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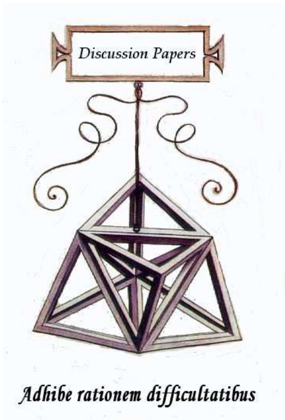

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Longevity, fertility and PAYG pension systems sustainability

Luciano Fanti* **and Luca Gori****

Abstract Recently Fanti and Gori (2008) showed – in the basic overlapping generations (OLG) model of neoclassical growth with exogenous fertility (Diamond, 1965) – that a positive relationship between longevity and pay-as-you-go (PAYG) pensions may exist independently of the size of the contribution rate burdening on the currently active generation (the young workers). We extend such a model to analyse how the balanced PAYG pension budget is affected by an increasing longevity in a fairly standard Diamond-style OLG model with endogenous fertility. It is shown that the positive relationship between longevity and pensions may be thought to be a robust feature of OLG economies. In particular, (1) the demand for children may either increase or decrease along with an increased longevity, though the latter is not very likely, and (2) the endogeneisation of fertility rates may strengthen or weaken the beneficial effects that the reduction in adult mortality plays on PAYG pensions, and this result depends exclusively on the size of the households' preference for raising children. Therefore, even in a context in which agents choose endogenously the number of children raised, there would be room for an increase, rather than the often threatened reduction, in future pensions by keeping unaltered the contribution rate to the PAYG scheme paid by current workers, and this holds especially in the case in which parents have a strong preference for having children.

Keywords Pensions; Fertility; OLG model

JEL Classification J26; O41

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1 Introduction

In practically all countries governments are facing with a very striking population ageing and, since old-age pensions are generally financed on a PAYG-basis, with expected severe PAYG budget problems.[1] In fact a widespread belief shared both by economists and policymakers is the negative relationship between the reduction in adult mortality and the balance of the PAYG pension system in developed economies, especially in European Union countries. As a consequence, on the one hand, both advocated pension reforms including policies surely painful either for old-aged pension beneficiaries or for young-aged contributors,[2] such as the compulsory reduction in the lengthening of the retirement period and/or a reduction in government pension benefits and/or even an increase in the contribution rates bearing upon the current active generation. On the other hand, the other main contributor to the viability of the PAYG-based pension budgets in developed economies, namely the productivity growth, seems to be stagnant especially in those countries most plagued by population ageing such as, for instance, Spain and Italy.[3]

A variety of papers (e.g., Pecchenino and Pollard, 2002 and 2005; van Groezen and Meijdam, 2008; Yakita, 2001; Zhang et al., 2001) have theoretically addressed the issue of the relationship between longevity and economic growth, in both exogenous and endogenous growth contexts with PAYG pensions, but only the latter three of whom by considering endogenous fertility of households. These papers share the result that such a relation may – in some cases – be positive, and in addition Yakita (2001) and Zhang et al. (2001) argued that fertility rates and longevity are negatively related, but they did not focus on the fact that pension benefits might be increasing with population ageing.

In this article we pose the following questions: (1) is the negative trade-off between longevity and the viability of the PAYG pension system – exemplified, for instance, by Pecchenino and Pollard (2005, p. 450) who claimed that "to maintain benefit levels, tax rates and/or productivity growth will have to rise" – really warranted from a theoretical point of view? (2) do pension reforms advocated by several economists and politicians really represent a policy need? The answer, somewhat surprisingly, is that "it depends on".

Moreover, differently from Fanti and Gori (2008) who argued that pension benefits might increase with an increased population ageing when fertility rates are exogenously given, we investigate whether and how the endogenously determined number of children raised by individuals strengthen or weaken the sustainability of the balanced PAYG pension budgets. For doing this our model is simply the textbook Diamond (1965) style OLG framework extended to include an exogenous rate of longevity. The analysis of our simple model yields the following results: (1) the relationship between fertility and longevity is either positive or negative, though the latter is not very likely. In fact, as shown in Proposition 1 in the paper, the positive relationship between a reduced adult mortality and an increased pension is more likely to occur. In particular, (1.1) if the distributive capital share and the contribution rate to the PAYG scheme are both low enough then the rate of fertility is an inverted U-shaped function of the rate of longevity, and thus the fertility rate decrease monotonically with longevity only when the latter is sufficiently high; (1.2) by contrast, if either the distributive capital share or the contribution rate is high enough, the family size monotonically increases with the rate of longevity. (2) when the distributive capital share is sufficiently high then an increasing longevity always increases

[1] To have an idea of the measure of the expected population ageing in developed economies it is sufficient to say that while the age dependency ratio (over 65 in total population) was for most countries around 20 per-cent in 1995, it will be around 67 per-cent in Italy, around 57 per-cent in Japan, and around 49 per-cent in the Western Europe in 2040 (see United Nations, 1998). It is commonly retained that this steadily rising share of old-aged people in total population may cause serious concerns as regards the sustainability of the PAYG pension budgets.

[2] A recent literature aimed to document whether such a common belief is also shared by young and old people and, in any case, how to have success – for instance through appropriate informational campaigns – in popularising it. For instance, Boeri et al. (2001, 2002), drawing on surveys of European citizens, and Blinder and Krueger (2004) studying opinion polls in the US, noted that more informed individuals are more likely to support pension reforms and advise a more operative "advertising campaign".

[3] In fact those countries displayed – in recent decades – the lowest fertility (largely below-replacement) rates as well as the lowest productivity growth rates (around zero or even negative).

pensions no matter the size of the contribution rate to the PAYG scheme paid by current workers. This result holds especially in the case in which individuals have a strong preference for having children.

Therefore, our results may constitute a policy warning suggesting that the striking population ageing could not be harmful for the PAYG system viability and, thus, some commonly invoked (painful) policy measures could be unnecessary.[4] It must be emphasised that our results have been obtained by using as parsimonious a model as possible, that is, the standard Diamond-style OLG model of neoclassical growth with endogenous fertility decisions of individuals.

The paper is organised as follows. In Section 2 we develop the model; in Section 3 (4) the main steady-state results as regards the relationship between longevity and fertility (pensions) are analysed and discussed. Section 5 presents a numerical illustration of our theoretical findings; section 6 winds up with some concluding comments.

2 The model

2.1 Government

The government runs a balanced PAYG pension budget in every period according to the following constraint:

$$\pi p_t = \theta_w w_t n_{t-1}, \tag{1}$$

where the left-hand side represents the social security expenditure and the right-hand side the tax receipts, with $0 < \pi < 1$ being the individuals' (exogenous) probability of surviving to the second period of life, $p_t > 0$ is the benefit entitled to pensioners at time t, $0 < \theta_w < 1$ is the (constant) contribution rate paid by the young-adult contributors (workers), w is the pre-tax competitive wage (tax base) received by the currently active people and n is the rate of fertility in the whole economy.[5]

2.2 Individuals

Identical agents (N_t) are assumed to belong to an overlapping generations structure with finite lifetimes. Life is separated among three periods: childhood, young adulthood and old-age. Individuals belonging to generation t have a homothetic and separable utility function defined over young-aged consumption $(c_{1,t})$, old-aged consumption $(c_{2,t+1})$ and the number of children raised (n_t), as in Galor and Weil (1996).[6] Each young individual supplies inelastically one unit of labour in the labour market, and receives wage income at the competitive rate w_t. We assume that raising children requires a fixed amount $e > 0$ of resources (measured in units of market goods). During old-age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}. Moreover, we suppose old individuals survive to the second period with (constant) probability $0 < \pi < 1$. Therefore, the existence of a perfect annuity market implies old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased.[7] Furthermore, each old-age individual is entitled to a publicly provided pension benefit (p_{t+1}) financed at balanced budget by the government.

Thus, the representative individual born at time t is faced with the following programme:

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[4] Therefore, to the question posed by Boeri and Tabellini (2005, p. 2) "Why is it so difficult to reform the unsustainable and overly generous European pension systems?", our results could answer that fortunately, in some cases, it could not be a necessary measure.

[5] Notice that agents are assumed to have perfect foresight with respect to the level of the future public pension benefit.

[6] Notice that n represents the number of children with $n-1$ being the population growth rate, as usual in models with endogenous fertility.

[7] We note that even in the absence of a perfect annuity market, and therefore where adult mortality gives raise to accidental bequests, the qualitative results of this paper remain unchanged.

$$\max_{\{s_t, n_t\}} U_t = \ln(c_{1,t}) + \pi\gamma \ln(c_{2,t+1}) + \phi \ln(n_t), \tag{P}$$

subject to

$$c_{1,t} + s_t = w_t(1-\theta_w) - e\,n_t$$
$$c_{2,t+1} = \frac{1+r_{t+1}}{\pi} s_t + p_{t+1} \quad ,$$

where $0 < \gamma < 1$ is the subjective discount factor and $0 < \phi < 1$ captures the importance in the welfare function of consuming while young relative to the utility of children.

Solving programme (P) and exploiting the government budget (1) gives the saving function and the demand for children chosen optimally by individuals, that is:

$$s_t = \frac{w_t(1-\theta_w)\left[\pi\gamma e - \phi\theta_w \frac{w_{t+1}}{1+r_{t+1}}\right]}{(1+\pi\gamma+\phi)e - \phi\theta_w \frac{w_{t+1}}{1+r_{t+1}}}, \tag{2}$$

$$n_t = \frac{\phi w_t(1-\theta_w)}{(1+\pi\gamma+\phi)e - \phi\theta_w \frac{w_{t+1}}{1+r_{t+1}}}. \tag{3}$$

2.3 Firms

As regards the production sector, we suppose firms are identical and act competitively in both input and output markets. The (aggregate) constant returns to scale Cobb-Douglas[8] technology of production is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$, where Y_t, K_t and $L_t = N_t$ are output, capital and the time-t labour input respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the capital's share on total output. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function may be written as $y_t = Ak_t^{\alpha}$. Assuming total depreciation of physical capital at the end of each period and knowing that final output is treated at unit price, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{4}$$

$$w_t = (1-\alpha) A k_t^{\alpha}. \tag{5}$$

2.4 Equilibrium

Given the government budget (1) and knowing that population evolves according to $N_{t+1} = n_t N_t$, the market-clearing condition in goods as well as in capital markets is expressed by the equality $n_t k_{t+1} = s_t$. Substituting out for s and n according to Eqs. (2) and (3), respectively, exploiting (4) and (5), and assuming individuals are perfect foresighted, the equilibrium sequence of capital boils down to the following long-run per-capita stock of capital:

$$k^*(\pi) = \frac{\pi\gamma e \alpha}{\phi[\alpha + \theta_w(1-\alpha)]}. \tag{6}$$

From Eq. (6) it can easily be seen that an increase in longevity always increases the long-run per-capita stock of capital, that is:

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[8] Notice that we have chosen this usual functional form for analytical tractability. However, by using for instance a more general CES specification, the qualitative results of this paper, as numerical simulations not reported here revealed, are preserved as well.

$$\frac{\partial k^*(\pi)}{\partial \pi} = \frac{\gamma e \alpha}{\phi[\alpha + \theta_w(1-\alpha)]} > 0 . \tag{7}$$

In the next two sections we will analyse how a reduced adult mortality affects both the long-run individuals' fertility decisions and the balanced PAYG pension budget.

3 Fertility and longevity in the long-run

In this simple Diamond-style OLG model of neoclassical growth interesting insights can be derived firstly by analysing the effects on an increasing life span on the long-run fertility decisions of individuals. How does an increase in longevity (i.e., a reduction in adult mortality) affect the incentive to have children? The seminal paper by Ehrlich and Lui (1991) found that an increasing longevity promotes economic growth through both an increase in human capital investment in children and a reduction in fertility. Recently Soares (2005) – by developing a model to explain the Demographic Transition – found that a reduction in adult mortality is still associated with a lower fertility and a higher educational attainment. Even if in this model we abstract from the problem to analysing the relationship between individuals' fertility and human capital, we found – in the basic OLG model of neoclassical growth with endogenous fertility – that gains in adult longevity may be associated with higher fertility rates even in presence of a publicly provided PAYG social security program, that is, in a case in which parents do not expect to be cared for by their children during the retirement period. We will see that this result depends exclusively on the mutual relationship between the labour tax paid by young contributors to fund PAYG pensions to retired people and the capital's weight in technology.

Given Eq. (3), the long-run fertility rate as a generic function of the rate of longevity is the following:

$$n^* = n^*\{\pi, w^*[k^*(\pi)], r^*[k^*(\pi)]\}. \tag{8}$$

Thus, the total derivative of (8) with respect to π yields:[9]

$$\frac{dn^*}{d\pi} = \underbrace{\overbrace{\frac{\partial n^*}{\partial \pi}}^{-} + \underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial \pi}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial n^*}{\partial r^*}}^{-} \cdot \overbrace{\frac{\partial r^*}{\partial k^*}}^{-} \cdot \overbrace{\frac{\partial k^*}{\partial \pi}}^{+}}_{+}}_{+/-} . \tag{9}$$

As it can easily be seen the final effect of an increasing probability to surviving to the second period of life on the long-run fertility decisions of households is the result of three counterbalancing forces, and it appears to be ambiguous. In particular, Eq. (9) shows there exists a negative impact effect which directly reduces the number of children raised. However, such a negative relation is counterbalanced by the (positive) general equilibrium feedback effect played by the rate of longevity on both the wage and the interest rates through the increased long-run per-capita stock of capital, that is, a reduction in adult mortality increases (reduces) the wage rate (the interest rate) and thus, given the positive (negative) relationship between fertility and the wage rate (the interest rate), there exists an income effect which increases the demand for children. Moreover, contrary to van Groezen and Meijdam (2008), who stated that "The [positive] general equilibrium effects may eventually dominate the direct negative effect, though this is not very likely", we find specific threshold values both for the contribution rate and the share of capital in production with respect to which the relationship between longevity and fertility is either positive or negative; furthermore, as we will see below (Proposition 1), the positive relationship is more likely to occur. Interestingly, we will see that the negative relationship between fertility and longevity will arise – for sufficiently low values of adult mortality – if and only if both the contribution rate to the PAYG scheme paid by current workers and the share of capital in production are low enough.

To analyse ultimately which of the forces dominates, we exploit Eqs. (3), (4), (5) and (6) to obtain the long-run fertility rate as a function of the rate of longevity and the key parameters of the model, that is:

[9] Details are given in Appendix A.

$$n^*(\pi)=\frac{\phi(1-\theta_w)[\alpha+\theta_w(1-\alpha)](1-\alpha)A\left\{\frac{\pi\gamma e\alpha}{\phi[\alpha+\theta_w(1-\alpha)]}\right\}^{\alpha}}{e[\alpha(1+\pi\gamma+\phi)+\theta_w(1-\alpha)(1+\phi)]}. \tag{10}$$

From thus Eq. (10) the following proposition holds:

Proposition 1. *Let* $\theta_w<\hat{\theta}$ *hold. Then*
(1) if $\alpha<\hat{\alpha}$, $n^*(\pi)$ *is an inverted U-shaped function of* π *with* $\pi=\pi_n(\theta_w)$ *being an interior global maximum;*
(2) if $\alpha>\hat{\alpha}$, *an increase in longevity always increases fertility;*
Let $\theta_w>\hat{\theta}$ *hold. Then an increase in longevity always increases fertility.*

Proof. The proof uses the following derivative:

$$\frac{\partial n^*(\pi)}{\partial\pi}=\frac{\phi(1-\theta_w)[\alpha+\theta_w(1-\alpha)]\alpha(1-\alpha)A\left\{\frac{\pi\gamma e\alpha}{\phi[\alpha+\theta_w(1-\alpha)]}\right\}^{\alpha}\cdot\{[\alpha+\theta_w(1-\alpha)](1+\phi)-(1-\alpha)\pi\gamma\}}{e\pi[\alpha(1+\pi\gamma+\phi)+\theta_w(1-\alpha)(1+\phi)]^2}.$$

Therefore,

$$\frac{\partial n^*(\pi)}{\partial\pi}\gtrless 0\Leftrightarrow\pi\lessgtr\pi_n(\theta_w),$$

where

$$\pi_n(\theta_w)\equiv\frac{(1+\phi)[\alpha+\theta_w(1-\alpha)]}{(1-\alpha)\gamma}, \tag{M1}$$

so that $\pi_n(\theta_w)<1$ if and only if $\alpha<\hat{\alpha}\equiv\frac{\gamma-\theta_w(1+\phi)}{\gamma+(1-\theta_w)(1+\phi)}$, with $\hat{\alpha}>0$ if and only if $\theta_w<\hat{\theta}\equiv\frac{\gamma}{1+\phi}$.[10] Thus, if $\theta_w<\hat{\theta}$ and $\alpha<\hat{\alpha}$ then $\pi=\pi_n(\theta_w)<1$ represents the fertility-maximising rate of longevity. By contrast, if $\alpha>\hat{\alpha}$ then $\pi_n(\theta_w)>1$ and $\frac{\partial n^*(\pi)}{\partial\pi}>0$ for any $0<\pi<1$. Finally, if $\theta_w>\hat{\theta}$ then $\pi_n(\theta_w)>1$ and $\frac{\partial n^*(\pi)}{\partial\pi}>0$ for any $0<\pi<1$ independently of the value of the distributive capital share. **Q.E.D.**

Proposition 1 reveals that the direct negative impact effect of an increasing longevity on the rate of fertility may prevail only in the case in which both the contribution rate and the capital's weight in production are small enough. In such a case, Proposition 1 affirms there exists a positive monotonic relationship among fertility and longevity up to the point in which the demand for children is maximised ($\pi=\pi_n(\theta_w)$). Beyond such a level the relationship between fertility and longevity turns out to be monotonically negative. If the capital's weight in production and/or the contribution rate are high enough then an increasing longevity is always associated with a higher fertility (that is, the positive general equilibrium income effect always prevails over the direct negative impact effect).

In the next section we will see how a reduction in adult mortality affects the PAYG-based pension benefits. The result is that a higher longevity may be associated with an increased pension received by the elderly by keeping unaltered the contribution rate paid by the young workers.

4 Pensions and longevity in the long-run

[10] Notice that $\hat{\alpha}<1/2$ always holds.

A stylised fact emerged in recent decades in all developed countries is the population ageing problem. Furthermore, the effects of the reduction in birth rates as well as the raise in longevity rates have been always viewed by policymakers as a damage challenging the sustainability of the widespread pension schemes paid on a PAYG basis. The common belief has even suggested that, in order to keep balanced the public pension budget, an increase in longevity must necessarily be associated with a lower pension benefit received during old-age by the elderly (retirement period) and/or a higher contribution rate paid when young (working life) and/or a lengthening of the age retirement. As shown by Fanti and Gori (2008) – in the basic OLG model of neoclassical growth with exogenous fertility – this view may be theoretically incorrect whatever the value of the contribution rate burdening on the young generation, that is, it may be possible to increase pensions (with the PAYG budget being kept unaltered) even with an increased rate of longevity, and this result depends exclusively on the size of the capital's share in production. The question posed here is to test for the robustness of this somewhat surprising result in the case in which the demand for children is endogenously determined by households.

Which are the effects of a reduced adult mortality on pension payments in an OLG model of neoclassical growth when individuals choose optimally the number of children raised? Does the endogenisation of fertility rates strengthen and weaken the results found by Fanti and Gori (2008)? In what follows we will show that even when fertility decisions are endogenously determined rather than exogenously given, the common belief which views population ageing as a threat for the sustainability of the balanced PAYG pension budgets may be reversed and thus the positive relationship between pensions and longevity may be thought to be a robust feature of OLG economies.

Given Eq. (1), the long-run pension benefit as a generic function of the rate of longevity may be written as:

$$p^* = p^*\{\pi, w^*[k^*(\pi)], n^*\{\pi, w^*[k^*(\pi)], r^*[k^*(\pi)]\}\}. \tag{11}$$

Therefore, totally differentiating Eq. (11) with respect to π gives:[11]

$$\frac{dp^*}{d\pi} = \underbrace{\overbrace{\frac{\partial p^*}{\partial \pi}}^{-} + \underbrace{\overbrace{\frac{\partial p^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial \pi}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial p^*}{\partial n^*}}^{+} \cdot \overbrace{\frac{dn^*}{d\pi}}^{+/-}}_{+/-}}_{+/-}. \tag{12}$$

Eq. (12) reveals that the final effect of an increase in longevity on the long-run pension payment depends on three counterbalancing forces, and it appears to be ambiguous: (1) a negative (direct) effect which tends to reduce pensions owing to the higher number of old-aged individuals in the economy, (2) a positive (indirect) general equilibrium feedback effect which acts on pensions through an increased wage rate (tax base) owing to an increased stock of capital per-capita. Given the positive relationship between PAYG pensions and wages, the higher the wage rate the higher the pension benefit received by each retiree, and (3) an ambiguous general equilibrium effect which affects pensions owing to the effect played by the adult mortality on the rate of fertility (as discussed in the previous section). Notice that when fertility decisions are endogenously determined, since the last addendum of the right-hand side of Eq. (12) may be positive or negative, then the positive relationship between pensions and longevity may be associated either with an increased or a reduced fertility depending on whether both the contribution rate and the capital's share in production are low or high enough, respectively (see Proposition 1). Moreover, we will show (see Proposition 3 below) that the beneficial effect of an increasing longevity on PAYG pensions found by Fanti and Gori (2008) with exogenous fertility may be either weakened or strengthened by the endogenisation of fertility rates, and this results depends exclusively on the size of the households' preference for children.

To analyse ultimately the effect of a reduction in adult mortality on PAYG pensions, we now combine Eqs. (1), (4), (5), (6) and (10) to obtain the following steady-state pension formula:

[11] Details are given in Appendix B.

$$p^*(\pi)=\frac{\phi\theta_w(1-\theta_w)[\alpha+\theta_w(1-\alpha)]\left\{(1-\alpha)A\left\{\frac{\pi\gamma e\alpha}{\phi[\alpha+\theta_w(1-\alpha)]}\right\}^{\alpha}\right\}^2}{e\pi[\alpha(1+\pi\gamma+\phi)+\theta_w(1-\alpha)(1+\phi)]}. \tag{13}$$

From Eq. (13) we have the following proposition:

Proposition 2. *For any given value of* θ_w *we have the following:*
(1) let $\alpha<1/2$ *hold. Then an increase in longevity always reduces pensions;*
(2) let $1/2<\alpha<\alpha_2$ *hold.* $p^*(\pi)$ *is an inverted U-shaped function of* π *with* $\pi=\pi_p(\theta_w)$ *being an interior global maximum;*
(3) let $\alpha_2<\alpha<1$ *hold. Then an increase in longevity always increases pensions.*

Proof. The proof uses the following derivative:

$$\frac{\partial p^*(\pi)}{\partial\pi}=\frac{\phi\theta_w(1-\theta_w)[\alpha+\theta_w(1-\alpha)]\left\{(1-\alpha)A\left\{\frac{\pi\gamma e\alpha}{\phi[\alpha+\theta_w(1-\alpha)]}\right\}^{\alpha}\right\}^2}{e\pi^2[\alpha(1+\pi\gamma+\phi)+\theta_w(1-\alpha)(1+\phi)]^2}\times$$

$$\{-(1+\phi)(1-2\alpha)[\alpha+\theta_w(1-\alpha)]-2\alpha(1-\alpha)\gamma\pi\}.$$

Therefore, if $\alpha<1/2$, then $\frac{\partial p^*(\pi)}{\partial\pi}<0$ for any $0<\pi<1$ and the higher is rate of longevity the lower is the pension benefit received by each retiree. If $\alpha>1/2$, then

$$\frac{\partial p^*(\pi)}{\partial\pi}\gtrless 0\Leftrightarrow\pi\lessgtr\pi_p(\theta_w),$$

where

$$\pi_p(\theta_w)\equiv\frac{(1+\phi)(2\alpha-1)[\alpha+\theta_w(1-\alpha)]}{2\alpha(1-\alpha)\gamma}>0. \tag{M2}$$

Therefore, $\pi_p(\theta_w)<1$ if and only if:

$$-\Lambda_1\alpha^2+\Lambda_2\alpha+\Lambda_3>0, \tag{14}$$

where $\Lambda_1\equiv 2[\gamma+(1+\phi)(1-\theta_w)]>0$, $\Lambda_2\equiv 2\gamma+(1+\phi)(1-3\theta_w)$ and $\Lambda_3\equiv\theta_w(1+\phi)>0$. Since $\Delta_p(\theta_w)\equiv\Lambda_2^{\,2}+4\Lambda_1\Lambda_3>0$, then by applying the Descartes' rule of sign we find that, independently of the sing of Λ_2, there always exist two real roots $\alpha_1\equiv\frac{\Lambda_2-\sqrt{\Delta_p(\theta_w)}}{2\Lambda_1}<0$ and $\alpha_2\equiv\frac{\Lambda_2+\sqrt{\Delta_p(\theta_w)}}{2\Lambda_1}>0$ which solve (14).[12] Since $\alpha_1<0$ it is automatically ruled out. Thus, if $1/2<\alpha<\alpha_2$ then $\pi=\pi_p(\theta_w)<1$ represents the pension-maximising rate of longevity. Finally, $\alpha_2<\alpha<1$ implies $\frac{\partial p^*(\pi)}{\partial\pi}>0$ for any

[12] Notice that $1/2<\alpha_2<1$. The first inequality in fact implies $\sqrt{\Delta_p(\theta_w)}>\Lambda_1-\Lambda_2$ with $\Lambda_1-\Lambda_2>0$. Therefore, we can write $\Delta_p(\theta_w)>(\Lambda_1-\Lambda_2)^2$. Using the definition of $\Delta_p(\theta_w)$, Λ_1, Λ_2 and Λ_3 and rearranging terms, we get $2\gamma>0$. Therefore, α_2 is always higher than $1/2$. The second inequality yields $\sqrt{\Delta_p(\theta_w)}<2\Lambda_1-\Lambda_2$ with $2\Lambda_1-\Lambda_2>0$. Therefore, we can write $\Delta_p(\theta_w)<(2\Lambda_1-\Lambda_2)^2$, which implies $1>0$, so that α_2 is always smaller than unity.

$0 < \pi < 1$ and thus the higher is longevity the higher is the pension benefit whatever the value of the adult mortality and the contribution rate burdening on young-aged individuals. **Q.E.D.**

Proposition 2 reveals that the pension payment received by each retiree is negatively linked with longevity if and only if the capital's share in production is low enough ($\alpha < 1/2$). On the contrary, when $\alpha > 1/2$, then, an increase in longevity may increase pensions and in particular, for moderate values of the capital share in production,[13] that is, $1/2 < \alpha < \alpha_2$, there exists a pension-maximising longevity rate, that is $\pi = \pi_p(\theta_w)$. Finally, if the capital's share in technology is sufficiently high, then the higher longevity the higher pension payments independently of the value of the contribution rate, that is the indirect general equilibrium effect of an increasing longevity which acts positively on wages through an increased capital accumulation always dominates over the negative direct effects which tends to reduce pensions owing the lower number of old-aged people deceased. This result suggests that the idea that population ageing is always detrimental for pension benefits paid to retired people is not warranted from a theoretical point of view. In fact, the policymaker may even reduce the contribution rate by keeping the PAYG pension budget balanced in every period of time as the number of old-aged individuals increase.

Since for any $\alpha > \hat{\alpha}$ there exists a positive monotonic relationship between fertility and longevity and $\hat{\alpha} < 1/2$, then for any $\alpha > 1/2$ the higher is longevity the is higher the households' demand for children. As a corollary one may conclude that Proposition 2 also reveals that for any $\alpha > 1/2$, that is, in the case in which $p^*(\pi)$ is either an inverted U-shaped function ($1/2 < \alpha < \alpha_2$) or a positive monotonic function ($\alpha_2 < \alpha < 1$) of the rate of longevity, the endogenisation of fertility is always associated with an increased number of contributors to the PAYG scheme, that is, the last term of the right-hand side of Eq. (12) is positive. Moreover, in the case in which pensions and longevity are negatively related ($\alpha < 1/2$), such a negative relationship is counterweighted by the increased rate of fertility when either $\hat{\alpha} < \alpha < 1/2$ or $\theta_w > \hat{\theta}$, or both hold, as the last term of the right-hand side of Eq. (12) still remains positive.

The reduction in PAYG pensions owing to an increased longevity is strengthened by the reduction in the demand for children only when both the contribution rate and the capital's share in production are low enough (see point 1 in Proposition 1). In such a case thus, since for high values of π the relationship between longevity and fertility is found to be negative, the reduction in PAYG pensions due to a reduced adult mortality is associated with a reduced number of contributors to the PAYG scheme (that is, the last term of the right-hand side of Eq. 12 is negative).

In the next proposition we compare the effects of an increasing longevity on the viability of the balanced PAYG pension budget in both exogenous (see Fanti and Gori, 2008, for a review) and endogenous fertility economies. In particular, the beneficial effects of a reduced adult mortality on PAYG pensions found by Fanti and Gori (2008) in an exogenous fertility setting may be weakened or strengthened by the endogenisation of households' fertility behaviour depending on whether the preference for children is low or high enough, respectively. In fact, in the former case, the pension-maximising rate of longevity in an endogenous fertility economy is found to be always lower than the corresponding value in an exogenous fertility setting; that is, the maximisation of PAYG pensions occurs at a lower value of the rate of longevity implying that a reduction in adult mortality makes easily

[13] In order to better clarify the meaning of the coefficient α (the capital's share in production), it is worth noting that a possible interpretation is that the capital stock may be thought in its broad concept, including thus physical and human components, and that the labour input only includes non-specialised labour. In fact, as argued by Mankiw et al. (1992, p. 417), if we take a broad view of capital, then the physical capital's share of income is expected to be about 1/3 and the human capital's share of income should be between 1/3 and one half. In sum, the coefficient α may be fairly about 0.6 and 0.8. Indeed, for instance, Barro and Sala-i-Martin (2003, p. 110) used $\alpha = 0.75$ saying that: "Values in the neighbourhood of 0.75 accord better with the empirical evidence, and these high values of α are reasonable if we take a broad view of capital to include human components".

– when fertility is endogenously determined rather than exogenously given – to turn to a negative monotonic relationship between pensions and longevity.

Proposition 3. *Let* $\phi < 1$ *(*$\phi > 1$*) hold. Then the pension-maximising rate of longevity in an endogenous fertility economy is always lower (higher) than the pension-maximising rate of longevity in an exogenous fertility economy.*

Proof. From Proposition 2 we found that the pension-maximising rate of longevity in the case of endogenous fertility is given by:

$$\pi_{p,END}(\theta_w) \equiv \frac{(1+\phi)(2\alpha-1)[\alpha+\theta_w(1-\alpha)]}{2\alpha(1-\alpha)\gamma},$$

whereas in the case of exogenous fertility (see, Fanti and Gori, 2008, Proposition 1) it is represented by:

$$\pi_{p,EXO}(\theta_w) \equiv \frac{(2\alpha-1)[\alpha+\theta_w(1-\alpha)]}{\alpha(1-\alpha)\gamma}.$$

Therefore,

$$\pi_{p,END}(\theta_w) \equiv \frac{1+\phi}{2}\pi_{p,EXO}(\theta_w). \tag{15}$$

From (15) it can easily be seen that if $\phi < 1$ ($\phi > 1$) then $\pi_{p,END}(\theta_w) < \pi_{p,EXO}(\theta_w)$ ($\pi_{p,END}(\theta_w) > \pi_{p,EXO}(\theta_w)$). **Q.E.D.**

Proposition 3 presents a clear-cut result when comparing the effects of an increasing life span both in exogenous and endogenous fertility economies. Interestingly, it argues that if the parents' preference for children is not too high the beneficial effect of an increasing longevity on the viability of the balanced PAYG pension budget is always weakened by the endogenisation of fertility whatever the value of the adult mortality, that is, it is more likely that an increasing life span is associated with lower pensions, *ceteris paribus* as regards the other parameters of the model. By contrast, if parents assign a higher relative importance to raise children than to their own material consumption the result is dramatically reversed with the pension-maximising rate of fertility in an endogenous fertility economy being higher than the corresponding value when fertility is exogenously given. In this latter case, therefore, it is more likely that an increasing life span is associated with higher pensions, *ceteris paribus* as regards the other parameters of the model.

5 A numerical illustration

An example, chosen only for illustrative purposes, of the results stated in Propositions 1 and 2 is summarised in the following Tables 1 and 2. We take the following parameter values: $A = 10$ (simply a scale parameter in the Cobb-Douglas production function), $\gamma = 0.30$ (as in de la Croix and Michel, 2002, p. 50), and $\theta_w = 0.40$ (the contribution rate paid workers), $\phi = 0.30$ (the preference for having children). This parameter set generates $\alpha_2 = 0.5756$. As regards the value of the capital's share in production note that, as observed by Jones (2003, p. 8), countries such as Italy and Spain, which are strongly plagued by population ageing problems, show a value of α (without self-employment correction) between 0.5 and 0.6.[14] Therefore, in the following Tables 1 and 2 we used $\alpha = 0.56$ and $\alpha = 0.60$, respectively, to illustrate the effects of an increasing longevity on both fertility and pensions as stated in Propositions 1 and 2. In particular, in Table 1 we show there exists a pension-maximising value of the rate of longevity, whereas in Table 2 we illustrate the case in which the relationship between longevity and pensions is found to be monotonically positive. It must be noted that the

[14] Notice that in both tables the fixed amount of resources needed to take care of children has been assumed to be from around the 35 per-cent to around 10 per-cent of the steady-state wage rate, that is, as regards Table 1 (Table 2) $e = 0.15$ ($e = 0.05$).

beneficial effect of longevity on pensions is paralleled by a monotonic enhancement of the demand for children either.

Table 1. Effects of an increasing longevity on pension payments. Case $1/2 < \alpha < \alpha_2$ ($\alpha = 0.56$ and $\pi_p(\theta_w) = 0.7766$).

π	$n^*(\pi)$	$p^*(\pi)$
0.10	0.3260	0.4687
0.20	0.4725	0.5007
0.30	0.5830	0.5169
0.40	0.6737	0.5263
0.50	0.7511	0.5318
0.60	0.8186	0.5350
0.70	0.8785	0.5364
$\pi_p(\theta_w) = 0.7766$	0.9200	0.5367
0.85	0.9569	0.5365
0.90	0.9806	0.5361
0.99	1.0204	0.5350

Table 2. Effects of an increasing longevity on pension payments. Case $\alpha_2 < \alpha < 1$ ($\alpha = 0.60$ and $\pi_p(\theta_w) = 1.3722$).

π	$n^*(\pi)$	$p^*(\pi)$
0.10	0.3929	0.2271
0.20	0.5851	0.2563
0.30	0.7334	0.2731
0.40	0.8568	0.2844
0.50	0.9632	0.2924
0.60	1.0569	0.2983
0.70	1.1406	0.3026
0.80	1.2160	0.3059
0.90	1.2847	0.3083
0.99	1.3414	0.3098

6 Conclusions

In practically all countries governments – especially in developed economies – are facing with a very striking population ageing as birth rates have fallen and longevity has risen dramatically in recent times, and thus the fraction of current active individuals in total population is steadily declining. Since old-age pensions are generally financed on a PAYG-basis, policymakers will expect severe PAYG budget challenges in the near future since such a negative trend poses a threat on the sustainability of the public pension budgets. Moreover, both economists and policymakers share the belief that reforming the social security system in such a way that the viability of the PAYG scheme becomes primal in the political agenda, represents a necessary policy measure. However, these reforms will surely be painful either for old-aged beneficiaries or for young-aged contributors as they tend to increase the length of the working life and/or to lower the publicly provided pension benefits, or even to augment the contribution rates bearing upon the current active people.

In this paper we investigated how an increasing longevity affects the balanced PAYG pension budget by adopting the fairly standard Diamond-style OLG model of neoclassical growth augmented to account for endogenous fertility and a balanced PAYG pension budget. Interestingly, starting from the recent paper by Fanti and Gori (2008), we showed that a positive relationship between a reduced adult mortality and PAYG pensions may represent a robust feature of OLG economies, provided that the distributive capital share is sufficiently high. As a consequence, the common belief arguing that the increased longevity poses a threat to the PAYG pension system viability is not really warranted from a theoretical point of view. Moreover, we proved that a reduced adult mortality may promote population growth, thus weakening the process of population ageing. In particular, the higher the capital's share in production, the more likely an increase in longevity favours a fertility enhancement. As an illustrative example, we displayed that countries strongly plagued by population ageing and endowed with a high distributive capital share, such as for instance Italy and Spain, may obtain – along with an increasing longevity – both higher fertility rates and PAYG pension payments, thus increasing the number of contributors to the publicly provided pension system and avoiding the need to reform drastically and painfully the social security system. The policy implication is that the strongly advocated pension reforms aiming to keep balanced the PAYG pension budget (e.g., the compulsory reduction in the lengthening of the retirement period and/or a reduction in pension benefits received by old-aged individuals, or even an increase in the contribution rates paid by the young) might not be essential. To put this in another way, our findings show that when the family size is endogenously determined by households and parents are sufficiently children-interested, there could be, a "fortiori", room for an increase in future pensions in many countries when longevity increases by keeping unaltered the tax rate paid by the young contributors to provide pensions to the elderly at balanced budget.

An interesting extension of the present model could be to let the rate of longevity being determined endogenously rather than exogenously given and/or to study the interaction between child and adult mortality on both household fertility and the viability on an unfunded social security programme.

Appendix A

The effects of a marginal increase in the rate of longevity on the long-run fertility rate are:

$$\frac{\partial n^*}{\partial \pi} = \frac{-\phi\gamma e(1-\theta_w)w^*(1+r^*)^2}{\left[(1+\pi\gamma+\phi)e(1+r^*)-\phi\theta_w w^*\right]^2} < 0, \tag{A1}$$

$$\frac{\partial n^*}{\partial w^*} = \frac{\phi e(1-\theta_w)(1+\pi\gamma+\phi)(1+r^*)^2}{\left[(1+\pi\gamma+\phi)e(1+r^*)-\phi\theta_w w^*\right]^2} > 0, \tag{A2}$$

$$\frac{\partial n^*}{\partial r^*} = \frac{-\phi^2\theta_w(1-\theta_w)(w^*)^2}{\left[(1+\pi\gamma+\phi)e(1+r^*)-\phi\theta_w w^*\right]^2} < 0, \tag{A3}$$

$$\frac{\partial w^*}{\partial k^*} = \alpha(1-\alpha)A(k^*)^{\alpha-1} > 0, \tag{A4}$$

$$\frac{\partial r^*}{\partial k^*} = -\alpha(1-\alpha)A(k^*)^{\alpha-2} < 0. \tag{A5}$$

Appendix B

The effects of a marginal increase in the rate of longevity on the long-run PAYG pension benefit are:

$$\frac{\partial p^*}{\partial \pi} = \frac{-\theta_w w^* n^*}{\pi^2} < 0, \tag{B1}$$

$$\frac{\partial p^*}{\partial w^*} = \frac{\theta_w n^*}{\pi} > 0, \tag{B2}$$

$$\frac{\partial p^*}{\partial n^*} = \frac{\theta_w w^*}{\pi} > 0. \tag{B3}$$

Appendix C

An alternative way of financing the PAYG system

We examine here how an increasing lifespan affects both fertility and pension payments when the PAYG pension budget is funded entirely with a contribution rate burdening on firms. Therefore, the government budget (1) becomes:

$$\pi p_t = \theta_f w_t n_{t-1}, \tag{1'}$$

where $\theta_f > 0$ is the (constant) contribution rate to the PAYG scheme paid by each single firm in the economy and w is the after-tax competitive wage (i.e., the wage net of the contribution rate paid by firms) received by current workers.

The individual maximisation of utility (P) subject to the lifetime budget constraint

$$c_{1,t} + \frac{\pi c_{2,t+1}}{1+r_{t+1}} = w_t - e n_t + \frac{\pi p_{t+1}}{1+r_{t+1}},$$

gives the saving function as well as the demand for children, that is:

$$s_t = \frac{w_t \left[\pi \gamma e - \phi \theta_f \frac{w_{t+1}}{1+r_{t+1}} \right]}{(1+\pi\gamma+\phi)e - \phi\theta_f \frac{w_{t+1}}{1+r_{t+1}}}, \tag{2'}$$

$$n_t = \frac{\phi w_t}{(1+\pi\gamma+\phi)e - \phi\theta_f \frac{w_{t+1}}{1+r_{t+1}}}. \tag{3'}$$

As regards the production sector, the representative firm is now faced with

$$\max_{\{K_t, L_t\}} \pi_t = A K_t^{\alpha} L_t^{1-\alpha} - (1+r_t)K_t - w_t(1+\theta_f)L_t,$$

where the total labour cost is determined by the net hourly wage paid by firms to workers plus the amount of resources needed – by the government – to finance PAYG pensions to the old beneficiaries. Therefore, the marginal conditions for capital and labour are respectively given by (4) and

$$w_t = \frac{(1-\alpha)A}{1+\theta_f} k_t^{\alpha}. \tag{5'}$$

In order to close the model we exploit (1')-(3'), (4) and (5') to get the following long-run stock of capital per person:

$$k^*(\pi) = \frac{\pi \gamma e \alpha (1+\theta_f)}{\phi(\alpha+\theta_f)}. \tag{6'}$$

From Eq. (6') it can easily be seen that an increase in longevity always increases the long-run per-capita stock of capital, that is:

$$\frac{\partial k^*(\pi)}{\partial \pi} = \frac{\gamma e \alpha (1+\theta_f)}{\phi(\alpha+\theta_f)} > 0. \tag{7'}$$

In what follows we will analyse how a reduced adult mortality affects both the long-run individual fertility and the balanced PAYG pension budget when the social security system is funded entirely by a contribution rate levied on firms.

The long-run rate of fertility and the PAYG pension formula as a function of the rate of longevity and the basic parameters of the model in the case in which only firms contribute to fund the pension system – with current workers being untaxed – are respectively given by:

$$n^*(\pi)=\frac{\phi(\alpha+\theta_f)(1-\alpha)A\left[\frac{\pi\gamma e\alpha(1+\theta_f)}{\phi(\alpha+\theta_f)}\right]^{\alpha}}{(1+\theta_f)e[\alpha\pi\gamma(1+\theta_f)+(1+\phi)(\alpha+\theta_f)]}, \qquad (10')$$

$$p^*(\pi)=\frac{\phi\theta_f(\alpha+\theta_f)\left\{(1-\alpha)A\left[\frac{\pi\gamma e\alpha(1+\theta_f)}{\phi(\alpha+\theta_f)}\right]^{\alpha}\right\}^2}{e\pi(1+\theta_f)^2[\alpha\pi\gamma(1+\theta_f)+(1+\phi)(\alpha+\theta_f)]}. \qquad (13')$$

From (10') and (13') the following propositions hold:

Proposition 4. *Let* $\theta_f<\hat{\hat{\theta}}$ *hold. Then*

(1) if $\alpha<\hat{\hat{\alpha}}$, $n^*(\pi)$ *is an inverted U-shaped function of* π *with* $\pi=\pi_n(\theta_f)$ *being an interior global maximum;*

(2) if $\alpha>\hat{\hat{\alpha}}$, *an increase in longevity always increases fertility;*

Let $\theta_f>\hat{\hat{\theta}}$ *hold. Then an increase in longevity always increases fertility.*

Proof. The proof uses the following derivative:

$$\frac{\partial n^*(\pi)}{\partial\pi}=\frac{\phi(\alpha+\theta_f)\alpha(1-\alpha)A\left[\frac{\pi\gamma e\alpha(1+\theta_f)}{\phi(\alpha+\theta_f)}\right]^{\alpha}\cdot[(1+\phi)(\alpha+\theta_f)-\pi\gamma(1-\alpha)(1+\theta_f)]}{e\pi(1+\theta_f)[\alpha\pi\gamma(1+\theta_f)+(1+\phi)(\alpha+\theta_f)]^2}.$$

Therefore,

$$\frac{\partial n^*(\pi)}{\partial\pi}\gtrless 0\Leftrightarrow\pi\lessgtr\pi_n(\theta_f),$$

where

$$\pi_n(\theta_f)\equiv\frac{(1+\phi)(\alpha+\theta_f)}{\gamma(1-\alpha)(1+\theta_f)}, \qquad (M1')$$

so that $\pi_n(\theta_f)<1$ if and only if $\alpha<\hat{\hat{\alpha}}\equiv\frac{\gamma-\theta_f(1+\phi-\gamma)}{1+\phi+\gamma(1+\theta_f)}$, with $\hat{\hat{\alpha}}>0$ if and only if $\theta_f<\hat{\hat{\theta}}\equiv\frac{\gamma}{1+\phi-\gamma}$.[15] Thus, if $\theta_f<\hat{\hat{\theta}}$ and $\alpha<\hat{\hat{\alpha}}$ then $\pi=\pi_n(\theta_f)<1$ represents the fertility-maximising rate of longevity. If $\alpha>\hat{\hat{\alpha}}$ then $\pi_n(\theta_f)>1$ and $\frac{\partial n^*(\pi)}{\partial\pi}>0$ for any $0<\pi<1$. Finally, if $\theta_f>\hat{\hat{\theta}}$ then $\pi_n(\theta_f)>1$ and $\frac{\partial n^*(\pi)}{\partial\pi}>0$ for any $0<\pi<1$ independently of the value of the distributive capital share. **Q.E.D.**

Proposition 5. *For any given value of* θ_f *we have the following:*

(1) let $\alpha<1/2$ *hold. Then an increase in longevity always reduces pensions;*

(2) let $1/2<\alpha<\alpha_4$ *hold.* $p^*(\pi)$ *is an inverted U-shaped function of* π *with* $\pi=\pi_p(\theta_f)$ *being an interior global maximum;*

(3) let $\alpha_4<\alpha<1$ *hold. Then an increase in longevity always increases pensions.*

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[15] Notice that $\hat{\hat{\alpha}}<1/2$ always holds.

Proof. The proof uses the following derivative:

$$\frac{\partial p^*(\pi)}{\partial \pi}=\frac{\phi\theta_f(\alpha+\theta_f)\left\{(1-\alpha)A\left[\frac{\pi\gamma e\alpha(1+\theta_f)}{\phi(\alpha+\theta_f)}\right]^{\alpha}\right\}^2\{-(1+\phi)(1-2\alpha)(\alpha+\theta_f)-2\alpha(1-\alpha)(1+\theta_f)\gamma\pi\}}{e\pi^2(1+\theta_f)^2[\alpha\pi\gamma(1+\theta_f)+(1+\phi)(\alpha+\theta_f)]^2}.$$

Therefore, if $\alpha<1/2$, then $\frac{\partial p^*(\pi)}{\partial \pi}<0$ for any $0<\pi<1$ and the higher is longevity the lower is the pension benefit received by each retiree. If $\alpha>1/2$, then

$$\frac{\partial p^*(\pi)}{\partial \pi}\underset{<}{>}0 \Leftrightarrow \pi \underset{>}{<}\pi_p(\theta_f),$$

where

$$\pi_p(\theta_f)\equiv\frac{(1+\phi)(2\alpha-1)(\alpha+\theta_f)}{2\alpha(1-\alpha)\gamma(1+\theta_f)}>0. \qquad \text{(M2')}$$

Therefore, $\pi_p(\theta_f)<1$ if and only if

$$-\mathrm{B}_1\alpha^2+\mathrm{B}_2\alpha+\mathrm{B}_3>0, \qquad \text{(14')}$$

where $\mathrm{B}_1\equiv 2[1+\phi+\gamma(1+\theta_f)]>0$, $\mathrm{B}_2\equiv(1+\phi)(1-2\theta_f)+2\gamma(1+\theta_f)$ and $\mathrm{B}_3\equiv\theta_f(1+\phi)>0$. Since $\Delta_p(\theta_f)\equiv \mathrm{B}_2{}^2+4\mathrm{B}_1\mathrm{B}_3>0$, then by applying the Descartes' rule of sign we find that, independently of the sing of B_2, there always exist two real roots $\alpha_3\equiv\frac{\mathrm{B}_2-\sqrt{\Delta_p(\theta_f)}}{2\mathrm{B}_1}<0$ and $\alpha_4\equiv\frac{\mathrm{B}_2+\sqrt{\Delta_p(\theta_f)}}{2\mathrm{B}_1}>0$ which solve (14').[16] Since $\alpha_3<0$ it is automatically ruled out. Thus, if $1/2<\alpha<\alpha_4$ then $\pi=\pi_p(\theta_f)<1$ represents the pension-maximising rate of longevity in the case in which PAYG pensions are financed only by a contribution rate burdening on firms. Finally, if $\alpha_4<\alpha<1$ then $\frac{\partial p^*(\pi)}{\partial \pi}>0$ for any $0<\pi<1$ and thus the higher is longevity the higher is the pension benefit no matter the size of the contribution rate burdening on firms. **Q.E.D.**

Appendix D

How pension benefits are affected by a marginal increase in longevity if the PAYG budget is balanced with a contribution rate burdening entirely either on current workers or upon firms? By looking at the PAYG accounting rules (1) and (1'), and assuming the contribution rates to the PAYG budget to be exactly the same, it seems that apparently (i.e., without considering both the partial economic equilibrium and the general economic equilibrium effects of a marginal increase in longevity) the pension payments received by the elderly are identically affected by a higher life span independently of whether the PAYG budget is funded entirely by workers or entirely firms. However, since an increased longevity affects differently both the wage and the rate of fertility when pensions are funded either by

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[16] Notice that $1/2<\alpha_4<1$. The first inequality in fact implies $\sqrt{\Delta_p(\theta_f)}>\mathrm{B}_1-\mathrm{B}_2$ with $\mathrm{B}_1-\mathrm{B}_2>0$. Therefore, we can write $\Delta_p(\theta_f)>(\mathrm{B}_1-\mathrm{B}_2)^2$. Using the definition of $\Delta_p(\theta_f)$, B_1, B_2 and B_3 and rearranging terms, we get $2\gamma(1+\theta_f)>0$. Therefore, α_4 is always higher than $1/2$. The second inequality yields $\sqrt{\Delta_p(\theta_f)}<2\mathrm{B}_1-\mathrm{B}_2$ with $2\mathrm{B}_1-\mathrm{B}_2>0$. Therefore, we can write $\Delta_p(\theta_f)<(2\mathrm{B}_1-\mathrm{B}_2)^2$, which implies $\theta_f>-1$, so that α_4 is always smaller than unity.

workers or by firms, both in the short and the long run, then a higher life span can be more or less effective to uplift long-run PAYG pensions depending on the type of taxation used.

The following proposition compares how longevity affects fertility and PAYG pensions under the two different contributions we adopted to fund social security. Interestingly, both household fertility and PAYG pensions are differently affected by a marginal increase in longevity depending on the mutual relationship between the contribution rates to the PAYG scheme paid either by workers or by firms.

Define

$$\tilde{\theta}_f \equiv \frac{\theta_f}{1+\theta_f}.$$

then we have

Proposition 6. *Let the size of the contribution rate to the PAYG scheme paid by workers relative to that paid by firms be such that* $\theta_w < \tilde{\theta}_f$ ($\theta_w > \tilde{\theta}_f$)*. Then both the fertility-maximising and the pension-maximising rates of longevity in an endogenous fertility economy with the PAYG scheme being financed exclusively with a contribution rate levied on current workers are always lower (higher) than the corresponding values in the case in which the PAYG scheme is funded entirely with a contribution rate burdening on firms.*

Proof. Comparison of (M1) versus (M1') (M2 versus M2') gives $\pi_n(\theta_w) > \pi_n(\theta_f)$ ($\pi_p(\theta_w) > \pi_p(\theta_f)$) if and only if

$$\alpha\lfloor\theta_f - \theta_w(1+\theta_f)\rfloor > \theta_f - \theta_w(1+\theta_f).$$

If $\theta_w < \tilde{\theta}_f$ ($\theta_w > \tilde{\theta}_f$) then $\theta_f - \theta_w(1+\theta_f) > 0$ ($\theta_f - \theta_w(1+\theta_f) < 0$) and thus $\pi_p(\theta_w) < \pi_p(\theta_f)$ ($\pi_p(\theta_w) > \pi_p(\theta_f)$) always holds. **Q.E.D.**

Proposition 6 states that shifting the contribution rate used to fund the PAYG scheme from firms to workers promotes the beneficial effects of a reduced adult mortality on both fertility and PAYG pensions, other things being unchanged. This result holds because, given the same level of taxation, if the PAYG budget is balanced by firms' contribution then the wage received by the young workers will be smaller than that obtained in the case in which the PAYG budget is balanced by workers' contribution. Therefore, a higher life span – by uplifting both capital per person and wages – tends to be much more effective when the contribution to the PAYG scheme paid by workers is sufficiently high.[17]

In the following table (constructed using the same parameter set than that of Table 1) we will illustrate how an increasing longevity affects pension payments ceteris paribus as regards the size of the contribution rates to the PAYG system paid by workers and firms, i.e., $\theta_w = \theta_f = 0.40$. Notice that this is a special case of Proposition 6; in particular, assuming the contribution rate to the PAYG scheme paid by firms to be smaller than or equal to unity, this hypothesis implies that $\theta_w > \tilde{\theta}_f$ always holds.

We take the same parameter values than that used in Table 1, so that $\theta_w = \theta_f = 0.40$.

Table 3. Effects of an increasing longevity on pension payments under the same workers' and firms' contribution rates.

π	$p^*(\pi)$ Case $\theta_w > 0$ and $\theta_f = 0$	$p^*(\pi)$ Case $\theta_f > 0$ and $\theta_w = 0$
0.10	0.4687	0.4309
0.20	0.5007	0.4597

[17] Note that if $\theta_w = \tilde{\theta}_f$ then a marginal increase in longevity affects identically both fertility and PAYG pensions in the long run.

0.30	0.5169	0.4741
0.40	0.5263	0.4821
0.50	0.5318	0.4867
0.60	0.5350	0.4890
0.70	0.5364	0.4898
$\pi_p(\theta_f) = 0.7235$	0.5366	0.4899
$\pi_p(\theta_w) = 0.7766$	0.5367	0.4897
0.85	0.5365	0.4892
0.90	0.5361	0.4886
1	0.5348	0.4869

Table 3 shows that the rather unexpected beneficial effect of longevity on pensions, that is, the more individuals live longer the higher is the pension payment received by each pensioner in the long-run, holds in both cases of workers' and firms' contribution to the balanced PAYG scheme. However, given that $\theta_w = \theta_f$ implies $\theta_w > \tilde{\theta}_f$ then we may conclude that a higher life span is much more effective to enhance pensions when the PAYG budget is funded entirely by workers rather than entirely by firms, *ceteris paribus* as regards the size of the contribution rates.

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