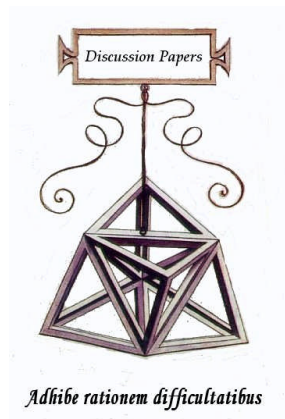




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Endogenous fertility, endogenous lifetime and economic growth:
the role of health and child policies

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Endogenous fertility, endogenous lifetime and economic growth: the role of health and child policies[°]

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Abstract In this paper we link endogenous fertility, endogenous longevity, economic growth and public policies – represented by public health investments and child policies – in a basic overlapping generations model. We found that there even exist four equilibria, and thus low and high development regimes, which may be, however, determined by government policies, and concluded that when fertility is endogenous, increasing public health is always beneficial allowing economies to escape from poverty and, hence, to prosper. The same conclusion holds for the child tax policy. In particular, the latter result may be in accord with, for instance, the tremendous development experienced by China where a restrictive one child per family policy forced by the government planned and restricted the size of Chinese families, probably allowing some geographic areas within China to escape from poverty.

Keywords Child policy; Endogenous fertility; Health; Life expectancy; OLG model

JEL Classification I1; J13; O4

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1. Introduction

Over the past century and, in particular, in the recent decades, the citizens of most countries of the world have experienced dramatic increases in their longevity (e.g., Livi-Bacci, 1997). The importance of longevity in determining the long-run outcome of the economy is the object of a growing body of economic literature, e.g., Blackburn and Cipriani (2002) and Chakraborty (2004). In particular, Blackburn and Cipriani conducted a research based on the overlapping generations (OLG) model, with three-period lived individuals that accumulate human capital through education. They determined both the macroeconomic and demographic outcomes the economy, depending on whether the rate of longevity is low or high, finding that multiple (three) steady states are possible. The higher the individual life span, the greater the time that agent may devote to education and, hence, the higher the human capital accumulation. The model by Blackburn and Cipriani predict, therefore, the possibility of both low and high development regimes, the former being characterised by low income, high fertility and a relatively low length of life, the latter by high income, low fertility and a relatively high length of life. Their model accords with the empirical aspects of the demographic transition.

Chakraborty, instead, introduced endogenous longevity – through public health expenditure – into an otherwise standard two-period overlapping generations model. He found that endogenous longevity may be responsible of the multiplicity of steady states (three). In particular, raising the public health tax may lead individuals to live longer and this, in turn, provides an incentive to greater capital accumulation and, hence, higher life span as well. According to Chakraborty, therefore, poverty and prosperity are explained on the basis of a public health measure, such as hospitals, vaccination programmes and so on.

The theoretical literature on endogenous fertility is of greater importance in the theory of economic growth (e.g., Barro and Becker, 1989; Becker and Barro, 1988), even as an explanation of multiple regimes of developments when human capital is considered (e.g., Becker et al., 1990;

Blackburn and Cipriani, 2002). In this paper, therefore, we extend the overlapping generations model by Chakraborty (2004) to allow for endogenous fertility¹ and then, different from the latter author, we study the effects of both health and child policies in determining both the transitional dynamics and steady states of the economy. We found that increasing either the health tax or the child tax produces a reduction in mortality sufficient to create a stimulus to eliminate poverty, while introducing child allowances may dramatically confine an economy into poverty, irrespective of initial conditions. The present paper is differentiated from each of the above mentioned contributions in terms its specific objectives, analysis and results.

From a broader perspective, our paper may be seen as belonging to the economic-demography literature which treats the key demographic variables – i.e. fertility and longevity – as being endogenous rather than exogenous, and links them to the process of economic growth, in the simple and intuitive context of the standard OLG model (e.g. Zhang et al., 2001). The paper may also be viewed as a contribution to the wider literature on multiple equilibria, poverty traps and demographic changes of economies over the very long-run (e.g. Azariadis and Drazen, 1990), focused on the role of health and child policies.

The remainder of the paper is organised as follows. In Section 2 we construct the model. In Section 3 we study the dynamic evolution of capital as well as the effects the health tax rate on capital accumulation, longevity and fertility. In Section 4 we analyse how a child subsidy/tax policy affects the stages of development, and, finally, Section 5 concludes.

¹ Note that even in Blackburn and Cipriani fertility is the result of a rational comparison of agents between benefits and costs of having children, but their paper focuses on human capital formation, with longevity being determined only on the accumulation of human capital through education instead of being dependant on a certain public health measure as in the present context (and in Chakraborty, 2004, as well). In contrast, we use the more flexible functional form used by Blackburn and Cipriani (for their relationship between longevity and human capital) – instead of the form used by Chakraborty – to model the relationship between longevity and public health spending.

2. The model

Consider a general equilibrium overlapping generations (OLG) closed economy populated by identical individuals. Life is divided into childhood and adulthood, the latter being, in turn, divided into working period and retirement period: youth and old age, respectively. As a child each agent does not make economic decisions. When adult she draws utility from material consumption over the life cycle and the number of children.²

Young individuals of generation t (N_t) are endowed with one unit of time which is supplied inelastically on the labour market, while receiving wage income at the competitive rate w_t .

It is assumed that the probability of surviving from youth (working period) to old age (retirement period) is endogenous and determined by the individual health level, which is, in turn, given by the public provision of a certain health measure such as, for instance, hospitals, vaccination programmes and so on (see Chakraborty, 2004). The survival probability of a young person born at t , π_t , depends upon her health capital, h_t , and is given by a non-decreasing concave function $\pi_t = \pi(h_t)$. Following Blackburn and Cipriani (2002), we specialise this relationship with the following function of health capital:³

$$\pi_t = \pi(h_t) = \frac{\pi_0 + \pi_1 h_t^\delta}{1 + h_t^\delta}, \quad (1)$$

² See Galor and Weil (1996).

³ Although the independent variable in the analysis conducted by Blackburn and Cipriani (2002) is human capital instead of public health capital, the line of reasoning to justify this formulation may be the same. Realistically, health investment has a more intense effect in reducing adult mortality often when a certain threshold level of public health expenditure has been reached, while becoming scarcely effective of course when longevity is close to its saturating value (e.g., the functional relationship between health investment and longevity may be S-shaped).

satisfying the following properties: $\delta \geq 1$, $0 < \pi_1 \leq 1$, $0 < \pi_0 < \pi_1$, $\pi(0) = \pi_0 > 0$,

$$\pi'_h(h) = \frac{\delta h_t^{\delta-1} (\pi_1 - \pi_0)}{(1 + h_t^\delta)^2} > 0, \quad \pi''_{hh}(h) < 0 \text{ if } \delta = 1 \text{ and } \pi''_{hh}(h) \begin{matrix} \geq 0 \\ < 0 \end{matrix} \text{ for any } h \begin{matrix} \leq \\ > \end{matrix} \left(\frac{\delta - 1}{1 + \delta} \right)^{\frac{1}{\delta}} \text{ if } \delta > 1,$$

$\lim_{h \rightarrow \infty} \pi(h) = \pi_1 \leq 1$, $\lim_{h \rightarrow 0} \pi'_h(h) < \infty$.⁴ We define π_0 as being the exogenous “natural” rate of longevity of individuals in a country. For instance, π_0 may be thought to be higher, the better the lifestyle of people (e.g. a higher attention to a healthy life), the higher the importance to the protection of the workers’ rights, the higher the investment in social infrastructures, education and so on. Thus, we may expect π_0 to be higher in developed rather than developing or under-developed countries, and the higher the natural rate of longevity, the smaller the reduction in adult mortality due to a rise in public health capital. This means that in economies where individuals naturally live longer (developed countries), the effectiveness of an additional increase in public health investment in raising life expectancy will be lower as that would be experienced in economies where individuals naturally live shorter (developing or under-developed countries).

We follow Chakraborty (2004) and assume that public health expenditure at time t is financed through a (constant) proportional tax $0 < \tau < 1$ according to the following technology

$$h_t = \tau w_t. \quad (2)$$

⁴ Note that the scenario $\pi_0 = 0$ and $\delta = 1$ resumes the case studied by Chakraborty (2004) in a neoclassical OLG growth model with exogenous fertility. The assumption of a positive natural rate of longevity, however, exposes the economy to a dramatic change: the zero equilibrium – which is a catching point when $\pi_0 = 0$ and the output elasticity of capital is high enough –, becomes unstable when $0 < \pi_0 < \pi_1$ and the number of equilibria moves from three to four, thus making certainly more plausible a comparison between different development regimes, e.g. low income-high fertility-high mortality (poor) economies versus high income-low fertility-low mortality (rich) economies. As a matter of fact, the existence of a stable zero equilibrium where the economy is collapsed may be an useful abstraction which, however, suffers from lack of realism, to represent poor economies. In this model, therefore, we concentrate exclusively on the case $0 < \pi_0 < \pi_1$.

Raising children is costly, and the amount of resources needed for parents to care about children is given by a monetary cost $q w_t$ per child, with $0 < q < 1$ being the percentage of child-rearing cost on working income.⁵ Moreover, we assume that in every period the government finances a child allowance programme at a balanced budget through wage income taxes. The per capita child policy budget of the government at t , therefore, reads as

$$b w_t n_t = \theta_t w_t, \quad (3)$$

the left-hand side being the child allowance expenditure and the right-hand side the tax receipt, where $0 < b < q$ is the fixed percentage of wage income entitled to each young parent as a subsidy for each additional newborn, $0 < \theta_t < 1$ the wage tax rate, and n_t the average number of children at time t .⁶

Therefore, in period t the budget constraint faced by an individual of the younger working-age (child-bearing) generation reads as:

$$c_{1,t} + s_t + (q - b) w_t n_t = w_t (1 - \tau - \theta_t), \quad (4.1)$$

i.e. wage income – net of contributions paid to finance health expenditure and child subsidies – is divided into material consumption when young, $c_{1,t}$, savings, s_t , and the (net) cost of raising n children.

When old adult individuals are retired and live uniquely with the amount of resources saved when young plus the interests accrued at the rate r_{t+1} . Moreover, to abstract from the risk associated with uncertain lifetimes, we assume the existence of a perfect annuity market where savings are intermediated through mutual funds. Each mutual fund invests these savings in the unique asset, i.e.

⁵ This child cost structure (which can also be seen as a proxy of the time-cost structure) is similar to that adopted, amongst many others, by Wigger (1999) and Boldrin and Jones (2002).

⁶ It is worthwhile noting that we allow even for the existence of child taxes ($b < 0$) to finance a wage income subsidy ($\theta_t < 0$).

capital, while guaranteeing a gross return of \hat{R}_{t+1} to the surviving old. If a fund earns a gross return R_{t+1} on its investment, then the condition $\hat{R}_{t+1} = R_{t+1} / \pi_t$ holds in a competitive equilibrium, where $R_{t+1} = 1 + r_{t+1}$. Hence, the budget constraint of a retired individual entered the working period at t can easily be expressed as

$$c_{2,t+1} = \frac{1 + r_{t+1}}{\pi_t} s_t, \quad (4.2)$$

where $c_{2,t+1}$ is old-aged consumption.

The representative individual entering the working period at time t must choose (i) how much to save out of her disposable income and (ii) how many children to raise in order to maximise the following lifetime logarithmic utility function

$$U_t = \ln(c_{1,t}) + \pi_t \ln(c_{2,t+1}) + \gamma \ln(n_t), \quad (5)$$

subject to Eqs. (4), with $\gamma > 0$ capturing the parents' relative taste for children.

The constrained maximisation of Eq. (5) gives the demand for children and the saving function, respectively:

$$n_t = \frac{\gamma(1 - \tau - \theta_t)}{(1 + \pi_t + \gamma)(q - b)}, \quad (6.1)$$

$$s_t = \frac{\pi_t w_t (1 - \tau - \theta_t)}{1 + \pi_t + \gamma}. \quad (6.2)$$

Now, using Eq. (3) to eliminate θ_t and rearranging terms, Eqs. (6) can definitively be written as:

$$n_t = \frac{\gamma(1 - \tau)}{(1 + \pi_t)(q - b) + \gamma q}, \quad (7.1)$$

$$s_t = \frac{\pi_t w_t (1 - \tau)(q - b)}{(1 + \pi_t)(q - b) + \gamma q}, \quad (7.2)$$

with π_t being determined by Eq. (1). Note that both fertility and saving still remain positive even in the absence of public health investment, and are respectively given by $n_t = \frac{\gamma}{(1 + \pi_0)(q - b) + \gamma q}$ and

$$s_t = \frac{\pi_0 w_t (q - b)}{(1 + \pi_0)(q - b) + \gamma q}, \text{ the former being constant.}$$

2.2. Production and equilibrium

Firms are identical and act competitively on the market. The (aggregate) constant returns to scale Cobb-Douglas technology is $Y_t = AK_t^\alpha L_t^{1-\alpha}$, where Y , K and $L = N$ are output, capital and the labour input respectively, $A > 0$ represents a scale parameter and $0 < \alpha < 1$ is the distributive capital share. Defining $k_t := K_t / N_t$ as being the per capita stock of capital and $y_t := Y_t / N_t$ per capita output, the intensive form production function can be written as $y_t = Ak_t^\alpha$. Standard profit maximisation implies that factor inputs are paid their marginal products, that is⁷

$$r_t = \alpha Ak_t^{\alpha-1} - 1, \quad (8)$$

$$w_t = (1 - \alpha)Ak_t^\alpha. \quad (9)$$

Knowing that $N_{t+1} = n_t N_t$, market-clearing in goods and capital market yields the usual condition $n_t k_{t+1} = s_t$, which is combined with (1), (7.1), (7.2) and (9) to obtain the following first order difference equation driving the dynamic evolution of capital in this stylised economy.

$$k_{t+1} = \frac{Dk_t^\alpha (\pi_0 + \pi_1 Bk_t^{\alpha\delta})}{\gamma(1 + Bk_t^{\alpha\delta})}, \quad (10)$$

where $B := [\tau(1 - \alpha)A]^\delta$ and $D := (q - b)(1 - \alpha)A$ are two positive constant used to simplify notation.

⁷ We assumed both full depreciation of capital at the end of each period and a unit price of final output.

A rise in the health tax rate may either rise or reduce the saving rate in the short run, while reducing monotonically the rate of fertility. However, as can be ascertained from Eq. (10), the future capital stock always depends positively on the health tax rate and, hence, on the rate of longevity. This means that an increase in public health expenditure, through health taxes, causes a rise in longevity and hence, in the capital stock, or, in other words, the reduction in fertility is always proportionally greater than the possible reduction in savings due to higher health taxes.

3. Dynamics

Analysis of Eq. (10) gives the following proposition:

Proposition 1. *The dynamic system described by Eq. (10) admits either two steady states $\{0, \bar{k}\}$, with $\bar{k} > 0$ (only the positive one being asymptotically stable) or four steady states $\{0, \bar{k}_1, \bar{k}_2, \bar{k}_3\}$, with $\bar{k}_3 > \bar{k}_2 > \bar{k}_1 > 0$ (only the second and the fourth being asymptotically stable).*

Proof. Let first the following lemma be established.

Lemma 1. *Define the right-hand side of (10) as $G(k)$. Then, we have: (1.i) $G(0)=0$, (1.ii) $G'_k(k) > 0$ for any $k > 0$, (1.iii) $\lim_{k \rightarrow 0^+} G'_k(k) = +\infty$, (1.iv) $\lim_{k \rightarrow +\infty} G'_k(k) = 0$, (1.v) $G''_{kk}(k)$ admits at most three roots and $G''_{kk}(0) \neq 0$.*

From Eq. (10), property (1.i) is straightforward. Differentiating the right-hand side of (10) with respect to k gives

$$G'_k(k) = \frac{\alpha D (Fk^{2\alpha\delta} + Ek^{\alpha\delta} + \pi_0)}{\gamma k^{1-\alpha} (1 + Bk^{\alpha\delta})^2}, \quad (11.1)$$

where $E := [\pi_0 + \pi_1 + \delta(\pi_1 - \pi_0)]B > 0$ and $F := \pi_1 B^2 > 0$. Defining $k^{\alpha\delta} := x$ as a new supporting variable, we may express (11.1) as

$$g(k, x) = \frac{\alpha D (Fx^2 + Ex + \pi_0)}{\gamma k^{1-\alpha} (1 + Bx)^2}. \quad (11.2)$$

Since no positive real roots of (11.2) can exist, then (11.1) implies that $G'_k(k) > 0$ for any $k > 0$.

This proves (1.ii).

Moreover,

$$\lim_{k \rightarrow 0^+} G'_k(k) = \frac{\alpha D}{\gamma} \lim_{k \rightarrow 0^+} \frac{Fk^{2\alpha\delta} + Ek^{\alpha\delta} + \pi_0}{k^{1-\alpha} (1 + Bk^{\alpha\delta})^2} = +\infty.$$

and

$$\lim_{k \rightarrow +\infty} G'_k(k) = \lim_{k \rightarrow +\infty} \frac{\alpha D (Fk^{2\alpha\delta} + Ek^{\alpha\delta} + \pi_0)}{\gamma k^{1-\alpha} (1 + Bk^{\alpha\delta})^2} = \frac{\alpha D}{\gamma} \lim_{k \rightarrow +\infty} \frac{\frac{\pi_0}{k^{2\alpha\delta}} + \frac{E}{k^{\alpha\delta}} + F}{k^{1-\alpha} \left(\frac{1}{k^{2\alpha\delta}} + \frac{2B}{k^{\alpha\delta}} + B^2 \right)} = 0.$$

which prove (1.iii) and (1.iv), respectively. Now, differentiating (11.1) with respect to k gives

$$G''_{kk}(k) = \frac{-\alpha D (\Lambda_1 k^{3\alpha\delta} + \Lambda_2 k^{2\alpha\delta} + \Lambda_3 k^{\alpha\delta} + \Lambda_4)}{\gamma k^{2-\alpha} (1 + Bk^{\alpha\delta})^3}, \quad (12.1)$$

where

$$\Lambda_1 := (1 - \alpha)BF > 0,$$

$$\Lambda_2 := (1 - \alpha)(BE + F) + \alpha\delta(BE - 2F),$$

$$\Lambda_3 := (1 - \alpha)(\pi_0 B + E) + \alpha\delta(2\pi_0 B - E),$$

$$\Lambda_4 := \pi_0(1 - \alpha) > 0.$$

Using $k^{\alpha\delta} := x$, Eq. (12.1) can be rewritten as

$$f(k, x) = \frac{-\alpha D (\Lambda_1 x^3 + \Lambda_2 x^2 + \Lambda_3 x + \Lambda_4)}{\gamma k^{2-\alpha} (1 + Bx)^3}. \quad (12.2)$$

From (12.2), it is straightforward to see that $f(k, x)$ admits at most three roots for x and $f(k, 0) \neq 0$, i.e. $x = 0$ cannot be an inflection point. Hence, from (12.1), $G''_{kk}(k)$ admits at most three roots for k and $G''_{kk}(0) \neq 0$, i.e. $k = 0$ cannot be an inflection point. This proves (1.v).

Proposition 1 therefore follows. In fact, by properties (1.i) and (1.iii), zero is always an unstable steady state of Eq. (10). By (1.ii)-(1.iv), $G(k)$ is a monotonic increasing function of k and eventually falls below the 45° line, so that at least one positive stable steady state exists for any $k > 0$.

Now, assume *ad absurdum* the existence of an odd number of equilibria. By (1.ii)-(1.iv), the inflection points cannot be odd-numbered for any $k > 0$. By property (1.v), therefore, the number of inflection points is either zero or two for any $k > 0$. Since at least one positive stable steady state exists, then for any $k > 0$ the phase map $G(k)$ may intersect the 45° line from below *at most* once before falling below it. Hence, an even number of equilibria must necessarily exist. In particular, the steady states are either two, with the positive one being the unique asymptotically stable equilibrium, or *at most* one positive steady state separates the lowest asymptotically stable steady state from the highest asymptotically stable one, and, thus, the number of equilibria is four. **Q.E.D.**

Proposition 1 says that endogenous longevity through public health investment may be responsible to the existence of multiple positive equilibria and, hence, low and high development regimes. The former being characterised by low income, high fertility and high mortality, the latter by high income, low fertility and low mortality, as shown in the following figure.

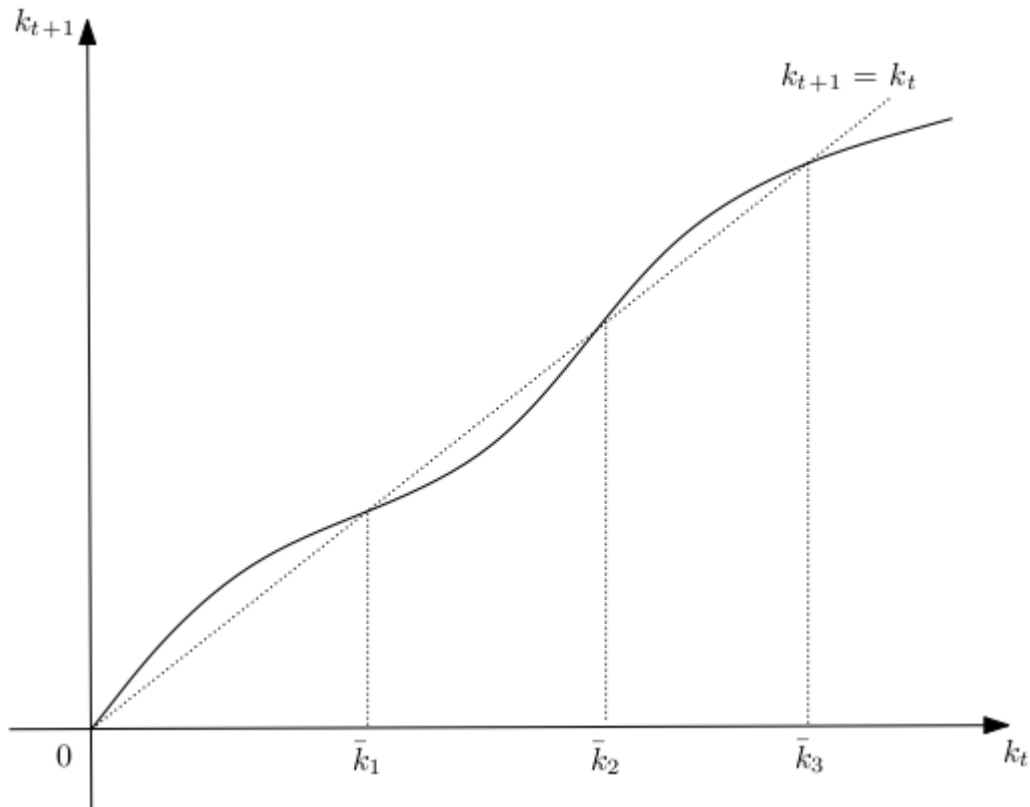


Figure 1. Multiple steady states.

Figure 1 depicts the dynamic evolution of capital as described by Eq. (10) in a stylised way, showing all the possible outcomes for an economy with endogenous longevity – determined by public health investment – and endogenous fertility. It is important to note that extensive numerical simulations revealed that the multiplicity of equilibria appears more likely when the output elasticity of capital, α , is relatively high, otherwise the standard result of the uniqueness of the equilibrium – as in Diamond (1965) – holds. In particular, in the case $\delta = 1$, multiple steady states appears when α exceeds $1/2$,⁸ and this threshold shrinks monotonically as δ raises. It is important to note that the preceding literature concentrated especially on the case of three equilibria with the poverty trap being represented by the zero equilibrium. This surely represents a useful abstraction in the theoretical literature, allowing to discriminate between poor and rich countries, but certainly it may suffer from lack of realism, especially as regards the empirical relevance of the result. It is

⁸ This result is in accord with the Chakraborty's result.

therefore important to stress that our finding permits us to be able to discriminate between low and high development regimes in the very basic overlapping generations model, and, hence, the comparison between different regimes becomes relevant even from an empirical viewpoint. Moreover, postulating the functional form adopted by Blackburn and Cipriani (2002) as regards the relationship longevity and public health services, we obtain that a wider range of economies may be described as being prone to be entrapped into poverty, since the threshold of the output elasticity of capital that allows multiple equilibria is empirically more plausible and smaller than one-half, given the importance of the parameters of the health technologies.

Now, assume that economies differ exclusively as regards the initial condition k_0 . Figure 1 shows that an economy that starts below the unstable equilibrium \bar{k}_2 is entrapped into poverty, \bar{k}_1 , and is characterised by low income, high fertility and high mortality. In contrast, an economy with an initial capital stock above \bar{k}_2 converges towards the highest possible stable steady state \bar{k}_3 , where income is high, fertility is low and mortality is low. Therefore exogenous shifts in k_0 may cause a change in development regimes.

However – *ceteris paribus* as regards the key parameters of the model –, the existence of multiple stable steady states depends upon the relative size of public health investment, as measured changes in the health tax rate τ , since it affects longevity, which in turn determines the dynamics of capital. It is therefore interesting studying the effects of the health tax rate τ on the transitional dynamics and the steady states of the economy. Below, we will see that – irrespective of whether the initial condition is higher or lower than the threshold that discriminates between low and high development regimes, \bar{k}_2 –, a relatively low health tax rate may be responsible of the loss of the highest stable steady state, thus confining permanently all the economies into the poverty trap, while a wide investment in public health may favour the loss of the poverty trap, thus creating the conditions to converge towards the high development regime even for economies that were entrapped in the low development regime, which will experience, therefore, a rise in both the

income per capita and the length of life and a reduction in fertility due to the improved health conditions.

In fact, numerical simulations revealed that a rise (reduction) in the health tax rate shifts upward (downward) the phase map $G(k)$, others things being equal. Therefore, a government whose purpose is to promote public health through higher health taxes causes a rise in both the lowest and the highest stable steady states, \bar{k}_1 and \bar{k}_3 respectively, while reducing the intermediate unstable one, \bar{k}_2 . This shrinks the basin of attraction to the poverty trap. Hence, an economy whose initial condition was to the left of \bar{k}_2 may still remain entrapped into poverty but converge faster towards the new relatively higher low equilibrium where mortality and fertility are slightly reduced, while an economy whose initial condition was close to the left of \bar{k}_2 may escape from poverty and move towards the new higher development regime, thus experiencing a large reduction in both fertility and mortality, the latter, in turn, providing an impetus to a wide increase in income per capita. In contrast, a government whose purpose is that of shrinking health taxes and, hence, public health capital, causes a reduction in both \bar{k}_1 and \bar{k}_3 , as well as a rise in the intermediate equilibrium \bar{k}_2 , thus enlarging the basin of attraction to the poverty trap.

But this is not the end of the story. Investing extensively in public health may represent a design that helps economies to escape permanently from poverty, irrespective of initial conditions. In contrast, a government whose purpose is that of reducing public medical aid, vaccination programmes and so on through lowering health taxes, may have dramatic negative effects since prosperity may disappear, and hence, even countries with huge differences in initial conditions are confined into poverty, as shown in Figure 2 below (which has been depicted in a stylised way exclusively for illustrative purposes).

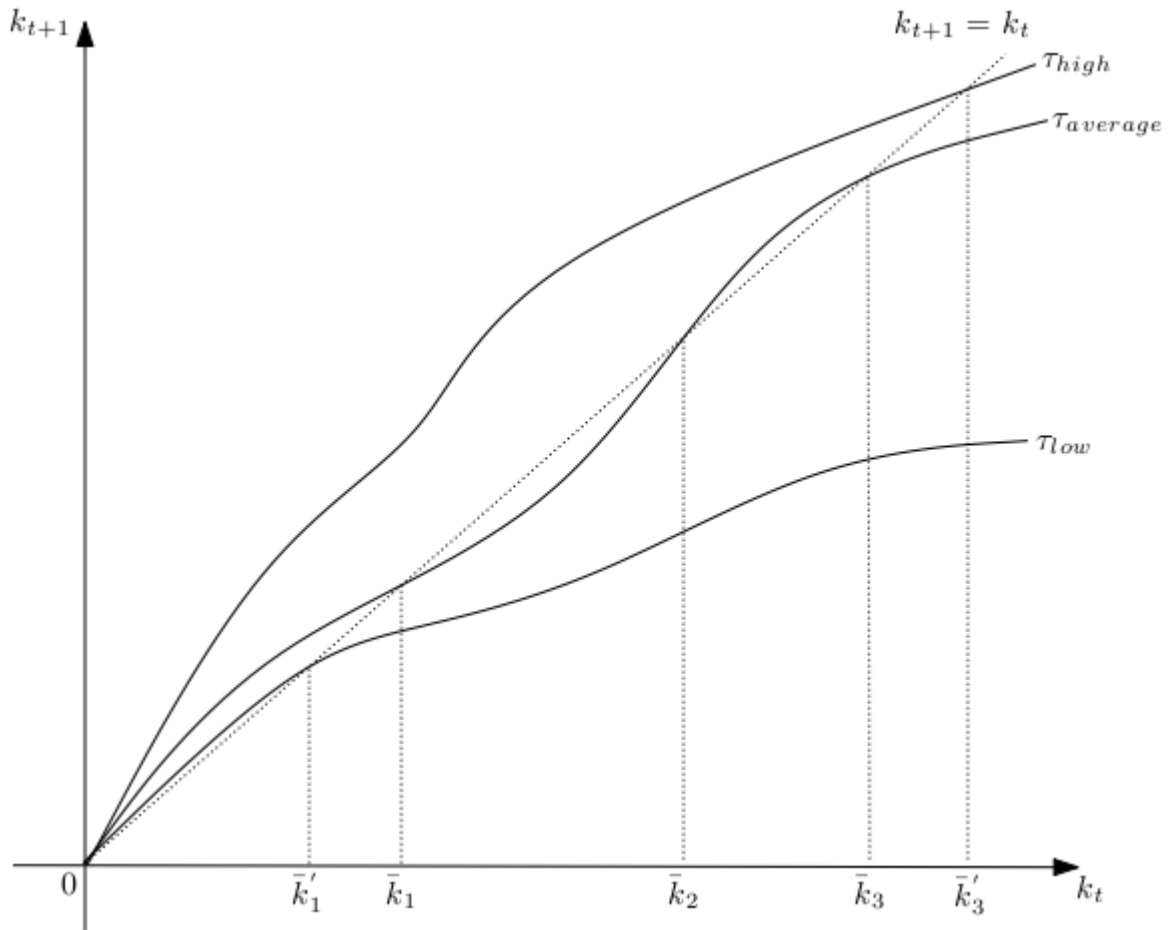


Figure 2. Health tax and development regimes.

In Figure 2 we have depicted the generic behaviour of the phase map $G(k)$ for three different values of the health tax rate τ , namely, τ_{low} , $\tau_{average}$ and τ_{high} , showing that the public health policy may be responsible of the existence of multiple steady states (the picture holds given the structural parameters of both the production and health technologies as well as the parents' relative taste for children and the cost of child bearing). The starting point is $\tau_{average}$, with respect to which there exist both low and high development regimes, as given by \bar{k}_1 and \bar{k}_3 respectively. An economy that starts below \bar{k}_2 is a poor country, with low per capita income, high fertility and high mortality, while an economy whose initial condition is above \bar{k}_2 is a rich country, with higher per capita income and both fertility and mortality rates lower than that obtained by the poorer country. Assume now that the government wants to stimulate public health investment through a percentage

increase in the health tax rate. This scenario is represented by τ_{high} . In that case, the upward shift of the phase map is such that poverty vanishes, or in other words, the impetus to capital accumulation created by the increased health capital implies that irrespective of whether an economy started either below or above the threshold \bar{k}_2 before the tax change, economies that were confined to the poverty trap now converge towards \bar{k}_3' , experiencing a huge increase in both the per capita income and the length of life of individuals as well as a reduction in fertility rates. In contrast, if for some exogenous reasons the government wishes to reduce public health through a reduction in the tax rate, the consequences can be dramatically negative. This scenario is described by τ_{low} . In that case economies move to the lowest possible equilibrium \bar{k}_1' without exceptions and are therefore constrained by poverty.

To illustrate numerically Figure 2, we take the following parameter set: $A = 10.5$, $\alpha = 0.33$, $\gamma = 0.30$, $\pi_0 = 0.30$, $\pi_1 = 0.95$, $\delta = 10$, $q = 0.15$ and, for simplicity we concentrate on the case $b = 0$ since the effects of the child policy variable will be analysed in the next section. Then, by considering $\tau_{average} = 0.10$ we find that the low development regime is characterised by the equilibrium values $\bar{k}_1 = 1.358$, $\pi(\bar{k}_1) = 0.348$ and $n(\bar{k}_1) = 1.091$, and the high development regime by $\bar{k}_3 = 5.338$, $\pi(\bar{k}_3) = 0.873$ and $n(\bar{k}_3) = 0.828$. The intermediate unstable equilibrium which discriminates between poverty and prosperity is $\bar{k}_2 = 2.125$. Therefore, an economy whose initial condition is below (above) $\bar{k}_2 = 2.125$ represents a poor (rich) country. Let us now see what happens if the health tax from $\tau_{average} = 0.10$ raises to $\tau_{high} = 0.20$. In that case, irrespective of initial conditions, economies converge to the high regime characterised by $\bar{k}_3' = 6.053$, $\pi(\bar{k}_3') = 0.949$ and $n(\bar{k}_3') = 0.711$, and the poverty trap disappears. In contrast, a reduction in the health tax to $\tau_{low} = 0.05$ implies the existence of a unique low stable equilibrium such that $\bar{k}_1' = 1.083$,

corresponding to which the equilibrium values of both longevity and fertility are $\pi(\bar{k}_1') = 0.30$ and

$n(\bar{k}_1') = 1.1874$, respectively. Therefore, the following result holds:

Result 1. *Poverty or prosperity might not depend on initial conditions, while being the result of a public health design! In particular poverty may be eliminated even with a slight increase in health taxes.*

It is worthwhile noting, however, that changes in some of the basic parameter of the model (e.g., either the natural rate of longevity or the output elasticity of capital) give rise to a change in the phase map $G(k)$, altering also the threshold value which discriminates between different development regimes.

4. Child policies

At this point, it is useful to study whether and how the child policy variable affects both the transitional dynamics and steady states of the economy as well as their implications on the rate of fertility in this stylised framework with endogenous longevity and endogenous fertility. The child subsidy in the form of direct monetary transfers entitled to families with children is an instrument often suggested both by politicians and economists as a remedy to low fertility.⁹ In contrast, the one child per family policy of China belonging to the Chinese family planning policy (see Coale, 1981) was introduced, among other things, as a stimulus to economic growth. What about the effects of

⁹ Policies consisting in cash subsidies for children are largely present in most countries. As an example, consider that in Italy a 1,000 euro child grant for each new born was introduced in the year 2005, while in Poland every woman will benefit from a one-off 258 euro payment for each child, and women from poorer families will receive double the previous amount.

the child policy variable b in this context? As expected, we find that a rise in the child subsidy (tax) increases (reduces) fertility, while reducing (increasing) savings. A child subsidy (tax), therefore, discourages (promotes) capital accumulation and, hence, through public health capital, provides a disincentive (incentive) to longevity, which in turn contributes to depress (rise) further the stock of capital and thus per capita income.

A value added of this paper therefore grounds on the importance of child policy variable in determining the behaviour of capital accumulation and the multiplicity of the steady states, as shown in the following proposition.

Proposition 2. *A rise in the child subsidy (tax) reduces (increases) the extreme stable steady states, increases (reduces) the intermediate unstable steady state, while enlarging (shrinking) the size of the basin of attraction of the poverty trap.*

Proof. Since

$$G'_b(k) = \frac{k^\alpha (\pi_0 + \pi_1 Bk^{\alpha\delta})}{\gamma(1 + Bk^{\alpha\delta})} \cdot \frac{\partial D}{\partial b},$$

and

$$G''_{kb}(k) = \frac{\alpha(Fk^{2\alpha\delta} + Ek^{\alpha\delta} + \pi_0)}{\gamma k^{1-\alpha}(1 + Bk^{\alpha\delta})^2} \cdot \frac{\partial D}{\partial b},$$

where $\partial D / \partial b = -(1-\alpha)A < 0$ ($(1-\alpha)A > 0$) for any $0 < b < q$ ($b < 0$), then $G'_b(k) < 0$ (> 0) and

$G''_{kb}(k) < 0$ (> 0) for any $0 < b < q$ ($b < 0$). Hence, a rise in the child subsidy (tax) always reduces

(increases) the slope of $G(k)$, while also shifting downward (upward) the phase map for any $k > 0$.

Proposition 2, therefore, follows.

Moreover, the following result can be established.

Result 2. (1) A too high child subsidy can produce the loss of the highest stable steady state, thus confining permanently the economy to the lowest stable steady state (the poverty trap). (2) A sufficiently high child tax can produce the loss of the lowest stable steady state, thus allowing poorer economies to escape permanently from the poverty trap and converge towards the highest stable steady state. Therefore, given the value of the health tax, poverty or prosperity may not depend on initial conditions, while being the result of a public family policy. In particular poverty may be eliminated even with a slight increase in child taxes.

The following figure describes both Proposition 2 and Result 2.

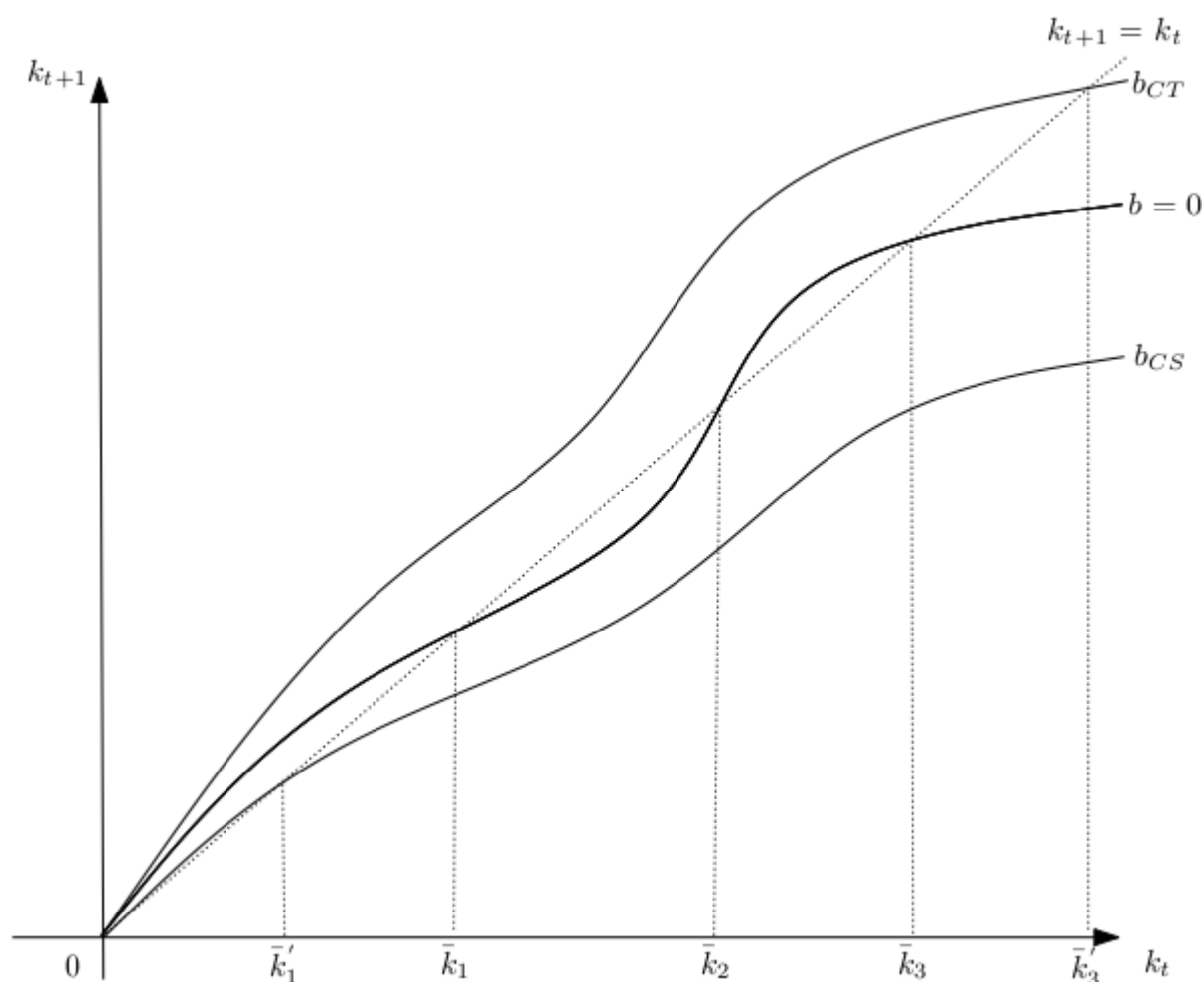


Figure 3. Child policy and development regimes.

A numerical experiment illustrating Figure 3 now follows. We take same parameter values as before, that is, $A=10.5$, $\alpha=0.33$, $\gamma=0.30$, $\pi_0=0.30$, $\pi_1=0.95$, $\delta=10$, $q=0.15$, while keeping unchanged the health tax at $\tau=0.10$ to look at the effects of the child policy variable b . Starting from the case $b=0$ where the world is divided into poor and rich countries, and characterised by the equilibrium values $\bar{k}_1=1.358$, $\pi(\bar{k}_1)=0.348$ and $n(\bar{k}_1)=1.091$, and $\bar{k}_3=5.338$, $\pi(\bar{k}_3)=0.873$ and $n(\bar{k}_3)=0.828$, respectively, the introduction of a child subsidy $b_{CS}=0.05$ implies that the new phase map will lie everywhere below the 45° line once the lowest equilibrium $\bar{k}'_1=0.602$ is reached, corresponding to which longevity and fertility are $\pi(\bar{k}'_1)=0.303$ and $n(\bar{k}'_1)=1.539$, respectively. Thus, a direct implication of introducing the child subsidy at too high a level to stimulate fertility is the entrapment into poverty, since adult mortality raises and then creates the condition for a further decrease in capital accumulation. In contrast, a child tax policy such that $b_{CT}=-0.05$ improves fertility, savings, capital accumulation, the wage rate earned by the young workers and life expectancy, which in turn provides a further stimulus to accumulate capital. As a consequence, a relatively slight increase in child taxes leads the phase map to lie everywhere above the 45° line and to fall below it only once the highest equilibrium $\bar{k}'_3=9.086$ is reached where longevity and fertility are, respectively, $\pi(\bar{k}'_3)=0.935$ and $n(\bar{k}'_3)=0.624$.

5. Conclusions

We studied the effect of both health and child policies in a basic overlapping generations model with endogenous fertility, extending the model by Chakraborty (2004). We assumed a sufficiently general form (see Blackburn and Cipriani, 2002) of the relationship between longevity and public

health spending, to allow a rather realistic features of a “natural” longevity and a possibly logistic form to co-exist.

We found the existence of four equilibria (instead of the usual three equilibria setup), and this enriched set of long-run outcomes is possible under more plausible economic conditions with respect to those described in Chakraborty (2004), i.e. even economies with a relatively low capital share might experience poverty traps. Therefore low and high development regimes do exist: however, they may be determined by government policies. In particular, irrespective of whether an economy starts with either a low or a high stock of capital, an increasing amount of public health expenditure (or a child tax policy) represents a stimulus to accumulate capital, increase per capita income and life expectancy and reduce fertility, thus allowing to escape permanently from poverty

In this paper, therefore, we link endogenous fertility, endogenous longevity, economic growth and public policies – represented by public health investments and child policies – in a basic overlapping generations framework. We conclude that when fertility is endogenous, increasing public health is always beneficial and allow economies to escape from poverty and to prosper. The same conclusion holds for the child tax policy. This result, therefore, may be a possible answer to the tremendous development experienced by China where a restrictive one child per family policy – forced by the government – planned and restricted the size of the Chinese families, probably allowing some geographic areas within China to escape from poverty.

Finally other lines along which the present model can be extended are to include in the analysis: (i) a private system of health provision, in comparison with public health provision; (ii) other public interventions in addition to child and health policies, such as public pensions and public education; (iii) improvements in health care can have important economic effects beyond those engendered by greater life expectancy: for instance lower morbidity and better functionality can raise (just like education) productivity and wages of individuals (e.g. Strauss and Thomas, 1998).¹⁰

¹⁰ All these possible extensions are current work in progress by authors.

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