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Tamara Fioroni

# **Life Expectancy, Health Spending and Saving**

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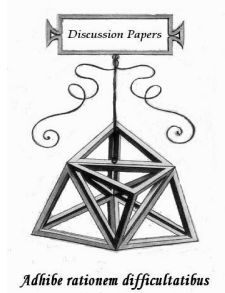
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Tamara Fioroni

## Life Expectancy, Health Spending and Saving.

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### Abstract

This paper investigates the relationship between saving and health spending in a two-period overlapping generations economy. Individuals work in the first period and live in retirement in the old age. Health investment is an activity that increases the quality of life and the probability of surviving from the first period to the next. Empirical evidence shows that both health spending and saving, i.e. the consumption when old, behave as luxury goods but their behavior is strongly different according to the level of per capita GDP. The share of saving on GDP is nearly concave with respect to per capita GDP whereas the share of health expenditure on GDP increases more than proportionally with respect to per capita GDP. The ratio of saving to health investment is nonlinear with respect to per capita GDP, i.e. first increasing and then decreasing. This ratio, in the proposed model, is equal to the ratio between the elasticity of the survival function and the elasticity of the utility function. The model can replicate empirical results if the utility function and the survival function presents a non-constant elasticity.

**Classificazione JEL:** D91, I12, E21

**Keywords:** Intertemporal Choice, Health Spending, Adult Mortality, Saving

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## *I. Introduction*

Through the last two centuries, economic development gradually contributed to the increase in the human life span. In 1840 life expectancy at birth was 40 years in England, 44 years in Denmark and 45 years in Sweden ( Livi-Bacci, 1997). According to recent life tables, in 2004 life expectancy at birth in England, Denmark and Sweden is 78, 77 and 80 years respectively. In particular, in most developed countries, life expectancy at birth is around 80 years (World Development Indicators 2006).

The increase in life expectancy has significant implications for various aspects of the society. In the literature, Bloom et al. (2003), Kageyama (2003), and Zhang et al. (2003), for example, show that increases in life expectancy lead to higher savings rates. This is because agents, in the working age, increase their saving to finance higher consumption needs in old age ( Modigliani and Brumberg (1980)). Blackburn and Cipriani (2002) analyze the relationship between life expectancy, human capital and fertility. However, in this literature life expectancy is exogenous or depends on the level of human capital. Thus the explicit role of health investment on life expectancy is not analyzed.

Some theoretical contributions focus on the willingness of people to pay to reduce mortality risk. The willingness to pay criterion is based on the principle that living is a generally enjoyable activity for which consumers should be willing to sacrifice other pleasures (Schelling, 1968; Murphy and Topel, 2003; and Enrich and Yin, 2004).

Strictly related to the willingness to pay criterion is the Grossman's (1972) paper which analyze the demand for the commodity "good health". In this model agents demand health since it increases the time available for market and non market activities. Indeed, a rise in the stock of health reduces the amount of time lost for these activities and the monetary value of this reduction is an index of the return to the investment in health ( Grossman, 1972). A central result of the Grossman model is that the consumer's demand for

health and medical care is positively correlated with his\her wage rate and his\her education level.

The recent paper of Jones and Hall (2006) consider the optimal choice between length of life and consumption. They show that health is a superior good, that is as income rises the marginal utility of consumption falls quickly more than the marginal utility of health spending.

The aim of our paper is to analyze the direct effect of health investment on life expectancy. This framework allows us to investigate the agent's decision on the allocation of total resources between saving and health investment, i.e the consumption in old age and the length of life. We analyze a two-period overlapping generations model in which agents work in the first period and live in retirement in the old age. Health investment is an activity that increases the quality of life and the probability of surviving from the first period of life to the next. Longevity depends on agent's specific health level which in turn offers an important contribution to agent's enjoyment of life ( Ehrlich and Chuma, 1990). On the other hand, agents can ensure a good quality of life in the old age by increasing the saving in the working age.

Empirical evidence shows that both health spending and saving, i.e. the consumption when old, appear to be luxury goods but their behavior is strongly different according to the level of per capita GDP. The share of saving on GDP appears to be concave with respect to per capita GDP. On the opposite, the share of health expenditure on GDP increases more than proportionally with respect to per capita GDP. The ratio of saving to health investment is non-linear with respect to per capita GDP, it is first increasing and then decreasing.

In the proposed model, the ratio of health spending to saving is equal to the ratio between the elasticity of the survival function and the elasticity of the utility function. We prove that the model can replicate empirical results if the utility function is HARA (hyperbolic absolute risk aversion) and the survival function presents a non-constant elasticity with respect to health investment. We show

that CES (constant elasticity of substitution) preferences don't allow to understand the different path of saving and health spending.

The structure of the paper is outlined as follows. Section 1 presents empirical analysis. Section 2 introduces the general model. Section 3 discusses some possible specification of the instantaneous utility function and the survival function. Section 4 demonstrates that using HARA (hyperbolic absolute risk aversion) preferences we can replicate empirical results. Finally, section 5 draws some concluding remarks.

## *II. Empirical evidence*

The data used in the analysis are taken from World Development Indicators (World Bank, 2006), they are for the period 1960-2005 and cover 208 countries. In Figure 1 we present a recent version of the Preston curve (1975), that is the international relationship between adult survival rate<sup>1</sup> and per capita GDP in purchasing power parity. Whereas Preston (1975) uses the data on life expectancy we use the data on the survival rate that is less sensitive to child mortality. This is because we are interested in adult's health investment decisions to improve his\her probability of surviving to old age<sup>2</sup>. We estimate the Preston curve using a cross-country non-parametric regression (year 2002, 158 countries).

We prefer to perform nonparametric regression since it allows us to investigate the relationship between dependent variable and one or more explanatory variables, without making any a priori explicit or implicit assumption about the shape of such relationship. The confidence interval in Figure 1 identifies clearly a positive relationship between survival rate and per capita GDP. In particular, the confidence interval is an indication of the degree of variability

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<sup>1</sup>The survival rate is the difference between 1 and adult mortality rate. The adult mortality rate is defined from the World Bank as the probability of dying between the ages of 15 and 60, that is, the probability of a 15-year-old dying before reaching age 60, if subject to current age specific mortality rates between ages 15 and 60.

<sup>2</sup>However if we use the data on life expectancy the path of the life expectancy with respect to the per capita income is very similar to the path of the survival rate.

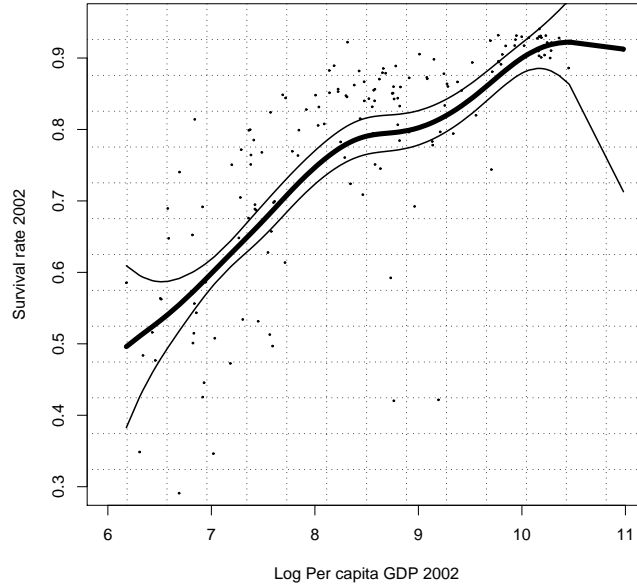


Figure 1: The Preston Curve: Survival Rate versus GDP Per Capita. Nonparametric kernel smoother (bandwidth = 0.45), year 2002,  $n = 158$ .

present in the estimate but it cannot be used to draw firm conclusions about the shape of the curve in particular regions <sup>3</sup>(Hardle et al. 2004; Bowman and Azzalini, 1997). To assess the shape of the curve we carry out a test which compares nonparametric regression with a simple linear regression. This test indicates that the relationship between survival rate and the per capita GDP, can be represented by a linear model, i.e. the significance test for the nonparametric regression shows a  $p$ -value = 0.119. However, in Figure 1 we can see that the relationship is not clearly linear, indeed, in low income countries, increases in the per capita GDP are strongly associated with increases in life expectancy, as income per head rises the relationship flattens out. This path reflects the influence of a

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<sup>3</sup>The confidence interval describes the level of variability present in the estimate without attempting to adjust for the inevitable presence of bias. The wideness of the confidence interval is determined by an estimate of the standard error (Hardle et al. 2004; Bowman and Azzalini, 1997). However the confidence interval cannot be used to assess the shape of the curve in particular regions. This is not only because of the presence of bias, but also because of the pointwise nature of the bands (Bowman and Azzalini, 1997).



country's own level of income on mortality through such factor as nutrition, education, leisure and health expenditure. With respect the latter factor Figure 2 shows the direct relationship between survival rate and per capita health investment in 2002 for 155 countries. Per capita health investment includes both public and private expenditures on health. It covers the provision of health services (preventive and curative), family planning activities, nutrition activities, and emergency aid designated for health but does not include provision of water and sanitation (World Bank, 2006). The relationship between survival rate and per capita health is clearly positive and can be represented by a linear model ( $p - value = 0.618$ ). However, like the Preston curve, figure 2 shows that countries with low health expenditure tend to gain more in life expectancy than countries starting with high level of health spending.

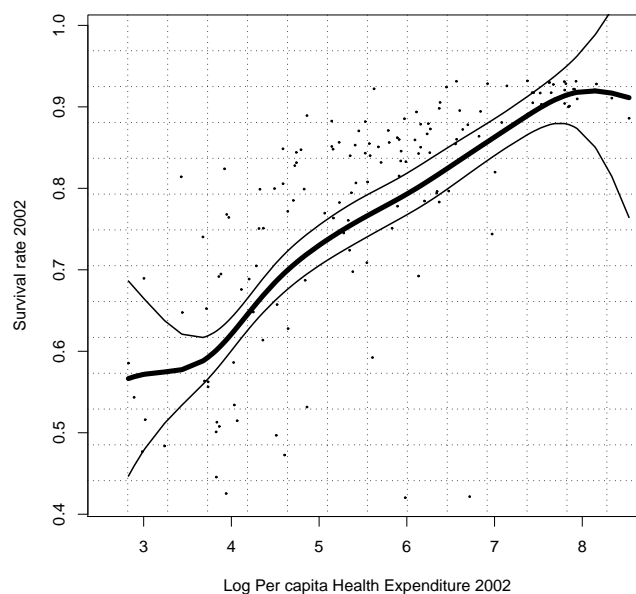


Figure 2: Survival rate versus per capita Health Expenditure. Nonparametric kernel smoother (bandwidth = 0.53), year 2002,  $n = 155$ .

In figures 3 and 4 we examine the path of health expenditure and saving with respect to income. The aim is to analyze the behaviour of health spending with respect to different level of income and the

relationship between health investment and saving; i.e. on one side agents, investing in health, can increase their length of life, on the other side a high saving imply more consumption in old age.

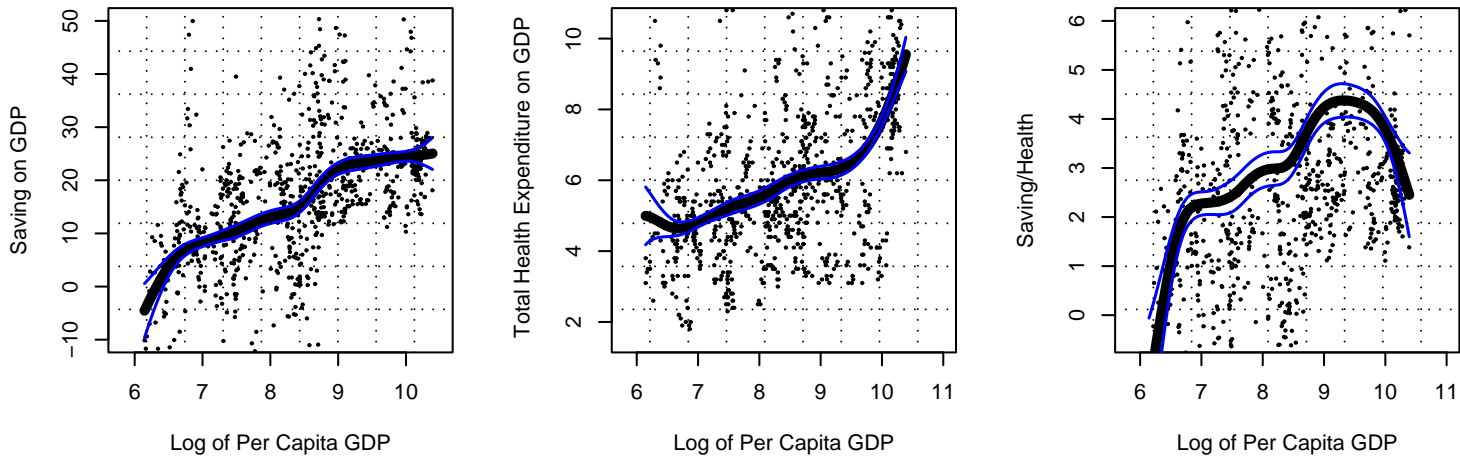


Figure 3: Saving and Health versus GDP Per Capita. Nonparametric kernel smoother (bandwidth = 0.31), years from 1997 to 2002,  $n = 863$ .

Figures 3 and 4 show nonparametric regressions for the saving on GDP, the health expenditure on GDP and the ratio between saving and health. In particular, we perform a pooling of all observation in the period 1997-2002 for 147 countries.

Figure 3 shows that both health expenditure on GDP and saving on GDP present a luxury goods behavior. However the path of health share and saving share is strongly different according to different levels of per capita GDP. The share of saving on GDP appears to be a concave function with respect to per capita GDP, i.e. the comparison between nonparametric regression and a simple linear model yields that the linear model can be refused ( $p - value = 0$ ). In the opposite, health spending on GDP increases more than proportionally with respect to per capita GDP. The test for a linear model provides indication that a simple linear model is inappropriate ( $p - value = 0$ ). The path of the ratio between saving and

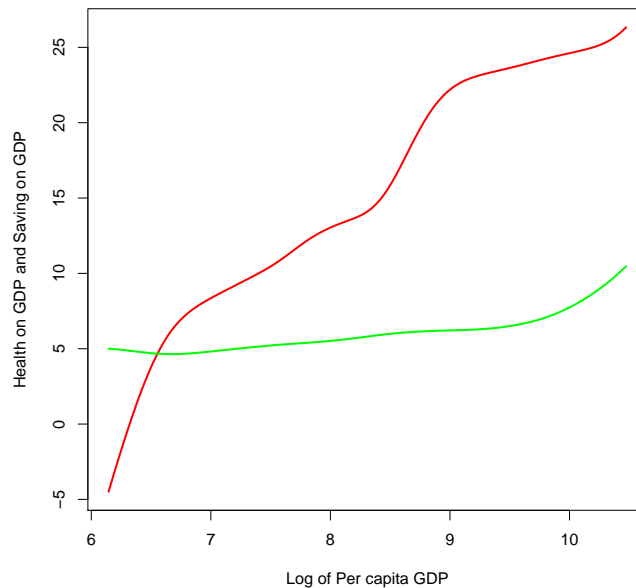


Figure 4: Saving Share and Health Share. Years from 1997 to 2002.

health expenditure is clearly nonlinear, it is first increasing and then decreasing ( $p - value = 0$ ). This suggests that the investment in health increases faster than the saving when a country is sufficiently developed. The intuition is that as income increases, the saturation occurs faster in saving than in health spending.

Figure 4 compares the path of saving share and health share. We can see that when the log of per capita GDP is very low (6 and 7) the saving share is below the health investment. This result can be explained by the fact that health investment covers a part of public expenditure as emergency aid. When income increases health on GDP grows more quickly rather than the saving share.

It is possible to give different explanations for this luxury good behavior of health expenditure. One explanation can be the progressiveness of the tax schedule since the average tax rate increases with income. Others explanations are based on individuals preferences. The idea is that as income grows individual preferences extend not only on the amount of the good consumed but also on the length of life which allows to enjoy additional period of utility

( Jones, 2004, Jones and Hall, 2006). In other words, when people became richer decide to increase the consumption of health services to extend their life expectancy. In the next section we propose a model based on the latter explanation.

### *III. A general model*

In this section we present a general model to analyze agent's decision about the allocation of total resources between saving and health spending.

We consider an overlapping generations economy in which agents live for two periods "youth" and "old age". At the end of the first period agents give birth to a single child. Parents are non altruistic and when they do not survive to the old age, their saving is passed on their offspring as unintended bequest. Hence in the first period of life agents inherit a certain amount of wealth as unintended bequest<sup>4</sup>,  $b_t \geq 0$ , and work receiving a constant wage equal to  $\bar{w}$ . The total resources of agents, i.e.  $y_t = \bar{w} + b_t$ , are allocated between current consumption, health expenditure and saving for the old age consumption. Thus, in the first period, the budget constraint of the representative agent is:

$$c_t = y_t - m_t - s_t, \quad (1)$$

where  $m_t$  is the health investment<sup>5</sup> and  $s_t$  is the saving.

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<sup>4</sup>The unintended bequest  $b_t$  is given by the saving of the parents that did not survive to the old age, that is:

$$b_t = (1 - p_{t-1})s_{t-1}.$$

This implies that in period  $t$  agents whose parents die prematurely have higher endowment. In the proposed model we assume that the initial distribution of wealth is given.

<sup>5</sup>We suppose a perfect substitutability between public health expenditure and private health spending. This implies that a higher proportion of government expenditure devoted to health services reduce private health spending. Indeed, health investment,  $m_t$ , in the consumer's budget constraint is the sum of private health investment,  $m_t^{PRI}$ , and public health investment,  $m_t^{PUB}$ . The latter is equal to a proportional tax on income that is  $m_t^{PUB} = \tau y_t$ . Thus the budget constraint in the first period is:

$$c_t = (1 - \tau) y_t - m_t^{PRI} - s_t,$$

In the second period agents live in retirement and consume entirely their savings, hence the budget constraint in the old age is:

$$c_{t+1} = s_t R, \quad (2)$$

where  $R$  is the constant interest rate in the period  $t + 1$ .

Agents have a probability of surviving to the second period which depends on the health investment undertaken in the working age. Following empirical evidence (see Figure 2), we suppose that the probability of surviving increases with health investment:

$$p_t = p(m_t), \quad (3)$$

where  $p_t \in (0, \bar{p}]$ ,  $p'_t > 0$ ,  $p''_t < 0$ .

We assume that health spending, beyond the increase in the length of life, allows agents to enjoy better life. Thus agent's health level,  $h_{t+1}$ , is a positive function of investment in health services<sup>6</sup> (Grossman, 1972):

$$h_{t+1} = h(m_t). \quad (4)$$

For simplicity we consider health level a linear function of health investment, that is:

$$h_{t+1} = m_t. \quad (5)$$

The lifetime utility of a representative agent is:

$$U_t = u(c_t) + \beta p(m_t) \hat{u}(c_{t+1}, h_{t+1}) + [1 - p(m_t)] M, \quad (6)$$

where substituting  $m_t^{PUB} = \tau y_t$  we obtain:

$$c_t = y_t - s_t - (m_t^{PRI} + m_t^{PUB}),$$

where  $m_t^{PRI} + m_t^{PUB} = m_t$  in Eq. (1).

The idea is that if agents pay high tax then receive high quality public health services and therefore decide to devote a low proportion of income to private health expenditure. Otherwise when public health sector is absent, health spending is private.

<sup>6</sup>In particular Grossman (1972, 1999) assume that individuals inherit an initial amount of health that depreciates with age and can be increased by investment in health services:

$$h_{t+1} = m_t + (1 - \delta_t) h_t$$

where  $m_t$  is the investment in health,  $\delta_t$  is the depreciation rate that depends on age, and  $h_t$  is the inherited health level.

where  $0 < \beta < 1$  is the psychological discount factor,  $M$  is the utility in the death state (Rosen, 1988),  $u(c_t)$  is the utility in the first period, and  $\hat{u}(c_{t+1}, h_{t+1})$  is the utility in the second period. In particular, if agents survive to the second period enjoys an utility which depends on consumption and health level.

Assuming zero utility from death, i.e.  $M = 0$ <sup>7</sup>, and substituting Eq. (5) into Eq. (6) we get:

$$U_t = u(c_t) + \beta p(m_t) \hat{u}(c_{t+1}, m_t). \quad (7)$$

### III.A. Optimal saving and health spending

Proposition 1 characterizes the optimal condition for saving and health spending:

**Proposition 1** *The optimal allocation of resources implies that the ratio of saving to health investment is:*

$$\frac{s_t}{m_t} = \frac{\varepsilon_{\hat{u}_c}}{\varepsilon_{\hat{u}_m} + \varepsilon_p}, \quad (8)$$

where  $\varepsilon_{\hat{u}_c} = \hat{u}_c(c_{t+1}, m_t) c_{t+1} / \hat{u}(c_{t+1}, m_t)$  is the elasticity of the instantaneous utility function with respect to consumption,  $\varepsilon_{\hat{u}_m} = \hat{u}_m(c_{t+1}, m_t) m_t / \hat{u}(c_{t+1}, m_t)$  is the elasticity of the instantaneous utility function with respect to health investment<sup>8</sup> and  $\varepsilon_p = p'(m_t) m_t / p(m_t)$  is the elasticity of the survival function with respect to health investment.

<sup>7</sup>Following Rosen (1998) the expected utility in the second period is:

$$EU = p(m_t) u(c_{t+1}, h_{t+1}) + (1 - p(m_t)) M.$$

Subtracting  $M$  from utility in each state we normalize the utility of nonsurvival to zero:

$$EU = p(m_t) [u(c_{t+1}, h_{t+1}) - M] + (1 - p(m_t)) [M - M].$$

Therefore the expected utility in the second period is given by the differences in utility between life and death, that is:

$$EU = p(m_t) [u(c_{t+1}, h_{t+1}) - M].$$

<sup>8</sup>We define:

$$\hat{u}_c = \frac{\partial \hat{u}(c_{t+1}, m_t)}{\partial c_{t+1}},$$

**Proof.** Given the budget constraints in Eq. (1) and Eq. (2), the first order conditions with respect to  $s_t$  and  $m_t$  are:

$$\frac{u'(c_t)}{\hat{u}_c(c_{t+1}, m_t)} = \beta p_t(m_t) R, \quad (9)$$

and:

$$u'(c_t) = \beta p'_t(m_t) \hat{u}(c_{t+1}, m_t) + \beta p(m_t) \hat{u}_m(c_{t+1}, m_t). \quad (10)$$

The substitution of Eq. (10) in Eq. (9) yields the ratio between the saving and health investment. ■

Eq. (9) is the usual condition that requires the marginal rate of substitution between current and future consumption should be equal to the expected return on saving. Eq. (10) captures the trade-off between the marginal cost and marginal benefit of health care spending. By investing in health care, agents renounce to the current consumption to increase their health level and the probability of surviving to the second period.

According to Proposition 1 the response of the ratio between saving and health spending to variations in the level of income depends on the behavior of the elasticities in Eq. (8). Empirical evidence (Figures 3 and 4) shows that both saving and health investment rise with income but, when income is high, health spending on GDP grows faster rather than the saving on GDP. The intuition is that when income becomes higher than a certain threshold, consumption elasticity falls relative to the health elasticity causing the ratio between saving and health to decrease.

#### ***IV. Alternative specifications of the Utility function and the Survival function***

In this section we analyze the effect of alternative specifications of instantaneous utility function and survival function on the ratio between saving and health investment in Eq.(8).

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and:

$$\hat{u}_m = \frac{\partial \hat{u}(c_{t+1}, m_t)}{\partial m_t}.$$

#### IV.A. Constant elasticity of utility function and survival function

The intuition from figures 3 and 4 is that when income is low people prefer to devote more income to the consumption rather than health spending, but when income rises the marginal utility of consumption appears to decrease faster than the marginal utility of health spending. We cannot replicate this empirical evidence using an utility function with constant elasticity with respect to consumption and health investment, e.g.  $\hat{u} = [c^\beta m^{1-\beta}]^{1-\gamma} / (1-\gamma)$ , and a survival function with constant elasticity with respect to health investment, i.e.  $p = m_t^\delta$ . Indeed using this specification the ratio  $s_t/m_t$  is constant. In particular, from Eq. (8) we obtain  $s_t/m_t = \beta(1-\gamma) / ((1-\beta)(1-\gamma) + \delta)$ .

#### IV.B. Constant elasticity of utility function with respect to consumption

Using an utility function with constant  $\varepsilon_{\hat{u}_c}$  and non-constant  $\varepsilon_{\hat{u}_m}$ , and a survival function  $p(m)$  with non-constant  $\varepsilon_p$ , we have that the ratio  $s_t/m_t$  is consistent with empirical evidence if the sum  $\varepsilon_{\hat{u}_m} + \varepsilon_p$  is first decreasing and then increasing. This specification implies that the model is intractable with analytical tools.

#### IV.C. Constant elasticity of utility with respect to investment in health

In a model with non-constant  $\varepsilon_{\hat{u}_c}$ , constant  $\varepsilon_{\hat{u}_m}$  and non-constant  $\varepsilon_p$  the path of the ratio  $s_t/m_t$  depends on the movements of  $\varepsilon_{\hat{u}_c}$ ,  $\varepsilon_p$  and on the value of the constant elasticity  $\varepsilon_{\hat{u}_m}$ .

In the next section we present a model where the utility function presents a zero elasticity with respect to health investment. This specification allows us to replicate the empirical results.



### V. *A Model with zero elasticity of utility with respect to investment in health*

In this section we present a simplified version of the general utility function displayed in Eq. (7). In particular, we suppose that health does not enter in the utility function and affects only the survival function. Thus, the lifetime utility takes the following form:

$$U_t = u(c_t) + \beta p(m_t)u(c_{t+1}), \quad (11)$$

subject to the budget constraints given by the Eq. (1) and Eq. (2).

Given zero utility from health level, the ratio between saving and health investment is equal to the ratio between the elasticity of the utility with respect to consumption in old age and the elasticity of the probability function with respect to health investment. Thus Eq. (8) becomes:

$$\frac{s_t}{m_t} = \frac{\varepsilon_{u_c}}{\varepsilon_p}. \quad (12)$$

#### V.A. Survival function

Given Eq. (3) we specify the following probability of surviving to old age:

$$p(m_t) = \begin{cases} \underline{p} + \lambda m_t^\delta, & \text{if } m_t \in [0, \hat{m}] \\ \bar{p} & \text{if } m_t > \hat{m} \end{cases} \quad (13)$$

where  $0 < \delta < 1$ ,  $\lambda > 0$ ,  $\underline{p}$  is the minimum agent's survival probability if they do not invest in health services and  $\bar{p}$  is the highest probability of surviving to old age<sup>9</sup>. This means that an increase in the level of health investment beyond  $\hat{m}$  cannot increase the probability of surviving<sup>10</sup>. In particular  $\hat{m}$  is given by:

$$\hat{m}_t = \left( \frac{\bar{p} - \underline{p}}{\lambda} \right)^{1/\delta}. \quad (14)$$

<sup>9</sup>Assuming  $\underline{p} = 0.1$ , non linear least square estimates of the parameters  $\lambda$  and  $\delta$  in Eq.(13) yields  $\lambda = 0.2$  and  $\delta = 0.6$ .

<sup>10</sup>Empirical analysis (figure 3) shows that in rich countries health investment is still increasing. This stylized fact support the idea that health investment did not yet reach its maximum level  $\hat{m}$ .

The elasticity of the survival function is concave with respect to health investment, that is:

$$\varepsilon_p(m_t) = \frac{\delta \lambda m_t^\delta}{\underline{p} + \lambda m_t^\delta}, \quad (15)$$

where:

$$\begin{aligned} \varepsilon_p(0) &= 0, \\ \lim_{m \rightarrow \infty} \varepsilon_p &= \delta. \end{aligned}$$

## V.B. Preferences

Jones and Hall (2006) to explain the luxury good behavior of health spending choose to add a constant term to the standard utility function with constant elasticity of substitution (C.E.S.). Using this specification in our model we obtain intractable results. Thus, we choose to use H.A.R.A (hyperbolic absolute risk aversion function)<sup>11</sup> preferences which present a non-constant elasticity with respect to the consumption. Hence the utility function is:

$$u(c) = \frac{(\theta + \sigma c)^{\frac{\sigma-1}{\sigma}}}{\sigma - 1}, \quad (16)$$

where<sup>12</sup> the constant  $\theta > 0$  can be considered as the minimum required consumption at the end of the horizon. We assume that

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<sup>11</sup>The HARA family is rich, in the sense that by suitable adjustment of the parameters we can have an utility function with absolute or relative risk aversion increasing, decreasing or constant (Merton, 1992). Thus, isoelastic (constant relative risk aversion for  $\theta = 0$ ), exponential (constant absolute risk aversion) and quadratic utility functions are subsets of HARA family. In particular:

$$\begin{aligned} \textit{if} \quad \sigma > 0 &\Rightarrow D.A.R.A \\ \textit{if} \quad \sigma < 0 &\Rightarrow I.A.R.A \\ \textit{if} \quad \sigma = \infty &\Rightarrow A.R.A = 0 \end{aligned}$$

In this paper we assume  $\sigma > 0$ .

<sup>12</sup>This utility function shows an elasticity which increases with the consumption, that is:

$$\varepsilon_{u_c} = \frac{c(\sigma - 1)}{\theta + \sigma c}$$

$\sigma > 1$ , which implies that the function is D.A.R.A like the standard utility function C.E.S.

Given Eq. (13) and (16), Eq. (12) yields the following relationship between saving and health investment:

$$\frac{s_t}{m_t} = \frac{1}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \left( 1 + \frac{\underline{p}}{\lambda m_t^\delta} \right) - \frac{\theta}{\sigma R m_t}, \quad (17)$$

which implies that the saving is concave in health investment, i.e.  $\partial s_t / \partial m_t > 0$  and  $\partial^2 s_t / \partial m_t^2 < 0$  (see Appendix AC.).

The first order conditions corresponding to Eq. (11) in the range  $[0, \hat{m}]$  are given by:

$$c_t = \frac{\theta + \sigma c_{t+1}}{\sigma [\beta R (\underline{p} + \lambda m_t^\delta)]^\sigma} - \frac{\theta}{\sigma}, \quad (18)$$

$$c_{t+1} = R \left( \frac{\sigma - 1}{\sigma} \right) \frac{m_t}{\delta} \left( 1 + \frac{\underline{p}}{\lambda m_t^\delta} \right) - \frac{\theta}{\sigma}. \quad (19)$$

From Eq. (1), Eq. (19) and Eq. (18) we obtain the following implicit relation between health investment and income, that is:

$$F(y_t, m_t) = 0,$$

where:

$$F(y_t, m_t) \equiv \left( \frac{\sigma - 1}{\sigma} \right) \frac{m_t}{\delta} \left( 1 + \frac{\underline{p}}{\lambda m_t^\delta} \right) \left[ \frac{R^{1-\sigma}}{[\beta (\underline{p} + \lambda m_t^\delta)]^\sigma} + 1 \right] + m_t - y_t - \frac{\theta}{\sigma} \left[ 1 + \frac{1}{R} \right]. \quad (20)$$

We are interested in analyzing the behavior of saving and health investment according to different levels of per capita income. The aim is to show that the elasticity of saving falls more rapidly than the elasticity of health investment, that is as people became richer, saving rises but they prefer to devote an increasing share of income to additional years of life. The following propositions define the properties of health share and saving share.

**Proposition 2** *In the range  $[0, \hat{m}]$ , a sufficient condition to have health investment increasing and convex in income, i.e.  $\partial m_t / \partial y_t > 0$  and  $\partial^2 m_t / \partial y_t^2 > 0$ , is  $\delta \leq \frac{1}{\sigma}$ . When this condition is satisfied optimal health share presents the following properties<sup>13</sup> (see figure 5):*

- (1)  $\lim_{m \rightarrow m_0} \frac{m_t}{y_t} = \infty$ ,
- (2)  $\lim_{m \rightarrow \hat{m}} \frac{m_t}{y_t} = \frac{\hat{m}}{\hat{y}} > 0$ ,
- (3)  $\frac{\partial(m_t/y_t)}{\partial y_t} = 0$  for  $y_t = y_m$ ;  $\frac{\partial(m_t/y_t)}{\partial y_t} < 0$  for  $y_t < y_m$ ;  $\frac{\partial(m_t/y_t)}{\partial y_t} > 0$  for  $y_t > y_m$ .

**Proof.** The technical part of this proposition is proved in Appendix AB.. ■

**Proposition 3** *Given the condition  $\delta \leq \frac{1}{\sigma}$ , optimal saving share in income satisfies the following properties (see figure 5):*

- (1)  $\lim_{m_t \rightarrow m_0} \frac{s_t}{y_t} = -\infty$  if  $s_0 < 0$
- (2)  $\lim_{m_t \rightarrow \hat{m}} \frac{s_t}{y_t} = \frac{\hat{s}}{\hat{y}} > 0$
- (3)  $\frac{\partial s_t / y_t}{\partial y_t} > 0$  if  $R > \frac{\sigma \delta}{(\sigma - 1)}$

**Proof.** See Appendix AC. ■

Proposition 2 and 3 imply that both saving and health investment behave like luxury goods. In particular, when income is low, i.e.  $y_t < y_m$ , health share is decreasing and presents an elasticity with respect to income  $\varepsilon_m < 1$  (see figure 5<sup>14</sup>). When income increases, i.e.  $y_t > y_m$ , the elasticity of health with respect to income rises, i.e.  $\varepsilon_m > 1$ . (see Appendix AB.). This results support some theoretical contributions which shows that the income elasticity of demand for health care is larger than one. In particular Blomqvist and Carter, 1997 estimate that the income elasticity of health care spending, for

<sup>13</sup>The value  $m_0$  define the value of  $m_t$  so that  $y_t$  is equal to zero (see appendix AA.).

<sup>14</sup>Our calibration is  $\sigma = 2$ ,  $\beta = 0.7$ ,  $R = 3$ ,  $\delta = 0.5$ ,  $\theta = 1$ ,  $\lambda = 0.2$ ,  $\underline{p} = 0.3$ .

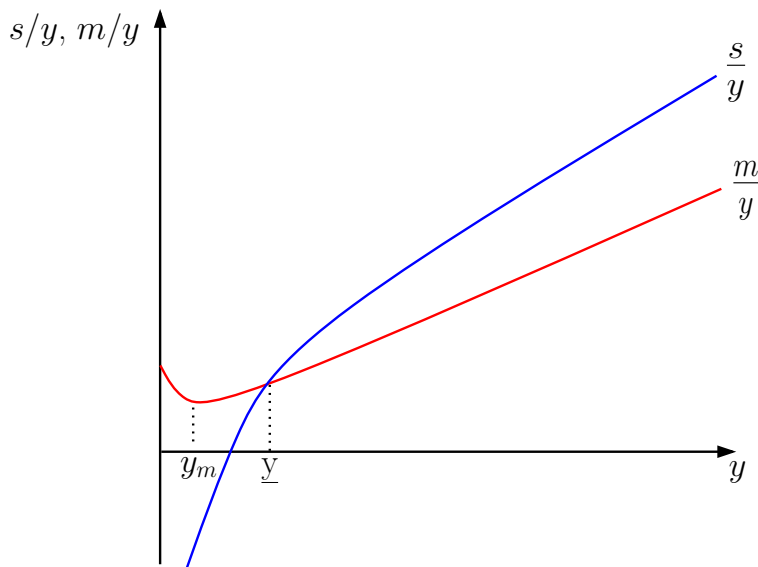


Figure 5: saving share and health share versus income.

OECD countries in the period 1960 to 1991, is significantly above one.

In figure 5 we can see that there exist a value of  $\underline{y}$  so that the saving share is equal to the health share (for the technical part see appendix AD.). Thus when the income is equal to  $\underline{y}$  the elasticity of utility function is equal to the elasticity of the survival function.

Figure 6 illustrates the results of our calibration for the ratio between optimal saving and optimal health investment with respect to different income levels (our baseline parameters values are  $\sigma = 2$ ,  $\beta = 0.7$ ,  $R = 3$ ,  $\delta = 0.5$ ,  $\theta = 1$ ,  $\lambda = 0.2$ ,  $\underline{p} = 0.3$ ). The following proposition characterizes the properties of the ratio between the saving share and health share.

**Proposition 4** *When  $y_t < \tilde{y}$  the saving grows more quickly than health investment; hence the ratio  $s_t/m_t$  is increasing as income increases. For  $y_t > \tilde{y}$  the ratio between saving and health investment decreases as income increases (see figure 6).*

**Proof.** See Appendix AD. ■

Proposition 4 implies that when income is low people devote more resources to the consumption, when income becomes higher than a

certain threshold agents spend more income to increase their probability of surviving to old age. Thus for  $y_t > \tilde{y}$  while the marginal utility of consumption decreases the marginal utility of additional years of life does not decrease. This implies that as income grows the optimal composition of spending shifts toward health investment (see appendix AD.).

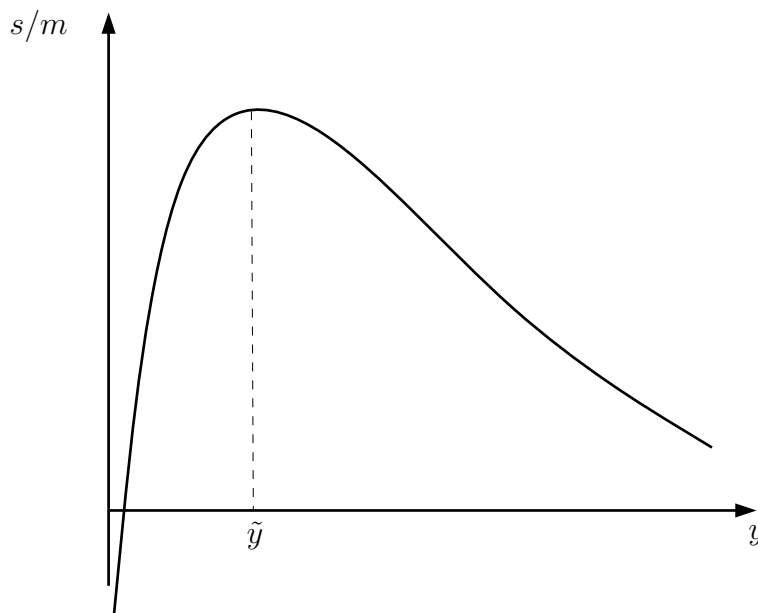


Figure 6: the ratio between saving and health expenditure versus income.

## ***VI. Conclusion***

This paper analyze agent's decision on the allocation of total resources between health investment and saving. Empirical evidence shows that when income is low agents devote more income to saving to assure consumption in the old age. As income rises the saving continues to rise but health spending increases more quickly. This indicates that for low levels of income, the elasticity of the utility function with respect to consumption is greater than the elasticity of the survival function with respect to health investment. When income rises the opposite occurs. The intuition for this results is

that as income grows people become saturated in non-health consumption and choose to spend more income to purchase additional years of life. This mechanism is supported with a theoretical model in which agents present HARA preferences and the survival function shows a non-constant elasticity with respect to health investment.

In the future, we plan to specify a model in which health level directly enters in the utility function. We need to know health inequality within countries and the effect of public and private health investment on health inequality. This determines whether and by how much income redistribution can improve population health.

## A Appendix

### AA. Proof of the existence of $m_0$

When  $y_t = 0$ , from Eq.(1) we have that:

$$m_t = -(c_t + s_t),$$

which, from Eq. (18) and Eq. (19), yields :

$$m_t + \frac{m_t}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \left( 1 + \frac{\underline{p}}{\lambda m_t^\delta} \right) \left[ \frac{R}{[\beta R (\underline{p} + \lambda m_t^\delta)]^\sigma} + 1 \right] - \frac{\theta}{\sigma} \left[ \frac{1}{R} + 1 \right] = 0 \quad (21)$$

We show here the existence of a value of  $m_t$ , i.e.  $m_0$ , so that the income is equal to zero. The value  $m_0$  can be considered as the activities that agents undertake to survive when they do not have resources. Moreover if we consider health spending as the sum of public health investment and private health investment, we can think that when income is equal zero agents receive a subsistence amount of resources to survive (see note 5).

From Eq.(21) we can define the two functions:

$$\Phi_1(m_t) = \frac{m_t}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \left( 1 + \frac{\underline{p}}{\lambda m_t^\delta} \right) \left[ \frac{R}{[\beta R (\underline{p} + \lambda m_t^\delta)]^\sigma} \right], \quad (22)$$

$$\Phi_2(m_t) = \frac{\theta}{\sigma} \left[ 1 + \frac{1}{R} \right] - m_t \left[ 1 + \frac{1}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \left( 1 + \frac{\underline{p}}{\lambda m_t^\delta} \right) \right]. \quad (23)$$

The function in Eq. (22) increases with respect to health investment, that is:

$$\frac{\partial \Phi_1(m_t)}{\partial m_t} = \frac{1}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \frac{R}{(\beta R)^\sigma} \left[ \frac{\lambda m_t^\delta (1 - \sigma\delta) + \underline{p}(1 - \delta)}{\lambda m_t^\delta (\underline{p} + \lambda m_t^\delta)^\sigma} \right] > 0,$$

since  $1 - \sigma\delta$  is assumed positive from proposition 2, and  $\Phi_1(0) = 0$ .

The function  $\Phi_2(m_t)$  in Eq. (23) is decreasing with respect to health investment, that is:

$$\frac{\partial \Phi_2(m_t)}{\partial m_t} = - \left[ \frac{\lambda m_t^\delta (\sigma - 1 + \delta\sigma) + \underline{p}(\sigma - 1)(1 - \delta)}{\sigma\delta\lambda m_t^\delta} \right] < 0$$

and  $\Phi_2(0) = \frac{\theta}{\sigma} \left[ 1 + \frac{1}{R} \right]$ .

Thus since  $\Phi_1(m_t)$  and  $\Phi_2(m_t)$  have different intercept, i.e.  $\Phi_1(0) = 0$  and  $\Phi_2(0) = \frac{\theta}{\sigma} \left[ 1 + \frac{1}{R} \right]$ , and  $\Phi_1(m_t)$  is increasing in health and  $\Phi_2(m_t)$  is decreasing in health, we obtain that there exist a value of  $m_t$ , i.e  $m_0$ , such that the two functions intersect.

## AB. Proof of proposition 2

Equation (20) implicitly defines optimal health investment as a function of income. Applying the implicit function theorem to Eq. (20) we get:

$$\frac{\partial m_t}{\partial y_t} = \frac{\sigma\delta\lambda m_t^\delta G(m_t)}{(1 - \delta)(\sigma - 1)\underline{p}[G(m_t) + R] + \lambda m_t^\delta [R(1 - \sigma\delta)(\sigma - 1) + (\sigma - 1 + \sigma\delta)G(m_t)]} \quad (24)$$

where:

$$G(m_t) = [\beta R(\underline{p} + \lambda m_t^\delta)]^\sigma$$

A sufficient condition to have health increasing in income is that:

$$\delta \leq \frac{1}{\sigma}. \quad (25)$$

We have that  $\partial^2 m_t / \partial y_t^2 > 0$  if:

$$R\sigma\lambda m_t^\delta [\lambda m_t^\delta (1 - \sigma\delta) + \underline{p}(1 - \delta)] + \underline{p}(1 - \delta) (\lambda m_t^\delta + \underline{p}) [G(m_t) + R] > 0.$$



which is satisfied when inequality (25) holds.

### Analysis of Health Share

Eq. (20) the following expression defines the health share:

$$\frac{m_t}{y_t} = m_t \left\{ \left( \frac{\sigma - 1}{\sigma} \right) \frac{m_t}{\delta} \left( 1 + \frac{\underline{p}}{\lambda m_t^\delta} \right) \left[ \frac{R^{1-\sigma}}{[\beta (\underline{p} + \lambda m_t^\delta)]^\sigma} + 1 \right] + m_t - \frac{\theta}{\sigma} \left[ 1 + \frac{1}{R} \right] \right\}^{-1}. \quad (26)$$

When income tends to zero, i.e.  $m_t \rightarrow m_0$ , we get:

$$\lim_{m \rightarrow m_0} \frac{m_t}{y_t} = \frac{m_0}{0} = \infty. \quad (27)$$

When  $y_t \rightarrow \infty$  which, from Eq. (13), implies that  $m_t \rightarrow \hat{m}$ , health share is equal to a positive constant:

$$\lim_{m \rightarrow \hat{m}} \frac{m_t}{y_t} = \frac{\hat{m}}{\hat{y}} > 0.$$

Deriving Eq. (26) with respect to income we obtain:

$$\frac{\partial (m_t/y_t)}{\partial y_t} = \frac{(\partial m_t / \partial y_t) y_t - m_t}{y_t^2}, \quad (28)$$

where  $\partial (m_t/y_t) / \partial y_t > 0$  if:

$$\varepsilon_m = \frac{(\partial m_t / \partial y_t) y_t}{m_t} > 1, \quad (29)$$

where  $\varepsilon_m$  is the elasticity of health spending with respect to income. Thus health share behaves like a luxury good if presents an income elasticity larger than one.

Since the denominator of Eq. (28) is always positive we study the numerator that is given by the following expression:

$$(\sigma - 1) \delta m_t (\sigma \lambda m_t^\delta + \underline{p}) R^2 + \delta G(m_t) [(\sigma - 1) R m_t \underline{p} - \lambda m_t^\delta \theta (1 + R)],$$

from which  $\varepsilon_m = 1$  if:

$$\frac{(\sigma - 1) m_t^{1-\delta} (\sigma \lambda m_t^\delta + \underline{p}) R^2}{G(m_t)} = -m_t^{1-\delta} (\sigma - 1) R \underline{p} + \lambda \theta (1 + R). \quad (30)$$

Thus we can analyze the two functions:

$$\psi_1(m_t) = \frac{(\sigma - 1) m_t^{1-\delta} (\sigma \lambda m_t^\delta + \underline{p}) R^2}{G(m_t)},$$

$$\psi_2(m_t) = -m_t^{1-\delta} (\sigma - 1) R \underline{p} + \lambda \theta (1 + R).$$

From condition in Eq. (25) we have that the function  $\psi_1(m_t)$  is increasing in health investment, that is:

$$\frac{\partial \psi_1}{\partial m_t} = \frac{(\sigma - 1)}{G(m_t)} \left\{ \sigma \lambda [\lambda m_t^\delta (1 - \sigma \delta) + \underline{p} (1 - \delta)] + \frac{(1 - \delta) \underline{p}}{m_t^\delta} \right\} > 0,$$

and:

$$\psi_1(0) = 0,$$

$$\lim_{m \rightarrow \infty} \psi_1(m_t) = \infty.$$

The function  $\psi_2$  decreases in health investment, that is:

$$\frac{\partial \psi_2}{\partial m_t} = -\frac{(\sigma - 1) (1 - \delta) R \underline{p}}{m_t^\delta} < 0,$$

and:

$$\psi_2(0) = \lambda \theta (1 + R),$$

$$\lim_{m \rightarrow \infty} \psi_2(m_t) = -\infty.$$

Thus there exist a value  $\bar{m}$  so that Eq. 30 is satisfied, that is  $\varepsilon_m = 1$ . Substituting this value  $\bar{m}$  to the Eq. (20) we obtain the value  $y_m$  so that  $\varepsilon_m = 1$ . When  $y_t < y_m$  then  $\psi_2(m_t) > \psi_1(m_t)$ , that is  $\varepsilon_m < 1$  and the health share is decreasing in income. When  $y_t > y_m$  then  $\psi_2(m_t) < \psi_1(m_t)$  and  $\varepsilon_m > 1$ , that is the health share increases.

### AC. Proof of proposition 3

The relationship between saving and health is positive and concave. That is, differentiation of Eq. (19) with respect to health investment give us:

$$\frac{\partial s_t}{\partial m_t} = \frac{1}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \left[ \frac{p(1 - \delta) + \lambda m_t^\delta}{\lambda m_t^\delta} \right], \quad (31)$$

and :

$$\frac{\partial^2 s_t}{\partial m_t^2} = - \left( \frac{\sigma - 1}{\sigma} \right) \frac{p(1 - \delta)}{m_t^{(\delta+1)}}.$$

Thus  $\partial s_t / \partial m_t > 0$  and  $\partial^2 s_t / \partial m_t^2 < 0$  since  $0 < \delta < 1$  and  $\sigma > 1$ .

When  $m_t = 0$  we have that the saving is negative, that is:

$$s_t = -\frac{\theta}{\sigma R}$$

We suppose that when  $m_t = m_0$  the saving is negative, that is:

$$s_0 = m_0^{1-\delta} (\lambda m_0^\delta + p) - \frac{\theta \delta}{R(\sigma - 1)} < 0 \quad (32)$$

From condition in Eq.(25) we obtain that the saving behaves like a luxury good, that is:

$$\frac{\partial s_t}{\partial y_t} = \frac{\partial s_t}{\partial m_t} \frac{\partial m_t}{\partial y_t} > 0$$

### Analysis of Saving Share

Eq. (19) and Eq. (20) yield the following expression for the saving share on income:

$$\frac{s_t}{y_t} = \frac{1}{y_t} \left[ \frac{m_t}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \left( 1 + \frac{p}{\lambda m_t^\delta} \right) - \frac{\theta}{\sigma R} \right]. \quad (33)$$

From Eq. (21), when  $y_t = 0$ , i.e  $m_t = m_0$ , given the condition in Eq. (32), we obtain:

$$\lim_{m \rightarrow m_0} \frac{s_t}{y_t} = -\infty. \quad (34)$$

When  $m_t \rightarrow \hat{m}$  we have that:

$$\lim_{m \rightarrow \hat{m}} \frac{s_t}{y_t} = \frac{\hat{s}}{\hat{y}} > 0$$

Deriving the saving share with respect to income we get:

$$\frac{\partial(s_t/y_t)}{\partial y_t} = \frac{1}{y_t^2} \left[ \frac{\partial s_t}{\partial y_t} y_t - s_t \right], \quad (35)$$

Eq. (35) is given by the following expression:

$$(\sigma - 1) R \left\{ \theta \left[ (1 - \sigma\delta) \lambda m_t^\delta + \underline{p} (1 - \delta) \right] + (\sigma - 1) \sigma m_t (\underline{p} + \lambda m_t^\delta) R \right\} + G(m_t) \left[ (\sigma - 1) (\theta\delta - \theta - \sigma m_t \delta) \underline{p} R + \theta \lambda m_t^\delta (\sigma (\delta - R) + R) \right],$$

from which  $\partial(s_t/y_t)/\partial y_t > 0$  if :

$$(\sigma - 1) R \left\{ \theta \left[ (1 - \sigma\delta) \lambda m_t^\delta + \underline{p} (1 - \delta) \right] + (\sigma - 1) \sigma m_t (\underline{p} + \lambda m_t^\delta) R \right\} + G(m_t) \left[ \frac{(\sigma - 1) (\theta\delta - \theta - \sigma m_t \delta) \underline{p} R}{(\sigma (\delta - R) + R)} \right] > -G(m_t) \theta \lambda m_t^\delta.$$

We define the function in left side  $\Upsilon_1(m_t)$  and the function in the right side  $\Upsilon_2(m_t)$ . The function  $\Upsilon_1(m_t)$  at  $m = 0$  is positive:

$$\Upsilon_1(0) = (\sigma - 1) R \left\{ \theta \left[ \underline{p} (1 - \delta) \right] \right\} + \left[ \frac{(\sigma - 1) \theta (1 - \delta) \underline{p} R}{R (\sigma - 1) - \sigma \delta} \right] > 0,$$

if:

$$R > \frac{\sigma \delta}{(\sigma - 1)}. \quad (36)$$

We obtain that  $\Upsilon_1(m_t)$  is increasing in  $m_t$ , that is:

$$\begin{aligned} \frac{\partial \Upsilon_1}{\partial m_t} &= (\sigma - 1) \sigma \left[ (1 + \delta) \lambda m_t^\delta + \underline{p} \right] + \frac{\sigma \beta \delta \underline{p} \left[ (1 + \sigma \delta) \lambda m_t^\delta + \underline{p} \right] R G(m_t)}{(R (\sigma - 1) - \sigma \delta) G(m_t)^{1/\sigma}} + \\ &+ \theta \delta \lambda m_t^{\delta-1} \left[ 1 - \sigma \delta + \frac{\sigma \beta (1 - \delta) \underline{p} R G(m_t)}{(R (\sigma - 1) - \sigma \delta) G(m_t)^{1/\sigma}} \right] > 0, \end{aligned}$$

if condition in Eq. (25) and condition in Eq. (36) are satisfied.

The function  $\Upsilon_2(m_t)$  is decreasing in  $m_t$  and at  $m_t = 0$  is equal to zero, that is:

$$\Upsilon_2(0) = 0,$$

and:

$$\frac{\partial \Upsilon_2}{\partial m_t} = -G(m_t)\theta\lambda m_t^\delta < 0$$

Thus since the function  $\Upsilon_1(m_t) > \Upsilon_2(m_t)$  if the condition in Eq. (25) and the condition in Eq. (36) are satisfied, then the saving share increases with income.

#### AD. Proof of proposition 4

Given Eq. (19) we get that:

$$\frac{\partial(s_t/m_t)}{\partial y_t} = \frac{1}{m_t^2} \frac{\partial m_t}{\partial y_t} \left[ \frac{\partial s_t}{\partial m_t} m_t - s_t \right]$$

where from Eq. (25)  $\partial m_t / \partial y_t > 0$ . From Eq. (31) and Eq. (19) we obtain:

$$\frac{\partial s_t}{\partial m_t} m_t - s_t = \frac{\theta}{\sigma R} - \left( \frac{\sigma - 1}{\sigma} \right) \left( \frac{p}{\lambda} m_t^{1-\delta} \right) = 0,$$

when:

$$\tilde{m} = \left[ \frac{\lambda \theta}{p R (\sigma - 1)} \right]^{\frac{1}{1-\delta}}. \quad (37)$$

Substituting Eq. (37) in Eq. (20) we get:

$$\tilde{y} = y(\tilde{m}).$$

If  $y < \tilde{y}$  then  $\partial(s_t/m_t)/\partial y_t > 0$ , that is:

$$\frac{\theta}{\sigma R} - \left( \frac{\sigma - 1}{\sigma} \right) \left[ \frac{p}{\lambda} m_t^{1-\delta} \right] > 0,$$

when:

$$m_t < \tilde{m}.$$

When  $y_t > \tilde{y}$  the ratio  $s_t/m_t$  is decreasing, that is:

$$\frac{\theta}{\sigma R} - \left( \frac{\sigma - 1}{\sigma} \right) \left[ \frac{p}{\lambda} m_t^{1-\delta} \right] < 0$$

if:

$$m_t > \tilde{m}$$

Thus the ratio  $s_t/m_t$ , for  $y < \tilde{y}$  is increasing and for  $y > \tilde{y}$  is decreasing.

Given Eq. (17) we show that there exist a value of  $y$  such that health share is equal to the saving share:

$$\frac{s_t}{m_t} = 1, \quad (38)$$

thus:

$$\frac{1}{\delta} \left( \frac{\sigma - 1}{\sigma} \right) \left( 1 + \frac{p}{\lambda m_t^\delta} \right) - \frac{\theta}{\sigma R m_t} - 1 = 0,$$

from which we study the two function:

$$m_t [(\sigma - 1) - \delta\sigma] - \theta\delta = -\frac{R(\sigma - 1)}{\lambda} m_t^{1-\delta}$$

where the function in the left side for  $m_t = 0$  is equal  $-\theta\delta$ , for  $m_t \rightarrow \infty$  it goes to infinity and finally it increases with  $m_t$  if the following condition is satisfied:

$$\sigma > \frac{1}{1 - \delta}$$

The function in the right is decreasing and for  $m_t = 0$  it is equal to zero and  $m_t \rightarrow \infty$  it is equal to  $-\infty$ . Thus since the function in the left increases and the function in the right decreases, the two functions cross at  $\underline{m}$ . Substituting  $\underline{m}$  in Eq. (20) we obtain the value  $\underline{y}$  so that the saving share is equal to the health share, that is  $\varepsilon_{u_c} = \varepsilon_p$ .

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