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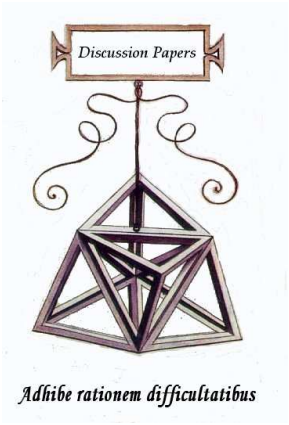


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# On economic growth and minimum wages

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**Abstract** We offer an analysis of the existence of a positive relationship between minimum wages and economic growth in a fairly standard general equilibrium, one-sector, two-period overlapping generations model, where the usual Romer-typed knowledge spill-over mechanism in production represents the engine of endogenous growth. It is shown that – contrary to the conventional view which has failed to pay due attention to dynamic contexts with labour market rigidities – the minimum-wage economy may grow faster than the competitive-wage economy in spite of a reduced employment rate and, in particular, a growth-maximising minimum wage does exist. A straightforward message is therefore that policymakers may appropriately use minimum wage policies to promote economic growth and individuals' welfare.

**Keywords** Endogenous growth; Minimum wage; Unemployment; OLG model

**JEL Classification** H24; J60; O41

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## 1. Introduction

An important and largely debated argument in the economic literature concerns the effects of minimum wages in aggregate macroeconomic models both in static and dynamic contexts. In particular, as regards the causes and consequences of labour market rigidities, policy debates around the effects of legislated wage minima traditionally have focused on two themes. Opponents argued that binding minimum wages negatively affect employment and output and thus they are viewed mainly as bad social policies. Proponents, instead, typically did not deal with employment effects; rather, they focused on redistributive goals that minimum wages might imply trading off thus between efficiency and equity.

Our knowledge of the effects of minimum wages in aggregate macroeconomic models has been improved firstly by the ancient and pioneering Stigler (1946). The basic (one-sector, static, partial equilibrium) model of minimum wage effects on employment and unemployment focuses on a single competitive labour market with homogeneous workers (all covered by the legislated wage) whose wage income would otherwise fall below the legally set minimum wage, i.e., a traditional neoclassical model of labour supply and demand (as that one hypothesised in this paper but assuming a dynamic context). Once the minimum wage has been legally set above the level that clears the labour market, employment falls (or, alternatively, unemployment arises) due to the excess supply of labour represented by the number (or work-hours) of those persons who are actually unemployed. Simple static neoclassical models, thus, clearly predict that the imposition of wage minima will lead those workers whose productivity levels are below the minimum wage being laid off: that is, introducing minimum wages in an otherwise competitive labour market will result (without doubts) in job losses and output reduction.

As regards the effects of legislated wage minima in dynamic contexts, the long-run relationship between the unemployment created by the minimum wage and the rate of per-capita income growth is essentially negative both in closed (Cahuc and Michel, 1996) and open (Irmen and Wigger, 2006) economies. In particular, Cahuc and Michel (1996)[1] considered a two-sector economy with a Lucas-typed externality in production and also added a positive externality of minimum wages to induce unskilled workers to become skilled and thus to accumulate human capital period by period in order to avoid unemployment: notwithstanding, they emphasised the depressing role of minimum wages on economic growth arguing that "it is generally recognized that minimum wage legislation induces distortions which have adverse effects on the efficiency of the economy" (Cahuc and Michel, 1996, pp. 1463–1464). Irmen and Wigger (2006), instead, built up a two-country endogenous growth model with capital mobility and claimed that, depending on the mutual relationship between the propensity to save in both countries, "… in the country with minimum wages, the national minimum wage policy is likely to reduce economic growth in both countries" (Irmen and Wigger, 2006, p. 286).

Last but not least, minimum wages may also have important interactions with the publicly organised social welfare system. Several countries in fact have accompanied minimum wage policies either with labour subsidies or unemployment insurance benefits or, alternatively, with tax reductions benefited mainly by low-wage workers. The purpose of the first two policies (e.g., labour subsidies and unemployment benefits) is essentially to increase the labour demand for low skilled workers, whereas the third policy (e.g., payroll tax reductions) is implemented to raise work incentives. On the one hand, countries with relatively low wage minima tended to complement minimum wage policies by introducing labour subsidies (e.g., the United States). On the other hands, countries with high wage minima opted either to facilitate the hiring of human resources in firms by reducing the tax burden charged on employers (e.g., the Netherlands) or to provide generous unemployment benefits to hired workers (e.g., Scandinavian countries).

Both minimum wages and labour-subsidy policies are implemented essentially to mitigate the disincentive to work arising from high unemployment compensations relative to labour earnings. This objective is directly achieved by statutory minimum wages since increasing the level of wage minima raises the gross labour earnings from unskilled jobs, whereas the introduction of in-work policies

[1] This paper may be considered the benchmark model as to this issue in overlapping generations economies, in that other subsequent papers such as Ravn and Sorensen (1999) and Askenazy (2003) do not depart from the line of reasoning adopted by Cahuc and Michel (1996).

(either in the form of labour subsidies or as tax reductions) aims to increase net incomes from low-paid jobs. Generally speaking, the two schemes may complement each other even if they have rather different implications in terms of financial costs for policymakers as well as in terms of economic incentives both for workers and firms. Even if minimum wages and unemployment compensations cannot probably be expected to greatly reduce household's poverty, their effectiveness in reducing income inequality among households with at least one person in work is recognised to be greater, that is, minimum wage legislation has been prevalently introduced for equity reasons trading-off with efficiency goals.

Minimum wage policies, thus, surely imply both benefits and costs. They can be used in preventing labour earnings from dropping under socially unacceptable levels, and also to raise incentives for unskilled workers to accept low skilled jobs. However, according to the common wisdom in the economic literature, since they give rise to large unemployment effects, the costs of introducing wage minima may exceed the benefits, and their effectiveness in strengthening low-paid workers' earnings will also depend on their interactions with other policy measures designed to support low-income people. In particular, in order to reduce social iniquities, minimum wages should be used in a broader context including other social policies such as, for instance, unemployment compensations.

Therefore, to sum up, the previous literature up to now seemed to retain that the introduction of national or statutory minimum wages in an otherwise competitive labour market context – even by assuming a positive externality on the accumulation of human capital – negatively affects employment and economic growth while raising the unemployment rate at the aggregate level; thus, if minimum wages could be used efficiently to reduce income inequality and to mitigate social iniquities, are essentially viewed as ineffective in promoting a more rapid per-capita income growth,[2] and thus, according to the prevailing common wisdom they can only be used as an instrument to trade off some efficiency for equity given the costs in terms of high rates of unemployment they create.

In this paper we address two simple questions: (1) do wage minima always deteriorate economic growth and individuals' welfare in the basic OLG (Diamond, 1965) one-sector endogenous growth model (that is, even without introducing the questionable assumption of positive externality created by minimum wage legislation)? (2) Can unemployment promote a more rapid per-capita income growth in modern economies? In contrast with the past literature, we argue that the negative relationship between the unemployment created by the minimum wage and the rate of per-capita income growth – often asserted as inevitable – is not absolutely warranted. In particular, we establish another theoretical channel – so far overlooked and based on a combination of regulated wages and publicly provided unemployment compensations financed at balanced budget with a non-distorting and inter-generationally-neutral consumption tax –, through which unemployment and economic growth may be positively linked along the balanced growth path. If the replacement rate is high enough in fact (in particular, if it is higher than the weight of the labour input in the production function), then the regulation of wages always promotes a more rapid per-capita income growth in comparison with the standard competitive labour market context. Minimum wages in fact play a twofold role on the accumulation of capital over time: on the one hand, they contribute to raise labour earnings and thus to increase individuals' savings; on the other hand, they give raise to unemployment and thus they also push down savings and the accumulation of capital.

If the replacement rate is high enough, then the positive effect played by the increased wage income on individuals' savings more than counterbalances the negative unemployment effect and definitely minimum wages lead to higher growth rates is spite of a reduced employment rate. Moreover, and most importantly, a growth-maximising value of the minimum wage does exist, that is, once the government fixes the minimum wage at the growth maximising level, the minimum-wage economy jumps instantaneously (i.e., without transition) over the highest possible balanced growth path.

[2] Moreover, policymakers around the world – at least starting from the early 90s of the last century –, share with economists the belief that unemployment must be narrowed as much as possible to improve the human well-being, and to this purpose they often recommend policies designed mainly to make labour markets flexible (for instance by eliminating minimum wage legislation or by reducing the union's power in fixing wages above the prevailing market-clearing level).

To the best of our knowledge, this result is rather novel to the long lasting debate around the effects of minimum wages and unemployment benefits on economic growth in one-sector endogenous[3] growth models and contributes to fill a gap in the existing endogenous growth literature.
The paper is organised as follows. In Section 2 we develop the model and the main results as regards the relationship between unemployment and economic growth (and individuals' welfare) along the balanced growth path are analysed and discussed. Section 3 winds up some concluding remarks.

## 2. The model

### *2.1. Individuals*

Identical agents are assumed to belong to an overlapping generations economy with a two-period-separated finite lifetime (Diamond, 1965): youth (working period) and old-age (retirement period) and, for simplicity, population is supposed to stay constant over time (i.e., $N_{t+1} = N_t$). Individuals belonging to generation $t$ have a homothetic and separable utility function defined over the first (young-aged) and the second (old-aged) period of life consumption bundles, $c_{1,t}$ and $c_{2,t+1}$, respectively. Each young individual supplies inelastically one unit of labour in the labour market and receives an hourly minimum wage income at the rate $w_{m,t}$ (fixed by the government as a mark up over the competitive wage, i.e., $w_{m,t} = \mu w_{c,t}$ with $\mu > 1$).[4] Therefore, in each period the labour market does not clear and involuntary unemployment occurs. The aggregate unemployment rate (defined in terms of hours not worked) is $u_t = (N_t - L_t)/N_t$, where $L_t$ is the labour demand.[5] Moreover, young-aged individuals are entitled to a publicly provided unemployment compensation – defined to be a fraction of the prevailing competitive wage, that is, $b_t = \gamma w_{c,t}$ with $0 < \gamma < 1$ being the so-called replacement ratio – for the hours left unemployed by the regulation of wages.[6] Notice that we consider both the mark up and the replacement rate to be exogenous policy parameters, whereas the unemployment rate is endogenously determined, as it will be clear below. We also assume that only a proportional (non-distorting) tax on the consumption of the young people at the rate $\tau_t > 0$ is levied by the government and used to fund the unemployment benefit system at balanced budget.[7]

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[3] As usual, a Romer-typed knowledge spill-over mechanism in production represents the engine of balanced per-capita income growth.

[4] It is worth noting that in this model where, for simplicity, only one type of labour has been hypothesised, a binding minimum wage simply indicates a regulated wage fixed by law as a mark up over the prevailing market-clearing level. In the case of more than one type of labour with uniformly distributed wages, this assumption would simply mean a regulated wage fixed over the average market wage.

[5] Notice that in this model there is no uncertainty, and, as assumed by Fanti and Gori (2007) in a simple OLG model of neoclassical growth with minimum wages and unemployment benefits, each young-adult agent will be employed for $1-u_t$ hours and unemployed for the remaining $u_t$ hours. In other words, we are assuming that the minimum wage reduces the number of hours employed by each household, rather than causing some households to be employed at the higher legislated wage and other households to be unemployed. Moreover, it is important to emphasise also that the theoretical structure we adopted here is qualitatively and quantitatively the same as assuming two types of individuals, i.e., employed and unemployed who earn wage income at the legislated non-competitive rate and the publicly provided unemployment insurance benefit, respectively.

[6] Of course $w_{m,t} > b_t$ holds by definition.

[7] We have deliberately chosen a tax rate levied only upon the young people's consumption for two reasons: (1) a better analytical tractability, and (2) in this way the nature of the unemployment benefit policy is purely redistributive, that is, consumption taxed away from the young rebated to the same individuals as a benefit for the hours left unemployed by the regulation of wages, and thus the old people are not affected by the tax policy. Notice that our findings are robust to other tax systems, such as capital income taxes, taxes levied on both periods consumption, lump-sum taxes on the young people and so on.

During old-age agents are retired and live on the proceeds of their savings ($s_t$) plus the accrued interest at the rate $r_{t+1}$.

The representative individual born at time $t$ is therefore faced with the following program:

$$\max_{\{c_{1,t},c_{2,t+1}\}} U_t = \ln(c_{1,t}) + \beta \ln(c_{2,t+1}), \tag{P}$$

subject to the intra-temporal constraints

$$\begin{aligned} c_{1,t}(1+\tau_t) + s_t &= w_{m,t}(1-u_t) + b_t u_t \\ c_{2,t+1} &= (1+r_{t+1}) s_t \end{aligned},$$

where $0 < \beta < 1$ represents the degree of individual's impatience to consume over the life cycle.

Solving program (P) thus gives the following first and second period of life consumption functions:

$$c_{1,t} = \frac{1}{(1+\beta)(1+\tau_t)}\left[w_{m,t}(1-u_t) + b_t u_t\right], \tag{1}$$

$$c_{2,t+1} = \frac{\beta}{1+\beta}(1+r_{t+1})\left[w_{m,t}(1-u_t) + b_t u_t\right], \tag{2}$$

whereas the savings path chosen optimally by individuals is:

$$s_t = \frac{\beta}{1+\beta}\left[w_{m,t}(1-u_t) + b_t u_t\right]. \tag{3}$$

*2.2. Firms*

As in Romer (1986) and Daveri and Tabellini (2000), we assume the technology of production faced by each firm as:

$$Y_{i,t} = B k_t^{1-\alpha} K_{i,t}^{\alpha} L_{i,t}^{1-\alpha}, \tag{4}$$

where the index $i$ denotes the typical firm, $Y_i$ is total output produced by firm $i$, $K_{i,t}$ and $L_{i,t}$ are capital and the labour input hired in that firm, respectively, $k_t = K_t / N_t$ is the average capital per-capita, which is taken as given by each single firm in the market economy, $B > 0$ represents a scale parameter and $0 < \alpha < 1$ is the capital's weight in technology. By setting $L_{i,t} = L_t$, $K_{i,t} = K_t$ and $Y_{i,t} = Y_t$, the aggregate time-$t$ production function takes place according to $Y_t = B k_t^{1-\alpha} K_t^{\alpha} L_t^{1-\alpha}$, where $L_t = (1-u_t)N_t$ is the total labour force employed at the aggregate level. Therefore, the intensive-form (per-capita) aggregate production function may be written as $y_t = B k_t (1-u_t)^{1-\alpha}$.

Assuming total depreciation of capital at the end of each period and knowing that final output is treated at unit price, profits maximisation leads to the following marginal conditions for capital and labour, respectively:

$$r_t = \alpha B (1-u_t)^{1-\alpha} - 1, \tag{5}$$

$$w_{m,t} = (1-\alpha) B k_t (1-u_t)^{-\alpha}. \tag{6}$$

Therefore, by using Eq. (6) and knowing that the minimum wage $w_{m,t} = \mu w_{c,t}$ (with $w_{c,t} = (1-\alpha) B k_t$ being the equilibrium competitive wage) is fixed by the government, the unemployment rate as a function of the mark up is simply:

$$u(\mu) = 1 - \mu^{\frac{-1}{\alpha}}. \tag{7}$$

As it can readily be seen by looking at Eq. (7), unemployment stays constant over time and $0 < u < 1$ for any $\mu > 1$, while $u = 0$ if and only if $\mu = 1$ (i.e., a competitive labour market context). Therefore, the interest rate, as expressed by Eq. (5) is constant either, and it is always lower than the corresponding value in the case of competitive wage.

*2.3. Government*

The government runs a balanced budget unemployment benefit policy in every period by levying and adjusting over time a proportional tax on the individuals' first period of life consumption bundle at the rate $\tau_t > 0$. Therefore, the per-capita time-$t$ government constraint is simply:

$$b_t u_t = \tau_t c_{1,t}. \tag{8}$$

where the left-hand side represents the total unemployment benefit expenditure and the right-hand side the tax receipts.

Exploiting Eqs. (1), (7) and (8) and using both the definitions of the minimum wage and the unemployment benefit (see Section 2.1), the (constant) equilibrium consumption tax which balances the unemployment benefit expenditure in every period of time may be expressed as:[8]

$$\tau(\mu) = \frac{(1+\beta)\gamma\left(\mu^{\frac{1}{\alpha}} - 1\right)}{\mu - \beta\gamma\left(\mu^{\frac{1}{\alpha}} - 1\right)}. \tag{9}$$

*2.4. Balanced growth*

We now close the model with the analysis of the balanced equilibrium growth. Given the government budget constraint (8) and knowing that population is constant over time, the market-clearing condition in goods as well as in capital markets may easily be expressed by the equality $k_{t+1} = s_t$, that is, the per-capita stock of capital in period $t+1$ equals the amount of resources saved in period $t$. Using Eq. (3) to substitute out for $s_t$ into the latter equation, equilibrium implies

$$k_{t+1} = \frac{\beta}{1+\beta}\left[w_{m,t}(1-u_t) + b_t u_t\right]. \tag{10}$$

We are now wondering about how a legislated wage minimum affects the accumulation of capital per-capita and thus the growth rate of economy. To this purpose, let us rewrite Eq. (10) as a generic function of the mark up $\mu$ in the following way:

$$k_{t+1} = k_{t+1}\left[w_{m,t}(\mu), u(\mu)\right], \tag{11}$$

where we used the definition of the minimum wage as well as the relationship between the unemployment rate and the mark up as expressed by Eq. (7). Therefore, totally differentiating (11) with respect to $\mu$ yields:[9]

$$\frac{dk_{t+1}}{d\mu} = \underbrace{\overbrace{\frac{\partial k_{t+1}}{\partial w_{m,t}}}^{+} \cdot \overbrace{\frac{\partial w_{m,t}}{\partial \mu}}^{+}}_{+} + \underbrace{\overbrace{\frac{\partial k_{t+1}}{\partial u}}^{-} \cdot \overbrace{\frac{\partial u}{\partial \mu}}^{+}}_{-}. \tag{12}$$

Eq. (12) reveals that an increase in the legislated minimum wage (and thus in the rate of unemployment) is not absolutely linked in a negative unambiguous way with capital accumulation as it ambiguously affects capital per-capita in subsequent periods. In particular, the final effect of changing $\mu$ on $k_{t+1}$ (and then on the rate of per-capita income growth) is twofold: firstly, it promotes the accumulation of capital through an increased hourly wage perceived by the young when employed which, in a life-cycle structured economy, pushes up savings and thus capital accumulation; secondly, given our neoclassical labour market context of supply and demand, it increases the unemployment rate (or, alternatively it lowers the employment rate) which, on the contrary, negatively affects individuals'

[8] It can easily be shown that the denominator of Eq. (9) is always positive for any $\mu > 1$.

[9] Details are given in Appendix A.

savings and then the accumulation of capital. To analyse ultimately which of the two forces dominates, and then to characterise the role played by policy and technology parameters on the long-run rate of per-capita income growth in this simple stylised endogenous growth economy, we combine Eqs. (7) and (10) to eliminate $u(\mu)$ and then we exploit the definitions of minimum wage and unemployment benefit, as given in Section 2.1, so that the dynamic equilibrium sequence of capital is determined by the following first order linear difference equation with constant coefficients:

$$k_{t+1} = \frac{\beta}{1+\beta}(1-\alpha)B \cdot \mathrm{H}(\mu) k_t , \tag{13}$$

where $\mathrm{H}(\mu) \equiv \mu^{\frac{\alpha-1}{\alpha}} + \gamma\left(1-\mu^{-\frac{1}{\alpha}}\right)$.

Therefore, the constant growth rate of the per-capita stock of capital in the minimum-wage economy (which obviously coincides with the long-run rate of per-capita income growth since unemployment stays constant whatever the value of the mark up fixed by the government)[10] may be expressed as:

$$g_m(\mu) = (1+g_c)\cdot \mathrm{H}(\mu) - 1, \tag{14}$$

with $g_c = \frac{\beta}{1+\beta}(1-\alpha)B - 1$ being the endogenously determined rate of economic growth in the competitive-wage economy. Notice that $g_m(\mu)$ is independent of time so that the model does not show transitional dynamics, and thus a change in the minimum wage – as expressed by a change in the mark up $\mu$ fixed by the policymaker according to an appropriate legislation –, automatically results in an instantaneous adjustment of the economy to a new balanced growth equilibrium.

By looking at Eq. (14) it can readily be seen that if $\mu = 1$, then the growth rate in the minimum-wage wage economy exactly coincides with the growth rate in the competitive-wage economy independently of the value of the replacement rate, that is, $\mathrm{H}(\mu) = 1$ and $g_m(1) = g_c$ for any $0 < \gamma < 1$. It what follows we will show that depending on the size of the wage-mark-up, the growth rate of the economy when the labour market does not clear and there is unemployment may be higher than the growth rate in the competitive labour market economy, in spite of a reduced employment rate.

From Eq. (14), thus, the following propositions hold:

**Proposition 1**. *Let $\gamma < \bar{\gamma}$ ($\gamma > \bar{\gamma}$) hold. Then the introduction of minimum wages reduces (promotes) economic growth.*

**Proof**. The proof straightforwardly derives by differentiating Eq. (14) with respect to $\mu$ and evaluating it at $\mu = 1$, that is:

$$\frac{\partial g_m(\mu)}{\partial \mu}\Big|_{\mu=1} = \frac{\beta(1-\alpha)B(\gamma+\alpha-1)}{(1+\beta)\alpha}.$$

Therefore,

$$\begin{cases} \dfrac{\partial g_m(\mu)}{\partial \mu}\Big|_{\mu=1} < 0 \quad \textit{iff} \quad \gamma < \bar{\gamma} \\ \dfrac{\partial g_m(\mu)}{\partial \mu}\Big|_{\mu=1} > 0 \quad \textit{iff} \quad \gamma > \bar{\gamma} \end{cases}.$$

where $\bar{\gamma} \equiv 1-\alpha$. **Q.E.D.**

[10] In fact, the rate of per-capita income growth in the minimum wage economy may easily be expressed as

$$g_{m,y}(\mu) = \frac{y_{t+1}-y_t}{y_t} = \frac{B[1-u(\mu)]^{1-\alpha}\cdot(k_{t+1}-k_t)}{B[1-u(\mu)]^{1-\alpha}\cdot k_t} = \frac{k_{t+1}-k_t}{k_t} = g_{m,k}(\mu) = g_m(\mu).$$

**Proposition 2**. *(1) Let* $\gamma \leq \bar{\gamma}$ *hold. Then* $g_m(\mu) < g_c$ *for any* $\mu > 1$. *(2) Let* $\gamma > \bar{\gamma}$ *hold. Then* $g_m(\mu)$ *is maximised at* $\mu = \hat{\mu}$ *with* $g_m(\mu) > g_c$ *for any* $1 < \mu < \mu^\circ$ *and* $g_m(\mu) < g_c$ *for any* $\mu > \mu^\circ$ *where* $\mu^\circ > \hat{\mu}$.

**Proof**. The proof uses the following derivative:

$$\frac{\partial g_m(\mu)}{\partial \mu} = \frac{\beta(1-\alpha)B\mu^{-\frac{1}{\alpha}}[\gamma - \mu(1-\alpha)]}{(1+\beta)\alpha\mu}.$$

Therefore, if $\gamma \leq \bar{\gamma}$ then $\frac{\partial g_m(\mu)}{\partial \mu} < 0$ for any $\mu > 1$. If, instead, $\gamma > \bar{\gamma}$ then

$$\frac{\partial g_m(\mu)}{\partial \mu} \underset{<}{\overset{>}{}} 0 \Leftrightarrow \mu \underset{>}{\overset{<}{}} \hat{\mu},$$

where

$$\hat{\mu} \equiv \frac{\gamma}{1-\alpha}, \tag{15}$$

represents the growth-maximising wage-mark-up. Notice that $\hat{\mu} > 1$ for any $\gamma > 1-\alpha$. Since $g_m(1) = g_c$, $g_m(\mu)$ is a positive (negative) monotonic function of $\mu$ for any $1 < \mu < \hat{\mu}$ ($\mu > \hat{\mu}$) and $\lim_{\mu \to +\infty}[1 + g_m(\mu)] = (1+g_c)\gamma < 1 + g_c$, then there always exists a threshold value $\mu^\circ > \hat{\mu}$ such that $g_m(\mu^\circ) = g_c$, and thus $g_m(\mu) > g_c$ for any $1 < \mu < \mu^\circ$ and $g_m(\mu) < g_c$ for any $\mu > \mu^\circ$. **Q.E.D.**

Proposition 1 reveals that – contrary to the prevailing common wisdom in the economic literature (see, for instance, Daveri and Tabellini, 2000) – the rate of per-capita income growth in the case of non-competitive labour markets may be higher than the rate of per-capita income growth in the case of competitive labour markets in spite of a positive unemployment rate, i.e., the relationship between economic growth and unemployment has found to be positive, and this result depends exclusively on the mutual relationship between the exogenous replacement rate $\gamma$ and the technology parameter $\alpha$. In particular, there exists a threshold value of the replacement rate which discriminates against the ineffectiveness of the introduction of minimum wages on economic growth: if the replacement ratio is high enough, in fact, (i.e., if it is higher than the weight of the labour input in the production function), then the positive effects on individuals' savings and then on the accumulation of capital played by the introduction of minimum wages always dominates over the negative unemployment effect, and then it definitely leads to a positive relationship between unemployment and economic growth; if, on the contrary, the replacement rate is fixed at too low a level (i.e., it is lower than the weight of the labour input in the production function), then the negative effect of the increased unemployment on savings overcompensates the positive effect on capital accumulation due to the increased labour income, and thus the rate of per-capita income growth in the minimum-wage economy becomes lower than the rate of per-capita income growth in the competitive-wage economy.

In order to stress the importance of Proposition 1, it must be noted that a sufficiently high replacement rate (which is a part of the unemployment benefit system financed at balanced budget according to rule described by Eq. (9)) fixed by the government in such a way that the introduction of minimum wages ensures a positive link between unemployment and economic growth does always exists whatever the value of the technology parameter $\alpha$.

Proposition 2 instead reveals that a growth-maximising minimum wage does exist. Given the weight of capital in production, it is always possible to fund at balanced budget (with a non-distorting proportional tax on the young-aged consumption bundle) a sufficiently high replacement rate for each hour left unemployed by the regulation of wages in such a way to trigger a virtuous growth mechanism such that the positive effect on growth due to the increased wage perceived during the working period of life more than counterbalances the negative unemployment effect of such a legislated wage which, on the contrary, tends to reduce savings and economic growth. Definitely, unemployment promotes balanced growth, and, in particular, there exists a growth-maximising minimum wage which allows the

non-competitive wage economy to growth faster than the competitive-wage economy and to instantaneously jumps (i.e., without transition) along the highest possible balanced growth path.

*2.5. Welfare*

Since both young-aged and old-aged consumption grow without transition at the (constant) rate $g_m(\mu)$, the lifetime welfare grows without showing transitional dynamics at the same constant rate along the balanced growth path in this stylised non-competitive-wage economy. Therefore, the growth-maximising minimum wage even represents the welfare-maximising minimum wage, and the higher the replacement rate the more likely a positive relationship between growth and unemployment (i.e., welfare and unemployment) does exist. Therefore, given Propositions 1 and 2, individuals are better off in an economy with legislated wage minima and unemployment rather than in a competitive labour market economy for any $\gamma > \bar{\gamma}$ and $1 < \mu < \mu^{\circ}$.[11]

## 3. Concluding remarks

Labour market rigidities represent relevant aspects of real phenomena (for instance, national or statutory minimum wages and the trade unions' power in setting wages and/or the employment level), and, especially in European Union countries, the unemployment theme became one the most important challenge to solve both for theoretical and empirical economists. Analysis of labour market imperfections and the effects of unemployment in aggregate macroeconomic models have been widely studied in the economic literature; however few investigations have focused on the role played by minimum wages in determining growth rates and, notwithstanding the often mentioned and questionable assumption that minimum wages create a positive externality, for instance on the accumulation of human capital,[12] concluded for a negative overall role.

In this paper we studied how legislated wage minima affect economic growth by using as parsimonious a growth model as possible, that is, a fairly standard general equilibrium, one-sector, two-period overlapping generations model extended to account for a publicly provided unemployment benefit mechanism financed by proportional taxes levied only upon young-aged consumption. We showed that, in spite of a reduced employment rate, the regulation of wages may enhance economic growth and individuals' welfare as compared with the basic competitive-wage economy. This result sharply contrasts the previous literature. It is important to note that the possibility of a positive relationship between minimum wages (i.e., unemployment) and economic growth depends exclusively on the mutual relationship between policy and technology parameters, i.e., the replacement rate ($\gamma$) and the weight of capital in the production function ($\alpha$). In particular, if the replacement rate is high enough (i.e., if it is higher than the weight of the labour input in production), then the regulation of wages always promotes a more rapid per-capita income growth in comparison with the standard competitive labour market context, since the positive effect exerted by the increased labour earning on individuals' savings outweighs the negative unemployment effect which, on the contrary, tends to reduce savings and capital accumulation. Moreover, interestingly, we have shown that a growth-maximising value of the minimum wage does exist and we have analytically picked up the algebraic necessary and sufficient conditions for the existence of it. The policy implication is straightforward: governments are able, managing appropriately both minimum wages and unemployment benefits, to bring instantaneously (i.e., without transition) the minimum-wage economy to the highest possible balanced growth path.

The essential message of this thus paper is that legislated wage minima may not be introduced only to trade off some efficiency for equity – as often suggested by the previous literature –, but even to further encourage the long-run rate of per-capita income growth in modern economies, and, as regards the lifetime welfare of individuals, we found that people are better off in a minimum-wage economy than in a competitive-wage economy in spite of the increased unemployment.

[11] Details are given in Appendix B.

[12] See Cahuc and Michel (1996).

**Appendix A**

In this appendix we clarify the role played by the mark up $\mu$ on the accumulation of capital period by period. In particular, we have that:

$$\frac{\partial k_{t+1}}{\partial w_{m,t}} = \frac{\beta}{1+\beta}\left[1-u(\mu)\right] > 0, \tag{A1}$$

$$\frac{\partial w_{m,t}}{\partial \mu} = w_{c,t} > 0, \tag{A2}$$

$$\frac{\partial k_{t+1}}{\partial u} = -\frac{\beta}{1+\beta}\left[w_{m,t}(\mu) - b_t\right] < 0, \tag{A3}$$

$$\frac{\partial u}{\partial \mu} = \frac{\mu^{-\frac{1}{\alpha}}}{\alpha\mu} > 0. \tag{A4}$$

**Appendix B**

We prove here that young-aged and old-aged consumption grow without transition at the constant rate $g_m(\mu)$ and thus the statement of Propositions 1 and 2 hold even as regards the individuals' lifetime welfare function as the one typified in the main text (Section 2.1).
Since the non-distorting consumption tax which balances the publicly provided unemployment benefit expenditure is constant over time (see Eq. (9) in the main text), the growth rate of young-aged consumption ($c_1$) becomes:

$$g_{m,c_1}(\mu) = \frac{c_{1,t+1} - c_{1,t}}{c_{1,t}} = \frac{\frac{(1-\alpha)B\mathrm{H}(\mu)}{(1+\beta)(1+\tau)} \cdot (k_{t+1} - k_t)}{\frac{(1-\alpha)B\mathrm{H}(\mu)}{(1+\beta)(1+\tau)} \cdot k_t} = \frac{k_{t+1} - k_t}{k_t} = g_{m,k}(\mu) = g_m(\mu), \tag{B1}$$

whilst, since unemployment stays constant (i.e., it does not depend the current level of capital per-capita) and thus the interest rate, as expressed by Eq. (5) in the main text is constant either, the growth rate of the retirement period consumption ($c_2$) may easily be expressed as:

$$g_{m,c_2}(\mu) = \frac{c_{2,t+2} - c_{2,t+1}}{c_{2,t+1}} = \frac{\frac{\beta(1-\alpha)B\mathrm{H}(\mu)(1+r)}{1+\beta} \cdot (k_{t+1} - k_t)}{\frac{\beta(1-\alpha)B\mathrm{H}(\mu)(1+r)}{1+\beta} \cdot k_t} = \frac{k_{t+1} - k_t}{k_t} = g_{m,k}(\mu) = g_m(\mu). \tag{B2}$$

Therefore, knowing that $c_1$ and $c_2$ grow without transition at the constant rate $g_m(\mu)$, then the representative individual's lifetime welfare (defined by the function (P) in the main text) grows steadily at the same rate along the balanced growth path. To prove this statement, consider first the following homogeneous of degree one utility function, which represents homothetic individual's preferences over the life cycle consumption bundles of the generation born at time $t$:

$$U_t \equiv (1+\beta)\ln\left[V_t(c_{1,t}, c_{2,t+1})\right] = \ln c_{1,t} + \beta \ln c_{2,t+1},$$

where

$$V_t(c_{1,t}, c_{2,t+1}) = c_{1,t}^{\frac{1}{1+\beta}} \cdot c_{2,t+1}^{\frac{\beta}{1+\beta}}, \tag{B3}$$

and $\beta \equiv \frac{1}{1+\sigma}$ with $\sigma \in (0,+\infty)$ being the subjective discount rate.
Therefore, exploiting (B3) the growth rate of individuals' welfare is determined by:

$$g_{m,U}(\mu) = g_{m,V}(\mu) = \frac{V_{t+1} - V_t}{V_t} = \frac{c_{1,t+1}^{\frac{1}{1+\beta}} \cdot c_{2,t+2}^{\frac{\beta}{1+\beta}} - c_{1,t}^{\frac{1}{1+\beta}} \cdot c_{2,t+1}^{\frac{\beta}{1+\beta}}}{c_{1,t}^{\frac{1}{1+\beta}} \cdot c_{2,t+1}^{\frac{\beta}{1+\beta}}}$$

$$= \left(\frac{c_{1,t+1}}{c_{1,t}}\right)^{\frac{1}{1+\beta}} \cdot \left(\frac{c_{2,t+2}}{c_{2,t+1}}\right)^{\frac{\beta}{1+\beta}} - 1$$

$$= [1 + g_m(\mu)]^{\frac{1}{1+\beta}} \cdot [1 + g_m(\mu)]^{\frac{\beta}{1+\beta}} - 1 \quad .$$

$$= 1 + g_m(\mu) - 1$$

$$= g_m(\mu)$$

Consequently, Propositions 1 and 2 can be extended to individuals' welfare and thus the growth-maximising wage-mark-up $\hat{\mu}$ represents the welfare-maximising wage-mark-up either along the balanced growth equilibrium. Therefore, individuals who live in the minimum-wage economy are better off than those who live in the competitive-wage economy is spite of a positive unemployment rate for any $\gamma > \bar{\gamma}$ and $1 < \mu < \mu^{\circ}$, and in particular choosing $\mu = \hat{\mu}$ implies an instantaneous adjustments of the equilibrium welfare level to the highest possible balanced growth path.

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