Abstract

The paper analyses a model where the fight for the appropriation of rents from natural resources between two groups leads to multiple equilibria. The possibility to be trapped into the low-income equilibrium, characterized by strong social conflict (civil war) and stagnation of income, increases with the weakness of political institutions, the population growth rate, the amount of rents from natural resources and the rate of depletion of natural resources and decreases with the level of per capita income, the investment rate and the length of life expectancy of individuals. The size of minority has an ambiguous effect, widening the range of income leading to low-income equilibrium, but also raising incentives to reach an agreement, i.e. a social contract, without any social conflict. Empirical evidence appears to support these findings.

Classificazione JEL: O11, O43, Q34, D74

Keywords: natural resources, social conflict, poverty trap, institutions, civil war.
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I. Introduction

Many countries whose output is concentrated in primary sectors show low growth rates (see Auty (2001); Sachs and Warner (2001); Mehlum et al. (2006) and Humphreys et al. (2007a)). The literature has proposed many complementary explanations of the phenomenon, denoted as the curse of natural resources, among which: i) strong exports of natural resources changes the terms of trade, crowding out the traded-manufacturing activities (Sachs and Warner (2001)), ii) the rents from natural resources distort the allocation of investments (e.g. less incentives to invest in education, see Gylfason (2001)), and iii) the rents from natural resources encourage strong rent-seeking activities and/or social conflict in countries with weak institutions (see Olsson (2007) and Mehlum et al. (2006)).

In the paper the stagnation of these economies is mainly explained by their high level of social conflict. Here, in particular, social conflict is caused by the joint effect of abundance of natural resources, low initial level of per capita GDP, social fractionalization, weakness of political institutions, low investment rates, high population growth rates, high rates of depletion of natural resources and low life expectancy.

A theoretical model is built on Olsson (2007) and Mehlum et al. (2003). It aims to identify the conditions under which countries can be trapped into a permanent underdevelopment regime. In the economy there are two sectors; in the natural resources sector output only depends on natural resources, while in the productive sector output depends on labour and on capital. The economy is populated by two groups of individuals (i.e. society is polarized into two homogeneous ethnic/religious groups). Formally, government owns the property rights on natural resources, but it can only partially appropriate rents from them (i.e. institutions are weak). The two groups compete for the appropriation of the residual rents. Grossman and Kim (1996) argue that the social conflict (predation in their terms) is particularly fierce when the level of rents does not
crucially depend on social conflict. Rents from natural resources are therefore assumed to be independent of social conflict and output in industrial sector cannot be predated. The source of capital accumulation is the non-consumed output of the productive sector.

The competition for the appropriation of residual rents between the two groups is first modelled as a one-shot game, where both groups simultaneously choose how to allocate their time between the productive sector and the fighting for the appropriation of rents from natural resources. Technology in the productive sector is linear in capital; therefore, given a sufficiently high level of investment rate, the economy without conflict would grow in the long run. But the waste of resources caused by the social conflict can generate a poverty trap, i.e. countries with a low initial level of capital can be trapped in a low-income equilibrium. The long-run behaviour of economy crucially depends on the quality of institutions: fewer appropriable rents means less incentive to compete for them, as in Olsson (2007) and Mehlum et al. (2003). But, differing from the latter, the level of actual stock of per capita capital is also a crucial factor; capital determines the outside option of the competition for rents (see Collier, Hoeffler and Sambanis (2005) for a similar point). The curse of natural resources is therefore the result of the joint effect of weak institutions and of a low level of per capita income (capital). A counterintuitive example explains the importance of considering institutions and level of per capita income jointly: an increase in resources harvested by government, which are then entirely consumed by the same government, thereby reducing the rents to be shared between the two groups, could help the country to escape from poverty trap. However, in the empirical analysis this could also signal weak institutions. The model, moreover, points out that high population growth rate and low investment rates may also increase the probability of being trapped in a stagnant regime. Finally, given the linear technology of conflict, the intensity of social conflict positively depends on the size of minority.

If the two groups realize that they will be interacting for many periods, in equilibrium an agreement for the sharing of rents, i.e. a
social contract, without any social conflict becomes feasible; the self-enforcing agreement between the two groups is indeed supported by the threat of future social conflict. This extension confirms all the findings of the case of one-period time horizon, but the impact of the size of minority on the intensity of social conflict is ambiguous, and it highlights two additional explaining factors of the onset of social conflict: the speed of exploitation of natural resources and the life expectancy of individuals. In particular, a bigger size of minority now favours the emergence of a social contract by decreasing the gain of every individual of deviating from agreement, but increases the range of income leading to low-income equilibrium. Moreover, higher speed of exploitation, reducing the future rents, increases the incentive to fight for the current rents, while lower life expectancy, reducing the time-horizon of individuals, increases their discount rate of future rents. Given the relationship between social conflict and long-run growth, the speed of exploitation of natural resources and life expectancy should be therefore considered among the possible causes of low growth of countries.

From an empirical point of view Auty (2001) contains many historical examples of collapses of economies caused by a social conflict for the appropriation of rents. Civil wars can be considered the fiercest type of social conflict: there is a general consensus on the definition of a civil war, while other type of social conflicts, as riots and coups, are more difficulty to measure. Many scholars, therefore, focus on the determinants of civil wars to study social conflict within a country. Empirical analysis by Collier and Hoeffler (2004) and Collier et al. (2006) of the onset of a civil war and its long-run effects support the theoretical results of the model. Collier, Hoeffler and Sambanis (2005)’s finding that sub-Saharan countries have the highest probability of civil war onset gives further empirical support to present approach. To test the relationship between the abundance of natural resources and civil war onset Sambanis (2003) presents many case studies of countries, including many sub-Saharan countries. Finally, Olsson (2006) and Olsson (2007) documents how diamond production has directly triggered civil war in
many sub-Saharan countries.

Fiaschi (2008) presents a first analysis of the case of one-shot game, and focuses on the explanation of the dynamics of sub-Saharan countries in the last thirty years. In spirit the model follows the game-theoretic approach to social conflict exposed in Hirshleifer (2001). The approach adopted by Dixit (2004) is also close; in the limiting case of no existence of government the issue analysed in the paper is equivalent to the definition of property rights on natural resources in an economy without any legal system. Binmore (2005) presents a similar point of view on the emergence and characteristics of social contract. Esteban e Ray (2008) analyze the determinants of the onset of class or ethnical conflicts and argue that the latter are generally more likely, especially in economies with high inequality. Besley and Persson (2008) analyze a model with social conflict (civil wars) but without capital accumulation; another crucial difference with respect to the present analysis is the assumption that groups are playing a one-shot Stackelberg game. Gonzalez (2006) analyses a growth model with social conflict but in equilibrium economy is not growing and therefore the opportunity cost of social conflict is constant; moreover his focus is on the welfare implication of social conflict. Benhabib and Rustichini (1996) deal with the determinants of social conflict but in a very different framework. Finally, models with occupational choice by Acemoglu (1995), Murphy et al. (1993) and Mehlum et al. (2003) are close to the present model in their focus on the incentives to individuals to become producers or predators and how their choice affects the development of an economy.

II. Empirical Evidence on Growth, Natural Resources and Social Conflict

With the negative effect of social conflict on the development of countries taken for granted, this section discusses two empirical regularities showed by cross-country analyses: i) the negative relationship between growth and abundance of natural resources and
ii) the positive relationship between the latter and social conflict, in particular with the civil war onset.\(^1\) Two such regularities suggest a natural explanation of the stagnant growth regime of sub-Saharan countries in the last 30 years. The theoretical model discussed in Section III. will aim at providing an economic framework to account for these two such regularities.

II.A. Cross-country evidence

The sample includes 108 countries, among which there are 30 sub-Saharan countries.\(^2\)

Table 1 reports the averages values of the following variables: average growth rate of per capita GDP in 1975-2004, \(AV.GR\), the log of per capita GDP in 1975, \(LOG.GDP\), average share of non-manufactures export on total export in the period 1975-2004, \(NM.EXP\), average life expectancy at birth in the period 1975-2004, \(LIFE.EXP\), the average investment rate in 1975-2004, \(INV.RATE\), the average growth of population in 1975-2004, \(GR.POP\), the average enrolment in secondary education in 1975-2004, \(EN.SEC\) for sub-Saharan countries and for the rest of the sample.

Table 1 shows that sub-Saharan countries’ per capita GDP stagnated in the period (0 per cent on average); by contrast, the average growth rate of per capita GDP of all the other countries is equal to 1.9 per cent. Moreover, sub-Saharan countries on average have lower initial levels of per capita GDP, higher population growth rates, lower investment rates, lower life expectancy at birth and lower education levels. Finally, the share of non-manufactured exports of total exports, which should capture the importance of natural resources in the country’s economy, is higher in sub-Saharan countries, as well as the share of countries with a civil wars in the period (10 out of 30 versus 19 out of 78).

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\(^1\) Ross (2004) surveys empirical literature on the relationship between civil wars and natural resources.

\(^2\) All the variables are drawn by World Development Indicators (2006). The country list is in Appendix A.

\(^3\) More precisely, \(NM.EXP\) includes agricultural commodities, raw materials, ores, metals, fuels and food.
Table 1: Descriptive statistics of the variables in the sample (sub-Saharan countries versus all the other countries). Source: WDI (2006)

<table>
<thead>
<tr>
<th>Variable</th>
<th>Subsaharian Countries</th>
<th>Other Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>30</td>
<td>78</td>
</tr>
<tr>
<td>AV.GR</td>
<td>0%</td>
<td>1.9%</td>
</tr>
<tr>
<td>LOG.GDP.1975</td>
<td>1919</td>
<td>7825</td>
</tr>
<tr>
<td>INV.RATE</td>
<td>10.7</td>
<td>21.2</td>
</tr>
<tr>
<td>GR.POP</td>
<td>2.7%</td>
<td>1.4%</td>
</tr>
<tr>
<td>EN.SEC</td>
<td>27.8</td>
<td>81.8</td>
</tr>
<tr>
<td>NM.EXP</td>
<td>79.9</td>
<td>51.8</td>
</tr>
<tr>
<td>LIFE.EXP</td>
<td>48.1</td>
<td>70.29</td>
</tr>
<tr>
<td>CIVIL.WAR</td>
<td>10</td>
<td>19</td>
</tr>
</tbody>
</table>

To show that a stagnant growth regime characterizes almost all sub-Saharan countries in the period, Figure 1 reports the average growth rate of per capita GDP for the period 1975-2004 against its initial level.

In Figure 1 each circle represents a country. The radius of circle is proportional to NM.EXP and dark grey circles represent sub-Saharan countries. The horizontal line represents the average growth rate of the sample equal to 1.4 per cent. The figure reports a nonparametric estimate and its confidence bands of the relationship between the average growth rate and the log of initial per capita GDP. The estimate shows that there is no convergence among the per capita GDP of the countries in the sample (confidence bands include the average growth rate of sample for all the range of the log of per capita GDP), and that, overall, sub-Saharan countries had particularly low growth rates and a substantial higher share of non-manufactures on exports.

To investigate the relationship between the latter two variables, Figure 2 reports the relationship between the average growth rate of per capita GDP versus the share of non-manufactured exports on total exports. The radius of the circle is proportional to the log of initial per capita GDP.

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4The estimate is made with R using package sm with standard settings by Bowman and Azzalini (2005).
Figure 1: Average growth rate of per capita GDP for the period 1975-2004 against the level of the log of per capita GDP in 1975. The circle radius is proportional to the share of non-manufactured exports on total exports ($NM.EXP$). Dark grey circles represent sub-Saharan countries. The horizontal line represents the average growth rate of the sample.
Figure 2: Average growth rate of per capita GDP for the period 1975-2004 against the share of non-manufactured exports on total exports ($NX.EXP$). The circle radius is proportional to the log of per capita GDP in 1975. Dark grey circles represent sub-Saharan countries. The horizontal line represents the average growth rate of the sample. Source: WDI (2006).
Figure 2 suggests that the abundance of natural resources could have a negative effect on growth rate. Moreover, the negative effect appears particularly severe for countries with a low initial income, e.g. the sub-Saharan countries.

The variables reported in Table 1 show a high collinearity among them and sound causal relationships are hardly detectable (i.e. possible regressions are biased by endogeneity and spuriousness). Nonetheless, only for explorative purpose, Table 2 reports the estimate of four models: Model 1 is the usual cross-country growth regression; Model 2 includes also the export of primary goods \(NM.EXP\); Model 3 includes many interaction terms and, finally, Model 4 is the best model (in terms of the highest \(\bar{R}^2\)).

<table>
<thead>
<tr>
<th>Dependent variable: (AV.GR)</th>
<th>Model 1</th>
<th>Model 2</th>
<th>Model 3</th>
<th>Model 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>(Intercept)</td>
<td>0.0385**</td>
<td>0.0457***</td>
<td>-0.0899**</td>
<td>-0.0567***</td>
</tr>
<tr>
<td>(INI.GDP)</td>
<td>-0.0136***</td>
<td>-0.0136***</td>
<td>-0.0133***</td>
<td>-0.0149***</td>
</tr>
<tr>
<td>(INV.RATE)</td>
<td>0.0014***</td>
<td>0.0013***</td>
<td>0.0018***</td>
<td>0.0018***</td>
</tr>
<tr>
<td>(POP.GR)</td>
<td>-0.4434**</td>
<td>-2.871</td>
<td>0.5780</td>
<td></td>
</tr>
<tr>
<td>(EN.SEC)</td>
<td>0.0277***</td>
<td>0.0273***</td>
<td>0.0295*</td>
<td>0.0160*</td>
</tr>
<tr>
<td>(LIFE.EXP)</td>
<td>-0.4434**</td>
<td>-2.871</td>
<td>0.5780</td>
<td></td>
</tr>
<tr>
<td>(NM.EXP)</td>
<td>-0.0120*</td>
<td>0.0701.</td>
<td>0.0252*</td>
<td></td>
</tr>
<tr>
<td>(INI.GDP : NM.EXP)</td>
<td>-0.0025</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(INV.RATE : NM.EXP)</td>
<td>-0.0013***</td>
<td>-0.0013***</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(POP.GR : NM.EXP)</td>
<td>-1.306*</td>
<td>-0.4346*</td>
<td></td>
<td></td>
</tr>
<tr>
<td>(EN.SEC : NM.EXP)</td>
<td>-0.0239</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(LIFE.EXP : NM.EXP)</td>
<td>0.0001</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(R^2)</td>
<td>0.67</td>
<td>0.69</td>
<td>0.76</td>
<td>0.76</td>
</tr>
<tr>
<td>Number of countries</td>
<td>108</td>
<td>108</td>
<td>108</td>
<td>108</td>
</tr>
</tbody>
</table>

Table 2: The best estimate of a standard growth model with the addition of life expectancy at birth and of the share of non-manufactured exports on total exports and interaction terms among variables. Significance codes: 0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1.

Table 2 shows the remarkable result that in Model 4 the coefficient of \(NM.EXP\) becomes positive once controlled for the interactions among variables, i.e. abundance of natural resources is good for growth. \(NM.EXP\) has, on the contrary, a negative impact when it interacts with the investment rate \(INV.RATE\) and
the growth rate of population $GR.POP$; the latter sign signals how social conflict could be at work (see Brückner (2008)), while the negative sign of the interaction term of $NM.EXP$ with $INV.RATE$ suggests that abundance of natural resources can crowd-out manufactures investment by changing the terms of trade (see Sachs and Warner (2001)).

The lack of data does not allow to control for the quality of institutions in the above regressions. For a limited number of countries (i.e. 26), and only for period 2001-2005, the World Bank provides three indices based on enterprise surveys in which managers surveyed ranked the respect of property right ($PROPERTY.RIGHTS$), corruption ($CORRUPTION$) and crime ($CRIME$) as a major business constraint in a given country (in particular the indices represent the per cent of managers surveyed which say that property rights are respected and corruption and crime are a problem for business activity in the country). Table 3 reports such indices.

<table>
<thead>
<tr>
<th></th>
<th>Sub-Saharan Countries</th>
<th>Other Countries</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of countries</td>
<td>5</td>
<td>21</td>
</tr>
<tr>
<td>$PROPERTY.RIGHTS$</td>
<td>36.3%</td>
<td>44.3%</td>
</tr>
<tr>
<td>$CORRUPTION$</td>
<td>45%</td>
<td>40.6%</td>
</tr>
<tr>
<td>$CRIME$</td>
<td>36.9%</td>
<td>29.25%</td>
</tr>
</tbody>
</table>

Table 3: Descriptive statistics of quality of institutions for 2001-2005 for a restricted number of countries. Source: WDI (2006).

Sub-Saharan countries seems to have low-quality institutions. The result is, however, not conclusive partly due to the small number of countries and, overall, because the quality of institutions is probably endogenous (i.e. the evidence reported in Table 3 could be the results of the stagnant growth regime of sub-Saharan countries).

In general, as regards civil wars, the fiercest type of social conflict, Collier, Hoeffler and Sambanis (2005) consider a sample of 161 countries in the period 1960-1999; they identifies 78 cases of civil wars regarding 50 countries, among which there are 18 sub-Saharan countries. Their analysis shows that the amount of rents from natural resources has significant explanatory power in predicting civil
war onsets. Collier and Hoeffler (2002), however, shows that the higher probability of civil war onsets in the sub-Saharan countries disappears after controlling for the initial level of per capita GDP, the importance of primary sector and social fractionalization. All these three aspects are consistent with the theoretical model presented below.

II.B. Case Studied of Sub-Saharan Countries

The empirical evidence discussed above provide a macro evidence of the relationships between growth, natural resources and social conflict for a large cross-section of countries. Case studies regarding some sub-Saharan countries provide complementary information to evaluate whether such macro relationships are effectively casual relationships. As noted by Olsson (2006) sub-Saharan countries have a large diamond production, and in such countries, with the exception of Botswana and South Africa, diamond production appears to fuel endemic social conflict. Indeed, civil wars in Angola, Sierra Leone, Liberia and Democratic Republic of Congo appear the result of the fighting for the appropriation of their diamond production. For other sub-Saharan countries, like Central African Republic, Cote d’Ivoire, Guinea, Uganda and the Republic of Congo, with no production of diamonds, but bordering diamond-producing countries, diamonds have been a source of intense illegal activities (e.g. corruption, smuggling, etc.). All these sub-Saharan countries appears to have been in a stagnant growth regime in the last 30 years. In particular, Minter (1994) discussed the collapse of Angola’s economy after the onset of civil war in 1975; first of civil war Angola’s economy showed a high growth rates and the primary sector was not dominant in the composition of output; after 1975 the primary sector became the main source of income. The dynamics appears the result of a mix of political instability caused by foreign countries and the strong increase in the rents from natural resources due to the increase in the oil price after the first oil shock.

Sambanis (2003) reviews 22 case studies of civil war onset or
war avoidance, including the following 10 sub-Saharan countries: Burundi, Democratic Republic of the Congo, Kenya, Cote d’Ivoire, Mali, Mozambique, Nigeria, Senegal, Sierra Leone and Sudan. Civil war in Burundi seems only partially linked to natural resources, social fractionalization being the main factor, while civil war in Mozambique was mainly financed by the diaspora in Rhodesia. By contrast, civil wars in Democratic Republic of the Congo seem to be caused by the abundance of natural resources (many different types of minerals); the same in Kenya (rich agricultural production), Mali (gold and diamonds), Nigeria (oil), Senegal (cannabis and timber), Sierra Leone (diamonds), Sudan (oil). Interestingly, in Cote d’Ivoire, which is a resource-abundant country, strong redistributive policies (good institutions) were used to mitigate conflict risk.

III. The model

Suppose that economy is composed by two groups of individuals, Citizens and Rebels. The existence of these two groups is taken as given and they will differ only for their cardinality (Citizens will be assumed more numerous than Rebels). At period $t$ the cardinality of the groups of Citizens ($C$) and Rebels ($R$) are equal to $N^C$ and $N^R$, and $N = N^C + N^R$ is the total population (in the following, time index is omitted if this is not source of confusion). In the economy there exists a flow of income (rents) from natural resources $F$. A share equal to $1 - \gamma \geq 0$ is appropriate by government and the remaining part $\gamma$ is appropriate by the two groups.

Parameter $\gamma$ should measure the quality of institutions, i.e. higher $\gamma$ means less efficient institutions. Given the purposes of the paper, it is out of the analysis the potential role of government expenditure in preventing/alleviating social conflict and/or the incentives

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5See Esteban e Ray (2008) for a discussion of why ethinical factors and not the levels of individual wealth are generally at the basis of the formation of groups.

6Section VI.C. analyses the case where the stock of natural resources is fixed and not (or partially) renewable; there $F$ is therefore the result of an exploitation whose length is limited over time (see Humphreys et al. (2007)).
of groups of individuals to capture the government in order to appropriate rents from natural resources (see, e.g., Esteban e Ray (2008) and Besley and Persson (2008)). In particular, the possibility that the collected resources by government are consumed by the same government and/or used to provide public goods and/or redistributed by lump-sum transfers to all individuals does not change the results discussed below because such policies does not affect the incentives to social conflict for the two groups (see Eqn. (1) and (2)).

In each period every rebel has to decide how to allocate her time between the productive sector, \( l^R \geq 0 \), and fighting for the appropriation of income from natural resources, \( p \geq 0 \). Total endowment of time is normalized to 1, i.e. \( l^R + p = 1 \). The time employed in the productive sector has a reward proportional to the per capita capital of economy \( k_t = K_t/N_t \); in particular a rebel gets \( Al^R k_t \) from productive sector, i.e. there is not difference in the reward of time between citizens and rebels in the productive sector. By the same token, every citizen has to decide how to employ her time between the productive sector, \( l^C \geq 0 \), and fighting for the appropriation of income from natural resources, \( d \geq 0 \), with the total endowment of time again normalized to 1, i.e. \( l^C + d = 1 \). Symmetrically, a citizen gets \( Al^C k_t \) from the productive sector.

The intensity of social conflict is measured by the share of population engaged in the fight for the appropriation of income from natural resources, i.e. \( \beta^R p + \beta^C d \), where \( \beta^R = N^R/N \) is the share of Rebels on total population and \( \beta^C = N^C/N \) is the share of Citizens on total population (\( \beta^R + \beta^C = 1 \)). We assume that \( \beta^R \) and \( \beta^C \) are constant over time.

Within each group appropriate income is equally shared among the members of the group, so that each member of the same group adopts the same decision on personal time allocation. Given \( p \)

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7Differently, Fiaschi (2008) analyzes an economy where government uses the collected resources to increase the capital accumulation of economy.

8The implicit assumption is that individual allocation of time is perfectly observable; differently the equal-sharing rule of appropriate income within a group would incentive free-riding behaviour.
and \( d \), assume the simplest formulation of the technology of conflict discussed in Hirshleifer (2001), that is \( \frac{N^R p}{N^R p + N^C d} \) and \( \frac{N^C d}{N^R p + N^E d} \) are the shares of \( \gamma F \) respectively accruing to Rebels and Citizens. The technology of conflict is therefore assumed to be linear in the number of individuals of the two groups engaged in the conflict.\(^9\) With this technology of conflict the model also includes the "the-winner-takes-all" case (the shares \( \frac{N^R p}{N^R p + N^C d} \) and \( \frac{N^C d}{N^R p + N^E d} \) would be the probabilities of the two groups of getting the total amount of income from natural resources). Finally, for the sake of simplicity fighting does not change the total amount of income from natural resources accruing to the two groups (see Grossman and Kim (1996)).

The utility of the representative rebel is linear in its income:

\[
U^R = A (1 - p_t) k_t + \left( \frac{N^R p_t}{N^R p_t + N^C d_t} \right) \frac{\gamma F}{N^R} = A (1 - p_t) k_t + \left( \frac{p_t}{\beta^R p_t + \beta^C d_t} \right) \gamma f, \tag{1}
\]

where \( f = F/N \) are the per capita rents from natural resource, assumed constant over time, while the utility of the representative citizen is given by:

\[
U^C = A (1 - d_t) k_t + \left( \frac{d_t}{\beta^R p_t + \beta^C d_t} \right) \gamma f. \tag{2}
\]

The accumulation of the aggregate stock of capital in the economy depends on the output of productive sector \( Y^P \); in particular:

\[
K_{t+1} = (1 - \delta) K_t + s Y^P = (1 - \delta) K_t + s \left[ A (1 - p_t) k_t N^R + A (1 - p_t) k_t N^C \right], \tag{3}
\]

where \( \delta > 0 \) is the depreciation rate of capital, \( s \geq 0 \) is the constant investment rate from the output of productive sector. Income from natural resources does not contribute to the accumulation of capital. This amounts to assume that income from natural resources is entirely consumed or that income from natural resources cannot be used to increase capital in the productive sector; for example,

\(^9\) Hirshleifer (2001) deeply discusses alternative technologies of conflict and their implications for the individuals’ equilibrium strategies.
if capital accumulation is indeed knowledge accumulation and the latter is the result of externalities and/or a by-product of the production activity.\textsuperscript{10} Investment rate is assumed to be constant in order to focus only on the effects of the individuals’ choices between predation and production (see, e.g., Benhabib and Rustichini (1996) for an analysis of the effects of social conflict on the choice investment/consumption).\textsuperscript{11}

Eq. (3) can be expressed in term of per capita capital, i.e.:

\begin{equation}
k_{t+1} = \frac{(1 - \delta) k_t + s \left[ A \left(1 - p_t \right) k_t \beta^R + A \left(1 - d_t \right) k_t \beta^C \right]}{1 + n},
\end{equation}

where \( n = N_{t+1}/N_t - 1 \geq 0 \) is the constant growth rate of population.

Income from the productive sector cannot be predated, i.e. it cannot be source of dispute between the two groups (see Grossman and Kim (1996)).

The framework can be extended to consider \( L \) different groups; the extension would not provide any additional insight with respect to the issue analysed here, but as \( L \) increases the time devoted to fighting would tend to decrease as its marginal effect on the share of appropriate rents from natural resources tends to decrease.\textsuperscript{12}

\section*{IV. Optimal Strategies of One-Shot Game}

In every period Citizens and Rebels choose their time allocation by playing a one-shot game. Proposition 1 states the Nash equilibrium of game.

\textsuperscript{10}The dynamics of capital accumulation would be qualitatively the same (i.e. rents from natural resources do not affect capital accumulation at low level of income) under the plausible assumption that total rents were decreasing with the intensity of social conflict; but the analysis would be much more involved.

\textsuperscript{11}It is worth noting that under these assumptions by an appropriate redefinition of parameter \( A \) the individual utility becomes a function of consumption (and not of income).

\textsuperscript{12}In a more general setting the number of groups should be endogenously determined by the trade-off between the advantage to coordinate actions within a group (due to the technology of conflict) and the free-riding behaviour within the same group (the benefit of fight is equally shared among all the members of the group). This extension should help to understand the empirical evidence that groups are generally composed by members who share some cultural, economic and/or social characteristics and their number derives from how such factors are distributed among the population (see Weinstein (2005)).
Proposition 1 Assume that $\beta^R \leq 1/2$; then in the Nash equilibrium of the one-shot game between Citizens and Rebels:

- $p^* = d^* = 1$ when $k_t \in [0, \bar{k}^d]$;
- $p^* = 1$ and $d^* = \left(\frac{1}{1-\beta^R}\right) \sqrt{\frac{\beta^R \gamma f}{A k_t}} - \frac{\beta^R}{1-\beta^R}$ when $k_t \in [\bar{k}^d, \bar{k}^p]$ and
- $p^* = \frac{\gamma f}{\beta^R A k_t}$ and $d^* = \frac{1 - \beta^R}{4(1-\beta^R) A k_t}$ when $k_t \in [\bar{k}^p, +\infty)$,

where $\bar{k}^d = \gamma f \beta^R / A$ and $\bar{k}^p = \gamma f / (4 \beta^R A)$ with $\bar{k}^d \leq \bar{k}^p$.

Proof. See Appendix B. ■

Proposition 1 shows that the intensity of social conflict depends on the ratio between $f$ and $k_t$: for low level of $k_t$ ($k_t \leq \bar{k}^d$) all population is engaged in the fight for the appropriation of natural resources, i.e. there is a fierce civil war; for a higher but always low level of $k_t$ ($k_t \in (\bar{k}^d, \bar{k}^p]$) only Rebels are fully engaged in the fight, while a part of Citizens are employed in the productive sector; finally for sufficiently high level of capital ($k_t > \bar{k}^p$) some Rebels also stop fighting and shift to productive sector. Therefore, ceteris paribus, social conflict, measured by the share of population engaged in predation $\beta^R p_t^* + \beta^C d_t^*$, monotonically decreases with the level of per capita capital $k_t$. The result is expected, given that the opportunity cost of fighting is proportional to $k_t$. If $\beta^R > 1/2$ in the intermediate range of capital all Citizens would engaged in the fight, while some Rebels would be in the productive sector.

It is straightforward to prove that if $k_t > \bar{k}^d$ social conflict decreases with $A$ (the opportunity cost of fighting) and increases with $\gamma f$ (the reward of fighting). Finally, if $k_t \in [\bar{k}^d, \bar{k}^p]$ social conflict increases with $\beta^R$ (the size of minority in the country); in fact, when minority is entirely engaged in fighting, the linear technology of conflict incentives the majority to fight more as its relative size $1 - \beta^R$ decreases. Moreover, given a certain level of per capita capital, an increase in $A$ decreases the probability of social conflicts with high intensity in the sense of decreasing both capital thresholds $\bar{k}^d$ and $\bar{k}^p$, while the opposite holds for $\gamma f$. From the same point of view an increase in $\beta^R$ increase the probability of civil war ($\bar{k}^d$ increases), but decreases the probability of social conflict in which
the minority is entirely engaged in fighting ($\bar{k}^p$ decreases). All these findings broadly agree with the empirical evidence on the causes and intensity of civil wars discussed in Collier and Hoeffler (2004) and in Collier et al. (2006).

The social optimal allocation of time $p^* = d^* = 0$ cannot be reached because there is no self-enforcing agreement on the sharing of rents from natural resources in the one-shot game. On the contrary, in the repeated-game where individuals’ time horizon is indefinite, the simple application of Folk Theorems suggests that social optimal allocation could be reached. The issue will be discussed in Section VI.

V. Long-run Equilibrium with One-period Time Horizon

Eq. (4) and Proposition 1 give the dynamics of per capita capital when individuals have a time horizon of one period:

$$k_{t+1} = \begin{cases} 
\frac{1}{1+n} (1 - \delta) k_t & \text{when } k_t \in [0, \bar{k}^d] \\
\frac{1}{1+n} [(1 - \delta + sA) k_t - s\sqrt{\beta R \gamma f A k_t}] & \text{when } k_t \in [\bar{k}^d, \bar{k}^p] \\
\frac{1}{1+n} [(1 - \delta + sA) k_t - s\gamma f / 2] & \text{when } k_t \in [\bar{k}^p, +\infty) 
\end{cases}$$

(5)

Proposition 2 states under which configuration of parameters economy displays multiple (two) equilibria.

**Proposition 2** Assume that $\beta R \leq 1/2$ and

$$sA > \delta + n.$$  

(6)

Then there exists two equilibria $k^{Es} = 0$ and $k^{Ev} > 0$, the first stable and the second unstable. Moreover, if

$$sA \in \left( \delta + n, \frac{\delta + n}{1 - 2\beta R} \right)$$

(7)

then

$$k^{Ev} = \frac{s\gamma f}{2(sA - \delta - n)}$$

(8)
where $k^{E_U} > \bar{k}^p$; otherwise if

$$sA \in \left(\frac{\delta + n}{1 - 2\beta R}, +\infty\right)$$

(9)

then:

$$k^{E_U} = \frac{\beta R s^2 A \gamma f}{(sA - \delta - n)^2}$$

(10)

where $0 < k^{E_U} < \bar{k}^p$.

**Proof.** See Appendix C. ■

Figure 3 reports a graphical illustration of Proposition 2 when $k^{E_U} > \bar{k}^p$ (i.e. Condition (7) holds).

Figure 3 shows that an economy displays two different dynamics according to its initial level of capital; an economy with a low initial level of per capita capital, i.e. $k_0 < k^{E_U}$, will be converging toward equilibrium $E_S$ with zero capital, while an economy with a sufficiently high initial level of per capita capital, i.e. $k_0 > k^{E_U}$, will grow forever. The model therefore exhibits (absolute) poverty trap.

Proposition 3 characterizes the long-run dynamics of economy.
Proposition 3 Assume that $\beta^R \leq 1/2$ and Condition (6) holds. If $k_0 < k^{EU}$ then the per capita capital of economy will be converging towards $k^{ES}$, while if $k_0 > k^{EU} = 0$ then $\lim_{t \to \infty} g_k \equiv k_{t+1}/k_t - 1 = sA - \delta - n$.

Proof. See Appendix D.

Figure 4 shows the growth path of the economy, $g_k$, when $k^{EU} > \bar{k}$ (i.e. Condition (7) holds).

Figure 4: The growth path of economy with two equilibria

The level of $k^{EU}$ is therefore the threshold of per capita capital which determines the long-run dynamics of economy. Remark 4 shows the relationship between $k^{EU}$ and the most relevant parameters of the economy.

Remark 4 The threshold level of per capita capital $k^{EU}$ increases with $f$, $n$, $\gamma$ and decreases with $A$ and $s$. If Condition (7) holds then $k^{EU}$ increases with $\beta^R$.

Proof. Derivatives of $k^{EU}$ reported in Proposition 2 with respect to $f$, $n$, $\gamma$, $A$ and $s$ directly prove the results. If Condition (7) holds then $k^{EU}$ is defined by Eq. (10) and the first derivative of $k^{EU}$ directly proves the result. QED

Remark 4 says that, given a certain level of initial per capita capital $k_0$, the probability of a country being trapped in an equilibrium
with low income and strong social conflict increases with rents from natural resources \((f)\), population growth rate \((n)\), weakness of institutions \((\gamma)\), size of minority \((\beta R)\) and decreases with investment rate \((s)\) and productivity of productive sector \((A)\). Higher rents from natural resources, higher weakness of institutions, larger size of minority, lower productivity of productive sector means higher social conflict and therefore a higher waste of resources; on the other hand, higher population growth rate decreases the opportunity cost of fighting by diluting the per capita capital, while higher investment rate has the opposite effect.

V.A. Dynamics of Per Capita Income

In the model the dynamics of per capita capital drives the dynamics of economy. However, the country’s per capita income depends both on the level of per capita capital and on the rents from natural resources. Proposition 5 shows the per capita income for different levels of per capita capital.

**Proposition 5** Assume that \(\beta R \leq 1/2\); then in the Nash equilibrium of the one-shot game between Citizens and Rebels the per capita income of the economy is given by:

\[
\begin{align*}
y_t &= f & \text{when } k_t \in [0, \bar{k}^d]; \\
y_t &= (1 + \beta R) \, Ak_t - \sqrt{\gamma f \beta R A k_t} + f & \text{when } k_t \in [k^d, \bar{k}^p] \text{ and} \\
y_t &= Ak_t + f \,(1 - \gamma/2) & \text{when } k_t \in [\bar{k}^p, +\infty),
\end{align*}
\]

where \(\bar{k}^d = \gamma f \beta R / A\) and \(\bar{k}^p = \gamma f / (4\beta R A)\).

**Proof.** At period \(t\) per capita income \(y_t\) is given by:

\[
y_t = (1 - d_t^*) \, Ak_t \beta^C + (1 - p_t^*) \, Ak_t \beta^R + f; \tag{11}
\]

Proposition 1 and Eq. (11) prove the results. QED

Proposition 6 states the dynamics of per capita income with poverty trap.
Proposition 6 Assume that $\beta^R \leq 1/2$ and $sA > \delta + n$. If $y_0 < y^{EU}$ then the per capita income will be converging towards $y^{ES} = f$; otherwise, if $y_0 > y^{EU}$ then in the long run the per capita capital will be growing at rate $sA - \delta - n$ where:

$$
\text{if } sA \in \left( \delta + n, \frac{\delta + n}{1 - 2\beta^R} \right) \text{ then } y^{EU} = f \left[ 1 + \frac{\gamma (n + \delta)}{2(sA - n - \delta)} \right] \quad \text{(12)}
$$

$$
\text{if } sA \in \left( \frac{\delta + n}{1 - 2\beta^R}, +\infty \right) \text{ then } y^{EU} = f \left[ 1 + \frac{\gamma \beta^R sA (sA \beta^R + n + \delta)}{2(sA - n - \delta)^2} \right] \quad \text{(13)}
$$

Proof. See Appendix E. ■

Propositions 2 and 6 show that in the poverty trap equilibrium capital is zero but per capita income is positive and entirely deriving from natural resources. Positive shocks to the income from natural resources (e.g. an increase in the real price of raw materials) causes an increase in the level of per capita output but it makes less probable the escape from the poverty trap, increasing also the threshold level of per capita income (note that the difference $y^{EU} - y^{ES}$ is proportional to $f$).

It is straightforward to show that $y^{EU}$ (like $k^{EU}$) increases with $f$, $n$, $\gamma$ and decreases with $s$ and $A$; moreover, if Condition (7) holds then $y^{EU}$ increases with $\beta^R$.

Finally, from Eq. (5) and Proposition 5 the overall investment rate of economy (total investment over aggregate income) is increasing in $k_t$, being equal to 0 for $k_t \in [0, k^d]$ and converging to $s$ for $k_t \to +\infty$; the positive relationship between the growth rate of per capita income and the investment rate is a by product of the different output composition of economy as per capita capital increases (a direct result of the assumption that rents from natural resource cannot affect the accumulation of capital).
V.B. Government consumption

In the model government consumption could have a positive impact on the development of a country if it dissipates rents from natural resources. For the sake of simplicity, assume that government consumes all income it collects from natural resources and consider the share of government consumption on total income $c_t^G$ around but below the threshold of poverty trap $y^{EU}$ in Eq. (12), i.e.:

$$c_t^G = \frac{(1 - \gamma) f}{y_t} = \frac{(1 - \gamma) f}{Ak_t + f (1 - \gamma/2)};$$

given a some level of per capita capital, a decrease in $\gamma$, i.e. a higher capacity of government to appropriate the rents from natural resources, causes an increase in $c_t^G$ and in $y_t$, but also a decrease in $y^{EU}$ (see Eq. (12)). An increase in the government consumption, therefore, increases the probability of a country of escaping from a poverty trap (i.e. $y_t - y^{EU}$ decreases). The intuition of the result is straightforward: in this economy government consumption is a waste of resources, but if such a waste derives from a decrease in $\gamma$, it reduces the incentives to fight; more resources will be consequently allocated to productive sector. The latter positive effect outweighs the former negative effect (at least in the long run).

VI. Indefinite Time Horizon

So far groups take their decisions with a time horizon of one period. This excludes the possibility that the two groups can find a self-enforcing agreement on the share of rents from natural resources, i.e. a social contract. Theory of Repeated Games however suggests that when the individuals’ horizon is indefinite (or infinite) a self-enforcing agreement on the sharing of rents without social conflict could be reached. The following analysis is indeed closed to the issue of the emergence of property rights and, consequently, of a social contract in a primitive economy discussed in Muthoo (2004).

Heuristically, the payoff of the deviation from an agreement be-
comes relatively lower as the per capita capital of economy increases; consider therefore the worst situation for reaching a self-enforcing agreement, i.e. \( k_0 \in [0, k^d] \), where social conflict is at its maximum in the one-shot game, i.e. \( d^* = p^* = 1 \). From Eqq. (1) and (2) the utility of representative citizen and rebel, \( U_{SC}^C \) and \( U_{SC}^R \), are respectively equal to:

\[
(U_{SC}^C, U_{SC}^R) = (\gamma f, \gamma f).
\]  

(14)

Suppose that at beginning of every period the two groups can bargain to decide how to share rents from natural resources. If such bargaining fails then they play their optimal strategies reported in Proposition 1. Otherwise, if they reach an agreement, there is not any social conflict in the period, i.e. \( d^* = p^* = 0 \), even if such agreement is not automatically enforceable. As in Muthoo (2004) assume that the bargaining process is such that the equilibrium is characterized by a Nash bargaining solution (NBS) with disagreement point \((U_{SC}^C, U_{SC}^R)\). The NBS is characterized by the Split-the-difference rule, i.e. each individual receives her disagreement payoff plus a share of surplus deriving from the agreement and allocate all its time to productive activity. The surplus deriving from agreement is given by:

\[
\Delta_t = \left( \frac{NAk_t + \gamma F}{\text{Total output of economy with agreement}} - \frac{\gamma F}{\text{Total output of economy without agreement}} \right) = NAk_t.
\]  

(15)

Assume that such output is equally shared among all the individuals of economy. This amounts to assume that the bargaining power of the two groups is proportional to their population size \( \beta^C \) and \( \beta^R \). Therefore the utilities of representative citizen and rebel in the NBS, \( U_{NBS}^C \) and \( U_{NBS}^R \), are respectively equal to:

\[
(U_{NBS}^C, U_{NBS}^R) = (Ak_t + \gamma f, Ak_t + \gamma f).
\]  

(16)

The comparison between payoffs in Eqq. (14) and (16) confirms the intuition that the incentive to deviate from the agreement decreases with per capita capital.
In order to evaluate under which conditions the NBS can be effectively played by both groups consider the ”trigger-strategy” approach, i.e. a strategy in which each group respects the agreement of the NBS until the other group makes the same and starts fighting and continues forever according its optimal strategies when the other group does not respect the agreement.

Proposition 7 states the conditions under which there exists a trigger strategy equilibrium (TSE) where both groups respect the agreement of the NBS.

**Proposition 7** Assume that \( \beta^R \leq 1/2 \), \( sA > \delta + n \) and that the initial per capita capital \( k_0 \in [0, \bar{k}^f] \), where

\[
\bar{k}^f = \frac{\beta^R A(1 + n)}{A(1 + sA - \delta)} < \bar{k}^d. \tag{17}
\]

Let \( \rho \in [0, 1] \) be the discount factor of individuals; then using the trigger strategies the agreement of the NBS can be sustained as a subgame perfect equilibrium if i) \( \rho \in [\hat{\rho}, 1] \) or ii) \( \rho \in [0, \hat{\rho}) \) and \( k_0 \geq \bar{k}^{TSE} \); on the contrary, using the trigger strategies the agreement of the NBS cannot be sustained as a subgame perfect equilibrium if \( \rho \in [0, \hat{\rho}) \) and \( k_0 \in [0, \bar{k}^{TSE}) \), where:

\[
\bar{k}^{TSE} = \frac{(1 - \beta^R)}{\beta^R A} \left[ \frac{(1 + n)}{\rho(1 + sA - \delta)} - 1 \right] \gamma_f, \tag{18}
\]

and

\[
\hat{\rho} = \frac{1 + n}{1 - \delta + sA}. \tag{19}
\]

**Proof.** See Appendix F. ■

Proposition 7 says that if \( \rho \in [0, \hat{\rho}) \) there always exists a range of low per capita capital where, using the trigger strategies, a self-enforcing agreement is not attainable. In the limiting case of \( \rho = 0 \), i.e. when the time horizon of individuals is one period, \( \bar{k}^{TSE} \to +\infty \): for all levels of per capita capital in the range \( [0, \bar{k}^f] \) fighting is the only equilibrium strategy for both groups (as in the one-shot game). Otherwise Proposition 7 says that a social contract between
the two groups can emerge; such social contract states that rents from natural resources is equally shared across all individuals in the economy (see Eq. (16)) without any social conflict. Of course, many other different equilibria are possible (e.g. not using the trigger strategies), i.e. many different social contracts can represent an equilibrium of the repeated game just analysed; however, Binmore (2005) suggests that the fairness of this social contract, even though based on the NBS, makes it the natural candidate to emerge in the long run.

VI.A. The Long-Run Equilibrium with Indefinite Time Horizon

Proposition 8 characterizes the long-run dynamics of economy.

**Proposition 8** Assume that $\beta^R \leq 1/2$, $sA > \delta + n$ and the initial per capita capital $k_0 \in [0, \bar{k}F]$. In the long run per capita capital of economy will grow at rate $g_k = sA - \delta - n$ when i) $\rho \in [\hat{\rho}, 1]$ or ii) $\rho \in [0, \hat{\rho})$ and $k_0 \geq \bar{k}^{TSE}$, while $\lim_{t \to \infty} k_t = k^{Es} = 0$ (poverty trap equilibrium) when $\rho \in [0, \hat{\rho})$ and $k_0 < \bar{k}^{TSE}$.

**Proof.** See Appendix G. ■

The results in Propositions 7 and 8 are summarized in Figure 5, which reports the loci in the space $(\rho, k_0)$ where self-enforcing agreement is sustainable, i.e. there a social contract emerges and economy is growing in the long run (area $CDGH$), and where it is not sustainable, i.e. there social conflict is at work and economy is in a poverty trap (area $OBCD$). The frontier of these two areas is defined by $\bar{k}^{TSE}$ in Eq. (18) and

$$\hat{\rho} = \left[ \frac{1 + sA - \delta}{1 + n} + \frac{(\beta^R)^2}{1 - \beta^R} \right]^{-1}, \quad (20)$$

is the level of $\rho$ such that $\bar{k}^F = \bar{k}^{TSE}$.

Figure 6 reports the dynamics of economy with a possible fierce social conflict in the long run (i.e. $\rho \in [0, \hat{\rho}]$). In particular, it shows
that if $k_0 < \bar{k}^{TSE} < \bar{k}^f$ (i.e. combination $(k_0, \rho)$ belongs to area $OBCD$ in Figure 5) then economy will converge towards $k^{Es} = 0$, where social conflict is permanent and economy will be trapped into the low-income equilibrium $E_S$. On the contrary, if $k_0 > \bar{k}^{TSE}$ (i.e. combination $(k_0, \rho)$ belongs to area $CDGH$ in Figure 5) using the trigger strategies the agreement of the NBS is self-enforcing, hence social contract emerges, and economy will grow forever.

VI.B. Comparative statics

Remark 9 shows the relationships between $\bar{k}^{TSE}$ and $\hat{\rho}$ in Proposition 7 and the most relevant parameters of the economy.

**Remark 9** $\bar{k}^{TSE}$ increases with $\gamma$, $n$, $\delta$ and $f$ and decreases with $\beta^R$, $s$, $A$ and $\rho$, while $\hat{\rho}$ increases with $n$ and $\delta$ and decreases with $s$ and $A$.

**Proof.** Simple derivatives of $\bar{k}^{TSE}$ and $\hat{\rho}$ with respect to parameters $\gamma$, $n$, $\delta$, $f$, $\beta^R$, $s$ and $A$ prove the results. □
A comparison of Remark 4 with Remark 9 shows how changes in $\gamma$, $n$, $f$, $s$, and $A$ has qualitatively the same impact on the dynamics of economy when the time-horizon of individuals is one or indefinite (infinite) with the additional feature for $n$, $s$, and $A$ to change the threshold also for $\rho$. Indeed, Figure 7 shows that a rise in $f$ and/or $\gamma$ increases the combinations of $(k_0, \rho)$ leading to social conflict and poverty trap from $OBCD$ to $OB'C'D$ (in particular $\overline{k}f$ shifts upwards while $\tilde{\rho}$ is constant, see Eqq. (17) and (20)). The result is caused by the higher gains of deviating form the NBS, which are proportional to $\gamma f$.

Figure 8 shows that an increase in $n$ or $\delta$ and/or a decrease in $s$ and $A$ has the same effect of widening the area leading to social conflict and poverty trap from $OBCD$ to $OB'C'D'$ (in particular, $\overline{k}f$, $\tilde{\rho}$ and $\hat{\rho}$ rise). The result is caused by the lower benefits of escaping from poverty trap, being the growth rate of capital with social contract a negative function of $n$ and $\delta$ and a positive function of $s$ and $A$.

Figure 7: The effect on the long-run behaviour of economy of an increase in $f$ and/or in $\gamma$.

Figure 8: The effect on the long-run behaviour of economy of an increase in $n$ or $\delta$ and/or a decrease in $s$ or $A$.

Figure 9: The effect on the long-run behaviour of economy of an increase in the size of minority $\beta^R$.

The remarkable difference with respect to the case with individual time-horizon of one period is the impact of an increase in the size of minority $\beta^R$ on social conflict. Figure 9 highlights that the
overall effect of such increase on the area leading to social conflict and poverty trap is indeed indeterminate (it goes from $OBCD$ to $OB'C'D$). In particular, it widens the range of per capita capital leading to the low-income equilibrium (i.e. $\bar{k}^f$ goes up, see Eq. (17)), but it also increases the incentive to reach an agreement by decreasing the rebel’s (one-period) gain of deviating from the agreement of the $NBS$; in fact, each rebel must divide the total amount of income from natural resources $\gamma F$ with all the other rebels, that is each single rebel would gain $\gamma F/N^R = \gamma f/\beta^R$ from deviating. Therefore for any given level of $\rho$ an increase of $\beta^R$ will cause a decrease in $\bar{k}^{TSE}$ (in Figure 9 this means a counter-clock rotation of curve $CD$ with pivot in $D$).

Sections VI.C. and VI.D. propose two other explanatory variables of the threshold of capital $\bar{k}^{TSE}$: the rate of depletion of natural resources and the life expectancy of individuals.

VI.C. Natural Resources as an Asset

Individuals with an indefinite time horizon should consider natural resources not only for their current flow of rents. In particular, Humphreys et al. (2007) argue that natural resources should be considered as an asset because a relevant part of them are non-renewable (e.g. oil) and/or their rate of regeneration is very low compared to their rate of depletion (e.g. wood from virgin forests). This means that the flows of rents from natural resources $F$ cannot be considered constant over time.

For the sake of simplicity suppose that the stock of natural resources $W^F$ follows:

$$W^F_t = W^F_0 \left(1 - \pi^D\right)^t,$$

where $W^F_0$ is the initial stock of natural resources and $\pi^D > 0$ is the constant rate of depletion (the rate of regeneration is therefore assumed to be equal to zero). If $\pi^D > 0$ then $\lim_{t \to +\infty} W^F_t = 0$. 

Given that population follows $N_t = N_0 (1 + n)^t$, then from Eq. (21):

$$w_t^F = w_0^F \left( \frac{1 - \pi^D}{1 + n} \right)^t,$$

(22)

where $w_t^F$ is the per capita stock of natural resources at period $t$. The flow of per capita rents from natural resources at period $t$, $f_t$, is therefore equal to:

$$f_t = \pi^D w_t^F = \pi^D w_0^F \left( \frac{1 - \pi^D}{1 + n} \right)^t.$$  

(23)

An increase in $\pi^D$ changes the time path of $f$, increasing the flow of income in the first $\hat{t} = (1 - \pi^D) / \pi^D$ periods and decreasing the flow in all the remaining periods.\(^{13}\) Eq. (23) therefore suggests that an increase in the rate of depletion $\pi^D$ makes more difficult to reach a self-enforcing agreement by decreasing the future gains from the agreement. Proposition 10 confirms the intuition.

**Proposition 10** Assume that $\beta^R \leq 1/2$, $sA > \delta + n$ and that $k_0 \in \left[ 0, \bar{k}_f^T \right]$, where

$$\bar{k}_f^T = \frac{\beta^R (1 + n) \gamma \pi^D w_0^F}{A (1 - \delta + sA)}.$$  

Then using the trigger strategies the agreement of the NBS can be sustained as a subgame perfect equilibrium if i) $\rho \in [\hat{\rho}, 1]$ or ii) $\rho \in [0, \hat{\rho})$ and $k_0 \geq \bar{k}_W^{TSE}$; on the contrary using the trigger strategies the agreement of the NBS cannot be sustained as a subgame perfect equilibrium if $\rho \in [0, \hat{\rho})$ and $k_0 < \bar{k}_W^{TSE}$, where:

$$\bar{k}_W^{TSE} = \frac{(1 - \beta^R)}{\beta^R A} \left[ \frac{(1 + n)}{\rho (1 + sA - \delta)} - 1 \right] \gamma \pi^D w_0^F.$$  

(24)

**Proof.** The proof of Proposition 10 follows the same steps of the proof reported in Appendix F but taking into account that $f = f_t$, where $f_t$ is defined in Eq. (23). \[\square\]

\(^{13}\)The first derivative of $f_t$ with respect to $\pi^D$ easily proves the statement.
An increase in $\bar{k}_{WSE}$, i.e. in $\pi^D$ and/or in $w^F_0$, has the same effect of an increase of $\gamma$ and/or $f$ reported in Figure 7, i.e. to widen the area of social conflict in the space $(\rho, k_0)$. Indeed, Proposition 10 shows that an increase in the rate of depletion $\pi^D$ increases both $\bar{k}_0^f$ and $\bar{k}_{WSE}^T$ for any given $\rho$. This finding suggests that not only the amount of rents from natural resources, but also the rate of depletion should be considered as a potential explanation of social conflicts. In the empirical analysis, however, to disentangle the two possible determinants of an increase in rents from natural resources, i.e. an increase in the stock of natural resources $w^F_0$ or in its rate of depletion $\pi^D$, appears very difficult. There exists only anecdotal evidence that a "supposed" increase in the rate of depletion of natural resources can trig a strong social conflict (e.g. of wood from virgin forests in Brazil and of oil in Niger Delta).\footnote{See, e.g., http://en.wikipedia.org/wiki/Movement_for_the_Emancipation_of_the_Niger_Delta.}

Finally, the level of $\pi^D$ could be the result of bargaining between the two groups; heuristically, with linear utilities in income the optimal level of $\pi^D$ would be 1, being no incentive to smoothing utilities over time. Otherwise, if utilities were concave in income, $\pi^D$ would negatively depend on the discount rate of individuals $\rho$: more patient individuals (i.e. with high $\rho$) would prefer higher smoothing of income from natural resources (i.e. low $\pi^D$). This means that when $\rho$ is low the emergence of a social contract would be further contrasted by a high level of $\pi^D$.

\section*{VI.D. Life Expectancy}

Discount factor reflects the intertemporal preferences of individuals; however it should also reflect the expected "length" of their life, i.e. their life expectancy. For the sake of simplicity suppose that $S_t$, the probability calculated at period 0 that an individual is still alive at period $t$, is given by:

$$S_t = (1 - \pi^M)^t,$$
where $\pi^M > 0$ is the constant probability to die in every period (therefore $\lim_{t \to +\infty} S_t = 0$). Life expectancy of an individual with an indefinite (infinite) time horizon, $LE$, is therefore equal to:

$$LE = \sum_{t=0}^{\infty} t \left(1 - \pi^M\right)^t = \frac{1 - \pi^M}{(\pi^M)^2};$$

(25)

$LE$ is therefore decreasing with $\pi^M$. Thus, if an individual has a positive probability to die in each period equal to $\pi^M$ then the discount factor $\rho$ should reflect such probability. But, the change of the discount factor also affects the conditions under which a social contract between the two groups can emerge. Proposition 11 confirms such intuition.

**Proposition 11** Assume that $\beta^R \leq 1/2$, $sA > \delta + n$ and that $k_0 \in [0, \bar{k}^f]$, where

$$\bar{k}^f = \frac{\beta^R (1 + n) \gamma f}{A \left(1 - \delta + sA\right)}.$$

then using the trigger strategies the agreement of the NBS can be sustained as a subgame perfect equilibrium if i) $\rho \in \left[\hat{\rho} / \left(1 - \pi^M\right), 1\right]$ or ii) $\rho \in [0, \hat{\rho} / \left(1 - \pi^M\right)]$ and $k_0 \geq \bar{k}^{TSE}_{LE}$; on the contrary using the trigger strategies the agreement of the NBS cannot be sustained as a subgame perfect equilibrium if $\rho \in [0, \hat{\rho} / \left(1 - \pi^M\right)]$ and $k_0 < \bar{k}^{TSE}_{LE}$, where:

$$\bar{k}^{TSE}_{LE} = \frac{(1 - \beta^R)}{\beta^R A} \left[\frac{(1 + n)}{\rho \left(1 - \pi^M\right) \left(1 + sA - \delta\right)} - 1\right] \gamma f.$$

(26)

**Proof.** The proof of Proposition 11 follows the same steps of the proof reported in Appendix F but taking into account that the discount factor net of the probability to survive is equal to $\rho \left(1 - \pi^M\right)$.}

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15In the calculation it is used the following property of geometric series:

$$\sum_{i=0}^{T} iq^i = \frac{1}{1-q} \left[\frac{q(1-q^T)}{1-q} - Tq^{T+1}\right].$$
Figure 10 shows that an increase from $\pi^M$ to $\pi^M'$, i.e. a decrease in the life expectancy of individuals $LE$ (see Eq. (25)), widens the area in the space $(\rho, k_0)$ leading to social conflict and poverty trap.

Indeed, Proposition 11 shows that an increase in $\pi^M$ causes an increase in $\bar{k}_{TSE}$ (see Eq. (26)) for any given $\rho$ (in Figure 10 this means a shift from $CD$ to $C'D'$). Among all the countries Sub-Saharan countries could particularly suffer of the very short life expectancy of their inhabitants.

Finally, social conflict could, in turn, negatively affect life expectancy; the resulting increase in $\pi^M$ would further lower the possibility of a country to escape from poverty trap. Note, however, that if the decrease in life expectancy caused by social conflict were expected, individuals could have a higher incentive to reach an agreement; indeed, social conflict would hurt future incomes and, additionally, lower life expectancy.\footnote{I thank to Nicola Meccheri for pointing me such possibility.}
VII. Concluding Remarks

The analysis has shown how social conflict can lead to a poverty trap. Collier and Hoeffler (2004), indeed, find that the stagnant growth regime caused by civil war is persistent over time. The explanatory factors of the onset of social conflict individuated in the analysis, as low level of per capita income, high level of population growth rate and high rents from natural resources, find an empirical corroboration (see, e.g., Auty (2001), Collier and Hoeffler (2004) and Collier, Hoeffler and Sambanis (2005)). The size of minority has, on the contrary, an ambiguous effect on social conflict by increasing, from one hand, the range of per capita income leading to poverty trap, and, from the other hand, the incentive to reach an agreement without fight, i.e. the emergence of a social contract. This could explain why many empirical analysis do not find any statistically significance relationship between ethnic/religious/linguistic fractionalization and social conflict (see, e.g., Collier and Hoeffler (2004)). In addition, the model suggests that high rates of depletion of natural resource and low life expectancy could be two further explanatory variables of social conflict and persistence in a low-income equilibrium. The growth patterns and anecdotal evidence of sub-Saharan countries provides some empirical support. The next steps in the analysis should aim at i) endogenizing the strengh of each group by allowing different fertility rates between the two groups (like, e.g., in De la Croix and Dottori (2008)); ii) deepening the factors causing the emergence/disruption of social contract following the insights by Binmore (2005); and iii) endogenizing the group formation following the suggestions by Weinstein (2005).

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References


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Appendix

A Country List

Algeria, Argentina, Bangladesh, Benin, Bolivia, Botswana, Brazil, Burkina Faso, Burundi, Cameroon, Central African Republic, Chad, Chile, China, Colombia, Congo, Dem. Rep., Congo, Rep., Costa Rica, Cote d’Ivoire, Cyprus, Dominican Republic, Ecuador, Egypt, El Salvador, Fiji, Gambia, Ghana, Greece, Guatemala, Guyana, Honduras, Hungary, India, Indonesia, Iran, Islamic Rep., Israel, Italy, Jamaica, Jordan, Kenya, Latvia, Madagascar, Malawi, Malaysia, Mali, Malta, Mauritania, Mexico, Morocco, Nepal, Nicaragua, Niger, Nigeria, Pakistan, Panama, Papua New Guinea, Paraguay, Peru, Philippines, Rwanda, Saudi Arabia, Senegal, Seychelles, Sierra Leone, Singapore, South Africa, Sri Lanka, Sudan, Swaziland, Switzerland, Syrian Arab Republic, Thailand, Togo, Trinidad and Tobago, Tunisia, Turkey, Uruguay, Venezuela, RB, Zambia, Zimbabwe.

B Proof of Proposition 1

Suppose first that in the Nash equilibrium \( p^*, d^* \in (0, 1) \). Then the first order conditions of the maximization of \( U^R \) and \( U^C \) are given by:

\[
\frac{\partial U^R}{\partial p_t} = -Ak_t + \frac{\gamma f \beta^C d_t}{(\beta^R p_t + \beta^C d_t)^2} = 0 \quad \text{and} \\
\frac{\partial U^C}{\partial d_t} = -Ak_t + \frac{\gamma f \beta^R p_t}{(\beta^R p_t + \beta^C d_t)^2} = 0,
\]

from which:

\[
p^*_t = \frac{\gamma f}{4\beta^R Ak_t} \quad \text{and} \quad d^*_t = \frac{\gamma f}{4\beta^C Ak_t},
\]

(27)

Since \( \beta^R \leq 1/2 \) then \( p^*_t \geq d^*_t \). From Eq. (27) the constraint on \( p^*_t \) (i.e. \( p^*_t \in [0, 1] \)) becomes binding for \( k_t \leq \bar{k}^p = \gamma f / (4\beta^R A) \), i.e. \( p^*_t < 1 \) for \( k_t > \bar{k}^p \) and \( p^*_t = 1 \) for \( k_t \leq \bar{k}^p \). Taking \( p^* = 1 \), the first
order condition for the maximization of $U^C$ becomes:

$$\frac{\partial U^C}{\partial d_t} = -Ak_t + \frac{\gamma f \beta R}{(\beta^R + \beta^C d_t)^2} = 0,$$

from which:

$$d^*_t = \left(\frac{1}{1 - \beta^R}\right) \sqrt{\frac{\beta^R \gamma f}{Ak_t} - \frac{\beta^R}{1 - \beta^R}} \text{ for } k_t > \bar{k}^d \text{ and }$$

$$d^*_t = 1 \text{ for } k_t \leq \bar{k}^d,$$

where $\bar{k}^d = \gamma f \beta R / A$ and $\beta^C = 1 - \beta^R$. Finally since $\beta^R \leq 1/2$ then $\bar{k}^d \leq \bar{k}^p$. QED

\section{Proof of Proposition 2}

First note that if $sA > \delta + n$ then:

$$\frac{\partial k_{t+1}}{\partial k_t} = \begin{cases} 
\frac{1 - \delta}{1 + n} < 1 & \text{when } k_t \in [0, \bar{k}^d) ; \\
(\frac{1}{1 + n}) \left[ 1 - \delta + sA - \left(\frac{s\sqrt{\beta^R \gamma f A}}{2}\right) k_t^{-1/2} \right] > 0 & \text{when } k_t \in (\bar{k}^d, \bar{k}^p) \text{ and } (28) \\
\frac{1 - \delta + sA}{1 + n} > 1 & \text{when } k_t \in (\bar{k}^p, +\infty), \end{cases}$$

and:

$$\frac{\partial^2 k_{t+1}}{\partial k_t^2} = \begin{cases} 
0 & \text{when } k_t \in [0, \bar{k}^d) ; \\
\left[ \frac{s\sqrt{\beta^R \gamma f A}}{4(1+n)} \right] k_t^{-3/2} > 0 & \text{when } k_t \in (\bar{k}^d, \bar{k}^p) \text{ and } (29) \\
0 & \text{when } k_t \in (\bar{k}^p, +\infty). \end{cases}$$

Suppose that there exists an equilibrium in the range $k_t \in (\bar{k}^p, +\infty)$; then in this equilibrium per capita capital is given by

$$k_{EU} = \frac{s\gamma f}{2(sA - \delta - n)}.$$

Condition for the existence of this equilibrium is that $\bar{k}^p < k_{EU}$ (for the sake of simplicity in the proof of stability of $k_{EU}$ the frontier of the range is excluded), i.e.

$$sA < \frac{\delta + n}{1 - 2\beta^R}. \quad (30)$$
This equilibrium is locally unstable since $sA > \delta + n$ (see Eq. (28)). If Condition (30) holds then $k_{t+1} < k_t$ in $k_t = \bar{k}^p$, while $k_{t+1} = 0$ in $k_t = 0$: at least an equilibrium in the range $[0, \bar{k}^p]$ therefore exists. The monotonicity and convexity of $k_{t+1}$ with respect to $k_t$ in the range $[0, \bar{k}^p]$ (see Eqq. (28) and (29)) ensures that there exists only the stable equilibrium $k^{Es} = 0$ in the range, i.e. $\partial k_{t+1}/\partial k_t < 1$ in $k^{Es} = 0$. Indeed, in a possible second equilibrium it should be that $\partial k_{t+1}/\partial k_t > 1$, i.e. $k_{t+1}$ should cross from below the bisector; in such a case, Eq. (29) implies that $\partial k_{t+1}/\partial k_t > 1$ for all levels of capital higher than the capital of equilibrium. But this contrasts with the fact that $k_{t+1}$ must be below the bisector in $k = \bar{k}^p$.

If $sA < (\delta + n) / (1 - 2\beta R)$ then $k_{t+1} > k_t$ in $k_t = \bar{k}^p$ and therefore $\bar{k}^p > k^{Ev} > \bar{k}^d$. In fact, being $k_{t+1}$ monotone and convex in $k_t$ (see Eqq. (28) and (29)) there is not any other equilibrium in $k_t \in (\bar{k}^p, +\infty)$ ($k_{t+1} > k_t$ in $k_t = \bar{k}^p$ and $\partial k_{t+1}/\partial k_t > 1$ in $k_t \in (\bar{k}^p, +\infty)$) and in $k_t \in (0, \bar{k}^d)$ ($k_{t+1} = 0$ in $k_t = 0$ and $\partial k_{t+1}/\partial k_t < 1$ for $k_t \in (0, \bar{k}^d)$ hence $k_{t+1} < k_t$ in $k_t \in (0, \bar{k}^d)$). The monotonicity, convexity and continuity of $k_{t+1}$ with respect to $k_t$ in the range $(\bar{k}^d, \bar{k}^p)$ ensures that $k^{Ev} \in (\bar{k}^d, \bar{k}^p)$ is unique and is given by:

$$k^{Ev} = \frac{\beta R s^2 A \gamma f}{(sA - \delta - n)^2}.$$  

QED

**D Proof of Proposition 3**

Under Conditions (6) Proposition 2 states the existence of two equilibria, $k^{Es} = 0$ and $k^{Ev} > 0$, the first locally stable and the second locally unstable, and that growth path is continuous. A simple graphical inspection of Figure 3 reveals that if $k_0 < k^{Ev}$ then economy will be converging towards $k^{Es} = 0$ while if $k_0 > k^{Ev}$ then economy will be growing forever. In the latter case $\lim_{t\to+\infty} g_k = k_{t+1}/k_t - 1 = sA - \delta - n$. QED
\section*{E Proof of Proposition 3}

Proposition 5 shows that the dynamics of per capita income is driven by the dynamics of per capita capital. If \( sA > \delta + n \) Proposition 2 states that there exists a threshold in per capita capital \( k^{E_U} \); the corresponding value in term of per capita income, denote it \( y^{E_U} \), is such that all the countries with a per capita income under \( y^{E_U} \) will see their income converge to \( f (k^{E_S} = 0) \); on the contrary all the countries with a per capita income over \( y^{E_U} \) will see their income to grow at the same rate of per capita capital, i.e. \( sA - \delta - n \), in the long run. In fact as \( k_t \) increases \( y_t / k_t \) tends to constant \( A \) (the term \( f (1 - \gamma / 2) / k_t \) tends to vanish as \( k_t \to +\infty \)). The threshold of per capita income \( y^{E_U} \) is calculated from the threshold of per capita capital \( k^{E_U} \) reported in Proposition 2, i.e. \( y^{E_U} \) depends on which range of capital the threshold of per capita capital \( k^{E_U} \) belongs to.

\section*{F Proof of Proposition 7}

The use of trigger strategies means that the group of \textit{Rebels} will respect the agreement of the \textit{NBS} when the group of \textit{Citizens} will make the same action; otherwise, if the group of \textit{Citizens} deviates from agreement, i.e. it takes all the rents from natural resources by allocating a small but positive \( \varepsilon \) of its time to predate, then in the next period the group of \textit{Rebels} will play its optimal strategy given in Proposition 1. The same rule symmetrically applies for the group of \textit{Citizens}. Suppose that the level of capital \( k_0 \) is sufficiently low (such level will be calculated exactly below) that if the group of \textit{Rebels} does not respect the agreement of \textit{NBS}, the next period the optimal strategy of the group of \textit{Citizens} will be \( d^* = 1 \) and therefore also the group of \textit{Rebels} will play \( p^* = 1 \). Then the representative
rebel will respect the agreement of the NBS if:

\[
\sum_{t=0}^{\infty} \left( \gamma f + A k_t^{NBS} \right) \rho^t \geq A k_0 + \frac{\gamma f}{\beta R} + \sum_{t=1}^{\infty} \gamma f \rho^t
\]

Total payoff from NBS  Payoff at period 0 from not respecting the NBS  Total payoff from fighting from period 1

(31)

where \( \rho \in [0, 1] \) is the discount factor. The left-hand side of Eq. (31) is the sum of discounted payoffs in the NBS, where \( k_t^{NBS} \) is the per capita capital at period \( t \) in the NBS. The payoff at period 0 to deviate from the NBS for the representative rebel is equal to \( A k_0 + \gamma f / \beta R \) (total rents from natural resources is appropriate by Rebels and equally shared among all rebels). From period 1 both to \( \infty \) groups will allocate all their time in the social conflict. From Eq. (31) it yields:

\[
A \sum_{t=0}^{\infty} k_t^{NBS} \rho^t \geq A k_0 + \gamma f \left( \frac{1 - \beta R}{\beta R} \right).
\]

Since \( \beta R \leq \beta C \) Eq. (32) shows that if the representative rebel has not incentive to deviate then also the representative citizens will have not incentive to deviate. From Eq. (4) in the NBS the per capita capital follows:

\[
k_t^{NBS} = \frac{(1 - \delta + sA) k_{t-1}^{NBS}}{1 + n},
\]

from which:

\[
k_t^{NBS} = \left( \frac{1 - \delta + sA}{1 + n} \right)^t k_0.
\]

Therefore:

\[
A \sum_{t=0}^{\infty} k_t^{NBS} \rho^t = A \sum_{t=0}^{\infty} \left[ \frac{(1 - \delta + sA) \rho}{1 + n} \right]^t k_0,
\]

from which if \( \rho \in [(1 + n) / (1 - \delta + sA), 1] \) then \( A \sum_{t=0}^{\infty} k_t^{NBS} \rho^t \to +\infty \) and therefore Condition (32) will be always satisfied. If \( \rho \in \)
\[0, \frac{(1 + n)}{(1 - \delta + sA)}\) then:

\[A \sum_{t=0}^{\infty} k_t^{NBS} \rho^t = \left[\frac{A (1 + n)}{1 + n - \rho (1 - \delta + sA)}\right] k_0\]

and Condition (32) becomes:

\[\left[\frac{A (1 + n)}{1 + n - \rho (1 - \delta + sA)}\right] k_0 \geq A k_0 + \gamma f \left(\frac{1 - \beta^R}{\beta^R}\right),\]

that is:

\[k_0 \geq \left(\frac{1 - \beta^R}{\beta^R A}\right) \left[\frac{(1 + n)}{\rho (1 + sA - \delta)} - 1\right] \gamma f = \bar{k}^{TSE}._{SE} \tag{33}\]

Eq. (33) shows that if \(\rho \in \left[0, \frac{(1 + n)}{(1 - \delta + sA)}\right]\) then \(\bar{k}^{TSE}\) > 0.

Summarizing for \(\rho \in \left[\frac{(1 + n)}{(1 - \delta + sA)}, 1\right]\) the NBS can be always sustained as a subgame perfect equilibrium (independent of \(k_0\)), while for \(\rho \in \left[0, \frac{(1 + n)}{(1 - \delta + sA)}\right]\) then \(k_0\) must be not lower than \(\bar{k}^{TSE}\), otherwise for \(k_0 < \bar{k}^{TSE}\) social conflict is the outcome.

Under Condition (6) Proposition 3 states that if \(k_0 \in [0, \bar{k}^d]\) then \(\forall t > 0 \ k_t\) will be in \([0, \bar{k}^d]\) when \(d^* = p^* = 1\). However, Condition (31) is more binding and it requires that also after one period with \(d^* = p^* = 0\) economy has not to reach a level of per capita capital higher than \(\bar{k}^d\). This is guarantees by Condition \(0 \leq k_0 \leq \bar{k}^f < \bar{k}^d\), where

\[\bar{k}^f = \frac{\beta^R (1 + n) \gamma f}{A (1 - \delta + sA)} < \bar{k}^d\]

QED

\section*{G Proof of Proposition 8}

Under assumptions \(\beta^R \leq 1/2, sA > \delta + n\) and \(k_0 \in [0, \bar{k}^f]\) Proposition 7 states that if \(\rho \in \left[\frac{(1 + n)}{(1 - \delta + sA)}, 1\right]\) then there
is no social conflict in the economy, i.e. $d^* = p^* = 0$; therefore from Eq. (4)

$$k_{t+1} = \frac{(1 - \delta + sA) k_t}{1 + n}, \quad (34)$$

from which $\lim_{t \to +\infty} g_k = sA - \delta - n$. Likewise, if $\rho \in [0, (1 + n) / (1 - \delta + sA))$ and $k_0 \geq \bar{k}^{TSE}$ there is no social conflict, i.e. $d^* = p^* = 0$ and $k_{t+1}$ is given by Eq. (34) (the increase in the per capita capital implies that in the future periods using the trigger strategies the agreement of the $NBS$ will be always self-enforcing); therefore again $\lim_{t \to \infty} g_k = sA - \delta - n$. Otherwise, if $\rho \in [0, (1 + n) / (1 - \delta + sA))$ and $k_0 < \bar{k}^{TSE}$ then $d^* = p^* = 1$ and $\Delta k_{t+1} < 0 \forall k_t \in (0, \bar{k}_f]$ in all periods (see Figure 6); therefore $\lim_{t \to \infty} k_t = k^{ES} = 0$. QED