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Nonlinear Dynamics in Welfare and the Evolution of World Inequality

Abstract

We propose a methodology to measure countries' welfare based on the lifetime utility of individuals and apply it to a large sample of countries. In the period 1960-2000 welfare inequality across countries appears stable as the result of increasing inequality in per capita GDP and decreasing inequality in life expectancy. However, the estimated distribution dynamics of welfare points out the emergence of three clusters of countries in 2000: one composed by low-income and low-life expectancy countries (mainly sub-Saharan countries); one by low-income but medium life expectancy countries (most of the highly populated Asian and Latin American countries); and, finally, the last one by high-income and high-life expectancy countries (almost all OECD countries). Such tendencies to polarisation are expected to strengthen in the future. In terms of the world population distribution, from 1960 to 2000 welfare inequality has been decreasing as the result of the falling inequality of both per capita GDP and life expectancy; this fall is mostly explained by the outstanding performance of the highly populated countries, mainly China and India. However, the decreasing trend is expected to be reverted (at most stabilise) in the future. Finally, the estimated distribution dynamics of welfare shows the emergence of two clusters of population, already detected in the distribution of 2000; such polarisation dynamics is expected to further intensify in the future, with the possible emergence of a cluster of populations from sub-Saharan countries.

Classificazione JEL: C13, D30, D63, O5

Keywords: distribution of welfare, nonparametric methods, polarisation, distribution dynamics, inequality

Contents

What we cannot speak about we must pass over in silence.
Ludwig Wittgenstein, Tractatus Logico-Philosophicus

I would rather be vaguely right than precisely wrong.
John Maynard Keynes

I. Introduction

The analysis of the dynamics of world inequality mainly focuses on the distribution of per capita GDP. Bourguignon and Morrisson (2002) and Becker et al. (2005), however, has stressed how a more meaningful analysis of welfare inequality across countries/world citizens should jointly consider the dynamics of per capita GDP and life expectancy. This paper proposes a methodology to measure welfare based on lifetime utility of individuals and apply it to a large cross-section of countries to assess the evolution of world inequality in welfare.

In a seminal contribution, Bourguignon and Morrisson (2002) observe that inequality in per capita GDP across world population increased from the beginning of the 19th century to World War II, and then stabilised (or slightly increased). On the contrary inequality in life expectancy strongly decreased after 1920-1930. Moreover, taking lifetime income as a proxy of welfare, they find that welfare inequality is increasing over time. Becker et al. (2005) propose a more sophisticated approach to the measurement of welfare based on the concept of lifetime utility as previously discussed in Rosen (1988); for the period 1960-2000 they find out indication of convergence across countries' populations.

Following the same approach as Becker et al. (2005) but allowing for the presence of nonlinearities, we find evidence of the emergence of clusters of countries and populations in the period 1960-2000. Moreover, taking into account in the calculation of welfare the possible cross-country heterogeneity in growth rates, a feature neglected in Becker et al. (2005), such pattern of polarisation results confirmed but with greater welfare inequality.

In particular, we present both cross-country and cross-population estimates for the period 1960-2000.[1] The cross-country estimates aim to evaluate whether countries are converging in their welfare levels, while the cross-population estimates to approximate the evolution of the world distribution of welfare by weighting observations by countries' populations.[2] Unfortunately, being information on the within-country distributions of life expectancy missing, there is the possibility to underestimate the true global inequality.[3] The use of nonparametric methods allows to detect the nonlinearities of the dynamics of per capita GDP, life expectancy and welfare and to highlight the crucial role of India and China in driving the evolution of inequality and polarisation across world citizens.

Summarising our findings, in the period 1960-2000 welfare inequality across countries appears stable as the result of an increase in inequality of per capita GDP and a decrease in inequality of life expectancy. However, the estimated distribution dynamics of welfare points out the emergence of three clusters of countries: one composed by low-income and low-life expectancy countries (mainly sub-Saharan); one by low-income but medium life expectancy countries (most of the highly populated Asian and Latin American countries); and, finally, the last one by high-income and high-life expectancy countries (almost all OECD countries). Such tendencies to polarisation are expected to strengthen in the future, with further convergence of countries around these three clusters.

Differently, from 1960 to 2000 welfare inequality across world population has been decreasing as the result of the falling inequality of both per capita GDP and life expectancy; such a fall is mostly explained by the outstanding performance of the most highly-populated countries, mainly China and India. However, the decreasing trend is expected to be reverted (at most stabilise) in

[1]As in Becker et al. (2005), the lack of data on the joint distribution of income and age leads to consider the welfare of a representative newborn as a proxy of the country's welfare. The obvious drawback is to neglect the country's population age structure.

[2]These two different approaches correspond to *Concept 1 inequality* and *Concept 2 inequality* defined in Milanovic (2005). He also discusses a third approach, *Concept 3 inequality*, which considers all world population, ranking the individuals from the poorest to the richest independently of their nationality.

[3] Becker et al. (2005), indeed, have the same problem.

the future. The estimated distribution dynamics of welfare shows the emergence of two clusters of population, already detected in the distribution of 2000. The first cluster is composed by population from highly populated countries, while the second mainly by population of OECD countries. Such polarisation dynamics is expected to further intensify in the future, with the possible emergence of a new cluster of populations from sub-Saharan countries.

Bourguignon and Morrisson (2002) and Becker et al. (2005) are the main sources of inspiration of the paper. Our theoretical model is built on Rosen (1988), while the empirical analysis is inspired by the work of Danny Quah on income distribution and convergence-club dynamics (see, e.g., Quah (1993) and Quah (1997)).

In the estimate of individual welfare by lifetime utility we adopt a point of view close to Murphy and Topel (2006); their goal is however different, being to value improvements in health and life expectancy. Anderson (2005) presents a similar framework, but he limits his empirical analysis to African countries and consider a zero growth rate of consumption. Milanovic (2005) and Sala-i-Martin (2006) present estimates of the world distribution of per capita GDP in the period 1970-2000 focusing both on poverty and inequality. Our approach is also close to the literature on the value of statistical life (see Viscusi and Aldy (2003)). Finally, Nordhaus (2003) and Hall and Jones (2007) provide stimulating discussions on the evaluation of welfare associated to extensions in life expectancy.

The nonparametric methodology used in the empirical analysis is based on Fiaschi and Lavezzi (2003). The estimate of the long-run distribution follows Johnson (2000), thus avoiding the discretization of state space. In addition, we propose a novel bootstrap procedure to identify confidence intervals for the estimated long-run (ergodic) distributions.

The paper is organised as follows. Section II. presents the theoretical model, Sections III. and IV. report and discuss the empirical results and Section V. concludes. Appendix contains proofs and other technicalities.

II. The Model

The model is built on Rosen (1988). Consider an agent born at time 0 with maximum length of life equal to T and a positive probability to die before $T > 0$. Given her initial wealth, $\bar{p}_0$, and a flow of potential labour incomes $(yl_0, yl_1, ..., yl_T)$, the intertemporal budget constraint of the agent is:

$$\int_0^T c_t \exp(-rt) S_t dt \leq w, \tag{1}$$

where r is the interest rate, S_t the probability to survive at age t, and w is the lifetime wealth of the agent, given by:

$$w = \bar{p}_0 + \int_0^T yl_t \exp(-rt) S_t dt. \tag{2}$$

We assume that r is constant over time and non-negative.

Budget constraint (1) assumes full annuity insurance, or the existence of a complete contingent claims market (see Becker et al. (2005)): the agent can borrow in perfect capital markets all her potential future labour incomes at the current interest rate r, and the survival function S is common knowledge across all the agents in the economy.

When the agent is alive, her preferences are described by the following *CIES* instantaneous utility function:[4]

$$u(c) = \begin{cases} \frac{c^{1-\sigma}}{1-\sigma} - M & \text{for } \sigma > 0 \text{ and } \sigma \neq 1; \\ \log(c) - M & \text{for } \sigma = 1, \end{cases} \tag{3}$$

Preferences (3) depends on two additive components: a constant term, M, which represents the utility of the state "dead",[5] and the term $c^{1-\sigma}/(1-\sigma)$ describing the utility of the state "alive".[6] Subtracting M from utility in each

[4]The form of the utility function for $\sigma \to 1$ in Eq. (3) is obtained adding the constant term $-1/(1-\sigma)$ to the term $c^{1-\sigma}/(1-\sigma)$.

[5]The presence of the constant term M allows the utility elasticity to decline with consumption. Under reasonable assumptions on the parameters' values, this implies that an agent would eventually prefer to substitute consumption with additional years of life (see Hall and Jones (2007)).

[6]The latter term is commonly used in the literature on economic growth, because it ensures constant growth

state (both "dead" and "alive") normalises the utility of nonsurvival to zero.

If $\sigma \in (0,1)$ and $M < 0$ being alive has a positive utility *per se*; the agent would prefer a longer life independently of her consumption level. On the contrary, if $\sigma > 1$, then M should be negative, otherwise $u(c) < 0$ for all c and therefore "dead" would be always the preferred state of the agent. We therefore assume that:[7]

$$\begin{aligned}&1.\ \text{if } \sigma \in (0,1) \text{ then } M > 0;\\ &2.\ \text{if } \sigma = 1 \text{ then } M \in (-\infty, +\infty) \text{ and}\\ &3.\ \text{if } \sigma > 1 \text{ then } M < 0.\end{aligned} \tag{4}$$

Under Assumption (4) there exists a *zero utility consumption*, c^{ZUC}, such that $u\left(c^{ZUC}\right) = 0$, i.e.

$$c^{ZUC} = \left[(1-\sigma) M\right]^{\frac{1}{1-\sigma}}; \tag{5}$$

The expected utility of the agent is given by:[8]

$$E[U] = \int_0^T \left(\frac{c^{1-\sigma}}{1-\sigma} - M\right) \exp(-\rho t) S dt, \tag{6}$$

where ρ is the discount rate.

Assume that:[9]

$$\dot{S}/S = -\pi^D, \tag{7}$$

where $\pi^D > 0$ is the mortality rate. Under Assumption 7 life expectancy at birth (i.e. at time $t = 0$) is given by:

$$LE = \frac{1 - \exp\left(-\pi^D T\right)}{\pi^D}. \tag{8}$$

If $T \to \infty$ then $LE = 1/\pi^D$, while if $\pi^D = 0$ then $LE = T$.

We also assume that the agent's expected labour income grows at a rate

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rates in steady state.

[7] Rosen (1988), p.287, argues that the economically interesting cases are those for which the elasticity of the instantaneous utility function $\varepsilon \in (0,1]$. This corresponds to the cases: i) if $\sigma \in (0,1)$ then $M > 0$ or ii) if $\sigma > 1$ then $M < 0$.

[8] In the following, we omit time index whenever it does not cause confusion.

[9] See Nordhaus (2003) for a similar framework.

equal to the steady-state growth rate g, i.e.[10]

$$yl_t = yl_0 \exp(gt) \ \text{ for } t \in [0, T]. \tag{9}$$

When the agent has no initial wealth, i.e. $\bar{p}_0 = 0$, her indirect lifetime utility is given by:[11]

$$V(T, yl_0, g) = \left(\frac{1}{1-\sigma}\right)\left\{ yl_0^{1-\sigma}\left[\frac{\exp((g-\hat{r})T)-1}{g-\hat{r}}\right] + \frac{(1-\sigma)M[\exp(-\hat{\rho}T)-1]}{\hat{\rho}}\right\}, \tag{10}$$

where $\hat{r} = r + \pi^D$ and $\hat{\rho} = \rho + \pi^D$ are respectively the interest rate and the discount rate adjusted for the instantaneous probability to die before T.[12]

III. Empirical Evidence

This section studies the evolution of world inequality in welfare, per capita GDP and life expectancy and their distribution dynamics.

III.A. Methodology of the Empirical Investigation

As in Becker et al. (2005) the welfare of a given country is assumed to be equal to the (indirect) lifetime utility of a representative agent with no initial wealth, $\bar{p}_0 = 0$, whose first yearly income, yl_0, is proxied by the per capita GDP of that country[13] and whose life expectancy, LE, is equal to the average life expectancy at birth of its citizens; country's welfare is therefore equal to the utility of a representative newborn.

We estimate the dynamics both of cross-country and of world population distributions. On one hand the dynamics of the cross-country distribution allows to identify possible clusters of countries with similar pattern of growth. These

[10] For sake of simplicity, in Eq. (9) we are considering that the agent works over the whole life; however, the analysis could be easily extended to the case in which the agent retires at age T^R, with $T^R \in (0, T]$.

[11] See Appendix A for the details.

[12] Lifetime utility V can be a non-monotonic function of life expectancy. The parameters' setting adopted in the paper (the same of Becker et al. (2005)) excludes such possibility. We refer to Fiaschi and Romanelli (2009) for a more detailed analysis of this point.

[13] The goodness of such a proxy relies on the constancy of factor income shares over time and across countries. Gollin (2002) provides an interesting cross-country analysis on factor shares.

findings can help to understand the drivers of economic growth/stagnation and to elaborate policy implications (see Sala-i-Martin (2006)). On the other hand, the analysis of the world population distribution provides a picture of the dynamics of inequality across individuals. Unfortunately, the unavailability of the joint distribution of income *and* life expectancy impedes to perform a complete analysis of world population distribution as in Bourguignon and Morrisson (2002), Milanovic (2005) and Sala-i-Martin (2006) for income inequality. [14] In the cross-population estimates we therefore use population-weighted observations, being aware that such estimates contain a bias neglecting the *within-country* distribution of welfare.[15]

From a methodological point of view the present analysis depart from the Becker et al. (2005)'s one in two points. Firstly, the focus on nonparametric techniques in the empirical analysis, which crucially affects the results because of the presence of nonlinearities in the distribution dynamics. Secondly, Eq. (10) shows that the Becker et al. (2005)'s decomposition of changes in welfare into two additive components, namely changes in income and changes in life expectancy, could bias the estimate of the welfare distribution given the nonlinear relationship between growth rate, income and life expectancy with welfare. Moreover, such bias might be further worsen by the high cross-country heterogeneity in the per capita GDP growth rates. However, the estimate of g for a certain country at a given year is not a simple task, because it should represent the *expected* growth rate of the newborn in that country at that year. This suggests to analyse the baseline case $g = 0$ and to devote Section IV. to

[14] In particular, Bourguignon and Morrisson (2002) and Sala-i-Martin (2006) overcome the lack of data on the within-country distribution of income assuming that similar countries have similar income distributions. However, we cannot follow such method given that, at least to our knowledge, the joint distribution of income and life expectancy is available for almost no country. Other scholars follow a different approach (e.g. see Chotikapanich *et al.* (1997) and Schultz (1998)). They estimate the countries' income distributions assuming a lognormal density function whose first two moments are inferred by the countries' mean income (or per capita GDP) and by a summary of inequality statistics. Milanovic (2002) relies on microdata drawn by Household Surveys to estimate the countries' income distributions.

[15] Bourguignon and Morrisson (2002) show that in modern economic history the within-country component was the main source of inequality in per capita GDP until World War II, accounting for almost 3/4 of total inequality on average. However, since the mid-20th century, its contribution to world inequality was halved, being the dynamics of between-country inequality the leading factor in determining inequality across world citizens.

investigate the implications on the distribution dynamics of welfare of non-null growth rates.

III.B. Calibration of the Model

As in Becker et al. (2005) the parameters' values used in the paper are estimated from the U.S. economy; in particular $\rho = 0.005$,[16] $\pi^D = 0$, so that $LE = T$,[17] $\sigma = 1/1.250$, $\varepsilon = u'(c)\,c/u(c) = 0.346$ and $c = 26,365\$$ from which $M = 16.2$;[18] The zero utility consumption, c^{ZUC}, is equal to 357$ (see Eq. (5)): an individual whose per capita income is in every period equal to 357$ is therefore indifferent between living or dying independently of her life expectancy. Appendix G shows how the next results are robust to alternative specification of the model's parameters. Finally, as stated above, country's welfare is computed by Eq. (10) assuming $g = 0$.[19]

The sample in the empirical analysis includes 97 countries. Countries' GDP is measured by the gross domestic income adjusted for terms of trade in 1996 international prices (I$) taken from Penn World Table 6.1; population is taken from the same dataset, while life expectancy at birth is drawn from World Development Indicators 2004.[20]

In order to gain an intuition of the relationships between per capita GDP, life expectancy and welfare, Figure 1 displays a series of level curves for welfare in the space *(per capita GDP, life expectancy)*. It also reports the positions of some representative countries in 1980 (diamond) and in 2000 (grey circle).

Since $g = 0$ differences in countries' welfare amount to differences in life

[16] Jones and Hall (2007) adopt similar parameters' values.

[17] An alternative specification could cxpectancy, in the estimates of the agent's utility. All the empirical results reported below are robust to this alternative specification.

[18] Indeed, from Eq. (3) $M = c^{(1-\sigma)}\left[1/\left(1-\sigma\right) - 1/\varepsilon\right]$.

[19] For example, the expected welfare of an American newborn in 2000 is:

$$V_{US} = \left(\frac{1}{1-\sigma}\right)\left\{\frac{\exp\left(-\rho LE_{US}\right) - 1}{\rho}\left[\left(1-\sigma\right)M - yl_{US}^{1-\sigma}\right]\right\} = 1533.2,$$

where $yl_{US} = I\$33523$ and $LE_{US} = 77.03$.

[20] Appendix B reports the country list; gross domestic income adjusted for terms of trade in 1996 international prices: variable *rgdptt* in Penn World Table 6.1, see http://pwt.econ.upenn.edu/; population: variable *pop* in Penn World Table 6.1; life expectancy at birth: see http://www.worldbank.org/.

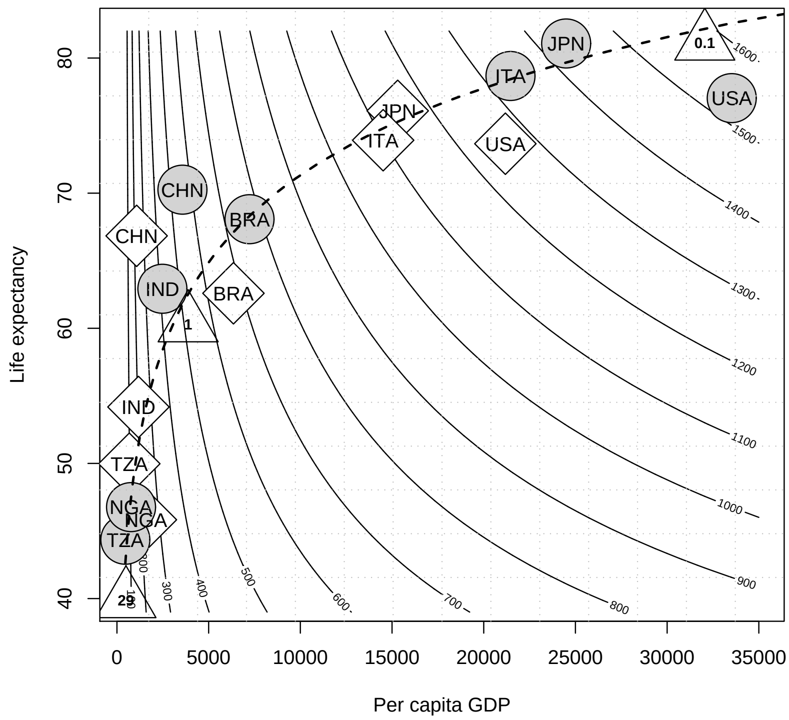

Figure 1: Welfare calculated with $g = 0$ for a sample of countries in 1980 (diamond) and in 2000 (grey circle). Country codes: Tanzania (TZA), China (CHN), Nigeria (NGA), India (IND), Brazil (BRA), Italy (ITA), United States (USA), Japan (JPN). Numbers in triangles are the marginal rate of substitution between life expectancy and per capita GDP (expressed in one hundred international dollars).

expectancy and in per capita GDP. Between 1980 and 2000, Nigeria and Tanzania show a marked decrease in their welfare, while China and India a large increase. Some developed countries present a relatively high increase in their life expectancy (Italy and Japan), while others a relatively marked increase in their per capita GDP (i.e. United States). The numbers reported in the three triangles along the dashed line are the marginal rates of substitution between life expectancy and per capita GDP (expressed in one hundred international dollars). As expected, at very low levels of life expectancy and per capita GDP, individuals relatively value income more than life expectancy (i.e. individuals value one hundred dollars per year equal to 29 years of life expectancy at birth). Instead at very high level of life expectancy and per capita GDP, the opposite occurs (i.e. individuals value a hundred dollars per year equal to 0.1 years of life expectancy at birth).

III.C. A First Exploration of the Sample

Table 1 reports some descriptive statistics of the sample, including a set of inequality indices for selected years (1960, 1980 and 2000).

Inequality in per capita GDP across countries strongly increased from 1960 to 2000, with the Gini index rising from 0.47 in 1960 to 0.55 in 2000 (Theil index followed the same pattern). Interestingly, the share of the top decile was almost stable at 32% of total income, while the share of bottom 20% decreased from 4% to 2%, suggesting that the change in inequality could be caused by changes in the bottom of the distribution. Inequality in life expectancy across countries fell, with a Gini index decreasing from 0.14 in 1960 to 0.11 in 2000. Welfare inequality across countries was fairly stable in the period 1960-2000 (Gini index was 0.39 in 1960 and 0.38 in 2000), as the result of the two competing distribution dynamics of income and life expectancy.

Inequality in both per capita GDP and life expectancy across world population strongly decreased from 1960 to 2000, with the Gini index respectively diminishing from 0.57 in 1960 to 0.54 in 2000 and from 0.14 to 0.07. Accord-

Table 1: Descriptive statistics for the sample's variables

		Across countries			Across world pop.		
Year		1960	1980	2000	1960	1980	2000
Per capita GDP							
	Mean	3564	6520	9413	2985	4949	7207
	Gini	0.47	0.49	0.55	0.57	0.59	0.54
	Theil	0.36	0.40	0.51	0.59	0.63	0.54
	Top 10%	0.32	0.29	0.32	0.45	0.40	0.42
	Bottom 20%	0.04	0.03	0.02	0.00	0.01	0.03
Life expectancy							
	Mean	53	61	65	49	62	67
	Gini	0.14	0.11	0.11	0.14	0.08	0.07
	Theil	0.03	0.02	0.02	0.03	0.01	0.01
Welfare ($g = 0$)							
	Mean	402	594	713	316	471	647
	Gini	0.39	0.36	0.38	0.51	0.40	0.30
	Theil	0.24	0.21	0.24	0.43	0.28	0.16
	Top 10%	0.25	0.21	0.21	0.36	0.30	0.23
	Bottom 20%	0.04	0.05	0.03	0.00	0.02	0.05
Pop							
	Total (millions)				2467	3690	5099

ingly, we also observe a strong reduction in the inequality of welfare, with a Gini index falling from 0.51 in 1960 to 0.30 in 2000. As for the cross-country distribution, welfare inequality across world population was lower than income inequality (0.30 vs. 0.54 in 2000). Finally, while income inequality in 2000 was almost at the same level across countries and across world population (0.55 vs. 0.54), inequality in life expectancy and, consequently, in welfare (0.11 vs. 0.07 and 0.38 vs. 0.30 respectively) differ considerably.

Figures 2 and 3 report the joint dynamics of per capita GDP and life expectancy in 1960-2000 across countries and across world population. In particular, they depict a vector field, where the arrows indicate direction and magnitude of the dynamics of per capita GDP and life expectancy at different points in the space *(per capita GDP, life expectancy)*.[21]

[21]For each point of the grid direction and magnitude is calculated as the weighted mean of all the observations' variations over a 5-year interval. Weights are calculated by means of an Epanechnikov kernel with an optimal normal bandwidth, and reflect the distance of each observation from the considered point of the grid and the relative size of countries' population (with respect to the average of the sample). In particular, the direction

and magnitude associated to the grid point (GDP_i, LE_i) is:

$$(\Delta GDP, \Delta LE)_{GDP_i, LE_i} = \left(\sum_{j=1}^{n} w_j K \left(\frac{GDP_i - GDP_j}{h_{GDP}^{opt}} \right) / n h_{GDP}^{opt}, \sum_{j=1}^{n} w_j K \left(\frac{LE_i - LE_j}{h_{LE}^{opt}} \right) / n h_{LE}^{opt} \right)$$

where n is the number of observations (i.e. the ordered couples (GDP_j, LE_j) with $j = 1, ..., n$) and w_j is the weight of observation j. In the calculation of cross-country dynamics $w_j = 1\ \forall j$, while in the cross-population calculation w_j is equal to the relative size of country j's population with respect to the average of the sample. The direction is calculated only for those points of the grid whose neighbourhood contains more than two observations for cross-country dynamics and more than 2/97 of the total population of the sample for the cross-population dynamics (the bottom-right hand side of the grid therefore presents no arrows for the absence of observations in that region).

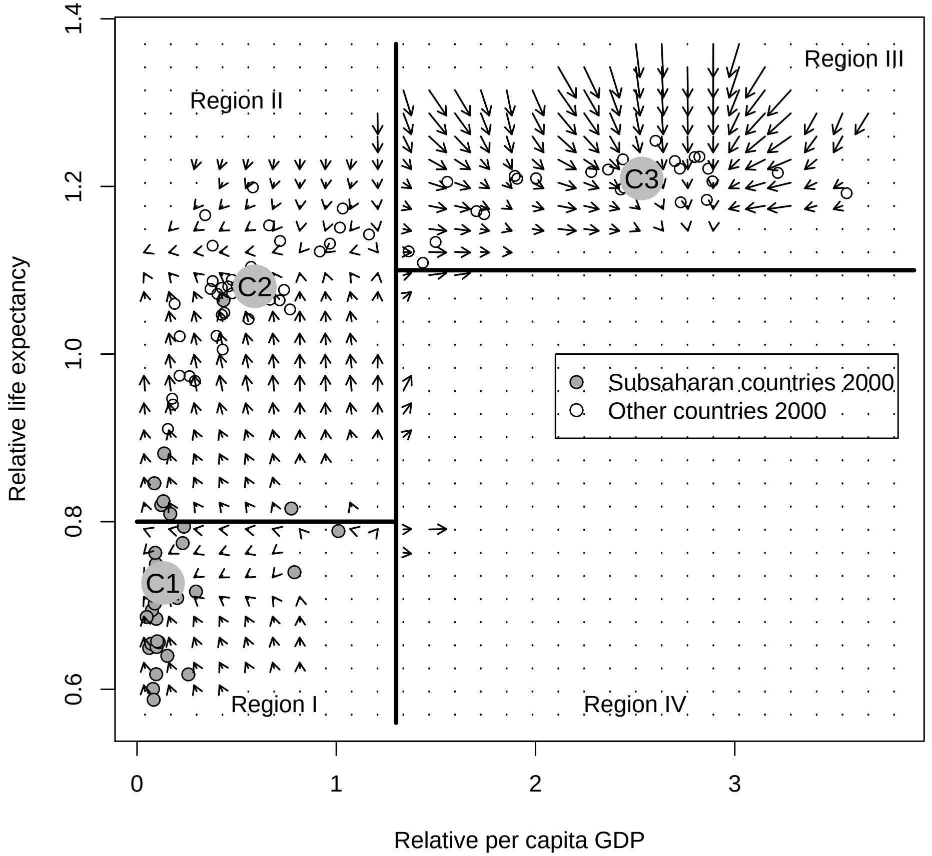

Figure 2: The joint dynamics of per capita GDP and life expectancy from 1960 to 2000 (cross-country). Circles represent countries observations in 2000.

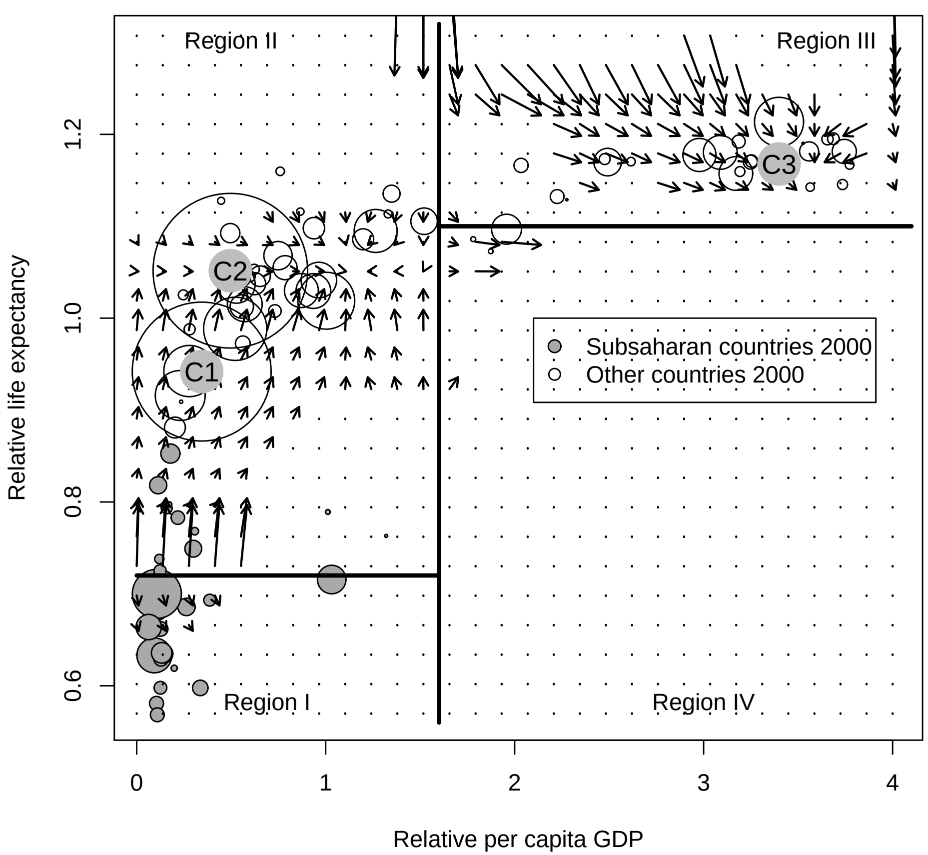

Figure 3: The joint dynamics of relative per capita GDP and relative life expectancy from 1960 to 2000 (cross-population). Circles represent countries observations in 2000 and they are proportional to countries'populations.

The dynamics in Figure 2 suggests the formation of three clusters of countries. For descriptive purposes only, we apply the *k-medians algorithm* to the joint distribution of per capita GDP and life expectancy in 2000, identifying three possible clusters centred in $C1 = (0.13, 0.73)$, $C2 = (0.59, 1.08)$ and $C3 = (2.53, 1.21)$.[22] Moreover, in Figure 2 four regions are defined on the basis of the pattern of the arrows, where Regions I, II and III should define the possible manifolds of the three clusters.[23] Cluster $C1$ is located in Region I and centred at very low levels of per capita GDP (about 13% of the average) and life expectancy (about 73% of the average); it is mainly composed by sub-Saharan countries. Cluster $C2$ is located in Region II and centred at low levels of per capita GDP (about 59% of the average) and intermediate values of life expectancy (about 108% of the average); the cluster is composed by highly populated countries as, for example, Brazil, China, India, Indonesia and Mexico. Finally, Cluster $C3$ is located in Region III and centred at high levels of per capita GDP and life expectancy (both variables are largely above the average, i.e. 253% and 121% of the average); the cluster is mainly composed by OECD countries. No country, with the only exception of Equatorial Guinea, is located in Region IV, suggesting that a high per capita GDP is always associated with a long life expectancy. From 1960 to 2000 the distribution of countries across the four regions is almost constant: the probability mass changes from $(0.29, 0.45, 0.25, 0.01)$, respectively, in Region I, II, III and IV in 1960 to $(0.25, 0.45, 0.30, 0)$ in 2000. Moreover, mobility across regions from 1960 to 2000 is very low (except for Region IV): the probabilities that a country in Region I, II, III and IV were in the same region in 1960 and in 2000 are respectively equal to $(0.68, 0.75, 0.92, 0)$.

In terms of the distribution of per capita GDP in 2000 Figure 2 suggests the existence of two main clusters of countries, one composed by countries in Regions I and II (i.e. those with a per capita GDP around 0.5) and the other

[22]The objective of *k-medians algorithm* is to minimise the total intra-cluster absolute distance and it results more robust with respect to outliers than the more popular *k-means algorithm*; for more details see Leisch (2006).

[23]The limits of Regions I, II, III and IV in term of relative per capita GDP and relative life expectancy are respectively given by (0,1.3)-(0,0.8), (0,1.3)-(0.8,+∞), (1.3,+∞)-(1.1,+∞) and (1.3,+∞)-(0,1.1).

one composed by countries in Region III (i.e. countries with per capita GDP around 2.5).[24] Analogously, in 2000 we observe the existence of two clusters in the distribution of life expectancy, one composed by countries in Region I (i.e. countries with relative life expectancy around 0.75) and the other one in Region II and III (i.e. those with life expectancy around 1.1).[25]

Figure 3 reports the dynamics of the joint distribution of relative per capita GDP and relative life expectancy across the world population. Circles, representing countries observations in 2000, are now proportional to countries' populations. Four regions are again defined on the basis of the dynamics of the vector field.[26] Region I contains populations from sub-Saharan countries, Region II populations from the largest countries (i.e. China and India) and Region III populations from OECD countries (Region IV is almost empty).

Applying the *k-medians algorithm* but weighting the observations by their population size we identify three clusters in $C1 = (0.34, 0.94)$, $C2 = (0.50, 1.05)$ and $C3 = (3.40, 1.17)$ in the cross-population distribution of 2000. With respect to the cross-country distribution the presence of high populated countries in Region II makes Clusters $C1$ and $C2$ very close and both in Region II (indeed the two clusters are around China and India), while Region I only contains the relatively low-populated sub-Saharan countries. Finally, with respect to the cross-country distribution the distance between Clusters $C1$-$C2$ and $C3$ is larger.

From 1960 to 2000 the distribution of populations across the four regions changes in favour of Region I: the probability mass varies from $(0, 0.82, 0.18, 0)$, respectively, in Region I, II, III and IV in 1960 to $(0.09, 0.75, 0.15, 0.01))$ in 2000. The change mainly reflects the increase in the population of the sub-Saharan countries (in Region I) with respect to the population in OECD countries (in

[24] Quah (1997) finds a similar feature. The result in Easterly (2006) partially differs, likely because of the different definition of the observed variable, which is computed with respect to the US income and not to the world average income.

[25] Indeed, Ram (2006) finds a reversal in the dynamics of convergence of the cross-country distribution of life expectancy after 1980.

[26] The limits of Regions I, II, III and IV in term of relative per capita GDP and relative life expectancy are respectively given by (0,1.6)-(0,0.72), (0,1.6)-(0.72,+∞), (1.6,+∞)-(1.1,+∞) and (1.6,+∞)-(0,1.1).

Region III). Mobility across regions is even lower than in the distribution dynamics across countries: the probabilities that an individual in Region I, II, III and IV were in the same region in 1960 and in 2000 are respectively equal to $(0.66, 0.84, 0.95, 0)$.

In terms of the per capita GDP two clusters of populations seems to exists in 2000, one in Region II (i.e. populations with relative per capita GDP around 0.5) and the other one in Region III (i.e. populations with relative per capita GDP around 3.2). Also the distribution of life expectancy shows two clusters of populations in 2000, one in Region II (around 0.9) and one in Region III (around 1.15).

In the following the observations just discussed will be investigated by non-parametric methods.

III.D. Distribution Dynamics of Per Capita GDP, Life Expectancy and Welfare

This section applies the methodology proposed in Fiaschi and Lavezzi (2003) in order to study the distribution dynamics of per capita GDP, life expectancy and welfare. In particular, Section III.D.i. reports the estimated growth path of the three variables so to detect possible nonlinearities, a necessary condition for the presence of polarisation; Section III.D.ii. then analyses their distribution dynamics by estimating stochastic kernels; and, finally, Section III.D.iii. discusses their long-run tendencies by the comparison between the actual distributions and the estimated ergodic distributions.

III.D.i. Growth Paths

The estimate of the growth paths of per capita GDP, life expectancy and welfare are reported in Figures 4-12. In particular, they show the estimate of Model (11), where x is alternatively the log of per capita GDP, life expectancy and the log of welfare level:

$$\overline{GR}_i^x = m\left(x_i^{INI}\right) + \epsilon_i; \tag{11}$$

$\overline{GR}_i^x$ is the average growth rate (or average difference for life expectancy) of x of country i in a given period, x_i^{INI} is the initial value of x and ϵ_i is a i.i.d. random variable with zero mean. The estimate of $m(.)$ is made using the Nadaraya-Watson estimator with the optimal normal bandwidth (see, Bowman and Azzalini (1997) for more details). [27]

The growth path of the three variables is estimated for the whole period 1960-2000 and for two subperiods 1960-1980 and 1980-2000. All figures report the cross-country estimate (thin line) and the cross-population estimate (thick line); the weights used in the cross-population estimates are the population sizes at the initial year. Dotted lines represent the pointwise confidence intervals at 95% (see Härdle *et al.* (2004)). We also report countries' observations by circles, whose area is proportional to the population at the initial year (the countries' codes reported in the figures refer to the top 10 countries in terms of population). Finally, sub-Saharan countries are represented by grey circles.

Per capita GDP and Life Expectancy In the period 1960-2000 there was no convergence across countries in terms of per capita GDP (see Figure 4); indeed, the slope of the growth path is not statistical different from zero in the whole range. Subperiods 1960-1980 and 1980-2000 have the same pattern (see Figures 6 and 8). However, at low levels of per capita GDP Figure 4 highlights both the bad performance of the sub-Saharan countries and the relevant growth of China and India. In the second subperiod (1980-2000) such a pattern is even clearer, with zero or negative growth rates for almost all sub-Saharan countries with respect to the extraordinary performance of China and India.

Over the whole period 1960-2000, convergence across world population is

[27] All the calculations and estimates in the paper are made using R. The estimate of nonparametric regression is made by the package *sm* (see Bowman and Azzalini (2005)). All codes are available on the following website: http://www.dse.ec.unipi.it/persone/docenti/fiaschi.

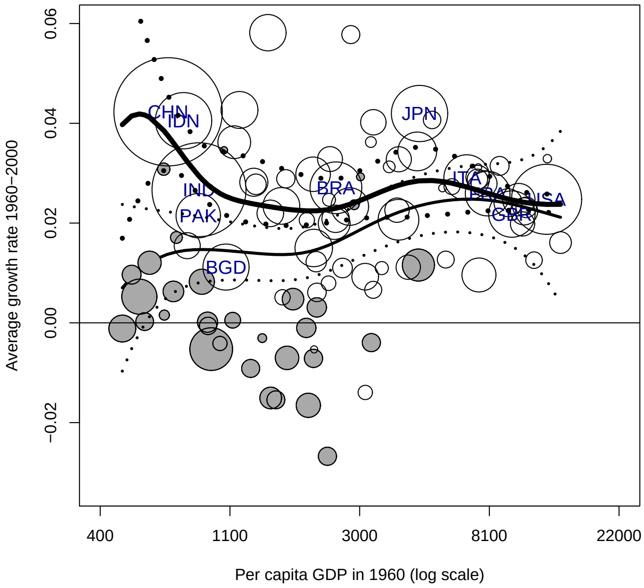

Figure 4: Growth path of per capita GDP in 1960-2000 (thin line: cross-country estimate, thick line: cross-population estimate).

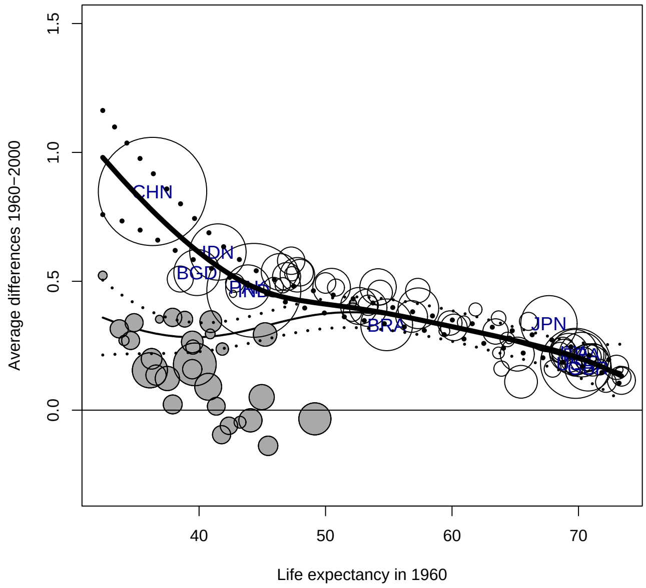

Figure 5: Growth path of life expectancy in 1960-2000 (thin line: cross-country estimate, thick line: cross-population estimate).

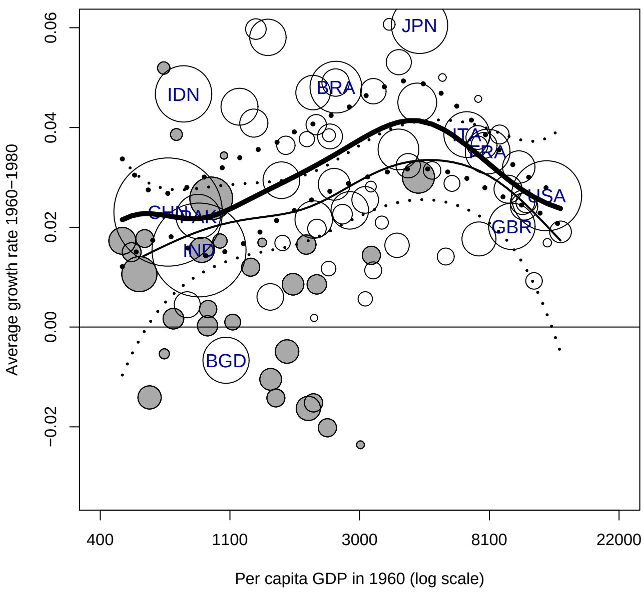

Figure 6: Growth path of per capita GDP in 1960-1980 (thin line: cross-country estimate, thick line: cross-population estimate).

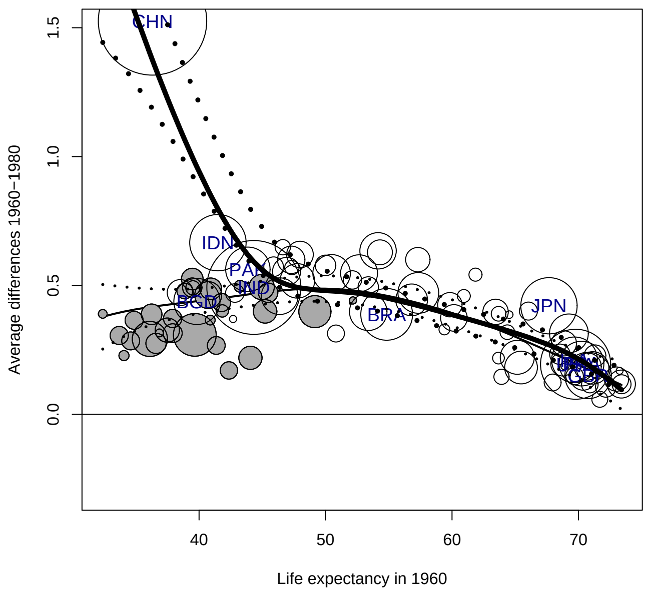

Figure 7: Growth path of life expectancy in 1960-1980 (thin line: cross-country estimate, thick line: cross-population estimate).

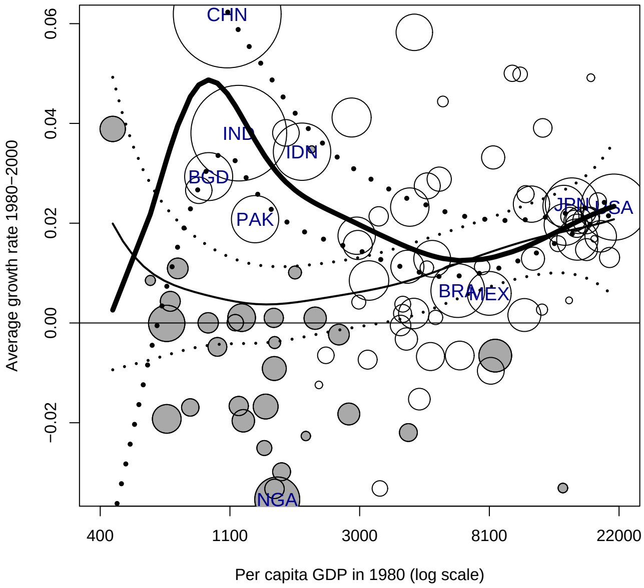

Figure 8: Growth path of per capita GDP in 1980-2000 (thin line: cross-country estimate, thick line: cross-population estimate).

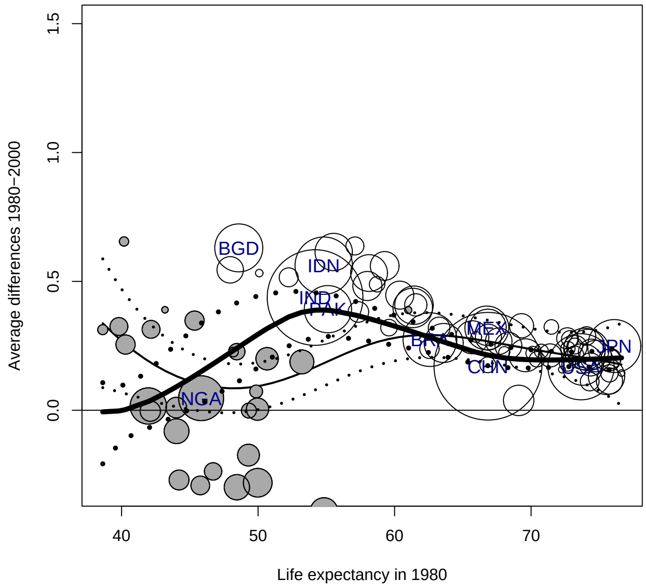

Figure 9: Growth path of life expectancy in 1980-2000 (thin line: cross-country estimate, thick line: cross-population estimate).

observable only at low levels of per capita GDP (below 1100I$ in 1960), as reported in Figure 4.[28] The patterns in the two subperiods, 1960-1980 and 1980-2000, are, however, strongly different. In the first time span population with medium and high levels of per capita GDP (above 5000I$ in 1960) tended to converge, while the low income countries had very low growth rates. The opposite holds for the second time span, where convergence only happens across population with low/medium levels of per capita GDP. Such path is mainly due to the high growth of four big Asian countries, Bangladesh (BGD), China (CHN), India (IND) and Indonesia (IDN)[29]. Finally, very populated countries with a medium level of per capita GDP (around 8000I$ in 1980), i.e. Brazil (BRA) and Mexico (MEX), had low performance with respect to high income countries.

For what concerns the dynamics of life expectancy, in the whole period 1960-2000 the decreasing growth path from 50 years of age on would suggest a dy-

[28]The negative slope of the growth path is statistically significant only at low level of per capita GDP
[29]In 2000 they represent more than 51% of the population in the sample.

namic of convergence across the countries in the sample (see Figure 5). However, at low initial levels of life expectancy two groups of countries can be identified: the sub-Saharan countries, with a very small increase in life expectancy, and the other ones (e.g. Bangladesh, China, India and Indonesia) with a large increase. The dynamics in the sub-periods 1960-1980 and 1980-2000 confirm this intuition (see Figures 7 and 9). In fact, in the second time span, sub-Saharan countries has experienced on average only a slight increase in their life expectancy, while the other countries with low life expectancy (i.e. those with a life expectancy in 1980 around 55 years) have been converging towards the high life expectancy countries. Moreover, the flat right-hand section of the growth path 1980-2000 indicates the absence of convergence also across those countries whose life expectancy in 1980 were higher than 60.

Convergence is much more evident across world population, being the growth path 1960-2000 strongly decreasing (see Figure 5). Again the main actors of this overall pattern are the large populated countries, with low initial level of life expectancy, as Bangladesh, China, India and Indonesia. However, looking closer at the sub-period 1980-2000, again convergence emerges only for the people living in countries with life expectancy higher than 55 years, while, on the contrary, the population of the sub-Saharan countries are left behind (the growth path has a positive slope, see Figure 9).

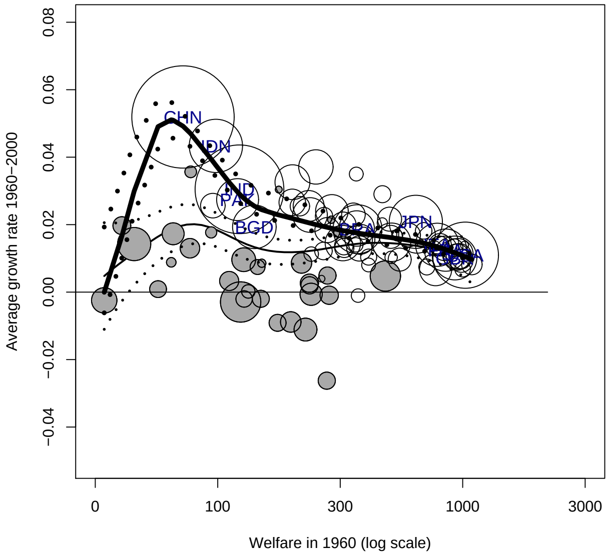

Figure 10: Growth path for welfare (g=0) in 1960-2000 (thin line: cross-country estimate, thick line: cross-population estimate).

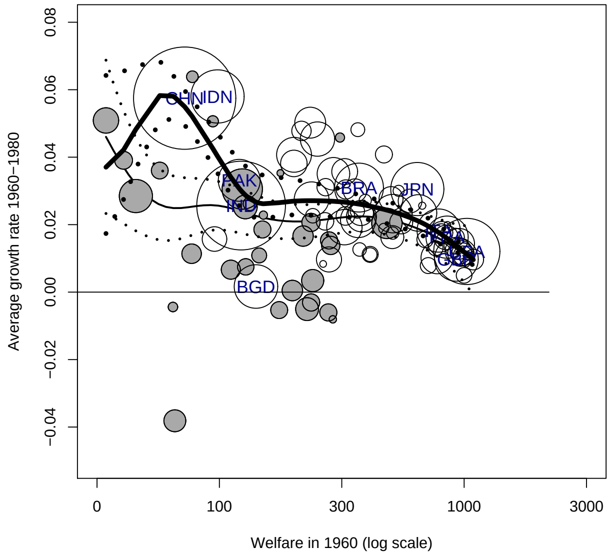

Figure 11: Growth path for welfare (g=0) in 1960-1980 (thin line: cross-country estimate, thick line: cross-population estimate).

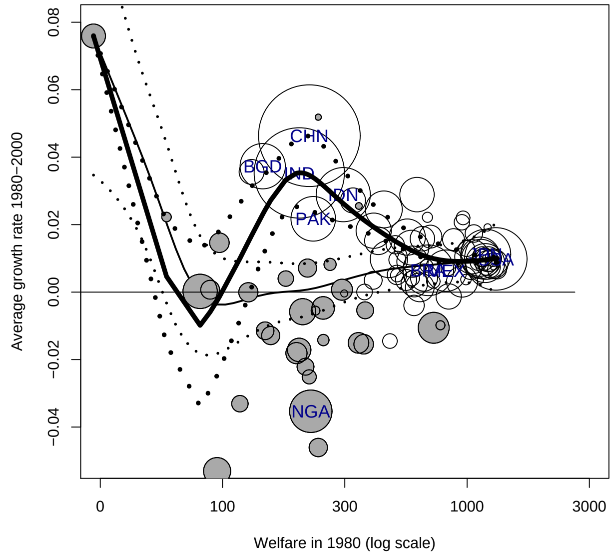

Figure 12: Growth path for welfare (g=0) in 1980-2000 (thin line: cross-country estimate, thick line: cross-population estimate).

Welfare In the period 1960-2000 there is a weak (or even null) convergence across the welfare of the countries in the sample (see Figure 10). Subperiods 1960-1980 and 1980-2000, however, present opposite patterns: in the first one convergence prevails, in the second divergence (see Figure 11 and 12). The different performances of sub-Saharan countries in the two sub-periods is the main explanation of such dynamics. The other countries with low/medium welfare (e.g. China and India) tend to converge to higher levels, while countries in the upper tail of the distribution of welfare (above 600 in 1960) do not show any convergence.

With respect to the world population, the picture partially changes. In fact, in the period 1960-2000 there is a strong convergence path (see Figure 10). As expected, the determinants of the dynamics are the population of the largest (and still poor in 1960) countries, as Bangladesh, China, India and Indonesia. The subperiod 1960-1980 was a period of strong convergence for most of the populations with low levels of welfare; on the opposite, in period 1980-2000, while the largest countries continued to follow their convergence path towards higher levels of welfare, the welfare of the population of sub-Saharan countries started diverging, with general stagnant/negative growth rates.

Overall the dynamics of welfare appear highly nonlinear and affected by a strong cross-country heterogeneity. The next section discusses the implications for the distribution dynamics.

III.D.ii. The Evolution of the Distribution of Per Capita GDP, Life Expectancy and Welfare from 1960 to 2000

The distribution dynamics is estimated by the stochastic kernel, which takes into account the nonlinearities and overcomes the bias in the estimate of the growth paths caused by the presence of cross-country heterogeneity.

Stochastic kernel indicates for each level of x at time t the probability distribution of x at time $t+\tau$, while the ergodic distribution represents the long-run tendency of the current distribution (see Quah (1997) and Durlauf and Quah

(1999) for more details).[30] In the estimate lag τ is set equal to 10 years to reduce the influence of short-run fluctuations. Observations of per capita GDP are available for every year from 1960 to 2000 (the total number of observations is therefore equal to 3977), while the observations on life expectancy and, consequently, welfare are available in 1960, 1962, 1965, 1967, 1970, 1972, 1975, 1977, 1980, 1982, 1985, 1987, 1990, 1992, 1995, 1997, 2000 (the total number of observations is equal to 1649).

In the estimate of densities and stochastic kernels we use the *adaptive kernel estimation* with the Gaussian kernel as suggested by Silverman (1986).[31]

All the figures displaying the estimates of stochastic kernel also report a solid line representing the estimated median value at $t+\tau$ conditional on the value at time t, a dotted line indicating the "ridge" of the stochastic kernel, and the 45° line.

Cross-Country Distribution Dynamics From 1960 to 2000 both inequality and polarisation of the cross-country distribution of per capita GDP increased. The Gini index significantly rose from 0.47 in 1960 to 0.55 in 2000 (the increase is statistically significant with a p-value less than 1%, see Table 2).

Table 2: Gini index of the cross-country distribution of per capita GDP, life expectancy and welfare ($g=0$) (standard errors are reported in parentheses). The results of the test on the equality between Gini indices (base-year 2000) are reported as it follows: "#" 15% significance level, "*" 10% significance level, "**" 5% and "***" 1%.

Year	GDP	Life exp.	Welfare ($g=0$)
1960	0.47 ***	0.14 ***	0.39
	(0.021)	(0.005)	(0.019)
1980	0.49 **	0.11	0.36
	(0.020)	(0.006)	(0.022)
2000	0.55	0.11	0.38
	(0.023)	(0.009)	(0.023)

The estimate of the stochastic kernel reported in Figure 14 provides the crucial information on the dynamics of polarisation: countries with a relative

[30] More formally, let $q(x_t, x_{t-\tau})$ be the joint distribution of $(x_t, x_{t-\tau})$ and $f(x_{t-\tau})$ be the marginal distribution of $x_{t-\tau}$, then the stochastic kernel is defined as $g_\tau(x_t|x_{t-\tau}) = q(x_t, x_{t-\tau})/f(x_{t-\tau})$. The ergodic distribution $f_\infty(x)$ is implicitly defined as $f_\infty(x) = \int_0^\infty g_\tau(x|z) f_\infty(z)\, dz$.

[31] See Appendix C.

per capita GDP lower than 1.4 (the point where the curve of the median value crosses the bisector from above) tend to converge towards a relative per capita GDP of about 0.5 (the first point where the curve of the median value crosses the bisector from below); countries with a relative per capita GDP higher than 1.4 tend to converge towards a relative per capita GDP of about 2.5 (the second point where the curve of the median value crosses the bisector from below).[32] These findings agree with the identification of the frontiers of regions in Figure 2.

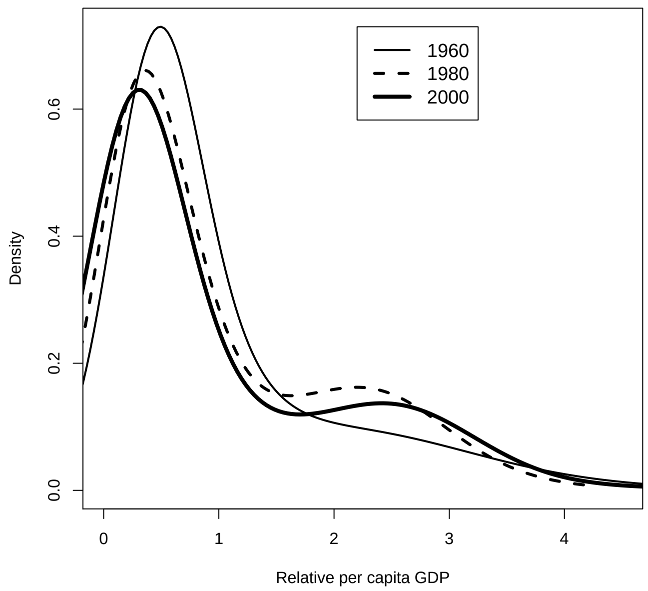

Figure 13: Cross-country distribution of relative (with respect to the average of the period) per capita GDP

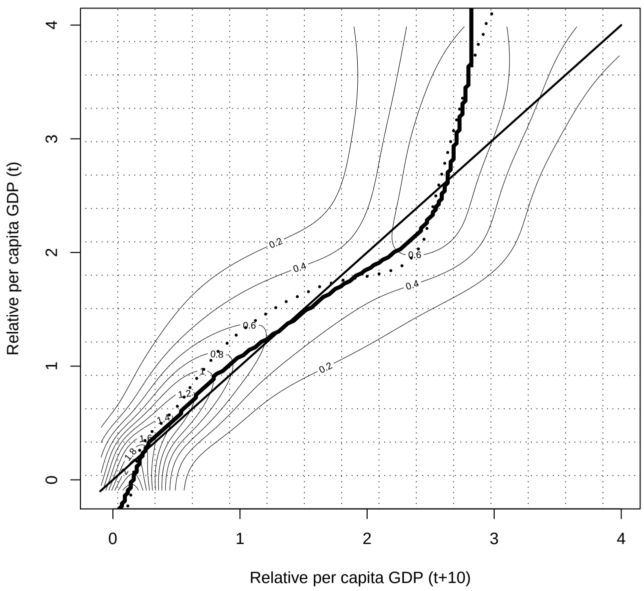

Figure 14: Stochastic kernel estimation of the relative (with respect to the average of the period) per capita GDP

Accordingly, two clusters of countries emerged in 2000 around 0.5 and 2.5 (see Figure 13), which broadly correspond to Clusters $C1$-$C2$ and $C3$ in Figure 2. Tests of multimodality state that the distribution is bimodal in 2000 (the null-hypothesis of unimodality is reject with a p-value equal to 0.03, while the null-hypothesis of bimodality cannot be rejected with a p-value equal to 0.78,

[32]The possible oversmoothing in the estimate of the stochastic kernel could make imprecise the identification of the thresholds.

see Tables 3).[33]

Table 3: P-value of the null-hypothesis of unimodality and bimodality of the cross-country distribution of per capita GDP, life expectancy and welfare ($g = 0$)

	Unimodality test			Bimodality test		
Year	GDP	Life exp.	Welfare ($g = 0$)	GDP	Life exp.	Welfare ($g = 0$)
1960	0.822	0.009	0.470	0.454	0.894	0.522
1980	0.043	0.062	0.138	0.236	0.386	0.249
2000	0.031	0.013	0.058	0.779	0.176	0.003

While the inequality of the cross-country distribution of life expectancy decreased, polarisation increased (at least from 1980). The Gini index fell from 0.14 in 1960 to 0.11 in 2000 (the decrease is statistically significant with a p-value less than 1%, see Table 2).

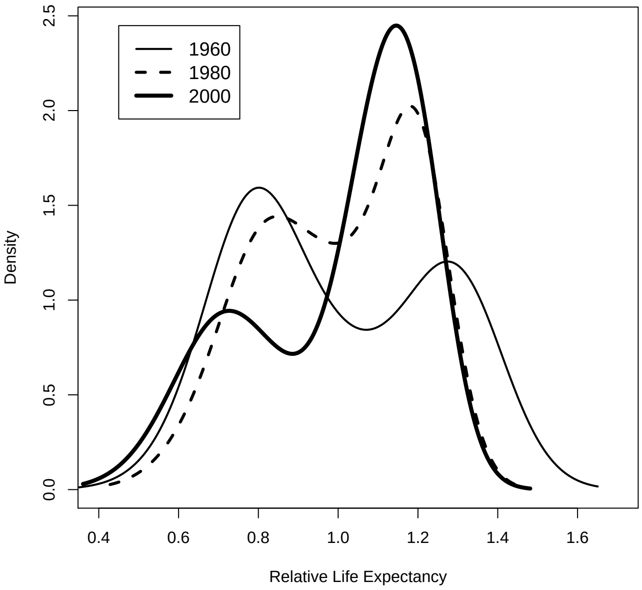

Figure 15: Cross-country distribution of relative (with respect to the average of the period) life expectancy

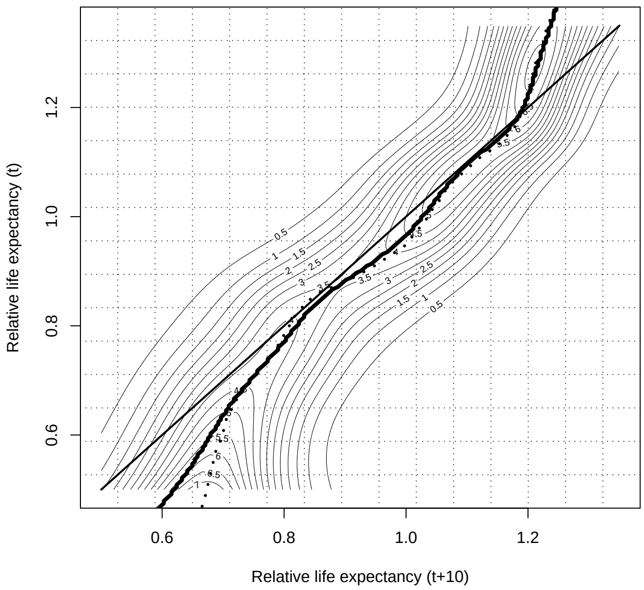

Figure 16: Stochastic kernel estimation of the relative (with respect to the average of the period) life expectancy

The estimated stochastic kernel reported in Figure 16 identifies around 0.85 the threshold for the dynamics of life expectancy. Countries with a relative life expectancy higher than 0.85 converge towards a relative life expectancy

[33]Details of tests of multimodality are gathered in Appendix D.

of about 1.2; countries with a relative life expectancy lower than 0.85 remain around that value. This agrees with the identification of the frontiers of regions in Figure 2. Accordingly, in Figure 15 two clusters of countries emerged in 2000 around 0.7 and 1.2, which broadly correspond to Clusters $C1$ and $C2$-$C3$ in Figure 2. Tests on multimodality confirms that the distribution is at least bimodal in 2000 (see Table 3).

Inequality across countries' welfare was fairly constant over the period. The Gini index fell from 0.39 in 1960 to 0.38 in 2000 (the variation is not statistically significant, see Table 2). On the contrary, the polarisation of the cross-country distribution of welfare increased from 1960 to 2000.

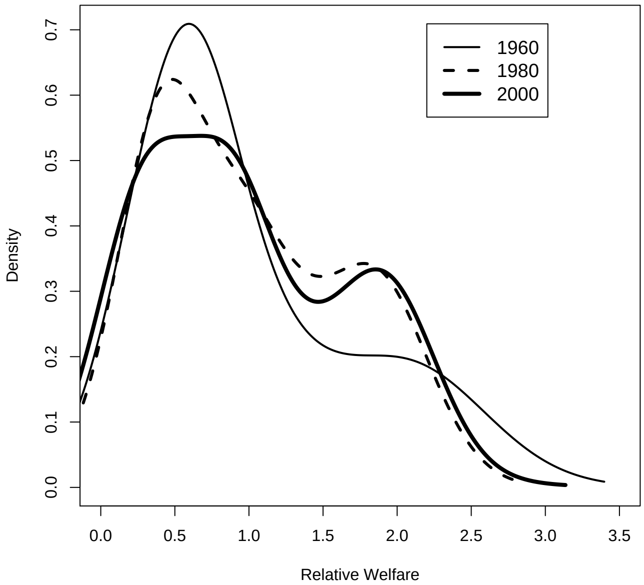

Figure 17: Cross-country distribution of relative (with respect to the average of the period) welfare ($g = 0$)

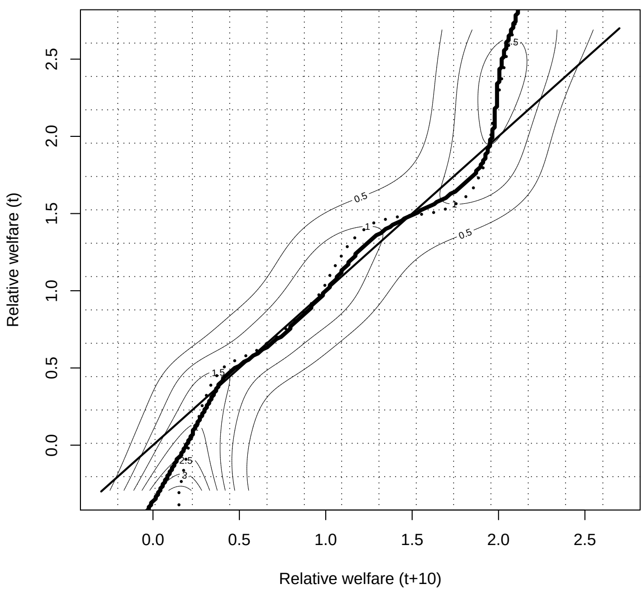

Figure 18: Stochastic kernel estimation of the relative (with respect to the average of the period) welfare (g =0)

The estimate of the stochastic kernel in Figure 18 indicates that three clusters of countries should emerge around 0.3, 1 and 2. The distribution in 2000 reported in Figure 17 displays a clear peak around 2, while the other two clusters of countries should be in correspondence of the plateau in the range (0.3,1). These figures agree with the position of Clusters $C1$, $C2$ and $C3$ in Figure 2; indeed, in terms of relative welfare, they respectively correspond to 0.27, 0.98

and 1.94.

Tests on multimodality confirm that the distribution of welfare in 2000 is (at least) trimodal (see Table 3).

The Distribution Dynamics of World Population From 1960 to 2000 inequality of per capita GDP among world population decreased, while polarisation increased. The Gini index significantly fell from 0.57 in 1960 to 0.54 in 2000 (see Table 4).

Table 4: Gini index of the cross-population distribution per capita GDP, life expectancy and welfare ($g = 0$) (standard errors are reported in parentheses). The results of the test on the equality between Gini indices (base-year 2000) are reported as it follows: "#" 15% significance level, "*" 10% significance level, "**" 5% and "***" 1%.

Year	GDP	Life exp.	Welfare ($g = 0$)
1960	0.57 * (0.012)	0.14 *** (0.006)	0.51 *** (0.013)
1980	0.59 ** (0.013)	0.08 (0.005)	0.40 *** (0.011)
2000	0.54 (0.024)	0.07 (0.008)	0.30 (0.022)

The estimate of the stochastic kernel reported in Figure 20 indicates that populations with a relative per capita GDP lower than 2 are converging towards the range $[0.4, 1]$. On the contrary, populations with a relative per capita GDP higher than 2 are converging towards 3.8. Accordingly, the distribution in 2000 shows a peak around 0.7, where are located the most populated countries and a non negligible mass around 3.5 (see Figure 19). This evidence broadly supports the definition of the regions and the identification of two clusters (Clusters $C1$-$C2$ against Cluster $C3$) in the cross-population distribution of per capita GDP reported in Figure 3.

Tests on multimodality suggest that distribution is indeed bimodal in 2000 (the null-hypothesis of unimodality is reject with a p-value equal to 0.045, while the null-hypothesis of bimodality is rejected only with a p-value equal to 0.327, see Table 5).

From 1960 to 2000 both inequality and polarisation of the cross-population distribution of life expectancy decreased. The Gini index significantly fell from

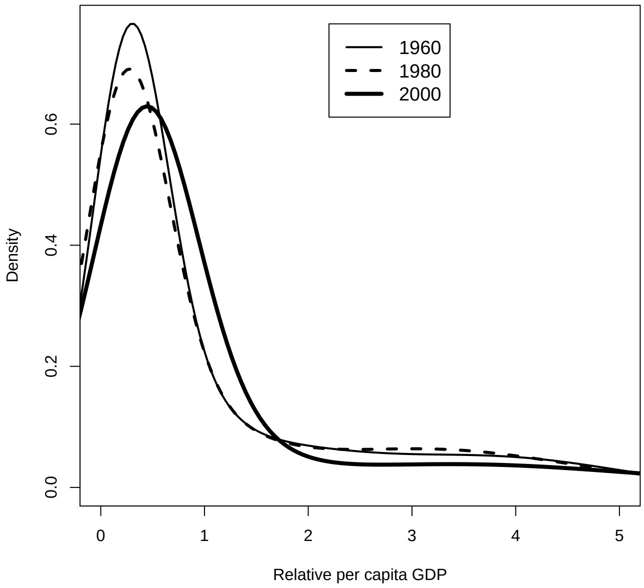

Figure 19: Cross-population distribution of relative (with respect to the average of the period) per capita GDP

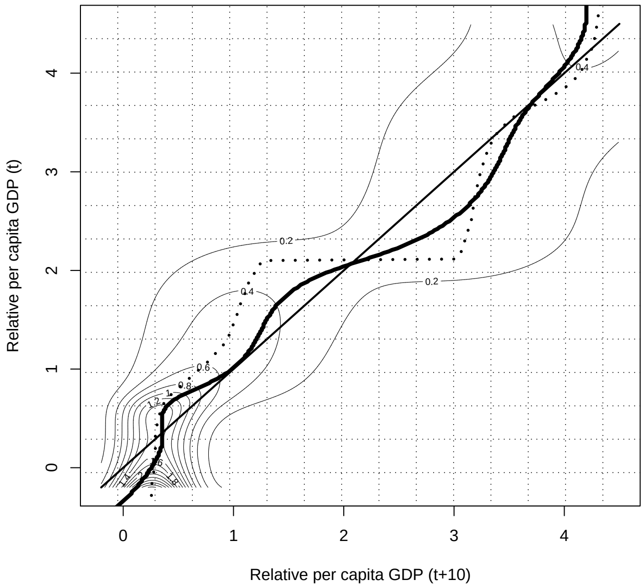

Figure 20: Cross-population distribution of relative (with respect to the average of the period) per capita GDP

Table 5: P-value of the null-hypothesis of unimodality and bimodality of the cross-population distribution of per capita GDP, life expectancy and welfare ($g = 0$)

	Unimodality test			Bimodality test		
Year	GDP	Life exp.	Welfare ($g = 0$)	GDP	Life exp.	Welfare ($g = 0$)
1960	0.721	0.013	0.055	0.600	0.093	0.539
1980	0.350	0.012	0.069	0.399	0.055	0.092
2000	0.045	0.047	0.025	0.327	0.057	0.164

0.14 in 1960 to 0.07 in 2000 (see Table 4,)

The estimate of the stochastic kernel reported in Figure 22 indicates that two clusters of populations should emerge: populations with a relative life expectancy lower than 0.9 are converging around 0.9. On the contrary, populations with a relative life expectancy higher than 0.9 are converging towards 1.1. The estimated distribution in 2000 does not appear twin-peaked (see Figure 21), probably because the formation of two clusters is still at work and, overall, the two clusters of populations are very near. However, tests of multimodality supports the multimodality of the distribution in 2000 (the null-hypothesis of unimodality is reject with a p-value equal to 0.047 as well as the null-hypothesis of bimodality, rejected with a p-value equal to 0.057, see Table 5). In Figure 3, Clusters $C1$-$C2$ and Cluster $C3$ should represent these two *clubs*. Finally, the diverging dynamics of sub-Saharan countries observed in Region I of Figure 3 is reflected by the non negligible (and increasing over time) probability mass of the left tail of the estimated distribution in 2000 (see Figure 21).

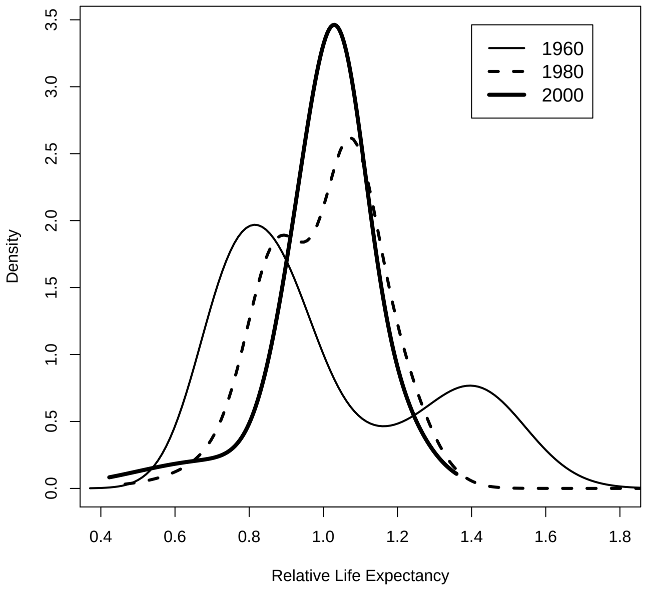

Figure 21: Cross-population distribution of relative (with respect to the average of the period) life expectancy

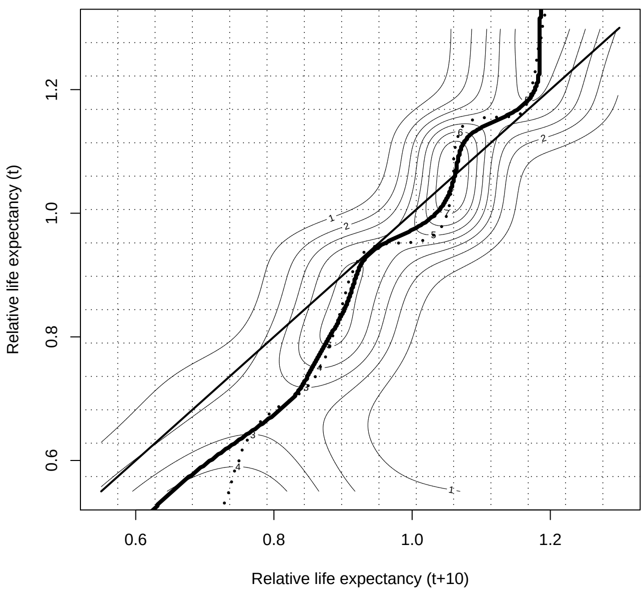

Figure 22: Cross-population distribution of relative (with respect to the average of the period) life expectancy

While inequality of the cross-population distribution of welfare decreased, polarisation increased. The Gini index significantly fell from 0.51 in 1960 to 0.30 in 2000 (see Table 4).

The estimate of the stochastic kernel indicates that the two clusters of populations should emerge around 0.7 and 2.4 (see Figure 24). The existence of two peaks around 0.8 and 2 is already clearly evident in the distribution of 2000 (see Figure 23). Accordingly, in terms of relative welfare in 2000, Clusters $C1$-$C2$ and $C3$ of Figure 3 respectively corresponds to 0.66-0.88 and 2.08.

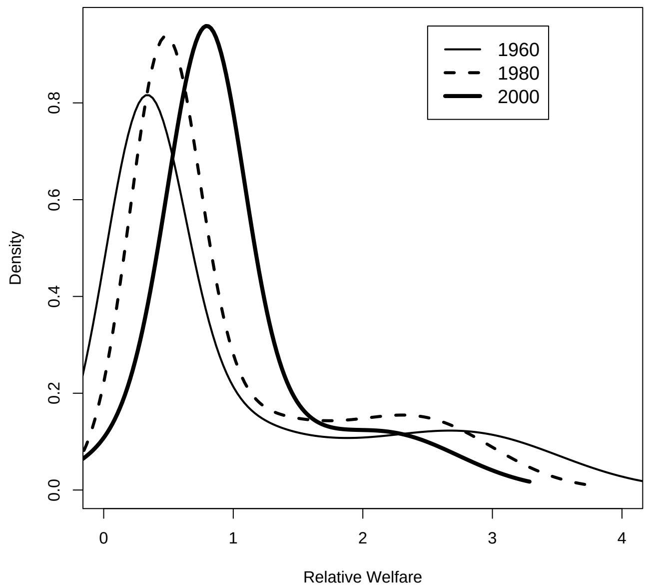

Figure 23: Cross-population distribution of relative (with respect to the average of the period) welfare ($g = 0$)

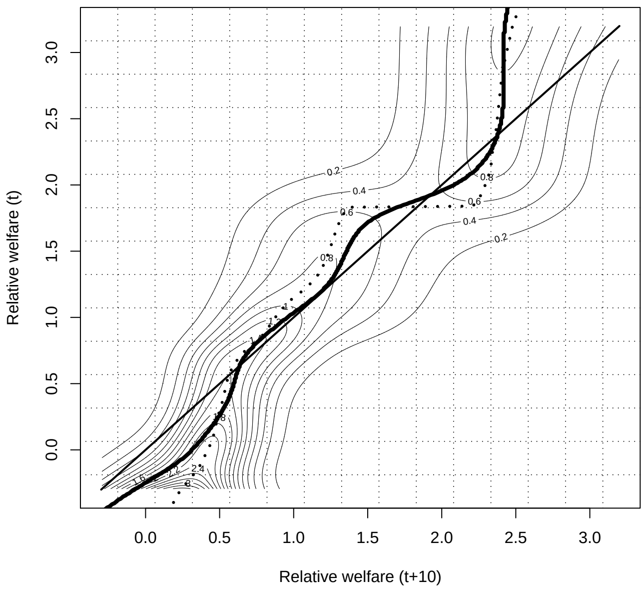

Figure 24: Cross-population distribution of relative (with respect to the average of the period) welfare ($g = 0$)

Tests of multimodality states the bimodality of the distribution in 2000 (see Table 5).

III.D.iii. The Ergodic Distribution of Per Capita GDP, Life Expectancy and Welfare: the Ergodic Distribution

The estimate of the ergodic distribution of per capita GDP, life expectancy and welfare by stochastic kernel aims at assessing the long-run tendencies resulting from the distribution dynamics discussed above. In other words, the ergodic distribution shows if the estimated distribution dynamics in the period 1960-2000 has completely exhausted its effect on the distribution in 2000 or, otherwise, significant distributional changes are expected in the future. This interpretation clearly does not take into account any structural shocks, such as the diffusion of technology and the spread of education worldwide, which might lead to non-stationary processes.

The ergodic distributions are estimated following the procedure in Johnson (2005), adjusted for the use of normalised variables (with respect to the average) in the estimate.[34] Both the ergodic distribution and the distribution in 2000 are depicted with their confidence intervals at 95% significance level, computed via a bootstrap procedure suggested in Bowman and Azzalini (1997) (see Appendix F for more details).

The Ergodic Cross-Country Distribution The inequality of the cross-country distribution of per capita GDP should remain stable. The Gini index of the ergodic distribution is equal to 0.55, the same level as in 2000 (see Table 6).[35] The dynamics of polarisation with the emergence of two clusters of countries around 0.3 and 2.5 in 2000 should persist and further increase, as highlighted in Figure 25.

Table 6: Gini index of the estimated ergodic cross-country distributions of per capita GDP, life expectancy and welfare ($g = 0$); standard errors are reported in parentheses. The results of the test on the equality between the Gini index of ergodic distribution and the one in 2000 are reported as it follows: "#"15% significance level, "*" 10% significance level, "**" 5% and "***" 1%.

Year	GDP	Life exp.	Welfare ($g = 0$)
2000	0.55 (0.023)	0.11 (0.009)	0.38 (0.023)
Ergodic	0.55 (0.010)	0.09 (0.004)	0.37 (0.013)

[34]See Appendix E for more details.

[35]Following Dorfman (1979) the Gini index of the estimated ergodic distribution, $\hat{G}$, is computed by:

$$\hat{G} = 1 - \frac{1}{\hat{\mu}} \int_0^{z^{max}} \left(1 - \hat{F}_\infty (z)\right)^2 dz, \tag{12}$$

where $\hat{f}_\infty$ is the estimate of the ergodic distribution, $\hat{F}_\infty$ its cumulative, $\hat{\mu} = \int_0^{z^{max}} \hat{f}_\infty (z)\, z dz$ and z^{max} the maximum value in the sample. Standard errors are calculated by the bootstrap procedure described in Appendix F.

Both inequality and polarisation of the cross-country distribution of life expectancy shall slightly decrease. The Gini index of the ergodic distribution is equal to 0.09 against 0.11 in 2000, but the difference is not statistically significant (see Table 6).[36] The two clusters of countries around 0.7 and 1.2, already present in the distribution of 2000, should persist, with the two modes moving closer towards the centre of distribution (see Figure 26).

[36]The hypothesis test of equality is based on the distribution of the Gini indices of the year 2000 and the ergodic distribution derived by the bootstrap procedure described in Appendix F. Via numerical integration we calculate the area of intersection of the these two distributions of Gini indices, i.e. the probability mass of the null hypothesis of equality; if such probability mass is greater than a given significance level (e.g. 1%, 5% or 10%) the null hypothesis is not rejected.

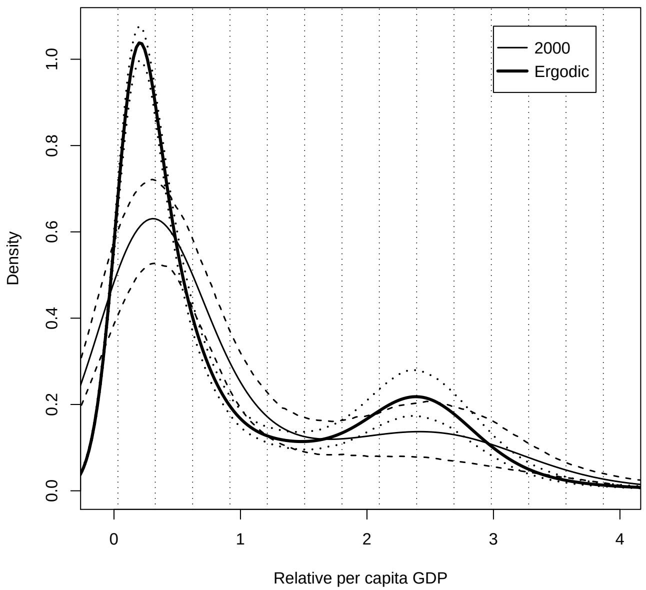

Figure 25: 2000 and ergodic distributions of relative per capita GDP across countries.

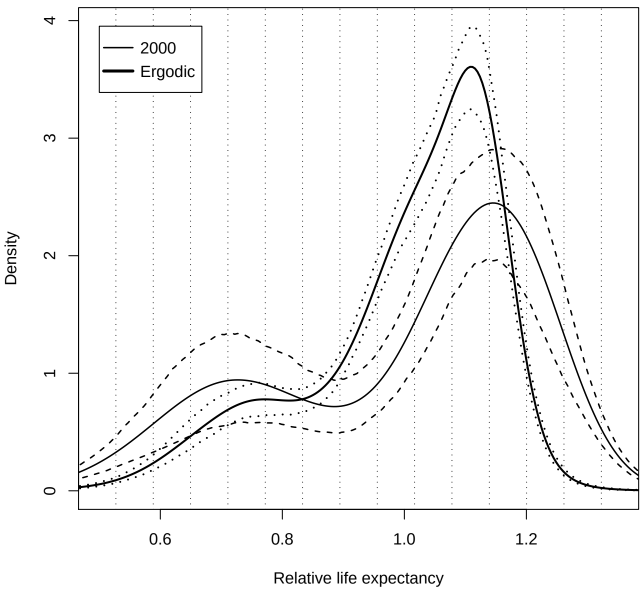

Figure 26: 2000 and ergodic distribution of the relative life expectancy across countries

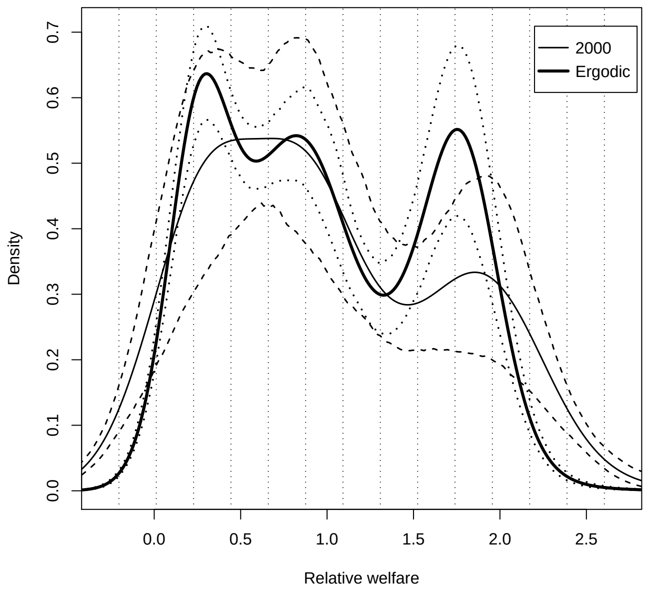

Figure 27: 2000 and ergodic distribution of the relative welfare (g =0) across countries

Finally, the cross-country distribution of welfare further increases its polarisation around the three clusters of countries already emerged in 2000 (located around 0.3, 1 and 2, see Figure 27). Such dynamics should be the result of the expected increase in polarisation of per capita GDP, being the polarisation in life expectancy slightly decreasing. Differently, welfare inequality is expected to be stable: the Gini index of the ergodic distribution is equal to 0.37 against 0.38 of distribution of 2000 (the difference is not statistically significant , see Table 6).

The Ergodic Cross-Population Distribution Both inequality and polarisation of the cross-population distribution of per capita GDP should increase. The Gini index of the ergodic distribution is indeed equal to 0.59 versus 0.54 in 2000 (see Table 7). Polarisation already present in the distribution of 2000 with two clusters of populations should persist and further reinforce with a shift of the two clusters towards 0.4 and 2.5 (see Figure 28).

Table 7: Gini index of the estimated ergodic cross-population distributions of per capita GDP, life expectancy and welfare ($g = 0$); standard errors are reported in parentheses. The results of the test on the equality between the long-term Gini index and the one in 2000 are reported as it follows: "#"15% significance level, "*" 10% significance level, "**" 5% and "***" 1%.

Year	GDP	Life exp.	Welfare ($g = 0$)
2000	0.54 (0.024)	0.07 (0.008)	0.30 (0.022)
Ergodic	0.59 # (0.008)	0.06 (0.001)	0.36 # (0.006)

Differently, the inequality of the cross-population distribution of life expectancy should remain stable, being the Gini index of the ergodic distribution equal to 0.06 against 0.07 in 2000 (see Table 7). Polarisation should decrease with the two clusters of populations already present in 2000 converging towards 1; however, the probability mass in the bottom tail of the distribution (around 0.8) tends to thicken (see Figure 29). This confirms our previous intuition that a new cluster of populations, mainly composed by the inhabitants of sub-Saharan countries, should emerge in the future.

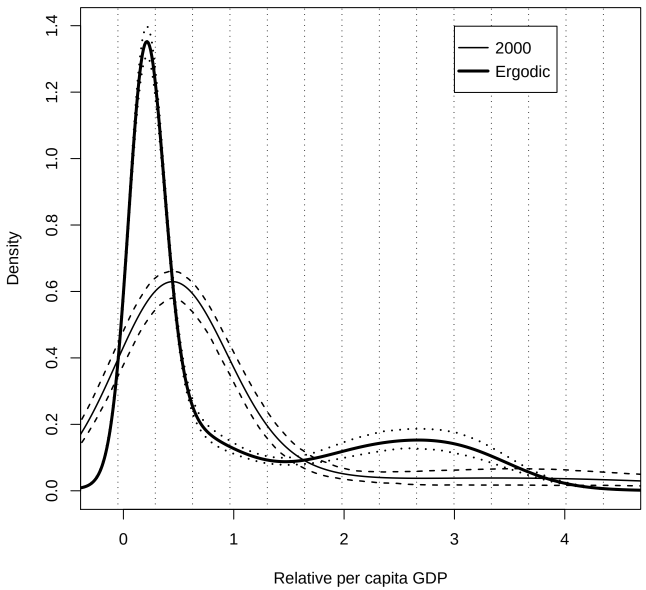

Figure 28: 2000 and ergodic distributions of relative per capita GDP across world population.

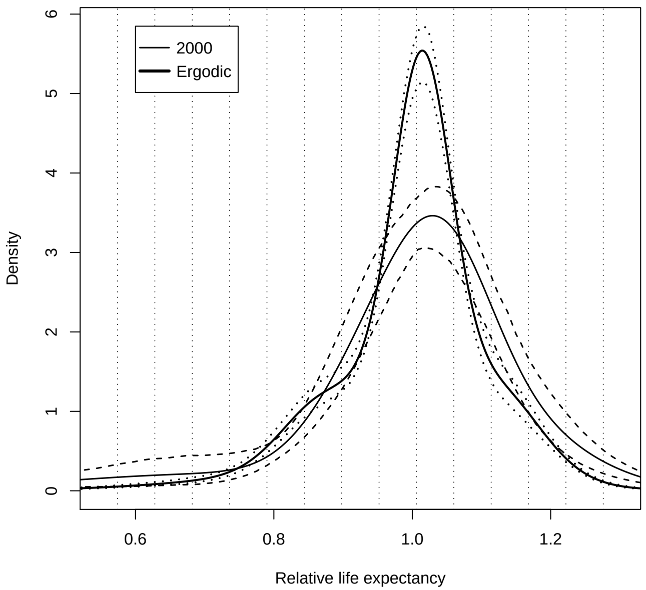

Figure 29: 2000 and ergodic distribution of relative life expectancy across world population

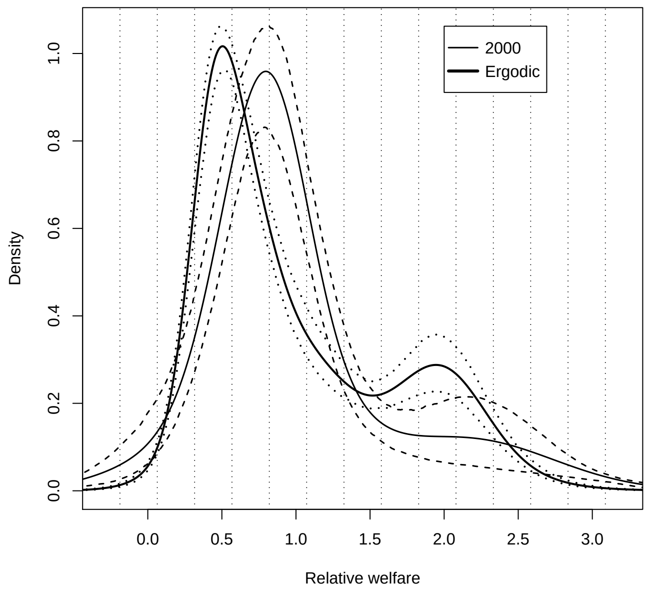

Figure 30: 2000 and ergodic distribution of relative welfare (g=0) across world population

Finally, both inequality and polarisation of the cross-population distribution of welfare should increase. The Gini index is equal to 0.36 against 0.30 in 2000 (see Table 7). Polarisation around the two clusters already present in 2000 should increase, with a shift of the modes towards 0.5 and 2.0 respectively (see Figure 27). The expected dynamics of welfare is the result of the strong (expected) increase in inequality and polarisation of per capita GDP, only marginally counterbalanced by the slight decrease (or stability) in inequality and polarisation of life expectancy.

IV. Welfare and the Growth Rate of Per Capita GDP

So far the analysis has been conducted on the assumption that all countries had the same expected zero-growth rate of income. However, setting $g = 0$ for all countries may introduce a bias because of i) the nonlinear relationships between individual welfare and the growth rate of income, the level of income and the life expectancy (see Eq.(10)); and ii) the high heterogeneity of growth rates across countries.[37]

The sensitivity of the results to the assumption of zero-growth rate of income is tested under two alternative scenarios. In the first scenario g is assumed time-constant and equal to the average growth rate of the per capita GDP of the period 1960-2000 in each country (denote it $g =$ 40y-av). In the second scenario, in each country g at time t is estimated by a moving average of its growth rates of per capita GDP in the previous $t - 20$ years (denote it $g =$ 20y-av). The choice of a 20-year period is the result of a trade-off: a longer period might reduce the impact of business cycle fluctuations; a shorter period, however, in presence of a long-run decreasing/increasing trend in growth rates, diminishes the possibility to overestimate/underestimate the expected growth rate of the

[37]For example, compare the expected welfare of a newborn in 2000 of two very different countries like US and Ghana under alternative hypotheses on g. With $g = 0$ for both countries the ratio of welfare in Ghana over the US is about 0.15, while with $g = 1.7\%$ (the average growth rate of per capita GDP of the sample) the ratio becomes 0.18. Finally, if we consider a country-specific g, equal to the average growth rate of per capita GDP experienced by each country in 1960-2000 ($g = 2.5\%$ for U.S. against $g = -0.7\%$ for Ghana), the ratio is equal to 0.09.

country.[38]

To summarise the results of this section, as regards the cross-country distribution in both scenarios inequality is slightly increasing from 1960 to 2000 (but the increase is not statistically significant). The estimated distribution dynamics suggests the emergence of three clusters of countries; the distribution in 2000 already shows three (g = 40y-av)/two peaks(g = 20y-av). The long-run (ergodic) distribution is expected to show the same level of inequality as in 2000, but with an increasing polarisation. This evidence is broadly consistent with the results with $g = 0$ reported in Section III.D., except for the higher inequality in 2000 and the more marked polarisation (at least with g = 40y-av).

As regards the distribution of welfare across world population, in both scenarios inequality is strongly decreasing from 1960 to 2000 and the estimated distribution dynamics suggests the emergence of two clusters of populations with g = 40y-av. The distribution in 2000 already appears twin-peaked. The long-run distribution is expected to show the same level of inequality as in 2000, but with an increasing polarisation (at least under g = 40y-av). These findings are broadly consistent with the results under $g = 0$, except for a weaker evidence in support of a strong polarisation with g = 20y-av.

IV.A. A First Glance at Welfare

In order to illustrate the impact of the income growth rate on welfare consider Figures 31 and 32, which report the level of welfare calculated in 1980 (circle) and 2000 (grey circle) respectively with g = 40y-av and g = 20y-av for a subsample of countries. The size of the circles are proportional to countries' welfare.[39]

The comparison of the Brazilian welfare in 1980 and 2000 with g = 20y-av provides an example of the impact on welfare of a decrease in g: both per capita

[38]It is worth to notice that the GDP growth series of the two biggest developing countries, India and China, exhibit a structural break respectively at the beginning of the '80s and the '90s (see Basu and Maertens (2007) and Smyth and Inder (2004)).

[39]The differences in the income growth rates across countries introduce an additional dimension. Hence, with respect to Figure 1 the level of welfare cannot be represented by level curves.

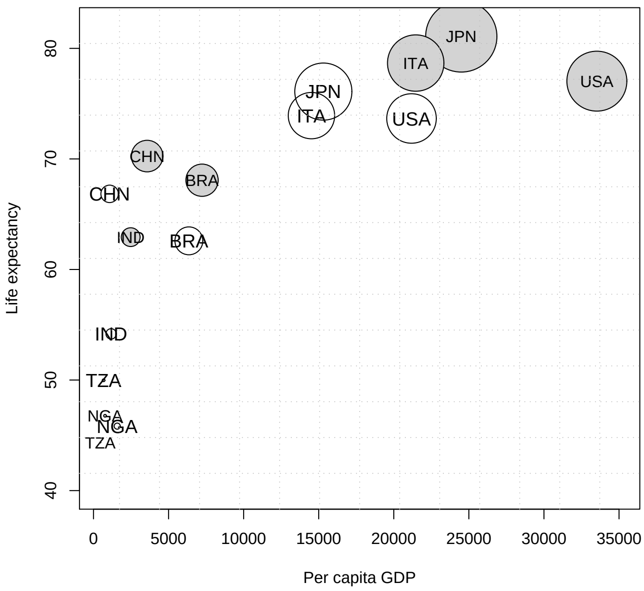

Figure 31: Welfare calculated with g equal to the average growth rate of the 40-year period 1960-2000 for a subsample of countries in 1980 (circle) and in 2000 (grey circle). The size of the circles is proportional to countries' welfare (log of).

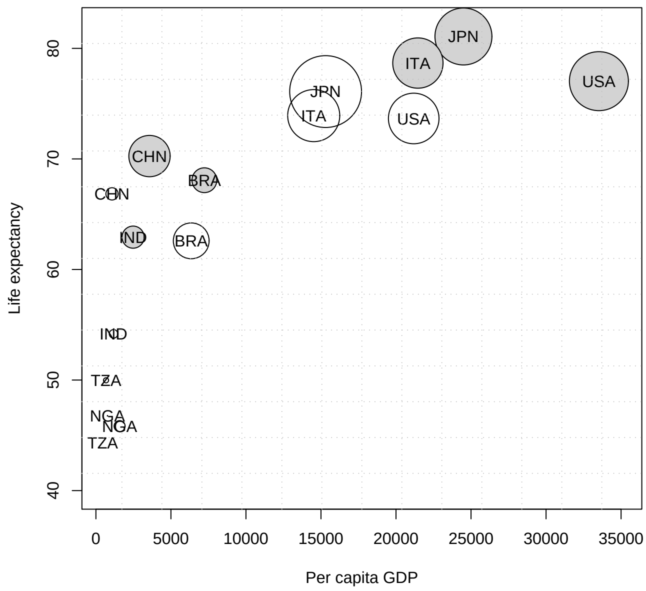

Figure 32: Welfare calculated with g equal to the moving average growth rate of the previous 20 years for a subsample of countries in 1980 (circle) and in 2000 (grey circle). The size of the circles is proportional to countries' welfare (log of).

GDP and life expectancy of Brazil increased over the period (respectively from I\$6353 to I\$7229 and from 62.6 to 68.1), but welfare decreased (from 1228 to 851) because g fell from 4.9% in 1980 to 0.7% in 2000 (see Figure 32). Moreover, Italian welfare in 2000 and US welfare in 1980 with g = 20y-av were about equal (1725 vs 1734, see Figure 32); however, both life expectancy and per capita GDP were higher in Italy in 2000 than in the US in 1980 (respectively 78.7 vs 73.7 and I\$21459 vs I\$21180). The equality in welfare was the result of the difference in the income growth rate g, which was 1.3 times higher in the US in 1980 than in Italy in 2000.

Differences between $g = 0$ and g = 40y-av are less evident, but still relevant. Compare Japan in 1980 and Italy in 2000: Italy in 2000 had both life expectancy and per capita GDP higher than Japan in 1980 (76.1 years in Japan 1980 vs 78.7 in Italy 2000; I\$15309 in Japan 2000 vs I\$21459 in Italy 2000). Nevertheless, welfare in Japan in 1980 was higher than in Italy in 2000 (1957 in Japan 1980 vs 1933 in Italy 2000), being the Japanese constant 40-year average growth rate almost 1.4 times the Italian one.

Table 8 reports some descriptives statistics of welfare distribution calculated with g = 40y-av and g = 20y-av.

Table 8: Descriptive statistics of welfare distribution (g = 40y-av and g = 20y-av)

		Across countries			Across world pop.		
Year		1960	1980	2000	1960	1980	2000
Welfare (g =40y-av)							
	Mean	563	826	1008	483	757	1008
	Gini	0.41	0.40	0.43	0.45	0.35	0.30
Welfare (g =20y-av)							
	Mean		928	921		758	1052
	Gini		0.41	0.44		0.42	0.29

Because g is positive for almost all countries, the average welfare is consistently higher than the one with $g = 0$ (compare Tables 1 and 8). However, the time-pattern are very similar, except for the average welfare of countries with g = 20y-av, which is not always increasing from 1980 to 2000.

With respect to the case $g = 0$, welfare inequality in 2000 is generally higher in the cross-country distribution and roughly equal in the cross-population distribution (compare Tables 1 and 8). On the contrary, the time-patterns appear similar, increasing in the cross-country distribution and decreasing in the cross-population one (such a declining trend is driven by the performance of China and India; without these two countries the Gini index would have been increasing from 0.35 in 1980 to 0.37 in 2000).

IV.B. Growth paths

Figures 33-38 report the growth path of welfare over the whole period (1960-2000) with g = 40y-av and in the subperiod 1980-2000 for both scenarios, g = 40y-av and g = 20y-av,[40] both for the cross-country and cross population analyses.

[40]Welfare with g = 20y-av in Uganda in 1980 and in Tanzania in 2000 is slightly negative (about -9), while the average welfare is about 950 in both years. Being the regressor the logarithm of welfare, in the estimates of growth path we set the two negative levels of welfare at a small but positive value (i.e. 5).

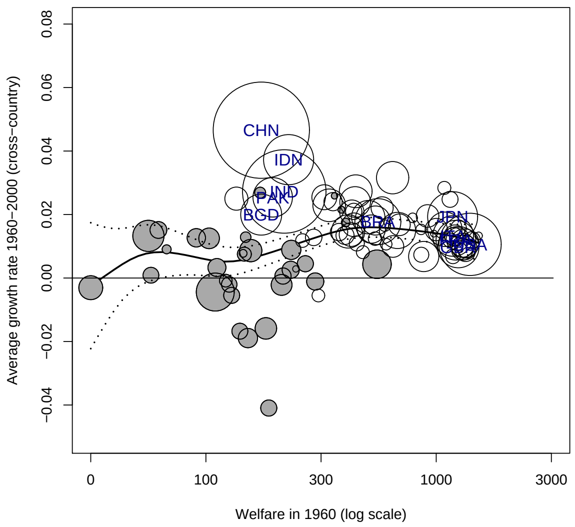

Figure 33: Growth path of welfare (g = 40y-av) in 1960-2000 (cross-country).

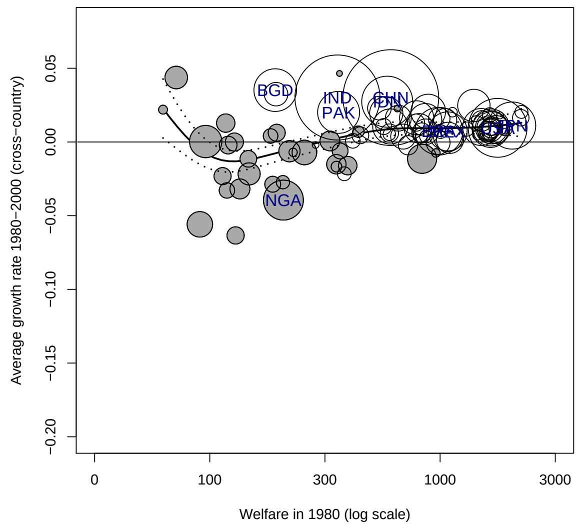

Figure 34: Growth path of welfare (g = 40y-av) in 1980-2000 (cross-country).

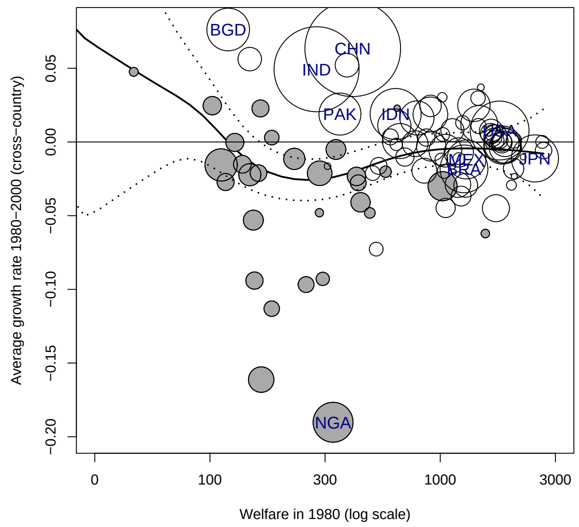

Figure 35: Growth path of welfare (g = 20y-av) in 1980-2000 (cross-country).

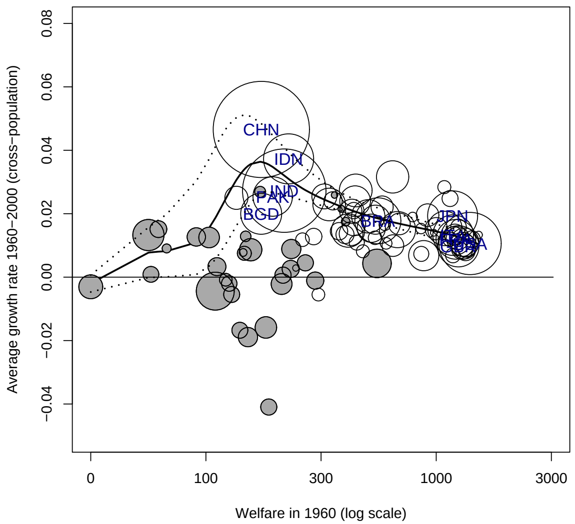

Figure 36: Growth path of welfare (g = 40y-av) in 1960-2000 (cross-population).

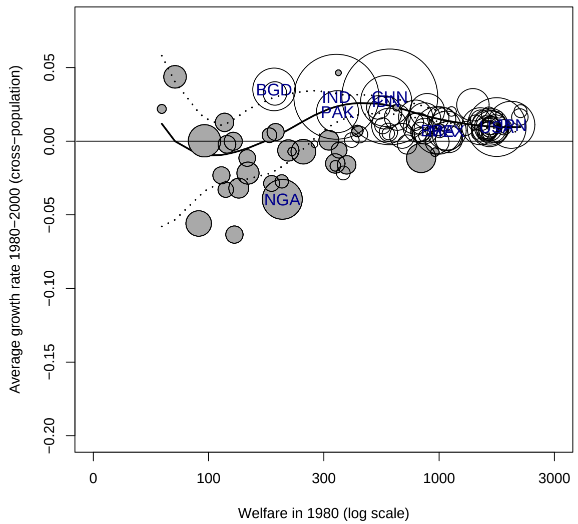

Figure 37: Growth path of welfare (g = 40y-av) in 1980-2000 (cross-population).

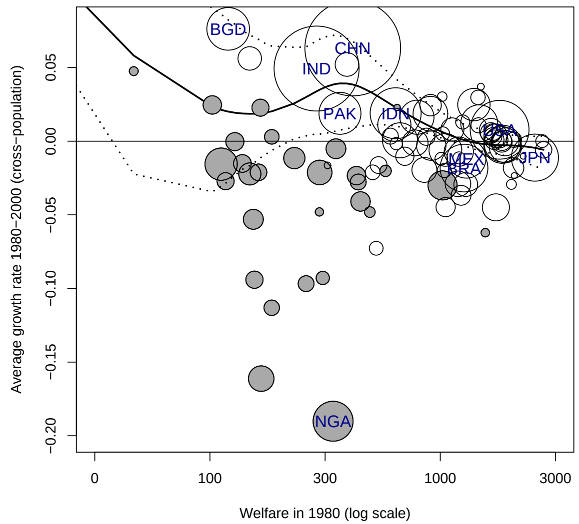

Figure 38: Growth path of welfare (g = 20y-av) in 1980-2000 (cross-population).

As with $g = 0$, cross-country welfare is not converging and, in particular, the welfare of sub-Saharan countries is diverging. Indeed, the growth path for welfare with $g =$ 40y-av is significantly increasing over the whole period 1960-2000. The same holds for the subperiod 1980-2000 for both scenarios of g. The diverging dynamics appears stronger than setting $g = 0$ and the estimates over the time-interval 1980-2000 suggests that such dynamics is accelerating (compare Figures 34 and 35 with Figure 12). In an overall downward trend of income growth rates, sub-Saharan countries show the most remarkable decline, while China and India the most remarkable rise.[41] This increase in the cross-country heterogeneity of growth rates explains the wide difference between Figures 34 and 35.

The growth path of cross-population welfare has an inverted-U shape, similar to the one with $g = 0$, but the gap between the sub-Saharan populations and the rest of the world population is even wider (see Figures 36-38).

IV.C. Cross-Country Distribution Dynamics

In both scenarios the Gini index of the welfare distribution increased steadily, even though this rise is not statistically significant (see Table 9). The cross-country heterogeneity in growth rates affects both the level of the Gini index and the dynamics: with respect to the case $g = 0$, the Gini index is always higher by 3-6 percentage points and displays an increasing trend (instead of being almost constant) over the period 1960-2000 (see Table 2).[42]

Overall the distribution dynamics of welfare seems to be robust to the different assumptions on g. As with $g = 0$, in both scenarios the estimate of the stochastic kernels suggests the emergence of three clusters of countries around 0.3, 1 and 2 (compare Figures 14, 40 and 48). In 2000 the distribution of wel-

[41] For 79 countries out of 97, the average growth rates of per capita GDP were higher in the period 1960-1980 than in 1980-2000. Moreover, the average growth of welfare in 1980-2000 is equal to -0.73% with $g =$ 20y-av and 0.48% with $g =$ 40y-av.

[42] A one-sided test where the null hypothesis is that the Gini index with $g =$ 40y-av or $g =$ 20y-av is equal to the Gini index with $g = 0$ is always rejected at 10% significance level for $g =$ 40y-av, except in 1960; for the case $g =$ 20y-av the null hypothesis is rejected at 10% significance level in 1980 and at 5% significance level in 1990 and 2000.

Table 9: Gini index of the cross-country distribution of welfare (g = 40y-av and g = 20y-av); standard errors are reported in parentheses. The results of the test on the equality between Gini indices (base-year 2000) are reported as it follows: "*" 10% significance level, "**" 5% and "***" 1%.

Year	Welfare (g = 40y-av)	Welfare (g = 20y-av)
1960	0.41 (0.019)	
1980	0.40 (0.023)	0.41 (0.026)
2000	0.43 (0.027)	0.44 (0.029)

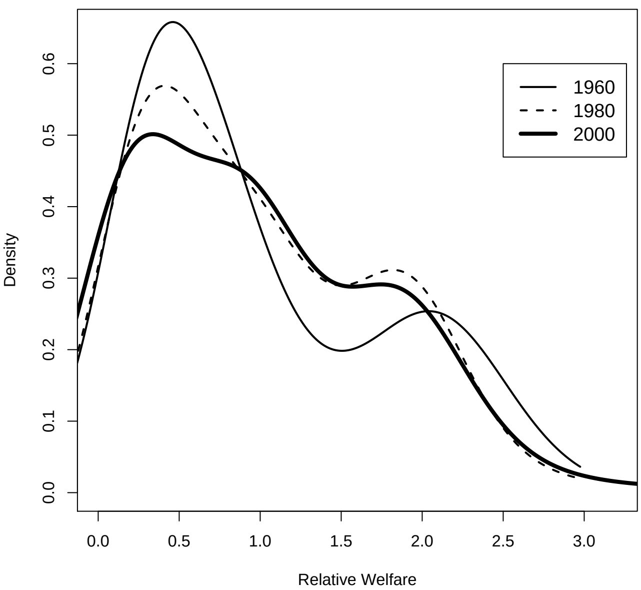

Figure 39: Cross-country distribution of relative (with respect to the average of the period) welfare (g = 40y-av)

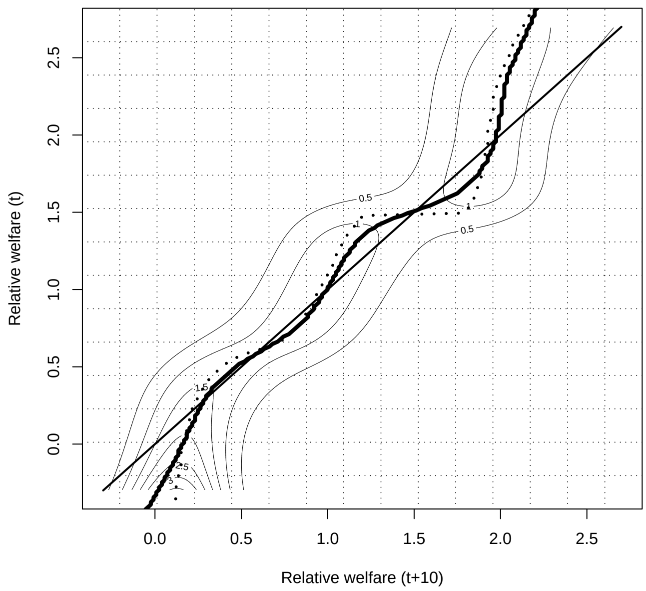

Figure 40: Stochastic kernel estimation of the relative welfare (g = 40y-av) across world population

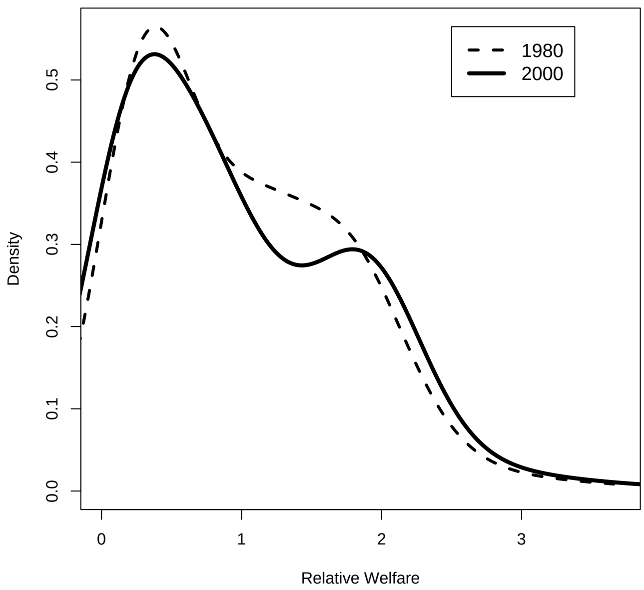

Figure 41: Cross-country distribution of relative (with respect to the average of the period) welfare (g = 20y-av)

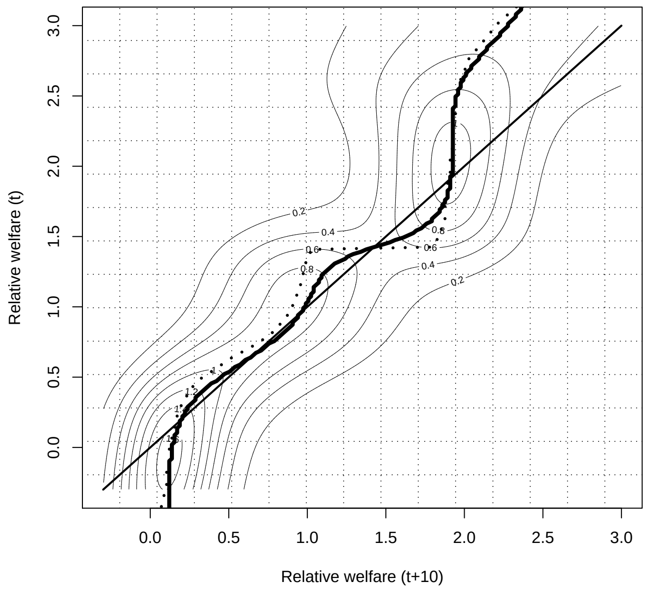

Figure 42: Stochastic kernel estimation of the relative welfare (g = 20y-av) across world population

fare already appears to be characterised by multiple peaks (compare Figures 17, 39 and 41), as confirmed also by the results of the tests of multimodality (see Table 10).

Table 10: P-value of the null-hypothesis of unimodality and bimodality of the cross-country distribution of welfare with g = 40y-av and with g = 20y-av

	Unimodality test		Bimodality test	
Year	Welfare (g = 40y-av)	Welfare (g = 20y-av)	Welfare (g = 40y-av)	Welfare (g = 20y-av)
1960	0.012		0.360	
1980	0.092	0.459	0.258	0.106
2000	0.262	0.082	0.003	0.366

IV.D. The Distribution Dynamics of World Population

The dynamics of inequality is robust to the assumptions on g also across populations, being the Gini index always decreasing from 1960 to 2000 in both scenarios (see Tables 4 and 11). The magnitude of inequality is similar as well, at least in 2000.

Table 11: Gini index of the cross-population distribution of welfare (g = 40y-av and g = 20y-av); standard errors are reported in parentheses. The results of the test of the equality between Gini indices (base-year 2000) are reported as it follows: "*" 10% significance level, "**" 5% and "***" 1%.

Year	Welfare (g = 40y-av)	Welfare (g = 20y-av)
1960	0.45^{***}	
	(0.011)	
1980	0.35^{**}	0.42^{***}
	(0.019)	(0.016)
2000	0.30	0.29
	(0.024)	(0.023)

The estimate of the stochastic kernel with g = 40y-av is close to the one with $g = 0$, suggesting the emergence of two clusters of population around 1 and 2.1 (compare Figure 20 and Figure 44). Differently, the distribution with g = 20y-av seems to concentrate around 1 (see Figure 46).

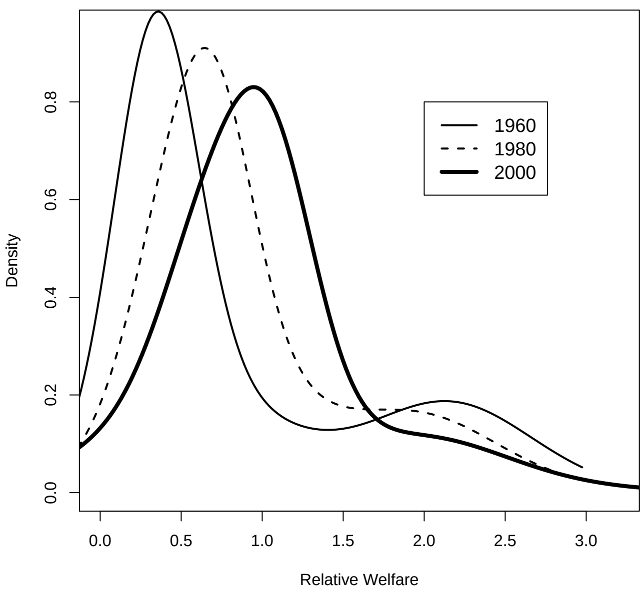

Figure 43: Cross-population distribution of relative (with respect to the average of the period) welfare (g = 40y-av)

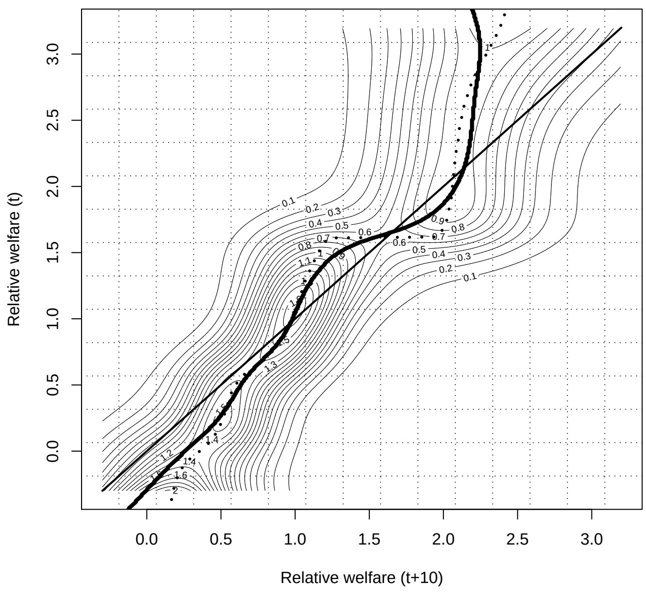

Figure 44: Stochastic kernel estimation of the relative welfare (g = 40y-av) across world population

As with $g = 0$, in both scenarios, from 1960 to 2000, the mode of the distribution shifts towards 1 (see Figures 43 and 45); on the other hand, distribution in 2000 is at least bimodal (the hypothesis of unimodality is rejected at 1% significance level, see Table 12). Indeed, independently of g, there is always a relevant probability mass around 2.

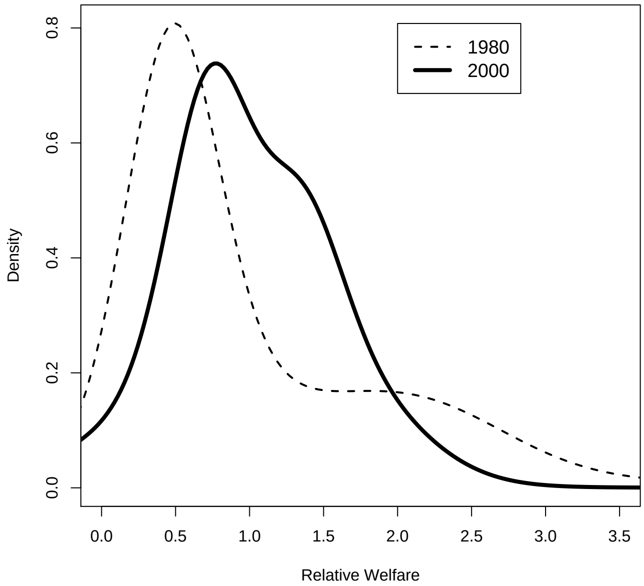

Figure 45: Cross-population distribution of relative (with respect to the average of the period) welfare ($g = 20$y-av)

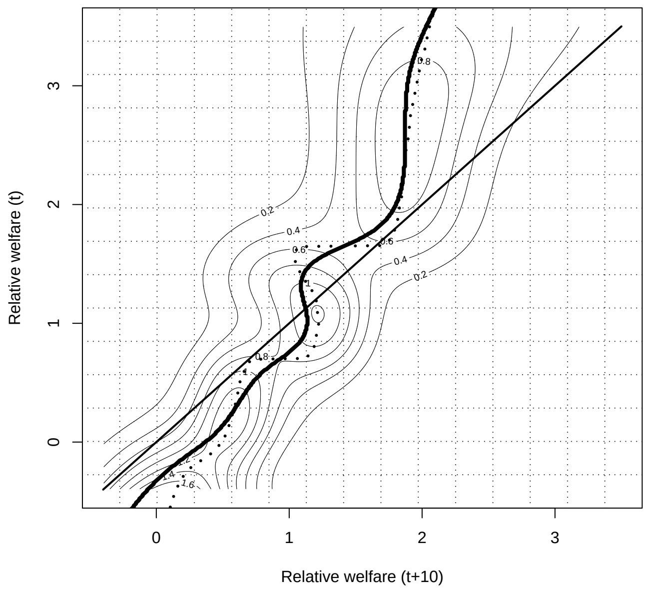

Figure 46: Stochastic kernel estimation of the relative welfare ($g = 20$y-av) across world population

Table 12: P-value of the null-hypothesis of unimodality and bimodality of the cross-population distribution of welfare with $g = 40$y-av and with $g = 20$y-av

	Unimodality test		Bimodality test	
Year	Welfare ($g = 40$y-av)	Welfare ($g = 20$y-av)	Welfare ($g = 40$y-av)	Welfare ($g = 20$y-av)
1960	0.006		0.067	
1980	0.043	0.074	0.108	0.149
2000	0.000	0.003	0.014	0.138

IV.E. The Ergodic Distribution

In the following we discuss the long-run tendencies of welfare distribution.

IV.E.i. The Ergodic Cross-Country Distribution

The inequality of cross-country distribution of welfare is expected to be slightly decreasing with g = 40y-av and slightly increasing with g = 20y-av; but, as with $g = 0$, differences with respect to inequality in 2000 are not statistically significant (see Table 13). In both scenarios the Gini index of the ergodic distribution is remarkably higher than in the case $g = 0$ (0.41 and 0.49 against 0.37, see Tables 6 and 13).

Table 13: Gini index of the estimated ergodic distributions of welfare (g = 40y-av and g = 20y-av); standard errors are reported in parentheses.

Year	Welf. (g = 40y-av)	Welf. (g = 20y-av)
2000	0.43 (0.027)	0.44 (0.029)
Ergodic	0.41 (0.011)	0.49 (0.023)

The estimated ergodic distribution with g = 40y-av shows a clear tendency to polarisation and the emergence of three peaks around 0.3, 1 and 2 (see Figure 47), the same as with $g = 0$ (see Figure 27). Also the distribution with g = 20y-av has a similar shape, even though instead of a peak there is a relevant probability mass around 1 (suggesting the possible presence of a cluster of countries, see Figure 48).

IV.E.ii. The Ergodic Cross-Population Distribution

In both scenarios the inequality of cross-population distribution of welfare is expected to be at the same level as in 2000 (see Table 14). This contrasts with the expected rise in the inequality of the distribution with $g = 0$. Accordingly, the Gini index of the ergodic distributions is expected to be lower than in the case with $g = 0$ (0.31 and 0.29 against 0.36, see Tables 7 and 14).

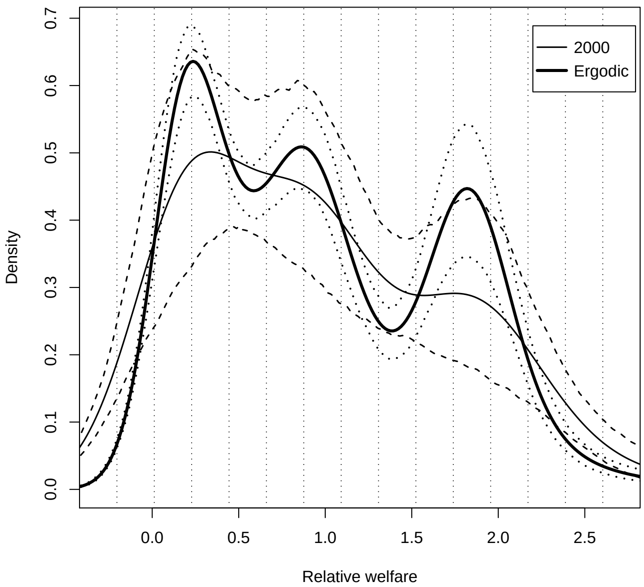

Figure 47: 2000 and ergodic distribution of the relative welfare (g = 40y-av) across countries

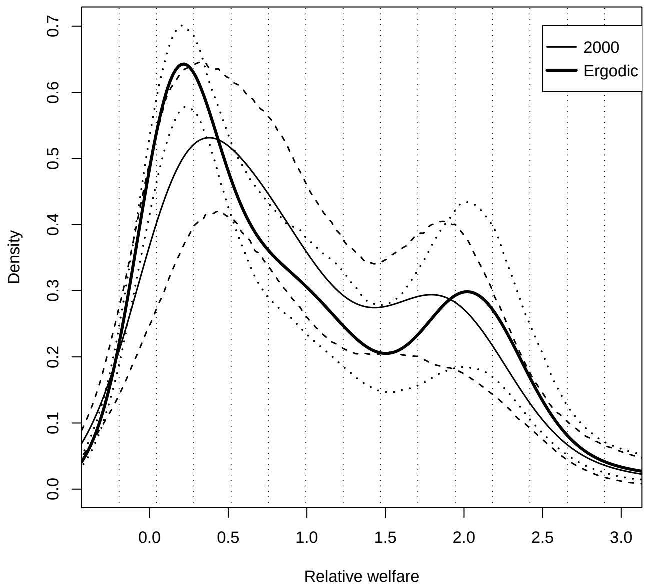

Figure 48: 2000 distribution of the relative welfare (g = 20y-av) across countries

Table 14: Gini index of the estimated ergodic distributions of welfare (g = 40y-av and g = 20y-av); standard errors are reported in parentheses.

Year	Welf. (g = 40y-av)	Welf. (g = 20y-av)
2000	0.30 (0.024)	0.29 (0.023)
Ergodic	0.31 (0.005)	0.29 (0.009)

As with $g = 0$, the estimated ergodic distribution with $g =$ 40y-av shows a clear tendency towards polarisation and the emergence of two peaks around 0.7 and 1.7 (see Figures 30 and 49). On the contrary, the ergodic distribution with $g =$ 20y-av does not show a clear pattern: a large probability mass is expected to persist around 0.7, which prevents the emergence of a clear single peak around 1 (see Figure 50).

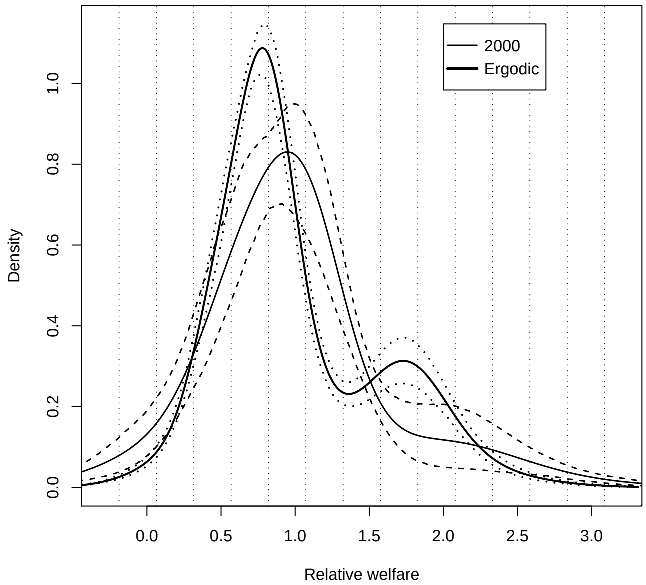

Figure 49: 2000 and ergodic distribution of the relative welfare ($g =$ 40y-av) across world population

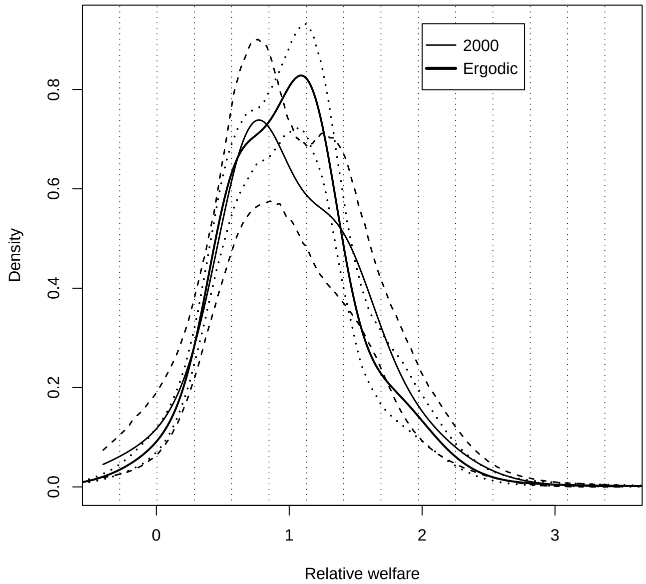

Figure 50: 2000 distribution of the relative welfare ($g =$ 20y-av) across world population

V. Concluding Remarks

The paper presents two main contributions to growth empirics literature: i) it provides a methodology to measure the welfare of a country/individual and ii) it highlights the nonlinearities in the distribution dynamics of per capita GDP, life expectancy and welfare.

The comparison with Becker et al. (2005)'s results shows the importance of considering the nonlinear relationship between levels and growth rates of welfare and of using nonparametric methods in order to detect possible dynamics of polarisation: indeed, while Becker et al. (2005) identify convergence across world population, we find strong evidence of polarisation. Moreover, the estimates of the long-run tendencies indicate that polarisation appears to be a persistent phenomenon.

Two aspects need to be further investigated. First, the methodology used to measure welfare might be extended to account for factors which appear very different across countries, such as labour market structure, provision of public goods and level of taxation, and market incompleteness. Second, in the empirical analysis the within-country distribution should be considered. Indeed, the role of within country inequality on assessing the dynamics of the world income distribution could be non-negligible, as shown by Milanovic (2005); even though the non availability of microdata on the relationship between income and life expectancy represents the main obstacle.

Acknowledgements

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A Solution of the agent's problem

The agent solves the following problem:

$$V = \max_{\{c_t\}_{t=0}^{T}} \int_0^T \left(\frac{c^{1-\sigma}}{1-\sigma} - M \right) \exp\left(-\rho t\right) S dt \tag{13}$$

$$s.t. \begin{cases} \dot{p} = p\hat{r} + yl - c; \\ p_0 = \bar{p}_0; \\ \lim_{t \to T} p \exp\left(-\hat{r}t\right) \geq 0; \end{cases}$$

where $\hat{r} = r + \pi^D$ is the interest rate adjusted for the instantaneous probability to die before T. Dynamic constraint $\dot{p} = p\hat{r} + yl - c$ in Problem 13 is directly derived by the intertemporal budget constraint given in Eq. (1).

The Hamiltonian of Problem (13) is given by:

$$\mathcal{H} = \left(\frac{c^{1-\sigma}}{1-\sigma} - M \right) \exp\left(-\rho t\right) S + \lambda \left(p\hat{r} + yl - c\right) \tag{14}$$

and the necessary and sufficient conditions of Problem (13) are the following:

$$\lambda = c^{-\sigma} \exp\left(-\rho t\right) S; \tag{15}$$

$$\dot{\lambda} = -\lambda \hat{r}; \tag{16}$$

$$\lim_{t \to T} \lambda p = 0, \tag{17}$$

from which:

$$\frac{\dot{c}}{c} = \frac{r - \rho}{\sigma} = g. \tag{18}$$

Given $\lambda\left(0\right) > 0$ and the constraints in Problem 13, Eq. (17) is always satisfied. Since r is assumed constant over time, we have:

$$c_t = c_0 \exp\left(gt\right). \tag{19}$$

The growth rate of consumption g is independent of T and S and it represents the steady-state growth rate.

Because of strict monotonicity of $u(c)$, budget constraint (1) holds with strict equality. Hence, the initial consumption level c_0 is given by:

$$c_0\left(T, w\right) = w \left[\frac{g - \hat{r}}{\exp\left(\left(g - \hat{r}\right) T\right) - 1} \right]. \tag{20}$$

Substituting Eq. (19) into Eq. (13) yields the agent's (indirect) utility:

$$V\left(T, w\right) = \frac{1}{\left(1-\sigma\right)} \left\{ c_0\left(T, w\right)^{1-\sigma} \left[\frac{\exp\left[\left(\left(1-\sigma\right) g - \hat{\rho}\right) T\right] - 1}{\left(1-\sigma\right) g - \hat{\rho}} \right] + \frac{\left(1-\sigma\right) M \left[\exp\left(-\hat{\rho} T\right) -\right.}{\hat{\rho}} \right. \tag{21}$$

where $\hat{\rho} = \rho + \pi^D$. V in Problem (13) is an improper integral for $T \to +\infty$ if $\left(g - \hat{r}\right) \geq 0$. Therefore if $T \to +\infty$ we must assume that $\left(g - \hat{r}\right) < 0$ in order to have a well-defined maximisation problem.

The agent's lifetime wealth w is therefore given by:

$$w = \frac{yl_0 \left[\exp\left(\left(g - \hat{r}\right) T\right) - 1\right]}{g - \hat{r}} + \bar{p}_0, \tag{22}$$

which substituted in Eq. (21) yields:

$$\begin{aligned} V\left(T, yl_0, g\right) = \frac{1}{1-\sigma} \Bigg\{ & \left(\frac{yl_0 \left[\exp\left(\left(g - \hat{r}\right) T\right) - 1\right]}{g - \hat{r}} + \bar{p}_0 \right)^{1-\sigma} \left(\frac{\exp\left(\left(g - \hat{r}\right) T\right) - 1}{g - \hat{r}} \right)^{\sigma} \\ & + \frac{\left(1-\sigma\right) M \left[\exp\left(-\hat{\rho} T\right) - 1\right]}{\hat{\rho}} \Bigg\}. \end{aligned} \tag{23}$$

B Country list

Algeria, Arab Rep. of Egypt, Argentina, Australia, Austria, Bangladesh, Barbados, Belgium, Benin, Bolivia, Brazil, Burkina Faso, Burundi, Cameroon, Canada, Cape Verde, Chad, Chile, China, Colombia, Comoros, Congo Rep., Costa Rica, Cote d'Ivoire, Denmark, Dominican Republic, Ecuador, El Salvador, Equatorial Guinea, Ethiopia, Finland, France, Gabon, Ghana, Greece, Guatemala, Guinea, Guinea-Bissau, Honduras, Hong Kong-China, Iceland, India, Indonesia, Ireland, Islamic Rep. of Iran, Israel, Italy, Jamaica, Japan, Kenya, Lesotho, Luxembourg, Madagascar, Malawi, Malaysia, Mali, Mauritius, Mexico, Morocco, Mozambique, Nepal, Netherlands, New Zealand, Nicaragua, Niger, Nigeria, Norway, Pakistan, Panama, Paraguay, Peru, Philippines, Portugal, Rep. of Korea, Romania, Rwanda, Senegal, Singapore, South Africa, Spain, Sri Lanka, Sweden, Switzerland, Syrian Arab Republic, Tanzania, Thailand, The Gambia, Togo, Trinidad and Tobago, Turkey, Uganda, United Kingdom, United States, Uruguay, Venezuela, Zambia, Zimbabwe.

C Adaptive Kernel Estimation

When observations vary in sparseness over the support of the distribution, the adaptive kernel estimation is a two-stage procedure which mitigates the drawbacks of a fixed bandwidth in density estimation (see Silverman (1986), p.

101). In general, given a multivariate data set $\boldsymbol{X} = \{\boldsymbol{X}_1, ..., \boldsymbol{X}_n\}$ and a vector of sample weights $\boldsymbol{W} = \{\omega_1, ..., \omega_n\}$, where $\boldsymbol{X}_i$ is a vector of dimension d and $\sum_{i=1}^{n} \omega_i = 1$, we first run the pilot estimate:

$$\tilde{f}(\boldsymbol{x}) = \frac{1}{n \det(\boldsymbol{H})} \sum_{i=1}^{n} \omega_i k \left\{ \boldsymbol{H}^{-1} (\boldsymbol{x} - \boldsymbol{X}_i) \right\}, \tag{24}$$

where $k(\boldsymbol{u}) = (2\pi)^{-1} exp(-1/2\boldsymbol{u})$ is Gaussian kernel and *bandwidth matrix* $\boldsymbol{H}$ is a diagonal matrix $(d \times d)$ with diagonal elements $(h_1, ..., h_d)$ given by the optimal normal bandwidths, i.e. $h_i = [4/(d+2)]^{1/(d+4)} \hat{\sigma}_i n^{-1/(d+4)}$; $\hat{\sigma}_i$ is the estimated standard error of the distribution of $\boldsymbol{X_i}$. The use of a diagonal bandwidth matrix instead of a full covariance matrix follows the suggestions in Wand and Jones (1993). In the case of $d = 1$ we have $\boldsymbol{H} = \det(\boldsymbol{H}) = (4/3)^{1/5} n^{-1/5} \hat{\sigma}$. In the cross-country estimate we consider $\boldsymbol{W} = \{1, ..., 1\}$, while in the cross-population estimate $\boldsymbol{W} = \{p_i, ..., p_n\}$, where p_i is the population of country i. We then define local bandwidth factors λ_i by:

$$\lambda_i = \left[\tilde{f}(\boldsymbol{X}_i) / g \right]^{-\alpha}, \tag{25}$$

where $\log(g) = \sum_{i=1}^{n} \omega_i \log\left(\tilde{f}(\boldsymbol{X}_i)\right)$ and $\alpha \in [0, 1]$ is a sensitivity parameter. We set $\alpha = 1/2$ as suggested by Silverman (1986), p. 103. Finally the adaptive kernel estimate $\hat{f}(x)$ is defined as:

$$\hat{f}(\boldsymbol{x}) = \frac{1}{n \det(\boldsymbol{H})} \sum_{i=1}^{n} \lambda_i^{-d} \omega_i k \left\{ \lambda_i^{-1} \boldsymbol{H}^{-1} (\boldsymbol{x} - \boldsymbol{X}_i) \right\}. \tag{26}$$

The Gaussian kernel guarantees that the number of modes is a decreasing function of the bandwidth; such a property is at the basis of the test for unimodality (see Silverman (1986), p. 139). In all the estimates we use package *sm* (see Bowman and Azzalini (2005)).

D Test of multimodality

Test of multimodality follows the bootstrap procedure described in Silverman (1986), p. 146. Given a data set $\boldsymbol{X} = \{x_1, \ldots, x_n\}$ and a vector of sample weights $\boldsymbol{W} = \{\omega_1, \ldots, \omega_n\}$, we calculate the smallest value of bandwidth, $\hat{h}_0$, for which the estimated distribution is unimodal and the corresponding local bandwidth factors $\boldsymbol{\Lambda} = \lambda_1, \ldots, \lambda_n$. We then perform a *smoothed bootstrap* from the estimated density of observed data set. Since we use the Gaussian kernel, it amounts to: i) draw (with replacement) a vector $I = \{i_1, \ldots, i_n\}$ of size n from $\{1, \ldots, n\}$, given the sample weights $\boldsymbol{W}$; ii) define $Y = \{x_{i_1}, \ldots, x_{i_n}\}$ and $W^* = \{\omega_{i_1}, \ldots, \omega_{i_n}\}$, calculate

$$x_j^* = \bar{\boldsymbol{Y}} + \left(1 + \left(\hat{h}_0 \lambda_{i_j}\right)^2 / \hat{\sigma}_{\boldsymbol{Y}}^2\right)^{-\frac{1}{2}} \left(y_j - \bar{\boldsymbol{Y}} + \hat{h}_0 \lambda_{i_j} \epsilon_j\right); \qquad j = 1, \ldots, n; \quad (27)$$

where $\bar{\boldsymbol{Y}}$ and $\hat{\sigma}_{\boldsymbol{Y}}^2$ are respectively the mean and the estimate variance of sample $\boldsymbol{Y}$ and ϵ_j are standard normal random variables; iii) find the minimum value of bandwidth, $\hat{h}_1^*$, for which the estimated density of $\boldsymbol{X}^*$ is unimodal; iv) repeat point i)-iii) B times in order to obtain a vector of critical values of bandwidth $\left\{\hat{h}_1^*, \ldots, \hat{h}_B^*\right\}$. Finally, p-value of null-hypothesis of unimodality is given by $\#\left\{\hat{h}_b^* \geq \hat{h}_0\right\}/B$. For testing the bimodality, point iii) has to be modified accordingly. We set $B = 1000$.

E The Estimate of Ergodic Distribution

The ergodic distribution solves:

$$f_\infty(x) = \int_0^\infty g_\tau(x|z)\, f_\infty(z)\, dz, \quad (28)$$

where x and z are two levels of the variable, $g_\tau(x|z)$ is the density of x, given z, τ periods ahead, under the constraint

$$\int_0^\infty f_\infty(x)\, dx = 1. \quad (29)$$

Since in our estimates all variables are normalized with respect to their average, the ergodic distribution, moreover, must respect the additional constraint:

$$\int_0^\infty f_\infty(x)\, x dx = 1. \tag{30}$$

Following the methodology proposed by Johnson (2005) we first estimate the distribution $\tilde{f}_\infty(x)$, which satisfies Constraints 28 and 29, but not Constraint 30. We then calculate $f_\infty(x) = \tilde{\mu}_x \tilde{f}_\infty(x)$, where $\tilde{\mu}_x = \int_0^\infty \tilde{f}_\infty(x)\, x dx$, which will satisfy all Constraints 28, 29 and 30. In particular, Theorems 11 and 13 in Mood et al. (1974), p. 200 and 205 prove that if $\tilde{f}_\infty(x)$ satisfies Constraints 28 and 29 then $f_\infty(x)$ satisfies Constraints 28, 29 and 30. In fact, $g_\tau(z|x) = f_{z,x}(z,x)/f_x(x)$ and $f_{y,q}(y,q) = \mu_z \mu_x f_{z,x}(z,x)$, where $y = z/\mu_z$ and $q = x/\mu_x$. In all computations we set $\tau = 10$.

F Bootstrap procedure to calculate confidence intervals for density estimation

In the following we describe the bootstrap procedure used to calculate the confidence intervals for the estimates of densities and ergodic distributions; this is based on the procedure reported in Bowman and Azzalini (1997), p. 41. Given a sample $\boldsymbol{X} = \{\boldsymbol{X}_1, ..., \boldsymbol{X}_n\}$ of observations and a vector of sample weights $\boldsymbol{W} = \{\omega_1, ..., \omega_n\}$, where $\sum_{i=1}^n \omega_i = 1$ and $\boldsymbol{X}_i$ is a vector of d dimensions, the bootstrap procedure is as follows.

1. Construct a density estimate $\hat{\phi}$ from sample $\boldsymbol{X}$, given the sample weights $\boldsymbol{W}$.

2. Resample $\boldsymbol{X}$ with replacement, taking into account the sample weights $\boldsymbol{W}$, to produce a bootstrap sample $\boldsymbol{X}^*$.

3. Construct a density estimate $\hat{\phi}^*$ from $\boldsymbol{X}^*$.

4. Repeat steps 2. and 3. B times in order to create a collection of bootstrap density estimates $\left\{\hat{\phi}_1^*, ..., \hat{\phi}_n^*\right\}$.

The distribution of $\hat{\phi}_i^*$ about $\hat{\phi}$ therefore can be used to mimic the distribution of $\hat{\phi}$ about ϕ, as discussed by Bowman and Azzalini (1997), p. 41, i.e. to calculate confidence intervals for the estimates. In particular, the confidence interval for the distribution in 2000 corresponds to the case $\hat{\phi} = \hat{f}$, while for the ergodic distribution to the case $\hat{\phi} = \hat{f_\infty}$. In the bootstrap procedure $\hat{\phi}^*$ are calculated taking the bandwidth(s) equal to the bandwidth(s) calculated for the observed sample $\boldsymbol{X}$, as suggested in Bowman and Azzalini (1997), p. 41. We set $B = 300$.

G Sensitivity Analysis

This section examines how the choice of parameters used in the calculation of welfare, i.e. ρ, σ and M, affects our findings (see Eq. (10)).

In order to accomplish this task we run three sets of experiments for both cross-country and cross-population distributions of welfare. In the first set, taking the values of σ and M used in the analysis (i.e. $\sigma = 0.8$ and $M = 16.2$), the distribution of welfare is calculated for the following alternative values of ρ: $(0.004, 0.0045, 0.005, 0.0055, 0.006)$. In the second set, taking the values of ρ and M used in the analysis (i.e. $\rho = 0.0.005$ and $M = 16.2$), the distribution of welfare is calculated for the following values of σ: $(0.64, 0.72, 0.8, 0.88, 0.96)$. In this second set of experiments we are implicitly considering alternative values of c^{ZUC} (about $(134, 221, 357, 255, 0)$ respectively, see Eq. (5)). This suggests the third set of experiments, where the distribution of welfare is calculated for five combinations of σ and M such that c^{ZUC} is at the level used in the analysis (i.e. equal to 357); in particular, taking $\rho = 0.005$, we consider the following couples of σ and M:

$$[(0.64, 23.05), (0.72, 18.52), (0.8, 16.2), (0.88, 16.87), (0.96, 31.63)].$$

The robustness of our findings is tested in terms of Gini index of the distribution of welfare (for every Gini index it is reported also its standard error) and of the tests of unimodality and bimodality of the distribution of welfare in 1960, 1980 and 2000.[43] Figures 51- 68 report the outcomes of the three experiments.

Our findings appear broadly robust to changes in parameters. In particular, we remark that:

- ρ does not appear to affect neither the magnitude of Gini index neither the tests of unimodality and bimodality for both cross-country and cross-population distributions (see Figures 51-53 and 60-62);
- σ does not appear to affect neither the magnitude of Gini index neither the tests of unimodality and bimodality for both cross-country and cross-population distributions (see Figures 54-56 and 63-65) except for the cases with $\sigma = 0.96$ and $\sigma = 0.64$. With $\sigma = 0.96$ in all three years 1960, 1980 and 2000 Gini index is remarkable reduced for both the cross-country and cross-population estimates (see Figures 54 and 63) and, less important, cross-country distribution appears to be at least bimodal already in 1960 at 10% significance level (see Figure 56). The decrease in Gini index reflects the fact that zero utility consumption c^{ZUC} is equal to 0 with this setting of parameters (in all the other cases c^{ZUC} is at least higher than 100). Heuristically, a decrease in c^{ZUC} means an upward shift of utility function; given the concavity of utility function, *ceteris paribus*, it should lead to more equal distribution of welfare. The other exception regards the case with $\sigma = 0.64$ for the cross-population estimates in 2000: the test of bimodality is rejected at 15% significance level (instead of at 10% with $\sigma = 0.80$, see Figure 65);
- different combinations of σ and M which maintain the level of c^{ZUC} equal to 357 do not appear to affect the results (see Figures 57-59 and 66-68, where the results are reported in terms of σ). Two minor exceptions are: i) the level of Gini index is always decreasing with the level of σ (see Figures 57 and66); however, the time evolution of Gini index appears unchanged (in the cross-country estimates Gini index has not any statistical significant

[43]Calculation of standard errors of Gini indexes and tests of unimodality and bimodality follow the same procedure used in Section III.D.ii..

change from 1960 to 2000, while in the cross-population estimates Gini index had a statistically significant fall from 1960 to 2000); and ii) the tests of bimodality of cross-population distribution with $\sigma = 0.64$ and $\sigma = 0.72$, rejected at 15% significance level (instead of at 10% with $\sigma = 0.80$, see Figure 68).

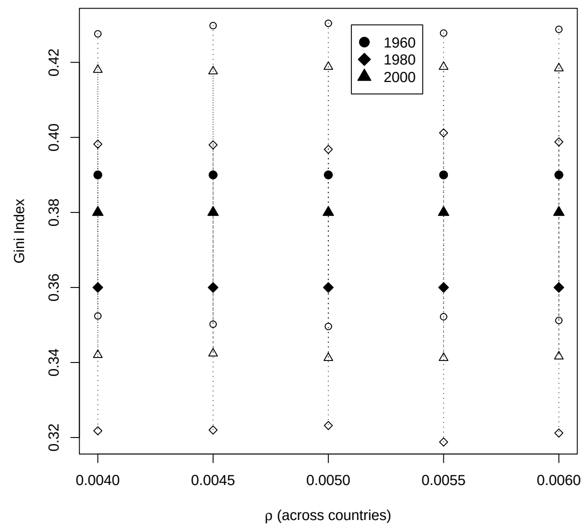

Figure 51: Gini index of the cross-country distribution of welfare with alternative values of ρ. Full points (or diamonds and triangles) represents Gini indexes in 1960 (or 1980 or 2000 respectively). The dotted line between two empty points (or triangles or diamonds) represents the range of +/- 2 standard errors around Gini index in 1960 (or 1980 or 2000 respectively).

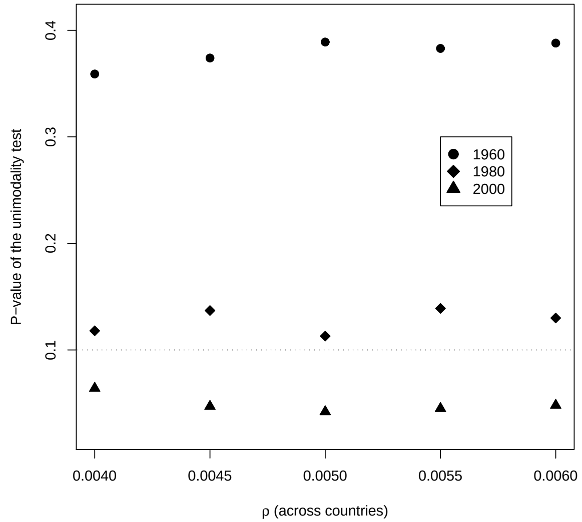

Figure 52: Unimodality test on welfare distribution with alternative values of ρ (across countries).

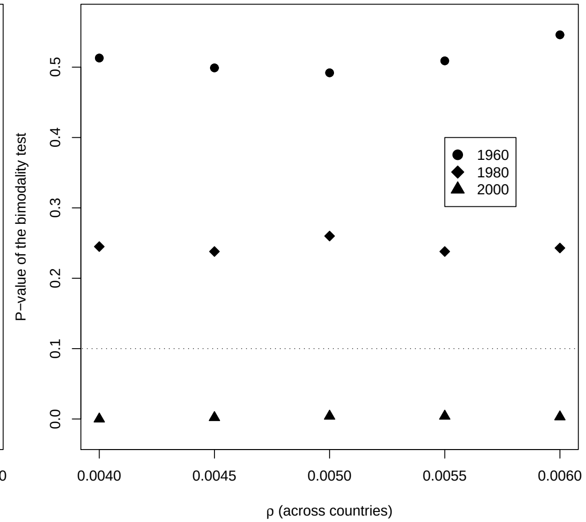

Figure 53: Bimodality test on welfare distribution with alternative values of ρ (across countries).

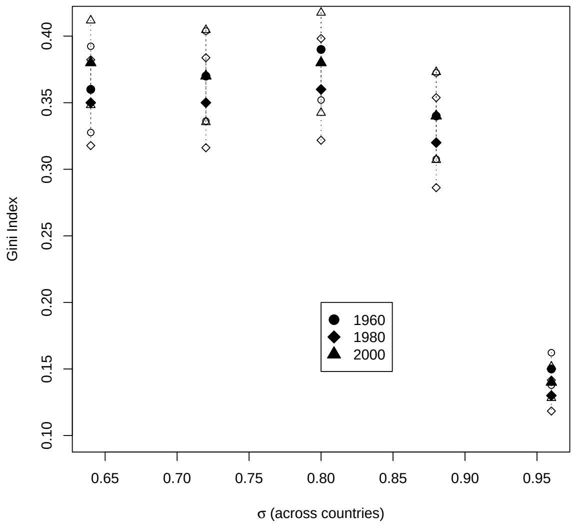

Figure 54: Gini index of welfare distribution with alternative values of σ (across countries). Full points (or diamonds and triangles) represents Gini indexes in 1960 (or 1980 or 2000 respectively). The dotted line between two empty points (or triangles or diamonds) represents the range of +/- 2 standard errors around Gini index in 1960 (or 1980 or 2000 respectively).

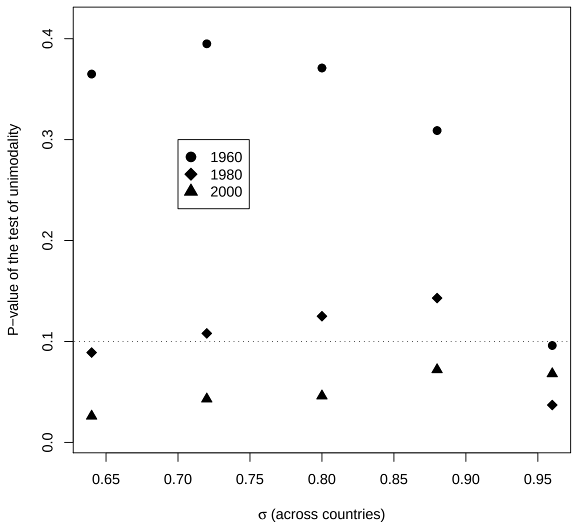

Figure 55: Unimodality test on welfare distribution with alternative values of σ (across countries). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

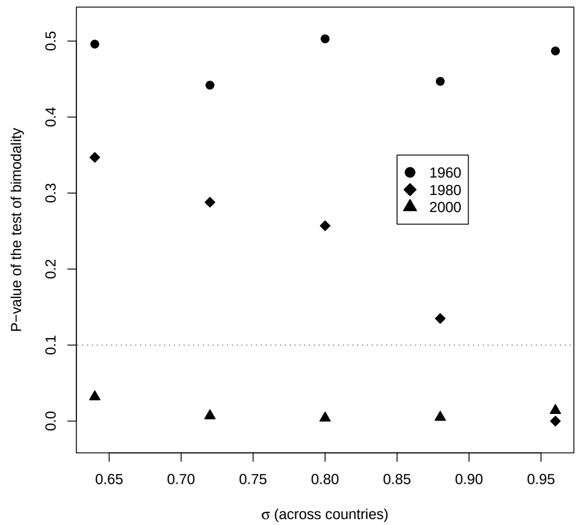

Figure 56: Bimodality test on welfare distribution with alternative values of σ (across countries). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

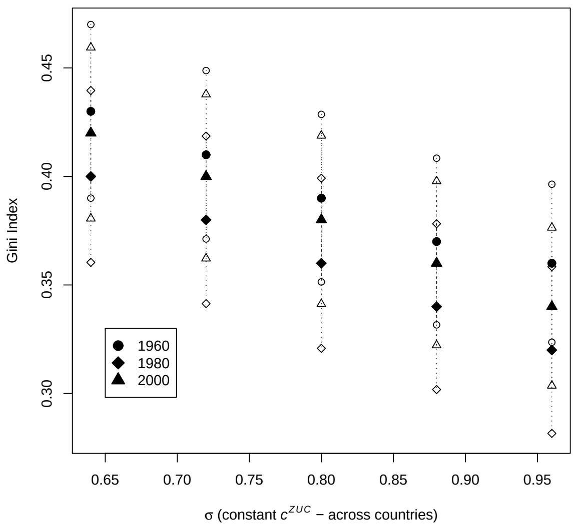

Figure 57: Gini index of welfare distribution with alternative values of σ and M such that $c^{ZUC} = 357$ (across countries). Full points (or diamonds and triangles) represents Gini indexes in 1960 (or 1980 or 2000 respectively). The dotted line between two empty points (or triangles or diamonds) represents the range of +/- 2 standard errors around Gini index in 1960 (or 1980 or 2000 respectively).

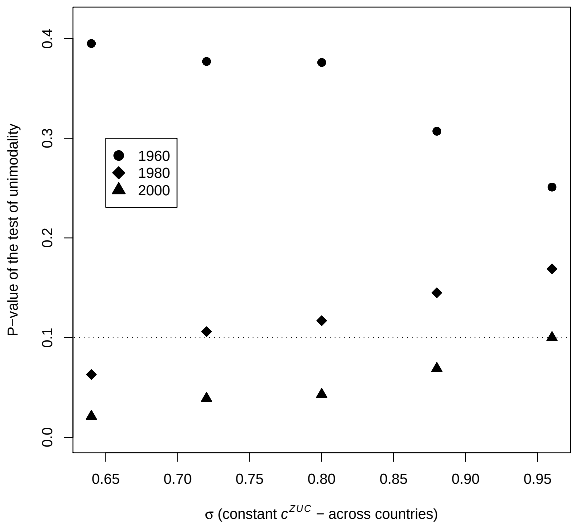

Figure 58: Unimodality test on welfare distribution with alternative values of σ and M such that $c^{ZUC} = 357$ (across countries). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

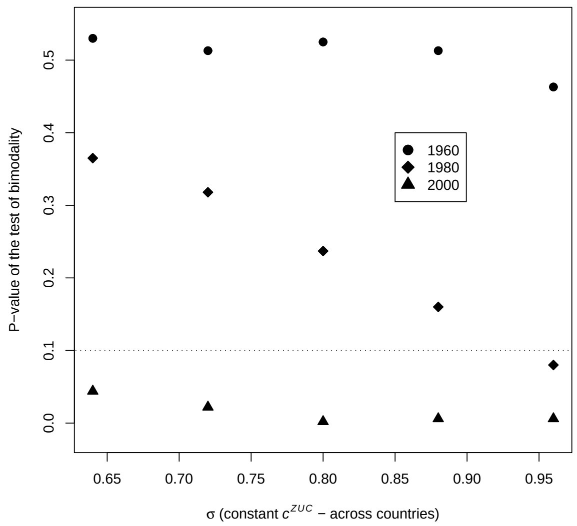

Figure 59: Bimodality test on welfare distribution with alternative values of σ and M such that $c^{ZUC} = 357$ (across countries). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

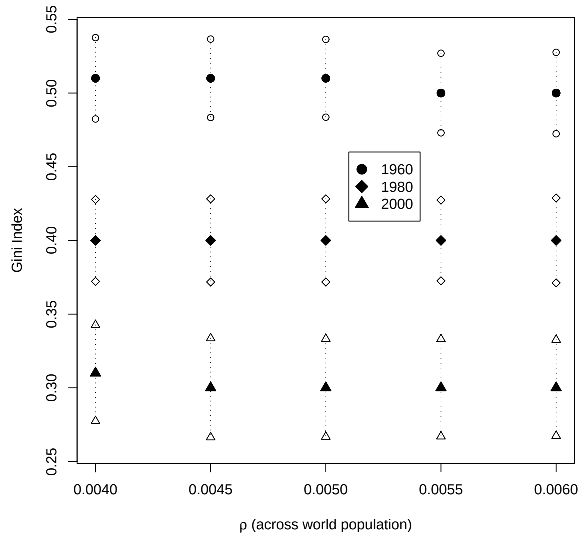

Figure 60: Gini index of welfare distribution with alternative values of ρ (across world population). Full points (or diamonds and triangles) represents Gini indexes in 1960 (or 1980 or 2000 respectively). The dotted line between two empty points (or triangles or diamonds) represents the range of +/- 2 standard errors around Gini index in 1960 (or 1980 or 2000 respectively).

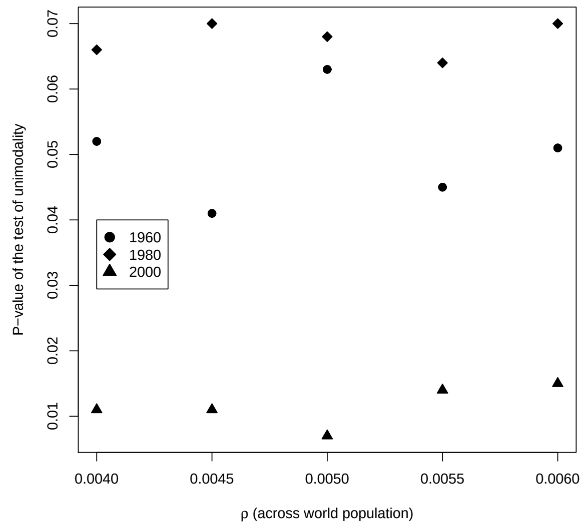

Figure 61: Unimodality test on welfare distribution with alternative values of ρ (across world population). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

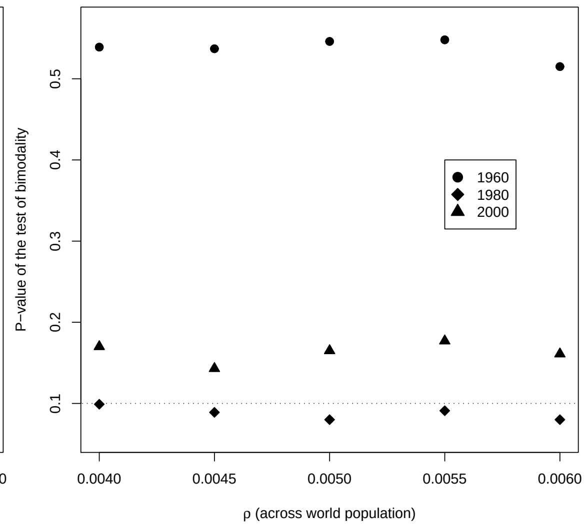

Figure 62: Bimodality test on welfare distribution with alternative values of ρ (across world population). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

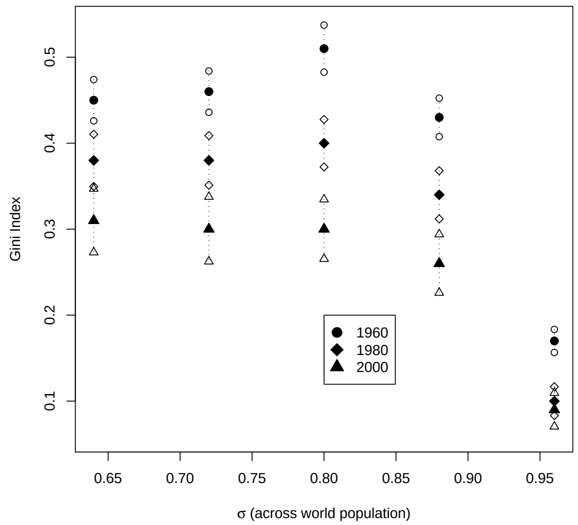

Figure 63: Gini index of welfare distribution with alternative values of σ (across world population). Full points (or diamonds and triangles) represents Gini indexes in 1960 (or 1980 or 2000 respectively). The dotted line between two empty points (or triangles or diamonds) represents the range of +/- 2 standard errors around Gini index in 1960 (or 1980 or 2000 respectively).

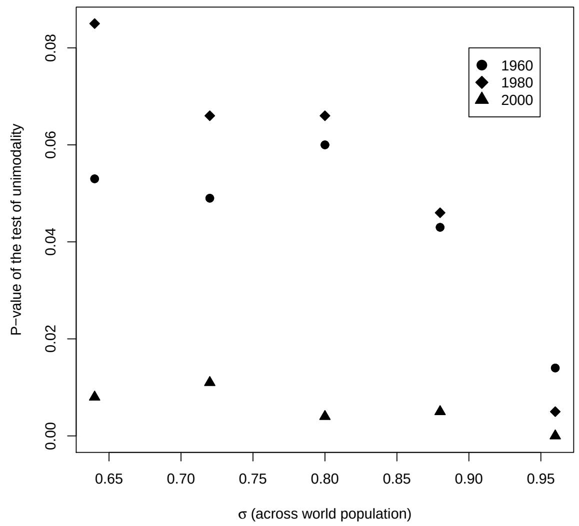

Figure 64: Unimodality test on welfare distribution with alternative values of σ (across world population). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

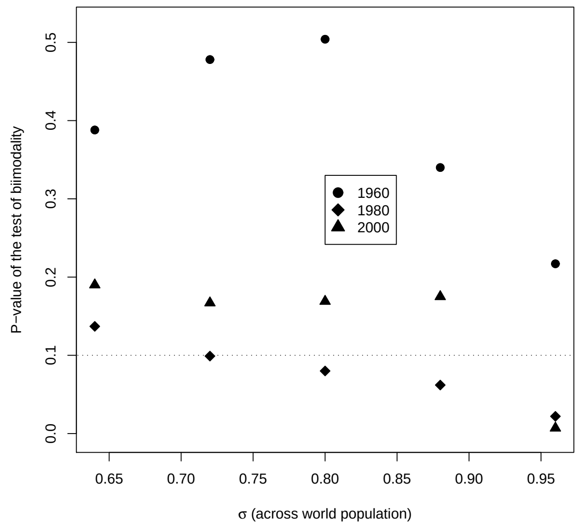

Figure 65: Bimodality test on welfare distribution with alternative values of σ (across world population). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

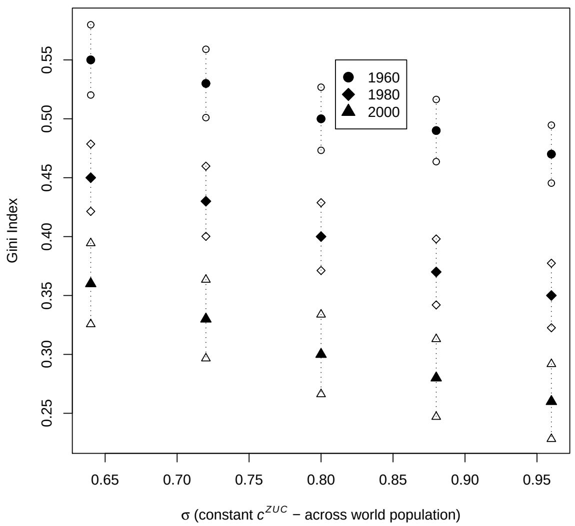

Figure 66: Gini index of welfare distribution with alternative values of σ and M such that $c^{ZUC} = 357$ (across world population). Full points (or diamonds and triangles) represents Gini indexes in 1960 (or 1980 or 2000 respectively). The dotted line between two empty points (or triangles or diamonds) represents the range of +/- 2 standard errors around Gini index in 1960 (or 1980 or 2000 respectively).

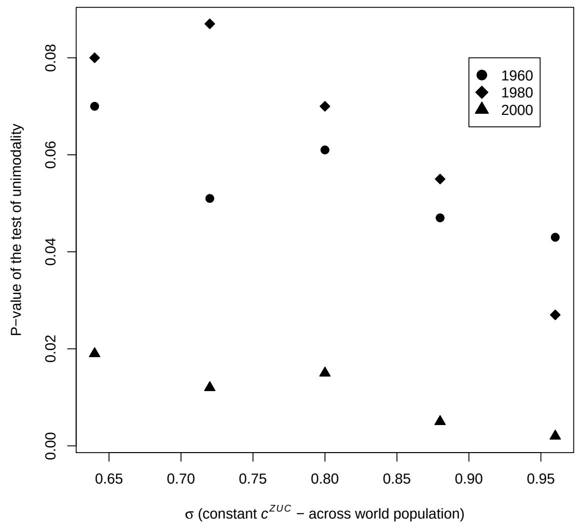

Figure 67: Unimodality test on welfare distribution with alternative values of σ and M such that $c^{ZUC} = 357$ (across world population). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).

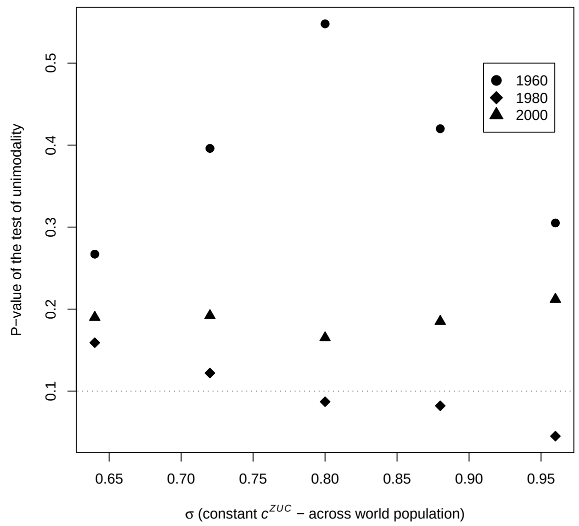

Figure 68: Bimodality test on welfare distribution with alternative values of σ and M such that $c^{ZUC} = 357$ (across world population). Full points (or diamonds and triangles) represents p-value of test in 1960 (or 1980 or 2000 respectively).