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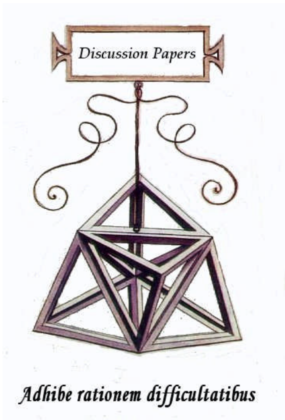

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Scale Effects in Idea-Based Growth Models:

a Critical Survey

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Discussion Paper
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Scale Effects in Idea-Based Growth Models: a Critical Survey

Abstract

This paper gives an overview on the topic of scale effects in idea-based growth models in a closed economy. I deal with weak and strong scale effects and with the crucial distinctive features of the three strata of idea-based models. I comment third-generation models emphasizing their fragile framework because of treating the potential R&D "spillover space" in a too simplified way. Finally I argue according to this and other shortcomings of third-generation models that a definite mapping of the relationship between population size and economic growth requires further research: strong scale effect is still on agenda!

Keywords: idea-based growth models, scale effects, R&D spillovers

1. Introduction

Does the population size (scale) matter for the economy? The answer is yes if idea-based growth models are realistic representation of the world, as Jones (2004) emphasized. However in the last decade there was a flourishing debate about the way scale can influence the performance of the economy. The starting point are the early idea-based growth models of Romer (1990) and Aghion – Howitt (1992) which exhibit an empirically uncomfortable strong scale effect, that is a positive relationship between the steady-state growth rate of the economy and the population size. Though the second-generation idea-based growth model of Jones (1995 b) and Segerstrom (1998) solved the problem by incorporating imperfect knowledge spillovers, they changed an empirically uncomfortable feature for an even so uncomfortable characteristic from the viewpoint of the theory. In these models long-run growth is semi-endogenous because it is ultimately pinned down by variables that are usually regarded as being outside of the range of economic policy. This is really an unappealing statement not denying that increasing complexity of technology can induce imperfect spillovers. This was the primary inspiration for economists to create a new stratum of idea-based models incorporating the seminal contribution of Young (1998) that the entry of new firms to the market can perfectly dilute the larger pool of rents for successful innovations originating from a larger aggregate market-(population-) size. Based on that technique, though with significantly different microstructures, with third-generation models (Aghion – Howitt 1998 ch. 3,, Dinopoulos – Thompson 1998, Peretto 1998 a) strong scale effect vanishes while the endogenity of the steady state growth rate is preserved. Although preliminary empirical observations (Ha-Howitt 2005, Laincz-Peretto 2004) do not contradict their theoretical suggestions, they have a fragile framework by handling the potential R&D "spillover space" in a stripped down way. Namely, third-generation models practically omit inter-firm R&D spillovers, which is a crucial point in them when eliminating strong scale effect.

The aim of this paper is to give an overview on the issue of scale effect *in a closed-economy context* – which is not the first trial in the literature, see Jones (1999), Dinopoulos-Thompson (1999), Ha-Howitt (2005) – and to shed light on the fragile framework of third-generation models and its consequences. The paper is organized as follow. Section 2. introduces the partial equilibrium framework of the analysis. Section 3. discusses the problem of weak and strong scale effects in idea-based growth models. Section 4. elaborates the comment on third-generation models by stressing the absence of inter-firm R&D spillovers and its consequences from the viewpoint of strong scale effect and (non-)accelerating growth rate. The paper is closed with some final remarks suggesting that concerning the previous and other shortcomings of third-generation models a definite mapping of the relationship between population size and economic growth requires further research: strong scale effect is still on agenda!

2. The Framework of the Analysis

To investigate the difference between the three main strata of idea-based growth models regarding their predictions on scale effects it is useful to consider a kind of stripped down version economic model incorporating expanding product variety and improving product- (process-) quality. Three basic questions arise before starting with that kind of set up. Which way to interpret product variety: the way Dixit-Stiglitz did or the one Either did follow; that is to introduce it in to the final or to the intermediate sector[1]? How to handle quality improvement in the model: as a process that improves production-technology or the technology of the product? To work with a Schumpeterian quality-ladder model with creative destruction or with a model with infinitely living firms? From one point of view these are only questions of modelling without any crucial importance and I refer the reader to the third strata of idea-based growth models already cited in the introduction to find an example for all of them. In this paper based on Peretto (1999 and 1998 a) I have chosen a framework with a Dixit-Stiglitz consumption index, infinitely living firms producing differentiated consumption goods and improving their production-technology, which is a continuous Romer-like state variable. The reason of my choice is that it is a more tractable way to present the relationship between the R&D-decision of a firm and the demand for its product (the scale of its market) and to make it shown how product diversity may dilute the increasing aggregate demand coming from the population growth. However I stress that the model below is a partial equilibrium one. It concentrates only on the equilibrium decisions of households and incumbent firms and omits the discussion of the decision problem for entrants and the general equilibrium condition of labour market clearing. Product-market clearing is always present by the production decision of the individual firms (see Eq. (5)). Capital-market clearing prevails because of the fixed market interest rate and the decreasing returns on R&D when R&D employment increases (see Eq. (7) and (15)). I would like to emphasize that my unique goal was to sketch *the link between the size of population and firms R&D decision*, which *is one of the main core of strong scale effect*. For a more detailed discussion of this set up in a general equilibrium context see eg. Peretto (1999 and 1998 a).

2.1 Households' Optimisation Problem

The households are the usual Ramsey-type ones with infinite time horizon and the same preferences. The decision problem of the representative household is

(1) $$\max U_t = \int_t^{\infty} e^{-(\rho-n)(\tau-t)} \ln C_\tau \, d\tau$$

according to the flow-budget constraint, where ρ is the subjective discount factor, n is the population growth rate and C is a Dixit-Stiglitz consumption index taking into account the utility gain from product variety. More precisely:

(2) $$C = \left(\int_0^N c_i^{\sigma} di \right)^{\frac{1}{\sigma}} (= N^{\frac{1}{\sigma}} c) \quad , 0 < \sigma < 1 ,$$

[1] See eg. Grossman-Helpman (1991, ch.3)

where N is the number of final-products[2], σ is the CES parameter and c_i is the quantity consumed per capita from product i. The equation in parenthesis presents the case of symmetry, as it will turn out to prevail in this paper. For simplicity of notations I dropped the time indexes, as I will do in the rest of the paper when no confusion emerges.

The solution for this optimisation problem is the well-known time-path for the expenditure per capita

$$\frac{\dot{E}}{E} = r - \rho$$

and the demand function for product i

(3) $$c_i = \frac{EP_i^{-\varepsilon}}{\int_0^N P_j^{1-\varepsilon} dj} \quad , 1 < \varepsilon < \infty \, ,$$

where P_i is the price of product i, E is the expenditure per capita, r is the market interest rate and ε is the elasticity of substitution in Eq. (2). One can set the value of r equal to the subjective discount rate without loss of any information from our point of view in order to get a constant expenditure per capita time-path.

From Eq. (3) we can easily derive the aggregate demand function for product i

(4) $$C_i = \frac{LEP_i^{-\varepsilon}}{\int_0^N P_j^{1-\varepsilon} dj} \left(= \frac{LE}{NP} \right) ,$$

where the second equation in parenthesis represents again the case of symmetry.

2.2 The Incumbents' Optimisation Problem

There is only one type of goods: the final consumption good; which consists of differentiated products each one produced in a monopolistic competition by separate firms. Every product has only one producer either because of patent-rights or the secrecy of production technology, which is internal to the firms and largely tacit – not codifiable – as well. In the production only raw labour and the firm's own technology are used

[2] In this paper firms, products and product-lines are treated as equal terms.

(5) $C_i = A_i L_{Ci}$, where A_i is the firm's production technology and L_{Ci} is labour.

The production technology is a state variable according to the following deterministic accumulation equation

(6) $\dot{A}_i = L_{Ai}^{\gamma} A_i^{\phi}$, $0< \gamma <1$ and $\phi = 1$,

where the dot means the time-derivative and L_{Ai} is labour.

Before proceeding I have to stress three things related to Eq. (5) and (6). *First*, as Peretto (1998 a) emphasized technology is used in a non-rival manner only at the firm level, which is the case in Eq. (5). The reasons for this can be an effective patent-protection system or just the tacit and "firm-specific"[3] feature of a large part of technology. The consequence is that increasing returns to scale are present at the firm level but not at the aggregate level. So if one constructs an aggregate production function, the technology level of a representative firm has to be put in the place of the "technology variable". This is a crucial and non-arguable point in improving quality multi-firms model, like the third-generation ones.

Second, though third generation idea-based growth models assume that "firm-specific" technology advance contributes to the "pool of public knowledge", which can be used in a non-rival manner in the R&D process of the firms allowing to grow by *standing on each other's shoulder,* this is only the *surface*! To realize it recall the way these models define the "pool of public knowledge". In Dinopoulos-Thompson (1998) and Peretto (1998 a) it is regarded as an (weighted) average technology level of the separate firms. In Aghion-Howitt (1998 ch 3.) it is the technology frontier to which every innovation in any sector contributes with a spillover parameter ($\ln\delta/N$) linearly decreasing in the number of sectors[4]. So disregarding now the emerging possible theoretical or empirical weaknesses, the fact is that in third-generation models the point is about *standing on its own shoulders*[5]. Dinopoulos-Thompson (1999) referred to this essential feature of these models as "*localized intertemporal R&D spillovers*". Saying other, what solely matters in the R&D process of a firm is its own past and present research effort. This is embodied in Eq.(6).

Third, the γ parameter in Eq. (6) has in our case no special economic significance – though it is common in the literature to interpret it as a congestion parameter –, the only purpose it serves now is to ensure that in the absence of entry capital market clearing can be possible without changing the market interest rate (see below).

The decision problem of the incumbents is

$$\max_{Pi,\tau;LAi,\tau} V_{i,t} = \int_t^{\infty} e^{-(r\tau)} \pi_{i,\tau} \, d\tau \quad ,$$

[3] Peretto (1998 a)

[4] $\dot{Q}_{\max} = \frac{\ln\delta}{N} INQ_{\max} = (\ln\delta) IQ_{\max}$, where $Q_{\max}$ is the technology frontier, I is the number of innovations in a sector and ($\ln\delta/N$) is the spillover parameter.

[5] The "pool of public knowledge" assumption in models with stochastic R&D processes (Aghion-Howitt 1998 ch.3, Dinopoulos-Thompson 1998) has the practical role to ensure symmetry in R&D efforts of the various sectors by providing the same pay-off for any successful innovation independent of the relative technology level of the given sector.

where $V_{i,t}$ is the present value of firm i at time t and π_i is the instantaneous profit according to

$$\pi_i = P_i C_i - L_{ci} - L_{Ai} = (P_i - A_i^{-1})C_i - L_{Ai}$$, where the wage is the numeraire.

In this dynamic optimisation problem (see Appendix 1. for more details) the control variables are the price and the R&D input, while the accumulated technology level is the state variable. Now if we pin down the initial level of the state variable at an equal value for every firm, symmetry prevails and at every point in time prices charged, R&D worker employed and productivity achieved will be the same across the firms. From the first-order conditions when maximizing the objective function we are able to derive the capital market clearing condition

$$r = \frac{LE(\varepsilon - 1)\gamma}{N\varepsilon L_{Ai}^{1-\gamma}} + (1-\gamma)\frac{\dot{L}_{Ai}}{L_{Ai}}. \tag{7}$$

The RHS of Eq. (7) is the returns to R&D for firm i. In constructing Eq. (7) I made use of the symmetry feature of the model[6].

2.3 Growth

In models with differentiated final consumption goods the proper measure of real income per capita *(y)* is expenditure per capita divided by the unit price of the composite consumption good embodied in Eq. (2), that is

$$y = E / P_C = C \quad .$$

So real income per capita equals the Dixit-Stiglitz consumption index. Substituting Eq. (5) into Eq. (2) and taking into account the symmetry we get

$$y = C = N^{\frac{1}{\sigma}} A_i \frac{L_C / N}{L}$$, where L_C is the aggregate labour input in production.

In steady-state the share of aggregate labour force employed in the production (s_{Lc}) must be constant, so the steady-state income per capita is

$$y^* = (N^*)^{\frac{1-\sigma}{\sigma}} A_i^* s_{Lc}^* \quad . \tag{8}$$

Now taking logs and time derivatives of Eq. (8) we obtain the steady-state growth rate for the income per capita

[6] The i index is left at the case of L_{Ai} to avoid confusion with aggregate R&D employment. The difference between the two will have crucial role when eliminating strong scale effect. The index will be left at the case of A_i as well throughout the paper just to stress that the point is about firm-level technology.

(9) $$g_y^* = \frac{1-\sigma}{\sigma} g_N^* + g_{Ai}^* \quad .$$

Eq. (9) says that it is the sum of the steady-state growth rate of product variety – multiplied by a constant – and the firm-level technology. According to Eq. (6) the latter depends on the steady-state level of the firm's R&D input

(10) $$g_{Ai}^* = \left(L_{Ai}^*\right)^{\gamma} .$$

2.4 The Evolution of the Number of Firms and the Average Firm Size

In the previous section we have seen that the steady-state per capita growth depends on the steady-state variety growth and firm-level technology advance. From Eq. (10) we have the result that the latter depends on the steady-state firm-level R&D employment, which is determined by the equilibrium decision of the representative firm. According to Eq. (7) this decision is strongly influenced by the evolution of the number of product lines. If the number of products is treated as a variable, that is entry takes place in the model, the decision problem of entrants should be investigated too. Concerning the goal of the paper this is not a crucial point and I omit the analysis here to spare space. I refer the reader to Peretto (1998 a, and 1999) and Howitt (1999) for a discussion of the problem. At that point it is enough to set the following expression for the steady-state firm size

(11) $$\theta^* = \frac{L}{N^*} = \frac{1}{\lambda} L^{1-\beta} , \qquad \text{where } 0 \le \beta \le 1 \text{ and } \lambda \text{ is a constant.}$$

From Eq.(11) we can derive the steady-state firm number

(12) $$N^* = \lambda L^{\beta} .$$

3. Scale Effects: an Overview

Scale effect is defined in the growth literature as the effect of population size on steady-state output per capita. We can distinguish two types of it: *weak* and *strong scale effect* (Jones 2004). The former attributes to population size level-effect in the long-run, while the latter rate-effect. As Jones pointed out weak-scale effect is always present in idea-based growth models, which treat technology as an endogenous productive variable in the final production function. To realize it intuitively, remember that technology is non-rival that makes output per capita positively dependent on the absolute stock of technology and not on its relative stock to the population size (i.e. technology/capita)[7] as in the case of capital. However, technology is endogenous as well and is the result of the innovative activity of talented persons, so its stock depends positively on the population size. As Jones (2004, 14 p.) notes: "A larger population

[7] The most evident example if you regard a Solow-like production function $Y = AK^{\alpha} L^{1-\alpha}$. Then output per capita is $y = A(K/L)^{\alpha}$.

means more Mozarts and Newtons, and more Wright brothers, Sam Waltons, and William Shockleys."

Weak scale effect is present in our case as well. Because of symmetry $L_{Ai} = L_A/N$. In steady-state the share of R&D employment (s_{LA}) must be constant so $L^*_{Ai} = (s^*_{L_A} L/N^*)$. Substituting Eq. (12) in place of N^* and then the whole expression into Eq. (6) results

$$(13) \qquad \dot{A}_i^* = \left(\frac{s^*_{L_A}}{\lambda} L^{1-\beta} \right)^{\gamma} (A_i^*)^{\phi} \ .$$

According to Eq. (13) and (12) the size of population affects in a positive manner the steady-state output/capita (Eq. (8)).

An empirical proof for weak scale effect is Kremer's (1993) famous cross-sectional evidence on the different regions of the world which became separated at the end of the last ice-age (10.000 B.C.), when the melting of the polar ice cancelled the land-bridges between the Old World, the continent of America, Australia, Tasmania and the Flinders Island. Connection has been restored again around 1500 A.C.. At that time the estimated population density and technology level of these regions followed the rank of their land-area: 1. Old World, 2. America, 3. Australia, 4. Tasmania, 5. Flinders Island. If population size was determined in a Malthusian context as Kremer argues, that is the evolution of output per capita ultimately pinned down by the technological advance constrained the evolution of population, then assuming a more or less same initial population density and an aggregate technology function similar to Eq. (6) it is a support on weak scale effect. Consider the output function in Kremer:

$$Y_t = A_t L_t^{\alpha} T_t^{1-\alpha}$$, where T is land, A is technology, L is population size and $0< \alpha <1$.

Then the Malthusian population determination is

$$\bar{y} = \left(\frac{A_t^{\frac{1}{1-\alpha}} T}{L_t} \right)^{1-\alpha}$$, where $\bar{y}$ is the subsistence level of output per capita.

Now if we assume that technology was nearly the same at the end of the ice-age[8], the population density (L/T) must have been the same in each region. However, greater initial land size meant higher initial population level, which resulted in a higher technology level at the time of 1500 A.C. according to the technology function. Based on the Malthusian population determination equation population density must have been higher as well, as it turned out to be the case quite significantly at Kremer[9]. If one assumes, that population adjusts slower than technology advance evolves, the output per capita had to be also higher in regions with higher initial land area and population.

The presence of weak scale effect doesn't mean that countries with larger population should be wealthier, like e.g. China. Because of the intense cross-boarder flow of information and ideas, we have to think about weak scale effect rather in a global dimension.

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[8] If one thinks about the possible range of technologies at that time (10.000 B.C.), it should be noticed that this is not a binding assumption.

[9] Kremer (1993) Table VII, 710 p.

3.1 Strong Scale Effect

Contrary to the weak scale effect, strong scale effect is a much more doubtful prediction of first generation idea-based growth models since the criticism of Jones (1995 a, b). The intuition of strong scale effect comes from the non-rival nature of technology that is innovation/invention costs must be incurred only once and after success the discovered new idea can be used for the production of infinite units. Because technology is partly excludable as well, the innovator earns a certain monopoly mark-up on each unit of product. So the more a successful innovator is able to sell, the higher will be its aggregate profit spreading more the usually high innovation costs. But the demand on the innovator's product is ultimately determined by the size or saying other the *scale of its market,* which is manifested in L/N in the framework of this paper (see Eq. 4). All in all larger scale of the market means larger incentives for R&D (see Eq. 7). From then on it depends only on the *form of the technology production function* whether the resulting larger firm level R&D inputs raise the rate of technological advance in the long-run or not?

In first-generation models (Romer 1990, Aghion-Howitt 1992) the *relevant scale of the market for a firm is identified with the population size* and so changes proportionally with it. According to the previous reasoning this imposes higher returns on R&D when population is larger implying larger firm-level R&D intensity as well. The latter leads naturally to a faster technology and output per capita steady-state growth in the theory if one uses a Romer-like technology function with perfect standing on shoulders. To show the mechanism in the framework of the paper, notice that this interpretation of the scale of the market is just the case of $\beta = 0$ in Eq. (11). This parameter value causes the number of products to be fixed at λ (Eq. (12)). Now consider the case when L increases. The scale of the market (L/N) increases as well and results higher demand for the product of firm i (Eq. (4)). This higher demand makes R&D more profitable and the returns to R&D rise above the market interest rate. To realize it, notice that with growing L/N the RHS in Eq. (7) increases. Because of the arbitrary assumption on market interest rate ($r = \rho$) E is a constant, the only way to restore capital market equilibrium is an increased firm-level R&D employment, which finally dissipates the additional returns on R&D. In the new capital market equilibrium L_{Ai} and according to Eq. (10) the rate of firm-level technology advance will be higher. The later generates a higher steady-state growth rate for output per capita. To sum up, the ultimate effect of a larger population is higher long-run growth rate, which accelerates when population growth is present. This however lacks clear evidence. Moreover, according to Jones' critique (1995 a, b) the aggregate R&D employment have heavily risen in developed countries during the second-half of the last century without any increasing trend in their TFP growth rates.

Growth theorists responded to the empirical weaknesses of the predictions of first-generation models by constructing a new stratum of idea-based growth models (Jones 1995 b, Segerstrom 1998) that relies on the assumption of imperfect intertemporal knowledge spillovers. According to these models the key to Jones' observations on the rising R&D inputs and trendless TFP growth rate in the USA during the second half of the last century is a technology function less than linear in the previously accumulated technology. Saying other, the intertemporal spillover parameter in Eq. (6) is $0 < \phi < 1$. Although they *continued to link the scale of market for a given firm to the scale of population* (so $\beta = 0$ because of the same reasoning as before), this seemingly slight parameter change cancels any relationship between the population size and long-run growth rate. Furthermore, in these models steady-state growth becomes semi-endogenous in the sense that usual R&D enhancing policy instruments (e.g. R&D subsidies) have no effect on long-run technological change, which is pinned down by the rate of population growth. This statement follows from the shape of the technology function. Divide Eq. (13) by A_i and take logs and time-derivatives:

$$\frac{\dot{g}_{Ai}}{g_{Ai}} = \gamma n - (1-\phi) g_{Ai} .$$

This is a stable differential equation, and so the steady-state technological growth rate is

$$(14) \qquad g^{*}_{Ai} = \frac{\gamma}{1-\phi} n .$$

Eq. (14) means according to Eq. (9) that the steady-state growth rate of output per capita is determined by variables, which are usually regarded as being external to the economic policy. That prediction was highly criticized by many growth economists. Although treating population growth rate as endogenous to economic policy (Jones 1997) might save second-generation models from this criticism, they still share another theoretical weakness: growth is zero in the absence of population growth. The reason is the special assumption on the value of ϕ, which causes the productivity of R&D labour to increase less rapidly than technology (A_i) driving ceteris paribus the marginal cost of R&D above its marginal revenue. This would lead naturally to zero R&D input and technological advance, if something didn't balanced out the detrimental effect of imperfect spillovers. This is done by the rising population, which has an advantageous effect on the returns to firm's R&D from the same reason as at the case of first-generation models: linking the market size for the individual firms directly to the population size.

To show it in the framework of the paper, consider the decision problem of the representative firm (2.2 chapter) when the moving equation (Eq. (6)) is non-linear in A_i with a parameter of $0 < \phi < 1$. From the first-order conditions when maximizing the objective function we are able again to derive the capital market clearing condition (see Appendix 1.)

$$(15) \qquad r = \frac{LE(\varepsilon - 1)\gamma}{N\varepsilon L_{Ai}^{1-\gamma} A_i^{1-\phi}} + (1-\gamma)\frac{\dot{L}_{Ai}}{L_{Ai}} .$$

The RHS of Eq. (15) is the return to R&D for firm *i*, which differs only in $A_i^{1-\phi}$ from the one with perfect intertemporal spillovers (Eq. (7)). From Eq. (15) it is clear that in the absence of population growth with growing A_i the RHS finally must fall below the fixed market interest rate no matter what the growth rate of R&D employment is, because the later is harmed in the long-run by the fixed resource constraint (population size): $g^{*}_{L_{Ai}} = 0$ if n=0. The economical reason of the falling returns to R&D is the one discussed above. Note, that the marginal cost of technology is $MC_{Ai} = \frac{1}{\gamma} L_{Ai}^{1-\gamma} A_i^{-\phi}$ and in case of n = 0 (that is $g^{*}_{L_{Ai}} = 0$) the marginal revenue is $MR_{Ai} = \frac{LE(\varepsilon - 1)}{N\varepsilon A_i}$ (see Appendix 2.). Marginal cost decreases in the level of technology less than linearly because marginal productivity of R&D labour improves through imperfect intertemporal spillovers – remember that the wage rate is the numeraire. On the contrary, marginal revenue decreases linearly with technology. In order to have a steady-state with positive technological advance Eq. (15) must hold, which is only possible if the population growth balances out the previous divergent dynamics of marginal cost and revenue along the path of technological change. Suppose that this is the case and Eq. (15) holds in the long-run, taking into account that in steady-state $\dot{L}_{Ai} / L_{Ai} = n$ we can calculate easily that the required population growth rate is $n = \frac{1-\phi}{\gamma} g^*_{Ai}$ so as the RHS of Eq. (15) not to decline below the market interest rate[10]. But this is the case in Eq. (14), which derives the steady-state technological change solely from the dynamic properties of the technology function.

The theoretical inconvenience of semi-endogenous long run growth with zero rate in the absence of population growth inspired scientists to construct models, which lack strong scale effect without the shortcomings of second-generation models. The third-generation idea-based growth models (Aghion – Howitt 1998 Ch. 12, Dinopoulos –Thompson 1998, Peretto 1998 a) preserve the perfect intertemporal spillovers in the firm-level technology function (ϕ=1) Eq. (6), however *they separate the size of the firm's market totally from the scale of the population* taking the seminal contribution of Young (1998) serious by incorporating entry of new firms. Entry (horizontal innovations) dilutes perfectly in the long-run the glorious effect of a population rise on the size of market for a given firm. Saying other the scale of the market for a firm is independent of population size in steady-state. For that reason firms equilibrium R&D-decision is not influenced by any change in population, contrary to the case at first- and second-generation models. Firms continue to employ the same number of researchers even in the presence of population growth, which keeps the rate of firms' technological advance constant because of the linear differential equation in the technology accumulation process. However, the number of firms rises with the population size and so does aggregate R&D employment as well.

To show how the mechanism works in a formal way note that from population size independent market size means a demand on firm *i*'s product, which is not influenced by the value of L in Eq. (4). Since E is a constant and L does not impact P, the only way to dissipate the positive effect of larger aggregate market is, if the number of firms (N) evolves proportionally to the population size. According to Eq. (12) then β=1. Unchanged scale of the market keeps returns to R&D unchanged even in the presence of population growth: the R&D enhancing effect of a greater aggregate market size is fully dissipated in Eq. (7) by new entries, L/N is constant with a value of $1/\lambda$ (Eq. (11)). This is the perfect dilution of a greater pool of rents available due larger population size by horizontal innovations, suggested originally by Young in the literature. Unchanged returns on R&D leaves L^*_{Ai} unchanged as well, which results constant firm-level technological advance (Eq. (10)). Then the steady-state growth rate will also be constant in Eq. (9) if the population growth rate is constant, since $g^*_N = n$ when β=1 (Eq. (12)). The *perfect dilution* embodied in the value of parameter β may seem an arbitrary assumption, but it is just because the discussion of the decision problem of entry was neglected throughout this paper. In Peretto (1998 a) and Howitt (1999) however the case turns out to be the same with the number of firms increasing proportionally with population size in steady-state (β=1), which is an outcome of long-run general equilibrium incorporating the entry decision problem as well.

As I argued at the beginning of this chapter, the scale of the market matters concerning the returns on the firm's R&D activity. This arises naturally from the non-rival character of

[10] which is fixed in our case

technology. The question is, that is there any connection between the population size and the market size and how intense are spillovers in the technology accumulation process? These two points are crucial whether growth exhibits strong scale effect or not. If population size affects the scale of the market for a representative firm in a positive manner in the long-run ($0 \leq \beta < 1$), than firm level R&D intensity will increase, and from then on it depends only on the extent of spillovers whether the outcome will be a higher steady-state growth rate or not. The three generations of idea-based growth models differ mainly along these two dimensions, not regarding here the different approaches in their rich microeconomic structures. This is well embodied in the values of parameter β and ϕ:

TABLE 1.

	β	ϕ
First-Generation models	=0	=1
Second-Generation models	=0	<1
Third-Generation models	=1	=1

That crucial distinctive characteristic was revealed in Jones (1999), Dinopoulos – Thompson (1999) and Ha-Howitt (2005) as well.

Recall, that according to the discussion above $\beta = 0$ means that the market size for a firm increases proportionally with the population, while at the case of $\beta = 1$ there is no connection between them (Eq. (11)). On the other hand, when $\phi = 1$ the intensity of spillovers suffices to have a linear differential equation in the technology accumulation (Eq. (6)), while at the case of $\phi < 1$ constant technological growth is possible only with growing R&D input.

3.2 Some Empirical Observations

The three strata of idea-based growth models provide empirically testable predictions according to their different assumptions on the value of ϕ and β. If $\phi = 1$ then productivity-growth is sustainable with constant R&D input. In the context of quality-ladder models – with constant innovation size – it would mean that innovations per unit R&D input is constant. If a steady part of innovations are patented, then *patent statistics* can be a natural judge on these models. Third-generation models argued that Jones' criticism, increasing aggregate R&D employment and constant TFP-growth, is consistent with a Romer-like firm-level technology function because of the perfect dilution of aggregate resources. However, with perfect standing on shoulders the innovations/researcher, and so the patents/researcher rate, should still remain constant, which is clearly not the case. In the USA in the second half of the last century the patents granted to residents showed a certain degree of stability, it fluctuated around 40.000-50.000 per year, while the number of researchers increased heavily implying a sharp fall in patents/researcher (Kortum 1997). This could be considered as a testimony for second-generation models with decreasing relative productivity of R&D inputs. Notwithstanding Dinopoulos-Thompson (1999) call our attention on some ambiguities related to this result. First, the fairly stable behaviour of granted patents covers the divergent tendency of patents granted to individuals and patents granted to corporations. While the former declined significantly from the beginning of the century, the latter realized a considerable upsurge. Because firms constitute a predominant part of R&D and innovative activity, it is questionable how to interpret the absence of significant trends in aggregate data? Second, patent applications per year by domestic residents have doubled until 1995 (more than 120.000) comparing to 1985 in the USA, which wasn't matched by the tendency of patents granted. Naturally arises the question to what extent does this jump in the applications cover real innovations?

Beside patent statistics another way of testing was suggested by Ha-Howitt (2005). As they pointed out, it is easy to derive the *"ultimate source of growth"* in the given models, if we substitute the relevant values of ϕ and β in Eq. (13)

First-generation models: $$g_{Ai}^{*} = \left(\frac{s_{L_A}^{*}}{\lambda} L \right)^{\gamma};$$

Second-generation models: $$g_{Ai}^{*} = \frac{\gamma}{1-\phi} n \quad \text{and}$$

Third-generation models: $$g_{Ai}^{*} = \left(\frac{s_{L_A}^{*}}{\lambda} \right)^{\gamma}.$$

According to this the "ultimate source" is aggregate R&D input but from a different point of view, in first-generation models its absolute value matters, in second-generation models its growth rate and in third-generation models its share to the aggregate stock of that resource - or equivalently the long-run firm level R&D intensity (Eq. (10)).

Based on Jones' critique the first-generation models are refuted. But testing the prediction of the second- and third-generation models on the "ultimate source of growth" against the trendless behaviour of TFP growth in the USA during the second-half of the 20th century can be interesting. A constant long-run technology (TFP) growth rate requires a constant growth rate of aggregate R&D employment in second-generation models, while at the case of third-generation ones only a constant share of it. If one works with a one-sector lab-equipment model[11] these conditions are respectively constant growth rate of (average) technology adjusted aggregate R&D expenditure and constant share of R&D expenditure in GDP. Further indicators can be derived to test third-generation models on the basis that the "ultimate source of growth" at their case is firm-level R&D input. Because the number of firms increases in accordance with population size in these models, any aggregate R&D input measure adjusted by any variable that evolves proportionally with population can serve as a proxy for firm level

[11] See e.g. Barro – Sala-i-Martin (2004 ch 6. and 7.)

R&D intensity. These proxies are L_A/L, L_A/hL, R&D/AL, R&D/AhL, R&D/Y in Ha and Howitt[12].

The results of Ha and Howitt confirm that both the growth rate of R&D employment and technology adjusted R&D expenditures have decreased significantly throughout the second-half of the last century in the USA. This contradicts to second-generation models. On the contrary two of the indicators related to third-generation models: R&D/AhL and R&D/Y; behaved as they must have in the light of the TFP growth rate, that is they had only a statistically insignificant trend in that period. The rest of them had small upward trends, however their intensities were far from the ones experienced at the downward trends for the growth rates of R&D employment and technology adjusted R&D expenditures.

Third- and second-generation models have strong *predictions on the long-run firm size* as well. Recall that because of $\beta = 1$ the number of firms grow proportionally with the population size (Eq. 12)) at the former, and so in the long-run firm size is stable. On the other hand when the dilution of larger aggregate market is zero or imperfect ($0 \leq \beta < 1$) firm size (L/N) grows with population, like in second-generation models. Laincz and Peretto (2004) provide some empirical support in favour of third-generation models by reporting a fairly stable employment/establishment ratio in the USA for the period 1964-2001. Moreover, they observed a similar trendless R&D employment/establishment ratio[13], which matches the prediction of these models under the circumstances of constant market size for a firm (saying other a constant firm size like above) and constant long-run TFP growth rate[14].

4. A Comment on Third-Generation Models

Third-generation models eliminate strong scale effect in a way, which seems to conform to preliminary empirical findings. They have a fragile framework however, because they work with a strongly stripped down version of the potential R&D "spillover space". As I argued before only "localized vertical spillovers" are involved in them. The set-up is clear and simple: greater aggregate market size is perfectly diluted by new entries in an economy where firms are totally separated and independent from each other having any interactions and spillovers among them. There are two kinds of possible inter-firm R&D spillovers: inter-temporal and contemporary. The former corresponds to the original concept of Romer about "standing on each other's shoulder", or saying other benefiting from the accumulated technology of other firms, that is from their past research effort, as well when accomplishing its own R&D. Contemporary spillovers are productivity gains in the R&D process originating

[12] Respectively the share of aggregate R&D employment, R&D employment divided by aggregate human capital (hL), (average) quality adjusted R&D expenditures (R&D/A) divided by the labour force, by the aggregate human capital, and by the quality adjusted GDP (Y/A). Note that the later is equal to the share of R&D in GDP (R&D/Y).

[13] Throughout this paper product–firms and entry of firms–horizontal innovations were used interchangeable. However, when testing the predictions of third-generation models one has to define the proper unit of observation. This is not simple because of multi-product and/or multi-plant firms. Laincz and Peretto suggest two alternatives: the establishment (plant), or the firm. They regarded the former as a better measure for testing third-generation models.

[14] The results of Laincz and Peretto must be considered carefully because as they report, behind the scenes the employment/manufacturing establishment has fallen, while the employment/service sector establishment has risen.

from other firms' current R&D effort. This later can be a result of interaction between the researches.

The presence of these spillovers can drive to explosive growth. The underlying intuition is that with growing number of firms the potential sources of these spillovers grow parallel. Peretto and Smulders (2002) argue however that the range of useful sources is constant in the limit because as entry evolves totally new technologies can be introduced suited only for that particular product or production process, and as the number of firms reaches a high level the probability that the new entrant will have a similar technology to the ones of a given incumbent decreases to zero. This reasoning is convincing and if one starts thinking about the potential sources of inter-firm spillovers the primacy of "innovation networks" – consisting of firms involved in familiar industries and similar technologies – can be easily accepted. Notwithstanding, even if we implement the idea of Peretto – Smulders about the constant size of "innovation networks" in the limit, inter-temporal spillovers from that limited number of firms still can induce explosive growth supposing that "standing on its own and the others' shoulder" is intense enough.

According to the above an augmented firm-level R&D function meriting from Peretto (1998 b) and Peretto-Smulders (2002) would look like as the follow:

(16) $\dot{A}_i = (L_{Ai})^{\gamma} A_i^{\phi} (s_{in}\mu\overline{A})^{\psi} (s_{in}\omega\overline{L}_{Aj})^{\xi}$; $\gamma, \phi, \psi, \xi \geq 0$; $1 > \mu > 0$ [16]; $s_{in} \geq 1$[16]; $\omega, \overline{L}_{Aj} > 0$[15];

where s_{in} is the size of innovation network for firm *i* (that is the number of firms in the network except firm *i*), $\overline{A}$ is the average technology level of the firms in the network, μ is the proportion of each firm's technology that spills over, ψ is the intensity of standing on the others' shoulder, $\overline{L}_{Aj}$ is the average R&D employment of the firms in the network, ω is a simple efficiency parameter of interaction of researchers in the network and ξ is the measure for contemporary spillovers.

From Eq. (16) we can conclude, that if $\phi = 1$ and $\psi > 0$ the intensity of standing on "my and your" shoulders is large enough to have an accelerating technological growth even with constant innovation network size. Note that $\overline{A}$ increases as well in accordance with the research effort of the other firms in the innovation network!

In third generation models the two knife-edge conditions of ψ, $\xi = 0$ imply that externalities are absent[16] in R&D. This assumption could be realistic only, if firms involved in similar technologies were totally separated from each other for example because of geographical distance paired with insufficient communication networks or with completely uncodifiable technologies. This is truly not the case, especially not in the age of globalisation.

Moreover, the R&D incentives for a given firm also rise as the size of externalities increases. Substitute K for $(s_{in}\mu\overline{A})^{\psi} (s_{in}\omega\overline{L}_{Aj})^{\xi}$ in Eq.(16) to get

(17) $\dot{A}_i = (L_{Ai})^{\gamma} A_i^{\phi} K$.

[15] These conditions ensure that Eq. (16) is not zero when $L_{Ai} > 0$! One can easily accept that they are not binding in an economy.

[16] Although Peretto and Smulders (2002) incorporate intertemporal spillovers, their model behaves just like the other third-generation ones from that point of view when the size of innovation networks becomes constant in the limit. The reason is their special assumption about the intensity of intertemporal spillovers: $\psi + \phi = 1$.

K represents the externality factor in the technology function. If we solve the decision problem of the representative firm (see the appendix) with this new technology function in the more general case of not restricting the value of ϕ to 1, we have

$$(18) \qquad r = \frac{LE(\varepsilon - 1)\gamma K}{N\varepsilon L_{Ai}^{1-\gamma} A_i^{1-\phi}} + (1-\gamma)\frac{\dot{L}_{Ai}}{L_{Ai}}.$$

From Eq.(18) it is obvious that the rise of K increases the RHS, which corresponds to the returns to R&D for firm i as before. The economic explanation is that larger positive externalities reduce the marginal cost of technology. The resulting larger firm-level R&D employment leads to higher technological growth rate in the case of $\phi = 1$.

One could argue that the GDP/capita statistics don't support accelerating growth rates, which is true. However we can be curious about if it is not just a matter of aggregation over space? In industry agglomerations the inter-firm spillovers above are more intense just because the geographical vicinity allows a larger frequency of interactions of researchers and more efficient technology spillovers. The later derives from the fact that a part of the technology is tacit – it can't be codified – and needs personal contact to spill over. So it is reasonable to consider industry agglomerations where the standing on its own shoulders together with inter-firm spillovers can suffice to have an accelerating firm-level technological advance even with constant R&D input. The rush upsurge of the Silicon Valley is just one of the many examples supporting the idea – which is clearly not a new one – that industry agglomerations matter.

This kind of comment on third-generation models arguing with the over-simplifying treatment of knowledge spillovers is by any case not novel in the literature. Chol Won Li (2000) pointed out that these models dismiss the possibility of inter-knowledge spillovers by cancelling any relationship between the two dimensions of knowledge (technology): product-variety and product- (or process-) quality. He concluded, that in the presence of these spillovers semi-endogenous steady-state growth emphasized by Jones is a more general case even in third-generation models comparing to endogenous one! Furthermore, when the dimension of knowledge increases endogenous steady-state growth becomes harder and harder to prevail by requiring a rising number of knife-edge conditions (Li 2002).

5. Final Remarks

The aim of this paper was not to judge third-generation models on the ground of the usual argument on imposing knife-edge conditions (see e.g. Jones 1999). I agree with Jonathan Temple (2003), that the presence or absence of knife-edge conditions is not "…a useful criterion in discriminating between rival explanations of the growth process."[17]. It is widely common that even the Solow-model requires them for having a steady-state growth path (Barro – Sala-i-Martin 2004 ch.1, Jones 2004). Knife-edge conditions are useful to keep the analysis tractable, and to abstract from problems not directly relevant to the purpose of the model. And this was clearly the point at third-generation models, which had the basic motivation of preserving the legitimacy of economic policy in determining the evolution of a country's economy. Notwithstanding, the theoretical shortcomings of them suggest that further questions are left unanswered related to strong scale effect.

[17] Temple (2003, p. 500)

In this paper I dealt only with the absence of inter-firm spillovers and its consequences. According to my arguments even if strong scale effect is absent when investigating country (or world) data, it might be present at a regional level in case of agglomerations: larger population induces a larger number of firms, which can result in accelerating growth rate in industry agglomerations.

One could also doubt that the population and the market size are really completely independent from each other, as they predict. Though the theoretical models and preliminary empirical findings support it, further empirical investigations are needed for clear evidence.

Another shortcoming of third-generation models, which has strong implications, is the unrealistic "closed economy" assumption. In fact this is general for all idea-based models cited throughout this paper, independent of which generation they belong to. Therefore these models are more suited to investigate the relation of population and economic growth for the world, than for single countries.[18] But is strong scale effect really a problem emerging solely at a global level? That would require effectively zero transportation costs and perfect international knowledge spillovers. None of them prevail. The extent to which transportation costs in the different industries limit the effective market size for a firm[19] and to which international knowledge spillovers are imperfect are crucial determinants of the right observational unit for strong scale effect.

We can conclude that a definite mapping of the "population - economic growth" relationship – if possible – requires further research: strong scale effect is still on agenda!

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[18] I also worked with a closed economy framework for two reasons. First, as the title of the working paper shows, this is "only" a critical survey, so my hands were tied by the existing models devoted to the topic of scale effects. Second, it made the analysis simpler, and had not any special importance during the investigation of the absence of inter-firm spillovers in third-generation models and its implications.
[19] I am indebted to Erich W. Streissler (University of Wien) fo this suggestion.

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Appendix

Appendix 1.
The equilibrium conditions for firm optimisation

The optimisation problem of firm i according to chapter 2.2. is

(A.1) $\max_{Pi,\tau;LAi,\tau} V_{i,t} = \int_t^\infty e^{-(r\tau)}[(P_{i,\tau} - A_{i,\tau}^{-1})C_{i,\tau} - L_{Ai,\tau}]d\tau$

(A.2a) $\dot{A}_{i,\tau} = L_{Ai,\tau}^{\gamma} A_{i,\tau}^{\phi}$

(A.2b) $C_{i,\tau} = \dfrac{L_\tau E P_{i,\tau}^{-\varepsilon}}{\int_0^N P_{j,\tau}^{1-\varepsilon} dj}$

(A.3) $A_{i,t} = A > 0 \quad \forall$ i (the initial technology level is the same for every firm)

(A.4) $A_{j,\tau}$ $\forall$ $\tau \geq$ t and j $\neq$ i (firm i takes as given the time-path of the rivals' technology level)

(A.5) $\dot{A}_{i,\tau} \geq 0 \;\; \forall \;\tau \geq$ t (technology is irreversible)

where (A.2a)-(A.5) are the constraints. The notations are the same as in the text. The control variables are the price and the R&D employment. The problem is truly a composition of a dynamic and static one. The optimal time-path of P_i is the result of a sequence of static maximisation of profit with respect to the price and according to the constraint (A.2b) taking as given the prevailing level of the firm's technology at the relevant point in time

$$\max_{Pi,\tau} \pi_{i,\tau} = (P_{i,\tau} - A_{i,\tau}^{-1})C_{i,\tau} - L_{Ai,\tau}.$$

The first-order condition yields the usual mark-up rule price strategy

(A.6) $P_{i,\tau} = \dfrac{\varepsilon}{\varepsilon - 1} A_{i,\tau}^{-1}$.

The optimal time-path of L_{Ai} is however a dynamic optimisation problem with the state-variable A_i. The current-value Hamiltonian is then

$CVH = (P_i - A_i^{-1})C_i - L_{Ai} + q_i(L_{Ai}^{\gamma} A_i^{\phi})$, where q_i is the current value shadow price.

For simplifying notation I dropped time-indexes, as I will do during the remaining part of the Appendix. *The first-order conditions are the followings in the case of $\phi = 1$*

$$FOC_1 = \frac{dCVH}{dL_{Ai}} = 0 = -1 + q_i \gamma L_{Ai}^{\gamma-1} A_i^{\phi} \Rightarrow q_i = \frac{1}{\gamma}\frac{L_{Ai}^{1-\gamma}}{A_i^{\phi}}$$

$$FOC_2 = \frac{dCVH}{dA_i} = rq_i - \dot{q}_i = A_i^{-2}C_i + q_i L_{Ai}^{\gamma} \Rightarrow r = \frac{A_i^{-2}C_i}{q_i} + L_{Ai}^{\gamma} + \frac{\dot{q}_i}{q_i}$$

FOC_3: the transversality condition

Now it is evident that with same initial stock of technology the time-path for P_i ,L_{Ai} and A_i will be the same for all *i* and symmetry prevails. To derive Eq. (7) substitute Eq. (4) in place of C_i in FOC_2 and make use of the equilibrium price definition (A.6)

$$\text{(A.7)}\quad r = \frac{LE(\varepsilon-1)}{N\varepsilon A_i q_i} + L_{Ai}^{\gamma} + \frac{\dot{q}_i}{q_i}.$$

If we substitute for q_i and $\dot{q}_i / q_i$ according to FOC_1 we get Eq. (7).

The economic meaning of FOC_1 is that the marginal benefit of A_i must equal the marginal cost. The marginal benefit is the current value shadow price. It is easy to check according to Eq. (6) that the marginal cost is instead $\frac{1}{\gamma}\frac{L_{Ai}^{1-\gamma}}{A_i^{\phi}}$, when the wage is the numeraire. FOC_2 represents the equilibrium condition that the returns to R&D (the RHS in FOC_2 and (A.7)) must equal the market interest rate.

We can derive Eq. (15), when $0 < \phi < 1$, in the same way.

Appendix 2.
The components of the firm's returns on R&D

It is worth to investigate the components of the returns to R&D in (A.7). The $\dot{q}/q$ is asset appreciation (depreciation), while L_{Ai}^{γ} represents the cost-reducing benefit of present research on future research[20] because of "standing on its own shoulders". The $\frac{LE(\varepsilon-1)}{N\varepsilon A_i q_i}$ part is the cash-flow from an additional unit of A_i divided by its cost (q_i). To see it note, that (LE/ε) is the economy-wide gross-profit

$$\pi^G = \int_0^N \pi_i^G di = \int_0^N (P_i - A_i^{-1})C_i di = \int_0^N \frac{LE}{N\varepsilon} di = \frac{LE}{\varepsilon}$$

, where at the third equation I made use of the condition for equilibrium price (A.6), the definition of C_i (Eq. (4)) and the symmetry property. $\frac{\varepsilon-1}{NA_i}$ is the increase in the market-share when firm *i* improves A_i by one unit holding the technology level of the other firms constant. To realize it multiply both sides of Eq. (4) by P_i and divide them by LE to get the expression for the market share of firm *i*, which always equals 1/N in symmetry

$$S_i = \frac{P_i^{1-\varepsilon}}{\int_0^N P_j^{1-\varepsilon} dj}\left(=\frac{1}{N}\right).$$

[20] Peretto (1999) referred to it as „dynamic learning economies in R&D".

Now we can easily derive the elasticity of market share respect to the price: $\varepsilon_{Si,Pi} = 1 - \varepsilon$; and from Eq. (A.6) the elasticity of price respect to the technology level: $\varepsilon_{Pi,Ai} = -1$. Then the elasticity of firm *i*'s market share respect to its technology level is $\varepsilon_{Si,Ai} = \varepsilon - 1$. According to these, if the technology level of firm *i* rises by one unit, that is with $1/A_i$ percentage, when the levels of the other firms are held constant, its market share will rise with $(\varepsilon-1)/A_i$ percentage. The absolute value of this percentage change is

$$\dot{S}_i = S_i \frac{\dot{S}_i}{S_i} = \frac{1}{N} \frac{(\varepsilon - 1)}{A_i}.$$

Now it is evident that the cash-flow from one unit of technology is

$$\pi^G \dot{S}_i = \frac{LE(\varepsilon - 1)}{N\varepsilon A_i}.$$

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