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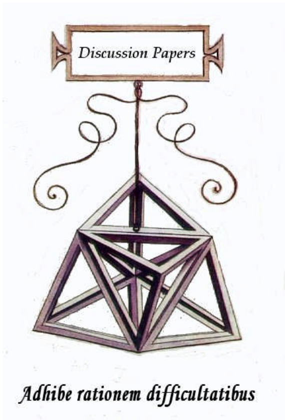

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Fertility and regulated wages in an OLG model of neoclassical growth: Pensions and old age support

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Fertility and regulated wages in an OLG model of neoclassical growth: Pensions and old age support

Luciano Fanti · Luca Gori

Abstract Since little attention has been paid to the effects of the regulation of wages on individuals' fertility choice, this paper investigates such effects within a standard OLG model of neoclassical growth. Some new results, so far escaped closer scrutiny by the increasing literature investigating economic growth and fertility, and which may have interesting policy implications, emerge: introducing minimum wages may, under suitable conditions, (1) have a favourable impact on the long-run outcomes of the economy; and (2) reduce the population growth rate. This occurs more likely when both sufficiently high capital's weight in technology and unemployment benefits do exist. Interestingly, these results are robust to different extensions introducing pensions and intra-family transfers. Therefore, to the extent that the absence of unions and of any regulation of wages are related with low wages, and the weight of capital in the distribution is increasing, the findings of this work also offer some policy implications which are, especially for developing countries in which the population growth may be too high, very interesting.

Keywords Minimum wage; Unemployment; Endogenous fertility

JEL classification E24; J13; O41

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1 Introduction

The relation between fertility choices and economic growth has been largely debated in literature, since the works of Becker (1960), Razin and Ben-Zion (1975) and Becker and Barro (1988).[1]

In particular, many recent works have tackled issues such as: (1) the demographic transition and the change in the fertility rates over historical times and between countries (Becker et al., 1990; Galor and Tsiddon, 1997; Galor and Weil, 1999; Doepke, 2004); (2) the female labour force participation (Galor and Weil, 1996; Martinez and Iza, 2004); (3) alternate periods of baby boom and baby bust (Greenwood et al., 2002); (4) the existence of a Malthusian trap (Galor and Weil, 2000; Fernandez-Villaverde, 2001; Strulik, 2004).

However, none of these models is concerned with regulated wage economies. Therefore, to our knowledge there is no literature that formally explores the joint roles played by policy interventions on the labour market, on the one side – e.g., the regulation of wages and the introduction of unemployment insurance schemes – and on endogenous fertility, on the other side. In this paper we will try to fill the gap by developing a standard OLG model of neoclassical growth (Samuelson, 1958; Diamond, 1965) embodying such features.

The plan of the paper is as follows. In Sections 2 and 3 we present the competitive-wage and the regulated-wage models, respectively, and the corresponding steady state results, as regards both capital and income per capita, are derived and discussed. Section 4 discusses the long run welfare properties of the models presented in Sections 2 and 3. Section 5 compares the fertility rates in both the competitive-wage and regulated-wage economies. Section 6 presents four different extensions of the model with regulated wages by introducing pensions and intra-family transfers, to test for the robustness of the results. Finally, section 7 bears the conclusions.

2 The competitive-wage economy

In this section we consider a simple dynamic general equilibrium OLG economy à la Samuelson – Diamond with young population N_t growing at the rate $n_t - 1$, i.e. $N_{t+1} = n_t N_t$.[2] The economy is closed to international trade, and goods, capital and labour markets are assumed to be competitive.[3]

Individuals. Each generation is represented by identical individuals belonging to an overlapping generations structure with finite lifetime. Life is separated among three periods: childhood, young-adulthood and old age. During childhood individuals do not make economic decisions and thus they consume a fixed fraction of the time endowment from their parents. During adulthood, individuals draw utility from working period consumption (c_t^y), retirement period consumption (c_{t+1}^o), and from having children (n_t).[4] We assume young-adult individuals are endowed with one unit of time which is supplied inelastically to the labour market. They receive wage income at the competitive rate w_t used for consumption and saving purposes. In their second period of life agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}. Furthermore, we suppose that raising children requires a variable cost indexed with labour earnings, that is, the cost of having a child is simply qw_t with $q \in (0,1)$.[5] Each generation takes the time-t real wage and the interest rate on savings as given.

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[1] See Nerlove and Raut (1997) for a survey of the literature focused on population and endogenous growth.

[2] If $n = 1$ ($n > 1$) population is stationary (increasing) at the steady-state.

[3] Two reference textbooks are Azariadis (1993) and De La Croix and Michel (2002).

[4] Since the scope of the paper is to isolate the relation between minimum wages and fertility, as a first attempt we ignore both the trade-off between child quantity and quality and the assumption that parents maximise utility of their offspring, which has been employed to explain economic growth and stagnation by among others (see Becker et al., 1990; Ehrlich and Lui, 1991). Last but not least, the choice to ignore any type of altruistic bond between parents and their children runs is in accord with to the empirical results by Cigno and Rosati (1996), who find that parents are self-interested and choose saving and fertility without regard to their offspring.

[5] A cost of child rearing linked with working income (in a model with unemployment working income means income derived from participating to the labour market while receiving either wage or benefit payments), is coherent with the microeconomic dependence of such a cost on the opportunity cost of the parents' home time which is increasing in their working income (see Cigno, 1991). Likewise, it is natural to conjecture that even the component of the cost of child-rearing due to expenditure for children consumption is positively linked with the working income. Moreover, this assumption is rather usual in literature (see, for instance, Strulik, 2004).

Therefore, the representative individual born at time t is faced with the problem of maximising the logarithmic utility function:[6]

$$\max_{\{c_t^y, c_{t+1}^o\}} U_t\left(c_t^y, c_{t+1}^o, n_t\right) = (1-\phi)\ln\left(c_t^y\right) + \phi\ln\left(c_{t+1}^o\right) + \rho\ln(n_t), \tag{P}$$

subject to the following intra-temporal budget constraints:

$$\begin{aligned} c_t^y + s_t &= w_t(1 - qn_t) \\ c_{t+1}^o &= (1 + r_{t+1})s_t \end{aligned},$$

where $0 < \phi < 1$ captures the importance in the welfare function of consuming while young relative to retirement period consumption, and $\rho > 0$ is an index reflecting the preference for children in the welfare evaluation.
The first order conditions (FOC's henceforth) for an interior solution are given by:

$$\frac{c_{t+1}^o}{c_t^y} = \frac{\phi}{1-\phi}(1 + r_{t+1}), \tag{1}$$

$$\frac{\rho}{n_t} = \frac{1-\phi}{c_t^y} \cdot qw_t. \tag{2}$$

Eq. (1) equates marginal utility of current and future consumption in terms of current consumption, whereas Eq. (2) equates the marginal utility of having a child with the involved marginal costs in terms of forgone utility of consumption. Therefore, exploiting Eqs. (1) and (2) along with the individual's budget constraints, we find that young-aged consumption, old-aged consumption and the number of children are respectively given by:

$$c_t^y = \frac{1-\phi}{1+\rho} w_t, \tag{3}$$

$$c_{t+1}^o = \frac{\phi}{1+\rho}(1 + r_{t+1})w_t, \tag{4}$$

$$n^*_c = \frac{\rho}{(1+\rho)q}, \text{[7]} \tag{5}$$

while the savings path is determined by:

$$s_t = \frac{\phi}{1+\rho} w_t, \tag{6}$$

with $\phi/(1+\rho) < 1$ being the propensity to save.

Eq. (5) implies that the demand for children is constant over time, that is it does not depend on wages, and $n^*_c > 1$ if and only if $\rho/(1+\rho) > q$, that is ρ must be sufficiently high and q sufficiently small for ensuring a positive long-run population growth rate.

Firms. All the firms in the economy are identical and act competitively. The (aggregate) constant returns to scale Cobb-Douglas technology of production is $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the labour input respectively, $A > 0$ represents a scale parameter and $\alpha \in (0,1)$ is the capital's weight in technology.[8] Assuming that the capital stock used in production totally depreciates at the end of each period ($\delta = 1$), profits maximisation leads to the following marginal conditions for capital and labour:[9]

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{7}$$

$$w_t = (1-\alpha)Ak_t^\alpha. \tag{8}$$

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[6] A caveat concerns the assumption of separability of preferences between goods consumption and "children" consumption. Although this separability is a standard practice in literature, one should expect that the results of the paper will be affected by relaxing this hypothesis: separability should be relaxed if, for instance, time devoted to children activities reduces time for consumption, because of a "shopping" time constraint.

[7] The subscript c means competitive-wage economy.

[8] Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as per capita capital and output per-capita respectively, the intensive form production function may be written as $y_t = Ak_t^\alpha$.

[9] The price of final output has been normalised to unity.

Equilibrium. Knowing that $N_{t+1} = n_t N_t$, the market-clearing condition in goods as well as in capital markets is expressed by the equality between savings and investment: $n_t k_{t+1} = s_t$, that is, the per-capita stock of capital in period $t+1$ equals the amount of resources saved in period t discounted by the number of individuals. Substituting out for n and s according to Eqs. (5) and (6) respectively, and using Eq. (8), the dynamic equilibrium sequence of capital is given by:

$$k_{t+1} = \frac{\phi q}{\rho}(1-\alpha)A k_t^{\alpha}. \tag{9}$$

Steady-state implies $k_{t+1} = k_t := k^*$. Hence, the long-run (per-capita) stock of capital is determined by:[10]

$$k^*_c = \left[\frac{\phi q}{\rho}(1-\alpha)A\right]^{\frac{1}{1-\alpha}}. \tag{10}$$

Using Eq. (10), the long-run income per-capita, and the long-run interest rate and market-clearing wage are respectively given by:

$$y^*_c = A\left[\frac{\phi q}{\rho}(1-\alpha)A\right]^{\frac{\alpha}{1-\alpha}}, \tag{11}$$

$$r_c^* = \alpha A\left(k^*_c\right)^{\alpha-1} - 1 = \frac{\rho\alpha}{(1-\alpha)\phi q} - 1, \tag{12}$$

$$w^*_c = (1-\alpha)A\left(k^*_c\right)^{\alpha} = \left(\frac{\phi q}{\rho}\right)^{\frac{\alpha}{1-\alpha}} \cdot \left((1-\alpha)A\right)^{\frac{1}{1-\alpha}}. \tag{13}$$

3 The regulated-wage economy

We now extend the model typified in the previous section by introducing a (constant) binding minimum wage per hour worked legally set by the policymaker beyond the prevailing market-clearing level.[11] Therefore, the labour market does not clear and unemployment occurs.

Government. A straightforward effect of the regulation of wages is to cause a positive level of unemployment. Furthermore, the presence of an unemployment insurance mechanism raises the need to finance the payment of benefits. There are many ways to raise revenues to finance the benefit scheme. In particular, we assume the government chooses to levy and adjust over time a lump-sum tax upon young-aged individuals such as to balance out unemployment benefit expenditure with tax receipt in every period, according to the following constraint:

$$\gamma \underline{w} u_t = \tau_t, \tag{14}$$

where $0 < \gamma < 1$ is the so-called replacement ratio, $\underline{w}$ the hourly minimum wage, u_t the unemployment rate and $\tau_t > 0$ the (lump-sum) tax receipt.[12]

It is important to emphasise the purely redistributive nature of this intra-generational policy (which does not cause any transfer from old-aged to young-aged people), that is income taxed away from the young rebated to the same individuals as a benefit for the hours of unemployment. Therefore, (1) problems of inter-generational equity do not arise; and (2) the inter-generational tax transfer channel which may be considered a typical feature of the OLG framework favouring capital accumulation (see, for instance, Bertola, 1996 and Uhlig and Yanagawa, 1996) is not working here. As for the latter point, we remark that the positive effect of both minimum wage and unemployment benefits on long-run income per-capita and the lifetime welfare shown in the next sections may be considered to be an intrinsic feature of the minimum wage model rather than to the OLG frame.

Individuals. Young-aged individuals earn a constant minimum wage ($\underline{w}$) for each employed hour, while they receive an hourly unemployment benefit indexed with the minimum wage ($b(\underline{w}) := \gamma\, \underline{w}$) when unemployed. The total income

[10] Using Eq. (9), it can easily be shown that steady-state is always stable.

[11] It is worth noting that in this model where, for simplicity, only one type of labour does exist, a binding minimum wage simply indicates a regulated wage fixed by law over the market-clearing wage. In the case of more than one type of labour with uniformly distributed wages, this assumption would simply mean a regulated wage fixed over the average market wage.

[12] We assume that individuals act atomistically and do not take into account the government budget constraint when deciding on the desired number of children and the savings path.

of each young-aged individual is therefore given by the sum of the working income plus the unemployment benefit, that is $W_t := \underline{w}(1-u_t)+\gamma \underline{w} u_t$.[13] We treat $\underline{w}$ and γ as policy parameters, whereas the quantity of employed labour force is endogenous. The aggregate unemployment rate, defined in terms of hours not worked, is $u_t = (N_t - L_t)/N_t$, where L_t is the labour demand.

The representative individual thus faces with the problem of maximising (P) subject to

$$c_t^{\,y} + s_t = W_t \cdot (1-qn_t) - \tau_t \qquad c_{t+1}^{\,o} = (1+r_{t+1})s_t ,$$

The F.O.C.'s are given by Eq. (1) and

$$\frac{\rho}{n_t} = \frac{1-\phi}{c_t^{\,y}} \cdot qW_t . \tag{15}$$

Eq. (15) has the same economic interpretation than Eq. (2). However, in this case the opportunity cost of having an additional child depends upon the replacement ratio and the unemployment rate.

Combining (1) and (15) with the individual's budget constraints, and exploiting Eq. (14), we find:

$$c_t^{\,y}(\underline{w}, u_t) = \frac{1-\phi}{1+\rho}\underline{w}(1-u_t), \tag{16}$$

$$c_{t+1}^{\,o}(\underline{w}, u_t) = \frac{\phi}{1+\rho}(1+r_{t+1})\underline{w}(1-u_t), \tag{17}$$

$$n_t(\gamma, u_t) = \frac{\rho(1-u_t)}{(1+\rho)q[1-u_t \cdot (1-\gamma)]}, \tag{18}$$

whereas the saving function is determined by:

$$s_t(\underline{w}, u_t) = \frac{\phi}{1+\rho}\underline{w}(1-u_t). \tag{19}$$

Eq. (18) implies that $n_t(u_t) > 1$ if and only if $\rho/(1+\rho) > q\left[1+\frac{\gamma u_t(k_t, \underline{w})}{1-u_t(k_t, \underline{w})}\right]$. The latter inequality shows that the term in square brackets is higher the higher both the unemployment rate and the replacement ratio.

Firms. As regards the production side, knowing that goods and capital markets are both competitive while the labour demand is now given by $L_t = (1-u_t)N_t$, the per-capita Cobb-Douglas technology of production transforms to:

$$y_t = Ak_t^{\,\alpha}(1-u_t)^{1-\alpha} . \tag{20}$$

Standard profit maximisation leads to the following marginal conditions for capital and labour:

$$r_t = \alpha A\left(\frac{k_t}{1-u_t}\right)^{\alpha-1} - 1, \tag{21}$$

$$\underline{w} = (1-\alpha)A\left(\frac{k_t}{1-u_t}\right)^{\alpha} . \tag{22}$$

As far as labour is concerned, the marginal product of capital will adjust to meet the fixed minimum wage with the unemployment rate being an endogenous variable.[14] Therefore, making use of (22), the current rate of unemployment is determined by:

$$u_t(k_t, \underline{w}) = 1 - \left[\frac{(1-\alpha)A}{\underline{w}}\right]^{\frac{1}{\alpha}} \cdot k_t , \tag{23}$$

[13] Note that in this model there is no uncertainty, so that each young-adult agent will be employed for $1-u_t$ hours and unemployed for u_t hours.

[14] It is worth noting that firms are assumed to have the right to hire workers according to the perceived labour demand curve at the minimum wage level preset by the policymaker.

which is positively related with the minimum wage and strictly decreasing in the per-capita stock of capital. Moreover, it is important to note that once the wage has been fixed the real interest rate is exogenous, that is, capital returns are independent of the capital stock. In fact, substituting out (23) into (21) to eliminate u_t yields:

$$r(\underline{w}) = \alpha A\left[\frac{(1-\alpha)A}{\underline{w}}\right]^{\frac{1-\alpha}{\alpha}} - 1, \tag{24}$$

so that increasing the minimum wage always pushes down the real interest rate below its competitive level, and $r(\underline{w}) = r^*{}_c$ if and only if $\underline{w} = w^*{}_c$.

Equilibrium. We now combine all the pieces of the model to characterise the long-run equilibrium. Given the government budget (14), equilibrium implies $n_t k_{t+1} = s_t$. Substituting out for n_t and s_t from (18) and (19) into the latter equation we get:

$$k_{t+1} = \frac{\phi q}{\rho} W_t. \tag{25}$$

Using Eq. (23) to substitute out for the current rate of unemployment, the dynamic equilibrium sequence of capital is driven by the following first order linear difference equation:

$$k_{t+1} = \frac{\phi q}{\rho}\gamma\,\underline{w} + \frac{\phi q}{\rho}(1-\gamma)\underline{w}^{\frac{\alpha-1}{\alpha}}\left((1-\alpha)A\right)^{\frac{1}{\alpha}} k_t. \tag{26}$$

Steady-state implies $k_{t+1} = k_t = k^*$. Therefore, the long-run per-capita stock of capital as a function of the minimum wage is:

$$k^*(\underline{w}) = \frac{\frac{\phi q}{\rho}\gamma\,\underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \frac{\phi q}{\rho}(1-\gamma)\left((1-\alpha)A\right)^{\frac{1}{\alpha}}}, \tag{27}$$

while the long-run unemployment rate and the long-run per-capita income are determined by the following equations:

$$u^*(\underline{w}) = \frac{\underline{w}^{\frac{1-\alpha}{\alpha}} - \frac{\phi q}{\rho}\left((1-\alpha)A\right)^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \frac{\phi q}{\rho}(1-\gamma)\left((1-\alpha)A\right)^{\frac{1}{\alpha}}}, \tag{28}$$

$$y^*(\underline{w}) = \frac{\frac{\phi q}{\rho} A\left((1-\alpha)A\right)^{\frac{1-\alpha}{\alpha}}\gamma\,\underline{w}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \frac{\phi q}{\rho}(1-\gamma)\left((1-\alpha)A\right)^{\frac{1}{\alpha}}}, \tag{29}$$

which are defined for any $\underline{w} \neq w_T$, where $w_T := (1-\gamma)^{\frac{\alpha}{1-\alpha}} \cdot w^*{}_c < w^*{}_c$.[15]

Substituting out (28) into (18) to eliminate $u^*(\underline{w})$, the steady-state number of children as a function of the minimum wage is simply:

$$n^*(\underline{w}) = \frac{\phi}{1+\rho}\cdot\frac{\left((1-\alpha)A\right)^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}}. \tag{30}$$

It is possible to demonstrate the following results:[16]

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[15] Note that stability requires $\underline{w} > w_T$. Since $w_T < w^*{}_c$, then the steady-state equilibrium is globally stable whatever the minimum wage. If the minimum wage is not binding, i.e. $\underline{w} = w^*{}_c$, then Eqs. (27) and (29) collapse to Eqs. (10) and (11) respectively, and $u^*(w^*{}_c) = 0$. Finally, it is worth to note that $u^*(\underline{w}) > 0$ for any $\underline{w} > w^*{}_c$ and $\lim_{\underline{w}\to+\infty} u^*(\underline{w}) = 1$.

[16] The results derive straightforwardly by investigating eqs. (27) and (29). For economy of space we do not report here the complete proof, which is of course available on request. Since in this paper we focus only on fertility behaviour, we do not pursue here a formal investigation of capital and income per-capita, which is beyond the scope of the present paper.

Result (1) a value of the minimum wage which implies a higher steady-state stock of capital per-capita than in the competitive-wage economy does always exist. In particular, it can be shown that if the minimum wage is sufficiently high, the per-capita long-run capital accumulation in the minimum-wage economy is always higher than in the market-wage economy;

Result (2) $\alpha > 1-\gamma$ is a sufficient condition to have $k^*(\underline{w}) > k^*_c$ for any $\underline{w} > w^*_c$;

Result (3) $\alpha > 1/(1+\gamma)$ is a sufficient condition to have $y^*(\underline{w}) > y^*_c$ for any $\underline{w} > w^*_c$.[17]

Therefore, if the latter inequality is satisfied, capital and income per-capita in the minimum-wage economy are both higher than the corresponding values in the competitive-wage economy.

The basic story behind the unexpected beneficial effect of the minimum wage on the long-run capital accumulation and income per-capita stated in Results 1-3 is the following: a labour market imperfection which gives rise to a wage hike, on the one side would increase the income of the currently young generation, but on the other side, since the hourly wage is fixed at too a high level for the labour market to be cleared, there will be equilibrium unemployment in equilibrium. This leads, in the short-run, to a decrease of the overall income of the younger generation (despite the presence of unemployment insurance benefits) and, given the constant propensity to save, a decrease in savings as well. This fact would be coherent with the negative belief on the minimum wage effects dating back to Stigler (1946). However, since in this model fertility is an endogenous variable, the unemployment created by the minimum wage reduces the short-run fertility rate and, in particular, fertility reduces more than savings. In the light of the latter effect, the accumulation of capital (which depends on the ratio between savings and the number of children) may be higher than in the competitive wage economy. This possible increase in the per-capita stock of capital is, however, only a first part of the story: a technological condition, according to which the output elasticity to capital intensity is larger than the output elasticity to labour employed, is another necessary component to obtain a higher long-run income. Indeed, in this dynamic context the effect of minimum wages depends on two counterbalancing forces acting on output: the induced increase in the stock of capital on the one hand, and the induced reduction in the labour input on the other hand. Whether (transitional) economic growth and long-run output will be reduced or increased by the introduction of a minimum wage will depend ultimately, given our Cobb-Douglas technology, on the capital distributive share, which should be larger than the labour share.

Finally, in the next section we will see that even a long run lifetime utility higher than in the competitive wage economy may be obtained.

4 Welfare analysis

We now turn to the welfare analysis, which has been carried out in terms of comparing steady state paths of the lifetime welfare of the representative generation, following, among many others, Samuelson (1975).[18]

The benevolent government is supposed to be a Stackelberg leader with respect to individuals and firms (Stackelberg followers). Given the followers' behaviour and knowing also that the lump-sum tax is an endogenous variable, the government chooses the minimum wage such as to maximise the steady-state indirect utility index of the representative generation,[19] i.e.:

$$\max_{\{\underline{w}\}} V^*(\underline{w}) = (1-\phi)\ln\left(c^{*y}(\underline{w})\right) + \phi\ln\left(c^{*o}(\underline{w})\right) + \rho\ln\left(n^*(\underline{w})\right), \tag{31}$$

subject to (30) and

$$c^{*y}(\underline{w}) = \frac{1-\phi}{1+\rho}\left((1-\alpha)A\right)^{\frac{1}{\alpha}} \underline{w}^{\frac{\alpha-1}{\alpha}} k^*(\underline{w}),$$

$$c^{*o}(\underline{w}) = \frac{\phi}{1+\rho}\left(1+r(\underline{w})\right)\left((1-\alpha)A\right)^{\frac{1}{\alpha}} \underline{w}^{\frac{\alpha-1}{\alpha}} k^*(\underline{w}),$$

with $r(\underline{w})$ and $k^*(\underline{w})$ being determined by (24) and (27), respectively. Differentiation of (30) with respect to $\underline{w}$ yields:

[17] Note that this condition may be satisfied for many real economies (for a discussion on the empirical measure of α see Barro and Sala-i-Martin, 2003 and Jones, 2003).

[18] It is worth noting that in this paper we perform only a positive rather than a normative analysis. It should be clear that we only show that in some circumstances regulated wage economies might perform better than the competitive-wage economy, but minimum wage legislation may be probably dominated by other second-best policies.

[19] We recall the only "positivist" approach of our paper, which does not involve the issue of "optimal policy", although we recognise that such an issue, when fertility is endogenous, involves very controversial concerns as regards population ethics. For example, the Pareto criterion rests on the condition that the number and identity of individuals is unaffected by the choice of the policy, which hardly holds in a dynamic model with endogenous population, for instance when the policy, as in our model, reduces fertility. In any case, the discussion of normative criteria with endogenous population would be far beyond the scope of this paper (among various authors attempting to tackle this controversial problem, Razin and Sadka, 1995, and more recently Michel and Wigniolle, 2007).

$$\frac{\partial V^*(\underline{w})}{\partial \underline{w}} := \Lambda(z) = \frac{[\alpha(2+\phi+\rho)-(1+\phi+\rho)]z - [\alpha(1+\phi+\rho)-(\phi+\rho)]z_T}{z^{\frac{\alpha}{1-\alpha}}\alpha(z-z_T)},$$

where $z := \underline{w}^{\frac{1-\alpha}{\alpha}}$, $z_c := \frac{\phi q}{\rho}((1-\alpha)A)^{\frac{1}{\alpha}}$ and $z_T := (1-\gamma)z_c < z_c$.

It is easy to verify that $\frac{\partial V^*(\underline{w})}{\partial \underline{w}} < 0$ for any $\alpha < \frac{1+\gamma(\phi+\rho)}{1+\gamma(1+\phi+\rho)}$ and $\frac{\partial V^*(\underline{w})}{\partial \underline{w}} > 0$ for any $\alpha > \frac{1+\gamma(\phi+\rho)}{1+\gamma(1+\phi+\rho)}$. Therefore, *Result (4)* a sufficient condition for ensuring the lifetime welfare of the representative generation in the minimum-wage economy to be higher than the corresponding value in the market-wage economy whatever the minimum-wage is $\alpha > \frac{1+\gamma(\phi+\rho)}{1+\gamma(1+\phi+\rho)}$. Result (4) confirms that, coherently with results (2) and (3), the minimum wage legislation is beneficial when α and γ are sufficiently large.

Finally, another important question can be asked when not only the long-run, but also the short-run is considered: is there any possibility for the introduction of the minimum wage reform not to imply any welfare loss for the generations bearing it? It is easy to see that, on the one hand, the welfare of the young generation may be enhanced when the capital accumulation effect is sufficiently positive, but, on the other hand, even if the welfare of the younger generation is increased, the older generation living at the moment of the introduction of the reform always incurs in a welfare loss due to the decreased interest rate. We also conjecture that some form of inter-generational transfer or debt policies from current and future young-aged individuals to current old-aged individuals might be implemented for compensating the old agents' loss. This is our current direction of research.

5 Endogenous fertility: Regulated-wage versus market-wage

We now proceed with the analysis of fertility decisions of individuals in the minimum-wage economy, both in the short and in the long-run, and then we compare fertility behaviour in competitive-wage and regulated-wage economies.

In Section 1 we have seen that if the labour market is competitive, population is constant over time (see Eq. (5)). However, the introduction of minimum wages brings about completely different results. It is of interest to derive both the current and the steady-state fertility rate as a function of the different parameters.

In the short-run, the demand for children in the minimum-wage economy is represented by Eq. (18), which then identifies some of the properties of the fertility rate, including how it might change because of a change in the economy's parameters. Expression (18) shows how the short-run fertility rate is directly affected by the replacement ratio, the preference for children and the cost of child-rearing on the one hand, and by the unemployment rate on the other hand (and, thus, it is indirectly affected by the current stock of capital per-capita and the level of the minimum wage). For instance, the short-run fertility is lower if (1) the current rate of unemployment is higher; (2) the replacement ratio is higher; (3) the stock of capital per-capita is lower (given the inverse relationship between unemployment and capital accumulation); (4) the minimum wage is higher (given the positive relationship between unemployment and minimum wage); (5) the preference for children is lower; (6) the cost of bearing children is higher.

More in detail, the following propositions hold:

Proposition 1. *In the short-run, the fertility rate in the minimum-wage economy depends negatively on the rate of unemployment.*

Proof. The proof is straightforward by differentiating (18) with respect to u_t, that is $\frac{\partial n_t(\gamma, u_t)}{\partial u_t} = \frac{-\rho\gamma}{(1+\rho)q[1-u_t(1-\gamma)]^2} < 0$. **Q.E.D.**

Proposition 2. *In the short-run, the fertility rate in the minimum-wage economy depends negatively on the replacement ratio.*

Proof. The proof is straightforward by differentiating (18) with respect to γ, that is $\frac{\partial n_t}{\partial \gamma} = \frac{-\rho u_t(1-u_t)}{(1+\rho)q[1-u_t(1-\gamma)]^2} < 0$. **Q.E.D.**

Proposition 3. *Both in the competitive wage and minimum-wage economies, the short-run rate of fertility rate depends negatively on the cost of child-rearing and positively on the preference for children.*

Proof. Differentiation of (5) and (18) with respect to q and ρ yields $\frac{\partial n^*_c}{\partial q} = \frac{-n^*_c}{q} < 0$, $\frac{\partial n^*_c}{\partial \rho} = \frac{1}{(1+\rho)^2 q} < 0$, $\frac{\partial n_t(\gamma, u_t)}{\partial q} = \frac{-\rho(1-u_t)}{(1+\rho)q^2[1-u_t(1-\gamma)]} < 0$ and $\frac{\partial n_t(\gamma, u_t)}{\partial \rho} = \frac{1-u_t}{(1+\rho)^2 q[1-u_t(1-\gamma)]} > 0$. **Q.E.D.**

To examine the impact of a change in the economy's parameters upon the fertility rate, it is useful to look at the steady state, Eq. (30). For comparative static analyses, as known, it is sufficient to investigate the perturbation of the stationary state equation with respect to a sustained change in a given parameter. We note that at the steady state both the replacement ratio (γ) and the child-rearing (q) do not affect fertility choices. A clarification for this is needed here: by observing the capital accumulation equation, Eq. (26), and the current values of n and u, Eqs. (18) and (23) respectively, we can see that, although the first-round impact of an increase in the replacement ratio (child-rearing cost) is that to reduce the current demand for children, there also exists the effect of the replacement ratio (child-rearing cost) to raise the accumulation of capital, so that when the steady state is approached, both (direct and indirect) effects of γ and q on fertility are counterbalancing: in the long run thus the negative direct effect on the current rate of fertility is exactly counterbalanced by the indirect effect which acts positively on the per-capita stock of capital. The final result therefore is that the long run rate of fertility is independent of both the unemployment benefit and the cost of raising children.

As regards the crucial relationship between minimum wages and long-run fertility behaviour, from Eq. (30) the following proposition holds:

Proposition 4. *In the minimum-wage economy, the long-run fertility rate depends negatively on the minimum wage (as against of the case of the competitive-wage economy where fertility is always constant), and* $n^*(\underline{w}) < n^*_c$ *for any* $\underline{w} < w^*_c$.

Proof. Differentiating (30) with respect to $\underline{w}$, that is, $\frac{\partial n^*(\underline{w})}{\partial \underline{w}} = -\frac{1-\alpha}{\alpha} \cdot \frac{n^*(\underline{w})}{\underline{w}} < 0$. Since $n^*(\underline{w}) = n^*_c$ if and only if $\underline{w} = w^*_c$, then Proposition 4 follows. **Q.E.D.**

The economic intuition behind the result stated in Proposition 4 is the following: (1) monetary child-rearing costs may be thought to be a proxy of a time cost of children, which means that a fraction $q \cdot n$ of the individual's time endowment is devoted to child-rearing activities, and consequently in our economy each individual obtains an income W for a unit of supplied time (net of the time spent to raising children); (2) therefore, W includes an unemployment benefits (UB) component, and since W affects the cost of children, then both UB and the unemployment rate affect fertility decisions of individuals. The final effect of the unemployment rate on fertility is negative because, while first period consumption of the adult is affected by income net of taxes (or, in other words, net of UB given the balanced budget government policy, see Eq. (14)), the consumption of the "children" good is affected by income comprehensive of UB (this because, as we said before, the cost of raising children is also affected by UB). The fact that in the minimum-wage economy, the income net of UB affects first period consumption and income comprehensive of UB affects fertility decisions, while in the competitive-wage economy the relevant income is the same for both decisions, is responsible for having fertility rates dependant (independent) of income in the minimum-wage (competitive-wage) economy. Ultimately, this means that an increase in the cost of bearing children (i.e., the opportunity cost, comprehensive of UB) is always higher than the increase in individuals' income (net of UB), implying that the demand for children in minimum-wage economy is always lower than in the competitive-wage economy.[20] Finally, it should be remarked that in this economy, where unemployment benefits are assumed to be financed with a lump-sum tax levied on each young-adult individual, minimum wage legislation may be beneficial for both the long-run capital and income per-capita only because of the effect of reducing fertility more than savings (in the short-run). The underlying economic reason is that the assumed unemployment benefit financing scheme implies a simple income redistribution between the young generation: that is income taxed away from the young rebated to the same individuals as a benefit for the hours

[20] While our assumption of monetary child-rearing costs is only a proxy – which allows us for a full analytical tractability – of a time cost, investigating more formally individuals' time allocation between labour market and child rearing in the presence and absence of a regulation of wages is a further direction of research. We thank an anonymous referee for having pointed out it.

of unemployment, so that the income of young adult agents is, because of the emerged unemployment, always lower than the competitive wage and, consequently, even the savings behaviour of individuals is lower than in the competitive-wage economy. Therefore, it is the stronger negative effect of such a legislation on fertility[21] which permits a long-run income improvement.[22]
Finally, the analysis of Eq. (30) gives:

Proposition 5. *In the minimum-wage economy, an increase in the individual's preference for children reduces the long-run fertility rate.*

Proof. Differentiating (30) with respect to ρ we find $\frac{\partial n^*(\underline{w})}{\partial \rho} = -\frac{n^*(\underline{w})}{1+\rho} < 0$. **Q.E.D.**

The nature of the solutions examined may be further clarified by a numerical example. The example makes clear that, on the one hand, for increasing minimum wages the per-capita stock of capita is increased in the long-run, and, on the other hand, the fertility rate decreases up to the point in which population becomes stationary (and even below for further increasing values of the minimum wage). The parameter set (chosen only for illustrative purposes) is the following: the production scale parameter is $A = 10$, the capital's weight in technology, in line with the recent increasing trend in many countries and with the broad concept of capital[23] is $\alpha = 0.60$ (which has been chosen to be around the value for the year 2000 for Italy according to the official statistics, see Jones, 2003), the cost of child-rearing has been fixed to be the 10 per-cent of the young total income ($q = 0.10$), the rate of thrift is $\phi = 0.10$ and the preference for children parameter is $\rho = 0.20$, thus these latter two parameters generate the following propensity to save: 0.083. Finally, the replacement ratio is $\gamma = 0.90$.

[21] However, other different unemployment benefit financing schemes, such as using either consumption taxes or capital income taxes, could permit to obtain positive effects of the regulation of wages on economic performances even without endogenous fertility, in that in such cases even young adults' income and savings might be higher than in the competitive-wage economy. This investigation, which is in our current research agenda, is beyond the scope of the present paper, which focuses on endogenous fertility behaviour of households.
[22] A positive role of government interventions on the labour market has also been argued in other contexts: e.g., employment protection policies rather than minimum wage legislation may endogenously emerge and be welfare improving in an OLG economy when credit market imperfections exist (Fogli, 2004).
[23] In fact Mankiw et al. (1992), p. 417, suggest that: (1) since the minimum wage is a proxy of the return to labour without human per capita capital, and (2) since the minimum wage has averaged about 30 to 50 percent of the average wage in manufacturing, then 50 to 70 percent of total labour income represents the return to human per capita capital, then if the physical per capita capital's share of income is expected to be about 1/3, the human per capita capital's share of income should be between 1/3 and one half. In sum, with the broad view of per capita capital the coefficient α may be fairly about 0.6 and 0.8.

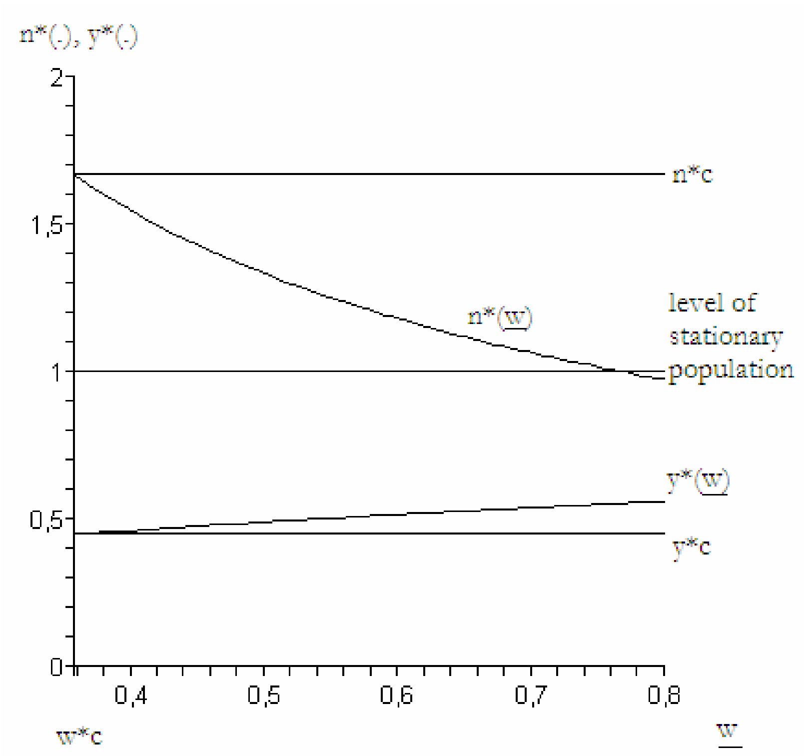

Figure 1. The long-run income per-capita and the long-run fertility rate both in the market-wage and regulated-wage economies. The starting point of the horizontal axis is the steady-state market-clearing wage, $w^*_c = 0.3577$. y^* is scaled 2:1.

Figure 1 shows that at the competitive-wage equilibrium ($w^*_c = 0.3577$) the long-run income is 0.8944 and the growth rate of population is $n - 1 = 0.667$.[24] Increasing the regulated wage over w^*_c from 0.3577 to around 0.76 permits to observe that income per-capita increases from the 0.8944 to 1.1014 while the fertility rate decreases to zero, that is to the stationary population level. The figure clearly illustrates that the introduction of minimum wages (as well as an unemployment benefit system) is beneficial for economic growth and reduces the growth rate of population. In other words, the government may use the sole tool of the regulation of wages to reach contemporary two objects of policy: (1) an enhancement in the long-run economic growth; and (2) a reduction in population growth.[25] The taxation system in fact has no effects on the long-run outcomes of the model, while the role played by the replacement ratio is very useful for policy purposes: that is, on the one hand it may enhance economic growth,[26] while on the other hand it does not affect the fertility rate. However we note that, while this may be of interest for developing countries where the fertility may be too high, for developed countries plagued by low or even negative population growth, the regulation of wages may cause an unpleasant trade-off between economic growth and population growth. The result that a rise in the regulated wage lowers the birth rate may be coherent with some empirical evidence (e.g., Cigno, 1991, pp. 113-115, in which wages and fertility evidence an inverse relationship). However, in our model, the low fertility would appear in concomitance with a high level of unemployment, in contrast with the prevailing evidence (see again Cigno, cit.); but, in contrast with the common wisdom, in our model the unemployed time should not be considered as a social damage, but rather as an additional resource which can be exploited for leisure, education, home production and so on. A further

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[24] The population growth rate ($n - 1$) is a long term rate calculated over the span of adulthood. For a better interpretation we transform it into a growth rate per year, p, applying the geometric mean: $p = (1 - n)^{\frac{1}{\pi}} - 1$, where π is the length of adulthood (fecundity period) in years. We set $\pi = 30$. Therefore, in the competitive wage economy $p = 0.017$.

[25] As regards population growth, various policies have been proposed and adopted: (1) in a pro-natalist view, such as family benefits, extensive parental (paid or unpaid) leave schemes, parents-friendly workplaces or the public provision of day-care; (2) in a control population view, such as information campaigns on contraception means, the liberalization of abortion, the levelling-up of women's educational level, the China's one-child policy. More recently, tradable procreation entitlements have been proposed as a measure that could serve both pro-natalist and anti-natalist purposes (see De La Croix and Gosseries, 2006).

[26] This can be easily ascertained by computing $\partial y^*(\underline{w}) / \partial \gamma > 0$.

direction of research should consider the possible use of the unemployed time, among other uses, for child-rearing activities.

6 Model extensions with inter-generational transfers

In this section we develop different specifications of the model, aiming to embed some realistic economic features such as those of inter-generational transfers: (1) the family as means of old-age insurance: voluntary intra-family transfers from young-aged to old-aged individuals, and (2) public inter-generational redistribution from young-aged to old-aged individuals (i.e., a pay-as-you-go (PAYG henceforth) pension scheme). More in detail, two different specifications for each type of inter-generational transfers have been considered: lump-sum transfers and proportional-to-wage transfers. In this way, these extensions permit to study the impact of a variety of different sizes and types of either intra-family transfers or public inter-generational policies on income per-capita, households fertility decisions and welfare, in order to compare the results of the extended models with those of the basic model presented in the previous sections.

E1 Pensions and lump-sum taxes

Competitive-wage economy. We introduce a publicly provided social security program and we assume the government runs a balanced budget policy by levying lump-sum taxes upon young-aged individuals used to finance a PAYG pension scheme according to the following per-capita budget constraint:[27]

$$\eta_t = \theta \cdot n_{t-1}, \tag{E1.1}$$

where the left-hand side represents the pension expenditure and the right-hand side the tax receipt, with $\theta > 0$ being a lump-sum tax levied on each young-aged individual.

Therefore, the problem for the representative individual born at time t is to maximise (P) subject to the following constraints

$$\begin{aligned} c_t^{\,y} + s_t &= w_t(1 - qn_t) - \theta \\ c_{t+1}^{\,o} &= (1 + r_{t+1})s_t + \eta_{t+1} \end{aligned}.$$

Using the first order conditions (1) and (2) together with the intra-temporal budget constrains and exploiting (E1.1), the demand for children and the saving function are respectively determined by:

$$n_t = \frac{\rho(w_t - \theta)}{(1+\rho)qw_t - \dfrac{\rho\theta}{1+r_{t+1}}}, \tag{E1.2}$$

$$s_t = \frac{w_t - \theta}{(1+\rho)qw_t - \dfrac{\rho\theta}{1+r_{t+1}}} \cdot \left\{ \phi q w_t - \frac{\rho\theta}{1+r_{t+1}} \right\}. \tag{E1.3}$$

Using (E1.2) and (E1.3), equilibrium in goods as well as in capital markets implies

$$k_{t+1} = \frac{\phi q}{\rho} w_t - \frac{\theta}{1+r_{t+1}}. \tag{E1.4}$$

Assuming individuals are perfect foresighted and exploiting (7) and (8), the dynamic equilibrium sequence of capital is determined by:

$$k_{t+1} = \frac{\phi q}{\rho}(1-\alpha)Ak_t^{\alpha} - \frac{\theta}{\alpha A k_{t+1}^{\ \alpha}}. \tag{E1.5}$$

As it can easily be seen by looking at Eq. (E1.5) closed-form solutions for the steady-state stock of capital per-capita are prevented. In what follows, we will study the effects of the PAYG social security program in a minimum wage context and then we shall run numerical simulations to compare competitive-wage and minimum-wage economies.

Minimum-wage economy. Following van Groezen et al. (2003) we suppose the government runs two distinct balanced budget policies to provide both unemployment benefits to young-aged individuals and pensions to old-aged individuals. In the former case, the government balances the unemployment benefit expenditure by levying and adjusting over time lump-sum taxes on the younger generation and thus Eq. (14) still holds. In the latter case, we suppose Eq. (E1.1) is satisfied. Therefore, the representative individual maximises (P) subject to:

[27] Agents are assumed to be perfect foresighted with respect to the level of future public pension benefits. Furthermore, we suppose agents act atomistically and do not internalise the government budget (E1.1) when deciding on the desired number of children. These hypotheses will hold throughout the paper.

$$c_t^{\ y} + s_t = \left[\underline{w}(1-u_t) + \gamma\ \underline{w} u_t\right] \cdot (1 - q n_t) - \tau_t - \theta$$
$$c_{t+1}^{\ o} = (1 + r_{t+1}) s_t + \eta_{t+1}$$
.

The first order conditions for an interior solution are exactly the same than those presented in Section 3. Thus, exploiting (1) and (15) along with the individual's budget constraints, and making use of Eqs. (14) and (E1.1), yields:

$$n_t(\underline{w}) = \frac{\rho\left[\underline{w}(1-u_t) - \theta\right]}{(1+\rho) q\left[\underline{w}(1-u_t) + \gamma\ \underline{w} u_t\right] - \frac{\rho\theta}{1 + r_{t+1}}}, \tag{E1.6}$$

$$s_t(\underline{w}) = \frac{\underline{w}(1-u_t) - \theta}{(1+\rho) q\left[\underline{w}(1-u_t) + \gamma\ \underline{w} u_t\right] - \frac{\rho\theta}{1 + r_{t+1}}} \cdot \left\{\phi q\left[\underline{w}(1-u_t) + \gamma\ \underline{w} u_t\right] - \frac{\rho\theta}{1 + r_{t+1}}\right\}. \tag{E1.7}$$

Thus, combining (E1.6) and (E1.7) equilibrium implies:

$$k_{t+1} = \frac{\phi q}{\rho}\left[\underline{w}(1-u_t) + \gamma\ \underline{w} u_t\right] - \frac{\theta}{1 + r_{t+1}}. \tag{E1.8}$$

Substituting out for u_t and r_{t+1} according to Eqs. (23) and (24), capital dynamics is driven by:

$$k_{t+1} = \frac{\phi q}{\rho}\gamma\ \underline{w} - \frac{\theta \underline{w}^{\frac{1-\alpha}{\alpha}}}{\alpha A\left((1-\alpha)A\right)^{\frac{1}{\alpha}}} + \frac{\phi q}{\rho}(1-\gamma)\underline{w}^{\frac{\alpha-1}{\alpha}}\left((1-\alpha)A\right)^{\frac{1}{\alpha}} k_t\ , \tag{E1.9}$$

so that the steady-state per-capita stock of capital as a function of the minimum wage is:[28]

$$k^*(\underline{w}) = \frac{\frac{\phi q}{\rho}\gamma\ \underline{w}^{\frac{1}{\alpha}} - \frac{\theta\ \underline{w}^{2\left(\frac{1-\alpha}{\alpha}\right)}}{\alpha A\left((1-\alpha)A\right)^{\frac{1-\alpha}{\alpha}}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \frac{\phi q}{\rho}(1-\gamma)\left((1-\alpha)A\right)^{\frac{1}{\alpha}}}. \tag{E1.10}$$

Competitive-wage versus minimum-wage. In the following Table 1[29] we summarise the steady-state results of both competitive-wage and minimum-wage economies by showing the effects of the regulation of wages on capital per-capita, income per-capita, fertility choices and the lifetime welfare. By looking at Table 1.B it can easily be seen that the introduction of a minimum wage (which has been chosen to be the 20 per-cent higher then the competitive wage)[30] boosts capital accumulation, long-run income and the lifetime welfare in spite of a positive rate of unemployment, while it has a negative effect on fertility decisions of individuals as compared with the corresponding variables in the case of competitive labour market (Table 1.A). The lump-sum tax used to finance the pension expenditure has been fixed to be around $1/4$ of the competitive wage.

Table 1.A. Competitive-wage economy. Parameter values: $A = 10$, $\alpha = 0.60$, $\phi = 0.10$, $\rho = 0.20$, $q = 0.10$ and $\theta = 0.07$.

w_c^*	k_c^*	y_c^*	n_c^*	V_c^*
0.2849	0.0122	0.7124	1.2721	-1.6293

Table 1.B. Minimum-wage economy. Parameter values: $A = 10$, $\alpha = 0.60$, $\phi = 0.10$, $\rho = 0.20$, $q = 0.10$, $\gamma = 0.90$ and $\theta = 0.07$.

$\underline{w}$	$k^*(\underline{w})$	$u^*(\underline{w})$	$y^*(\underline{w})$	$n^*(\underline{w})$	$V^*(\underline{w})$
0.3420	0.0146	0.1181	0.7540	1.1550	-1.5869

[28] It can easily be shown that the steady-state equilibrium is globally stable whatever the minimum wage. For the sake of brevity, we do not report here the complete proof which is of course available on request. Stability of equilibrium always holds throughout the paper.

[29] Note that, in order to compare the results, the parameter set used in Tables 1-4 is exactly the same than the one adopted in Section 5.

[30] Note that in Table 1 as well as in Tables 2-4 below, the minimum wage is supposed to be fixed around the 20 per-cent over the market-clearing wage.

E2 Pensions and proportional-to-wage taxes

Competitive-wage economy. We now suppose that the publicly provided inter-generational redistribution from young-aged to old-aged individuals is exclusively financed with a constant (proportional-to-wage) tax on labour earnings ($0 < \theta_w < 1$). Therefore, the government budget (E1.1) modifies to become:

$$\eta_t = \theta_w \, w_t \cdot n_{t-1} , \tag{E2.1}$$

In this case the representative individual maximises (P) subject to

$$\begin{aligned} c_t^{\,y} + s_t &= w_t(1 - qn_t) - \theta_w \, w_t \\ c_{t+1}^{\,o} &= (1 + r_{t+1})s_t + \eta_{t+1} \end{aligned} .$$

Using the first order conditions (1) and (2), the intra-temporal budget constraints and Eq. (E2.1), the demand for children and the savings path can be expressed by the following equations:

$$n_t = \frac{\rho \, w_t(1 - \theta_w)}{(1 + \rho)qw_t - \dfrac{\rho \theta_w w_{t+1}}{1 + r_{t+1}}} , \tag{E2.2}$$

$$s_t = \frac{w_t(1 - \theta_w)}{(1 + \rho)qw_t - \dfrac{\rho \theta_w w_{t+1}}{1 + r_{t+1}}} \cdot \left\{ \phi q w_t - \frac{\rho \theta_w w_{t+1}}{1 + r_{t+1}} \right\} , \tag{E2.3}$$

and thus in equilibrium we get:

$$k_{t+1} = \frac{\phi q}{\rho} w_t - \frac{\theta_w w_{t+1}}{1 + r_{t+1}} . \tag{E2.4}$$

Using (7) and (8) and knowing that agents are perfect foresighted, the dynamic equilibrium sequence of capital is determined by:

$$k_{t+1} = \frac{\phi q \alpha (1 - \alpha) A}{\rho[\alpha + \theta_w(1 - \alpha)]} k_t^{\,\alpha} , \tag{E2.5}$$

so that the steady-state per-capita stock of capital in the competitive-wage economy is:

$$k^*_c = \left\{ \frac{\phi q \alpha (1 - \alpha) A}{\rho[\alpha + \theta_w(1 - \alpha)]} \right\}^{\frac{1}{1-\alpha}} , \tag{E2.6}$$

Minimum-wage economy. Here we assume the government levies a proportional-to-wage tax for each hour worked by young individuals to provide pensions toward old individuals. Therefore, the government balances its budget according to:

$$\eta_t = \theta_w \, \underline{w}(1 - u_t) \cdot n_{t-1} , \tag{E2.7}$$

while unemployment benefits are balanced by the government according to Eq. (14).

The representative individual thus maximises (P) subject to

$$\begin{aligned} c_t^{\,y} + s_t &= [\underline{w}(1 - u_t) + \gamma \, \underline{w} u_t] \cdot (1 - qn_t) - \tau_t - \theta_w \, \underline{w}(1 - u_t) \\ c_{t+1}^{\,o} &= (1 + r_{t+1})s_t + \eta_{t+1} \end{aligned} ,$$

Exploiting the first order conditions (1) and (15) along with the individual's budget constraints and making use of (14) and (E2.7) we get:

$$n_t(\underline{w}) = \frac{\rho(1 - u_t)(1 - \theta_w)}{(1 + \rho)q[1 - u_t(1 - \gamma)] - \rho \theta_w \cdot \dfrac{1 - u_{t+1}}{1 + r_{t+1}}} , \tag{E2.8}$$

$$s_t(\underline{w}) = \frac{(1 - u_t)(1 - \theta_w)}{(1 + \rho)q[1 - u_t(1 - \gamma)] - \rho \theta_w \cdot \dfrac{1 - u_{t+1}}{1 + r_{t+1}}} \cdot \left\{ \phi q[\underline{w}(1 - u_t) + \gamma \, \underline{w} u_t] - \rho \theta_w \, \underline{w} \cdot \frac{1 - u_{t+1}}{1 + r_{t+1}} \right\} . \tag{E2.9}$$

Using (E2.8) and (E2.9) equilibrium implies:

$$k_{t+1} = \frac{\phi q}{\rho}[\underline{w}(1 - u_t) + \gamma \, \underline{w} u_t] - \theta_w \, \underline{w} \cdot \frac{1 - u_{t+1}}{1 + r_{t+1}} . \tag{E2.10}$$

Substituting out both for u_{t+1}, according to the one-period in advance (23), and r_{t+1}, according to (24), and rearranging terms we get:

$$k_{t+1} = \frac{\phi q \alpha}{\rho[\alpha + \theta_w(1-\alpha)]}\left[\underline{w}(1-u_t) + \gamma\, \underline{w} u_t\right], \tag{E2.11}$$

so that the dynamic equilibrium sequence of capital and the steady-state per-capita stock of capital as a function of the minimum wage are respectively given by the following equations:

$$k_{t+1} = \frac{\phi q \alpha}{\rho[\alpha + \theta_w(1-\alpha)]}\gamma\, \underline{w} + \frac{\phi q \alpha}{\rho[\alpha + \theta_w(1-\alpha)]}\underline{w}^{\frac{\alpha-1}{\alpha}}(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}} k_t, \tag{E2.12}$$

$$k^*(\underline{w}) = \frac{\frac{\phi q \alpha}{\rho[\alpha + \theta_w(1-\alpha)]}\gamma\, \underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \frac{\phi q \alpha}{\rho[\alpha + \theta_w(1-\alpha)]}(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}}. \tag{E2.13}$$

Competitive-wage versus minimum-wage. In the following Table 2 we can see that the introduction of a minimum wage increases the steady-state values of capital per-capita, income per-capita and welfare whilst it depresses the demand for children.

Table 2.A. Competitive-wage economy. Parameter values: $A = 10$, $\alpha = 0.60$, $\phi = 0.10$, $\rho = 0.20$, $q = 0.10$ and $\theta_w = 0.25$.

w_c^*	k_c^*	y_c^*	n_c^*	V_c^*
0.2839	0.0121	0.7097	1.2650	-1.6396

Table 2.B. Minimum-wage economy. Parameter values: $A = 10$, $\alpha = 0.60$, $\phi = 0.10$, $\rho = 0.20$, $q = 0.10$, $\gamma = 0.90$ and $\theta_w = 0.25$.

$\underline{w}$	$k^*(\underline{w})$	$u^*(\underline{w})$	$y^*(\underline{w})$	$n^*(\underline{w})$	$V^*(\underline{w})$
0.3406	0.0144	0.1255	0.7447	1.1202	-1.6279

E3 Old-age support and lump-sum taxes

Competitive-wage economy. In this section we relax the assumption of a publicly provided social security scheme (financed at balanced budget by the government) and we allow for the possibility of a form of inter-generational transfer privately provided by young-aged individuals to support consumption of old-aged individuals. In particular, in this section we assume that each young agent supplies a constant amount of resources ($d > 0$) to each old-age individual in every period.

Thus, the problem for the representative agent born at time t is to maximise (P) subject to:[31]

$$\begin{aligned} c_t^{\,y} + s_t &= w_t(1 - qn_t) - d \\ c_{t+1}^{\,o} &= (1 + r_{t+1})s_t + dn_t \end{aligned}.$$

The first order conditions for an interior maximising solution are given by Eq. (1) and

$$\frac{\rho}{n_t} = \frac{1-\phi}{c_t^{\,y}} \cdot \left[qw_t - \frac{d}{1 + r_{t+1}} \right]. \tag{E3.1}$$

Using (1) and (E3.1) and the individual's budget constraints, the demand for children and the savings path are given by:

$$n_t = \frac{\rho}{1+\rho} \cdot \frac{w_t - d}{qw_t - \frac{d}{1 + r_{t+1}}}, \tag{E3.2}$$

[31] In the case of a privately provided transfer from young-aged to old-aged people we assume that each young agent knows the number of individuals when old. Therefore, the amount of resources transferred to the old generation is taken into account by the young when deciding on the desired number of children and the savings path.

$$s_t = \frac{w_t - d}{1+\rho} \cdot \left[\frac{\phi q w_t (1 + r_{t+1}) - d(\phi + \rho)}{q w_t (1 + r_{t+1}) - d} \right]. \quad \text{(E3.3)}$$

Using (E3.2) and (E3.3) we find that:

$$k_{t+1} = \frac{\phi q}{\rho} w_t - \frac{\phi + \rho}{\rho} \cdot \frac{d}{1 + r_{t+1}}. \quad \text{(E3.4)}$$

Exploiting (7) and (8) and assuming individuals are perfect foresighted, the dynamic equilibrium sequence of capital is determined by:

$$k_{t+1} = \frac{\phi q}{\rho}(1-\alpha) A k_t^{\alpha} - \frac{\phi + \rho}{\rho} \cdot \frac{d}{\alpha A k_{t+1}^{\alpha}}. \quad \text{(E3.5)}$$

Minimum-wage economies. The unemployment benefit expenditure is still balanced according to Eq. (14). Therefore, the representative individual maximises (P) subject to:

$$c_t^{y} + s_t = [\underline{w}(1-u_t) + \gamma \underline{w} u_t] \cdot (1 - q n_t) - \tau_t - d$$
$$c_{t+1}^{o} = (1 + r_{t+1}) s_t + d n_t \quad ,$$

The first order conditions are represented by Eq. (1) and

$$\frac{\rho}{n_t} = \frac{1-\phi}{c_t^{y}} \cdot \left\{ q[\underline{w}(1-u_t) + \gamma \underline{w} u_t] - \frac{d}{1 + r_{t+1}} \right\}. \quad \text{(E3.6)}$$

Combining (1), (E3.6), the individual's budget constraints and (14) yields:

$$n_t(\underline{w}) = \frac{\rho}{1+\rho} \cdot \frac{\underline{w}(1-u_t) - d}{q[\underline{w}(1-u_t) + \gamma \underline{w} u_t] - \dfrac{d}{1 + r_{t+1}}}, \quad \text{(E3.7)}$$

$$s_t(\underline{w}) = \frac{\underline{w}(1-u_t) - d}{1+\rho} \cdot \left\{ \frac{\phi q[\underline{w}(1-u_t) + \gamma \underline{w} u_t](1 + r_{t+1}) - d(\phi + \rho)}{q[\underline{w}(1-u_t) + \gamma \underline{w} u_t](1 + r_{t+1}) - d} \right\}. \quad \text{(E3.8)}$$

Using (E3.7) and (E3.8) equilibrium implies:

$$k_{t+1} = \frac{\phi q}{\rho}[\underline{w}(1-u_t) + \gamma \underline{w} u_t] - \frac{d}{1 + r_{t+1}} \cdot \frac{\phi + \rho}{\rho}. \quad \text{(E3.9)}$$

Substituting out both for u_t and r_{t+1} according to Eqs. (23) and (24) respectively, the dynamic equilibrium sequence of capital and the steady-state stock of capital per-capita are respectively the following:

$$k_{t+1} = \frac{\phi q}{\rho} \gamma \underline{w} - \frac{\phi + \rho}{\rho} \cdot \frac{d \underline{w}^{\frac{1-\alpha}{\alpha}}}{\alpha A((1-\alpha)A)^{\frac{1}{\alpha}}} + \frac{\phi q}{\rho} \underline{w}^{\frac{\alpha - 1}{\alpha}} (1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}} k_t , \quad \text{(E3.10)}$$

$$k^*(\underline{w}) = \frac{\dfrac{\phi q}{\rho} \gamma \underline{w}^{\frac{1}{\alpha}} - \dfrac{d(\phi + \rho) \underline{w}^{2\left(\frac{1-\alpha}{\alpha}\right)}}{\rho \alpha A((1-\alpha)A)^{\frac{1}{\alpha}}}}{\underline{w}^{\frac{1-\alpha}{\alpha}} - \dfrac{\phi q}{\rho}(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}}. \quad \text{(E3.11)}$$

Competitive-wage versus minimum-wage. In the following Table 3 we can see that the introduction of a minimum wage increases capital per-capita, income per-capita and welfare, while it depresses the demand for children, even in the case in which a privately provided form of inter-generational transfer from the young to sustain consumption of the old is introduced.[32]

Table 3.A. Competitive-wage economy. Parameter values: $A = 10$, $\alpha = 0.60$, $\phi = 0.10$, $\rho = 0.20$, $q = 0.10$ and $d = 0.065$.

w_c^*	k_c^*	y_c^*	n_c^*	V_c^*

[32] Note that in Table 3 we assumed d to be around $1/4$ of the competitive wage.

0.2542	0.0101	0.6357	1.3309	-1.7518

Table 3.B. Minimum-wage economy. Parameter values: $A = 10$, $\alpha = 0.60$, $\phi = 0.10$, $\rho = 0.20$, $q = 0.10$, $\gamma = 0.90$ and $d = 0.065$.

$\underline{w}$	$k^*(\underline{w})$	$u^*(\underline{w})$	$y^*(\underline{w})$	$n^*(\underline{w})$	$V^*(\underline{w})$
0.3051	0.0121	0.1136	0.6760	1.2134	-1.7006

E4 Old-age support and proportional-to-wage taxes

Competitive-wage economy. We now suppose that each young-aged individual provides an amount of resources proportional to the wage rate perceived for each hour worked to support old-aged people consumption. Hence, a fraction $0 < d_w < 1$ of the working income when young is privately supplied to the old generation.

The maximisation of (P) subject to the following intra-temporal budget constraints

$$c_t^{\ y} + s_t = w_t(1 - qn_t) - d_w w_t$$

$$c_{t+1}^{\ o} = (1 + r_{t+1})s_t + d_w w_{t+1} n_t$$

gives the first order conditions (1) and

$$\frac{\rho}{n_t} = \frac{1-\phi}{c_t^{\ y}} \cdot \left[qw_t - \frac{d_w w_{t+1}}{1 + r_{t+1}} \right]. \tag{E4.1}$$

Making use of (1) and (E4.1) and the individual's budget constraints we get:

$$n_t = \frac{\rho}{1+\rho} \cdot \frac{w_t(1-d_w)}{qw_t - \frac{d_w w_{t+1}}{1+r_{t+1}}}, \tag{E4.2}$$

$$s_t = \frac{w_t(1-d_w)}{1+\rho} \cdot \left[\frac{\phi q w_t(1+r_{t+1}) - d_w w_{t+1}(\phi+\rho)}{qw_t(1+r_{t+1}) - d_w w_{t+1}} \right]. \tag{E4.3}$$

After having substituted (E4.2) and (E4.3), equilibrium in goods as well as in capital markets is:

$$k_{t+1} = \frac{\phi q}{\rho} w_t - \frac{\phi + \rho}{\rho} \cdot \frac{d_w w_{t+1}}{1 + r_{t+1}}. \tag{E4.4}$$

Exploiting (7) and (8) and assuming individuals are perfect foresighted, the dynamic equilibrium sequence of capital and the steady-state stock of capital are respectively determined by:

$$k_{t+1} = \frac{\phi q \alpha(1-\alpha)A}{\rho\alpha + d(1-\alpha)(\phi+\rho)} k_t^{\ \alpha}, \tag{E4.5}$$

$$k^*_{\ c} = \left[\frac{\phi q \alpha(1-\alpha)A}{\rho\alpha + d(1-\alpha)(\phi+\rho)} \right]^{\frac{1}{1-\alpha}}. \tag{E4.6}$$

Minimum-wage economy. Assuming a fixed fraction $0 < d_w < 1$ of the working income when young is privately supplied to the old-age generation and knowing that Eq. (14) is satisfied, the intra-temporal budget constraints of the representative individual modifies to:

$$c_t^{\ y} + s_t = [\underline{w}(1-u_t) + \gamma \underline{w} u_t] \cdot (1 - qn_t) - \tau_t - d_w \underline{w}(1-u_t)$$

$$c_{t+1}^{\ o} = (1 + r_{t+1})s_t + d_w \underline{w}(1 - u_{t+1})n_t .$$

Therefore, the maximisation of (P) under the above constraints gives the first order conditions (1) and

$$\frac{\rho}{n_t} = \frac{1-\phi}{c_t^{\ y}} \cdot \left\{ q[\underline{w}(1-u_t) + \gamma \underline{w} u_t] - d_w \underline{w} \cdot \frac{1-u_{t+1}}{1+r_{t+1}} \right\}, \tag{E4.7}$$

which may be used together with the individual's budget constraints and Eq. (14) to obtain:

$$n_t(\underline{w}) = \frac{\rho}{1+\rho} \cdot \frac{(1-d_w)(1-u_t)}{q[1 - u_t(1-\gamma)] - d_w \cdot \frac{1-u_{t+1}}{1+r_{t+1}}}, \tag{E4.8}$$

$$s_t(\underline{w}) = \frac{\underline{w}(1-d_w)(1-u_t)}{1+\rho}\left\{\frac{\phi q[1-u_t(1-\gamma)](1+r_{t+1})-d_w(1-u_{t+1})(\phi+\rho)}{q[1-u_t(1-\gamma)](1+r_{t+1})-d_w(1-u_{t+1})}\right\}. \tag{E4.9}$$

Now, combining (E4.8) and (E4.9), equilibrium implies:

$$k_{t+1} = \frac{\phi q}{\rho}\left[\underline{w}(1-u_t)+\gamma\,\underline{w}u_t\right]-d_w\underline{w}\frac{1-u_{t+1}}{1+r_{t+1}}\cdot\frac{\phi+\rho}{\rho}. \tag{E4.10}$$

Substituting out both for u_{t+1}, according to the one-period in advance (23), and r_{t+1}, according to (24), and rearranging terms we get:

$$k_{t+1} = \frac{\phi q\alpha}{\rho\alpha+d(1-\alpha)(\phi+\rho)}\left[\underline{w}(1-u_t)+\gamma\,\underline{w}u_t\right], \tag{E4.11}$$

so that the dynamic equilibrium sequence of capital and the steady-state per-capita stock of capital as a function of the minimum wage are respectively given by the following equations:

$$k_{t+1} = \frac{\phi q\alpha}{\rho\alpha+d(1-\alpha)(\phi+\rho)}\gamma\,\underline{w}+\frac{\phi q\alpha}{\rho\alpha+d(1-\alpha)(\phi+\rho)}\underline{w}^{\frac{\alpha-1}{\alpha}}(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}k_t, \tag{E4.12}$$

$$k^*(\underline{w}) = \frac{\frac{\phi q\alpha}{\rho\alpha+d(1-\alpha)(\phi+\rho)}\gamma\,\underline{w}^{\frac{1}{\alpha}}}{\underline{w}^{\frac{1-\alpha}{\alpha}}-\frac{\phi q\alpha}{\rho\alpha+d(1-\alpha)(\phi+\rho)}(1-\gamma)((1-\alpha)A)^{\frac{1}{\alpha}}}. \tag{E4.13}$$

Competitive-wage versus market-wage. In the following Table 4 we can look at the introduction of a minimum wage increases capital accumulation, income per-capita, welfare whilst it depresses the demand for children.

Table 4.A. Competitive-wage economy. Parameter values: $A=10$, $\alpha=0.60$, $\phi=0.10$, $\rho=0.20$, $q=0.10$ and $d_w=0.25$.

w_c^*	k_c^*	y_c^*	n_c^*	V_c^*
0.2560	0.0102	0.6400	1.3392	-1.7368

Table 4.B. Minimum-wage economy. Parameter values: $A=10$, $\alpha=0.60$, $\phi=0.10$, $\rho=0.20$, $q=0.10$, $\gamma=0.90$ and $d_w=0.25$.

$\underline{w}$	$k^*(\underline{w})$	$u^*(\underline{w})$	$y^*(\underline{w})$	$n^*(\underline{w})$	$V^*(\underline{w})$
0.3072	0.0121	0.1255	0.6715	1.1860	-1.7251

To sum up, the results of the basic model concerning the effects of regulated wage on fertility choices and per-capita income are confirmed within all the extended models, so that we feel confident in arguing a strong robustness of such results.

7 Conclusion

In this paper we analysed the effects of the regulation of wages in an OLG model of neoclassical growth with endogenous fertility and usual Cobb-Douglas preferences and technology. The introduction of a regulated wage triggers an enhancement of the long run economic growth and a reduction of the population growth rate. This occurs more likely when sufficiently high capital's weight in technology and unemployment benefits do exist. Therefore, our findings also offer some policy implications which may be, especially for developing countries,[33] very interesting.[34]

Noteworthy, our conclusions: (1) are reached within a standard dynamic general equilibrium OLG model with endogenous fertility, and the only departure from the textbook OLG model is the assumption that a minimum wage is

[33] Indeed by considering some stylised facts emerged in the recent decades especially in developing countries such as: (1) absence of unions and of any regulation of wages, which penalises labour; (2) low wages, which are more and more recognised as a limiting factor for development; (3) an increase in the weight of capital in the distribution and in the production technology, and considering also that, for developing countries, a too high rate of fertility represents another long lasting stylised fact, our results seems to better attain these countries.

[34] However we note that any policy implication should be derived very cautiously, because of the numerous stringent requirements of our findings such as the existence of no credit market imperfection, lump-sum taxation, unemployment systems, high weight of capital per-capita in technology and so on.

legally set by the policymaker over the prevailing market-clearing level as well as an unemployment benefit system financed at balanced budget with levied only upon young-aged individuals; and (2) are robust to different extensions of the basic model introducing either privately provided intra-family transfers or publicly provided PAYG pensions schemes financed at balanced budget.

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