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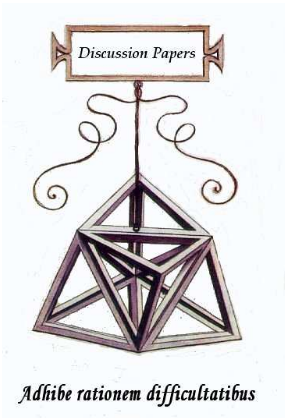

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A two-sector overlapping generations economy: economic growth and multiple equilibria

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A two-sector overlapping generations economy: economic growth and multiple equilibria

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Abstract We study an overlapping generations economy with two sectors of production, the commodities sector and the services sector and public health investments that affect the lifetime of people (as in Chakraborty, 2004). It is shown a reallocation of labour towards the services sector increases capital accumulation and longevity, thus allowing those economies that, for some exogenous reasons, were entrapped into poverty to prosper.

Keywords Life expectancy; OLG model; Poverty trap; Public health spending; Services market

JEL Classification I18; O41

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1. Introduction

This paper analyses two prominent problems on the political and economic debate in several countries: (*i*) population ageing, and (*ii*) the increasing demand for both health care provision and services for old age people. The productive structure of an economy can be broadly shared into two categories: capital-intensive commodities and labour-intensive services. A specific feature of the demand of the two categories is worth to be noted: the demand is different depending on the consumers' age and, in particular, the demand for services is an increasing function of it. The need of older people to use more services that the younger emerges from both anecdotic and statistical sources: (*i*) for instance, The Economist argued that: "Older people are likely to spend more on medical care and domestic services, such as those of gardeners and cleaners. Younger people are likely to spend more on new products, such as mobile phones or computers (The price of age, The Economist, 21 December, 2000); (*ii*) budget surveys tend to reveal a larger service expenditure for the elderly. For instance, from the ONS Family Spending 2000–01 for the United Kingdom and from the CBS Budgetonderzoek for The Netherlands may be seen that the elderly spend 1.5 times of their total expenditures on services more than younger people.[1] Therefore, to capture this feature of consumptions in this model we assume, for simplicity, that only the old-aged spend on services.[2]

In this paper the number of children is exogenous while the rate of longevity depends on public health spending, as in Chakraborty (2004).

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[1] Both instances are drawn by van Groezen et al. (2005, p. 648; 2007a, p. 736).

[2] This assumption captures the feature of the need of higher demand for services by the older people, standing in the middle between the assumption made by van Groezen et al. (2007b), where young- and old-aged consume both goods and services, and the assumption made by van Groezen et al. (2005, 2007a), where young people only consumes goods and old people only consume services. It must be noted, however, that while a similarity as regards the assumption of a two-sector economy exists between the present model and theirs, van Groezen et al. (2007a; 2007b) focused on pension problems, while van Groezen et al (2005) built on a model with exogenous demographic variables and analysed the effect of a rise in the rate of longevity on both the production structure of the economy and economic growth.

Most articles that investigated the mutual relationship between economic and demographic outcomes only considered either a static partial equilibrium context (e.g., Apps and Rees, 2004) or, in a general equilibrium context accounted for neither the existence of aggregate consumption of goods and services, nor the consequences that a two-sector economy may have on transitional dynamics and long-run demo-economic performances. In addition to the above mentioned assumption of a two-sector economy, and different from several papers that neglect, e.g., the production side of the economy (e.g. Blackburn and Cipriani, 2002), another distinguishing feature of the present paper is that a component of demographic behaviour (i.e., longevity) is endogenously determined in a dynamic general equilibrium context.

A crucial key ingredient for a change in the sector composition is represented by the change in individual savings. In fact, the saving rate directly affects the demand for services by augmenting the income of the elderly, and by determining capital and wages it affects the capital intensity in the commodity sector, the return on savings – and thus both the income of the elderly and demand for services – and the price of the service. Therefore, to the extent that changes in the individual length of life affect individual savings, the sector composition changes as well. But the effect of the demographic variable through savings is only a part of the story: indeed the dependency ratio also causes a direct effect on the demand for old-aged services, and thus population ageing in itself shifts the sector composition towards a larger share for the production of services.

The focus of the paper grounds on studying the effect of the demand for old-aged services within a neoclassical overlapping generations (OLG) growth model (Diamond, 1965) modified to account for two production sectors (i.e. the commodity and the service sectors), where an endogenous relationship between life expectancy, health care provision and output emerges.

We account for endogenous mortality by assuming that the probability of surviving from the first to the second period of life depends upon individual health status which is, in turn, augmented through public investment financed by a wage tax.[3] While the provision of health care services, on the one hand, competes resources away (through taxation) from consumption and savings, on the other hand it raises life expectancy and capital accumulation, while also increasing the demand of old-age services, so that the effect of health policies on economic growth may appear, a priori, uncertain. We show that the role played by the preference for old-aged services on economic growth is the following: since a rise in the preference for services tends to increase life expectancy (through the rise in wages and thus in the provision of health care services), then (*i*) the per worker GDP increases because of the positive effect of longevity on capital accumulation, but (*ii*) the per capita GDP shrinks due to the increased number of mature individuals. The overall effect, however, is positive and thus capital accumulation increases.

Moreover, multiple steady states and, hence, poverty traps may exist: unless a high-mortality society starts out with a high enough capital stock (above a certain threshold level), it is unable to escape the vicious cycle of poverty and ill-health. We show that the higher the preference for old age services, the likelier high-mortality societies will escape from poverty.

The remainder of the paper is organised as follows. Section 2 develops the model. Section 3 analyses the dynamics and long-run outcomes of the economy. Section 4 concludes.

2. The model

Consider a general equilibrium OLG closed economy populated by identical individuals, identical firms and a government that finances a public health programme by collecting wage income taxes.

Each generation is composed of a continuum of agents of measure N and population is stationary. Moreover, two sectors of production exist: the commodity sector and the services sector (see, e.g., van Groezen et al., 2007b).

[3] The assumption that the probability of surviving depends on a certain measure of public health care services follows Chakraborty (2004).

2.1. Firms

The commodity sector. At time t firms produce a homogeneous good (Y_t) by combining capital (K_t) and labour employed in the commodity sector ($L_{Y,t}$), through the constant returns to scale Cobb-Douglas technology $Y_t = AK_t^{\alpha} L_{Y,t}^{1-\alpha}$, where $A > 0$ and $0 < \alpha < 1$. Firms maximise profits and perfect competition guarantees that factor inputs are paid their marginal products, that is

$$r_t = \alpha A \left(\frac{k_t}{l_{Y,t}} \right)^{\alpha - 1} - 1, \tag{1}$$

$$w_t = (1-\alpha) A \left(\frac{k_t}{l_{Y,t}} \right)^{\alpha}, \tag{2}$$

where $k_t := K_t / N$ is the stock of capital per young and $l_{Y,t} := L_{Y,t} / N$ is the fraction of young people in the economically active population employed in the commodity sector.[4]

The services sector. Production takes place only with labour. The technology is linear and given by $D = L_{D,t}$ where $L_{D,t}$ is the number of people employed in such a sector. Since labour is homogenous and perfectly mobile, the price of services in terms of commodities (p_t) equals the wage in the commodity sector, that is $p_t = w_t$.

2.2. Individuals

Agents of each generation live for two periods: youth (working period) and old age (retirement period). Individuals of generation t inelastically supplies their time endowment of measure one to

[4] The price of final output is normalised to unity and capital totally depreciates at the end of each period.

firms and receives the wage w_t per unit of labour. The budget constraint of a young individual at t reads as:

$$c_{1,t} + s_t = w_t(1-\tau), \tag{3.1}$$

i.e. wage income – net of the contributions paid to finance the provision of health expenditure, where $0 < \tau < 1$ is the wage tax rate – is divided into young-age consumption for commodities, $c_{1,t}$, and savings, s_t. Following Chakraborty (2004), we assume that at the end of youth survival is uncertain and the survival probability from work time to retirement time, π_t, is an increasing – though bounded – function of the public health measure h_t. Different from Chakraborty (2004), however, we follow Blackburn and Cipriani (2002) and de la Croix and Ponthiere (2010) and specialise the relationship between public health investments and life expectancy with the following S-shaped function:

$$\pi_t = \pi(h_t) = \frac{\pi_0 + \pi_1 \Delta (h_t)^\delta}{1 + \Delta (h_t)^\delta}, \tag{4}$$

where $\delta, \Delta > 0$, $0 < \pi_1 \leq 1$, $0 \leq \pi_0 < \pi_1$, $\pi(0) = \pi_0 \geq 0$, $\pi'_h(h) = \dfrac{\delta \Delta h^{\delta-1}(\pi_1 - \pi_0)}{\left(1 + \Delta h^\delta\right)^2} > 0$, $\lim_{h \to \infty} \pi(h) = \pi_1 \leq 1$ and $\lim_{h \to 0} \pi'_h(h) = \pi_1 - \pi_0 < \infty$, $\pi''_{hh}(h) < 0$ if $\delta \leq 1$ and $\pi''_{hh}(h) \gtreqless 0$ for any $h \lesseqgtr h_T := \left[\dfrac{\delta - 1}{(1+\delta)\Delta}\right]^{\frac{1}{\delta}}$ if $\delta > 1$.

Eq. (5) allows us to capture several aspects of the timing of the length of life of the typical agent up to the retirement age. First, we define π_0 as the natural or biological rate of longevity of people (see, e.g., Ehrlich, 2000; Leung and Wang, 2010): it represents the fraction of time lived by individuals at the end of youth irrespective of public health spending; it may be affected by both economic and non-economic factors, e.g. the lifestyle of people, education, economic growth and the standards of living, the degree of culture and civilisation, weather and climate changes, ethnical and civil wars, endemic diseases and so on. Thus, we may expect π_0 to be higher in developed rather than developing or under-developed countries, and the more individuals naturally live longer, the smaller the reduction in adult mortality due to a rise in health expenditure. Second, the parameter π_1 captures the intensity of the efficiency of health investments on the rate of longevity. A rise in π_1 may be interpreted as exogenous medical advances due, for instance, to scientific research, vaccination programmes and so on. Third, we may think, realistically, that health investments have a more intense effect in reducing adult mortality often when a certain threshold level of public health expenditure has been reached, while becoming scarcely effective when longevity is close to its saturating value (e.g., the functional relationship between health investment and longevity may be S-shaped). The parameters δ and Δ both allow to capture such an idea and determine both the turning point of $\pi_h'(h)$ and speed of convergence from the natural length of life π_0 to the saturating value π_1. In particular, given the value of Δ, the parameter δ represents the degree of proportionality in the distribution of public health investments across population as an inducement to higher life expectancy, other things being unchanged. In other words, it measures how an additional unit of public health capital is transformed into higher longevity through the public health technology. If $\delta \leq 1$, $\pi(h)$ is concave for any h and, hence, no threshold effects of public health investments on longevity exist, i.e. the distribution of an additional unit of health capital is relatively widespread and smooth across population, so that longevity increases monotonically with decreasing returns from the starting point π_0 to the saturating value π_1 as h rises, and the more δ is close to zero the more efficiently and rapidly an additional unit of health capital is transformed into higher longevity when h is relatively low, while reaching the saturating value π_1 more slowly as h becomes larger. Figure 1 illustrates in a stylised way the evolution of the rate of longevity as a function of public health capital when $\delta \leq 1$: the solid (dashed) [dotted] line refers to the case $\delta = 1$ ($\delta < 1$) [$\delta \to 0$]. As can readily be seen the lower (higher) is δ the more (less) efficient is an additional unit of public health capital expenditure as an inducement to higher life expectancy until (once) a certain level of h is reached.

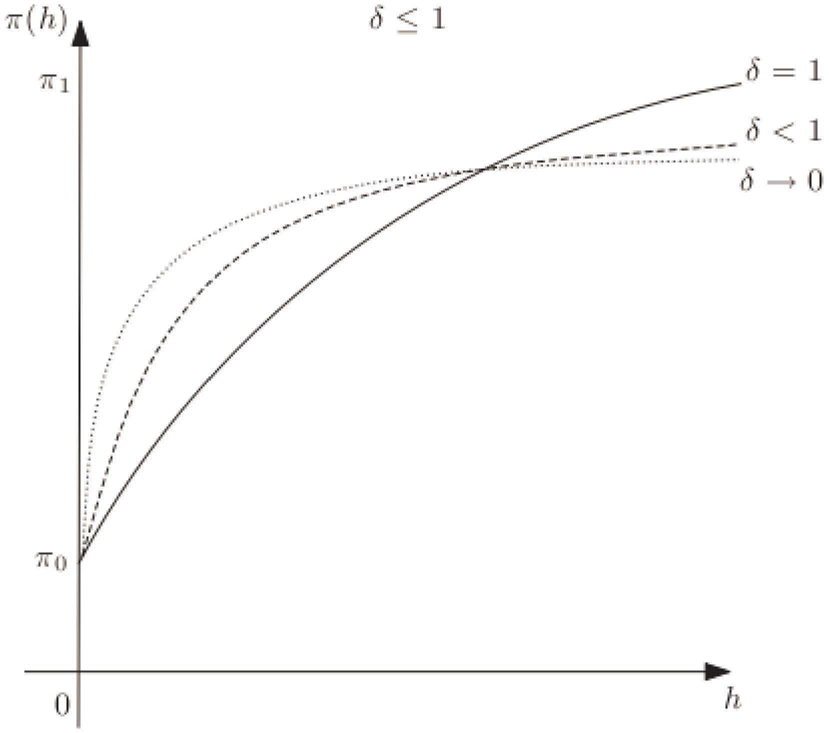

Figure 1. Longevity and public health capital when $\delta \leq 1$.

In contrast, when $\delta > 1$ the longevity function is S-shaped and, hence, threshold effects exist in the way in which public health investments are distributed across population and then transformed into higher longevity. In particular, longevity increases more (less) than proportionally when $h < h_T$ ($h > h_T$), i.e. it grows with increasing (decreasing) returns until (once) the turning point h_T is reached. However, a rise in δ shifts the longevity function to the right while also increasing the speed of convergence from π_0 to π_1, as clearly shown in Figure 2 below, where the solid (dashed) [dotted] line refers to δ_{low} (δ_{high}) [$\delta \to +\infty$]. The distribution of an additional unit of health capital is not smooth across population in that case and, in particular, an increasing amount of public health expenditure is required to trigger beneficial effects on longevity, and the higher δ is the more slowly an additional unit of health investments is transformed into higher life expectancy when h is relatively low, while reaching the saturating value π_1 more efficiently and rapidly as h becomes larger.

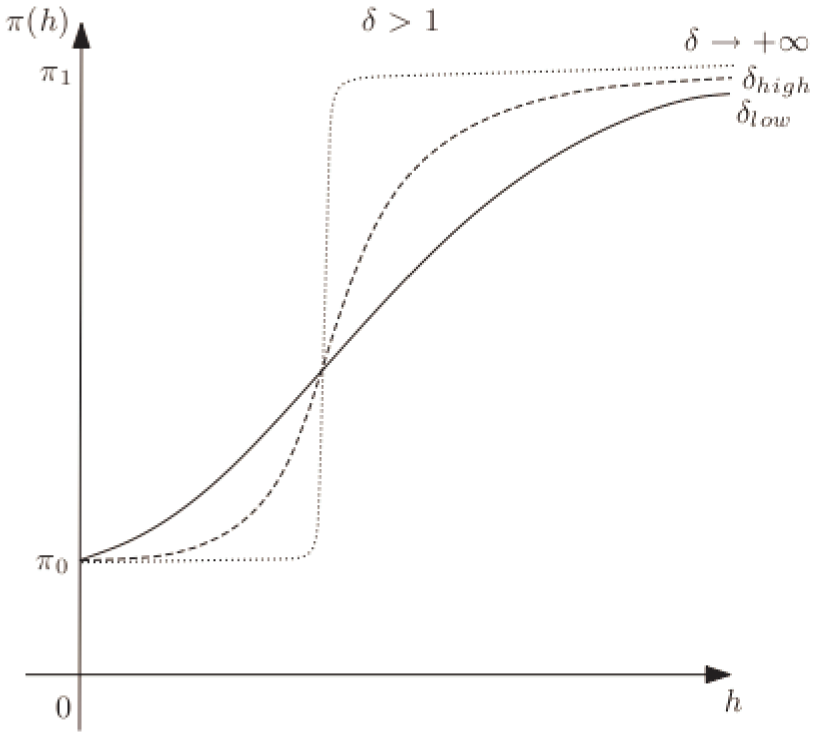

Figure 2. Longevity and public health capital when $\delta > 1$.

This means that increasing public health expenditure is not effective as an inducement to higher longevity until a threshold value of health capital is accumulated (and this value is larger the larger is δ), because, for instance, a certain degree of knowledge to enable health investments to be distributed and transformed efficiently into a higher longevity has not yet been reached. Beyond that threshold, however, a "sudden" effect exists that efficiently allows such a transformation. As an example, we may think about the existence of threshold effects in the accumulation of knowledge required for new medical advances and discoveries in the treatment of diseases (e.g. vaccines) to be effective and efficient: the public health expenditure to finance new research projects may be high and apparently useless up to a certain degree of knowledge is achieved. Once that threshold is reached, however, there exists a sudden effect that allows to trigger and bring to light the beneficial effects of the new discoveries, to make them efficient, usable and operative across population and eventually transformed into a higher life expectancy.

Old individuals are retired and live uniquely with the amount of resources saved when young plus the interest accrued from time t to time $t+1$ at the rate r_{t+1}. The existence of a perfect annuity market (where savings are intermediated through mutual funds) implies that old survivors will benefit not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased. Hence, the budget constraint of an old retired agent started working at t can be expressed as

$$c_{2,t+1} + p_{t+1} d_{2,t+1} = \frac{1+r_{t+1}}{\pi} s_t \, , \tag{3.2}$$

where $c_{2,t+1}$ is old-aged consumption for commodities and $d_{2,t+1}$ is the number of services used when old.

The representative individual born at t chooses consumption for commodities when young and old and the number of commodities and services when old in order to maximise the logarithmic utility function

$$U_t = \ln\left(c_{1,t}\right) + \pi_t \beta \left[(1-\lambda)\ln\left(c_{2,t+1}\right) + \lambda \ln\left(d_{2,t+1}\right)\right], \tag{5}$$

subject to Eqs. (3.1) and (3.2), where β is the psychological subjective discount factor and λ captures the relative importance of consuming services when old. The constrained maximisation of Eq. (5) gives the following demand functions:

$$c_{1,t} = \frac{w_t(1-\tau)}{1+\pi_t \beta}, \tag{6}$$

$$c_{2,t+1} = \frac{\beta(1-\lambda)(1+r_{t+1})w_t(1-\tau)}{1+\pi_t \beta}, \tag{7}$$

$$d_{2,t+1} = \frac{1+r_{t+1}}{w_{t+1}} \frac{\beta \lambda w_t (1-\tau)}{1+\pi_t \beta}, \tag{8}$$

while the saving rate is

$$s_t = \frac{\pi_t \beta w_t (1-\tau)}{1+\pi_t \beta}. \tag{9}$$

2.3. Government

The government finances health investments (e.g. hospitals, scientific research and so on) by collecting wage taxes at the constant rate $0 < \tau < 1$ (see Chakraborty, 2004). Therefore, the (per young) budget constraint faced by the government at t reads as

$$h_t = \tau w_t, \tag{10}$$

the left-hand side being the health expenditure and the right-hand side the tax receipt.

2.4. Equilibrium

The equilibrium in the services market is given by $L_{D,t} = \pi_{t-1} N d_{2,t}$, or

$$l_{D,t} = \pi_{t-1} d_{2,t}, \tag{11}$$

The equilibrium in the labour market is given by $N = L_{Y,t} + L_{D,t}$, or

$$1 = l_{Y,t} + l_{D,t}, \tag{12}$$

where $l_{D,t} = L_{D,t} / N$ is the fraction of young people in the economically active population employed in the services sector.

The equilibrium in the commodity sector is $AK_t^{\alpha} L_{Y,t}^{1-\alpha} = N(c_{1,t} + s_t) + \pi_{t-1} N c_{2,t}$, or $AK_t^{\alpha} L_{Y,t}^{1-\alpha} = N w_t (1-\tau) + \pi_{t-1} N c_{2,t}$, that is

$$Ak_t^{\alpha} l_{Y,t}^{1-\alpha} = w_t (1-\tau) + \pi_{t-1} c_{2,t}, \tag{13}$$

Given the government budget Eq. (10), market-clearing in goods and capital market leads to the usual equilibrium condition $N k_{t+1} = N s_t$, that is

$$k_{t+1} = s_t \,. \tag{14}$$

Combining Eqs. (14) and (9) equilibrium implies:

$$k_{t+1} = \frac{\pi_t \beta w_t (1-\tau)}{1+\pi_t \beta} \,. \tag{15}$$

Substituting out for w_t from Eq. (2), the dynamic path of capital accumulation is:

$$k_{t+1} = \frac{\pi_t \beta (1-\alpha) A (1-\tau)}{1+\pi_t \beta} \left(\frac{k_t}{l_{Y,t}} \right)^{\alpha} \,. \tag{16}$$

Now, using Eqs. (1), (2), (11) and (12) together with the one period backward Eqs. (8) and (16) we find that

$$l_D = \frac{\lambda \alpha}{1-\alpha(1-\lambda)} \,, \tag{17}$$

so that

$$l_Y = \frac{1-\alpha}{1-\alpha(1-\lambda)} \,. \tag{18}$$

Eqs. (17) and (18) reveal that the reallocation of labour between sectors depends on both the preference for old-aged services and the output elasticity of capital in the commodity sector. In particular, a rise in either λ or α reduces (increases) the fraction of young population employed in the commodity (services) sector, that is $\frac{\partial l_Y}{\partial \lambda} = \frac{-\alpha(1-\alpha)}{[1-\alpha(1-\lambda)]^2} < 0$ and $\frac{\partial l_Y}{\partial \alpha} = \frac{-\lambda}{[1-\alpha(1-\lambda)]^2} < 0$ ($\frac{\partial l_D}{\partial \lambda} = \frac{\alpha(1-\alpha)}{[1-\alpha(1-\lambda)]^2} > 0$ and $\frac{\partial l_D}{\partial \alpha} = \frac{\lambda}{[1-\alpha(1-\lambda)]^2} > 0$). Moreover, the higher the preference for old-aged services, the lower the intensity of labour reallocation in favour of the services sector, that is $\frac{\partial^2 l_D}{\partial \lambda^2} = \frac{-2\alpha^2(1-\alpha)}{[1-\alpha(1-\lambda)]^3} < 0$.

Using Eqs. (2), (4), (10), (16) and (18) the dynamic path of capital accumulation is:

$$k_{t+1} = \frac{Z_2(\lambda) k_t^{\alpha} \left[\pi_0 + \pi_1 Z_1(\lambda) k_t^{\alpha\delta}\right]}{1+\pi_0 \beta + (1+\pi_1 \beta) Z_1(\lambda) k_t^{\alpha\delta}} \,, \tag{19}$$

where $B := \Delta[\tau(1-\alpha)A]^{\delta}$, $E := \beta(1-\alpha)A(1-\tau)$, $M(\lambda) := \dfrac{1-\alpha(1-\lambda)}{1-\alpha}$, $Z_1(\lambda) := B \cdot [M(\lambda)]^{\alpha\delta}$ and $Z_2(\lambda) := E \cdot [M(\lambda)]^{\alpha}$ are positive constants.

In the next section we show that multiple regimes of development can appear and study how changes in the allocation of labour between sectors, through a change in the preference for old-aged services λ, affect both the transitional dynamics and steady-state outcomes.

3. Dynamics

Analysis of Eq. (19) gives the following proposition:

Proposition 1. *The dynamic system described by Eq. (19) admits either two steady states* $\{0, \bar{k}\}$, *with* $\bar{k} > 0$ *(only the positive one being asymptotically stable) or four steady states* $\{0, \bar{k}_1, \bar{k}_2, \bar{k}_3\}$, *with* $\bar{k}_3 > \bar{k}_2 > \bar{k}_1 > 0$ *(only the second and the forth being asymptotically stable). Moreover, (1) a sufficient condition to avoid development traps is* $\Lambda_2(\lambda) > 0$ *and* $\Lambda_3(\lambda) > 0$, *and (2) a necessary condition for the existence of multiple steady states is that at least either* $\Lambda_2(\lambda) < 0$ *or* $\Lambda_3(\lambda) < 0$ *holds, where* $\Lambda_2(\lambda) := Z_1(\lambda)Q_2(\lambda)(1+\pi_1\beta)[1-\alpha(1-\delta)] + Q_1(\lambda)(1+\pi_0\beta)[1-\alpha(1+2\delta)]$,

$\Lambda_3(\lambda) := Q_2(\lambda)(1+\pi_0\beta)[1-\alpha(1+\delta)] + Q_3 Z_1(\lambda)(1+\pi_1\beta)[1-\alpha(1-2\delta)]$,

$Q_1(\lambda) := \pi_1(1+\pi_1\beta)[Z_1(\lambda)]^2 > 0$ *and* $Q_2(\lambda) := [\pi_0\beta + \pi_1(1+2\pi_0\beta) + \delta(\pi_1 - \pi_0)]Z_1(\lambda) > 0$.

Proof. Let first the following lemma be established.

Lemma 1. *Define the right-hand side of (19) as* $G(k)$. *Then, we have: (1.i)* $G(0)=0$, *(1.ii)* $G'_k(k)>0$ *for any* $k>0$, *(1.iii)* $\lim_{k\to 0^+} G'_k(k)=+\infty$, *(1.iv)* $\lim_{k\to+\infty} G'_k(k)=0$, *(1.v)* $G''_{kk}(k)$ *admits at most three roots and* $G''_{kk}(0)\neq 0$.

From Eq. (19), property (1.*i*) is straightforward. Differentiating the right-hand side of Eq. (19) with respect to k gives

$$G'_k(k)=\frac{\alpha Z_2(\lambda)\left[Q_1(\lambda)k^{2\alpha\delta}+Q_2(\lambda)k^{\alpha\delta}+Q_3\right]}{k^{1-\alpha}\left[1+\pi_0\beta+(1+\pi_1\beta)Z_1(\lambda)k^{\alpha\delta}\right]^2}, \tag{20.1}$$

where $Q_3:=\pi_0\beta(1+\pi_0)>0$. Defining $k^{\alpha\delta}:=x$ as a new supporting variable, (11.1) can be transformed to:

$$g(k,x)=\frac{\alpha Z_2(\lambda)\left[Q_1(\lambda)x^2+Q_2(\lambda)x+Q_3\right]}{k^{1-\alpha}\left[1+\pi_0\beta+(1+\pi_1\beta)Z_1(\lambda)x\right]^2}. \tag{20.2}$$

Since no positive real roots of (20.2) can exist, then (20.1) implies that $G'_k(k)>0$ for any $k>0$. This proves (1.*ii*).

Moreover,

$$\lim_{k\to 0^+} G'_k(k)=\alpha Z_2(\lambda)\lim_{k\to 0^+}\frac{Q_1(\lambda)k^{2\alpha\delta}+Q_2(\lambda)k^{\alpha\delta}+Q_3}{k^{1-\alpha}\left[1+\pi_0\beta+(1+\pi_1\beta)Z_1(\lambda)k^{\alpha\delta}\right]^2}=+\infty.$$

and

$$\lim_{k\to+\infty} G'_k(k)=\lim_{k\to+\infty}\frac{\alpha Z_2(\lambda)\left[Q_1(\lambda)k^{2\alpha\delta}+Q_2(\lambda)k^{\alpha\delta}+Q_3\right]}{k^{1-\alpha}\left[1+\pi_0\beta+(1+\pi_1\beta)Z_1(\lambda)k^{\alpha\delta}\right]^2}$$
$$=\alpha Z_2(\lambda)\lim_{k\to+\infty}\frac{Q_1(\lambda)+\frac{Q_2(\lambda)}{k^{\alpha\delta}}+\frac{Q_3}{k^{2\alpha\delta}}}{k^{1-\alpha}\left\{\frac{(1+\pi_0\beta)^2}{k^{2\alpha\delta}}+\frac{2(1+\pi_0\beta)(1+\pi_1\beta)Z_1(\lambda)}{k^{\alpha\delta}}+(1+\pi_1\beta)^2[Z_1(\lambda)]^2\right\}}=0,$$

which prove (1.*iii*) and (1.*iv*), respectively.

Now, differentiating (20.1) with respect to k gives

$$G''_{kk}(k)=\frac{-\alpha Z_2(\lambda)\left[\Lambda_1(\lambda)k^{3\alpha\delta}+\Lambda_2(\lambda)k^{2\alpha\delta}+\Lambda_3(\lambda)k^{\alpha\delta}+\Lambda_4\right]}{k^{2-\alpha}\left[1+\pi_0\beta+(1+\pi_1\beta)Z_1(\lambda)k^{\alpha\delta}\right]^3}, \tag{21.1}$$

where $\Lambda_1(\lambda):=(1-\alpha)(1+\pi_1)Z_1(\lambda)Q_1(\lambda)>0$ and $\Lambda_4:=(1-\alpha)(1+\pi_0)Q_3>0$. Knowing that $k^{\alpha\delta}:=x$, Eq. (21.1) can be rewritten as

$$f(k,x)=\frac{-\alpha Z_2(\lambda)\left[\Lambda_1(\lambda)x^3+\Lambda_2(\lambda)x^2+\Lambda_3(\lambda)x+\Lambda_4\right]}{k^{2-\alpha}\left[1+\pi_0\beta+(1+\pi_1\beta)Z_1(\lambda)x\right]^3}. \tag{21.2}$$

From (21.2), it is straightforward to see that $f(k,x)$ admits at most three roots for x and $f(k,0)\neq 0$. Hence, from (21.1), $G''_{kk}(k)$ admits at most three roots for k and $G''_{kk}(0)\neq 0$. This proves (1.*v*).

Proposition 1 therefore follows. In fact, by properties (1.*i*) and (1.*iii*), zero is always an unstable steady state of Eq. (19). By (1.*ii*)-(1.*iv*), $G(k)$ is a monotonic increasing function of k and eventually falls below the 45° line, so that at least one positive stable steady state exists for any $k>0$.

Now, assume *ad absurdum* the existence of an odd number of equilibria. By (1.*ii*)-(1.*iv*), the inflection points cannot be odd-numbered for any $k>0$. By property (1.*v*), therefore, the number of inflection points of $G(k)$ is either zero or two for any $k>0$. Since at least one positive stable steady state exists, then for any $k>0$ the phase map $G(k)$ may intersect the 45° line from below *at most* once before falling below it. Hence, an even number of equilibria must necessarily exist. In particular, the steady states are either two, with the positive one being the unique asymptotically stable equilibrium, or *at most* one positive steady state separates the lowest asymptotically stable steady state from the highest asymptotically stable one, and, thus, the number of equilibria is four.

Moreover, from Eq. (21.1) we observe that if $\Lambda_2(\lambda)>0$ and $\Lambda_3(\lambda)>0$ then no inflection points of $G(k)$ exist for any $k>0$, $G''_{kk}(k)<0$ and two steady states exist in that case. In contrast, $G(k)$

has two inflection points for any $k > 0$ if at least either $\Lambda_2(\lambda) < 0$ or $\Lambda_3(\lambda) < 0$ is fulfilled and, hence, four steady states *can* exist in such a case. **Q.E.D.**

Proposition 1 provides sufficient conditions to avoid development traps and necessary but not sufficient conditions to have multiple steady states, which depend – as can be ascertained from Eq. (21.2) – on the key parameters of the problem and the policy variable τ and the preference parameter λ. However, as extensive numerical simulations revealed, the existence of multiple regimes of development crucially depends on the mutual relationship between the output elasticity of capital (α) and the degree of effectiveness of public health investments on longevity (δ). In particular, for any given value of δ development traps are likelier to occur when production is relatively capital-oriented (high values of α). In particular, when $\delta = 1$ multiple steady states appear when α exceeds $1/2$,[5] and this threshold monotonically shrinks as δ raises. Therefore, when threshold effects of health capital on longevity exist (i.e. $\delta > 1$), a wider range of economies is prone to be characterised by development traps, since the output elasticity of capital that discriminates between a unique regime and multiple regimes of development is empirically more plausible and smaller than one half (as shown in the numerical example below).

Now, assume that economies exclusively differ as regards the initial condition k_0. Figure 3 depicts in a stylised way all the possible outcomes of an economy with endogenous lifetime and a services sector. We use the following parameter values: $A = 14$, $\alpha = 0.33$ (see Gollin, 2002; Kehoe and Perri, 2002), $\beta = 0.3$ (see de la Croix and Michel, 2002), $\pi_0 = 0.2$, $\pi_1 = 0.95$, $\delta = 10$ (see de la Croix and Ponthiere, 2010), $\Delta = 1$, $\tau = 0.1$ and $\lambda = 0$ (solid line). The figure clearly shows that an economy that starts below the unstable equilibrium $\bar{k}_2$ is entrapped into the low regime ($\bar{k}_1$) where income per worker is low, fertility is high and mortality is high. In contrast, an economy that starts beyond the threshold $\bar{k}_2$ converges towards the high regime ($\bar{k}_3$) where income per worker is

[5] This result is in accord with Chakraborty (2004, Proposition 1, (i), p. 126).

high, fertility is low and mortality is low. Therefore, as expected, an exogenous shift in k_0 may cause a change in the development regime.

Therefore two development regimes are possible in this stylised two-sector economy. The high regime is characterised by low mortality and high capital accumulation. The low regime, instead, is characterised by high mortality and low capital accumulation. Below we show that a reallocation of labour towards the services sector (higher values of λ) allows poorer economies to escape from poverty.

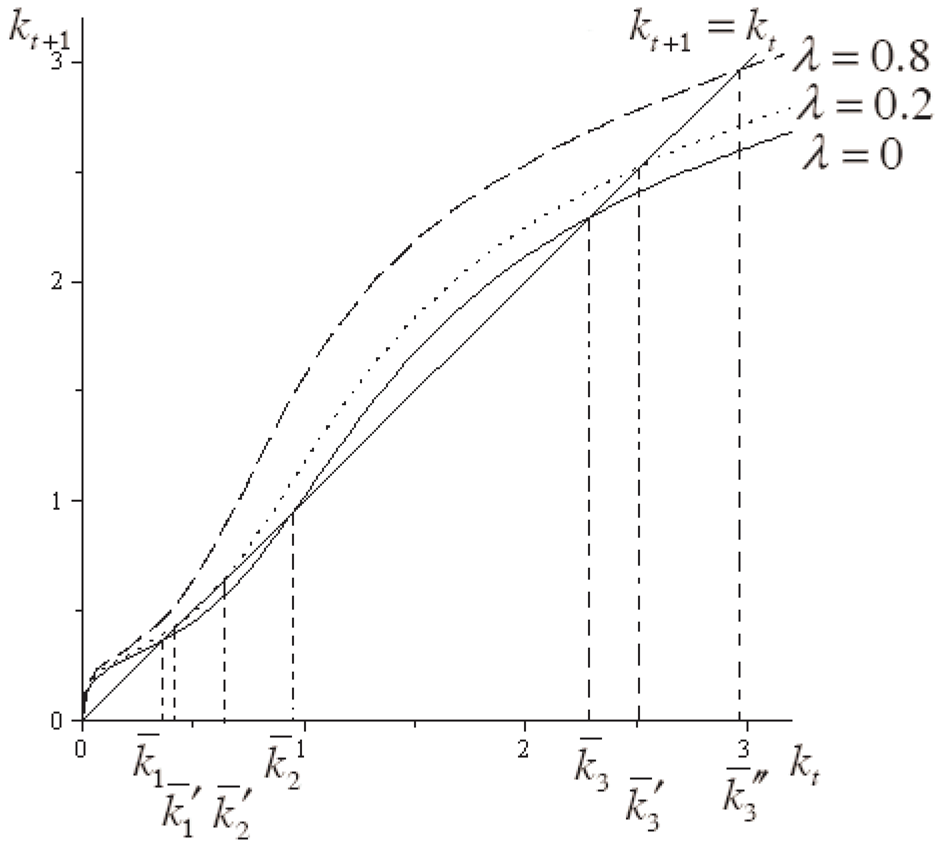

Figure 3. Multiple steady states in a two-sector economy.

3.1. Escaping from poverty: a numerical example

A change in the preference for the number of old-age services changes the allocation of the fraction of workers employed in both the commodity and services sector. In particular, a rise in λ increases the fraction of workers employed in the services sector while reducing those employed in the commodity sector. A reduction in l_Y causes a rise in wages that directly promotes savings and capital accumulation. Higher wages, however, means both higher fertility and longevity. In particular, the increased life expectancy causes an impetus to increase the saving rate further and this in turn implies a positive indirect effect on capital accumulation. Definitely, the shape of the function $G(k)$ raises as a consequence of a redistribution of labour in favour of the services sector, i.e. capital accumulation in both regimes of development increases. In particular, the following result holds:

Result 1. *Poverty or prosperity may not depend on initial conditions. Indeed, a rise in the preference for old-age services increases both the lowest and highest stable steady states, reduces the intermediate unstable steady state, while reducing the basin of attraction of the poverty trap. Moreover, a large enough increase in λ may cause the loss of the lowest stable steady state, thus allowing poorer economies to permanently escape from poverty and then converge towards the high regime of development.*

In order to illustrate Result 1 we resort to a numerical experiment based on Figure 3. It is shown that when λ is high enough the low equilibrium vanishes and all economies converge towards the high regime of development. We choose three values of the preference for old age services: $\lambda = 0$ (solid line), $\lambda = 0.2$ (dotted line) and $\lambda = 0.8$ (dashed line). Therefore, when $\lambda = 0$ (i.e., $l_Y = 1$) the low regime is characterised by the equilibrium values $\bar{k}_1 = 0.365$ and $\pi(\bar{k}_1) = 0.213$, and the high regime instead by $\bar{k}_3 = 2.294$ and $\pi(\bar{k}_3) = 0.868$. The unstable equilibrium that discriminates between poor and rich countries is $\bar{k}_2 = 0.949$. Therefore, an economy that for some exogenous reasons starts with a stock of capital below (beyond) such a threshold level of development will end up in the low (high) regime, where income per worker is low (high) and mortality and fertility rates are high (low), confirming some of the most striking aspects of the so-called demographic transition. However, as can be seen from Figure 3, a rise in λ causes a rise in capital accumulation, wages and life expectancy both regimes. In fact, when $\lambda = 0.2$ (i.e., $l_Y = 0.9103$), capital accumulation and longevity increase in both regimes and thus the basin of attraction towards the poverty trap shrink: the low regime now is characterised by the equilibrium values $\bar{k}_1' = 0.425$ and $\pi(\bar{k}_1') = 0.213$, and the high regime instead by $\bar{k}_3' = 2.519$ and $\pi(\bar{k}_3') = 0.903$. Finally, if $\lambda = 0.8$ (i.e., $l_Y = 0.713$), the low equilibrium vanishes and an economy approaches to the high regime, characterised by $\bar{k}_3'' = 2.955$ and $\pi(\bar{k}_3'') = 0.936$, irrespective of initial conditions.

4. Conclusions

We analysed an overlapping generations economy with two sectors of production: the commodities sector and the services sector. We assumed that these services are consumed only by the old age people. We studied how a reallocation of labour in favour of the labour-intensive services sector affects both the transitional dynamics and long-run outcomes of an economy where public health investments affect the lifetime of people. A crucial key ingredient for a change in the sector composition is represented by the change in individual savings. We showed that multiple regimes of development can exist: a regime where capital accumulation and longevity are low and a regime where capital accumulation and longevity are high. In this context, a rise in the preference for old age services increases capital accumulation and longevity thus helping to escape from poverty.

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