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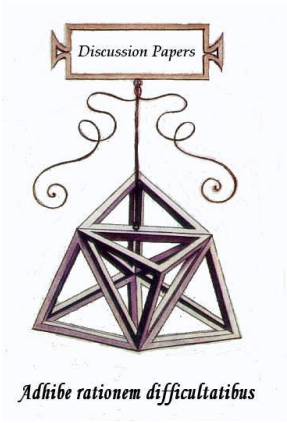

Luciano Fanti and Luca Gori

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Economic Growth and Welfare in a Simple Neoclassical OLG Model with Minimum Wage and Consumption Taxes

Luciano Fanti* and Luca Gori**

Abstract

Since little attention, despite a long lasting debate, has been paid to the effects of minimum wages on economic growth and welfare, this paper investigates such effects within a textbook OLG framework with an unemployment insurance scheme financed at balanced budget by the government with a consumption tax on the young individuals. Some new results, so far escaped closer scrutiny by the economic growth literature and which may have interesting policy implications, emerge: i) introducing minimum wages may have a favourable impact on the long run output levels; ii) under suitable conditions a regulated-wage economy performs always better than a competitive economy, iii) despite the fact that the tax rate tends to reduce the consumption of the young, the long run lifetime welfare may be greater than in the competitive economy; iv) a welfare-maximising value of the minimum wage (and thus of the proportional consumption tax rate) is picked up.

Keywords: Minimum wage; Unemployment; Consumption Tax; Neoclassical Economic Growth; Welfare

JEL classification: E24; H24; O41

* Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I-56124 Pisa (PI), Italy; e-mail address: lfanti@ec.unipi.it; tel.: +39 050 22 16 369; fax: +39 050 59 80 40.

** Corresponding author. Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I-56124 Pisa (PI), Italy; e-mail address: luca.gori@ec.unipi.it; tel.: +39 050 22 16 372; fax: +39 050 59 80 40.

Sections 1, 3, 5 and Appendix A should be attributed to Luciano Fanti; Sections 2, 4 and Appendix B should be attributed to Luca Gori.

1 Introduction

Although a vast debate about the macroeconomic consequences of minimum wages has been developed dating from Stigler (1946), less attention has been paid as regards the long run effects of the regulation of wages in a dynamic model (i.e. a simple OLG economy). So far the literature has generally believed that the introduction of the wage regulation in a simple competitive economy would have been always caused an output loss due to the unemployment occurrence.[1] Only few papers have investigated possible positive macroeconomic effects of the minimum wage law, but only postulating the existence of a relationship between the unemployment created by the minimum wage and the long-run productivity growth induced by schooling and on the-job-training: for instance Cahuc and Michel (1996) and Ravn and Sorensen (1999). In this paper we show that the regulation of wages could have a favourable impact on both economic growth[2] and lifetime welfare, and under suitable conditions a regulated wage economy may perform better than a market-wage economy. Noteworthy, our conclusions are reached within a textbook OLG model with the only departures of 1) the assumption of a minimum wage imposed by national law; 2) an unemployment insurance scheme financed at balanced budget with a consumption tax on the young generation.
The plan of the paper is as follows. In section 2 we develop the model and we analyse the effects of minimum wages on capital and production. In section 3 we present the analysis of the steady-state lifetime welfare and, finally, section 4 concludes.

2 The Market-Wage Economy

In this section we consider a standard dynamic general equilibrium OLG economy (as in Samuelson (1958) and Diamond (1965)) with young population N_t growing at the constant rate n and closed to international trade, and where goods, capital and labour markets are competitive.[3]

2.1 Individuals. Each generation is represented by identical individuals who live for two periods. Only young individuals work. In the first time-period they supply inelastically one unit of labour and receive wage income. This income is used to consume and to save. During the second period of life old-age individuals are retired and live on the proceeds of their savings, earning a return of $1+r_{t+1}$ on their investments when young, where r_{t+1} is the rate of return on savings (s_t) from t to $t+1$. Individuals are non-altruistic and have a homothetic and separable utility function defined over consumption when young and old: ${c_t}^y$ and ${c_{t+1}}^o$ respectively. The lifetime utility of the representative individual born at time t is $U_t\left({c_t}^y, {c_{t+1}}^o\right) = (1-\phi)\ln\left({c_t}^y\right) + \phi \ln\left({c_{t+1}}^o\right)$, where $\phi \in (0,1)$ is a consumption preference parameter (that is, $\phi/(1-\phi)$ is the rate of time preference). The higher ϕ the more individuals prefer to postpone consumption in the future. Each generation takes the time-t real wage (w_t) and the real interest rate on savings as given. Therefore, the maximisation of $U_t\left({c_t}^y, {c_{t+1}}^o\right)$ under the constraints ${c_t}^y + s_t = w_t$, ${c_{t+1}}^o = (1+r_{t+1})s_t$, ${c_t}^y \geq 0$ and ${c_{t+1}}^o \geq 0$ implies the optimal young and old age consumption functions are the following:

$$ {c_t}^y = (1-\phi)w_t, \tag{1} $$

$$ {c_{t+1}}^o = \phi(1+r_{t+1})w_t. \tag{2} $$

The solution of the problem may also be expressed in terms of the savings function as:[4]

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[1] For instance, "it is generally recognized that minimum wage legislation induces distortions which have adverse effects on the efficiency of the economy." (Cahuc and Michel (1996), p. 1464).

[2] In this paper the term economic growth always refers to the to the level (rather than to the rate of growth) of the long run income, according to the terminology of the neoclassical growth theory (e.g. Solow (1956) and Mankiw et al. (1992)). In any case, needless to say, an increase in the long run level of output, implies a transitional increase in the rate of growth as well.

[3] Two reference textbooks are Azariadis (1993) and De La Croix and Michel (2002).

$$s_t = \phi w_t . \tag{3}$$

It can be easily seen by eq. (3) that ϕ also represents the (constant) propensity to save.

2.2 Firms. All the firms on the economy are identical and own a constant returns to scale Cobb-Douglas production technology by which physical capital and labour are transformed into consumption good.[5] Thus, the representative profit-maximising firm hires aggregate capital stock (K_t) and demands labour supplied by young agents ($L_t = N_t$ in equilibrium) to determine aggregate production, that is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where $A > 0$ is a technology scale parameter and $\alpha \in (0,1)$ is the capital weight in technology.[6] Factor prices are taken as given. Having normalised the price of output to unity, profits maximisation leads to the following marginal conditions for capital and labour:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{4}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{5}$$

2.3 Equilibrium. Given the economy's resource constraint, $y_t = c_t^{y} + c_t^{o}/(1+n) + (1+n)k_{t+1}$, the market-clearing condition in goods as well as in capital markets is usually determined by the equality between savings and investments, i.e. with the hypothesis of total obsolescence of capital over time and knowing that $N_{t+1} = (1+n)N_t$, equilibrium implies:

$$(1+n)k_{t+1} = s_t , \tag{6}$$

and combining (6) with (3) and (5), capital evolves over time according to the following first order non-linear difference equation:

$$(1+n)k_{t+1} = \phi(1-\alpha) A k_t^{\alpha}. \tag{7}$$

Steady-state implies $k_{t+1} = k_t \coloneqq k^*$. Hence, the long-run (per-capita) stock of capital is:

$$k^*_{pc} = \left(\frac{\phi(1-\alpha)A}{1+n} \right)^{\frac{1}{1-\alpha}}. \tag{8}$$

Substitution of k^*_{pc} into the intensive form production function and into eqs. (4) and (5) yields the long-run per-capita output, and the long-run interest rate and market clearing wage respectively:

$$y^*_{pc} = A\left(\frac{\phi(1-\alpha)A}{1+n} \right)^{\frac{\alpha}{1-\alpha}}, \tag{9}$$

$$r_{pc} = \alpha A \left(k^*_{pc}\right)^{\alpha-1} - 1 = \frac{\alpha(1+n)}{(1-\alpha)\phi} - 1, \tag{10}$$

$$w_{pc} = (1-\alpha) A \left(k^*_{pc}\right)^{\alpha} = \left(\frac{\phi}{1+n} \right)^{\frac{\alpha}{1-\alpha}} \left((1-\alpha)A\right)^{\frac{1}{1-\alpha}}. \tag{11}$$

3 The Regulated-Wage Economy

We now characterise a two-period OLG economy where goods and capital markets are both competitive and where the only departure from the model typified in the previous section is the

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[4] Given our log utility specification, the elasticity of savings with respect to the interest rate is equal to zero. Anyway, by considering a more general CIES utility function, where the interest rate affects savings, it can be seen that the main findings of this paper are numerically confirmed. For the sake of analytical tractability we use here the usual Cobb-Douglas preferences.

[5] For simplicity we assume physical capital totally depreciates over time, i.e. $\delta = 1$.

[6] By defining $k_t \coloneqq K_t / N_t$ and $y_t \coloneqq Y_t / N_t$ as capital and output per-capita respectively, the intensive form production technology is simply $y_t \coloneqq A k_t^{\alpha}$.

existence of an imperfect labour market in which a binding minimum wage per hour worked ($\underline{w}$) is introduced by law.[7] Thus, the labour market does not clear and unemployment occurs. The model is outlined in what follows.

3.1 Individuals. Only young individuals work, assuming a unitary constant labour supply. Depending on the demand for labour, the supplied labour force may be partially unemployed. If employed, wage income is $\underline{w}$. If unemployed, the government pays an unemployment insurance benefit indexed with the minimum wage, i.e. $b(\underline{w}) := \gamma \underline{w}$ where $\gamma \in (0,1)$ is the so-called replacement ratio. We treat $\underline{w}$ and γ as policy parameters, whereas the quantity of employed labour force is endogenous. The aggregate unemployment rate is defined as $u_t = (N_t - L_t)/N_t$, where $L_t = (1-u_t)N_t$ is the total number of hours worked by agents during youth. We assume that only a proportional (non-distorting) tax on consumption of the young people at the rate τ^c is levied by the government[8] and used to finance the unemployment benefit system at balanced budget. The individual maximisation problem faced by agents of generation t modifies to:

$$\max_{\{c_t^{\,y}, c_{t+1}^{\,o}\}} U_t\left(c_t^{\,y}, c_{t+1}^{\,o}\right) = (1-\phi)\ln\left(c_t^{\,y}\right) + \phi \ln\left(c_{t+1}^{\,o}\right),$$

subject to

$$\begin{aligned} c_t^{\,y}\left(1+\tau_t^{\,c}\right) + s_t &= \underline{w}(1-u_t) + \gamma \underline{w} u_t \\ c_{t+1}^{\,o} &= (1+r_{t+1})s_t \\ c_t^{\,y}, c_{t+1}^{\,o} &\geq 0 \end{aligned} \quad .$$

The optimal young and old age consumption functions as a function of the unemployment rate become:

$$c_t^{\,y}(\underline{w}, u_t) = \frac{1-\phi}{1+\tau_t^{\,c}} W_t(\underline{w}, u_t), \tag{12}$$

$$c_{t+1}^{\,o}(\underline{w}, u_t) = \phi(1+r_{t+1}) W_t(\underline{w}, u_t), \tag{13}$$

where $W_t(\underline{w}, u_t) := \underline{w}[1 - u_t(1-\gamma)]$ represents the total income of the young (given by the sum of the labour income, $\underline{w}$, plus the unemployment insurance benefit, $b(\underline{w})$).
The savings function, instead, is the following:

$$s_t(\underline{w}, u_t) = \phi W_t(\underline{w}, u_t). \tag{14}$$

3.2 Firms. Goods and capital markets are both competitive. The labour market is imperfect and regulated via the introduction of a minimum wage per hour worked. Since firms hire labour according to their demand curve, and given that the temporary equilibrium condition in the labour

[7] We assume $\underline{w}$ to be constant over time. It is worth noting that in this model where, for simplicity, there exists one type of labour only, a binding minimum wage simply means a wage (regulated by law) higher than the one that clears the labour market. In the case of more than one type of labour with uniformly distributed wages, this assumption would simply mean a regulated-wage fixed over the prevailing average wage. It is easy to see that when the minimum wage is fixed over the steady state competitive level, it is always binding even out of the steady state. In any case, in this paper we focus only on the steady-state results.

[8] We have deliberately chosen a consumption tax only levied upon young people for two reasons: 1) a better analytical tractability; 2) in this way the nature of unemployment benefits is purely redistributionary, that is the consumption taxed away from the young is again given to the young as benefits when unemployed, and thus the old people are not affected by the taxation policy. For simplicity, we treat here neither the issue of the implementability of such a tax nor the corresponding administrative costs, but it is worth to note that the qualitative results of this paper also hold when the consumption tax is levied on both periods consumption (the proof is disposable on request).

market implies $L_t = (1-u_t)N_t$, the Cobb-Douglas intensive-form production function transforms to:[9]

$$y_t = A(1-u_t)\left(\frac{k_t}{1-u_t}\right)^{\alpha}. \tag{15}$$

Standard profit maximisation leads to the following marginal conditions for capital and labour:

$$r_t = \alpha A\left(\frac{k_t}{1-u_t}\right)^{\alpha-1} - 1, \tag{16}$$

$$\underline{w} = (1-\alpha)A\left(\frac{k_t}{1-u_t}\right)^{\alpha}. \tag{17}$$

It is worth noting that once the wage has been fixed the real interest rate is exogenous, that is it does not depend on the capital stock. Substituting out (17) into (16) for $k_t/(1-u_t)$ yields:

$$r(\underline{w}) = \alpha A(\psi/\underline{w})^{\frac{1-\alpha}{\alpha}} - 1, \tag{18}$$

where $\psi := (1-\alpha)A$. An increase in $\underline{w}$ always reduces the real interest rate. Moreover, $r(\underline{w}) < r(w_{pc})$ for any $\underline{w} > w_{pc}$, where w_{pc} is the steady-state market clearing wage. The short-run (current) unemployment rate is endogenous, and solving eq. (17) for u_t we get:

$$u_t(k_t, \underline{w}) = 1 - (\psi/\underline{w})^{\frac{1}{\alpha}} \cdot k_t, \tag{19}$$

which is positively related with the minimum wage and strictly decreasing in the capital per-capita.

3.3 Government. One effect of the regulation of wages is to cause a positive level of unemployment. Therefore, in presence of an unemployment benefit scheme, there exists the necessity to finance the payment of benefits. There are many ways to raise revenues for financing the benefits system. The extent to which a long run welfare improvement will be successful depends crucially upon the type of taxation used.[10] Since in this paper we have supposed that the revenues used to finance the unemployment benefit scheme under balanced budget only derives by proportional taxes on consumption of the young generation, we suppose that the government strategy is to adjust the consumption tax rate such as to balance out unemployment benefit expenditures with tax receipts in each period. Thus, the per-capita time-t government constraint is simply the following:

$$\gamma \underline{w} u_t = \tau_t^{\,c} c_t^{\,y}. \tag{20}$$

3.4 Equilibrium. We now combine all the pieces of the model to analyse the long run equilibrium. Given the economy's resource constraint and the government balanced budget equation (see eq. (20)), the market clearing condition in goods as well as in capital markets is simply given by the equality between savings and investment, that is:

$$(1+n)k_{t+1} = s_t(\underline{w}, u_t), \tag{21}$$

and combining (21) with (14) we find:

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[9] In order to better clarify the meaning of the coefficient α, it is worth noting that a possible interpretation is that the capital stock may be thought in its broad concept, including physical and human components and that the labour input only includes non-specialised labour. In fact, as argued by Mankiw et al. (1992), p. 417, the minimum wage is a proxy of the return to labour without human capital; they suggest that since the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing, then 50 to 70 percent of total labour income represents the return to human capital, so that if the physical capital's share of income is expected to be about 1/3, the human capital's share of income should be between 1/3 and one half. In sum, with the broad view of capital the coefficient α may be fairly about 0.6 and 0.8.

[10] For the sake of brevity we do not present here the investigation of the cases with taxes on the income from capital and lump-sum taxes on young and old people, which are analysed in two different companion papers.

$$(1+n)k_{t+1} = \phi \underline{w}[1 - u_t(k_t, \underline{w}) \cdot (1-\gamma)]. \tag{22}$$

Substituting out for $u_t(k_t, \underline{w})$ from eq. (19), capital evolves over time according to the following first order linear difference equation:

$$k_{t+1} = \frac{\phi}{1+n}(1-\gamma)\psi^{\frac{1}{\alpha}} \underline{w}^{-\frac{1-\alpha}{\alpha}} k_t + \frac{\phi}{1+n}\gamma \underline{w}. \tag{23}$$

Steady-state implies $k_{t+1} = k_t := k^*$. When the wage is regulated, the per-capita long-run unemployment rate, capital stock and income are given by:

$$u^*(\underline{w}) = \frac{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi\psi^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}, \tag{24}$$

$$k^*(\underline{w}) = \frac{\phi\gamma \underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}, \tag{25}$$

$$y^*(\underline{w}) = \frac{A\phi\gamma\psi^{\frac{1-\alpha}{\alpha}} \underline{w}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}, \tag{26}$$

which are defined for any $\underline{w} \neq w_T$, where $w_T := (1-\gamma)^{\frac{\alpha}{1-\alpha}} \cdot w_{pc} < w_{pc}$.

Solving eq. (23) yields:

$$k_t = k_0 h^t + k^*(\underline{w}). \tag{27}$$

with $k_0 > 0$ given and $h := \frac{\phi(1-\gamma)\psi^{\frac{1}{\alpha}}}{(1+n)\underline{w}^{\frac{1-\alpha}{\alpha}}}$. Stability requires $h < 1$, that is $\underline{w} > w_T$. Since $w_T < w_{pc} < \underline{w}$ holds true, then the regulated-wage economy always converges towards its steady-state equilibrium, $k^*(\underline{w})$, and $\lim_{t \to +\infty} k_t = 0^+$.

In the following figure we depict the capital accumulation loci - for a parametric set chosen only for illustrative purposes - in both the competitive-wage and regulated-wage economies, showing that the regulation of wages brings about to steady-states capital per-capita higher than in the standard market-wage economy.

[FIGURE 2.1 ABOUT HERE]

We want to study whether, and under which conditions, a minimum-wage regime could perform better than a market-wage economy from the point of view of the long-run output. In other words, the question is: is it always better to have a competitive labour market, or are there some cases in which the regulation of wages may bring, despite a positive unemployment rate, to higher levels of long-run per-capita income? In what follows we prove that, under some plausible conditions, a minimum wage economy not only could correspond to some target of equity but, through the effect of increasing the total income of the young, it may also raise the level of efficiency. This fact can be summarised by saying that there exist reasons for which a government could positively evaluate the unemployment rate, and consequently it could use it as a policy instrument not only for equity reasons but mostly for the efficiency of an economy.[11]

[11] Notice that in our model we are not taking account for the important leisure values associated with unemployment: for instance leisure time, self-enrichment activities, education, home production and so on. Indeed the unemployment is to be interpreted as hours of life which are work-free. In any case taking into account for the leisure values associated with unemployment (for instance introducing a home production positively dependant on the time of unemployment), would confirm, a fortiori, our results.

We now analyse how changes in the real wage do influence the long-run per-capita income in the regulated-wage economy, and we also compare the steady-state level of output in the minimum-wage economy, $y^*(\underline{w})$, with the one obtained in the case of competitive labour market, that is $y^*(w_{pc})$.[12] As regards the relation between the minimum wage and the long-run per-capita income, differentiation of eq. (16) with respect to $\underline{w}$ yields:

$$\operatorname{sgn}\left\{\frac{\partial y^*(\underline{w})}{\partial \underline{w}}\right\} = \operatorname{sgn}\left\{\left(\frac{2\alpha-1}{\alpha}\right)\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}\right\}. \tag{28}$$

By looking at the right-hand side (RHS) of (18), it may be seen that the sign of $\partial y^*(\underline{w})/\partial \underline{w}$ strongly depends on the technology parameter. In fact, we must have that: 1) if $\alpha \in (0, 0.5]$, i.e. the technology is not sufficiently capital intensive, then $\partial y^*(\underline{w})/\partial \underline{w} < 0$ for any $\underline{w} > w_{pc}$ and $y^*(\underline{w})$ is a monotonically decreasing function of the wage rate. Moreover, since $y^*(\underline{w}) = y^*(w_{pc})$ if and only if $\underline{w} = w_{pc}$, then the introduction of a binding minimum wage implies $y^*(\underline{w}) < y^*(w_{pc})$ for any $\underline{w} > w_{pc}$;[13] 2) on the contrary, if the technology is sufficiently capital intensive, that is $\alpha \in (0.5, 1)$, we can state the following proposition:

Proposition 1 $\alpha \in (0.5, 1)$ is a necessary condition and $h(\alpha, \gamma) \le 1$, that is $\alpha \ge 1/(1+\gamma)$, is a sufficient condition to have $y^*(\underline{w}) \ge y^*(w_{pc})$ for any $\underline{w} \ge w_{pc}$ and $\gamma \in (0,1)$.

Proof If $\alpha \in (0.5, 1)$, then the analysis of the RHS of (17) implies that $\partial y^*(\underline{w})/\partial \underline{w} < 0$ if $w_T < \underline{w} < w_{y^*(\underline{w})}$, $\partial y^*(\underline{w})/\partial \underline{w} = 0$ if and only if $\underline{w} = w_{y^*(\underline{w})}$ and $\partial y^*(\underline{w})/\partial \underline{w} > 0$ for any $\underline{w} > w_{y^*(\underline{w})}$, where $w_{y^*(\underline{w})} := h(\alpha, \gamma) \cdot w_{pc}$ and $h(\alpha, \gamma) := \left(\frac{\alpha(1-\gamma)}{2\alpha-1}\right)^{\frac{\alpha}{1-\alpha}}$. Therefore, if $\alpha \in (0.5, 1)$ the long-run per-capita income is a monotonically decreasing function of the wage rate if $w_T < \underline{w} < w_{y^*(\underline{w})}$, while it becomes a monotonically increasing function for any $\underline{w} > w_{y^*(\underline{w})}$. As a consequence $\underline{w} = w_{y^*(\underline{w})}$ is an inner relative lower bound of $y^*(\underline{w})$. Since $w_{y^*(\underline{w})}$ relates α, γ and the market-clearing wage, depending on the mutual relation between the technology parameter and the replacement ratio it can be easily seen that if $h(\alpha, \gamma) \le 1$, i.e. $\alpha \ge 1/(1+\gamma)$, then $w_{y^*(\underline{w})} \le w_{pc}$. In this case, $\partial y^*(\underline{w})/\partial \underline{w} \ge 0$ for any $\underline{w} > w_{pc}$ implying that $y^*(\underline{w})$ is a monotonically increasing function for any minimum wage level. Since $y^*(\underline{w}) = y^*(w_{pc})$ if and only if $\underline{w} = w_{pc}$, then for any $\alpha \in (0.5, 1)$ and $\gamma \in (0,1)$ it is sufficient to set $\alpha \ge 1/(1+\gamma)$ to obtain $y^*(\underline{w}) > y^*(w_{pc})$ for any $\underline{w} > w_{pc}$. Q.E.D.

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[12] The analysis of $u^*(\underline{w})$ is here omitted for the sake of brevity. Anyway, it can be proved that: 1) $0 \le u^*(\underline{w}) < 1$ for any $\underline{w} \in [w_{pc}, +\infty)$, that is $u^*(\underline{w}) = 0$ if and only if $\underline{w} = w_{pc}$, and $\partial u^*(\underline{w})/\partial \underline{w} > 0$ for any $\underline{w} \ge w_{pc}$, meaning that the long-run unemployment rate is a monotonically increasing function of the minimum wage, and $\lim_{\underline{w} \to +\infty} u^*(\underline{w}) = 1$.

[13] Note that $\lim_{\underline{w} \to +\infty} y^*(\underline{w}) = 0$ if $\alpha \in (0, 0.5)$ and $\lim_{\underline{w} \to +\infty} y^*(\underline{w}) = l$ if $\alpha = 0.5$, where $0 < l < y^*(w_{pc})$.

When $\underline{w} > w_{k^*(\underline{w})}$, the long-run capital stock increases with $\underline{w}$ for any $\alpha \in (0,1)$. This means that an increasing minimum wage raises savings and, therefore, implies a higher pace of accumulation of capital. But when $\alpha \in (0, 0.5]$, the production technology is relatively labour intensive and thus the accumulation of capital is relatively ineffective on the output side. In this case, the usual belief that a competitive-wage economy is preferable for economic growth with respect to a regulated-wage regime holds. However, for a technology sufficiently capital intensive, i.e. $\alpha \in (0.5, 1)$, the conclusions are extremely different. In particular, provided that $\alpha \geq 1/(1+\gamma)$, then $k^*(\underline{w})$ and $y^*(\underline{w})$ are increasing in the minimum wage for any $\underline{w} > w_{pc}$. Therefore, a technological capital intensity as well as an unemployment benefit sufficiently high are a sufficient condition for the introduction of a regulated wage to bring about values of savings, capital and income always higher than the ones obtained in the market-wage economy. Indeed, given the relatively capital intensive technology, the increasing accumulation induced by the increasing minimum wage leads to a rising output so to create a virtuous growth mechanism. Moreover, the higher the minimum wage the higher the long run capital stock and income.

To sum up, under some plausible conditions: 1) an increase in the regulated wage is always beneficial for the long-run per-capita level of activity of the economy; and 2) interestingly, this beneficial effect may lead to an economic growth (in the Solow sense) higher than in the market-wage economy.

These results appear in contrast with the common belief prevailing in the literature, according to which any departure of the wage over the competitive level (with the associated positive rate of unemployment) reduces economic growth. For instance, Daveri and Tabellini (2000) found that "... The fall in employment induces firms to reduce investment... the smaller capital stock increases the rate of return on capital until the growth effect vanishes and the economy is back to a lower steady-state level of output", p. 100.

The sole unpleasant effect of the introduction of a minimum wage is due to the need of financing the total unemployment benefit, $b(\underline{w})$. In the next section we will show that the unemployment benefit may be easily financed at balanced budget with a capital income tax. Moreover, we will prove that not only the long run growth but also the long run lifetime welfare may be enhanced by the introduction of a regulated wage. Furthermore, under some particular conditions, we will show the existence and uniqueness of a welfare-maximising minimum wage.

4 Welfare Analysis at Balanced Budget

In this context, we assume the government adjusts the consumption tax rate in each period such as to balance out benefit expenditures and tax receipts, showing that a welfare-maximising minimum wage may exist. [14]

The optimal steady-state young and old age consumption functions are simply given by:

$$c^y(\underline{w}) = \frac{1-\phi}{1+\tau^c} W(\underline{w}), \tag{29}$$

$$c^o(\underline{w}) = \phi(1 + r(\underline{w}))W(\underline{w}), \tag{30}$$

[14] The proposal of substituting income tax with consumption tax with the aim of exempting savings from the burden of double taxation and thus of stimulating, via enhanced capital accumulation, economic growth is dating back to, among others, Fisher (1937) and Kaldor (1955) and more recently taken up (see, for example, Bradford (1986) and McClure and Zodrow (1996)). In a companion paper, we studied the present model under different taxation systems financing the unemployment benefit system at balanced budget, i.e. i) a non-distorting consumption tax levied upon both periods consumption; ii) a non-distorting capital income tax; iii) a distorting lump-sum tax on old people, showing that, interestingly, a welfare-maximising minimum wage does also exist in such cases. The investigation of the optimal tax design (for instance an income-based versus an expenditure-based taxation system) for financing unemployment benefit expenditures is beyond the scope of this paper and is left for future research.

where $W(\underline{w}) := \underline{w}\left[1-u^*(\underline{w})\cdot(1-\gamma)\right] = \dfrac{\gamma(1+n)\underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)-\phi(1-\gamma)\psi^{\frac{1}{\alpha}}}$ is the long-run total income of the young. From eq. (9), the steady-state government balanced budget condition is $\tau^c = \gamma \underline{w} u^*(\underline{w})/c^y(\underline{w})$. Using (24) and (29), and solving for the tax rate yields:

$$\tau^c(\underline{w}) = \frac{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)-\phi\psi^{\frac{1}{\alpha}}}{\phi\left[\psi^{\frac{1}{\alpha}}-\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)\right]}. \tag{31}$$

Interestingly, eq. (31) shows that $\tau^c(\underline{w})$ does not depend on the replacement ratio, that is, increasing, ceteris paribus, the unemployment insurance benefit does not weigh upon the government budget. Thus once $\underline{w}$ has been fixed, the steady-state balanced budget consumption tax is fixed as well.[15]

As regards the representative individual's long-run lifetime welfare, it is given by the following indirect utility function:

$$V(\underline{w}) = (1-\phi)\ln\left(c^y(\underline{w})\right) + \phi\ln\left(c^o(\underline{w})\right), \tag{32}$$

where $V(\underline{w}) = \ln(U(\underline{w}))$. The government aims to maximise (32) with respect to $\underline{w}$ subject to the individual's optimal choices over consumption when young and old at balanced budget, that is eqs. (29) and (30).

The maximisation of (22) yields to the following first order conditions:

$$\frac{\partial V(\underline{w})}{\partial \underline{w}} = 0 \Leftrightarrow \frac{1-\phi}{c^y(\underline{w})}\frac{\partial c^y(\underline{w})}{\partial \underline{w}} + \frac{\phi}{c^o(\underline{w})}\frac{\partial c^o(\underline{w})}{\partial \underline{w}} = 0. \tag{33}$$

In what follows, we will show that, under some plausible conditions on the key parameters of the model (the technology parameter, the propensity to save and the replacement ratio), there exists a public policy prescribing to fix the minimum wage between two threshold values bringing about higher steady-state lifetime welfare levels than the market-wage economy does. Moreover, a welfare-maximising minimum wage does exist as well. The analysis of eqs. (32) and (33) leads to the following propositions:[16]

Proposition 2 Let α, ϕ and γ be such that $\Delta_V > 0$, $e > 0$, $0 < \xi_{1,V}(\alpha,\gamma,\phi) < 1$ and $\xi_{2,V}(\alpha,\gamma,\phi) > 1$, then 1) $w_T < w_{1,V} < w_{pc} < w_{2,V} < w^{\circ\circ} < w_0$; and 2) there always exists a level of minimum wage, $w_{2,V} > w_{pc}$, such that the difference $V(w_{2,V}) - V(w_{pc}) > 0$ is maximised. Moreover, $\underline{w} = w_{2,V}$ is a global maximum of the function $V(\underline{w})$ for any $\underline{w} \in (w_{pc}, w_0)$.

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[15] $\tau^c(\underline{w}) = 0$ if and only if $\underline{w} = w_{pc}$. Moreover, the numerator of (18) is positive for any $\underline{w} > w_{pc}$, while the denominator is greater than zero if and only $\underline{w} < w_0$, where $w_0 := (1/\phi)^{\frac{\alpha}{1-\alpha}} w_{pc} > w_{pc}$ is the wage-threshold value below which $c^y(\underline{w}) > 0$. Thus, $\tau^c(\underline{w}) > 0$ for any $w_{pc} < \underline{w} < w_0$ and $\tau_{\underline{w}}'^c(\underline{w}) = \underline{w}^{\frac{1-\alpha}{\alpha}-1}(1+n)(1-\phi)\dfrac{1-\alpha}{\alpha}\psi^{\frac{1}{\alpha}} \Big/ \left[\psi^{\frac{1}{\alpha}} - \underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)\right]^2 > 0$.

[16] In Appendix A we present the analysis of eqs. (32) and (33) together with details of Propositions 2 and 3 (see Case 3, sub-case iii)), while the proofs of such propositions are given in Appendix B.

Proposition 3 Under the hypotheses and the conditions stated in points 1) and 2) of Proposition 2, if the minimum wage is set between $w_{pc} < \underline{w} < w^{\circ\circ}$, then $V(\underline{w}) > V(w_{pc})$, i.e., in the long-run, the minimum wage regime is welfare-preferred to the market-wage economy.

The surprising results of Propositions 1, 2 and 3 contrast with the common wisdom of the regulated-wage literature, stating that in the regulated-wage economy there exists a public policy bringing about an increase in the efficiency of the economy despite a positive unemployment rate, and obtaining higher steady-state lifetime welfare levels than in the market-wage case. Moreover, our model prescribes to fix exactly the minimum wage at $\underline{w} = w_{2,V}$ to obtain a welfare maximum.

In the following figure we compare the lifetime welfare in the regulated-wage and in the market-wage economies and we show that the welfare-maximising level of the regulated wage, for the following - purely illustrative - parameter set: $A = 10$, $\alpha = 0.55$, $\phi = 0.15$, $\gamma = 0.99$ and $n = 0$.

The picture shows that the regulated-wage economy is welfare-preferred to the market-wage economy for values of the minimum wage set between w_{pc} and $w^{\circ\circ}$. In particular, the lifetime welfare is maximised at $w_{2,V} = 3.99$ corresponding to which the unemployment rate is 25.8% and the budget-balancing consumption tax is 43.1%.

[FIGURE 3.1 ABOUT HERE]

5 Conclusions

In this paper we have focused on the steady state effects of the introduction of the minimum wage on economic growth and welfare of the representative individual in a textbook neoclassical OLG growth model, with an unemployment insurance scheme financed at balanced budget by a consumption tax on the young individuals.

Our results differ markedly from the conventional wisdom which argues that the minimum wage creates a reduction in output. The reason for this wisdom is that it implicitly assumes a static context where production factors are fixed, while in a dynamic overlapping generations here capital accumulation is affected by wages such a wisdom could be incorrect.

The novelty of the results is that we provide three analytical conditions for which the minimum wage should be introduced for enhancing in the long run economic growth as well as welfare: i) a first condition establishes the threshold value of the regulated wage beyond which larger wages spur capital accumulation; ii) a second condition, instead, involving technological and policy parameters, fixes the threshold value of the minimum wage beyond which the production losses due to the unemployment created by the minimum wage are transformed in output gains, and in particular a sufficiently high capital intensity and a sufficiently high replacement ratio are required for the existence of such a threshold value; iii) a third condition shows the existence of a value of the minimum wage which is optimal from a welfarist point of view, and fixes the corresponding consumption tax rate to balance the government budget.

The policy implications of our results are direct and, in a some sense, unusual: 1) in many cases, which are analytically picked out, a regulated wage rate should be always introduced, since a regulated economy performs better than a competitive economy; 2) moreover, a simple young consumption tax (used for financing the unemployment benefit system) may be an efficient fiscal device for improving long run individual welfare.

The interest of these results lies in: 1) the relevance of their messages showing a new perspective for the regulation of wages and offering policy implications, and in 2) the simplicity with which are obtained, that is within a standard dynamic general equilibrium overlapping generations model where agents live two periods and the only departure from the textbook OLG model is the assumption that a minimum wage may be imposed by a government.

Many further extensions are possible in the pursuing of the analysis of a regulated wage economy: 1) utility and production functions can be generalised; 2) a context of endogenous growth can be

introduced; 3) the labour supply may be endogenized; and 4) a whole range of tax policies can be used to finance unemployment benefit either in balanced budget or in deficit.

Appendix A

In what follows we analyse the young and old people consumption functions, and the representative individual's indirect utility in the long-run.

Substituting eq. (31) into (29) for $\tau^c(\underline{w})$ and using the equation for $W(\underline{w})$, the long-run young age consumption function can be written as:

$$c^y(\underline{w}) = \frac{\phi\gamma\underline{w}\left[\psi^{\frac{1}{\alpha}} - \underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)\right]}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}. \tag{A1}$$

Eq. (A1) implies $c^y(\underline{w}) \geq 0$ if and only if $w_{pc} < \underline{w} \leq w_0$.

Substituting eq. (18) into (30) and using $W(\underline{w})$, the old agents consumption function becomes:

$$c^o(\underline{w}) = \frac{\phi\alpha A\gamma\psi^{\frac{1-\alpha}{\alpha}}(1+n)\underline{w}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}. \tag{A2}$$

By looking at eq. (A2), it can be easily seen that $c^o(\underline{w}) > 0$ for any $\underline{w} > w_{pc}$.

Using eqs. (A1) and (A2), the steady-state representative individual's indirect utility function, eq. (31) in the main text, may be written as follows:

$$V(\underline{w}) = (1-\phi)\ln\left\{\frac{\phi\gamma\underline{w}\left[\psi^{\frac{1-\alpha}{\alpha}} - \underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)\right]}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}\right\} + \phi\ln\left\{\frac{\phi\alpha A\gamma\psi^{\frac{1-\alpha}{\alpha}}(1+n)\underline{w}}{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}}\right\}. \tag{A3}$$

Assuming $\underline{w} \in (w_{pc}, w_0)$, we have $\lim_{\underline{w}\to w_0^-} V(\underline{w}) = -\infty$. If the minimum wage is not binding, i.e. $\underline{w} = w_{pc}$, then the standard results of the market-wage economy hold, that is: $c^y(\underline{w}) = c^y(w_{pc}) = (1-\phi)w_{pc}$, $c^o(\underline{w}) = c^o(w_{pc}) = \phi(1+r_{pc})w_{pc}$, where $r_{pc} = \alpha A\left(k^*(w_{pc})\right)^{\alpha-1} - 1 = \frac{\alpha(1+n)}{(1-\alpha)\phi} - 1$ is the real interest rate in the market-wage economy and $V(\underline{w}) = V(w_{pc})$.

We now proceed with the study of the behaviour of eq. (A3), and we analyse the conditions for the existence of internal upper and lower bounds of $V(\underline{w})$.

The maximisation of (A3) with respect to the wage rate yields:

$$\frac{\partial V(\underline{w})}{\partial \underline{w}} = \frac{-\underline{w}^{2\left(\frac{1-\alpha}{\alpha}\right)}(1+n)^2\alpha\phi(1-\phi) + \underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)(1-\phi)\phi\psi^{\frac{1}{\alpha}}\left(\phi(1-\gamma)+2\alpha-1\right) - \alpha(1-\gamma)(1-\phi)\phi^2\psi^{\frac{2}{\alpha}}}{\alpha\phi\underline{w}\left[\psi^{\frac{1}{\alpha}} - \underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)\right]\left[\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}\right]}$$

$$+ \frac{\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)(2\alpha-1)\phi - \alpha(1-\gamma)\phi^2\psi^{\frac{1}{\alpha}}}{\alpha\underline{w}\left[\underline{w}^{\frac{1-\alpha}{\alpha}}(1+n) - \phi(1-\gamma)\psi^{\frac{1}{\alpha}}\right]} = 0. \tag{A4}$$

Defining an auxiliary unknown variable, $\theta := \underline{w}^{\frac{1-\alpha}{\alpha}}(1+n)$ (which is a positive monotonic transformation of $\underline{w}$), and a new parameter, $\eta := \phi\psi^{\frac{1}{\alpha}}$, after some algebra eq. (A4) can be rewritten as:

$$\frac{\partial V(\underline{w})}{\partial \underline{w}} := \Lambda(\theta, \underline{w}) = \frac{-\alpha\phi(1-\phi)\theta^2 + (1-\phi)(\phi(1-\gamma)+2\alpha-1)\eta\theta - \alpha(1-\gamma)(1-\phi)\eta^2}{\alpha\,\underline{w}[\eta-\phi\theta][\theta-(1-\gamma)\eta]} + \frac{\phi(2\alpha-1)\theta - \alpha\phi(1-\gamma)\eta}{\alpha\,\underline{w}[\theta-(1-\gamma)\eta]} = 0. \tag{A5}$$

Straightforward but cumbersome algebra leads to:

$$\Lambda(\theta, \underline{w}) = \frac{-d\theta^2 + e\theta - f}{\alpha\,\underline{w}(\theta_0 - \theta)(\theta - \theta_T)} = 0, \tag{A6}$$

where, $d := \phi(\alpha - \phi(1-\alpha))$, $e := (\phi(1-\gamma)(1-\phi(1-\alpha)) + 2\alpha - 1)\eta$, $f := \alpha(1-\gamma)\eta^2 > 0$, $\theta_T := \eta(1-\gamma)$ and $\theta_0 := \eta/\phi$. The coefficient d is positive if and only if $\alpha/(1-\alpha) > \phi$, while $e > 0$ if and only if $\alpha > \frac{1-\phi(1-\phi)(1-\gamma)}{2+\phi^2(1-\gamma)}$. Eq. (A6) is defined for any $\theta \neq \theta_T$ and $\theta \neq \theta_0$, that is to say, for any $\underline{w} \neq w_T$ and $\underline{w} \neq w_0$, and its denominator is positive if and only if $\theta \in (\theta_T, \theta_0)$. Since we have assumed $\underline{w} \in (w_{pc}, w_0)$, then the positivity of the denominator of (A6) is always guaranteed.[17] $\Lambda(\theta, \underline{w}) = 0$ if and only if:

$$-d\theta^2 + e\theta - f. \tag{A7}$$

After some algebra, the roots of eq. (A7) may be written as follows:

$$\theta_{1,V} = \frac{\eta}{2\phi(\alpha - \phi(1-\alpha))}\left\{\phi(1-\gamma)(1-\phi(1-\alpha)) + 2\alpha - 1 - \sqrt{\Delta_V}\right\}, \tag{A8}$$

and

$$\theta_{2,V} = \frac{\eta}{2\phi(\alpha - \phi(1-\alpha))}\left\{\phi(1-\gamma)(1-\phi(1-\alpha)) + 2\alpha - 1 + \sqrt{\Delta_V}\right\}, \tag{A9}$$

where $\Delta_V := [\phi(1-\gamma)(1-\phi(1-\alpha)) + 2\alpha - 1]^2 - 4\alpha\phi(1-\gamma)(\alpha - \phi(1-\alpha))$ is the discriminant of (A7). Such a discriminant only depends on three parameters: the technology parameter, the replacement ratio and the propensity to save, α, γ and ϕ respectively. Therefore, depending on the mutual relation between such parameters, we may have the following three cases:

Case 1 $\Delta_V < 0$. The solutions of eq. (A7) are two complex conjugate roots. Thus, eqs. (A8) and (A9) modify to:

$$\theta_{1,V}^{\ complex} = \frac{\eta}{2\phi(\alpha - \phi(1-\alpha))}\left\{\phi(1-\gamma)(1-\phi(1-\alpha)) + 2\alpha - 1 - i\sqrt{\Delta_V}\right\}, \tag{A10}$$

and

$$\theta_{2,V}^{\ complex} = \frac{\eta}{2\phi(\alpha - \phi(1-\alpha))}\left\{\phi(1-\gamma)(1-\phi(1-\alpha)) + 2\alpha - 1 + i\sqrt{\Delta_V}\right\}. \tag{A11}$$

Substituting for θ and η into (A10) and (A11), and solving for the wage rate yields:

$$w_{1,V}^{\ complex} = \left[\frac{1}{2\phi(\alpha - \phi(1-\alpha))}\right]^{\frac{\alpha}{1-\alpha}}\left\{\phi(1-\gamma)(1-\phi(1-\alpha)) + 2\alpha - 1 - i\sqrt{\Delta_V}\right\}^{\frac{\alpha}{1-\alpha}} \cdot w_{pc}, \tag{A12}$$

and

[17] In the sequel we will always assume $d > 0$.

$$w_{2,V}^{\ complex} = \left[\frac{1}{2\phi(\alpha-\phi(1-\alpha))}\right]^{\frac{\alpha}{1-\alpha}} \left\{\phi(1-\gamma)(1-\phi(1-\alpha))+2\alpha-1+i\sqrt{\Delta_V}\right\}^{\frac{\alpha}{1-\alpha}} \cdot w_{pc}, \qquad \text{(A13)}$$

Eqs. (A12) and (A13) represent the two values of the wage rate for which $V'_{\underline{w}}\left(w_{1,V}^{\ complex}\right)=0$ and $V'_{\underline{w}}\left(w_{2,V}^{\ complex}\right)=0$ in the complex plane. Anyway, since $\Delta_V<0$ there do not exist minimum wages such that $V'_{\underline{w}}(\underline{w})=0$ in the real plane $\left(\underline{w},V(\underline{w})\right)$. Given that $-d<0$, we can conclude that for any $\underline{w}\in\left(w_{pc},w_0\right)$ we must have $V'_{\underline{w}}(\underline{w})<0$. As a consequence, the representative individual's lifetime welfare is a monotonically decreasing function of the minimum wage. Moreover, since $V(\underline{w})=V\left(w_{pc}\right)$ if and only if $\underline{w}=w_{pc}$, then $V(\underline{w})<V\left(w_{pc}\right)$ for any $\underline{w}>w_{pc}$.

[FIGURE A.1 ABOUT HERE]

Figure A.1 represents the general behaviour of the representative individual's indirect utility function in both the regulated-wage regime and in the market-wage economy, showing that, in the case in which $\Delta_V<0$, the introduction of a binding minimum wage always decreases the long-run lifetime welfare, and the competitive-wage economy is always welfare-preferred to the minimum-wage regime.

Case 2 $\Delta_V=0$. In this case, there only exists one real solution of eq. (A7) with algebraic multiplicity equal 2. Thus, eqs. (A8) and (A9) collapse to the following:

$$\theta_{1,V}=\theta_{2,V}=\theta_V=\frac{\eta\left[\phi(1-\gamma)(1-\phi(1-\alpha))+2\alpha-1\right]}{2\phi(\alpha-\phi(1-\alpha))}. \qquad \text{(A14)}$$

Substituting out for θ and η into (A14), and solving for $\underline{w}$ we find the unique solution of $V_{\underline{w}}'(\underline{w})$ as a function of the basic parameters of the model and the market-clearing wage, that is:[18]

$$w_{1,V}=w_{2,V}=w_V=\left[\frac{\phi(1-\gamma)(1-\phi(1-\alpha))+2\alpha-1}{2\phi(\alpha-\phi(1-\alpha))}\right]^{\frac{\alpha}{1-\alpha}}\cdot w_{pc}. \qquad \text{(A15)}$$

By using (A14), $\Lambda(\theta,\underline{w})$ may be written as:

$$\Lambda(\theta,\underline{w})=\frac{-(\theta-\theta_V)^2}{\alpha\,\underline{w}(\theta_0-\theta)(\theta-\theta_T)}=0. \qquad \text{(A16)}$$

The analysis of (A16) implies that $\Lambda(\theta,\underline{w})=0$ if and only if $\theta=\theta_V$ and $\Lambda(\theta,\underline{w})<0$ for any $\theta\in(\theta_{pc},\theta_0)$, where $\theta_{pc}=\eta>\theta_T$. Thus, $V_{\underline{w}}'(\underline{w})=0$ if and only if $\underline{w}=w_V$ and $V_{\underline{w}}'(\underline{w})<0$ for any $\underline{w}\in\left(w_{pc},w_0\right)$.[19] Thus when $\Delta_V=0$, the representative individual's indirect utility function is always a monotonically decreasing function of the minimum wage for any $\underline{w}\in\left(w_{pc},w_0\right)$. Since $V(\underline{w})=V\left(w_{pc}\right)$ if and only if $\underline{w}=w_{pc}$, then $V(\underline{w})<V\left(w_{pc}\right)$ for any $\underline{w}>w_{pc}$, meaning that introducing a binding minimum wage (plus a wage-indexed unemployment benefit for the hours left unemployed) is always welfare-worsening, and the market-wage economy is always welfare-preferred. The following figure represents the general behaviour of the lifetime welfare function in both the

[18] Notice that in order to guarantee $w_V>0$, the condition $e>0$ must hold. Furthermore w_V may be higher or smaller than the market-clearing wage depending if the term in square brackets in (A15) is higher or smaller than one.

[19] The condition $V_{\underline{w}}'(w_V)=0$ is a necessary but not a sufficient condition for w_V to be an inner relative upper or lower bound of the function $V(\underline{w})$. Since $V_{\underline{w}}'(\underline{w})<0$ for any $\underline{w}\in\left(w_{pc},w_0\right)$, then $\underline{w}=w_V$ is neither an inner relative upper bound nor an inner relative lower bound. In particular, given that $V_{\underline{w}}'(w_V)=0$ and $V_{\underline{w}\underline{w}}''(w_V)=0$, $\underline{w}=w_V$ is a horizontal inflection point of the function $V(\underline{w})$. The complete proof is, of course, available on request.

regulated-wage and in market-wage economies in the case in which $\Delta_{V(\underline{w})}=0$, depicting that the long-run lifetime welfare in the competitive-wage economy is always higher than that of the regulated-wage regime.

[FIGURE A.2 ABOUT HERE]

Case 3 $\Delta_V>0$. The solutions of eq. (A7) are two real roots with algebraic multiplicity equal 1 (see eqs. (A8) and (A9)). In order to guarantee the positivity of such two roots, by applying the Descartes' rule of sign, we must impose $e>0$ ($e<0$ involves two negative roots which are obviously not economically relevant). Thus, substituting for θ and η into eqs. (A8) and (A9) and solving for the wage rate, we find the two (positive) values of $\underline{w}$, $w_{1,V}$ and $w_{2,V}$, which are the zeros of eq. (A4), expressed as a function of three parameters and the market-clearing wage as well, i.e.:

$$w_{1,V}=\xi_{1,V}(\alpha,\gamma,\phi)\cdot w_{pc}, \tag{A17}$$

and

$$w_{2,V}=\xi_{2,V}(\alpha,\gamma,\phi)\cdot w_{pc}, \tag{A18}$$

where we have defined

$$\xi_{1,V}(\alpha,\gamma,\phi)=\left[\frac{1}{2\phi(\alpha-\phi(1-\alpha))}\right]^{\frac{\alpha}{1-\alpha}}\left\{\phi(1-\gamma)(1-\phi(1-\alpha))+2\alpha-1-\sqrt{\Delta_V}\right\}^{\frac{\alpha}{1-\alpha}}, \tag{A19}$$

and

$$\xi_{2,V}(\alpha,\gamma,\phi)=\left[\frac{1}{2\phi(\alpha-\phi(1-\alpha))}\right]^{\frac{\alpha}{1-\alpha}}\left\{\phi(1-\gamma)(1-\phi(1-\alpha))+2\alpha-1+\sqrt{\Delta_V}\right\}^{\frac{\alpha}{1-\alpha}}. \tag{A20}$$

By looking at (A19) and (A20) it can be easily seen that $0<\xi_{1,V}(\alpha,\gamma,\phi)<\xi_{2,V}(\alpha,\gamma,\phi)$, implying $0<w_{1,V}<w_{2,V}$.

Using eqs. (A8) and (A9) permits to rewrite (A6) as:

$$\Lambda(\theta,\underline{w})=\frac{-(\theta-\theta_{1,V})(\theta-\theta_{2,V})}{\alpha\,\underline{w}(\theta_0-\theta)(\theta-\theta_T)}=0. \tag{A21}$$

The denominator of eq. (A21) is positive if (and only if) $\theta\in(\theta_T,\theta_0)$, while the numerator is greater than zero for any $\theta\in(\theta_{1,V},\theta_{2,V})$. If $\theta<\theta_0$ (that is, $\underline{w}<w_0$), the analysis of the sign of (A21) implies that: $\Lambda(\theta,\underline{w})<0$ for any $\theta_T<\theta<\theta_{1,V}$ and $\theta_{2,V}<\theta<\theta_0$; $\Lambda(\theta,\underline{w})>0$ if and only if $\theta_{1,V}<\theta<\theta_{2,V}$; and $\Lambda(\theta,\underline{w})=0$ if and only if $\theta=\theta_{1,V}$ and $\theta=\theta_{2,V}$. It follows that: $V_{\underline{w}}'(\underline{w})<0$ for any $w_T<\underline{w}<w_{1,V}$ and $w_{2,V}<\underline{w}<w_0$; $V_{\underline{w}}'(\underline{w})>0$ for any $w_{1,V}<\underline{w}<w_{2,V}$; and $V_{\underline{w}}'(\underline{w})=0$ if and only if $\underline{w}=w_{1,V}$ and $\underline{w}=w_{2,V}$. Given that $\lim_{\underline{w}\to w_T^+}V(\underline{w})=+\infty$ and $\lim_{\underline{w}\to w_0^-}V(\underline{w})=-\infty$, and since in the neighbourhood of $w_{1,V}$, $V(\underline{w})$ is monotonically decreasing if $\underline{w}<w_{1,V}$ and monotonically increasing if $\underline{w}>w_{1,V}$ with $V_{\underline{w}}'(w_{1,V})=0$, then $\underline{w}=w_{1,V}$ is an inner relative lower bound of $V(\underline{w})$. On the contrary, knowing that in the neighbourhood of $w_{2,V}$, $V(\underline{w})$ is monotonically increasing if $\underline{w}<w_{2,V}$ and monotonically decreasing if $\underline{w}>w_{2,V}$, with $V_{\underline{w}}'(w_{2,V})=0$, then $\underline{w}=w_{2,V}$ is an inner relative upper bound of $V(\underline{w})$.

[FIGURE A.3 ABOUT HERE]

Figure A.3. shows that there exists a range of minimum wages for which the regulated-wage economy is welfare-preferred to the market-wage economy.

Depending on the value of the functions $\xi_{1,V}(\alpha,\gamma,\phi)$ and $\xi_{2,V}(\alpha,\gamma,\phi)$, it can be easily seen that $w_{1,V}$ and $w_{2,V}$ (see eqs. (A17) and (A18)) could be higher or smaller than the market-clearing wage.

We now study the conditions under which $w_{1,V}$ and $w_{2,V}$, that is the local minimum and the local maximum of $V(\underline{w})$, are higher or lower than w_{pc}. To this purpose, due to the mutual relations between α, γ and ϕ, the analysis of (A17) and (A18) implies that we may have the following three cases: i) $\xi_{2,V}(\alpha,\gamma,\phi) > \xi_{1,V}(\alpha,\gamma,\phi) > 1$, that is $w_{2,V} > w_{1,V} > w_{pc}$. In this case, introducing a minimum wage may be either welfare-reducing or welfare-improving as compared with the competitive economy; ii) $0 < \xi_{1,V}(\alpha,\gamma,\phi) < \xi_{2,V}(\alpha,\gamma,\phi) < 1$, implying $w_{1,V} < w_{2,V} < w_{pc}$ and $V(\underline{w}) < V(w_{pc})$ for any $\underline{w} > w_{pc}$. Thus, in this case, the introduction of a minimum wage is always welfare-worsening; iii) $0 < \xi_{1,V}(\alpha,\gamma,\phi) < 1$ and $\xi_{2,V}(\alpha,\gamma,\phi) > 1$, that is $w_{1,V} < w_{pc} < w_{2,V}$. Cases i) and ii) may be easily investigated by observing Figure A.1 and A.2, while case iii) is illustrated in Figures 3.1 and A.3. As we will prove in Appendix B, in the latter case iii): 1) there always exists a minimum wage such that the representative individual's lifetime welfare is maximised, and 2) the regulated-wage regime is always welfare-preferred than the competitive-wage economy (up to the limit value of $\underline{w} = w^{\circ\circ}$).

Appendix B

Since in this paper we are mostly interested in showing that the regulated-wage economy is always welfare-preferred as compared with the competitive-wage economy, we do not formally show cases i) and ii) as stated in Appendix A and we will only focus on case iii) above mentioned.[20]

In what follows we prove Propositions 2 and 3 as stated in section 3 of the main text.

Let $\underline{w} \in (w_T, w_0)$. In Appendix A we have shown that under the hypotheses $\Delta_V > 0$, $e > 0$, $0 < \xi_{1,V}(\alpha,\gamma,\phi) < 1$ and $\xi_{2,V}(\alpha,\gamma,\phi) > 1$, then $w_T < w_{1,V} < w_{pc} < w_{2,V} < w_0$ holds true.

1) When $\Delta_V > 0$ and $e > 0$, then in the neighbourhood of w_{pc} we have that $V'_{\underline{w}}(\underline{w}) > 0$ for any $\underline{w} > w_{pc}$, i.e. $V(\underline{w})$ is a monotonically increasing function of the minimum wage (see Case 3, Appendix A). Moreover, if the market-clearing wage prevails, i.e. $\underline{w} = w_{pc}$, then $V(\underline{w}) = V(w_{pc})$. Therefore, the introduction of minimum wages implies that (locally) $V(\underline{w}) > V(w_{pc})$.

2) $V'_{\underline{w}}(\underline{w}) = 0$ if and only if $\underline{w} = w_{1,V}$ and $\underline{w} = w_{2,V}$. When α, γ and ϕ are also such that $0 < \xi_{1,V}(\alpha,\gamma,\phi) < 1$ and $\xi_{2,V}(\alpha,\gamma,\phi) > 1$, then $w_{1,V} < w_{pc}$ and $w_{2,V} > w_{pc}$. Thus, there exists one and only one zero of $V'_{\underline{w}}(\underline{w})$ for any $\underline{w} \in (w_{pc}, w_0)$ as given by $\underline{w} = w_{2,V}$ (which is a local maximum of $V(\underline{w})$ in the range $w_T < \underline{w} < w_0$, as we have proved in Appendix A (Case 3)).

3) For any $w_{pc} \le \underline{w} < w_{2,V}$, then $V'_{\underline{w}}(\underline{w}) > 0$ and $V(\underline{w})$ is a monotonically increasing function of the minimum wage, while for any $w_{2,V} \le \underline{w} < w_0$, then $V'_{\underline{w}}(\underline{w}) < 0$ and $V(\underline{w})$ is a monotonically decreasing function of the minimum wage. Since for any $\underline{w} \in (w_{pc}, w_0)$, $V'_{\underline{w}}(\underline{w}) = 0$ if and only if $\underline{w} = w_{2,V}$ and $\lim_{\underline{w} \to w_0^-} V(\underline{w}) = -\infty$, then $\underline{w} = w_{2,V}$ is a global maximum of the function $V(\underline{w})$, and the difference $V(w_{2,V}) - V(w_{pc})$ is maximised. This proves Proposition 2.

4) Since $V'_{\underline{w}}(\underline{w}) > 0$ if $w_{pc} \le \underline{w} < w_{2,V}$, $V'_{\underline{w}}(\underline{w}) < 0$ if $w_{2,V} \le \underline{w} < w_0$, and $V'_{\underline{w}}(\underline{w}) = 0$ once and only once for any $\underline{w} \in (w_{pc}, w_0)$ at $\underline{w} = w_{2,V}$, and finally $\lim_{\underline{w} \to w_0^-} V(\underline{w}) = -\infty$, then there necessarily exists one and only one value of the binding minimum wage ($w^{\circ\circ} > w_{pc}$) corresponding to which

[20] The complete taxonomy including cases i) and ii), which are in any case economically interesting, is investigated in a companion paper.

$V(w^{\circ\circ}) - V(w_{pc}) = 0$, that is, the lifetime welfare in the regulated-wage economy equals the one of the market-wage economy. Point 4) implies that there always exists a range of minimum wages, $w_{pc} < \underline{w} < w^{\circ\circ}$ such that $V(\underline{w}) > V(w_{pc})$.[21] This proves Proposition 3.

References

[1] Azariadis, C. (1993). Intertemporal Macroeconomics. Basil Blackwell, Oxford.

[2] Barro, Robert J. and X. Sala-i-Martin (2003). Economic Growth, 2nd ed., MIT Press, Cambridge, Massachusetts.

[3] Bradford, D. (1986). Untangling the Income Tax, Harvard University Press, Cambridge, Massachusetts.

[4] Cahuc, P. and P. Michel (1996). "Minimum Wage, Unemployment and Growth". European Economic Review, vol. 40 (7), pp. 1463-1482.

[5] Daveri, F. and G. Tabellini (2000). "Unemployment, Growth and Taxation in Industrial Countries". Economic Policy, vol. 15 (30), pp. 49-104.

[6] De La Croix, D. and P. Michel (2002). A Theory of Economic Growth. Dynamics and Policy in Overlapping Generations. Cambridge University Press.

[7] Diamond, P. (1965). "National Debt in a Neoclassical Growth Model". American Economic Review, vol. 55, pp. 1126-1150.

[8] Fisher, I. (1937). "Income in Theory and Income Tax in Practice", Econometrica, vol. 5, pp. 1-55.

[9] Kaldor, N. (1955). An Expenditure Tax, Allen and Unwin, London.

[10] Mankiw, N.G., D. Romer and D.N. Weil (1992). "A Contribution to the Empirics of Economic Growth". Quarterly Journal of Economics, vol. 107 (2), pp. 407-437.

[11] McClure, C. and G. R. Zodrow (1996). "A Hybrid Approach to the Direct Taxation of Consumption". Frontiers of Tax Reform, M. Boskin (ed.), Hoover Press, Stanford, California.

[12] Ravn, M.O. and J.R. Sorensen (1999). "Schooling, Training, Growth and Minimum Wages". Scandinavian Journal of Economics, vol. 10 (3), pp. 441-457.

[13] Solow R.M. (1956). "A Contribution to the Theory of Economic Growth". Quarterly Journal of Economics, vol. 70, pp. 65-94.

[14] Stigler, G.J. (1946). "The Economics of Minimum Wage Legislation". American Economic Review, vol. 36, pp. 358-365.

[21] We do not present here a closed-form solution for $w^{\circ\circ}$, since it is cumbersome and not economically relevant.

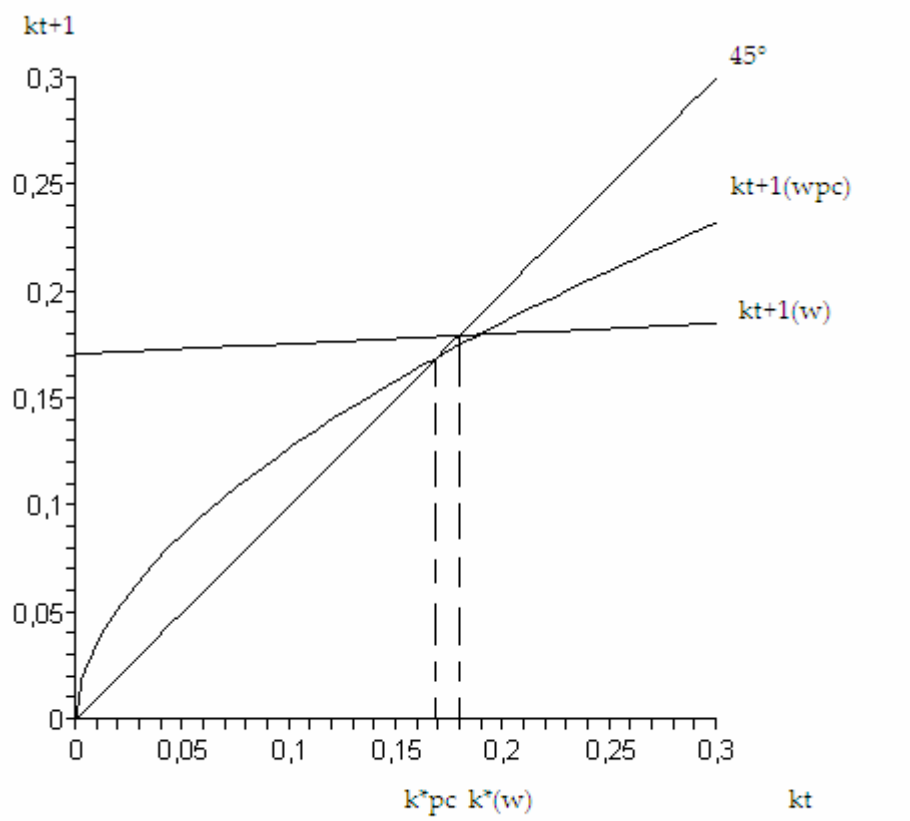

Figure 2.1. The capital accumulation equation in the case of both the competitive-wage and the regulated-wage economies, $k_{t+1}(w_{pc})$ and $k_{t+1}(\underline{w})$ respectively. Parameter set: $A = 10$, $\alpha = 0.55$, $\phi = 0.10$, $\gamma = 0.95$ and $n = 0$.

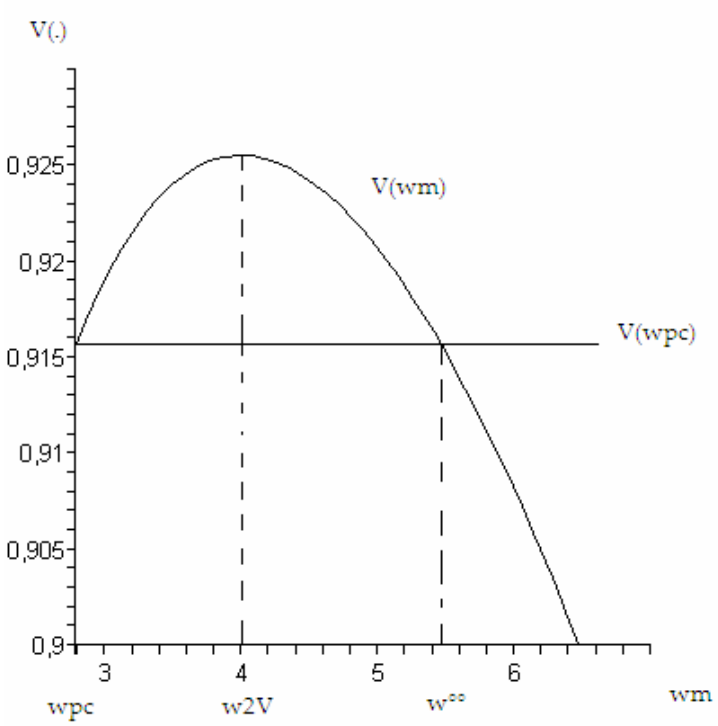

Figure 3.1. The lifetime welfare in the regulated-wage, V(wm), and in the market-wage economies, V(wpc), in the case in which $\Delta_V > 0$, $e > 0$, $0 < \xi_{1,V}(\alpha,\gamma,\phi) < 1$ and $\xi_{2,V}(\alpha,\gamma,\phi) > 1$, implying that $w_{1,V} < w_{pc} < w_{2,V}$. The starting point of the horizontal axis is $w_{pc} = 2.78$. Parameter set: $A = 10$, $\alpha = 0.55$, $\phi = 0.15$, $\gamma = 0.99$ and $n = 0$.

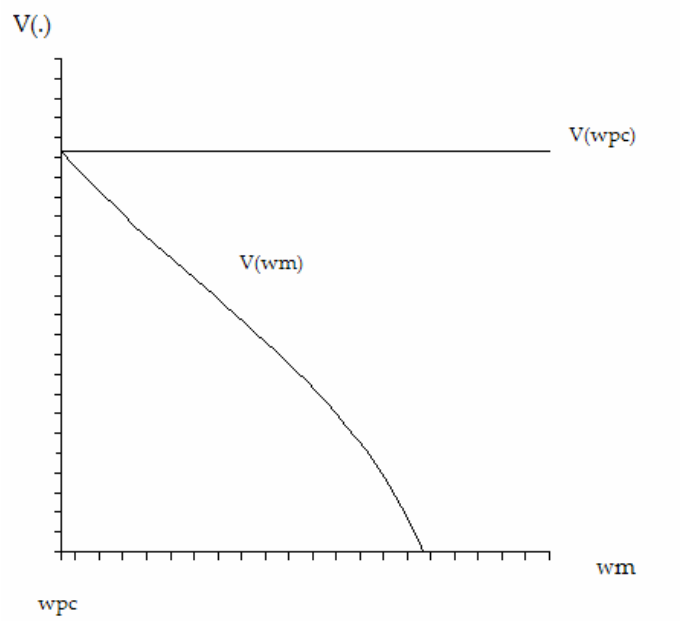

Figure A.1. $\Delta_V < 0$. The general behaviour of the lifetime welfare in the regulated-wage, V(wm), and market-wage economies, V(wpc), as a function of the wage rate.

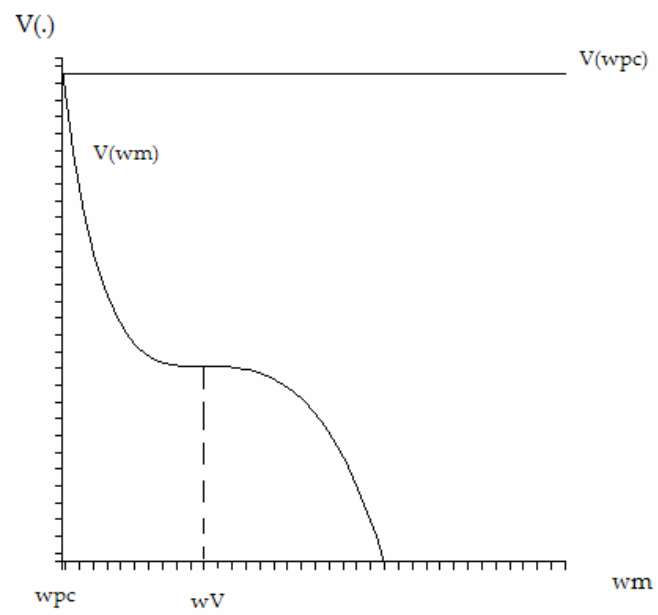

Figure A.2. $\Delta_V = 0$ and $e > 0$. The general behaviour of the lifetime welfare in both the regulated-wage, V(wm), and market-wage economies, V(wpc), as a function of the wage rate, where wV>wpc is the horizontal inflexion point.

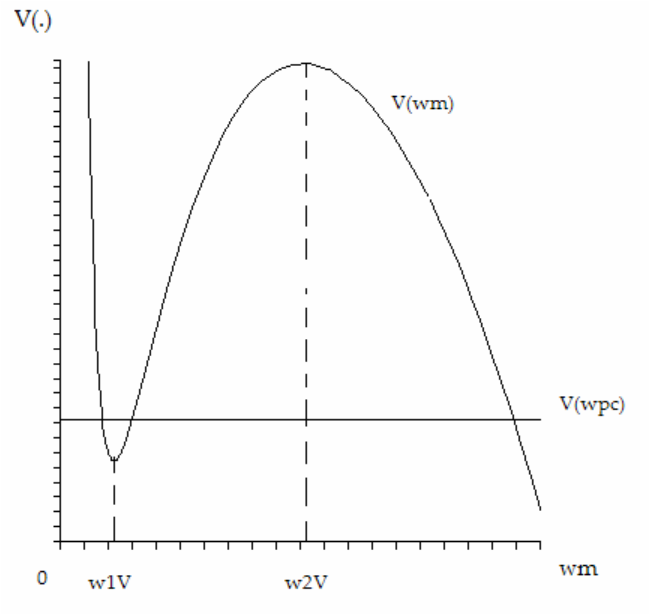

Figure A.3. $\Delta_V > 0$ and $e > 0$. A general representation of the lifetime welfare in the regulated-wage economy, V(wm), and in the market-wage economy, V(wpc), as a function of the wage rate, where w1V and w2V are the local minimum and local maximum of the indirect utility function.ù

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