Discussion Papers
Collana di E-papers del Dipartimento di Scienze Economiche – Università di Pisa

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Discussion Paper n. 76
2009
Discussion Paper n. 76: presentato Gennaio 2009

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Abstract

Unemployment and population ageing are probably two of the most important concerns in developed countries. Since reforming labour markets is high on the political agenda, a theoretical knowledge of the possible long-run interaction between unemployment and the childcare system may be highly valuable. Applying a fairly standard OLG model with endogenous fertility and minimum wages, we show that a child tax (rather than the more conventional child subsidy) can be used as an instrument (1) to promote population growth and (2) to reduce unemployment and, in particular, to restore the full employment equilibrium.

Keywords

Child Tax; Fertility; OLG model; Unemployment

JEL Classification

H24; J13; J18
1. Introduction

The present analysis is motivated by two important issues in contemporary developed economies (especially in Europe), namely population ageing and the persistently high rates of unemployment. The former is mainly originated by the reduced birth rates. The latter, instead, has several origins but surely many economists ascribe it to the high wage rates determined in non-competitive labour markets (e.g., unions and minimum wage legislation).\(^1\)

In order to solve the ageing problem, several remedies have been indicated (e.g., child subsidies); hence, the present analysis is motivated also by proposals suggested both by the political and the academic debates.\(^2\) As regards the unemployment problem, although a vast literature argued for different recipes and remedies, nobody has, to the best of our knowledge, so far considered the possible effects of the childcare system on the aggregate unemployment rate.

The current paper contributes to the efforts to remedy the plagues of high unemployment and low fertility by offering a model which incorporates some important institutional features. Thereby it provides an analytic framework in which relevant policy issues are addressed.

In the theoretical literature, unemployment and child policies have traditionally been studied in separated class of models. A recent valuable work focusing on the unemployment issue is Corneo and Marquardt (2000), while other papers have recently tackled out the role of child policies either in a static context (e.g., Apps and Rees, 2004) or by adopting a dynamic competitive overlapping generations (OLG) model (Momota, 2000; van Groezen et al., 2003). While the preceding papers assumed, either for some exogenous reasons (Apps and Rees, 2004) or for obtaining Pareto improvements in presence of other externalities such as pensions (van Groezen et al., 2003), that governments wish to increase fertility, and investigated the effects of the child allowance as an instrument to achieve such a goal, we argue that a child tax rather than the more traditional child subsidy may work properly both to promote population growth and to reduce unemployment.\(^3\) For doing this, we develop a fairly standard OLG model (Diamond, 1965), extended to account for endogenous fertility and minimum wage legislation to study the interaction between child tax, savings, fertility and unemployment.

A number of clear-cut results, which represent a novelty, can be obtained by investigating both child policy and unemployment in a unitary framework. In particular, a child tax can be used to promote population growth and to restore the full employment equilibrium.

The paper is structured as follows. In Section 2 we present the model and the steady-state results. Sections 3 (Section 4) investigates the relationship between child taxes and unemployment (fertility) in the long-run. Section 5 concludes.

2. The model

2.1. Government

The government runs a balanced child policy budget in every period. In particular, a fixed per child tax is collected and used to fund a labour income subsidy (adjusted to balance out the budget). Therefore, the per-capita time-\(t\) government constraint is simply:\(^4\)

\[
\tau_t w(1-u_t) = \beta n_t, \tag{1}
\]

where the left-hand side represents the total government expenditure and the right-hand side the child-tax receipt, with \(\beta > 0\) being the (constant) child tax, \(\tau_t > 0\) is the subsidy rate received by the young workers, \(w\) is the minimum wage (legally set over the market-clearing level), \(u_t\) is the aggregate unemployment rate (defined in terms of hours not worked)\(^5\) and \(n_t\) is the number of children.

2.2. Individuals

\(^1\) For instance “The observed increase in unemployment and the slowdown in economic growth in Europe are related, both stem from a common cause, an excessively rapid growth of the cost of labour… If labor markets are non-competitive, an exogenous and lasting increase in labor costs has two effects. On the one hand, it reduces labor demand, and thus creates unemployment. On the other hand, as firms substitute capital for labor, the marginal product of capital falls. Over long periods of time, this in turn diminishes the incentive to invest and thus to grow.” (Daveri and Tabellini 2000, p. 50).

\(^2\) See Neyer (2006) who surveyed the relationship between family policy, fertility, employment and child care. In particular we wish to note that recently in Italy a sensible lump-sum child benefit have been introduced.

\(^3\) In addition to the fundamental differences given by both the child-tax (rather than the child-subsidy) policy and the minimum wage law (rather than the competitive wages), many other differences can distinguish this paper from the current literature. For instance, Apps and Rees (2004) faced with a static context. Momota (2000) instead assumed two kinds of individuals, a gender wage gap, a time cost for childrearing and a rather special form of subsidy policy, while van Groezen et al. (2003) investigated the interactions between the childcare and the social security systems in an OLG a small open economy.

\(^4\) We suppose individuals act in an atomistic way and thus do not take (1) into account when deciding on the desired number of children and the saving function.

\(^5\) Notice that \(u_t = (N_t - L_t) / N_t\), where \(L_t\) is the labour demand and \(N_t\) the number of young-adult people.
Agents are identical and live a three-period OLG economy. During childhood individuals do not make economic decisions. Adults entering the workforce at time $t$ ($N_t$) are endowed with one unit of time which is supplied inelastically to the labour market, and draw utility over material consumption when young and old, $c_{1,t}$ and $c_{2,t+1}$, respectively, and the number of children raised, $n_t$ (Galor and Weil, 1996). We assume young-adult workers earn a constant minimum wage per hour worked which is used to consume, to pay taxes, to raise children and to save. Rearing children requires a fixed amount of resources $m$ per child. During oldness agents are retired and live exclusively on the proceeds of their savings ($s_t$) plus the accrued interest at the rate $r_{t+1}$.

The representative individual maximises the following logarithmic utility:

$$U_t(c_{1,t}, c_{2,t+1}, n_t) = (1-\phi)\ln(c_{1,t}) + \gamma \ln(c_{2,t+1}) + \phi \ln(n_t),$$

subject to the lifetime budget constraint

$$c_{1,t} + \frac{c_{2,t+1}}{1+r_{t+1}} = w(1-u_t)(1+r_t) - (m+\beta)n_t,$$

where $0 < \gamma < 1$ is the subjective discount factor and $0 < \phi < 1$ captures the importance in the welfare function of having children relative to material consumption when young.

Exploiting the first order conditions, the lifetime budget constraint and the government budget (1), the demand for children and the saving function are, respectively:

$$n_t = \frac{\phi w(1-u_t)}{(1+\gamma)m + (1+\gamma-\phi)\beta},$$

$$s_t = \frac{\gamma (m+\beta)w(1-u_t)}{(1+\gamma)m + (1+\gamma-\phi)\beta}.$$  

2.3. Firms

Identical firms act competitively. The representative firm produces according to the following (intensive form) aggregate constant returns to scale Cobb-Douglas technology:

$$y_t = k_t^\alpha (1-u_t)^{1-\alpha},$$

where $k_t := K_t/N_t$ and $y_t := Y_t/N_t$ are capital and output per-capita respectively and $0 < \alpha < 1$ is the capital’s weight in technology. Assume the stock of capital totally depreciates at the end of each period and the final output is traded at unit price. Therefore, profit maximisation leads to:

$$r_t = \alpha \left( \frac{k_t}{1-u_t} \right)^{\alpha-1} - 1,$$

$$w = (1-\alpha) \left( \frac{k_t}{1-u_t} \right)^{\alpha}.$$  

Since the minimum wage is binding and firms hire workers according to their labour demand curves, the marginal product of labour will adjust to meet the fixed real wage with unemployment being endogenously determined, that is:

$$u_t = 1 - \left( \frac{1-\alpha}{w} \right)^{\frac{1}{\alpha}} \cdot k_t.$$  

2.4. Equilibrium

Given the government budget (5), and knowing that $N_{t+1} = n_t N_t$, equilibrium implies $n_t k_{t+1} = s_t$. Using (2) and (3) we get the following (constant) long-run stock of capital per-capita:

$$k_{t+1} = k^* (\beta) = \frac{\gamma}{\phi} (m+\beta).$$

From Eq. (8) we find that a rise in the per child tax increases the long-run stock of capital per-capita, $\partial k^*(\beta)/\partial \beta = \gamma/\phi > 0$. This is the combining result of a twofold effect: (i) the positive income effect of the child
tax revenue – rebated as wage subsidy within the same child bearing generation – increases savings, and (ii) the higher is the child tax the higher is the cost of raising an extra child, \( m + \beta \). Hence, the number of children in the short-run is reduced. However, in the long-run, the higher capital stock installed contributes to reduce unemployment. Therefore, according to (2) and (3) fertility and saving tend to increase.

3. Unemployment

From Eqs. (7) and (8), the following proposition holds:

**Proposition 1.** For any given value of the minimum wage, a child tax reduces the long-run unemployment rate. In particular, there exists a child tax such that the full employment equilibrium is restored.

**Proof.** Knowing that

\[ u^*(\beta) = 1 - \left( \frac{1 - \alpha}{w} \right) \frac{1}{\alpha} \gamma \phi (m + \beta), \]  

(9)
differentiating (9) with respect to \( \beta \) gives:

\[ \frac{\partial u^*(\beta)}{\partial \beta} = -\left( \frac{1 - \alpha}{w} \right) \frac{1}{\alpha} \frac{\gamma}{\phi} < 0, \]

for any \( w > w^*_c \), where \( w^*_c \) is the equilibrium competitive wage.

Since \( \lim_{w \to w^*_c} u^*(\beta) = 1 \), \( 0 \leq u^*(\beta) < 1 \) holds for any \( w \geq w^*_c \), and by Proposition 1 \( \partial u^*(\beta)/\partial \beta < 0 \), then equating (9) to zero and solving for \( \beta \) gives:

\[ \beta_u := \phi \left( \frac{w}{1 - \alpha} \right) - m, \]  

(10)

which represents the limiting value of the child tax such that the full employment equilibrium is restored, that is \( u^*(\beta) = 0 \) for any \( w > w^*_c \). Q.E.D.

Proposition 1 stems directly from the offspring tax effect played on the long-run stock of capital. In particular, the higher is the child tax the higher is the total cost of raising children whatever the value of the minimum wage, and thus the higher is the capital stock. An increasing minimum wage rises unemployment. However, there exists a value of the child tax \( (\beta = \beta_u) \) such the stock of capital is high enough to eliminate completely unemployment in the long-run.

4. Fertility

Analysis of fertility decisions of households gives another interesting and unconventional result which may be carefully examined.

Define the long-run rate of fertility as a generic function of the child tax as:

\[ n^* = n \left\{ \beta, u^*[k^*(\beta)] \right\}. \]  

(11)

Now, totally differentiating (11) with respect to \( \beta \) gives:

\[ \frac{dn^*}{d\beta} = \frac{\partial n^*}{\partial \beta} + \frac{\partial n^*}{\partial u} \frac{\partial u}{\partial \beta} + \frac{\partial n^*}{\partial k} \left( \frac{\partial k^*}{\partial \beta} \right) \]  

(12)

Eq. (12) reveals that the final effect of a rise in the child tax appears to be ambiguous and depends on two counterbalancing forces: (i) a negative (direct) effect which, by increasing the cost of children, reduces the rate of fertility, and (ii) a positive (indirect) feedback effect which acts on fertility through the unemployment rate. In particular, a rise in the child tax increases savings and thus the pace of accumulation of capital, while reducing unemployment. Given the negative relationship between unemployment and fertility, a lower unemployment rate increase population growth.

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6 Details are given in the Appendix.
To analyse ultimately which of the two forces dominates, we now combine Eqs. (2) and (9) to obtain:

\[ n^*(\beta) = \frac{\gamma(m + \beta)(1 - \alpha)^{\frac{1}{\alpha}}}{[(1 + \gamma)m + (1 + \gamma - \phi)\beta]^{1-\alpha}}. \] (13)

From (13) the following proposition holds:

**Proposition 3.** For any given value of the minimum, a child-tax always increases the long-run rate of fertility.

**Proof.** Differentiating (13) with respect to \( \beta \) gives:

\[ \frac{\partial n^*(\beta)}{\partial \beta} = \frac{n^*(\beta)m\phi}{(m + \beta)[(1 + \gamma)m + (1 + \gamma - \phi)\beta]} > 0, \] (18)

for any \( w > w^*_c \) and \( 0 < \beta \leq \beta^*_u \). Q.E.D.

Proposition 3 reveals that the fertility rate is always higher than whether the child tax policy is absent. This result holds because the positive general equilibrium feedback effect which increases fertility by reducing unemployment always prevails over the negative direct effect of the child tax which – by contrast – tends to rise the cost of children.

The essential message of the paper is the following: although it is rather unusual to think about the possibility to reduce unemployment and to increase population growth by adopting offspring taxes, we may conclude that countries with imperfect labour markets (such as the most part of European Union countries) should consider the possibility to introduce a child tax as an instrument to decrease (or even nullify) unemployment and increase population growth.

5. Conclusions

This paper, by examining child policy and unemployment rate in a unitary framework, achieves a number of clear-cut results, which are, so far, surprisingly escaped closer scrutiny and which may have interesting policy implications. In particular, we show that a child tax (rather than the more traditional child benefit) policy (1) promotes population growth, (2) reduces the unemployment rate, and (3) may be used as instrument to restore the full employment equilibrium. These findings are, to the best of our knowledge, a novelty.

The present paper offers a manageable framework of analysis by incorporating a number of simplifying assumptions. For instance, time (opportunity) child costs, childcare facilities, home production technologies and unemployment insurance systems may be further considered. Incorporating these features is a promising direction for future research.

Appendix

Effects of the child tax on the long-run rate of fertility:

\[ \frac{\partial n^*}{\partial \beta} = -\frac{(1 + \gamma - \phi)n^*}{(1 + \gamma)m + (1 + \gamma - \phi)\beta} < 0, \quad \text{(A1)} \]

\[ \frac{\partial n^*}{\partial u} = \frac{-\phi w}{(1 + \gamma)m + (1 + \gamma - \phi)\beta} < 0, \quad \text{(A2)} \]

\[ \frac{\partial u^*}{\partial k^*} = \left(1 - \alpha \right)^{\frac{1}{\alpha}} \left(\frac{1}{w} \right) < 0. \quad \text{(A3)} \]

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