Luciano Fanti · Luca Gori

Neoclassical Economic Growth and Lifetime Welfare in a Simple OLG Model with Unions

Discussion Paper n. 71
2008

L. Fanti
Department of Economics, University of Pisa,
Via Cosimo Ridolfi, 10, 1-56124 Pisa (PI), Italy
e-mail: lfanti@ec.unipi.it tel.: +39 050 22 16 369; fax: +39 050 22 16 384

L. Gori (Corresponding author)
Department of Economics, University of Pisa,
Via Cosimo Ridolfi, 10, 1-56124 Pisa (PI), Italy
e-mail: luca.gori@ec.unipi.it tel.: +39 050 22 16 372; fax: +39 050 22 16 384

Sections 1, 2, 3, 6E1 and 6E2 should be attributed to Luciano Fanti; Sections 4, 5, 6E3, 6E4 and 7 should be attributed to Luca Gori.
Neoclassical Economic Growth and Lifetime Welfare in a Simple OLG Model with Unions

Luciano Fanti* and Luca Gori**

Abstract

We analyse the effects of the introduction of a unionised labour market in a simple Diamond's OLG framework. Interesting findings, so far escaped closer scrutiny, emerge. Under some particular conditions about the key parameters of the model, the unionised-wage economy may perform better than the standard market-clearing wage frame as regards economic growth and the lifetime welfare. We show that wages are set by the monopolistic union as a mark-up over the (constant) unemployment benefit such as to maximise the long-run stock of capital. Furthermore, given the union's wage, a benevolent government is able to pick up exactly a value of the unemployment bonus such as to obtain a welfare maximum. These results may have important policy implications.

Keywords: Unions; Unemployment; Economic Growth; Welfare

JEL Classification: E24; H24; J51; O41

* Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I-56124 Pisa (PI), Italy; email address: lfanti@ec.unipi.it; tel.: +39 050 22 16 369; fax: +39 050 22 16 384.

** Corresponding author. Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I-56124 Pisa (PI), Italy; email address: luca.gori@ec.unipi.it; tel.: +39 050 22 16 372; fax: +39 050 22 16 384.

Sections 1, 2 and 4 should be attributed to Luca Gori; Sections 3, 5 and the Appendix should be attributed to Luciano Fanti.
1 Introduction

The debate about the relation between economic growth and unemployment is long lasting. In particular, especially in the European Union countries, unemployment has become one of the major concerns and probably one of the most important challenges for both theoretical and empirical economists. Moreover, high rates of unemployment in European Union countries, both in historical terms and relative to those in the United States, have also been a major concern for economic policymakers since the early 90s. Unemployment is still disappointingly high in most Central and Eastern European countries, which may be a reflection of the ongoing adjustment to institutional shocks resulting from systemic transition towards capitalist economies, or it may be caused by high labor market rigidities or even due to a weak aggregate demand.

Further, there are some recent important contributions relating unemployment and growth. Most of them follow Aghion and Howitt (1994) and Pissarides (2000) and consider unemployment caused by search frictions within a growth framework. Pissarides (1998) studies the effects of employment tax cuts on unemployment and wages in four equilibrium models: competitive, union, bargaining, search and efficiency wage. In particular, he analyzes the role of unemployment benefits and the tax structure on employment and unemployment. Braunerger (2000a, 2000b and 2005) and Lingens (2003) examine the relation among unemployment caused by wage bargaining and economic growth. Comeo and Marquardt (2000) concentrate on the relation between social security, unemployment, and growth explicitly. Daveri and Tabellini (2000) argue that the rise in unemployment and the reduction in economic growth are caused by the increase in the tax on labor income. Even though their model is similar in spirit to the one presented in the following sections, their results differ remarkably to ours.

In general, in these models, unemployment deteriorates growth. Only few papers have tried to reverse the general negative view between unemployment and economic growth in the macroeconomic literature. Two examples are Cahuc and Michel (1996) and Ravn and Sorensen (1999), postulating a possible positive relationship between the unemployment created by the (regulated) minimum wage and the long-run productivity growth induced by schooling and on-the-job training.

In this paper, the macroeconomic link between economic growth and labor market imperfections is analyzed within a basic overlapping generations (OLG) framework. We present a basic dynamic general equilibrium OLG model (Diamond (1965)) where equilibrium unemployment is caused by the presence of a monopolistic trade union who wishes to fix a higher than the market-clearing wage rate. Thus, in the short-run, the reduction in labour input due to the unemployment occurrence increases the marginal product of labour. The main purpose of this paper is to build up a simple model in which the possible effects of imperfections of the labour market and social welfare policies on both unemployment and growth can be isolated. In particular, we show that unemployment may be positively linked with the long-run economic growth when unions fix monopolistically the wage once the capital accumulation effects are considered. The value added of the present article, in contrast with the prevailing literature, is the presence of trade unions - pushing the wage above the competitive level - together with the consequent diffusion of unemployment benefit mechanisms may, despite the unemployment occurrence, enhance economic growth. Moreover, it is shown that there is room for a benevolent government intervention aiming to maximize the representative individual's lifetime welfare via an appropriate choice over the unemployment bonus.

The paper proceeds as follows: Section 2 describes a simple two-period OLG model with unions. Section 3 adds the equilibrium growth analysis and discusses the steady-state results. In section 4 we characterize the government action. Finally, section 5 summarizes the main results.

2 The Model

We characterize a basic dynamic general equilibrium two-period OLG model (see, for instance, Samuelson (1958) and Diamond (1965)) with young population $N_t$ growing at the constant rate $\alpha$ and closed to international trade. Goods and capital markets are both competitive, and the only departure from the standard textbook model is the assumption that

---

1 In this paper the term economic growth always refers to the level (rather than to the rate of growth) of the long run income, according to the terminology of the neoclassical growth theory (e.g. Solow (1956) and Mankiw et al. (1992)). In any case, needless to say, an increase in the long run level of output, implies a transitional increase in the rate of growth as well.

2 To name a few, one may ask where the market power of unions originates from, why so little wage underbidding on the part of the unemployed is observed, and why unemployment exists even when unions are absent. Lindbeck (1993) summarizes many of these questions.

3 Two reference textbooks are Azariadis (1993) and De La Croix and Michel (2002).
the wage rate is set by a monopolistic trade union above the market clearing level. Thus, the labour market does not clear and involuntary unemployment occurs. The model is outlined as follows.

**Individuals.** Each generation is represented by non-altruistic identical individuals endowed with a homothetic and separable utility function defined over consumption when young and old: \( c_{t}^{y} \) and \( c_{t+1}^{o} \) respectively. Only young individuals work in their first period of life, assuming a unitary constant labour supply. Depending on the demand for labour, the supplied labour force may be partially unemployed. If employed, working income is given by the non-competitive wage \( w_{t} \). If unemployed, the government pays a constant unemployment subsidy \((b)\) such that \( b \in (0, w_{t}) \). The total income received when young (working income plus unemployment benefit) is used to consume and to save. The aggregate unemployment rate is \( u_{t} = (N_{t} - L_{t}) / N_{t} \), where \( L_{t} = (1 - u_{t})N_{t} \) represents the total number of hours worked by young agents. During their second period of life individuals are retired and live on the proceeds of their savings, earning a (net of tax) return \( 1 + r_{t+1}(1 - \tau_{t+1}) \) on their investments when young where \( r_{t+1} \) is the gross rate of return on savings \((s_{t})\) from \( t \) to \( t+1 \). We assume that only a proportional tax on the income from capital at the rate \( \tau \in (0,1) \) is levied by the government and used to finance the unemployment benefit system at balanced budget. Note that individuals take the non-competitive time real wage and the lump-sum unemployment bonus as given. The maximisation problem faced by agents of generation \( t \) is:

\[
\max_{\{c_{t}^{y}, c_{t+1}^{o}\}} U_{t}\left(c_{t}^{y}, c_{t+1}^{o}\right) = (1 - \phi) \ln(c_{t}^{y}) + \phi \ln(c_{t+1}^{o}),
\]

subject to:

\[
\begin{align*}
&c_{t}^{y} + s_{t} = w_{t}(1 - u_{t}) + bu_{t} \\
&c_{t+1}^{o} = (1 + r_{t+1}(1 - \tau_{t+1})s_{t}, \\
&c_{t}^{y}, c_{t+1}^{o} \geq 0
\end{align*}
\]

where \( \phi \in (0,1) \) is a preference parameter.

The optimal young and old age consumption and the savings function are given by:

\[
\begin{align*}
&c_{t}^{y}(w_{t}, b, u_{t}) = (1 - \phi)W_{t}, \\
&c_{t+1}^{o}(w_{t}, b, u_{t}) = \phi(1 + r_{t+1}(1 - \tau_{t+1}))W_{t}, \\
&s_{t}(w_{t}, b, u_{t}) = \phi W_{t}^{\beta}
\end{align*}
\]

where \( W_{t} := w_{t}(1 - u_{t}) + bu_{t} \) represents the total income of the young (given by the sum of the labour income \( w_{t} \), plus the unemployment insurance benefit \( b \)).

**Firms.** All the firms in the econo my are identical and own a constant returns to scale Cobb-Douglas production technology by which physical capital and labour are transformed into consumption good. The representative profit-maximising firm hires aggregate capital stock \((K_{t})\) and demands labour supplied by young agents \((L_{t} = (1 - u_{t})N_{t})\) to determine aggregate production according to \( Y_{t} = AK_{t}^{\alpha}L_{t}^{1-\alpha} \), where \( A > 0 \) is a technology scale parameter and \( \alpha \in (0,1) \) is the capital's weight in technology. Defining \( k_{t} := K_{t} / N_{t} \) and \( y_{t} := Y_{t} / N_{t} \) as capital and output per capita respectively, the intensive form production technology becomes.

---

4 The typical trade union setting here analysed is presented in Booth (2002) and Layard et al. (2005) and it is usual in many works on unemployment and economic growth (for instance, Daveri and Taddei (1997 and 2000) and Bräuninger (2005)). It assumes unions to be large enough to have market power and small enough to ignore the fiscal policy variables and on the interest rate the effect of their actions.

5 We treat the unemployment insurance benefit as a policy parameter, whereas the quantity of employed labour force is endogenous.

6 Note that in this model there is no uncertainty. Thus, individuals of generation \( t \) will be employed for \( 1 - u_{t} \) hours and unemployed for \( u_{t} \) hours. Furthermore, we also assume that unemployed hours are without economic value. The use of the unemployment time for self-enrichment activities or for exploiting home production technologies and so on is left for future research.

7 In this context, the rate of time preference is simply \( \phi / (1 - \phi) \).

8 As it can be easily seen \( \phi \) also represents the (constant) propensity to save. Moreover, due to the logarithmic preferences, the capital income tax is non-distorting.

9 For simplicity we assume physical capital totally depreciates over time i.e. \( \delta = 1 \).
\[ y_t = A(1-u_t)(k_t/(1-u_t))^{\alpha}. \]  \hspace{1cm} (4)

Standard profit maximisation leads to the following marginal conditions for capital and labour:

\[ r_t = \alpha A(k_t/(1-u_t))^{\alpha-1} - 1, \]  \hspace{1cm} (5)

\[ w_t = (1-\alpha)A(k_t/(1-u_t))^{\alpha}. \]  \hspace{1cm} (6)

The short-run (current) unemployment rate is endogenous and may be derived by using the equilibrium marginal condition on the labour market. Solving eq. (6) for \( u_t \) yields:

\[ u_t(k_t, w_t) = 1 - (1-\alpha)A/w_t \cdot k_t, \]  \hspace{1cm} (7)

which is positively related with the wage rate and strictly decreasing in the per-capita stock of capital.

In order to better clarify the meaning of the coefficient \( \alpha \) (the capital weight in technology), it is worth noting that a possible interpretation is that the capital stock may be thought in its broad concept, including physical and human components and that the labour input only includes non-specialised labour. In fact, as argued by Mankiw et al. (1992), p. 417, the non-competing wage may be thought to be a proxy of the return to labour without human capital; they suggest that since the non-competing wage (for example, a minimum wage) has averaged about 30 to 50 percent of the average wage in manufacturing, then 50 to 70 percent of total labour income represents the return to human capital, so that if the physical capital’s share of income is expected to be about 1/3, the human capital’s share of income should be between 1/3 and one half. In sum, with the broad view of capital\(^{11}\) the coefficient \( \alpha \) may be fairly about 0.6 and 0.8. Indeed, for instance, Bano and Sala-i-Martin (2003) used \( \alpha = 0.75 \) saying that: "Values in the neighbourhood of 0.75 accord better with the empirical evidence, and these high values of \( \alpha \) are reasonable if we take a broad view of capital to include human components", (p. 110).

Government. One effect of the union's wages is to cause a positive level of unemployment. Therefore, the necessity to finance an unemployment benefit scheme may occur. In this paper we assume that the government chooses to levy and adjust a proportional tax on the income from capital at the rate \( \tau \in (0,1) \) such as to balance out unemployment benefit expenditures with tax receipts in each period. Thus, the per-capita time-government constraint is simply the following:

\[ bu_t = \tau f(k_t). \]  \hspace{1cm} (8)

Unions. We now present the model of wage determination (see, for instance, Layard and Nickell (1990), Booth (2002) and Layard et al. (2005)). In particular, we closely follow the structure presented, among others, by Daveri and Tabellini (1997 and 2000). Workers of each firm are represented by a union. We assume that a given fraction \( q \leq 1 \) of young individuals belongs to a trade union. Note that the union membership is exogenously fixed, whereas the number of employed individuals is endogenous.\(^{12}\) The union maximises the utility of a representative worker under the following standard assumptions: 1) the union is large enough to have market power, but small enough to neglect the effects of their doing on the macroeconomic frame (e.g. fiscal policy variables and the interest rate are taken as given); 2) it operates at the firm level, so that the welfare of the current old cannot be affected by their doing; 3) moreover, it affects the welfare of the current young only through their current income; 4) finally, the union neglects risk aversion and maximises the expected income (rather than the expected utility) of the young members.

We assume that the union faces a static optimization problem, i.e. in each period it sets wages so as to maximise the following utility function of a risk neutral representative member:

\[ U_t = \frac{1-u_t(k_t, w_t)}{w_t} + \frac{q - (1-u_t(k_t, w_t))}{q} \]  \hspace{1cm} (9)

subject to eq. (7), where \( b \) and \( q \) are given, and the ratio \( (1-u_t(k_t, w_t))/q \) represents the fraction of union members that find a job.\(^{14}\)
The interpretation of the union’s utility is straightforward: the first term in (9) is the net wage times the probability of finding a job, while the second term is the unemployment subsidy times the probability of being unemployed.\(^{15}\)

Given our production structure, the optimal net wage for the union is a constant markup over the unemployment subsidy. Specifically, the maximisation of (9) with respect to the wage rate leads to the following (constant) optimal net wage for the monopolistic trade union.\(^{16}\)

\[
w^* = \frac{b}{1 - \alpha}.
\]  

As it can be easily seen by eq. (10), the union’s wage depends positively on both the unemployment insurance benefit and the capital’s weight in technology. Given this wage-setting formulation, firms have the right to hire as many workers as dictated by the perceived labour demand curve at the wage level preset by the monopolistic union. Thus, a positive equilibrium unemployment does occur.

It is important to note that since the wage set by the union is constant over time, the real interest rate is exogenous when \(w^*_t = w^*\), that is it does not depend on the capital stock. In fact, substituting out (6) into (5) for \(k_t / (1 - u_t)\) and using eq. (10) yields:

\[
r(b) = \alpha A \left( (1 - \alpha)^{-1} \alpha b \right)^{-\alpha} - 1.
\]  

### 3 Equilibrium Growth

We can now combine all the pieces of the model to analyse the dynamics of the economy. Let us assume, for the moment, that \(w^*_t\) represents a generic non-competitive wage rate fixed by the monopolistic union (or by national law) over the time-

\[^{17}\]  

market-clearing level \((w, p_c)\).\(^{17}\)

Given the government’s balanced budget equation (see eq. (8)) and the economy’s resource constraint, the market clearing condition in goods as well as in capital markets is given by the equality between savings and investment, that is the capital stock in period \(t + 1\) is equal to the amount saved by young individuals in period \(t\):

\[
(1 + n) k_{t+1} = s_t(w_t b, u_t(k_t, w_t)),
\]  

and combining (12) with (3) we find:

\[
(1 + n) k_{t+1} = \phi [w_t - u_t(k_t, w_t) \cdot (w_t - b)].
\]  

Substituting out \(u_t(k_t, w_t)\) from eq. (7), capital evolves over time according to the following first order linear difference equation:

\[
k_{t+1} = \frac{\phi b}{1 + n} + \frac{\phi}{1 + n} (w_t - b)((1 - \alpha) A)^{\frac{1}{\alpha}} w_t^{-\frac{1}{\alpha}} k_t.
\]  

Steady-state implies \(k_{t+1} = k^* = k^*\). Thus, the per capita long-run capital stock, income and unemployment rate are given by the following conditions:

\[
u^* \left( w \right) = \frac{\frac{1}{\alpha} (1 + n) - \phi ((1 - \alpha) A)^{\frac{1}{\alpha}} w}{\frac{1}{\alpha} (1 + n) - \phi (w - b) ((1 - \alpha) A)^{\frac{1}{\alpha}}},
\]  

\[
k^* \left( w \right) = \frac{\phi b w^{\frac{1}{\alpha}}}{\frac{1}{\alpha} (1 + n) - \phi (w - b) ((1 - \alpha) A)^{\frac{1}{\alpha}}},
\]  

\[^{15}\] As noticed by Daveri and Tabellini (2000) p. 98, “The assumption that trade unions maximize expected income (and not expected utility) of their members can also be interpreted as saying that there is an insurance scheme within the union against the risk of being unemployed”.

\[^{16}\] Details are presented in appendix.

\[^{17}\] It is worth noting that for the moment we are not interested in fixing the wage precisely at the constant value set by the union \((w^*_t = w^*)\) for reasons that will become clear in what follows.
\[ y^*(w) = \frac{A\phi((1-\alpha)A)^{\frac{\alpha}{1-\alpha}}bw}{w^a(1+n) - \phi(w-b)((1-\alpha)A)^{\frac{1}{\alpha}}} \]  

(17)

We now proceed with the study of the effects of changes in the real wage on the steady-state capital stock, income and the rate of unemployment in the non-competitive wage economy and we compare these results with the ones of the market-wage frame.

Differentiating eq. (15) with respect to \( w \) yields:

\[ \frac{\partial u^*(w)}{\partial w} = \frac{\phi b((1-\alpha)A)^{\frac{\alpha}{1-\alpha}} \left[ \frac{1}{w^a} (1+n) - \alpha \phi((1-\alpha)A)^{\frac{1}{\alpha}} \right]^2}{\alpha \left[ w^a(1+n) - \phi(w-b)((1-\alpha)A)^{\frac{1}{\alpha}} \right]^2} \]  

(18)

Eq. (18) shows that the rate of unemployment in the long-run is a monotonically increasing function of the non-competitive wage.

As regards savings, the general derivative of the savings function (see eq. (3)) with respect to \( w \) is:

\[ \frac{\partial s^*(w)}{\partial w} = \phi \left\{ -u^*(w) - (w-b) \frac{\partial u^*(w)}{\partial w} \right\} \]  

(19)

There exist two counterbalancing effects of the non-competitive wage on the long-run capital accumulation (recalling eq. (14)) as showed by eq. (19): i) a positive income effect, which depends not only on the level of the non-competitive wage but also on the difference between the wage and the unemployment benefit; ii) a negative unemployment effect. A further investigation allows us for a clear identification of the role of the three economic factors (non-competitive wage, unemployment bonus and the capital weight in technology) affecting the capital accumulation: by computing the derivative of eq. (16) with respect to \( w \) we get:

\[ \frac{\partial k^*(w)}{\partial w} = \text{sgn} \left\{ \frac{b}{w} - (1-\alpha) \right\} \]  

(20)

It is easy to see that \( k^*(w) \) is a humped function. The \( \lim_{w \to +\infty} k^*(w) = \lambda \), is a horizontal asymptote which may lie i) over or ii) below the steady state stock of capital in the competitive-wage frame. In the former case, for whatever value of the wage over the level that clears the labour market, the long-run capital accumulation will always be higher than in the competitive-wage economy; in the latter case, a threshold value of \( w \) beyond which \( k^*(w) \) becomes lower than \( k^*(w_c) \) does exist. \( \left( w^* \right) \). Therefore, in this case, the long-run accumulation-improving wage must lie in the interval \( w \in (w_c, w_0) \). In fact, the right-hand side of (20) tells us that the long-run stock of capital is increased by the non-competitive wage for any \( w \in (w_c, w_k) \), where \( w_k := b/(1-\alpha) \), and decreasing for \( w > w_k \). Thus, the following remark holds:

---

18 If the market-clearing wage prevails, then \( w_c = \left( \phi ((1+n))^{\frac{\alpha}{1-\alpha}} \right)^{\frac{1}{\alpha}} \) and eqs. (16) and (17) collapse to \( k^*(w_c) = \left( \phi ((1-\alpha)A((1+n))^{\frac{1}{\alpha}} \right)^{\frac{1}{\alpha}} \) and \( y^*(w_c) = A \left( \phi ((1-\alpha)A((1+n))^{\frac{1}{\alpha}} \right)^{\frac{1}{\alpha}} \) respectively, and the unemployment rate is zero.

19 Note that an increase in the non-competitive wage brings about to a raise in the long-run unemployment rate if and only if \( w > w_0 \), where \( w_0 := (\alpha) \cdot w_c < w_c \). Thus, for any \( w > w_c \), the rate of unemployment is always positively correlated with the wage and \( \lim_{w \to +\infty} u^*(w) = 1 \).

20 A simple inspection of (19) shows that when the benefit approaches the non-competitive wage (\( b \to w \)), the effect of the introduction of such a wage always increases savings independently of the reaction of the rate of unemployment.

---
Remark 1. Since \( k^*(w) \) is a monotonically increasing function of the wage for any \( w \in (w_{pc}, w_{y}) \), and it is monotonically decreasing for any \( w > w_{y} \), it follows that \( w_{y} = b / (1 - \alpha) \) is a local maximum of the function \( k^*(w) \).

The surprising fact suggested by Remark 1 is that the long run capital accumulation maximising wage is exactly that one resulting by the union’s optimal choice (see eq. (10)). Therefore, the presence of a monopolistic union together with the diffusion of unemployment insurance schemes may not only rise the long run stock of capital over the market-wage level but, interestingly, \( k^*(w) \) is maximised at the union’s wage precisely. It would seem that the union acts as it were a benevolent planner who wants to maximise the steady-state accumulation of capital obtained in a decentralised economy.

Finally, as far as the long run per-capita output, the general derivative of the production function with respect to the wage rate is

\[
\frac{\partial y^*(w)}{\partial w} = \alpha A \left( \frac{k^*(w)}{1 - u^*(w)} \right)^{\alpha - 1} \frac{6}{\alpha} \frac{\partial k^*(w)}{\partial w} - (1 - \alpha) \left( \frac{k^*(w)}{1 - u^*(w)} \right)^{\alpha} \frac{\partial u^*(w)}{\partial w} \tag{21}
\]

The introduction of the non-competitive wage affects the output through two channels: 1) the channel of the capital accumulation (accumulation effect), and 2) the channel of the demand for labour (unemployment effect). The latter one is always negative while the former can be positive or negative depending on the productivity of capital and the magnitude of the unemployment benefit. In the case in which the lump-sum benefit is too small relatively to the level of the technological capital intensity, the capital accumulation is always worsened by the introduction of the non-competitive wage. On the contrary, when the capital accumulation is improved by the introduction of such a wage, since both \( b \) and \( \alpha \) are sufficiently high, then the long run destiny of the output will depend on the two counterbalancing effects: the positive effect due to the higher capital in production versus the negative effect of the lower labour quantity in production. Whether the positive or the negative effect will prevail will depend ultimately on the relative weights of capital and labour, i.e., the capital weight in technology summarised by the parameter \( \alpha \).

Differentiating now eq. (17) with respect to \( w \) yields

\[
\text{sgn} \left( \frac{\partial y^*(w)}{\partial w} \right) = \text{sgn} \left( \frac{\partial w}{\partial w} \right) \left( (w_{pc} - w) / (w_{y} - w) \right)^{\alpha} \cdot (1 - \alpha) A \left( \frac{w_{pc}}{1 + n} \right) - \left( \frac{w_{y} - w}{1 - \alpha} \right) A \left( \frac{w_{y}}{1 + n} \right) \right). \tag{22}
\]

The right-hand side of (22) simply means that the per-capita long run output is increased by the non-competitive wage for all the values of \( w \in (w_{pc}, w_{y}) \), where \( w_{y} := \left( \frac{\phi \alpha b}{(1 + n)(1 - \alpha)} \right)^{\alpha} \cdot (1 - \alpha) A \). Given the feasibility constraint on the value of the bonus \( (b) \), the following condition emerges

\[
\frac{b}{w} = \frac{1 - \alpha}{\alpha} \left( \frac{\phi}{1 + n} \right)^{-1} \left( (1 - \alpha) A \frac{1 - \alpha}{\alpha} w_{pc} \right) < 1. \tag{23}
\]

The latter inequality boils down, after some simple algebraic manipulations, to

\[
\frac{w}{(1 - \alpha) A \frac{1 - \alpha}{\alpha} w_{pc}} < 1. \tag{24}
\]

However, if we assume a broad concept of capital, as discussed in Section 2, this condition should be always satisfied leading to conclude that a positive correlation between minimum wage and output holds for a possibly relevant set of economies. This would amount to say that minimum wage economies are more efficient than market-wage economies are the rule rather than the exception.

---

21 Notice that also the long run per-capita output and the representative individual’s lifetime welfare may be enhanced by fixing a union’s wage even if they are not at their maximum values. The complete proof is obviously available on request.

22 If we assume a broad concept of capital, as discussed in Section 2, this condition should be always satisfied leading to conclude that a positive correlation between minimum wage and output holds for a possibly relevant set of economies. This would amount to say that minimum wage economies are more efficient than market-wage economies are the rule rather than the exception.
If we relax the assumption that the individuals’ working income is assumed to be a generic non-competitive wage and we consider the prevailing wage is set by the monopolistic trade union according to eq. (10), then the above considerations about the steady-state imply:

\[ u^*(b) = \frac{1-\alpha}{b^\alpha - \alpha (1 + n) + \phi(1 - \alpha)^2 A^\alpha} \]

\[ k^*(b) = \frac{b^\alpha (1 - \alpha) (1 + n) + \phi(1 - \alpha)^2 A^\alpha}{b^\alpha (1 - \alpha) (1 + n) - \phi(1 - \alpha)^2 A^\alpha} \]

\[ y^*(b) = \frac{b^3 A (1 - \alpha) (1 + n) + \phi(1 - \alpha)^2 A^\alpha}{b^\alpha (1 - \alpha) (1 + n) - \phi(1 - \alpha)^2 A^\alpha} \]

We have seen that if \( w_{pc} < w < w^\alpha \), then the stock of capital is higher than in the competitive-wage economy.

In Figure 1 we show the locus of the long-run capital accumulation (as a function of the wage rate) in the decentralised economy. It can be easily seen that \( k^*(w) \) is maximised when the trade union sets the wage at its optimal value (given by a fixed mark-up over the unemployment insurance benefit). In the case depicted in the figure, \( \lim_{w \to +\infty} k^*(w) \) lies below \( k^*_{pc} \). Figure 2, instead, clearly depicts that a wage rate (lower than \( w^\alpha \)) such that the long-run income \( (y^*(w)) \) is maximised there also exists as well. Anyway, when \( w = w^\alpha \), the long-run income is higher than in the competitive wage case. Whether the union’s wage, which is capital accumulation maximising, brings about even a higher per-capita output than that of the market-clearing wage case or not depends on the productivity of net effect of two countervailing forces of the increased wage on output: the negative effect of the increased unemployment versus the positive effect of the increased capital accumulation.

To better understand the intuition of why the wage set by the monopolistic trade union is capable to enhance the long run economic growth, it is sufficient to observe that: higher wages, despite the corresponding unemployment, may augment – under the twofold conditions that on the one hand the weight of capital in technology is sufficiently high to guarantee a high productivity of labour and on the other hand the unemployment benefit are sufficiently high – the average income of young individuals which, in turn, in a lifecycle model leads to more savings. The effects of the unions may be loosely speaking assimilated to the implementation of the reverse of a social security scheme that, in each period, transfers resources from the old to the young. The young workers - and all future generations - are made better off by such a scheme because it raises their permanent income by shifting resources from the second to the first period at better terms than those offered by the market. In a nutshell, the higher unionised wage transfers resources over time from the older to younger individuals by raising the labour income and decreasing the interest rate.

4 The Government Action and the Steady-State Lifetime Welfare

We now proceed with the analysis of the representative individual’s lifetime welfare in terms of comparing steady-state paths (see Samuelson (1975)).

The representative individual, when deciding the optimal young and old age consumption and the saving path, takes the non-competitive time-real wage \( (w_r) \) and the unemployment benefit \( (b) \) as given. Wages are set (in each period) by a monopolistic trade union according to a mechanism such that a utility function of a risk neutral representative member is maximised by keeping the unemployment subsidy as given. In particular, the optimal net wage for the union \( (w^\alpha) \) is a
constant mark-up over $b$. Even if the union’s wage maximises the decentralised economy's long-run accumulation of capital, the maximisation of the lifetime welfare is not guaranteed as well. Thus, given the union's wage, the benevolent government has the opportunity to intervene directly in the economy by choosing $b$ such as to maximise the individual’s lifetime welfare. In particular, the government faces the following constrained maximisation:

$$\max_{[b]} V(b) = (1 - \phi) \ln(c^*(b)) + \phi \ln(c^{**}(b)),$$

subject to

$$w^u = b r(1 - \alpha),$$
$$\tau^*(b) = bu^*(b) / r(b) k^*(b),$$
$$c^{**}(b) = (1 - \phi)(w^u - u^*(b)(w^u - b)),
\quad c^*(b) = \phi[1 + r(b)(1 - \tau^*(b))](w^u - u^*(b)(w^u - b)).$$

The behaviour of the welfare function (26) under the above constraints is dependent on the unemployment benefit, the technology and the preference parameters, $\alpha$, $A$ and $\phi$ respectively and the constant growth rate of population. Unfortunately, such a dependence is highly non-linear, so that analytical results are prevented. Nevertheless, the first order conditions give the implicit solutions of the program for $b$, that is:

$$\frac{\partial V(b)}{\partial b} = 0 \iff \begin{cases} b_1 = b_1(\alpha, \phi, n, A) \\ b_2 = b_2(\alpha, \phi, n, A). \end{cases}$$

Eq. (27) gives the two solutions of the government program. In particular, $b_1$ is not economically relevant while $b_2$ represents the optimal (welfare-maximising) value of the unemployment bonus once the monopolistic trade union has fixed the wage at the (constant) rate $w^u$.

Using $b_2$ together with eq. (10) we find:

$$w^w = \frac{b_2}{1 - \alpha}.$$

Given that the unionised wage is consistent with whatever sufficiently high unemployment benefit, the maximisation of the long-run accumulation of capital in the decentralised economy is preserved even when the government sets $b$ at the point in which the representative individual’s lifetime welfare is maximised ($b_2$). Therefore, if $w = w^w$ two objectives are reached: the maximisation of both the long-run accumulation of capital in the decentralised economy and the individual’s lifetime welfare.

Since eq. (27) is difficult to handle analytically, we shall run simulations that basically make use of both eq. (27) and eq. (28). In what follows, our purpose is to show that a “calibrated” standard OLG economy with a market imperfection consisting in the existence of non-competitive wages could produce an improvement of the lifetime welfare in the long run. Now we are concerned with the choice of the parameter values for the simulations. Recently Jones (2003 and 2005) provides estimates of the capital’s share in OECD countries. He reports two types of measures for the capital’s share: 1) a measure constructed as one minus employee compensation divided by GDP and 2) the employee compensation share corrected for self-employment. As regards Italy, the evidence reported by Jones ((2003), Figure 1, p. 8) shows that in the recent period the capital’s share is among 0.37 and 0.42 according to the second measure.

Table 1 . Household Saving Rates 1990-2000 (in percentage points) for some OECD countries

<table>
<thead>
<tr>
<th>Household Saving Rates 1990-2000 (in percentage points)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Canada</strong></td>
</tr>
<tr>
<td>1990s peak</td>
</tr>
<tr>
<td>2000</td>
</tr>
</tbody>
</table>
In what follows we resort to numerical simulations to compare the market-wage and the unionised-wage representative individual’s steady-state lifetime welfare. We used for the technological capital weight in technology a value of \( \alpha = 0.50 \), which is an intermediate value among the two measures calculated by Jones (2003), since we think that in our model the labour input interested to the union’s program only includes non-specialised labour.

The following Figure 3 depicts the behaviour of the welfare function in both the market-wage and unionised-wage economies showing that if the unemployment insurance benefit is sufficiently high, then the lifetime welfare is higher in the union’s frame than in the competitive-wage regime. Furthermore, the government is able to pick up an appropriate value of the bonus, \( b_* \). In Figure 4, instead, we present the behaviour of the long-run stock of capital as a function of the wage rate, showing the point \( (w^*) \) in which the accumulation of capital is maximised.

When \( b = b_* \) and, therefore, \( w = w^* \) then the objectives of both the employees’ trade union and the benevolent government are satisfied. Thus, the long-run accumulation of capital in the decentralised economy and the representative individual’s lifetime welfare are maximised.

To sum up, numerical simulations using typical values of the weight of capital in technology and of the propensity to save concerning, for instance, Italy leads to clear cut results as regards the effects of the introduction of the union’s wage and the government intervention. In particular, we show that the welfare function may be either minimised \( (b_1) \) or maximised \( (b_2) \) by choosing appropriate values of \( b \). Therefore, even if the government is able to pick up the exact value of the benefit which is welfare-maximising, the choice of the unemployment bonus requires an accurate evaluation since an inappropriate value of \( b \) could also be harmful.

5 Conclusions

This paper has concerned about the effects on the neoclassical economic growth and the lifetime welfare of the presence of a monopolistic trade union in a simple two-period OLG model. We have showed that the steady-state capital accumulation may be enhanced by the introduction of non-competing wages despite the unemployment occurrence. In particular, the long-run stock of capital is maximised exactly at the optimal union’s wage, and both income and welfare may be enhanced as well. Further, we have also demonstrated that a benevolent government taking into account the wage set by the union, may obtain an individual’s welfare maximum by choosing an appropriate unemployment bonus. These findings may have important policy implications as regards the role played both by trade union and the government. The most important result of this paper is that we have proved that a simple two-period OLG frame with a non-competitive labour market may perform better than the standard market-clearing wage economy as regards capital accumulation, per-capita income and the lifetime welfare. These results, showing a possible positive effect of labour market imperfections on the long-run economic growth and welfare, contrast with the prevailing literature for which non-competitive wages and high rates of unemployment should be harmful. Finally, these results may also be generalised by considering more complex utility functions, such as the CIES one, or by introducing the possibility for endogenous growth. Moreover, other types of taxation systems may be considered. These arguments are left for future research.

Appendix

In this appendix we show that the optimal union’s wage is a maximum.

The maximisation of eq. (9) subject to eq. (7) leads to the following first order conditions

\[
\frac{\partial U^u}{\partial w_i} = 0 \Leftrightarrow \frac{(1 - \alpha)(1 - \alpha)A}{\alpha} w_i^{-\frac{1}{\alpha}} k_i + \frac{(1 - \alpha)A}{\alpha} w_i^{-\frac{1}{\alpha}} w_i^{-1} b k_i = 0. \quad (A1)
\]

After a simple algebra, eq. (A1) may be written as

\[
\frac{(1 - \alpha)A}{\alpha} w_i^{-\frac{1}{\alpha}} k_i \left[ b - (1 - \alpha) \right] = 0, \quad (A2)
\]
It is easy to see that eq. (A2) boils down into eq. (10) of the main text. By computing the second order derivative with respect to the wage rate, we find that:

\[
\frac{\partial^2 U^w}{\partial w^2} \bigg|_{w^*=w^*} = -\frac{1-\alpha}{\alpha q} \left( (1-\alpha) A \right) \left( \frac{b}{1-\alpha} \right)^{1-\alpha} k_t < 0. \quad (A3)
\]

Therefore, \( w_t = w^* \) represents a maximum point.

References


Figure 1. The steady-state accumulation of capital in both the non-competitive ($k^*(w)$) and market-clearing wage ($k^*(w_{pc})$) economies. The figure shows that $k^*(w)$ is maximised when the optimal trade union's wage rate ($w^*$). The starting point of the horizontal axis is the competitive wage, $w_{pc} = 8.63$. Parameter set: $A = 10$, $\alpha = 0.33$, $\phi = 0.25$, $b = 8$ and $n = 0$.

Figure 2. The long-run stock of capital and the long-run income (scaled by 35) as a function of the wage rate in both the non-competitive and market-clearing wage economies. The starting point of the horizontal axis is the competitive wage.
$w_{pc} = 0.025$. The union's wage is $w_{uc} = 0.082$. Parameter set: $A = 10$, $\alpha = 0.71$, $\phi = 0.05$, $b = 0.024$ and $n = 0$.

Figure 3. The behaviour of the welfare function in the case of both competitive-wage and non-competitive-wage economies. The starting point of the horizontal axis is the value of the unemployment benefit which guarantees a non-negative long-run rate of unemployment (see eq. (23)), $b_0 = 0.416$, while the welfare-maximising benefit is $b_2 = 0.685$. The

Note that this high value of the capital weight in technology is in line with Barro and Sala-i-Martin (2003), as above mentioned in section 2.
competitive wage is \( w_{pc} = 0.833 \) and the union's wage is \( w^u = 1.370 \). Parameter set: \( A = 10, \ \alpha = 0.50, \ \phi = 0.10 \) and \( n = 2 \).

**Figure 4.** The behaviour of the long-run stock of capital in the case of both competitive-wage and non-competitive-wage economies. The starting point of the horizontal axis is the market clearing wage \( w_{pc} = 0.833 \), while \( w^u = 1.370 \). Parameter set: \( A = 10, \ \alpha = 0.50, \ \phi = 0.10 \) and \( n = 2 \).
1. Luca Spataro, Social Security And Retirement Decisions In Italy, (luglio 2003)


5. Pompeo Della Posta, Vecchie e nuove teorie delle aree monetarie ottimali, (luglio 2003)


10. Gaetano Alfredo Minerva, Location and Horizontal Differentiation under Duopoly with Marshallian Externalities (settembre 2003)


13. Davide Fiaschi - Pier Mario Pacini, Growth and coalition formation (settembre 2003)

14. Davide Fiaschi - Andre Mario Lavezzi, Nonlinear economic growth; some theory and cross-country evidence (settembre 2003)


19. Luciano Fanti, Notes on Keynesian models of recession and depression (ottobre 2003)

20. Luciano Fanti, Technological Diffusion and Cyclical Growth (ottobre 2003)


30. Paolo Mariti, Costi di transazione e sviluppi dell’economia d’impresa (giugno 2004)
32. Francesco Drago, Redistributing opportunities in a job search model: the role of self-confidence and social norms (settembre 2004)
33. Paolo Di Martino, Was the Bank of England responsible for inflation during the Napoleonic wars (1897-1815)? Some preliminary evidence from old data and new econometric techniques (settembre 2004)
34. Luciano Fanti, Neo-classical labour market dynamics and uniform expectations: chaos and the “resurrection” of the Phillips Curve (settembre 2004)
35. Luciano Fanti – Luca Spataro, Welfare implications of national debt in a OLG model with endogenous fertility (settembre 2004)
36. Luciano Fanti – Luca Spataro, The optimal fiscal policy in a OLG model with endogenous fertility (settembre 2004)
38. Luciano Fanti – Luca Spataro, Dynamic inefficiency, public debt and endogenous fertility (settembre 2004)
40. Gaetano Alfredo Minerva, How Do Cost (or Demand) Asymmetries and Competitive Pressure Shape Trade Patterns and Location? (ottobre 2004)
42. Andrea Mario Lavezzi - Nicola Meccheri, Job Contact Networks, Inequality and Aggregate Output (ottobre 2004)


49. Marco Guerrazzi, Intertemporal Preferences, Distributive Shares, and Local Dynamics (dicembre 2004)

50. Valeria Pinchera, “Consumo d’arte a Firenze in età moderna. Le collezioni Martelli, Riccardi e Salviati nel XVII e XVIII secolo” (dicembre 2004)

51. Carlo Casarosa e Luca Spataro, “Propensione aggregata al risparmio, rapporto ricchezza-reddito e distribuzione della ricchezza nel modello del ciclo di vita "egualitario": il ruolo delle variabili demografiche” (aprile 2005)

52. Alga D. Foschi – Xavier Peraldi – Michel Rombaldi, “Inter – island links in Mediterranean Short Sea Shipping Networks” (aprile 2005)


55. Annetta Binotti e Enrico Ghiani, “Changes of the aggregate supply conditions in Italy: a small econometric model of wages and prices dynamics” (settembre 2005)


58. Mario Morroni (2006), “Complementarities among capability, transaction and scale-scope considerations in determining organisational boundaries”


64. Alga D. Foschi (2006), “La concentrazione industriale per i sistemi di trasporto sostenibile


67. Luciano Fanti and Luca Spataro (2007), “Neoclassical OLG growth and underdeveloped, developing and developed countries”


69. Carlo Brambilla and Giandomenico Piluso (2008), “Italian investment and merchant banking up to 1914: Hybridising international models and practices”