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How to Reduce Unemployment: Notes on Macro-Economic Stability and Dynamics

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Si prega di citare così:
Abstract

In this paper, I explore the out-of-equilibrium macro-economic dynamic behaviour of the Farmer’s (2010d) ME-NA model. Specifically, preserving the assumption that micro-economic adjustments are instantaneous, I build a dynamic model in continuous time that describes the macro-economic adjustments of the value of output and the interest rate. Within this framework, I show that the model economy has a unique stationary solution whose dynamics is locally stable. Moreover, simulating the model economy under the baseline calibration, I show that the adjustments towards the steady-state equilibrium occur through endogenous convergent oscillations while the most promising way out from a finance-induced recession combines a fiscal expansion with interventions aimed at altering the trade-off between holding risky and safe assets.

**JEL Classification:** E12, E32, E62.

**Keywords:** Old Keynesian Economics, ME-NA Schedules, Short-Run Macroeconomic Fluctuations.
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1 Introduction

In a recent work, Farmer (2010d) develops an overlapping generation (OLG) model with a perpetual-youth demographic framework along the lines put forward by Yaari (1965) and Blanchard (1985) aiming at exploring the effectiveness of fiscal policies as possible ways out from the 2008 finance-induced recession in the context of the US economy. There he proposes a non-linear apparatus similar to a closed IS-LM system able to catch the effects on the real economy produced by balanced-budget fiscal policies and variations in the value of the financial wealth due to the self-fulfilling beliefs of households about the stock market value. The major results achieved in this seminal contribution is that in the afterwards of a finance-induced recession an expansionary fiscal policy can be an effective way to increase the value of output and reduce unemployment but there are also better alternatives that do not crowd out private consumption such as public interventions that directly influence asset markets by preventing sudden booms and consequent disastrous crashes.\footnote{Those works are parts of a broader project aimed at providing a new micro-foundation of the General Theory grounded on search and business cycle theories. See Farmer (2008a-b, 2010a), Guerrazzi (2010) and Gelain and Guerrazzi (2010).}

Although it assumes instantaneous micro- and macro-economic adjustments, the non-linear apparatus suggested by Farmer (2010d) has some intriguing and non-standard features. First, consistently with the choice of units made by Keynes (1936) in the General Theory, the nominal wage is used as numeraire so that all the nominal variables are measured in money wage units. Second, on the side of the market for goods, the IS schedule is derived under the assumption that the only autonomous component of aggregate demand is public expenditure while private consumption (saving) depends positively (positively) on the value of output and negatively (positively) on the real interest rate.\footnote{The model provides a short-run framework that abstracts from investment by assuming that there is a fixed amount of productive capital. See Farmer (2008b).} As a consequence, a demand-constrained equilibrium in the market for goods implies a (non-linear) decreasing relationship between the value of output and the real interest rate denomi-
nated as ME schedule. Third, the monetary sector of the model economy, i.e., the LM schedule, is replaced by a no-arbitrage condition such that all interest bearing assets must pay the same return in equilibrium. Under the competitive assumption that the marginal productivity of capital is proportional to the value of output, this condition implies an upward equilibrium relationship between the value of output and the real interest rate denominated as NA schedule. Finally, the intersection between the ME and the NA schedule determines the short-run equilibrium value of output and the equilibrium real interest rate.

In this paper, following the theoretical contributions by Chang and Smyth (1972) and Varian (1977), I explore the out-of-equilibrium macro-economic dynamic behaviour of the Farmer's (2010d) ME-NA model. Specifically, preserving the assumption that micro-economic saddle-path adjustments implied by the OLG perpetual-youth framework are instantaneous, I build a dynamic model in continuous time that describes the macro-economic adjustments of the value of output and the real interest rate. Within this framework, I show that the model economy has a unique meaningful stationary solution whose dynamics is locally stable.

Moreover, I use the macro-dynamic model to perform some numerical simulations. Specifically, I show that exploiting the baseline calibration suggested by Farmer (2010d) in his seminal contribution, the adjustments to the short-run macro-economic equilibrium occur through endogenous convergent oscillations. Finally, I explore the effectiveness of a set of policy interventions that might be helpful in exiting from a finance-induced recession driven by the self-fulfilling beliefs of households about the stock market value. My computational experiments suggest that an expansionary fiscal policy can be an effective way to increase the value of output and reduce unemployment but there might be also welfare-enhancing companion interventions that provide for a lower crowding out of private consumption. While in the original contribution there is only a narrative account on

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3Non-linearity comes from the hyperbolic relationship between saving (or consumption) and the real interest rate.

4Interactions between micro- and macro-economic adjustments are a topic for future researches.
that matter, in the present work I show that those interventions rely on policies aimed at altering the trade-off between holding risky and safe financial assets and I provide a quantitative assessment of the wedge needed to restore the value of output (and employment) achieved before the finance-induced recession.

The paper is arranged as follows. Section 2 develops the theoretical model. Section 3 illustrates the results of some numerical simulations. Finally, section 4 concludes.

2 The Model

In the Farmer’s (2010d) OLG perpetual-youth model there is a unit mass of identical firms that produce a homogenous-perishable good whose value in wage units is indicated by $Z_t$. The production process of this good is described by a constant-returns-to-scale Cobb-Douglas production function in which the capital share is denoted with $\alpha \in (0,1)$. Each firm employs 1 unit of productive capital and a variable amount of labour hired on a competitive search market aiming at maximizing its profits.

Symmetrically with firms, in the Farmer’s (2010d) theoretical proposal there is a unit mass of identical households that discount the future period by period at the constant rate $\beta \in (0,1)$ and have a constant probability $\pi \in [0,1]$ to survive in the subsequent period. The problem of each household is to choose an optimal sequence of consumption expenditure $\{C_t\}_{t=0}^{+\infty}$ that maximizes an instantaneous logarithmic utility function under the dynamic constraint of a wealth accumulation path. In each period, the financial wealth of the economy is given by sum between the value of the public debt $(B_t)$, the price of a risky asset $(p_{k,t})$ and the value of the marginal return on the unit of employed capital $(\alpha Z_t)$. Finally, labour income is taxed at the proportional rate $\tau_t$ while $R_t$ is the interest factor.\(^5\)

Taking into account that each household consumes a fixed fraction of its wealth and that wealth is independent of the household age allows to derive the following modified

\(^5\)Obviously, the interest factor is 1 plus the real interest rate.
Euler equation:

\[ \frac{C_{t+1}}{C_t} = \tilde{\beta} R_t \left( \frac{C_t - \tilde{\alpha}(Z_t (1 - \tau_t (1 - \alpha)) + p_{k,t} + B_t)}{C_t} \right) \] (1)

where \( \tilde{\alpha} \equiv \frac{(1-\beta \pi)(1-\pi)}{1-\pi(1-\beta \pi)} \) and \( \tilde{\beta} \equiv \frac{1-\pi(1-\beta \pi)}{\pi} \) (see Farmer 2010d, p. 563).

It is worth noting that when the representative household is assumed to live forever, i.e., \( \pi = 1 \), so that \( \tilde{\alpha} = 0 \) and \( \tilde{\beta} = \beta \), the expression in (1) collapses to a standard Euler equation in which the growth rate of consumption is given by the interest factor corrected by the household subjective discount factor.

The dynamics of the value of public debt is described by the following first-order difference equation:

\[ B_{t+1} = R_t (B_t + G_t - \tau_t (1 - \alpha) Z_t) \] (2)

where \( G_t \) is the value of public expenditure.

Substituting (2) in (1) and assuming that that the micro-economic saddle-path adjustments of the household problem have achieved a stable solution allow to derive the following aggregate consumption function:

\[ C = \frac{\tilde{\alpha} \tilde{\beta} R}{R \beta - 1} \left( p_{k} + \frac{B}{R} + Z - G \right) \] (3)

The expression in (3) suggests that the households’ consumption depends positively (negatively) on disposable income (real interest rate). In addition, there is a wealth-effect that plays through the discounted value of public debt and the price of the risky asset.

Taking into account that public expenditure is the only autonomous component of aggregate demand and assuming that the value of income changes at a rate proportional to the excess demand in the goods market, the national account identity suggests that for any given level of \( R \) the macro-economic dynamics of \( Z \) can be described by the following differential equation:
\[
\dot{Z} = \gamma \left( G - \left( Z - \frac{\tilde{a}\beta R}{\beta R - 1} \left( p_k + \frac{B}{R} + Z - G \right) \right) \right)
\] 

(4)

where \( \gamma \) is a positive constant that conveys the speed of adjustment of the market for goods.

The differential equation in (4) suggests that the value of output increases (decreases) whenever the value of saving is lower (higher) than the value of public expenditure.\(^6\)

Substituting \( G \) with a constant level of investment, the macro-economic adjustment of \( Z \) is qualitatively identical to the one described by Chang and Smyth (1972) and Varian (1977) in the context of a dynamic IS-LM model.

In the Farmer (2010d) theoretical proposal the monetary sector of the model economy is replaced by a no-arbitrage condition such that in equilibrium all interest bearing assets must pay the same return. Specifically, there are two financial assets in the economy. The first is a risk-free asset issued on government debt that in each period yields with certainty the interest factor \( R_t \). By contrast, the second is a risky asset whose yield is given by the sum between its market price \( p_{k,t} \) augmented with the marginal return on the unit of employed capital.\(^7\) The idea that any discrepancy between the yields from bonds and capital goods would be eliminated by arbitrage was pioneered by Leijonhuvud (1968) in his own interpretation of the aggregative structure of the ‘basic model’ described by Keynes (1936) in the General Theory.

The assumption of no risk-less arbitrage opportunities implies that

\[
\frac{p_{k,t+1} + \alpha Z_{t+1}}{p_{k,t}} = R_t
\] 

(5)

\(^6\)When households live forever public expenditure completely crowds out private expenditure. As a consequence, with infinitely-lived households the value of output adjusts towards the corresponding level of public expenditure.

\(^7\)The price of the risky asset is assumed to be a fundamental of the model economy driven by the self-fulfilling beliefs of households about the value of the stock market. In principle, the dynamics of \( p_{k,t} \) could be described by a stochastic process with well-defined boundaries. However, in what follows I will consider only some particular values of such an ‘animal spirits’ variable.
Under the assumption that $p_k$, $Z$ and $R$ are taken as given from a micro-economic perspective, (5) implies that for any given level of $Z$ the macro-economic dynamics of $R$ can be described by the following differential equation:

$$\dot{R} = \delta (\alpha Z - p_k (R - 1))$$  \hspace{1cm} (6)

where $\delta$ is a positive constant that conveys the effectiveness speed of the no-arbitrage condition.

The differential equation in (6) implies that the interest factor increases (decreases) whenever the marginal return on the unit of employed capital is higher (lower) than the risk-free yield on the amount of resources invested in the risky asset.\(^8\) In other words, for given values of $\alpha$ and $p_k$, (6) defines the increase (decrease) of $R$ necessary to restore the no-arbitrage condition in the aftermath of an increase (decrease) in $Z$. Specifically, $R$ increases (decreases) whenever the risky asset earns more (less) than the safe asset.

### 2.1 Steady-State

Now I look for a meaningful pair $(Z^*, R^*)$ such that $\dot{Z} = \dot{R} = 0$. By ‘meaningful’ I mean a pair $(Z^*, R^*) \in \mathbb{R}^2_{++}$, i.e., a pair characterized by a positive value of output in wage units and a positive real interest rate that select a demand constrained equilibrium, i.e., an allocation in which all the demanded goods are produced while a fraction of the labour force can remain unemployed as a consequence of search frictions (see Farmer 2008a).\(^9\)

On the one hand, taking the result in (4) into account, $\dot{Z} = 0$ implies that

$$\hat{Z} = G + H \left( \hat{R} \right) \left( p_k + \frac{B}{R} \right)$$  \hspace{1cm} (7)

where $H \left( \hat{R} \right) \equiv \frac{\alpha \beta \hat{R}}{\hat{R} \beta (1 - \alpha) - 1}$.

\(^8\)Assuming a constant equity premium would not change the qualitative dynamics of $R$. The implications of counter-cyclical equity premia are discussed in Appendix.

\(^9\)The model does not contemplate the possibility of a negative equilibrium real interest rate.
The expression in (7) is the downward-sloped ME schedule and provides all the pairs \((\hat{Z}, \hat{R})\) such that the market for goods is in equilibrium. Obviously, the area on the right (left) of the ME schedule defines all the pairs \((Z, R)\) such that there is excess supply (demand) on the market for goods.

On the other hand, taking the result in (6) into account, \(\dot{R} = 0\) implies that

\[
\hat{Z} = \frac{p_k (\hat{R} - 1)}{\alpha}
\]  

(8)

The expression in (8) is the upward-sloped NA schedule and provides all the pairs \((\hat{Z}, \hat{R})\) such that there are no risk-less arbitrage opportunities. Obviously, the area above (below) the NA schedule defines all the pairs \((Z, R)\) such that the safe asset earns more (less) than the risky asset.

To facilitate the analysis of the stationary solution, I define the following constants:

\[
a \equiv \tilde{\beta} (1 - \tilde{\alpha}) p_k \\
b \equiv \tilde{\beta} (1 - \tilde{\alpha}) (p_k + \alpha G) + p_k \left(1 + \alpha \tilde{\alpha} \tilde{\beta}\right) \\
c \equiv p_k + \alpha \left(G - \tilde{\alpha} \tilde{\beta} B\right)
\]  

(9)

Taking the definitions in (9) into account, a positive stationary interest factor is given by

\[
R^* = \frac{b + \sqrt{b^2 - 4ac}}{2a}
\]  

(10)

Once \(R^*\) is determined, the corresponding value of \(Z^*\) can be found by substituting (10) alternatively in (7) or in (8). It is worth noting that in addition to all the model parameters, the steady-state solution \((Z^*, R^*)\) also depends on the price of the risky asset, on the value of public debt and on the value of public expenditure.\(^{10}\) Such a steady-state solution is illustrated in Fig. 1.

\(^{10}\)For each value of \(G\) and \(B\), the stationary solution of the difference equation in (2) allows to derive the tax rate on labour income that sustains \((Z^*, R^*)\) as a steady-state equilibrium. The corresponding tax equation is \(\tau = ((1 - \alpha) Z^*)^{-1} \left((R^* - 1) (R^*)^{-1} B + G\right)\).
Fig. 1: Steady-state

The diagram in Fig. 1 allows to explore the short-run equilibrium effects of fiscal policies and variations in the value of financial wealth. On the one hand, a fiscal expansion (restriction), i.e., an increase (decrease) of $G$, leads the ME schedule to shift outward (inward). As a consequence, the new steady-state solution will be characterized by a higher (lower) value of $Z^*$ and a higher (lower) value of $R^*$. Those causal relationships are perfectly consistent with a static and/or a dynamic IS-LM model (e.g. Chang and Smyth 1972 and Varian 1977). On the other hand, a financial boom (crash), i.e., a self-fulfilling increase (decrease) of $p_k$, leads to a movement in both schedules. Specifically, the ME schedule shifts outward (inward) while the NA schedule rotates in a clockwise (counter-clockwise) direction. As a consequence, the new steady-state solution will be characterized by a higher (lower) value of $Z^*$ while the effect on $R^*$ is ambiguous.

2.2 Local Dynamics

The first-order Taylor expansion of the non-linear dynamic system that describes the macro-economic adjustments of the model economy is given by
\[
\begin{pmatrix}
\dot{Z} \\
\dot{R}
\end{pmatrix} =
\begin{bmatrix}
\gamma \left( \frac{R^* \tilde{\beta}(\tilde{\alpha}-1)+1}{R^* \tilde{\beta}-1} \right) & -\gamma \frac{\tilde{\alpha} \tilde{\beta}(p_k + Z^* - G + B \tilde{\beta})}{(R^* \tilde{\beta}-1)^2} \\
\delta \alpha & -\delta p_k
\end{bmatrix}
\begin{pmatrix}
Z - Z^* \\
R - R^*
\end{pmatrix}
\] (11)

On the one hand, the trace of the Jacobian matrix in (11) is the following:

\[- \left( \gamma \left( \frac{R^* \tilde{\beta}(\tilde{\alpha}-1)+1}{1 - R^* \tilde{\beta}} \right) + \delta p_k \right) < 0 \] (12)

On the other hand, its determinant is given by

\[\gamma \delta \left( \frac{R^* \tilde{\beta}(\tilde{\alpha}-1)+1}{1 - R^* \tilde{\beta}} \right) p_k + \alpha \tilde{\alpha} \tilde{\beta} \frac{(p_k + Z^* - G + B \tilde{\beta})}{(R^* \tilde{\beta}-1)^2} > 0 \] (13)

Since the trace is negative while the determinant is positive, it is possible to conclude that the dynamic system in (11) has two negative roots so that the stationary solution is a sink. As a consequence, \((Z^*, R^*)\) is locally stable so that the model economy admits a multiplicity of macro-economic equilibrium paths that converge to the steady-state each of them indexed by the initial conditions of \(Z\) and \(R\).

### 3 Numerical Simulations

In this section I report the results of some numerical simulations of the macro-dynamic model in (11).\(^{11}\) Specifically, using the baseline calibration suggested by Farmer (2010d) I explore the macro-economic adjustment towards the social optimal allocation that in the model economy fulfils the role of the natural rate of (un)employment (e.g. Farmer 2010a-b) and it also assumed to describe the situation of the US economy before the 2008 crisis. Thereafter, I consider the effectiveness of a set of policy interventions such as balanced-budget fiscal expansions and financial market regulations aimed at exiting from a finance-induced recession driven by the self-fulfilling beliefs of households about the stock market value.

\(^{11}\)The MATLAB\textsuperscript{TM} 6.5 code is available from the author.
### 3.1 Calibration

The baseline calibration of the model economy suggested by Farmer (2010d) is illustrated in Table 1.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha$</td>
<td>capital share</td>
<td>0.34</td>
</tr>
<tr>
<td>$\beta$</td>
<td>discount rate</td>
<td>0.97</td>
</tr>
<tr>
<td>$\pi$</td>
<td>surviving probability</td>
<td>0.98</td>
</tr>
</tbody>
</table>

**Table 1:** Baseline calibration

The value of $\alpha$ implies a labour share of two-third, i.e., a figure consistent with a century of US data and widely exploited in real business cycle literature (e.g. Gordon 1988 and Kydland and Prescott 1982). Moreover, the value of $\beta$ is chosen to match an average annual real interest rate of 4%. Finally, the value of $\pi$ implies an expected average life-span for households of 50 years, i.e., a figure that lines up with the average life-span of an American citizen computed with the population distribution in 2008.

In addition to the parameter values collected in Table 1, I complete the calibration of the dynamic model by setting $\gamma = 1$ and $\delta = 0.5$. Those parameter values do not alter the steady-state solution. However, they convey the idea that the equilibrium in the market for goods is achieved faster than the no-arbitrage condition in (5).

### 3.2 Results

I begin my computational experiments by exploring the macro-economic adjustments towards the social-optimal allocation. Specifically, Farmer (2010d, p. 566) defines the social-optimal allocation by setting $B = G = 0$ and $p_k = 12.2$. Those figures, combined 12

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12Think to the Mundell-Fleming version of the IS-LM model. After a policy shock, the equality between the home and the foreign interest rate (which is a genuine no-arbitrage condition) happens at the end of all the adjustments so that it seems reasonable to assume that $\gamma$ is higher than $\delta$. However, changing the values of those parameters does not alter the simulations results described below.
with the parameter values in Table 1, implies that $Z^* = 1.44$ while $R^* = 1.04$. The macro-economic adjustments towards the social-optimal allocation are illustrated in the two panels of Fig. 2.

![Diagram](image_url)

**Fig. 2**: Macro-economic adjustments towards the social-optimal allocation

$B = G = 0$, $p_k = 12.2$, $Z(0) = 1.43$, $R(0) = 1.03$, $Z^* = 1.44$, $R^* = 1.04$

The two diagrams in Fig. 2 suggest that the adjustments to the short-run macro-economic equilibrium occur through endogenous convergent oscillations. Obviously, this is due to the fact that under the baseline calibration in Table 1 the two negative roots of the dynamic system in (11) are complex-conjugate.

Now I explore the effects of a fiscal expansion aimed at restoring the social-optimal level of $Z$ in the afterwards of a financial crash driven by a self-fulfilling permanent drop of 20% in $p_k$. Taking into account a government debt is normalized to zero, the drop in $p_k$ leads to a corresponding 20% fall in the value of output but leaves the real interest rate unaltered. According to the baseline calibration in Table 1, to restore the social-optimal level of $Z$ the government has to increase the level of $G$ from zero to 0.88 (see Farmer 2010d, pp. 566-567). This permanent increase in government purchases can be financed

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13 Taking into account that Farmer (2010d) imposes a social-optimal unemployment rate equal to 5%, the social-optimal value of output can also be conveyed as $(1 - 0.05)(1 - \alpha)^{-1}$.

14 Those figures imply a value of the multiplier equal to 0.32.
by raising the tax rate on labour income without affecting the value of public debt.\textsuperscript{15} The effects of such a fiscal expansion are illustrated in the two panels of Fig. 3.

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{fig3.png}
\caption{Effects of a pure fiscal expansion}
\end{figure}

$B = 0, G = 0.88, p_k = 9.76, Z(0) = 1.152, R(0) = 1.04, Z^* = 1.44, R^* = 1.05$

The two diagrams in Fig. 3 show that the suggested fiscal stimulus will restore the social-optimal value of output in about 4 units of computational time after having followed a transitional path characterized by a strong deflation. Such a bad-behaved path is due to the fact that this strong fiscal expansion - under a balanced-budget constraint - leads to a sudden severe reduction of disposable income that in the very short-run heavily depresses consumption. In addition, the new stationary solution is characterized by a permanent increase of the real interest rate of 1%. Comparing this outcome with the social-optimal allocation described in Fig. 2, this higher level of the real interest rate will crowd out a fraction of private consumption at the end of all adjustment processes. Obviously, this equilibrium reduction of $C$ is exacerbated by the lower (higher) value of $p_k$ ($G$).

I close my computational experiments by exploring the consequences of a milder fiscal expansion carried out in combination with the introduction of a positive wedge $\omega$ between the returns of the risky and the safe asset that reduces the increase (decrease) of the interest factor needed to restore the no-arbitrage condition after an increase (decrease) in $\omega$.

\textsuperscript{15}In the present version, the model does not catch the increase in public debt usually induced by a fiscal expansion.
the value of output.\textsuperscript{16} Specifically, I consider a fiscal stimulus equal to 0.28 joined with an equilibrium 0.97% wedge between the returns of the risky and the safe assets that could be introduced by means of a taxation on financial transactions like (inter alia) the so-called Tobin tax. The effects of such a policy package are illustrated in Fig. 4.

![Figure 4: Fiscal expansion plus wedge between the returns of risky and safe assets](image)

\[ B = 0, \ G = 0.28, \ \omega = 0.97\%, \ p_k = 9.76, \ Z(0) = 1.152, \ R(0) = 1.04, \ Z^* = 1.44, \]

\[ R^* = 1.04\]

The two diagrams in Fig. 4 suggest that the policy package outlined above is able to restore the social-optimal value of output by avoiding the strong deflation path implied by the pure fiscal expansion described in Fig. 3.\textsuperscript{17} Moreover, being characterized by the same level of the real interest rate of the social-optimal allocation described in Fig. 2, this policy package - in spite of the lower (higher) value of \( p_k \) (\( G \)) - will provide a lower crowding out of private consumption at the end of the macro-economic adjustment process. As a consequence, it is quite likely that the combination of those public interventions will be

\textsuperscript{16}In this case the no risk-less arbitrage condition is \((p_{k,t+1} + \alpha Z_{t+1})p_{k,t}^{-1}(1 - \omega) = R_t\) so that the NA schedule becomes \( Z = p_k (\overline{R} - 1 + \omega) (\alpha (1 - \omega))^{-1}\) where \((\overline{Z}, \overline{R})\) is the set of pairs such that \( R = 0 \). As a consequence, the introduction of the wedge acts like a tax on the risky asset that leads the NA schedule to shift downwards and rotate in clockwise direction. Moreover, as far as a positive risk compensation is concerned, this intervention can also be interpreted as a reduction of the equilibrium equity premium.

\textsuperscript{17}In this case, the multiplier is equal to 1.02.
strictly preferred to a pure fiscal expansion. A corroboration of this ex-post statement on an ex-ante perspective can be derived for an inspection of the two panels of Fig. 5.

![Fig. 5: Consumption and social welfare](image)

The first panel of Fig. 5 illustrates the path of relative consumption under the two policy scenarios described in Figs 3 and 4 while the second traces out the corresponding path of relative social welfare that includes public expenditure measured as $C(1+G)$ (see Farmer 2010d, p. 596). Straightforward observation reveals that the area above the horizontal dotted line is larger than the area below in both panels of Fig. 5. As a consequence, especially when households do not discount future too heavily, it is quite likely that the combination of a fiscal expansion with an intervention aimed at altering the trade-off between holding risky and safe financial assets will be preferred to a pure fiscal expansion also on an ex-ante perspective.\(^\text{18}\)

4 Concluding Remarks

In this paper, following the theoretical contributions by Chang and Smyth (1972) and Varian (1977), I explore the out-of-equilibrium macro-economic dynamic behaviour of

\(^{18}\text{This result is quite robust; indeed, it also survives by exploiting the book and the working paper parameterization of the model economy. See Farmer (2010a, Chapter 7) and Farmer (2010c).}\)
the Farmer’s (2010d) ME-NA model. Specifically, preserving the assumption that microeconomic saddle-path adjustments are instantaneous, I build a dynamic model in continuous time that describes the macro-economic adjustments of the value of output and the real interest rate. Within this framework, I show that the model economy has a unique meaningful stationary solution whose dynamics is locally stable.

Moreover, I use the macro-dynamic model to perform some numerical simulations by exploiting the baseline calibration suggested by Farmer (2010d). Those computational experiments suggest that the adjustments to the short-run macro-economic equilibrium occur through endogenous convergent oscillations. Moreover, the scrutiny of different policies aimed at exiting from a finance-induced recession driven by the self-fulfilling beliefs of households about the stock marker value suggests that an expansionary fiscal policy can be an effective way to increase the value of output and reduce unemployment but there might be also welfare-enhancing companion interventions that provide for a lower crowding out of private consumption. Specifically, I show that those interventions rely on policies aimed at altering the trade-off between holding risky and safe financial assets.

5 Appendix: Counter-Cyclical Equity Premia

The theoretical analysis developed in section 2 does not explicitly consider the possibility of an equity premium. Under the assumption of a positive differential between the return of risky and risk-free assets, the differential equation for the interest factor becomes as follows:

\[ \dot{R} = \delta \left( (p_k + \alpha Z)(1 + \theta) - p_k R \right) \quad \theta > 0 \]  

(A.1)

where \( \theta \) is the equity premium.

Theory and circumstantial evidence seem to suggest that risk compensations are counter-cyclical (e.g. Bansal 2008). A linear counter-cyclical equity premium can be
conveyed as

$$\theta(Z) = \rho - Z \frac{\rho}{Z_{\text{max}}} \quad \rho > 0$$  \hspace{1cm} (A.2)

where $\rho$ is the upper bound between the yields of risky and safe assets while $Z_{\text{max}} \equiv \frac{1}{1-\alpha}$.

The expression in (A.2) suggests that in the shut-down allocation the equity premium is at its maximum level. Thereafter, it linearly decreases vanishing in the full employment allocation.

Substituting (A.2) in (A.1) leads to the following $\cap$-shaped NA schedule:

$$\bar{R} = 1 + \rho + \frac{\alpha (1 + \rho) - (1 - \alpha) \rho p_k Z}{p_k Z} - \frac{\alpha (1 - \alpha) \rho Z^2}{p_k Z^2}$$  \hspace{1cm} (A.3)

where $(\bar{Z}, \bar{R})$ is the set of pairs such that $\bar{R} = 0$.

A non-linear NA schedule such as the one in (A.3) allows for the possibility of multiple stationary solutions. An example is illustrated in Fig. A.1.

![Diagram](image)

**Fig. A.1:** Multiple equilibria

The diagram in Fig. A.1 shows a situation in which there are two distinct stationary solutions. Specifically, there is a stable (unstable) stationary solution $(Z_1^*, R_1^*)$ ($(Z_2^*, R_2^*)$)
characterized by a low (high) value of output and a high (low) level of the real interest rate. This isomorphism with the Diamond (1982) model reminds the need of coordination among public interventions, i.e., fiscal policy and interventions on financial markets, already stressed with the simulation results in section 3.

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References


