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Luciano Fanti - Nicola Meccheri

The Cournot-Bertrand profit differential in a differentiated duopoly with unions and labour decreasing returns

Abstract

This paper compares Cournot and Bertrand equilibria in a differentiated duopoly (with imperfect substitutes), total wage bill maximizing unions and labour decreasing returns. It is shown that the standard result, that equilibrium profits are always higher under Cournot, may be reversed even for a fairly low degree of product differentiation. Moreover, the presence of labour decreasing returns tends to reinforce the mechanisms that contribute to the reversal result, making this event possible for a wider range of situations, with respect to those identified by the earlier literature.

Classificazione JEL: J43, J50, L13
Keywords: Cournot-Bertrand profit differential, unions, labour decreasing returns

Contents

I. Introduction

A cornerstone result in duopoly theory is that, when goods are imperfect substitutes, firms' profits are higher under competition *à la* Cournot than *à la* Bertrand. Singh and Vives (1984) first showed such result by developing the Dixit's (1979) differentiated duopoly model with linear demand structure and exogenous (constant) marginal costs.[1] More recently, the robustness of this classic result has been investigated by introducing, in the same framework of Singh and Vives (1984), a two-stage game. While in the second stage firms compete in the product market, in the first stage either sole duopolists or duopolists together with an upstream agent make choices that affect their production costs. In particular, Qiu (1997) analyzes the case in which, prior to the standard product market game, each duopolist chooses a level of cost-reducing research and development (R&D) investment and shows that the relative efficiency of Cournot and Bertrand competition depends on three factors: R&D productivity, the extent of spillovers and the degree of product market differentiation. Correa-López and Naylor (2004), instead, introduce upstream "suppliers" in the form of unions and consider a decentralized wage-bargaining game played between each firm and a firm-specific labour union. In this context, they find that, if unions are sufficiently powerful and care enough about wages, the standard result (i.e. firms' profits are higher under Cournot competition) may be reversed.

The aim of this paper is to address the issue of whether the standard result concerning the dominance of Cournot over Bertrand equilibrium profits remains valid with unions and labour decreasing returns. Hereof, it relates more closely to Correa-López and Naylor (2004), but with an important departure. In particular, while Correa-López and Naylor (2004), following the previous literature on differentiated duopoly, consider labour constant returns (or, in

[1] With Singh and Vives's (1984, p. 456) words, "[...] profits are larger, equal, or smaller in Cournot than in Bertrand competition, according to whether the goods are substitutes, independent, or complements". See also, among others, Vives (1985), Cheng (1985) and Okuguchi (1987).

other words, constant marginal costs), we introduce labour decreasing returns, which also imply increasing marginal costs, into the analysis. Indeed, although the latter feature as the most common hypothesis in microeconomic modelling (at least, with reference to the short-run), the effects that they produce in a duopolistic framework have not been previously investigated by the literature.

The rest of this paper is organized as follows. In Section II., we present the basic model, in which two firms compete in the product market by producing differentiated goods. Under Cournot and Bertrand competition, we derive equilibrium values for the key variables of interest. In Section 3, we compare Cournot and Bertrand equilibrium profits. Finally, Section 4 concludes, while in the Appendix the proof of a result is provided.

II. Model

Following Singh and Vives (1984) and, subsequently, Qiu (1997) and Correa-López and Naylor (2004) (CL&N), among others, we consider a model of differentiated product market duopoly, in which each firm sets its output, given pre-determined wages, to maximize profits. Preferences of the representative consumer are given by:

$$U(q_i, q_j) = \alpha(q_i + q_j) - \frac{(q_i^2 + 2\gamma q_i q_j + q_j^2)}{2} \tag{1}$$

where q_i and q_j are outputs by firm i and j, respectively, $\alpha > 0$ and $\gamma \in (0, 1)$ denotes the extent of product differentiation, with goods assumed to be imperfect substitutes. The derived product market demand is linear and, with reference to the representative firm i, is given by:

$$p_i(q_i, q_j) = \alpha - \gamma q_j - q_i. \tag{2}$$

Let assume that only labour input is used for production. As already discussed in the Introduction, another literature's standard assumption is that labour exhibits constant returns, which implies firms face constant marginal costs. In this paper, instead, we modify such hypothesis by introducing labour decreasing returns. In particular, we assume the following production technology:

$$q_i = \sqrt{l_i} \tag{3}$$

where $l_i = q_i^2$ represents the number of workers employed by the firm i to produce q_i output units of variety i. The choice of such specific technology, described by the functional form of Eq. (3), allows for analytical results and also implies that firms have quadratic costs, which is a typical example of increasing costs.

Hence, the firm i's profit can be written as:

$$\pi_i = p_i q_i - w_i l_i = p_i q_i - w_i q_i^2 \tag{4}$$

where w_i is the per-worker wage paid by firm i, with $w_i < \alpha$.

Following the established literature on unionized oligopolies (e.g. Horn and Wolinsky, 1988; Dowrick, 1989; Naylor, 1999; CL&N), production costs (i.e. wages) are no longer assumed to be as exogenously given for firms, but they are the outcome of a strategic game previously played between each firm and a labour union. In this paper, we consider the case in which firms' wages are fixed by (firm-specific) "monopolistic" unions, which are rent-maximizing (e.g. Sørensen, 1992). As well-known (e.g. Pencavel, 1985; Oswald, 1985; Dowrick and Spencer, 1994), this is consistent with the case of an union that can costlessly redistribute income among its members. Technically speaking, in this context, each union's utility function is given by $V_i = (w_i - \underline{w})l_i$, where $\underline{w}$ is the reservation wage (e.g. the wage that applies in a competitive labour market). Also note that the total rent V_i equals the total wage bill if $\underline{w} = 0$. Since assuming $\underline{w} = 0$ does not produce any qualitative changes in our final results, for algebraic simplicity, from here onwards we will concentrate on the case of "total wage bill maximizing" unions, more specifically.[2] Hence, taking also Eq. (3) into account, the union i's

[2] As well known, wage and employment choices in the presence of unionisation may be modelled according to different ways. In this regard, we have chosen to adopt a relatively

utility function can be written as:

$$V_i = w_i q_i^2. \tag{5}$$

In what follows, we will study, according to the different types of product market competition, two different two-stage games. In stage 1, due to the fact that both firms are unionized, unions' choices take place simultaneously across firms, with each union taking the wage of the other firm as given. In stage 2, by playing a non-cooperative oligopolistic game (which could be either Cournot-type or Bertrand-type), firms choose their levels of output and (given the technology) factor input, taking wages as determined in the prior stage. We proceed by backward induction beginning with the Cournot case.

II.A. Cournot equilibrium under labour decreasing returns

Taking Eqs. (2) and (4) into account, profit-maximization under Cournot competition leads to the following firm i's best-reply function:

$$q_i(q_j) = \frac{\alpha - \gamma q_j}{2(w_i + 1)}. \tag{6}$$

As $\gamma > 0$, the best-reply functions are downward-sloping, that is, under the Cournot assumption, the product market game is played in strategic substitutes. From Eq. (6), and its equivalent for firm j, we can obtain, for given w_i and w_j, the firm i's output as:

$$q_i(w_i, w_j) = \frac{\alpha\left[2(w_j + 1) - \gamma\right]}{4(w_i + 1)(w_j + 1) - \gamma^2} \tag{7}$$

and, by substituting Eq. (7) in Eq. (4), the firm i's profit as:

simple structure because our aim is that to provide a first analysis of the effects that labour decreasing returns produce in the study framework. Extensions to other hypotheses are left for future research.

$$\pi_i(w_i, w_j) = \frac{\alpha^2(w_i+1)[\gamma - 2(w_j+1)]^2}{[4(1+w_i)(1+w_j)-\gamma^2]^2}. \tag{8}$$

By substituting Eq. (7) in Eq. (5) and maximizing with respect to w_i, we get also the following expression, which defines (for the union-firm pair i) the sub-game perfect best-reply function in relation to the wage, under the assumption of a non-cooperative Cournot-Nash equilibrium in the product market:

$$w_i(w_j) = \frac{4(w_j+1)-\gamma^2}{4(w_j+1)}. \tag{9}$$

In symmetric sub-game perfect equilibrium, $w_i = w_j = w$ and hence, from Eq. (9), the equilibrium wage is given by:

$$w^C = \frac{\sqrt{4-\gamma^2}}{2} \tag{10}$$

where the apex C recalls that it is obtained under Cournot competition in the product market.

Finally, the sub-game perfect equilibrium quantity (after substitution of Eq. (10) in Eq. (7)) and profit (after substitution of Eq. (10) in Eq. (8)) under Cournot competition are given by, respectively:

$$q_i = q_j = q^C = \frac{\alpha}{2+\gamma+\sqrt{4-\gamma^2}} \tag{11}$$

$$\pi_i = \pi_j = \pi^C = \frac{\alpha^2\left(2+\sqrt{4-\gamma^2}\right)}{2\left(2+\gamma+\sqrt{4-\gamma^2}\right)^2}. \tag{12}$$

II.B. Bertrand equilibrium under labour decreasing returns

We consider now the case in which the product market game is characterized by price-setting behaviour by firms, i.e. competition occurs *à la* Bertrand. From Eq. (2) and its counterpart for the firm j, we can write product demand for the firm i as:

$$q_i(p_i, p_j) = \frac{\alpha(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2} \tag{13}$$

hence, using Eq. (4), the firm i's profit is given by:

$$\pi_i(p_i, p_j) = p_i \left[\frac{\alpha(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2}\right] - w_i \left[\frac{\alpha(1-\gamma) - p_i + \gamma p_j}{1-\gamma^2}\right]^2. \tag{14}$$

From Eq. (14), the first-order condition for profit-maximization gives the firm's i price choice, as a function of the price chosen by firm j, as:

$$p_i(p_j) = \frac{[\alpha(1-\gamma) + \gamma p_j]\left(2w_i + 1 - \gamma^2\right)}{2(w_i + 1 - \gamma^2)} \tag{15}$$

thus, for $\gamma > 0$, the Bertrand product market game is played in strategic complements. By substituting in Eq. (15) the corresponding equation for the firm j and solving for p_i, we get the Bertrand equlibrium price for given wages, w_i and w_j:

$$p_i(w_i, w_j) = \frac{\alpha\left(2w_i + 1 - \gamma^2\right)[2(w_j+1) - \gamma(1+\gamma)]}{4(w_i+1)(w_j+1) - \gamma^2\left[2(w_i + w_j) + 5 - \gamma^2\right]}. \tag{16}$$

Hence, by substituting in Eq. (13), we get the sub-game perfect output as a function of wages, which are fixed by unions in the first stage of the game, as:

$$q_i(w_i, w_j) = \frac{\alpha\left[2(w_j+1) - \gamma(1+\gamma)\right]}{4(w_i+1)(w_j+1) - \gamma^2\left[2(w_i + w_j) + 5 - \gamma^2\right]} \tag{17}$$

and, by using Eqs. (17), (16) and (4), the firm i's profit as:

$$\pi_i(w_i, w_j) = \frac{\alpha^2\left[2(w_j+1) - \gamma(1+\gamma)\right]^2\left(w_i + 1 - \gamma^2\right)}{\left[4(w_i+1)(w_j+1) - \gamma^2\left[2(w_i + w_j) + 5 - \gamma^2\right]\right]^2}. \tag{18}$$

Also in the Bertrand competition case, the union's utility function is given by Eq. (5). Hence, by substituting Eq. (17) in Eq. (5), and maximizing with respect to w_i, we get the following expression:

$$w_i(w_j) = \frac{4(w_j + 1) + \gamma^2 \left[\gamma^2 - (2w_j + 5)\right]}{2\left[2(w_j + 1) - \gamma^2\right]} \tag{19}$$

which defines, analogously to Eq. (9) of the Cournot case, the best-reply function in relation to the wage of the union-firm pair i. Solving for the symmetric equilibrium ($w_i = w_j = w$), from Eq. (19), we get:

$$w^B = \frac{\sqrt{4 - \gamma^2\left(5 - \gamma^2\right)}}{2} \tag{20}$$

where the apex B recalls that the equlibrium wage defined by Eq. (20) is obtained under Bertrand competition in the product market.

Finally, the sub-game perfect equilibrium quantity (after substitution of Eq. (20) in Eq. (17)) and profit (after substitution of Eq. (20) in Eq. (18)) under Bertrand competition are given by, respectively:

$$q_i = q_j = q^B = \frac{\alpha}{2 + \gamma(1-\gamma) + \sqrt{4 - \gamma^2\left(5 - \gamma^2\right)}} \tag{21}$$

$$\pi_i = \pi_j = \pi^B = \frac{\alpha^2 \left[2\left(1 - \gamma^2\right) + \sqrt{4 - \gamma^2\left(5 - \gamma^2\right)}\right]}{2\left[2 + \gamma(1-\gamma) + \sqrt{4 - \gamma^2\left(5 - \gamma^2\right)}\right]^2}. \tag{22}$$

III. Cournot-Bertrand profit differential under labour decreasing returns

In this section, we investigate if the conventional wisdom, according to which Bertrand competition yields, in equilibrium, lower profits with respect to Cournot competition, still holds in the presence of labour decreasing returns and total wage bill (or, recalling that results do not qualitatively change, rent) maximizing unions.

In particular, the Cournot-Bertrand profit differential (based on the comparison between Eqs. (12) and (22)) is given by:

$$\Delta\pi = \pi^C - \pi^B = \frac{\alpha^2\gamma^2\left(2+\sqrt{4-\gamma^2}\right)\left[\gamma(1+\gamma)-\sqrt{4-\gamma^2\left(5-\gamma^2\right)}\right]}{\left(2+\gamma+\sqrt{4-\gamma^2}\right)^2\left[2+\gamma(1-\gamma)+\sqrt{4-\gamma^2\left(5-\gamma^2\right)}\right]^2} \tag{23}$$

from which, the following result derives.

Result 1 *In a context with labour decreasing returns (increasing quadratic costs), total wage bill maximizing unions and (imperfect) substitutes goods, profits are greater under Bertrand than under Cournot competition if, and only if, the degree of product differentiation is sufficiently low. In particular, we have that* $\Delta\pi \gtreqless 0 \Leftrightarrow \gamma \gtreqless 0.732 \equiv \overline{\gamma}$.

Result 1 straightforwardly derives from the observation that the sign of $\Delta\pi$ only depends on the last term in squared brackets of the r.h.s.'s numerator. In particular, we have that:

$$\Delta\pi \gtreqless 0 \Leftrightarrow \gamma(1+\gamma) \gtreqless \sqrt{4-\gamma^2\left(5-\gamma^2\right)} \tag{24}$$

which, solving last inequality for the γ's values of interest, gives Result 1.

A graphical demonstration of Result 1 is provided in Figure 1, where the behaviour of the Cournot-Bertrand profit differential, according to the degree of substitutability between goods (i.e. γ), is represented. In particular, further than the case of interest in this paper (with "total wage bill" maximizing unions and labour decreasing returns), represented by the green solid line, also two other useful benchmark cases are shown: the Cournot-Bertrand profit differential with total wage bill maximizing unions and labour constant

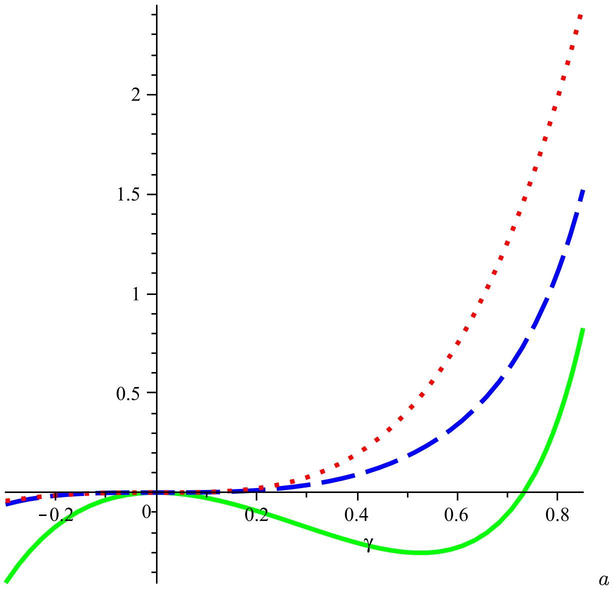

Figure 1: Cournot-Bertrand profit differentials

[a]Solid green line: unionized wage and decreasing returns; dashed blue line: unionized wage and constant returns; dotted red line: exogenous wage and decreasing returns. Parameters: $\alpha = 1$, exogenous wage $\omega = 0.1$. For graphical reasons, profit differentials of solid green and dashed blue lines have been multiplied by 100.

returns (dashed blue line)[3] and that with exogenous (non union-bargained) wages and labour decreasing returns (dotted red line).

From the figure clearly emerges that, when unions are total wage bill maximizing, the role they play in determining wages and the presence of labour diminishing returns are both necessary to get the "reversal result". In particular, Figure 1 neatly illustrates that, when both those requirements apply, it does exist a threshold value $\overline{\gamma}$, which is invariant with respect to the other economic parameters of the model, according to which profits can be lower, equal or higher with Bertrand competition according to $\gamma \lesseqgtr \overline{\gamma}$.

Instead, the behaviour of the dashed blue line confirms, accordingly with CL&N's results, that, in the presence of constant marginal costs and total wage bill maximizing unions, the weight the latter place on wages in their utility functions is not sufficiently high to

[3]Notice that this case applies in CL&N when the unions' relative bargaining power $\beta = 1$, the weight unions place on the wage $\theta = \frac{1}{2}$ and the reservation wage $\overline{w} = 0$. In particular, the dashed blue line of Figure 1 plots CL&N's Eq. (24) with β, θ and $\overline{w}$ parameters setted as specified above.

get the reversal result. This, however, no longer applies if we introduce labour decreasing returns into the analysis. However, as graphically displayed by the dotted red line (and formally shown in the Appendix), labour decreasing returns alone are not enough for equilibrium profits to be higher under Bertrand-type competition.[4]

Although a full understanding of this result deserves a more deep investigation, a first tentative explanation can be provide making reference to the CL&N's results. In particular, CL&N establish that the possibility of the reversal result rests on two facts: i) under Cournot competition unions bargain a higher wage than under Bertrand competition, because an increase in the wage rate determines a greater decrease in employment under the latter than under the former and this reduces the unions' incentives to settle for a higher wage when facing a Bertrand-type competitor in the product market; ii) equilibrium Cournot profits are more sensitive to the level of bargained wage than are Bertrand profits. However, CL&N also stress that "[T]he force of these arguments is strong enough to overturn the standard result – that profits are higher under Cournot – only if unions have sufficient influence over wages and are sufficiently wage-oriented. If unions do not exert a strong influence over wages, then the standard result obtains" (CL&N, p. 692). In our case, however, the presence of labour decreasing returns reinforces the facts i) and ii), independently by the degree of unions' wage-orientation. This is because, when wages increase, *ceteris paribus*, the employment reduction is more severe under decreasing returns. Furthermore, also strategic effects, which imply Cournot equlibrium profits decrease more steeply in wages than do Bertrand equilibrium profits,[5] are magnified by the presence of diminishing returns. This produces the reversal result notwithstanding that unions are not distinctly wage-oriented.

[4]Also notice that, although the case of interest here has been restricted to substitutes goods (i.e. $\gamma > 0$), from Figure 1 (partly) emerges that, if goods are complements ($\gamma < 0$), the standard Singh and Vives's (1984) result (see fn 1) applies, even with labour decreasing returns. This confirms also in this framework that, as emphasized by CL&N, the unionized oligopoly is not symmetric with respect to the effects of product differentiation.

[5]See CL&N, p. 691.

IV. Conclusion

In this paper we have investigated whether the conventional wisdom, according to which (with imperfect substitutes goods) the equilibrium profits under Cournot competition are higher than under Bertrand competition, still holds when there are decreasing returns to labour and wages are unilaterally fixed by a total wage bill (or rent) maximizing union.

It has been shown that the standard result may be reversed for a wide range of the degree of product differentiation. Moreover, the presence of labour decreasing returns tends to reinforce the mechanisms that contribute to the reversal result, making this event possible for a wider range of situations, with respect to those identified by the earlier literature.

Our result calls for further analyses that are deferred to future research. In particular, while it holds true for the cases of rent and total wage bill maximazing unions, extensions to other hypotheses concerning wage and employment determination in the presence of unionization (i.e. "right-to-manage" or efficient bargaining) deserve to be considered. Furthermore, we have not dealt with social welfare issues, which, nevertheless, may conduct to important results. Indeed, while previous literature suggests that, even when the Cournot-Bertrand profit differential reversal result is possible, welfare reversal never applies (Correa-López and Naylor, 2004), by introducing the (stronger) effects due to the presence of labour decreasing returns may produce, in this direction, a different outcome.

Appendix

Bertrand-Cournot profit differentials with labour decreasing returns and exogenous wages

We show here that, when wages are exogenously given, hence, they do not depend on different types of product market competition, profits are always greater under Cournot than under Bertrand competition, even in the presence of decreasing returns of labour.

Taking Eqs. (8) and (18) into account and exploiting the symmetry hypothesis, we get that equilibrium profits under Cournot-type and Bertrand-type competition (with exogenous wages and labour decreasing returns) are given by, respectively:

$$\overline{\pi_i}^C = \overline{\pi_j}^C = \overline{\pi}^C = \frac{\alpha^2(\omega+1)}{[(2(\omega+1)+\gamma]^2} \tag{A1}$$

$$\overline{\pi_i}^B = \overline{\pi_j}^B = \overline{\pi}^B = \frac{\alpha^2(\omega+1-\gamma^2)}{[(2(\omega+1)+\gamma(1-\gamma)]^2} \tag{A2}$$

where ω is the wage rate, which is assumed to be exogenous in this context.

Hence, by using Eqs. (A1) and (A2), we get that:

$$\Delta\overline{\pi} = \overline{\pi}^C - \overline{\pi}^B = \frac{\alpha^2\gamma^3\left[\gamma(\omega+2)+2(\omega+1)\right]}{[(2(\omega+1)+\gamma]^2\left[(2(\omega+1)+\gamma(1-\gamma)\right]^2} > 0 \tag{A3}$$

for any $\gamma > 0$.

Affiliazione ed indirizzo degli autori

Luciano Fanti and Nicola Meccheri, Dipartimento di Scienze Economiche (DSE), University of Pisa. E-mails: lfanti@ec.unipi.it (Fanti) and meccheri@ec.unipi.it (Meccheri)

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