedure and its results, a closer look at the expression for $\varphi(\beta, \delta)$ allows to state the following proposition:

Proposition 1 *In correspondence to the vertical asymptote for R^*, that is when $\xi = \xi_{\min}$, the model displays a locally indeterminate steady-state.*

Proof. In correspondence to the vertical asymptote for R^*, ξ is such that $A = 0$, therefore:

$$dR_t = \frac{\xi\beta^2 B(\xi-1)}{\xi\beta^2 B(\xi-1) + (B-\beta)} dR_{t-1}$$

Whenever $\xi = \xi_{\min}$, $(B-\beta) = 0$, hence

$$dR_t = dR_{t-1} \qquad Q.E.D.$$

Quite intuitively, proposition 1 suggests that when the steady-state level of the real interest rate approaches infinity, the local dynamics of the model is indeterminate.

Let us turn to the results of the numerical simulations. For given values of δ and β, in table 2 we find: (a) the interval for eligible ξs, (b) the interval in which the steady-state is stable or unstable, and (c) the interval in which the dynamics is monotone or oscillatory. Reasons of extension suggest to relegate table 2 in appendix. Such computations were obtained using specific MATLAB programs suited for the bounded and the unbounded case (see again the appendix). The most important conclusions that we drawn from them are the following:

- The simple OLG model we proposed might display *all* the variety of local dynamic patterns for parameter values that are not extreme;
- For a given value of δ, the higher is the value of β, the smaller is the interval of eligible ξ values such that the steady-state is locally stable;
- When β and δ approach their maximum level, there is a small interval of eligible values of ξ such that the steady-state is locally indeterminate;

The listed points deserve a wider argumentation. For example, consider the case in which $\delta = 0.6$ and $\beta = 0.3$. Obviously, this is an unbounded case: eligible ξs vary in the interval $(2.0714, +\infty)$. A plot of the steady-state level consumption by old $\left(c^{O*}\right)$ and young $\left(c^{Y*}\right)$ households against ξ is given figure 6.

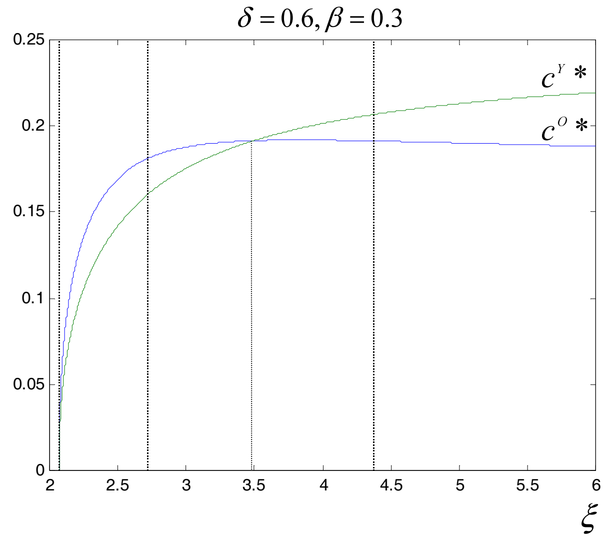

Figure 6: An example

From the results enclosed in table 2, we know that the steady-state is locally stable in the intervals $(2.0714, 2.7214)$ and $(25.471, +\infty)$. In the former the adjustment is monotone. In the latter the adjustment occurs through convergent oscillations. Clearly, 25.471 is the value of ξ such that the steady-state displays a "flip" instability. Furthermore, we realise that the steady-state is locally unstable in the interval $(2.7214, 25.471)$. However, until 4.3814 the adjustment is monotone (explosive or implosive). Beyond this value, the adjustment occurs through divergent oscillations.

Given the negative steady-state relationship between R^* and ξ, the consumption pattern can be easily rationalised. At the beginning, the steady-state real return on bonds is high, and this leads households to postpone consumption. Therefore, the difference $\left(c^{O*} - c^{Y*}\right)$ is positive. However, leading to lower levels of R^*, higher values of ξ are related to negative values in the difference between the consumption by old and the consumption by young households.[19].

In spite of their intriguing characteristics, an overall outlook to the numerical simulations suggests that the cases in which it occurs a "flip" instability are not very frequent. If we pass over them, we observe that the demand for

[19]This result, obviously, depends on the value of δ. More detailed computations (not enclosed in the paper) show that whenever $\delta < 0.6$, LCSs ($R^* > 1$) are characterised by a consumption in old age that is higher than the counterpart in young age. On the other hand, when $\delta \geqslant 0.6$, LCSs start to display negative values for the difference $(c^{O*} - c^{Y*})$. However, the rising of β - leading to a rise in R^* - contributes to restore positive values.

real cash balances seems to play a stability role: the steady-state becomes locally unstable when demand for money falls down a certain threshold. This statement should provide some good insights to explain the way in which intertemporal preferences and distributive shares interact in determining the local dynamic properties of the model.

Our thesis is that local stability could be related to consumption smoothing considerations. To this purpose, consider the difference between the consumption by old and young households per unit of output[20]:

$$H_t \equiv \frac{c_t^O - c_t^Y}{y_t} = \frac{\xi}{\xi - 1}\beta \left(\frac{R_{t-1}}{R_t}\right)^{\frac{\beta}{\beta-1}} - \delta\left(1-\beta\right) \tag{34}$$

Since $\beta \in (0,1)$, H_t is a decreasing curve if plotted against $\frac{R_{t-1}}{R_t}$. For a graphical exposition see figure 7.

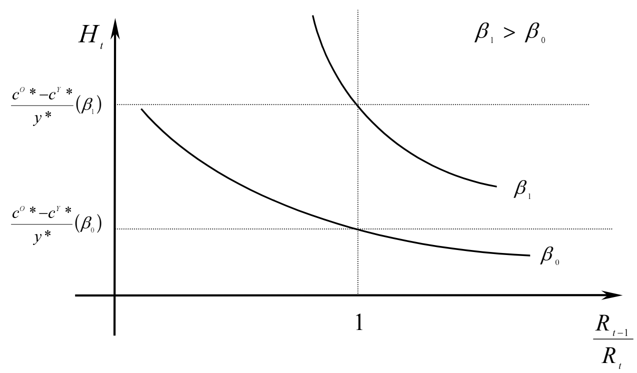

Figure 7: The consumption by old and young households

The slope of H_t in a steady state equilibrium has the following expression:

$$\left.\frac{\partial\left(H_t\right)}{\partial\left(\frac{R_{t-1}}{R_t}\right)}\right|_{R_{t-1}=R_t=R^*} = \frac{\xi\beta^2}{\left(\xi-1\right)\left(\beta-1\right)} < 0$$

An increase in β leads to an increase in the level of H^*, and in the slope that H_t has in a steady-state equilibrium. This means that when β is high, small deviations from 1 - that is, the steady-state ratio between the gross return on bonds in two successive periods - leads to big differences between H_t and its steady-state level. The simulations results suggest that the continuos

[20]The difference H_t should be thought as a measure of consumption smoothing.

market-clearing hypothesis involves some problems in the occurrence of local stability when these differences become too high. On the other hand, the parameter δ has no influence on the shape of H_t. This should explain why an increase in δ, even if it reduces H^*, makes for a more difficult local stability.

It possible to extend a similar argument even to the market for real debt. The equilibrium condition in that market is given by

$$(1-\delta)(1-\beta)\Psi_t = \frac{\beta}{R_t} \tag{35}$$

If plotted against R_t, the left-hand side of equation (35) (the amount of resources lent to the productive sector per unit of output) has always a lower slope than the right and side (the investment per unit of output). See figure 8.

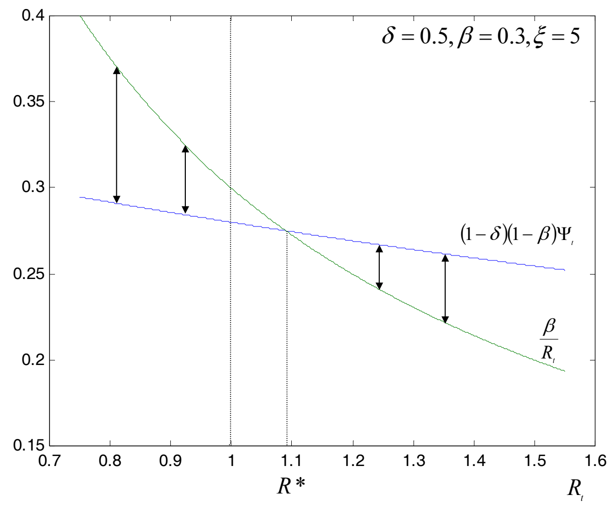

Figure 8: The market for real debt

An increase in β acts reducing the slope of the left-hand side and increasing the slope of the right-hand side. An increase in δ produces the same effect on the share of savings offered to the productive sector. Therefore, it is straightforward that high levels of δ and β leads small deviations from R^* to be associated with large disequilibria in the market for real debt. Even in this case, the simulations results suggests that wider disequilibria create some problems in the occurrence of local stability.

The role of the demand for money is more intricate. Transferring resources from young to old age, it contributes to rise H^*. Furthermore, an higher demand for money leads to a steeper H_t curve which, on turn, makes for large variations in the consumption smoothing measure outside the steady-state equilibrium. On the other hand, the effects on the slope of $(1-\delta)(1-\beta)\Psi_t$ are not univocal. Whenever $\xi + R^* < (>)\, 3$, a lower (higher) demand for money makes for a flatter (steeper) $(1-\delta)(1-\beta)\Psi_t$. These relationships - together with the simulation results - suggest that the way in which we introduce money in the model is quite *ad hoc*. Therefore, a precise role in determining local stability is hard to configure[21].

3 Conclusions

In this paper we analysed a two-periods OLG with perfect prices and wages flexibility following the lines of the framework proposed by Hahn and Solow (1995). As in the original contribution, our model encloses four markets which always clear: the labour market, the market for goods, the market for bonds, and the market for money.

The model was developed treating the labour market a little asymmetrically with respect to other markets. The real wage equals the marginal product of labour in each instant ensuring the continuos full employment. Given the real wage, we showed that the market for goods, the market for bonds, and the market for money are linked by a particular version of the Walras's law. This link allowed to ignore the market for money and to concentrate our attention on the equilibrium condition concerning the markets for goods and bonds.

Assuming a prices dynamics that is consistent with the quantity theory of money, we showed that in a situation in which households are liquidity constrained the parametric expression for the steady-state level of real interest rate is unique. The linearisation around the unique steady-state revealed that the local dynamic properties of the model depend only on households intertemporal preferences and distributive shares. Furthermore, the results of some numerical simulations suggested that is possible to establish a link between the steady-state local stability and the consumption smoothing operated by households. Whenever deviations from the steady-state are associated to large differences between the consumption by the old and by the

[21] Probably, more interesting insights could be obtained inserting money in the utility function or in the production function.

young, the hypothesis of continuos market-clearing seems to fail to pin down a locally stable equilibrium.

Our simple model could be developed in different directions. As suggested by Geanakoplos and Polemarchakis (1986), it could be interesting to analyse a "Keynesian" equilibrium in which labour market is permitted not to clear. This would lead to build a model with rationing in which unemployment is involuntary (Malinvaud, 1977).

Another development could be the analysis of a situation in which there are increasing returns in the production technology. As in the work of Benhabib and Farmer (1994), this should make the indeterminacy of the steady-state more likely to occur.

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A Appendix

Whenever agents are liquidity constrained $R_t x_t > 1$, that is, $R_t > \frac{1}{x_t}$. Furthermore, the equilibrium condition in the market for bonds requires that

$$k_t^{s*} = s_t^* \left[\frac{\xi - 1}{\xi + R_t x_t - 1} \right] = s_t^* \Psi_t = I_t^*$$

We know that $\theta_t = \frac{\xi}{\xi + R_t x_t - 1}$, therefore $\Psi_t = \frac{\xi - 1}{\xi} \theta_t$. Hence:

$$k_t^{s*} = s_t^* \frac{\xi - 1}{\xi} \theta_t = I_t^* \Rightarrow \theta_t s_t^* = \frac{\xi}{\xi - 1} I_t^*$$

Multiply each member by R_t

$$R_t \theta_t s_t^* = \frac{\xi}{\xi - 1} R_t I_t^*$$

Note that $s_t^* R_t \theta_t = c_{t+1}^*$, therefore:

$$c_t^O = \frac{\xi}{\xi - 1} R_{t-1} I_{t-1}^*$$

Finally, in terms of unit of output:

$$\frac{c_t^O}{y_t} = \frac{\xi}{\xi - 1} R_{t-1} \frac{I_{t-1}^*}{y_t} = \frac{\xi}{\xi - 1} \beta \left(\frac{R_{t-1}}{R_t} \right)^{\frac{\beta}{\beta - 1}} \qquad Q.E.D.$$

B Appendix

Now we derive the expression for the linearisation around the (unique) steady-state. Assuming $x_{t-1} = x_t = 1$, the expressions for the excess demand for goods and the excess demand for real debt per unit of output are given by

$$\widehat{\chi}_G \equiv \frac{\xi}{\xi - 1} \beta \left(\frac{R_{t-1}}{R_t} \right)^{\frac{\beta}{\beta - 1}} + \delta (1 - \beta) + \frac{\beta}{R_t} - 1$$

$$\widehat{\chi}_B \equiv (1 - \delta)(1 - \beta) \frac{\xi - 1}{\xi + R_t - 1} - \frac{\beta}{R_t}$$

Since $R^* = \frac{\beta(\xi - 1)}{A}$, it is straightforward to derive

$$\left. \frac{\partial \widehat{\chi}_G}{\partial R_{t-1}} \right|_{R_{t-1} = R_t = R^*} = \frac{\xi \beta}{(\beta - 1)(\xi - 1)^2} A$$

$$\left.\frac{\partial\widehat{\chi}_G}{\partial R_t}\right|_{R_{t-1}=R_t=R^*} = -\left[\frac{\xi\beta^2+(\beta-1)A}{\beta(\beta-1)(\xi-1)^2}\right]A$$

$$\left.\frac{\partial\widehat{\chi}_B}{\partial R_t}\right|_{R_{t-1}=R_t=R^*} = -\frac{\beta-(1-\beta)(\xi-1)(1-\delta)}{\beta(1-\beta)(\xi-1)^3(1-\delta)}A^2$$

where $A\equiv(1-\beta)[\delta+\xi(1-\delta)]-1$.

Total differentiation leads to

$$\xi\beta dR_{t-1}-\frac{\xi\beta^2+(\beta-1)A}{\beta}dR_t=0$$

$$-\frac{\beta-B}{\beta B(\xi-1)^2}dR_t=0$$

where $B\equiv(1-\beta)(\xi-1)(1-\delta)$.

Equalising

$$\xi\beta dR_{t-1}-\frac{\xi\beta^2+(\beta-1)A}{\beta}dR_t=-\frac{\beta-B}{\beta B(\xi-1)^2}dR_t$$

Finally, solving for dR_t

$$dR_t=\frac{\beta^2\xi B(\xi-1)^2}{B[\xi\beta^2+(\beta-1)A](\xi-1)^2+B-\beta}dR_{t-1}\qquad Q.E.D.$$

C Appendix

```
% MATLAB Program for bounded cases
clear all
format short g
sw=0;
delta=0.5
beta=0.3
disc=(1-delta)/(2-delta) % in bounded cases beta has to be lower than disc
vertasy=(1-(delta*(1-beta)))/((1-beta)*(1-delta)) % vertical asymptote for R*
(minimum value of xi)
ximax=((1-beta)*(delta-1))/(2*beta-1+delta*(1-beta))  % maximum value of xi
whenever the horizontal asymptote is lower than 1
for xi=vertasy:0.001:ximax
   sw=sw+1;
   A(sw)=((1-beta)*(delta+(xi*(1-delta)))-1);
   R(sw)=((beta*(xi-1))/A(sw));
   invest(sw)=((R(sw)/beta)^(1/(beta-1)));
   cons_old(sw)=((xi/(xi-1))*R(sw)*invest(sw));
   output(sw)=((R(sw)/beta)^(beta/(beta-1)));
   cons_young(sw)=(delta*(1-beta)*output(sw));
   real_wage(sw)=((1-beta)*output(sw));
   money(sw)=((1/xi)*cons_old(sw));
   saving(sw)=real_wage(sw)-cons_young(sw);
   conf(sw)=(cons_old(sw)-cons_young(sw));
   conf_asso(sw)=abs(conf(sw));
   conf_rel(sw)=conf(sw)/output(sw);
   conf_rel_asso(sw)=conf_asso(sw)/output(sw);
   phinum(sw)=((beta^2)*xi*(1-beta)*((xi-1)^3)*(1-delta));
   phidenB(sw)=((xi-1)*(1-delta)*(1-beta));
   phidenC(sw)=(((xi*((beta)^2)+((beta-1)*A(sw)))*((xi-1)^2))+1);
   phi(sw)=(phinum(sw)/((phidenB(sw)*phidenC(sw))-beta));
   XI(sw)=xi;
end
```

D Appendix

```
% MATLAB Program for unbounded cases
clear all
format short g
sw=0;
delta=0.6
beta=0.3
disc=(1-delta)/(2-delta) % in unbounded cases beta is higher or equal to disc
vertasy=(1-(delta*(1-beta)))/((1-beta)*(1-delta)) % vertical asymptote for R*
(minimum value of xi)
% In this case there is no upper bound for xi
for xi=vertasy:0.001:6
   sw=sw+1;
   A(sw)=((1-beta)*(delta+(xi*(1-delta)))-1);
   R(sw)=((beta*(xi-1))/A(sw));
   invest(sw)=((R(sw)/beta)^(1/(beta-1)));
   cons_old(sw)=((xi/(xi-1))*R(sw)*invest(sw));
   output(sw)=((R(sw)/beta)^(beta/(beta-1)));
   cons_young(sw)=(delta*(1-beta)*output(sw));
   real_wage(sw)=((1-beta)*output(sw));
   money(sw)=((1/xi)*cons_old(sw));
   saving(sw)=real_wage(sw)-cons_young(sw);
   conf(sw)=(cons_old(sw)-cons_young(sw));
   conf_asso(sw)=abs(conf(sw));
   conf_rel(sw)=(conf(sw)/output(sw));
   conf_rel_asso(sw)=conf_asso(sw)/output(sw);
   phinum(sw)=((beta^2)*xi*(1-beta)*((xi-1)^3)*(1-delta));
   phidenB(sw)=((xi-1)*(1-delta)*(1-beta));
   phidenC(sw)=(((xi*((beta)^2)+((beta-1)*A(sw)))*((xi-1)^2))+1);
   phi(sw)=(phinum(sw)/((phidenB(sw)*phidenC(sw))-beta));
   XI(sw)=xi;
end
```

E Appendix

Table 2: Simulations results

δ	β	ξ values			
		eligible	stability	monotonic	oscillations
0.1	0.1	1.1408, 1.235	*all*	*all*	$\varnothing$
0.1	0.2	1.2778, 1.3846	*all*	*all*	$\varnothing$
0.1	0.3	1.4762, 1.9091	*all*	*all*	$\varnothing$
0.1	0.4	1.7407, 3.8571	1.7407, 2.45	*all*	$\varnothing$
0.1	0.5	2.111, $+\infty$	2.111,2.65	*all*	$\varnothing$
0.1	0.6	2.6667,$+\infty$	2.6667, 2.9	*all*	$\varnothing$
0.1	0.7	3.5926, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.1	0.8	5.4444, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.1	0.9	11, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.2	0.1	1.1389, 1.1613	*all*	*all*	$\varnothing$
0.2	0.2	1.3125, 1.4545	*all*	*all*	$\varnothing$
0.2	0.3	1.5357, 2.1538	*all*	*all*	$\varnothing$
0.2	0.4	1.8333, 6	1.8333, 2.51	1.8333, 4.6	4.6, 6
0.2	0.5	2.25, $+\infty$	2.25, 2.7	*all*	$\varnothing$
0.2	0.6	2.875, $+\infty$	2.875, 2.985	*all*	$\varnothing$
0.2	0.7	3.9167, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.2	0.8	6, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.2	0.9	12.25, $+\infty$	$\varnothing$	12.31, $+\infty$	(*indeterminacy*) 12.25, 12.31
0.3	0.1	1.1587, 1.1887	*all*	*all*	$\varnothing$
0.3	0.2	1.3571, 1.5556	*all*	*all*	$\varnothing$
0.3	0.3	1.6122, 2.5789	1.6122, 2.428	*all*	$\varnothing$
0.3	0.4	1.9524, 21	1.9524, 2.6	1.952, 5.737	5.737, 6
0.3	0.5	2.4286, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.3	0.6	3.1429, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.3	0.7	4.333, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.3	0.8	6.7143, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.3	0.9	13.857, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.4	0.1	1.1852, 1.2273	*all*	*all*	$\varnothing$
0.4	0.2	1.4167, 1.7143	*all*	*all*	$\varnothing$
0.4	0.3	1.7143, 3.5	1.7143, 2.5037	1.714, 3.1687	3.1687, 3.5
0.4	0.4	2.111, $+\infty$	2.111, 2.6611	2.111, 8.4111	8.4111, $+\infty$
0.4	0.5	2.667,$+\infty$	2.667,2.8767	*all*	$\varnothing$
0.4	0.6	3.5, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.4	0.7	4.889, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.4	0.8	7.6667, $+\infty$	$\varnothing$	*all*	$\varnothing$
0.4	0.9	16, $+\infty$	$\varnothing$	*all*	(*small region of indeterminacy*)

					∅
0.5	0.1	1.222, 1.2857	*all*	*all*	∅
0.5	0.2	1.5, 2	*all*	*all*	∅
0.5	0.3	1.8571, 7	1.8571, 2.5971	1.857, 3.5811	3.5811, 7
0.5	0.4	2.333, +∞	2.333,2.7633	2.333,21.113	21.113,+∞
0.5	0.5	3, +∞	∅	*all*	∅
0.5	0.6	4, +∞	∅	*all*	∅
0.5	0.7	5.6667, +∞	∅	*all*	∅
0.5	0.8	9, +∞	∅	*all*	∅
0.5	0.9	19, +∞	∅	*all*	∅
0.6	0.1	1.2778, 1.3846	*all*	*all*	∅
0.6	0.2	1.625, 2.6667	1.625, 2.5745	*all*	∅
0.6	0.3	2.0714, +∞	2.0714, 2.7214 (25.471, +∞)	2.071,4.3814	4.3814, +∞ (ξ=25.471 “flip”)
0.6	0.4	2.6667, +∞	2.6667, 2.9067	*all*	∅
0.6	0.5	3.5, +∞	∅	*all*	∅
0.6	0.6	4.75, +∞	∅	*all*	∅
0.6	0.7	6.833, +∞	∅	*all*	∅
0.6	0.8	11, +∞	∅	*all*	∅
0.6	0.9	23.5, +∞	∅	*all*	(*small region of indeterminacy*) ∅
0.7	0.1	1.3703, 1.5882	*all*	*all*	∅
0.7	0.2	1.8333, 6	1.8333, 2.7333 (3.8993, 6)	1.833, 3.1743	3.1743, 6 (ξ=3.8993 “flip”)
0.7	0.3	2.4286, +∞	2.4286, 2.8886	2.428, 6.6686	6.6686, +∞
0.7	0.4	3.2222, +∞	∅	*all*	∅
0.7	0.5	4.3333, +∞	∅	*all*	∅
0.7	0.6	6, +∞	∅	*all*	∅
0.7	0.7	8.7778, +∞	∅	*all*	∅
0.7	0.8	14.333, +∞	∅	*all*	∅
0.7	0.9	31, +∞	∅	*all*	*small region of indeterminacy*
0.8	0.1	1.555, 2.25	*all*	*all*	∅
0.8	0.2	2.25, +∞	2.25, 2.98 (6.52, +∞)	2.25, 4	4, +∞ (ξ=6.52 “flip”)
0.8	0.3	3.1429, +∞	3.1429, 3.1629	3.142, 38.583	38.583 ,+∞
0.8	0.4	4.333,+∞	∅	*all*	∅
0.8	0.5	6, +∞	∅	*all*	∅
0.8	0.6	8.5, +∞	∅	*all*	∅
0.8	0.7	12.6667, +∞	∅	*all*	∅
0.8	0.8	21, +∞	∅	*all*	(*small region of indeterminacy*) ∅
0.8	0.9	46, +∞	∅	46.95, +∞	(*indeterminacy*) 46, 46.95
0.9	0.1	2.1111, +∞	2.1111, 3.3111 (3.9611, +∞)	2.1111, 3.591	3.591,+∞ (ξ=3.9611 “flip”)

0.9	0.2	3.5,+∞	∅	3.5, 9.72	9.72, +∞
0.9	0.3	5.2857, +∞	∅	*all*	∅
0.9	0.4	7.6667, +∞	∅	*all*	∅
0.9	0.5	11, +∞	∅	*all*	∅
0.9	0.6	16, +∞	∅	*all*	∅
0.9	0.7	24.333, +∞	∅	*all*	(*small region of indeterminacy*) ∅
0.9	0.8	41, +∞	∅	*all*	(*small region of indeterminacy*) ∅
0.9	0.9	91, +∞	∅	*all*	(*small region of indeterminacy*) ∅

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