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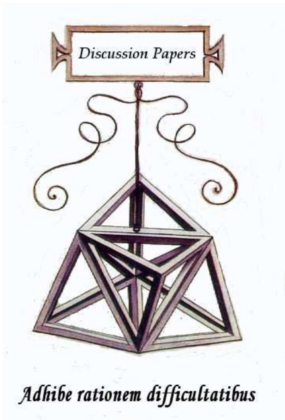

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Endogenous fertility, family policy and multiple equilibria

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Abstract In this paper we assess the role of direct monetary transfers to the benefit of households in raising children in a textbook Diamond (1965) style overlapping generations model. In particular, we examine how both the dynamics of capital and fertility of households are connected to a specific balanced budget policy to support child-rearing. We found that when the child allowance is higher than the fixed cost to children multiple equilibria are possible. As regards fertility, it is shown that increasing the child grant at too high a level may actually reduce the long-run population growth rate.

Keywords Childcare Policy; Endogenous fertility; Multiple equilibria; OLG model

JEL Classification H24; J13; J18

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1. Introduction

A recent and widespread decrease in fertility has occurred in several developed countries (especially in Europe, e.g., Germany, Italy and Spain), and the political debate around the effectiveness of family policies as an inducement to a higher population growth has risen dramatically in the last years. Moreover, there seems to be a general scepticism in the economic literature as regards the issue of whether public childcare policies may have an impact on fertility of households (most notably, as regards the effects that family policies may have on the viability of the widespread public pension system by increasing the number of workers-contributors). For instance, depending on whether either childcare facilities (e.g., investment in infrastructure for day-care centres, schools and so on) or direct monetary child payments are introduced by the policymaker, the final effects on fertility may be different (see Apps and Rees, 2004, who analysed a partial equilibrium model which accounts for endogenous fertility of households and differences among male and female wages). In particular, the they described the historical evidence of an inverse relationship between female labour force participation rates and fertility choices, and then assessed the effectiveness of family policies, in particular taxation and the system of child support, finding that: "the results suggest that countries which have individual rather than joint taxation, and which support families through improved availability of alternatives to domestic child care, rather than through direct child payments, are likely to have both higher female labour supply and higher fertility. These results are strengthened when we take into account the heterogeneity among households which undoubtedly exists" (Apps and Rees, 2004, p. 760).

Moreover, the importance of the argument has led some economists even to suggest – as a policy to reverse the decline in population growth – the introduction of tax penalties for households that choose low fertility, or, alternatively, a rise in the transfer payment for households that choose high fertility.

Although such a policy debate is high on the political agenda in the most part of developed countries, a few number of theoretical contributions have tried to give an answer to the question of the effectiveness of child support policies in a dynamic general equilibrium framework (see Momota, 2000, who focused on the gender gap in a context particularly close to that described by Galor and Weil, 1996). He introduced a rather special form of public services to support child-rearing of households and concluded that: "the subsidy policies have negative effects on the fertility rate if the ratio between male and female child-rearing productivities is low, the ratio between male and female labor share is low, or the tax rate [on labour earnings] is high" (Momota, 2000, p. 404).

In this paper we construct a parsimonious general equilibrium overlapping generations model of neoclassical growth with endogenous fertility, where parents draw utility from material consumption and the number of children raised (as in Galor and Weil, 1996), and then we analyse the effects on both the accumulation of capital the population growth rate in the long-run of child allowances. We dot not take into account inter-generational external effects which run from parents to children (Becker and Barro, 1988 and Barro and Becker, 1989) or from children to parents, that is children, in this latter case, are assumed to be thought as an investment in terms of old age consumption (Boldrin and Jones, 2002).

The Galor and Weil (1996) model allows for the possibility of multiple equilibria in a context in which production takes place according to a constant elasticity of substitution (CES) technology, where physical capital and mental labour exhibit complementarities, whereas physical labour is neither a complement nor a substitute for either of the other factors of production. In our model, instead, production takes place according to a constant return to scale Cobb-Douglas technology, and the possibility of multiple equilibria is allowed by the balanced budget child support policy.

The amount of resources needed to take care of each child is assumed to be made up of two components: a fixed part (i.e., a constant amount of resources measured in units of consumption goods) and a proportional-to-wage part (which is assumed to be a proxy of the time input required to raising children). Given this child cost structure, the government is assumed to provide a fixed child allowance – financed at balanced budget with a lump-sum tax levied on the same child bearing generation – to support families with children. We found that depending on the size of the child allowance relative to that of the fixed cost of children multiple equilibria are possible. Moreover, a further increase in the child grant may actually destabilise the economy.

As regards the fertility rate, we will show that population growth may be actually reduced by the public child allowance system. Furthermore, a value of the child grant beyond which the fertility rate from the Malthusian type (a positive relationship between fertility and wages) becomes of the Modern type (a negative relationship between fertility and wages) exists.

The remainder of the paper is organised as follows. In Section 2 we present the model and main results are analysed and discussed. In Section 3 we discuss the results of the model. finally, 3.4 bears the conclusions. Technical details are given in Appendix.

2. The model

2.1. Individuals

Agents have identical preferences and are assumed to belong to an overlapping generations structure with finite lifetimes. Life is separated among three periods: childhood, young adulthood and old-age. During childhood individuals do not make decisions. Adult individuals belonging to generation t (N_t) have a homothetic and separable utility function (U_t) defined over young-aged consumption ($c_{1,t}$), old-aged consumption ($c_{2,t+1}$) and from the number (quantity) of children they have (n_t), as in Galor and Weil (1996). As an adult, each young agent is endowed with one unit of time which is supplied inelastically to the labour market, while receiving wage income at the competitive rate w_t. This income is used to consume, to raise children, to pay taxes and to save. The total amount of resources needed to take care of one child (η_t) is assumed to be shared out between a fixed component m (measured in units of consumption goods) and a proportional-to-wage component $q w_t$, with $0 < q < 1$ being the percentage of child-rearing cost on working income. Therefore, the total cost of raising one child is $\eta_t := m + qw_t$.[1] Such a cost structure captures both the consumption and the time needed to take care of children. The former part says that to give birth and raise a child requires a certain amount of resources in terms of consumption goods. The latter part, instead, says that there exists an opportunity cost for the parents in terms of lost labour earnings for the time spent to bearing children. Moreover, the government provides a fixed child allowance ($\beta > 0$) for each child raised by families to support child-rearing. During old-age agents are retired and live on the proceeds of their savings (s_t) plus the accrued interest at the rate r_{t+1}.

The representative individual entering the working period at time t is faced with the problem of maximising the following logarithmic utility function:

$$\max_{\{c_{1,t}, c_{2,t+1}, n_t\}} U_t\left(c_{1,t}, c_{2,t+1}, n_t\right) = (1-\phi)\ln\left(c_{1,t}\right) + \gamma\ln\left(c_{2,t+1}\right) + \phi\ln\left(n_t\right),$$

subject to the lifetime budget constraint

$$c_{1,t} + \frac{c_{2,t+1}}{1+r_{t+1}} = w_t - \left(q\, w_t + m - \beta\right)n_t - \tau_t\,,$$

where $\tau_t > 0$ is a lump-sum tax levied on the working-age (child-bearing) generation, $0 < \gamma < 1$ is the subjective discount factor and $0 < \phi < 1$ captures the importance in the welfare function of raising children relative to material consumption when young.

[1] This child-cost structure is similar to that adopted by Boldrin and Jones (2002). In particular, they examined an overlapping generations model of fertility choice (assuming that agents do not derive utility from leisure) where individuals live for three periods, young, middle age and old age. Only middle aged individuals are endowed with one unit of productive time supplied inelastically to the labour market, while setting the labour supply of young and old aged people to be equal to zero. Therefore, the percentage of child-rearing cost on working income may be interpreted as an opportunity cost of the parents' home time which is increasing in their working income (see, among others, Cigno, 1991).

The (necessary and sufficient) first order conditions for an interior solution are given by:

$$\frac{c_{2,t+1}}{c_{1,t}} \cdot \frac{1-\phi}{\gamma} = 1 + r_{t+1}, \tag{1}$$

$$\frac{c_{1,t}}{n_t} \cdot \frac{\phi}{1-\phi} = q\,w_t + m - \beta\,. \tag{2}$$

Eq. (1) equates the marginal rate of substitution between working period and retirement period consumption to the interest rate, whereas Eq. (2) equates the marginal rate if substitution between material consumption when young and the number if children raised with the involved marginal of bearing an extra child. The higher id the child allowance (β) the lower is such a cost. It is worth to be noted that Eq. (2) requires that the net cost children must be positive in order to ensure a finite positive solution for n_t, that is, $q\,w_t + m - \beta > 0$.

Exploiting (1), (2) and the lifetime budget constraint, the demand for children and the saving function are respectively given by:

$$n_t = \frac{\phi}{1+\gamma} \cdot \frac{w_t - \tau_t}{q\,w_t + m - \beta}, \tag{3}$$

$$s_t = \frac{\gamma}{1+\gamma}\left(w_t - \tau_t\right). \tag{4}$$

2.2. Government

The government runs the following balanced budget child policy in every period:

$$\beta n_t = \tau_t\,, \tag{5}$$

where the left-hand side represents the total childcare expenditure and the right-hand side the tax receipts.

Therefore, combining (5) with (3) and (4) to eliminate τ_t, and rearranging terms gives:

$$n_t = \frac{\phi w_t}{(1+\gamma)(q\,w_t + m - \beta) + \phi\beta}, \tag{6}$$

$$s_t = \frac{\gamma(q\,w_t + m - \beta)w_t}{(1+\gamma)(q\,w_t + m - \beta) + \phi\beta}. \tag{7}$$

2.3. Firms

Firms are identical and act competitively. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^{\alpha} L_t^{1-\alpha}$,[2] where Y, K and $L = N$ are output, capital and the labour input respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the distributive capital's share. Defining $k_t := K_t / N_t$ and $y_t := Y_t / N_t$ as capital and output per-capita respectively, the intensive form production function may be written as $y_t = Ak_t^{\alpha}$. Then, assuming both total depreciation of capital at the end of each period and a unit price of final output, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

$$r_t = \alpha A k_t^{\alpha-1} - 1, \tag{8}$$

$$w_t = (1-\alpha)Ak_t^{\alpha}\,. \tag{9}$$

2.4. Equilibrium

[2] Adding exogenous growth in labour productivity does not alter any of the substantive conclusions of the model and, hence, it is not included here.

Given the government budget (5) and knowing that $N_{t+1} = n_t N_t$, the market-clearing condition in goods as well as in capital markets is expressed as $n_t k_{t+1} = s_t$. Substituting out for n_t and s_t form Eqs. (6) and (7) respectively, and using Eq. (9), the dynamic equilibrium sequence of capital is determined by:

$$k_{t+1} = f(k_t, \beta) = \frac{\gamma}{\phi}(m - \beta) + \frac{\gamma}{\phi} q(1-\alpha) A k_t^{\alpha} . \qquad (10)$$

Steady-state implies $k_{t+1} = k_t = k^*$. The steady-state equilibrium uniquely determines a stationary fertility rate as well as a stationary level of output per capita.

We now assume that the government would like to increase fertility, and we examine how the child allowance affects both the dynamics of capital and the population growth rate in the long-run. In particular, depending on whether the difference $m - \beta$ is positive or negative, things change dramatically as regards capital accumulation and fertility of households.

Case $m - \beta > 0$

If the child grant provided by the government is smaller than the fixed cost of bearing children, that is $m - \beta > 0$, then from Eq. (10) the following propositions hold:

Proposition 1. *Let* $\beta < m$ *hold. Then, there exists one and only one (locally asymptotically stable) steady-state equilibrium (*k^**).*

Proof. Provided $\beta < m$, the intercept of the capital accumulation path is strictly positive. Since k_{t+1} is a concave monotone increasing function of k_t, then there necessarily exists one and only one (locally asymptotically stable) steady-state equilibrium. **Q.E.D.**

Proposition 2. *Let* $\beta < m$ *hold. Then, an increase in the child grant (*β*) reduces* k^*.

Proof. Since the intercept of the k_{t+1} locus is negatively related with the child allowance, then a higher value of β shifts downward the capital accumulation path with its slope being unchanged by reducing the steady-state equilibrium. **Q.E.D.**

The following Figure 1 depicts the behaviour of the capital accumulation function in the case in which the child grant is lower than the fixed child rearing. The figure clearly shows that an increase in β shifts downward the locus of accumulation of capital with its slope being unchanged. The steady-state stock of capital thus from $k^*{}' = 0.6389$ ($\partial k_{t+1} / \partial k_t \big|_{k_t = k^*{}'} = 0.1492$) reduces to $k^*{}'' = 0.3330$ ($\partial k_{t+1} / \partial k_t \big|_{k_t = k^*{}''} = 0.2309$) as the child grant from $\beta' = 0.30$ increases to $\beta'' = 0.80$.

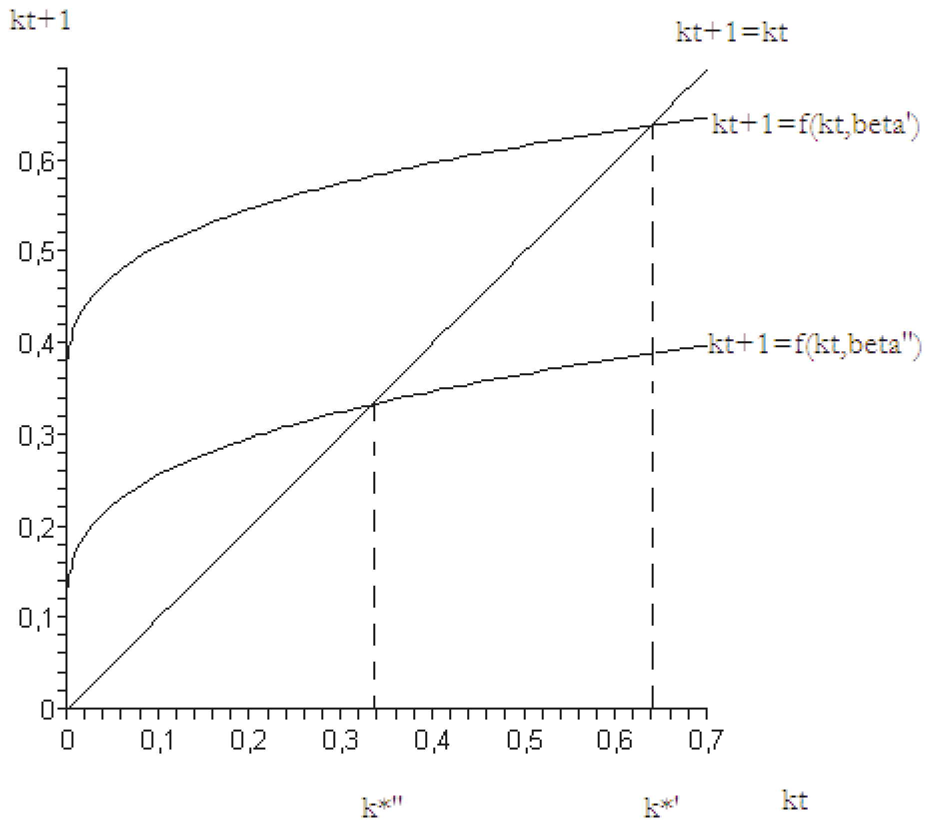

Figure 1. The capital accumulation path in the case $\beta < m$, where $\beta' = 0.30$ and $\beta'' = 0.80$. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.10$.

As regards fertility, the long-run demand for children is determined by:

$$n^* = n^*\{\beta, w^*[k^*(\beta)]\} = \frac{\phi w^*[k^*(\beta)]}{(1+\gamma)\{qw^*[k^*(\beta)] + m - \beta\} + \phi\beta}, \tag{11}$$

From (11), therefore, we have:

Proposition 3. *Let* $\beta < m$ *hold. Then the effect of an increase in the child grant on the long-run fertility rate is ambiguous.*

Proof. Consider the following total derivative of Eq. (11) with respect to β:

$$\frac{dn^*}{d\beta} = \underbrace{\overbrace{\frac{\partial n^*}{\partial \beta}}^{+} + \underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{+} \cdot \overbrace{\frac{\partial w^*}{\partial k^*}}^{+} \cdot \overbrace{\frac{\partial k^*}{\partial \beta}}^{-}}_{-}}_{+/-},$$

where $\frac{\partial k^*}{\partial \beta} < 0$ follows directly from Proposition 2.[3] Thus, Proposition 3 holds. **Q.E.D.**

Proposition 3 says that a rise in the child allowance ambiguously affects the long-run demand for children of households. In particular, there exists a twofold effect: (1) a direct (positive) effect of β which always increases the fertility rate by reducing the cost of children, and 2) an indirect (negative)

[3] Details are given in the Appendix.

general equilibrium feedback effect which, through the link wage-capital-child subsidy, tends to reduce the steady-state stock of capital. In particular, since in the case in which $\beta < m$, fertility is positively related with wages (namely, a Malthusian fertility effect) and the capital accumulation path is reduced as the child grant is increased, the higher is the child allowance the lower is the steady-state stock of capital per person. The final effect thus depends on which of two forces dominates. Therefore, depending on the mutual relationship between the key parameters of the model, the long-run fertility rate may be (i) a positive monotonic function of β, that is the direct positive effect always overcompensates the steady-state capital reduction, or (ii) an inverted U-shaped function of β. In this latter case, for low values of the child grant the direct positive effect dominates and the fertility rate increases as β raises, but further increases of β implies that the reduction of the stock of capital may dominates the positive effect of the child grant on fertility, with the consequence that the whole effect is negative, that is an increase in the subsidy to support child-rearing reduces fertility.

The following Table 1 shows (in the case $\beta < m$) the results of a numerical simulation for different values of both the child grant (β) and the percentage of child-rearing cost on working income (q), with preference and technology parameters being kept constant. In particular, it is shown that if q is high enough (Table 1.A) then the positive effect of the child grant on the fertility rate overcompensates the negative effect due to a reduced capital stock, and thus the higher the child grant the lower the steady-state stock of capital per-capita and the higher the long-run fertility rate. If, on the contrary, the percentage of child-rearing cost on working income is sufficiently low (Table 1.B), then n^* is an inverted U-shaped function of β. Therefore, for high values of the child grant the negative effect of β on the per-capita stock of capital prevails and the long-run fertility rate reduces as the child grant increases. This implies that a value of β for which n^* is maximised does exist.
Further, from Table 1 it is evident that, independently of the sign of the reaction of the long-run fertility rate to the child grant, the steady-state stock of capital per-capita is reduced as β is increased.

Table 1.A. Case $\beta < m$. The steady-state stock of capital and the long-run fertility rate as a function of the child grant. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.10$.

β	0	0.30	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.99
k^*	0.8128	0.6389	0.4591	0.3969	0.3330	0.3002	0.2665	0.2317	0.2174	0.2029
n^*	1.0040	1.1116	1.2578	1.3194	1.3899	1.4291	1.4713	1.5165	1.5355	1.5548

Table 1.B. Case $\beta < m$. The steady-state stock of capital and the long-run fertility rate as a function of the child grant. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.03$.

β	0	0.30	0.60	0.70	0.80	0.85	0.90	0.95	0.97	0.99
k^*	0.5841	0.4258	0.2648	0.2100	0.1542	0.1256	0.0964	0.0659	0.0531	0.0396
n^*	1.2528	1.4180	1.6430	1.7328	1.8237	1.8633	1.8888	1.8765	1.8467	1.7846

Case $m - \beta < 0$

We now consider the case in which the child grant is higher than the fixed cost of child-rearing, that is $m - \beta < 0$. We will show that depending on the size of β relative to that of m multiple equilibria are possible. Further, if β is fixed at too high a level, the risk of a destabilisation of the economy arises. Therefore, from eq. (10) the following proposition holds:

Proposition 4. *Let* $\beta > m$ *hold. Then we have the following:*

1) if $\beta < \beta_T$ *then there exist two positive equilibria,* k_L^* *and* k_H^**, where the subscripts* L *and* H *mean low and high respectively. The low equilibrium is locally unstable and the high equilibrium is locally asymptotically stable;*

2) if $\beta > \beta_T$ then no equilibrium does exist.

Proof. Provided $\beta > m$, the intercept of the capital accumulation path is strictly negative. Differentiating eq. (10) with respect to k_t yields:

$$\frac{\partial k_{t+1}}{\partial k_t} = \frac{\gamma}{\phi} q\alpha(1-\alpha)Ak_t^{\alpha-1}. \tag{12}$$

Equating the right-hand side of (12) to unity and solving for k_t we get:

$$\bar{k} = \left[\alpha(1-\alpha)Aq\frac{\gamma}{\phi}\right]^{\frac{1}{1-\alpha}}. \tag{13}$$

Eq. (13) represents the threshold value of the per-capita stock of capital such that

$$\begin{cases} \dfrac{\partial k_{t+1}}{\partial k_t} > 1 & for\,any \quad k_t < \bar{k} \\ \dfrac{\partial k_{t+1}}{\partial k_t} < 1 & for\,any \quad k_t > \bar{k} \end{cases}.$$

Substituting out (13) into (10) for k gives

$$\bar{k} = \frac{\gamma}{\phi}(m-\beta) + \frac{\gamma}{\phi}q(1-\alpha)A(\bar{k})^{\alpha}, \tag{14}$$

and solving (14) for the child grant we obtain

$$\beta_T = m - \frac{\phi}{\gamma}\bar{k} + q(1-\alpha)A(\bar{k})^{\alpha}. \tag{15}$$

Notice that using (14) and (15) it can be easily shown that $\beta_T > m$ always holds.
Combining now (10) with (15) yields

$$k_{t+1} = \frac{\gamma}{\phi}(m-\beta_T) + \frac{\gamma}{\phi}q(1-\alpha)Ak_t^{\alpha}, \tag{16}$$

which is the equation of the capital accumulation path tangent to the $k_{t+1} = k_t$ straight line at the point $\bar{k}$, where $\frac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=\bar{k}} = 1$. Since $\frac{\partial k_{t+1}}{\partial \beta} < 0$, then we may conclude that:

1) if $\beta < \beta_T$ then $k_{t+1} = f(k_t, \beta)$ lies above $k_{t+1} = f(k_t, \beta_T)$ for any k_t. Therefore, knowing that k_{t+1} is a concave monotone increasing function of k_t, two steady-state equilibria ($k_L^* < \bar{k}$ and $k_H^* > \bar{k}$) do exist. Since

$$\begin{cases} \dfrac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=k_L^*} > 1 \\ \dfrac{\partial k_{t+1}}{\partial k_t}\Big|_{k_t=k_H^*} < 1 \end{cases},$$

then the low equilibrium is locally unstable, and the high equilibrium is locally asymptotically stable;
2) if $\beta > \beta_T$ then $k_{t+1} = f(k_t, \beta)$ lies below $k_{t+1} = f(k_t, \beta_T)$ for any k_t, and thus no equilibrium does exists. **Q.E.D.**

In the following figure we summarise the behaviour of the capital accumulation function in the case $\beta > m$.

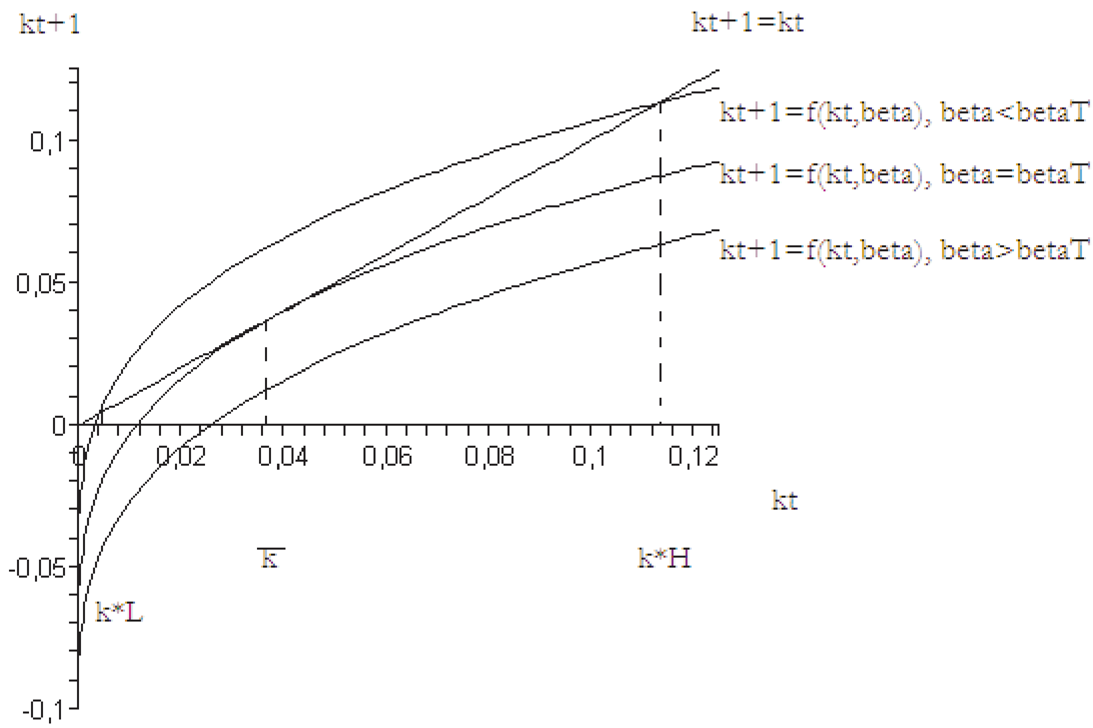

Figure 2. The locus of accumulation of capital in the case $\beta > m$, where $\beta_T = 1.1517$. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.10$.

Figure 2 shows that an increase in the child grant shifts always downward the locus of accumulation of capital with its slope being unchanged: if $\beta < \beta_T$ ($\beta = 1.10$) then two equilibria exist ($k_L^* = 0.0039$ with $\partial k_{t+1} / \partial k_t \big|_{k_t = k_L^*} = 4.5046$ and $k_H^* = 0.1132$ with $\partial k_{t+1} / \partial k_t \big|_{k_t = k_H^*} = 0.4756$); if $\beta = \beta_T = 1.1517$ the capital accumulation path is tangent to the $k_{t+1} = k_t$ straight line at the point in which its slope equals one ($\bar{k} = 0.0373$ with $\partial k_{t+1} / \partial k_t \big|_{k_t = \bar{k}} = 1$); and, finally, if $\beta > \beta_T$ ($\beta = 1.20$) then equilibrium vanishes and the economy shrinks.

Now, we are wondering about the effects of the policy parameter β on the long-run rate of fertility in the case in which the child grant fixed by the policymaker exceeds the fixed cost to raising children, that is $\beta > m$.[4] The analysis of (11) gives the following proposition:

Proposition 5. *Let $m < \beta < \beta_T$ hold. Then the effect of an increase in the child grant on $n^*\{\beta, w^*[k_H^*(\beta)]\}$ is ambiguous if and only if $\beta < \beta_C$, where the critical value $\beta_C := \dfrac{m(1+\gamma)}{1+\gamma-\phi} > m$.*

Proof. Let us consider the total derivative of the long-run rate of fertility with respect to β.[5]
If $\beta < \beta_C < \beta_T$ or $\beta < \beta_T < \beta_C$ then

[4] Note that the necessary and sufficient condition for the existence of a positive n, i.e. $qw_t + m - \beta > 0$, is satisfied.
[5] Details are given in Appendix.

$$\frac{dn^*}{d\beta} = \overbrace{\frac{\partial n^*}{\partial \beta}}^{+} + \underbrace{\underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{+}\overbrace{\frac{\partial w^*}{\partial k^*}}^{+}\overbrace{\frac{\partial k_H{}^*}{\partial \beta}}^{-}}_{-}}_{+/-},$$

where $\frac{\partial n^*}{\partial w^*} > 0$ for any $m < \beta < \beta_T$ (Malthusian Fertility Effect). It follows that the effect of an increase in the child grant on the long-run fertility rate is ambiguous.
If $\beta_C < \beta < \beta_T$ then

$$\frac{dn^*}{d\beta} = \overbrace{\frac{\partial n^*}{\partial \beta}}^{+} + \underbrace{\overbrace{\frac{\partial n^*}{\partial w^*}}^{-}\overbrace{\frac{\partial w^*}{\partial k^*}}^{+}\overbrace{\frac{\partial k_H{}^*}{\partial \beta}}^{-}}_{+} > 0,$$

where $\frac{\partial n^*}{\partial w^*} < 0$ for any $m < \beta < \beta_T$ (Modern Fertility Effect). **Q.E.D.**

Proposition 5 says that even in the case in which the child grant is higher than the fixed cost of child-rearing, the final result of an increase in β depends on two counterbalancing forces: a positive effect which directly increases fertility and a negative effect which acts on capital accumulation and tends to reduce the steady-state stock of capital. Furthermore, differently from the case stated in Proposition 3 ($\beta < m$), there exists now a critical value of the child grant ($\beta = \beta_C$) beyond which fertility from the Malthusian type becomes of the Modern type. This adds another element which help us to learn about the behaviour of the fertility rate in the long-run as for some exogenous reasons the child grant is increased by the government.

If β is slightly higher then the fixed cost of child-rearing ($m < \beta < \beta_C$), then an increase in the wage rate increases the demand for children (Malthusian Fertility Effect), and the net effect of an increase in the child grant on the long-run rate of fertility depends on two opposite forces. In particular, depending on the mutual relation between the key parameters of the model three cases are possible: 1) n^* is a positive monotonic function of β, that is the positive effect due to a direct increase in fertility always dominates the reduced stock of capital and thus fertility increases as the child grant raises; 2) n^* is a negative monotonic function of β, that is the negative effect of a reduced capital stock always dominates the positive effect due to a direct increase in fertility, so that the demand for children reduces as the child grant increases, and finally 3) n^* is an inverted U-shaped function of β. In this latter case, for low values of the child grant the direct positive effect dominates and the demand for children is enhanced as the child grant in increased. But if the child grant is fixed at too high a level the relationship fertility-child grant is reversed, that is the negative effect due to a reduced stock of capital prevails and thus n^* reduces as β increases.

If, on the contrary, β is sufficiently higher than the fixed cost of child-rearing, but smaller enough to preserve multiple equilibria ($m < \beta_C < \beta < \beta_T$), then an increase in the wage rate reduces the demand for children (Modern Fertility Effect), and the final effect of an increase in the child grant on the long-run rate of fertility is always positive, i.e. n^* is a positive monotonic function of β for any $m < \beta < \beta_T$.

In the following table we show (in the case $\beta > m$) the results of a numerical simulation for different values of both the child grant (β) and the percentage of child-rearing cost on working income (q), with preference and technology parameters being kept constant. In the first three tables it is shown the different behaviour of n^* as a function of β in the case in which the fixed child grant is moderately higher than the fixed child-rearing cost, that is $m < \beta < \beta_C$.

In particular, if q is sufficiently high (Table 2.A) then the fertility rate is positively related with the subsidy on child-rearing whatever the value of β, that is the direct positive effect of the child grant on the long-run fertility rate overcompensates the negative effect due to a reduced stock of capital.

If the percentage of child-rearing cost on working income reduces, Table 2.B shows that n^* is an inverted U-shaped function of β, that is a values of the child grant for which the long-run fertility rate is maximised does exist even in the case of multiple equilibria.

If q is low enough (Table 2.C) then the fertility rate in the long-run does always respond negatively to increases in the child-care subsidy up to the threshold value of the child grant for which two equilibria are preserved ($\beta \to \beta_T$). Beyond such a level, equilibrium vanishes and thus capital accumulation is always decreasing over time, i.e. if β is fixed at too high a level the child-subsidy support policy destabilises the economy.

Finally, the forth table (Table 2.D) shows that if β is sufficiently higher than the fixed child-rearing cost but smaller enough to preserve multiple equilibria ($m < \beta_C < \beta < \beta_T$), then the fertility rate does always increase as the child grant raises.

As it can be easily seen from Table 2, the higher the percentage of child rearing cost on working income ($q \uparrow$) the lower the risk of economic destabilisation ($\beta_T \uparrow$) due to the introduction of the family policy. As a consequence, a reduction in q lowers β_T and thus increases the risk of economic destabilisation as the policymaker increases the child grant. Further if q is sufficiently low, an increase in the child grant may have the undesired effect of reducing monotonically the long-run rate of fertility.

Table 2.A. Case $m < \beta < \beta_C < \beta_T$ (Malthusian Fertility), n^* is a positive monotonic function of β. The stable steady-state stock of capital and the long-run fertility rate as a function of the child grant, where $\beta_T = 1.4269$ and $\beta_C = 1.3529$. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.20$.

β	1.01	1.05	1.10	1.15	1.20	1.25	1.30	1.33	1.34	1.35
k_H^*	0.5425	0.5123	0.4735	0.4334	0.3917	0.3478	0.3006	0.2698	0.2590	0.2477
n^*	1.0591	1.0794	1.1067	1.1366	1.1698	1.2072	1.2500	1.2794	1.2889	1.3010

Table 2.B. Case $m < \beta < \beta_T < \beta_C$ (Malthusian Fertility), n^* is an inverted U-shaped function of β. The stable steady-state stock of capital and the long-run fertility rate as a function of the child grant, where $\beta_T = 1.1517$ and $\beta_C = 1.3529$. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.10$.

β	1.01	1.05	1.07	1.08	1.10	1.11	1.12	1.13	1.14	1.15
k_H^*	0.1879	0.1567	0.1401	0.1315	0.1132	0.1034	0.0929	0.0814	0.0678	0.0479
n^*	1.5745	1.6142	1.6335	1.6426	1.6588	1.6652	1.66925	1.66929	1.6608	1.6200

Table 2.C. Case $m < \beta < \beta_T < \beta_C$ (Malthusian Fertility), n^* is a negative monotonic function β. The stable steady-state stock of capital and the long-run fertility rate as a function of the child grant, where $\beta_T = 1.0251$ and $\beta_C = 1.3529$. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.03$.

β	1.010	1.013	1.015	1.019	1.020	1.021	1.022	1.023	1.024	1.025
k_H^*	0.0245	0.0220	0.0202	0.0163	0.0152	0.0141	0.0129	0.0115	0.0099	0.0074

n^*	1.6457	1.6095	1.5809	1.5069	1.4830	1.4555	1.4233	1.3836	1.3300	1.2297

Table 2.D. Case $m < \beta_C < \beta < \beta_T$ (Modern Fertility), n^* is a positive monotonic function β. The stable steady-state stock of capital and the long-run fertility rate as a function of the child grant, where $\beta_T = 1.4269$ and $\beta_C = 1.3529$. Parameter values: $A = 10$, $\alpha = 0.33$, $\phi = 0.30$, $\gamma = 0.15$, $m = 1$ and $q = 0.20$.

β	1.36	1.37	1.38	1.39	1.40	1.405	1.408	1.41	1.42	1.425
k_H^*	0.2360	0.2238	0.2108	0.1969	0.1815	0.1730	0.1676	0.1638	0.1410	0.1233
n^*	1.3125	1.3247	1.3377	1.3515	1.3666	1.3747	1.3799	1.3834	1.4034	1.4166

3. Discussion

From Eq. (10) it is easy to observe that the policy parameter β is responsible of the transition between three different regimes. In particular, 1) until β is lower than the fixed cost of child-rearing, one positive locally stable equilibrium does exist whatever the size of the child grant relative to that of the fixed child-rearing cost (see Figure 1); if, on the contrary, β exceeds the fixed child cost, then 2) for moderate values of the child grant ($\beta < \beta_T$) a fold bifurcation emerges and two steady-state equilibria appear. The low equilibrium is locally unstable and the high equilibrium is locally asymptotically stable; and 3) for higher values of the child grant, after a tangent bifurcation ($\beta = \beta_T$), both equilibria vanish ($\beta > \beta_T$) and the economy shrinks, that is capital accumulation is always decreasing over time (see Figure 2).

In particular, depending on the mutual relation between the basic parameters of the model, different consequences turn out to be possible in the pursuing of the implementation of the policy.

If the child grant is introduced in such a way to do not cover entirely the fixed cost of raising children, then only one steady-state equilibrium does exist and capital accumulation is reduced as the child grant is increased, but the effects of the family policy on the long-run fertility rate may change depending on whether the size of the percentage of the child-rearing cost on working income is high or low. If q is high enough, then the higher the child grant the higher the demand for children in the long-run. If, instead, q is sufficiently low the fertility rate is an inverted U-shaped function of β, so that a value of the child grant for which n^* is maximised does exist: beyond such a level the policymaker may obtain the undesired result to reduce fertility by increasing the subsidy to support child-rearing.

Things may change dramatically as the child grant introduced by the policymaker exceeds the fixed cost of raising children. In particular, depending on the size of the child grant relative to that of the fixed cost of child-rearing, the family policy introduces multiple equilibria or it may even destabilise the economy.

If the percentage of the cost of child rearing is high enough, then the essential message for the policymaker is that a trade-off between effectiveness (that is, higher fertility), on the one side, and risk of economic destabilisation of the introduction of a family policy, on the other side, does exist. If the child grant is not sufficiently high, the policymaker may obtain the undesired result (that is a lower fertility), but if the child grant is high enough for ensuring the effectiveness of the policy, then a destabilisation of the economy becomes more likely.

A reduction of q implies a decrease in the threshold value β_T, which separates the cases multiple equilibria-no equilibrium, and thus increases the risk of economic destabilisation as the child grant raises. However, the effects on the long-run fertility rate may be paradoxical in the sense that the long-run rate of fertility may be either an inverted U-shaped function or a negative monotonic function of the child grant, so that high values of β tend to reduce the demand for children in the long-run.

4. Conclusions

We evaluate the impact of child allowances in a theoretical textbook overlapping generations model of neoclassical growth where parents draw utility both from material consumption and the number of children raised.

As regards the dynamics of the OLG economy, we find that the economy evolves in two distinct environments depending on whether the fixed child allowance is higher or lower than the fixed cost of children. In particular, there exist (1) a unique steady-state equilibrium environment if the child allowance is lower than the fixed cost of child-rearing and (2) a multiple steady-state equilibria environment (the low equilibrium is unstable and the high equilibrium is locally asymptotically stable) if the child allowance is higher than the fixed cost of child-rearing. In this latter case, we show the existence of a threshold value of the child grant ($\beta = \beta_T$) beyond which the equilibrium vanishes and thus the economy is destabilised by the size of the balanced budget childcare policy.

As regards the long-run fertility decisions of households we find that in case (1) above mentioned, there exists a positive relationship between population growth and wages (namely, a Malthusian fertility effect), and, if the fraction of the variable (monetary) cost of child-rearing is high enough, then the response of the long-run fertility rate to a rise in the child allowance is always positive, that is, the direct positive effect of the child allowance on fertility always dominates the negative indirect general equilibrium effect which acts on capital accumulation. By contrast, if the variable cost of raising children is sufficiently low, then the long-run demand fir children is an inverted U-shaped function of the child allowance, and thus a value of β which maximises fertility does exist. Beyond such a level, fertility is reduced as the child allowance is raised. In this latter case, general equilibrium effect of the child allowance, which lowers capital accumulation dominates the positive direct effect. Definitely, fertility is lower than whether the childcare policy is not introduced at all.

In the multiple equilibria environment – case (2) above mentioned – there exists a value of the child grant ($\beta = \beta_C$) beyond which fertility from the Malthusian type (positive relationship between population growth and family income) becomes of the Modern type (negative relationship between population growth and family income). Moreover, in the case of Malthusian fertility, depending on the relative size between the fixed costs of raising children and the child allowance, the fertility rate may be a positive monotonic function, an inverted U-shaped function or a negative monotonic function of the child grant (high, moderate and low values of the variable cost of raising children, respectively). However, in the case of Modern fertility the relationship between population growth and child allowances is always positive.

Appendix

We give here details of Propositions 3 and 5 stated in the main text.

From Eq. (11) we have that:

$$\frac{\partial n^*}{\partial \beta} = \frac{(1+\gamma-\phi)\phi\, w^*[k^*(\beta)]}{\{(1+\gamma)[qw^*[k^*(\beta)]+m-\beta]+\phi\beta\}^2} > 0, \tag{A1}$$

and

$$\frac{\partial n^*}{\partial w^*} = \frac{\phi[(1+\gamma)m-\beta(1+\gamma-\phi)]}{\{(1+\gamma)[qw^*[k^*(\beta)]+m-\beta]+\phi\beta\}^2}, \tag{A2}$$

so that $\frac{\partial n^*}{\partial w^*} > 0$ if and only if $\beta < \beta_C$, and $\frac{\partial n^*}{\partial w^*} < 0$ if and only if $\beta > \beta_C$, where $\beta_C := \frac{m(1+\gamma)}{1+\gamma-\phi} > m$.

If $\beta < m$ then $\frac{\partial n^*}{\partial w^*} > 0$, that is in the case in which the child grant does not cover entirely the fixed cost of child-rearing, fertility is always of the Malthusian type.

If $m < \beta < \beta_T$ then the effect of an increase in the wage rate on the long-run fertility rate depends on the size of the child-care subsidy relative to that of the fixed child-rearing cost, and a critical value of the child grant ($\beta = \beta_C$) beyond which fertility from the Malthusian type becomes of the Modern type does exist.

In particular, provided $\beta_C < \beta_T$ two case are possible: 1) if the child grant is slightly higher than the fixed cost of child-rearing ($m < \beta < \beta_C$), then the long-run rate of fertility increases as the wage rate raises; and 2) if, on the contrary, β is sufficiently higher than m but smaller enough to preserve both equilibria ($m < \beta_C < \beta < \beta_T$), a raise in the wage rate lowers the long-run fertility rate.

In the case in which $\beta_C > \beta_T$, then for any $m < \beta < \beta_T$ the demand for children always increases as the wage rate raises.

Finally,

$$\frac{\partial w^*}{\partial k^*} = \alpha(1-\alpha)A\left(k^*\right)^{\alpha-1} > 0. \tag{A3}$$

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