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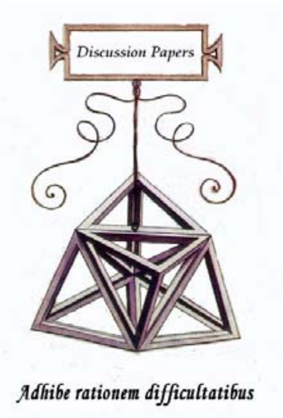

Marco Guerrazzi

Stochastic Dynamics and Matching in the Old Keynesian Economics: A Rationale for the Shimer's Puzzle

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Discussion Paper

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Stochastic Dynamics and Matching in the Old Keynesian Economics: A Rationale for the Shimer's Puzzle

Abstract

Following the Farmer's (2008a-b, 2010) micro-foundation of the *General Theory*, I build a competitive search model in which output and employment are demand-driven, prices are flexible, the nominal wage is used as numeraire and agents are divided in two categories: wage and profit earners. Within this framework, I show that the model economy has a continuum of demand constrained equilibria that might be consistent with a certain degree of endogenous real wage stickiness. Moreover, calibrating and simulating the model economy in order to fit the US first-moments data, I show that this setting can provide a rationale for the Shimer's (2005) puzzle, i.e., the relative stability of real wages in spite of the large volatility of labor market tightness.

JEL Classification: E12, E24, J63, J64.
Keywords: Stochastic Dynamics, Competitive Search, Old Keynesian Economics, Demand Constrained Equilibrium, Numerical Simulations.

Contents

1 Introduction

Addressing some criticisms raised after the publication of the *General Theory*, Keynes (1937, pp. 211-212) prophetically stated: "I'm more attached to the comparatively simple fundamental ideas which underlie my theory than to the particular forms in which I have embodied them, [...], time and experience and the collaboration of a number of minds will discover the best way of expressing them".

An earlier influential evaluation of the circulation of the Keynesian legacy in the economic profession was given thirty years later by Leijonhufvud (1968) in *On Keynesian Economics and the Economics of Keynes.* On the one hand, by *Keynesian Economics* Leijonhufvud (1968) meant the interpretation of the *General Theory* incorporated into the Hicksian IS-LM apparatus and more recently into the so-called "New-Keynesian" paradigm (e.g. Mankiw and Romer 1991). It is well known that this kind of modeling has a Neo-classical core through which deviations from the "natural" rate of unemployment can be interpreted as optimal reactions to nominal and/or real rigidity. On the other hand, Leijonhufvud (1968) correctly recognized that those rigidities were not a central argument of the *Economics of Keynes.*

In this paper I cause to breathe new life into the "Old Keynesian" economics by blending it with modern search and business cycle theories and then testing the resulting stochastic dynamic outcome against recent empirical evidence on macroeconomic fluctuations. Specifically, following the micro-foundation of the *General Theory* recently provided by Farmer (2008a-b, 2010), I build a competitive two-sided search model in which output and employment are driven by effective demand, the price level is perfectly flexible and the nominal wage rate is chosen as numeraire. As pioneered by Moen (1997), the "competitiveness" of the Farmer's (2008a-b, 2010) demand-driven search framework arises from the fact that agents take prices and matching probabilities as given when they decide their optimal behavior. However, in contrast to Moen (1997), in the model economy there are no "market makers" who compete each other to match unemployed workers with vacant jobs. On the contrary, the matching technology is assumed to enter the problem of economic agents as a proper search externality (and production) so that the resulting equilibrium allocation is not - in general - Pareto optimal.[1] As a consequence, the social-optimal allocation results from the solution of a well-defined planning problem in which search externalities are completely internalized.

Another Old Keynesian feature that I append to the Farmer's (2008a-b, 2010) proposal in order to ease aggregation and resume the Cambridge theory of distribution is the assumption that economic agents are divided into two broad income categories: wage and profit earners. As originally suggested by Kaldor (1955-1956, pg. 95), the wage-category comprises not only manual labor but salaries as well while the profit-category comprises the income of property owners and not only of entrepreneurs. Thereafter, I model both categories of agents with a discrete version of the perpetual youth overlapping-generations (OLG) framework pioneered by Yaari (1965) and Blanchard (1985) and I also assume that those two kinds of agents differ in their marginal propensities to consume and their tasks. On the one hand, wage earners - the owners of labor services - dislike savings and consume the whole income raised by supplying their fixed labor endowment. On the other hand, profit earners - the owners of the capital stock and/or overhead workers - are assumed to save the total income raised by employing a given amount of wage earners and arranging a production technology aimed at financing investments and capital accumulation.[2] In addition, following Farmer (2008a), I formalize the Keynesian "animal

[1]The underlying assumption is that that moral hazard factors prevents the creation of competitive markets for search inputs.

[2]Following the cash-flow criterion proposed by Abel et al. (1989), these two assumptions place the economy exactly in the border-line region between dynamic efficiency and inefficiency. However, in the analysis that follows nothing would be changed by assuming that profit earners consume a share of their planned investment expenditure. Finally, the "usual" situation in which investments are financed by the savings of workers is analyzed in the 2-period OLG model developed by Farmer (2010, Chapter 4).

spirits" of profit earners by assuming that their investment expenditure is driven by an autonomous stochastic process such as those usually exploited for total factor productivity (TFP) in conventional real business cycle (RBC) models.

Taking into account of those building blocks, I show that this framework has a continuum of demand constrained equilibria indexed by the employed physical capital and the stochastic realizations of demand and supply shocks. Each demand constrained equilibrium (DCE) is characterized by a different production level, a different real wage rate and a different (un)employment rate that are not necessarily Pareto-efficient. Furthermore, I emphasize that this theoretical setting might be consistent with a certain degree of endogenous real wage stickiness next to the social-optimum allocation.

Finally, calibrating and simulating the model economy in order to fit the US first-moments data, I show that under the hypothesis that the economy is hit simultaneously by demand and supply shocks the suggested framework can provide a rationale for the so-called Shimer's (2005) puzzle, i.e., the relative stability of real wage rates in spite of the large volatility of labor market tightness indicators. The added-value of my computational proposal is twofold. First, this striking US business cycle feature can hardly be explained by means of the standard matching model a là Mortensen and Pissarides's (1994) in which real wage rates are the outcome of a generalized Nash bargaining. Second, in sharp contrast with the contributions by Hall (2005a-b) and Shimer (2005), the puzzle is resolved without assuming any exogenous real wage rigidity. In fact, I show that the required real wage stickiness for the amplification of real shocks can endogenously arise from the combination of the Farmer's (2008a-b, 2010) demand-driven competitive search framework with a quite conventional calibration of the model economy. The intuition for this result is that there might be an interval for the equilibrium (un)employment rate in which the relevant Beveridge curve is steeper then the equilibrium wage function.

The paper is arranged as follows. Section 2 describes the theoretical framework. Section 3 provides some numerical simulations. Finally, section 4 concludes.

2 The Model

I develop a demand-driven perpetual-youth model with competitive search along the lines traced out by Farmer (2008a-b, 2010). Specifically, exploiting a discrete version of the OLG framework put forward by Yaari (1965) and Blanchard (1985), I consider a model economy in which a unit mass of two kinds of price-taking heterogeneous agents coexist over an infinite sequence of periods. Each kind of agents refers to an income earners' category which is assumed to be characterized by a specific marginal propensity to consume and a specific task. On the one hand, wage earners are saving-constrained and consume the whole income raised by supplying a fixed amount of labor services that might be allocated to production and/or recruiting activities. On the other hand, profit earners save the whole income raised by employing wage earners and arranging a production process randomly hit by productivity shocks. The net-of-wage payments saved by profit earners implicitly define the yield on employed capital and are exploited to finance investments.[3] Moreover, in order to pin down the equilibrium of the model economy, I follow Farmer (2008a) and I assume that the profit earners' investment expenditure evolves according to a stochastic AR(1) process.

This setting allows to derive all the properties of a continuum of demand constrained equilibria for a model economy with savings and productive investments in a straightforward manner. Finally, I exploit this framework to emphasize the possibility of a quite flat trade-off between real wage rates and equilibrium (un)employment due to a turning point in the concavity of the equilibrium wage function next to the social-optimal allocation.

[3]The resumption of the Cambridge theory of distribution is not meant to represent a society divided in proletarians and capitalists. On the contrary, it is meant to represent a model economy in which the equilibrium on the market for goods determines - inter alia - the real wage rate. This feature will result in being very important with a search labor market that does not deliver price signals.

2.1 Wage Earners

Without loss of generality, I assume the existence of a large number of identical wage earners. Thereafter, in each period, the problem of the representative wage earner is given by

$$\max_{C_t} \ \log(C_t) \\ s.to \\ p_t C_t \leq w_t L_t \tag{1}$$

where C_t is consumption in real terms, p_t is the price level, L_t is the employment rate and w_t is the nominal wage rate.

Wage earners are endowed with a single unit of time that they inelastically allocates to job-search activities. Among searching workers, the fraction L_t successfully find a job while the remaining U_t is unemployed. Therefore,

$$L_t + U_t = H_t \tag{2}$$

where $H_t \equiv 1$, for all t.

Current employment and labor supply are assumed to be linked by the following expression:

$$L_t = h_t H_t = h_t \tag{3}$$

where h_t is hiring effectiveness.

The hiring effectiveness - taken parametrically by the individual wage earner - can be determined as

$$h_t = \left(\frac{\overline{L}_t}{\overline{H}_t}\right) = \overline{L}_t \tag{4}$$

where $\overline{L}_t$ is aggregate employment while $\overline{H}_t \equiv 1$, for all t, is the aggregate labor supply.

The expression in (4) conveys a typical “thick market” externality, i.e., the higher (lower) the aggregate employment rate, the easier (harder) for a wage earner to find a job (e.g. Diamond 1982).

The solution of the problem in (1) is given by

$$C_t = \left(\frac{w_t}{p_t}\right) L_t \tag{5}$$

With equilibrium search unemployment ex-ante homogeneous wage earners might become heterogeneous in the absence of perfect income insurance because in this case the individual wealth would depend on the individual employment history (e.g. Merz 1995 and Andolfatto 1996). However, the assumption that in each period wage earners pool their income in order to subsidize who remain unemployed does not alter the main results achieved in the paper.

2.2 Profit Earners

Symmetrically with wage earners, I assume the existence of a large number of identical profit earners. Thereafter, in each period the representative profit earner is assumed to arrange the production of a homogeneous-perishable good by means of the following constant-returns-to-scale Cobb-Douglas production function:

$$Y_t = A_t K_t^{\alpha} X_t^{1-\alpha} \qquad\qquad 0 < \alpha < 1 \tag{6}$$

where Y_t is the output level, A_t is a common-knowledge productivity shock, K_t is the predetermined capital stock, X_t is the fraction of employed wage earners allocated to production activities and α is the capital share.

As in standard RBC models, the log of the productivity shock is assumed to follow a stochastic AR(1) process. Hence,

$$\ln A_t = \ln \mu + \xi \ln A_{t-1} + \eta_t \tag{7}$$

where μ is a positive drift, ξ measures the persistence of productivity shocks and $\eta_t \sim N\left(0, \sigma_\eta^2\right)$ is a stochastic productivity disturbance.

Given (6), the problem of the representative profit earner is given by

$$\begin{gathered}\max_{L_t} p_t A_t K_t^\alpha X_t^{1-\alpha} - w_t L_t \\ s.to \\ L_t = X_t + V_t\end{gathered} \tag{8}$$

where V_t is the fraction of employed wage earners allocated to recruiting activities by the representative profit earner.

In each period, current employment and the fraction of wage earners allocated to recruiting activities are assumed to be linked by

$$L_t = v_t V_t \tag{9}$$

where v_t is the recruiting efficiency parameter.

The expression in (9) is aimed at providing a micro-foundation for applications processing and it suggests that each profit earner knows that V_t recruiters can hire $v_t V_t$ wage earners X_t of whom will be employed in production activities.

The recruiting efficiency - taken parametrically by the individual profit earner - can be determined as

$$v_t = \left(\frac{\overline{L}_t}{\overline{V}_t}\right) \tag{10}$$

where $\overline{V}_t$ is the fraction of wage earners allocated to recruiting activities by all profit earners in the whole economy.

Taking into account of the results in (8) - (10), the first-order condition (FOC) for the problem in (8) can be written as

$$(1-\alpha)\frac{Y_t}{L_t} = \left(\frac{w_t}{p_t}\right) \tag{11}$$

where $Y_t = A_t K_t^{\alpha} L_t^{1-\alpha}\left(1-\frac{1}{v_t}\right)^{1-\alpha}$.

Finally, the capital stock evolves according to the usual dynamic accumulation law. As a consequence,

$$K_{t+1} = I_t + (1-\delta) K_t \qquad 0 < \delta < 1 \tag{12}$$

where $I_t = Y_t - \left(\frac{w_t}{p_t}\right) L_t$ are real investments while δ is the depreciation rate of capital.[4]

2.3 Matching

Now I describe how searching wage earners find jobs in the economy as a whole. Specifically, following Farmer (2008a-b, 2010), the aggregate search technology that moves wage earners towards profit earners is assumed to be given by the following constant-returns-to-scale Cobb-Douglas matching function:

$$\overline{L}_t = \overline{H}_t^{\gamma}\overline{V}_t^{1-\gamma} \qquad 0 < \gamma < 1 \tag{13}$$

where γ is the matching elasticity.

The expression in (13) suggests that in each period aggregate employment is the result of the matching between all the searching unemployed wage earners and the recruiters employed by all the profit earners displaced in the whole economy. Therefore, in contrast

[4]In the remainder of the paper, I will assume that the employment rate is a jump-variable for the model economy. In other words, as in standard RBC models, I will suppose that at the end of each period all the employed wage earners are fired and (possibly) re-hired at the beginning of the next. Obviously, this means a 100% of labor market turnover in each time period.

to the standard matching approach popularized by Pissarides (2000), Farmer (2008a-b, 2010) assumes that vacancies are posted by using labor instead of output.

Considering the fixed search allocation in (2), (13) simplifies to

$$\overline{L}_t = \overline{V}_t^{1-\gamma} \tag{14}$$

The expression in (14) describes how aggregate employment is related to the aggregate recruiting effort arranged by all the profit earners. Again, taking into account of the result in (2), a simple manipulation of (14) also allows to derive a stable trade-off between the aggregate level of recruiters and the total amount of unemployed wage earners which provides a version of the well-known Beveridge curve that, in turn, summarizes the operation of search externalities in the whole economy.[5] Specifically,

$$\overline{V}_t = \left(1 - \overline{U}_t\right)^{\frac{1}{1-\gamma}} \tag{15}$$

where $\overline{U}_t$ is the aggregate unemployment rate. A graphical outlook is given in figure 1.

Considering a situation of "symmetric" equilibrium, i.e., a situation in which $\overline{L}_t = L_t$ and $\overline{V}_t = V_t$, then it becomes possible to express the recruiting efficiency parameter as a function of aggregate employment. In fact, exploiting the results in (8), (9) and (14) it is possible to derive

$$v_t = L_t^{-\frac{\gamma}{1-\gamma}} \tag{16}$$

Conversely to (4), the expression in (16) conveys a typical "thin market" externality, i.e., the recruiting efficiency relevant for the profit earners' problem in (8) is higher (lower),

[5] Pissarides (2000) derives a Beveridge curve by considering the steady-state of the unemployment evolution law arising from a constant-returns to scale matching function coupled with an exogenous hazard rate for employment contracts. Closer to the Farmer's (2008a-b, 2010) proposal is the Hansen's (1970) approach who derived a negative equilibrium relationship between job vacancies (or recruiters) and unemployment from more primitive assumptions on the frictions of an auction labor market.

the lower (higher) is aggregate employment (e.g. Diamond 1982).

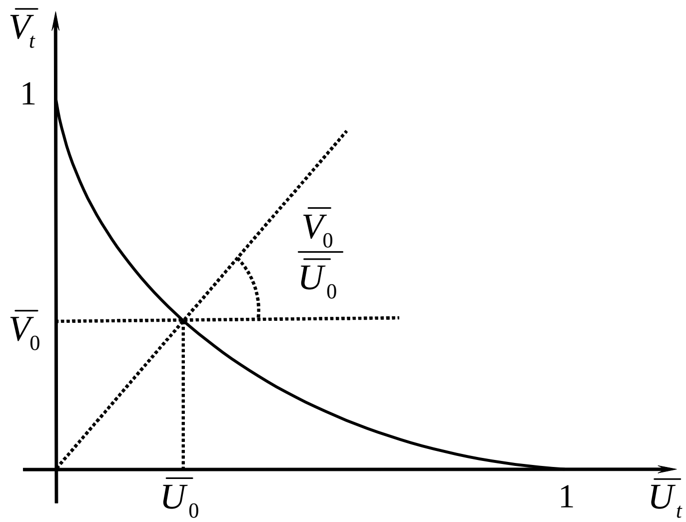

Figure 1: Beveridge curve

2.4 Social Optimum

In order to have an efficient benchmark for the evaluation of realized allocations, I analyze the problem of an omniscient-impartial social planner who can operate simultaneously the production and the matching technologies in (6) and (13) by internalizing the externality factor in (16). In a subsequent part of the paper, this exercise will provide some insights on the properties of the equilibrium wage function.

Assuming that the social planner attaches the same weight to each income category, the Pareto-optimal allocation is defined by the level of employment that maximizes the sum between wage income and realized profits in real terms, i.e., the level of L_t that maximizes the real amount of output for any given level of the productivity shock and the stock of employed capital.[6] As a consequence, taking into account of the results in (6), (8), (9) and (16) the social planner problem is the following:

[6]In a competitive (auction) model of the labor market, the Pareto-optimal allocation would be $L_S = 1$.

$$\max_{L_t} A_t K_t^{\alpha} \left(L_t \left(1 - L_t^{\frac{\gamma}{1-\gamma}} \right) \right)^{1-\alpha} \tag{17}$$

Therefore, the social-optimal level of employment is given by

$$L_S = (1-\gamma)^{\frac{1-\gamma}{\gamma}} \tag{18}$$

Obviously, $U_S = 1 - L_S$ provides the social-optimal unemployment rate.[7] Furthermore, it is worth noting that L_S and U_S depend only on the unique parameter of the matching technology that also conveys the steepness of the Beveridge curve.[8] A graphical outlook of the social planner problem is given in figure 2.

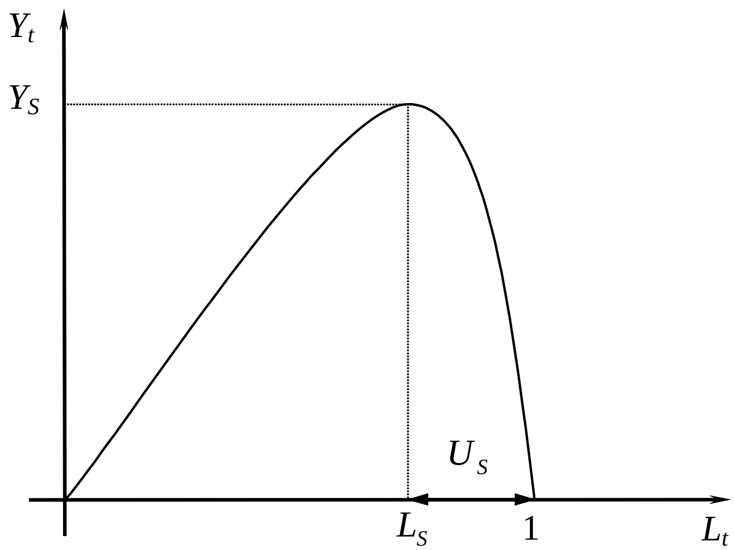

Figure 2: Social optimum level of (un)employment

The diagram in figure 2 allows to clarify some important features of the production and matching technologies. Obviously, whenever $L_t = 0$ there is no production because

[7]Under constant nominal wage rates, U_S is also a fair approximation of the NAIRU. A computational proof is sketched in Appendix.

[8]Farmer (2008b, 2010) sets $\gamma = \frac{1}{2}$, so that he derives $L_S = U_S = \frac{1}{2}$. A calibration that fits the average long-run US unemployment rate is given in section 3.

no wage earner is employed. However, there is no production even when $L_t = 1$. In fact, in this case, taking into account of the result in (16), the aggregate recruiting efficiency would be equal to one. Therefore, profit earners would be lead to allocate all the employees to recruiting while no wage earner would be allocated to production activities.[9] As a consequence, total output is at its maximum level Y_S whenever $L_t = L_S$. Indeed, any additional employed wage earner would not produce additional output but he would be simply employed in recruiting additional recruiters without improving the resulting allocation.

2.5 Aggregate Demand and Supply

In each period, taking into consideration the national account identity, the value of aggregate demand (AD_t) can be obtained by adding the aggregate nominal consumption ($p_t C_t$) and the nominal investment expenditure ($p_t I_t$). Hence,

$$AD_t = p_t \left(C_t + I_t \right) \tag{19}$$

Following the choice of units made by Keynes (1936) in the *General Theory* (Chapter 4, pg. 41) and resumed by Farmer (2008a-b, 2010), I use the nominal wage rate w_t as numeraire. Therefore, taking into account of the results in (5) and (19) the value of aggregate demand in wage units can be written as

$$\frac{AD_t}{w_t} = L_t + \widehat{I}_t \tag{20}$$

where $\widehat{I}_t \equiv \left(\frac{w_t}{p_t} \right)^{-1} I_t$.

In the model economy under examination, investments are not derived from (rational) optimization. Following Farmer (2008a, pg. 38), I formalize the Keynesian animal spirits

[9] In fact, whenever $\overline{U}_t = 0$ the corresponding point on the Beveridge curve in (15) is given by $\overline{V}_t = 1$.

of profit earners by assuming that their nominal investment expenditure measured in wage units is driven by an autonomous stochastic AR(1) process such as the one in (7). Hence,

$$\widehat{I}_t = \kappa + \rho \widehat{I}_{t-1} + \epsilon_t \tag{21}$$

where κ is a positive drift, ρ measures the persistence of the exogenous investment sequence and $\epsilon_t \sim N\left(0, \sigma_\epsilon^2\right)$ is a stochastic demand disturbance.

The expression in (21) pins down the equilibrium of the model economy and formalizes in a very simple way a central issue of the *General Theory*, i.e., the idea that investment expenditure evolves exogenously with no regard for expected profits. In fact, in the *Economics of Keynes* the main driving force of investments is given by the state of long-term expectations (e.g. Keynes 1936, Chapter 12, pg. 149). Along these lines, modeling a state variable for the beliefs' evolution process of traders committed in forecasting the true liquidation value of a risky asset, Kurz (2008, pp. 778-779) provides a piece of microfoundation for the expression in (21) by deriving a similar first-order autoregressive process as a limit posterior of a Bayesian learning inference in a non-stationary environment.[10]

Let me now turn to aggregate supply. In the *General Theory* (Chapter 3, pp. 24-25), Keynes (1936) defined the value of aggregate supply by having in mind the idea of entrepreneurs that compete one another for the production factors by means of price adjustments. As suggested by Farmer (2008a-b, 2010), in a one-good economy the equation that triggers competition for the labor factor is the FOC for L_t in (11). Therefore, if the nominal wage rate w_t is chosen as numeraire, then the value of aggregate supply in wage units is simply given by

$$\frac{AS_t}{w_t} = \frac{1}{1-\alpha} L_t \tag{22}$$

The equilibrium condition for the goods market, i.e., $AD_t = AS_t$, provides the follow-

[10]Different specifications with some heterogeneous microfoundation flavors are tested in Appendix.

ing solutions for the value of output in wage units and the level of employment:

$$\left(\frac{w_t^*}{p_t^*}\right)^{-1} Y_t^* = \frac{1}{\alpha}\widehat{I}_t \quad \text{and} \quad L_t^* = \frac{1-\alpha}{\alpha}\widehat{I}_t \tag{23}$$

Obviously, $U_t^* = 1 - L_t^*$ provides the corresponding rate of (actual) unemployment. A graphical outlook of aggregate demand and supply in wage units is given in figure 3.

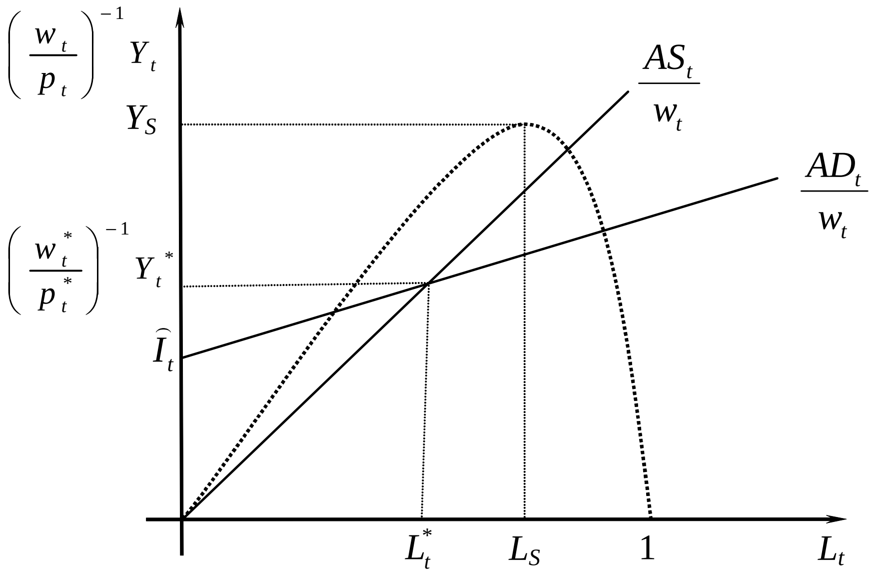

Figure 3: Aggregate demand and supply in wage units

Considering the erratic nature of the investment expenditure, there is no certainty that actual employment coincides with its social-optimal level. For example, the diagram in figure 3 shows a situation in which actual employment is lower than the social-optimal level so that the model economy is experiencing inefficient (and involuntary) unemployment.[11]

[11] As suggested by Farmer (2010, Chapter 4), this framework can also be exploited to design balanced fiscal policies that successfully implement the social-optimal level of (un)employment. Assuming that the public authority that designs the fiscal policy knows the value of L_S and that it is also able to observe the actual realization of $\widehat{I}_t$, then it becomes possible to derive a pair of fiscal parameters $\{\tau_t^S, T_t^S\}$ that fulfills the mentioned requirement. Suppose that that wage earners' nominal income is taxed at the

In addition, it is worth noting that (2) and (23) impose precise lower and upper bounds for the stochastic process followed by the investment expenditure in wage units. Specifically,

$$0 \leqslant \widehat{I}_t < \frac{\alpha}{1-\alpha} \quad \text{for all } t \tag{24}$$

The expression in (24) is straightforward and provides useful insights for the calibration of the stochastic process in (21). In fact, positive solutions for $\left(\frac{w_t^*}{p_t^*}\right)^{-1} Y_t^*$ and L_t^* are not consistent with a negative realization of $\widehat{I}_t$. Moreover, a value of $\widehat{I}_t$ higher than $\frac{\alpha}{1-\alpha}$ will result in an equilibrium employment higher than the available labor supply in the whole economy. Therefore, assuming that expectations are the driving force of the profit earners' investment expenditure, (23) and (24) suggest that animal spirits should be modeled as a bounded sequence of self-fulfilling beliefs that denotes the state of long-term expectations.[12]

2.6 Demand Constrained Equilibrium and Wage Function

Now I can provide a formal definition of equilibrium based on the building blocks outlined above. Following Farmer (2008a-b, 2010), I exploit the term "Demand Constrained Equilibrium" (DCE) in order to describe a demand-driven competitive search model closed by a balance material condition.[13] Hence,

Definition 1 *For each* $\widehat{I}_t \in [0, \frac{\alpha}{1-\alpha})$, $A_t > 0$ *and* $K_t > 0$ *a symmetric DCE is given by*

proportional tax rate τ_t and subsidized with a nominal lump-sum transfer T_t measured in wage units. In this case, the equilibrium in the goods market at the social-optimum level of employment implies that the social-optimal tax rate is given by $\tau_t^S = \frac{\widehat{I}_t}{L_S} - \frac{\alpha}{1-\alpha}$ while $T_t^S = \tau_t^S L_S$ is the corresponding social-optimal lump-sum transfer.

[12] In this demand-driven competitive search framework the equivalent of the Hosios's (1990) condition works just as a constraint on the actual realization of $\widehat{I}_t$. Specifically, whenever $\widehat{I}_t = \frac{\alpha}{1-\alpha} L_S$ the resulting equilibrium allocation is Pareto-optimal.

[13] The only common heritage with the fixed-price disequilibrium models developed, inter alia, by Benassy (1975), Drèze and Malinvaud (1977) is the Keynesian idea of effective demand.

(i) a real wage rate $\left(\frac{w_t^*}{p_t^*}\right)$

(ii) a production plan $\{Y_t^*, V_t^*, X_t^*, L_t^*, U_t^*\}$

(iii) a consumption allocation C_t

(iv) a pair $\{h_t^*, v_t^*\}$

with the following properties:

- *Feasibility and market-clearing in the market for goods:*

$$Y_t^* = A_t K_t^\alpha \left(X_t^*\right)^{1-\alpha} \tag{25}$$

$$L_t^* = X_t^* + V_t^* \tag{26}$$

$$U_t^* = 1 - L_t^* \tag{27}$$

$$C_t^* + \widehat{I}_t \left(\frac{w_t^*}{p_t^*}\right) = Y_t^* \tag{28}$$

- *Consistency with the optimal choices of wage and profit earners:*

$$C_t^* = \left(\frac{w_t^*}{p_t^*}\right) L_t^* \tag{29}$$

$$(1-\alpha)\frac{Y_t^*}{L_t^*} = \left(\frac{w_t^*}{p_t^*}\right) \tag{30}$$

- *Search market equilibrium:*

$$h_t^* = L_t^* \tag{31}$$

$$v_t^* = \left(\frac{L_t^*}{V_t^*}\right) \tag{32}$$

$$V_t^* = (1 - U_t^*)^{\frac{1}{1-\gamma}} \tag{33}$$

It is worth noting that in a DCE all nominal variables are expressed in money wage units and in sharp contrast with the competitive search equilibrium suggested by Moen (1997) no agent has incentives to change its behavior even if L_t^* is different from L_S. Furthermore, the results in (25), (26), (27) and (33) suggest that the FOC in (30) can be alternatively written as

$$\left(\frac{w_t^*}{p_t^*}\right) = (1-\alpha)\, A_t K_t^{\alpha} \frac{\left(1 - (L_t^*)^{\frac{\gamma}{1-\gamma}}\right)^{1-\alpha}}{(L_t^*)^{\alpha}} \tag{34}$$

where $L_t^* \in [0, 1]$.

The expression in (34) is the equilibrium wage function and it shows that for any eligible equilibrium employment rate, a positive (negative) productivity (or supply) shock leads to an increase (decrease) of the corresponding equilibrium real wage rate which is the genuine jump-variable of the model economy.[14] Moreover, the results in (23) imply that

$$\lim_{\widehat{I}t \to \left(\frac{\alpha}{1-\alpha}\right)^-} L_t^* = 1 \;\text{ so that } \lim_{\widehat{I}t \to \left(\frac{\alpha}{1-\alpha}\right)^-} \left(\frac{w_t^*}{p_t^*}\right) = 0 \tag{35}$$

$$\lim_{\widehat{I}t \to 0^+} L_t^* = 0 \text{ so that} \lim_{\widehat{I}t \to 0^+} \left(\frac{w_t^*}{p_t^*}\right) = +\infty \tag{36}$$

The expressions in (35) and (36) are straightforward. On the one hand, a full employment DCE, i.e., $L_t^* = H_t = 1$, is characterized by the fact that there is no production while

[14]Specularly, the DCE real interest rate consistent with a zero-profit condition on the profit earners' side would be equal to $\widehat{I}_t \left(\frac{w_t^*}{p_t^*}\right) K_t^{-1}$.

labor is a free-good.[15] In the other hand, a DCE in which no wage earner is employed is obviously characterized by the absence of production. However, given that labor is so scarce, profit earners would be willing to pay a real wage rate that tends to infinity.[16] A graphical outlook of the equilibrium wage function is given in figure 4.

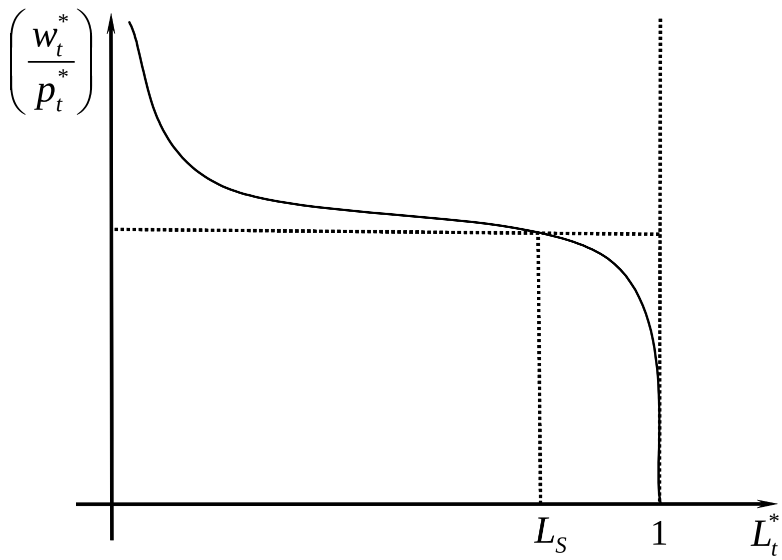

Figure 4: Equilibrium wage function

The diagram in figure 4 allows to clarify some important features of the non-linear expression in (34). First, the equilibrium wage function is strictly decreasing and this suggests a quite conventional trade-off between real wage rates and equilibrium employment.[17] Second, for $L_t^* = L_S$ there is an inflection point in the equilibrium wage function.

[15]In this case search (and production) externalities disappear. In fact, in a full employment DCE $h_t^* = 1$ while $v_t^* \to +\infty$.

[16]Conversely, in this case search (and production) externalities are at their maximum level, i.e., $h_t^* = v_t^* = 0$.

[17]This model economy fulfils what Keynes (1936) in the *General Theory* (Chapter 2, pg. 5) defined as the first postulate of classical economics. As a consequence, the real wage rate is always equal to the marginal productivity of labor. Moreover, ceteris paribus, an increase (decrease) in employment is always coupled with a decrease (increase) in the real wage rate.

Specifically, whenever the actual realization of L_t^* is lower (higher) than L_S, the equilibrium wage function is convex (concave).

Having in mind the social planner problem developed above and illustrated in figure 2, the reason why the inflection point of the equilibrium wage function has to coincide with the social-optimal (un)employment level is straightforward. In fact, equilibrium employment levels lower than L_S are associated with increasing total output because employing additional wage earners improves the resulting allocation at decreasing rates. Therefore, the average and marginal product of labor display the conventional convex decreasing path. However, beyond L_S total output starts to decrease because employing additional wage earners would not produce additional output but they would be simply employed in recruiting additional recruiters. As a consequence, this leads the average and marginal product of labor to follow a decreasing concave path that quickly converges to zero.

The possibility of a turning point in the concavity of the equilibrium wage function is an intriguing and not-yet-explored feature of the Farmer's (2008a-b, 2010) demand-driven competitive search framework. In fact, this means that equilibrium employment rates next to L_S imply a quite flat trade-off between equilibrium real wage rates and (un)employment. Therefore, even if all agents are price-takers and prices are free to adjust, this delivers an original possible endogenous source of real wage stickiness.

2.7 Stochastic Dynamics

Taking into account (12), (23), (26), (32) and (33), the global dynamics of the model economy is described - together with (7) and (21) - by the following stochastic first-order difference equation for the capital stock:

$$K_{t+1} = \alpha A_t K_t^{\alpha} \left(\frac{1-\alpha}{\alpha} \widehat{I}_t \right)^{1-\alpha} \left(1 - \left(\frac{1-\alpha}{\alpha} \widehat{I}_t \right)^{\frac{\gamma}{1-\gamma}} \right)^{1-\alpha} + (1-\delta) K_t \tag{37}$$

The stable stochastic steady-state solution of (37) is given by

$$\widehat{K} = \left(\frac{\alpha}{\delta}A_t\right)^{\frac{1}{1-\alpha}} \left(\frac{1-\alpha}{\alpha}\widehat{I}_t\right) \left(1 - \left(\frac{1-\alpha}{\alpha}\widehat{I}_t\right)^{\frac{\gamma}{1-\gamma}}\right) \tag{38}$$

It is worth noting that in the model economy under examination the realizations of the stochastic processes that summarize, respectively, productivity shocks and the Keynesian animal spirits hypothesis contribute to determine the steady-state equilibrium.[18]

3 Some Computational Experiments: A Rationale for the Shimer's Puzzle

In recent years, a lot of computational efforts have been addressed towards the explanation of the cyclical behavior of equilibrium unemployment and vacancies. Among the others, there is an influential paper by Shimer (2005) that questions the predictive power of the standard matching model a là Mortensen and Pissarides's (1994) by arguing that its theoretical framework cannot generate the observed business-cycle-frequency fluctuations in unemployment and job vacancies in response to shocks of plausible magnitude.[19]

Using quarterly data from different sources, Shimer (2005) measures, inter-alia, the autocorrelation and the volatility of unemployment, job vacancies and real wage rates for the US economy in the period from 1951 to 2003. Some of his empirical findings are summarized in table 1.

One of the most striking finding emphasized by Shimer (2005) is that the standard deviation of the vacancy-unemployment ratio is almost 20 times as large as the standard

[18]This is in sharp contrast with the previous literature on self-fulfilling prophecies in which the hypothesis of animal spirits (or sun-spots) leads to a multiplicity of equilibrium paths that converge to a unique stationary solution (e.g. Farmer 1993).

[19]In other words, the Mortensen and Pissarides's (1994) model would lack for an amplification mechanism for real shocks.

deviation of real wage rates over the period under examination. The so-called Shimer's (2005) puzzle arises from the fact that the matching model a là Mortensen and Pissarides (1994) in which real wage rates are the outcome of a generalized Nash bargaining predicts that those two variables should have nearly the same volatility.[20]

		$\ln U$	$\ln(\text{vacancies})$	$\ln\left(\frac{\text{vacancies}}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.190	0.202	0.382	0.020
Quarterly autocorrelation		0.936	0.940	0.941	0.878
Correlation matrix	$\ln U$	1	-0.894	-0.971	-0.408
	$\ln(\text{vacancies})$	–	1	0.975	0.364
	$\ln\left(\frac{\text{vacancies}}{U}\right)$	–	–	1	0.396
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1

Table 1: The Shimer's puzzle

An earlier important stream of contributions, such as Hall (2005a-b) and the very Shimer (2005), tried to reconcile the matching model to data by introducing some real wage rigidity in order to generate a stronger amplification of real shocks, i.e., in order to amplify the effects of productivity shocks on labor market tightness indicators. Specifically, the former gives up the Nash bargaining rule by postulating that wage payments follow a wage-norm constrained inside a well-determined bargaining set. By contrast, the latter preserves the Nash rule but it supposes that workers' bargaining power – usually assumed to be fixed – moves counter-cyclically.

In the remainder of this section, I explore the implications of a radically different approach. Specifically, I provide some computational experiments aimed at checking

[20]The intuition for this result is that real wage rates bargained according to the Nash rule absorbs a great deal of productivity shocks. As a consequence, vacancies and unemployment should be only partially affected by the stochastic disturbances that affect the value of produced output.

whether the demand-driven competitive two-sided search economy described in section 2 is able to provide a rationale for the Shimer's (2005) puzzle. Since in the Farmer's (2008a-b, 2010) framework labor instead of output is used to post job vacancies, my indicator of unfilled jobs will be the rate of recruiters employed by profit earners, while the indicator of labor market tightness will be the ratio between recruiters and unemployment. Moreover, my numerical experiments will be carried out by assuming that in each period private consumption and investments include government, net transfers and net exports.[21]

Finally, for the sake of comparability, the volatility of artificial series obtained with the suggested model economy will be computed with the same numerical procedure proposed by Shimer (2005). A short summary of this procedure is given in Appendix.

3.1 Calibration

The model economy is calibrated in order to fit the first-moments of US data. Specifically, I use the set of parameter values collected in table 2.

The parameter values illustrated in table 2 comes from different sources. First, the values of α and δ are calibrated as in Kydland and Prescott (1982). Second, the value of ρ is the quarterly figure provided by Farmer (2008a) for the US economy. Assuming that expectations are the main driver of profit earners' investment expenditure, this autocorrelation value fulfills the Kurz's (2008) Bayesian criterion for beliefs' inference. In fact, according to this criterion, what is actually learnable in a non-stationary economy can be described by a stable process and this explicitly suggests a value of ρ inside the unit circle.[22] Finally, the values of μ, ξ and σ_η are calibrated as in Chang (2000).

[21] The same simplifying assumption can also be found in Farmer and Guo (1995).

[22] In *A Treatise on Probability*, Keynes (1921, Chapter 32, pg. 391) had a similar intuition. In fact, he does explicitly stated: "No one supposes that a good induction can be arrived at merely by counting cases. The business of strengthening the argument chiefly consists in determining whether the alleged association is *stable*, when the accompanying conditions are varied." (italics from the original author).

Parameter	Description	Value
α	*capital share*	0.3000
δ	*capital depreciation*	0.0250
γ	*matching elasticity*	0.9862
μ	*productivity drift*	1.0000
ξ	*productivity persistence*	0.9500
σ_η	*productivity variance*	0.0076
κ	*investment drift*	0.2018
ρ	*investment persistence*	0.5000
σ_ϵ	*investment variance*	0.0030

Table 2: Calibration

The values of the other parameters have been chosen in order to fulfill the following requirements. First, the value of γ delivers a social-optimum unemployment rate around 5.8% a figure consistent with the historical average US unemployment rate. In fact, I computed this figure as a simple historical mean of the seasonally-adjusted US unemployment rate for the people of 16 years old and over provided by the Bureau of Labor Statistics.[23] Second, the value of κ combined with the Farmer's (2008a) value of ρ delivers an expected value of investment expenditure in wage units consistent with the social-optimal unemployment rate derived as described above. Therefore, the implicit hypothesis that drives my simulations is that profit earners' investment expenditure fluctuates around the level that provides the social-optimal unemployment rate implied by the chosen matching elasticity. Finally, the value of σ_ϵ is consistent with the restrictions on the stochastic process for $\widehat{I}_t$ in (24).

Moreover, using US annual data for the period 1960-1995, it is possible to derive a value of ρ around 65%, a figure consistent with the quarterly Farmer's (2008a) figure.

[23]Series identification: LNS14000000 (e.g. www.bls.gov).

3.2 Simulation Results

I begin my computational experiments by allowing only for shocks to the value of aggregate demand in wage units. In other words, using the set of parameters in table 2, I implement the Shimer's (2005) procedure to the artificial series generated by the model economy under the simplifying assumption that it is hit only by demand shocks (i.e. $A_t = A_{t-1} = 1$, for all t and $\sigma_\eta = 0$). The results are collected in table 3 (standard errors in parentheses).

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.047 (0.003)	0.137 (0.008)	0.502 (0.047)	0.014 (0.001)
Quarterly autocorrelation		0.486 (0.060)	0.489 (0.059)	0.489 (0.059)	0.460 (0.068)
Correlation matrix	$\ln U$	1	−0.994 (0.001)	−0.992 (0.004)	−0.974 (0.009)
	$\ln V$	–	1	0.995 (0.004)	0.950 (0.015)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.959 (0.012)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions: $K_0 = 32.331$; $\widehat{I}_0 = 0.403$					

Table 3: Demand shocks

The numerical results for a model economy driven only by the profit earners' animal spirits suggest some interesting but preliminary conclusions. First, the simple theoretical setting in section 2 is able to replicate the ranking of the observed standard deviations of all the involved variables: labor market tightness (real wage) is the most (less) volatile series. Second, the model matches the observed sign of the correlation coefficients of all the variables. Specifically, it is worth noting the negative correlation between unemployment and recruiters (Beveridge curve) and the pro-cyclicity of real wage rates. Third, the model with only demand shocks overstates real wage stickiness: the standard deviation of labor market tightness is about 35 times the standard deviation of real wage rates. Finally, the model tends to understate the volatility of unemployment and the autocorrelation coefficients of all the variables.

I close my computational experiments by augmenting the algorithm in order to allow for demand *and* productivity shocks both generated in order to have orthogonal disturbances (i.e., $E_t[\eta_t \epsilon_t] = 0$, for all t). Using the configuration of parameters in table 2 and implementing the Shimer's (2005) procedure, I derive the numerical results in table 4 (standard errors in parentheses).

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.047 (0.003)	0.137 (0.008)	0.502 (0.048)	0.026 (0.003)
Quarterly autocorrelation		0.485 (0.060)	0.488 (0.060)	0.488 (0.060)	0.854 (0.055)
Correlation matrix	$\ln U$	1	-0.994 (0.001)	-0.992 (0.004)	-0.537 (0.089)
	$\ln V$	–	1	0.995 (0.004)	0.523 (0.088)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.529 (0.088)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions: $K_0 = 32.331$; $\widehat{I}_0 = 0.403$; $A_0 = 1.000$					

Table 4: Demand and supply shocks

The simulations results for a model economy hit simultaneously by demand and supply shocks are straightforward. In the suggested framework productivity shocks affect only the volatility of real wage rates.[24] Therefore, whenever compared to the figure in table 3, the value of the standard deviation of $\left(\frac{w}{p}\right)$ becomes larger and closer to the Shimer's (2005) empirical findings.[25] In fact, taking a significance level of 10%, the hypothesis that

[24]See equation (34).

[25]The slight differences for the volatility and autocorrelation of the other variables with respect to the figures in table 3 are due only to the fact that the simulation were ran with a different seed.

my simulated value is equal to its observed counterpart cannot be rejected.[26] Moreover, the ratio between the volatility of the labor market tightness indicator and real wage rates becomes quite close to the puzzling value indicated in table 1.

3.3 Discussion

An intuitive rationale for my computational results can be given as follows. On the one hand, the inflection point in the equilibrium wage function in (34) is exactly in L_S. Obviously, this means that next to the social optimum level of (un)employment - which implicitly provides the barycentre of my simulations - the trade-off between real wage rates and equilibrium (un)employment is quite flat. On the other hand, the value of the matching elasticity in table 2 implies a steeper trade-off between equilibrium recruiters (vacancies) and unemployment.[27] As a consequence, the difference between the slopes of these two fundamental macroeconomic relationships arising from my model calibration suggests a straightforward explanation for the mechanism of amplification of real shocks needed to reconcile the Farmer's (2008a-b, 2010) micro-founded theoretical framework with the Shimer's (2005) empirical findings.

Before addressing the issue of robustness, it is worth noting that the insights of my simulation results are not completely new. In fact, augmenting the matching model à la Mortensen and Pissarides (1994) with a monopolistically competitive demand side and assuming that the model economy is hit simultaneously by monetary and productivity shocks, Barnichon (2007, 2009) finds that a great deal of the Shimer's (2005) puzzle might be due to a misidentification of the disturbances that drive (un)employment when firms are demand-constrained. In other words, taking into account of demand shocks vis-à-vis supply shocks can help in explaining the relative stability of real wage rates in spite of the large volatility of labor market tightness indicators even without invoking any exogenous

[26] The same also holds for the autocorrelation coefficient of real wage rates.

[27] See equation (15).

real wage rigidity.[28]

3.4 Robustness

In order to have an intuition about the robustness of the model calibration outlined in table 2, I simulate two different versions of model economy characterized by different parameter values but preserving the assumption of simultaneous demand and supply shocks. The first takes into consideration the matching elasticity value suggested for expositional purposes by Farmer (2008b, 2010) (i.e. $\gamma = \frac{1}{2}$) by leaving all the other parameters unaltered. In this way equilibrium unemployment fluctuates around its long-run historical mean while the social optimum level of (un)employment is one-half of the labor force. The second explores the consequences of assuming that the investment expenditure drift is such that the expected equilibrium unemployment is equal to the expected equilibrium rate of recruiters (i.e. $\kappa = 0.204\,7$) leaving all the other parameters unaltered.[29] In this way the social-optimum level of unemployment is consistent with its long-run historical mean while equilibrium unemployment fluctuates around a value of about 4.5%.

The results of those simulations can be briefly summarized as follows. In the first case, the labor market tightness indicator volatility is slightly higher than the volatility of real wage rates and in addition the model economy displays a counter-factual positive correlation between unemployment and real wage rates. In the second case, the model economy displays correlation coefficients with a sign consistent with their observed counterpart but the volatility of the labor market tightness indicator becomes explosive and

[28] In fact, Barnichon (2007) obtains artificial series closer to the Shimer's (2005) empirical findings when he does consider the flexible Nash-bargained real wage rate.

[29] This is a straightforward way to consider the Beveridge's (1944) definition of full employment inside the Farmer's (2008a-b, 2010) demand-driven competitive search framework. In fact, in *Full Employment in a Free Society*, Beveridge (1944) defined full employment as a state of affairs in which the number of vacant jobs (recruiters) is equal to the number of unemployed workers.

not statistically significant.[30]

In conclusion, the simulation results of these two different versions of the model economy suggest that the good quality of the figures in table 4 comes from an investment expenditure that fluctuates around a social optimum level of (un)employment that is fairly distant from the explosive (and inefficient) full employment allocation.

4 Concluding Remarks

This paper aims at providing a stochastic-dynamic matching model with an Old Keynesian flavor and then testing it against recent empirical evidence on macroeconomic fluctuations. Specifically, following the micro-foundation of the *General Theory* provided by Farmer (2008a-b, 2010), I build a competitive two-sided search model in which output and employment are driven by effective demand, the price level is perfectly flexible and the nominal wage rate is chosen as numeraire. Moreover, an additional Old Keynesian feature that I append to the Farmer's (2008a-b, 2010) proposal is the assumption that agents are divided into two income categories with different propensity to consume and different tasks: wage and profit earners (e.g. Kaldor 1955-1956). Wage earners are assumed to consume the whole wage income raised by supplying a fixed amount of labor services. By contrast, profit earners are assumed to save the whole income raised by arranging the production of goods in order to finance an investment expenditure driven by an autonomous stochastic process such as those exploited for TFP in RBC models.

This simple setting with stochastic dynamics and matching allow to derive in a straightforward manner all the properties of a continuum of demand constrained equilibria for a model economy with savings and productive investments. Furthermore, I emphasize that this framework might be consistent with a certain degree of endogenous real wage stickiness next to the social-optimal allocation.

[30]The complete results of those simulations are available from the author.

Finally, I show that under the assumption that the economy is hit simultaneously by demand and supply shocks, this demand-driven competitive two-sided search model can provide a rationale for the Shimer's (2005) puzzle. Specifically, calibrating and simulating the model economy in order to fit the US first-moments data, I show that the suggested theoretical framework can well be consistent with a degree of wage stickiness that easily rationalizes the relative stability of real wage rates in spite of the large volatility of labor market tightness indicators. In fact, I show that the real wage stickiness required to resolve the Shimer's (2005) puzzle can endogenously arise from the combination of the Farmer's (2008a-b, 2010) demand-driven competitive search framework with a quite conventional calibration of the model economy.

5 Appendixes

In this section, I provide a short description of the Shimer's (2005) numerical procedure implemented in section 3. Moreover, I explore the macroeconomic consequences of constant nominal wage rates for the relationship between inflation and unemployment. Finally, I test the implication for the cyclical volatility of real wage rates and labor market tightness of two additional hypotheses for the evolution of the investment expenditure measured in wage units, i.e., the accelerator and the chaotic dynamics.

5.1 Shimer's Procedure

The numerical procedure proposed by Shimer (2005) to assess the volatility of unemployment, vacancies, labor market tightness and real wages runs as follows. First, generate $1,212$ observations for each of the involved variables, i.e., unemployment, recruiters, labor market tightness and real wage rates. Second, throw away the first $1,000$ observations for each simulated series in order to have 212 data points which correspond to data for the 53 quarters from 1951 to 2003. Third, take the log of each series and compute their autocorrelation. Forth, detrend the log of the model-generated data by using the Hodrick-Prescott (HP) filter with a smoothing parameter set at $100,000$. Fifth, compute the deviations of all the artificial series from their respective HP trend and take the respective standard errors and correlation. Sixth, repeat the previous steps for $10,000$ times. Finally, take the mean and the standard deviation of all the variables obtained in the third and the fifth step.[31]

5.2 Phillips Curve

Following the choice of units made by Keynes (1936) in the *General Theory* (Chapter 4, pg. 41) and resumed by Farmer (2008a-b, 2010), the theoretical model in section 2 is developed by using the nominal wage rate as numeraire. An almost "natural" simplification is to consider the possibility that the nominal wage is completely fixed and assume that all the adjustments to real wage rates occur through changes in the price level. Hence,

$$w_t = \overline{w} \text{ for all } t \tag{A.1}$$

where $\overline{w}$ is a positive constant.

Given (A.1), the Farmer's (2008a-b, 2010) theoretical framework can be used to test the soundness of another fundamental macroeconomic relationship, i.e., the Phillips curve. As it is well-known, the most common representation of a Phillips curve is a relationship between inflation and the unemployment rate such as

$$\frac{p_t - p_{t-1}}{p_{t-1}} = f(U_t) + \chi_t \tag{A.2}$$

where χ_t is an erratic component while $f(\cdot)$ is a given function.

[31] The MAT LAB$^{\text{TM}}$ 6.5 code that implements this procedure to the theoretical model developed in section 2 is available from the author.

Using the set of parameters in table 2, I derive a non-parametric estimation of $f(\cdot)$ obtained with a normal kernel.[32] Retrieving artificial data from a typical replication a là Shimer (2005) obtained with demand and supply shocks, I derive the results illustrated in figure A.1.

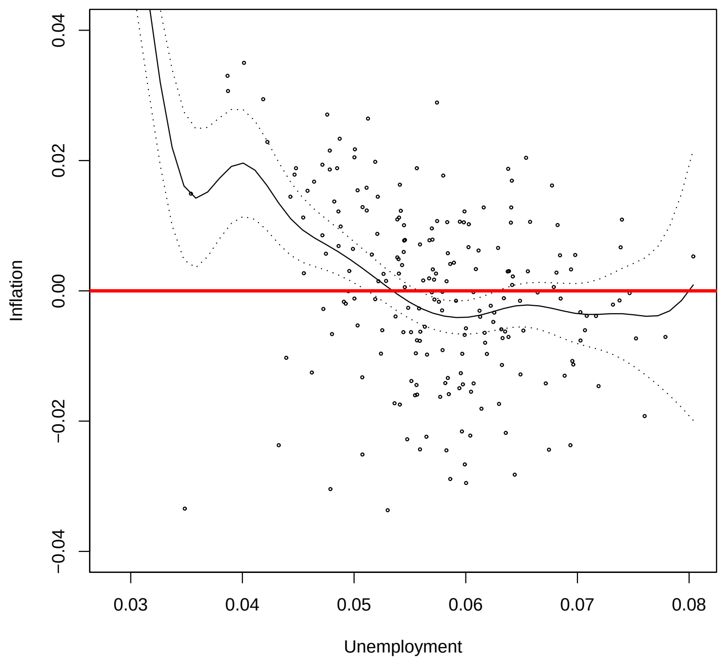

Figure A.1: Phillips curve

The non-parametric regression in figure A.1 shows a Phillips curve with a quite conventional shape, i.e., the unemployment and the inflation rate are linked by a weak decreasing relationship. Moreover, it is worth noting that given the value of the matching elasticity, the social-optimal unemployment rate corresponds to a nearly null inflation rate. As a consequence, under fixed nominal wage rates U_S is a fair approximation of the NAIRU.

[32] The estimation is carried out with R 2.4.0 by exploiting the statistical package *sm* (e.g. Bowman and Azzalini 1997). Moreover, the kernel bandwidth is obtained by assuming the normality of the probability density function of the regressor.

5.3 Investment Accelerator

Following Farmer (2008a), the theoretical model in section 2 is developed by assuming that the profit earners' investment expenditure measured in wage units is driven by a stochastic AR(1) process. An intriguing alternative could be the acceleration business cycle hypothesis pioneered by Clark (1917) and recently revisited by Puu et al. (2005). A convenient specification is the following:

$$\widehat{I}_t = d\left(Y_{t-1} - Y_{t-2}\right) + \widehat{I}_a \tag{B.1}$$

where d is a positive constant while $\widehat{I}_a$ is the autonomous investment expenditure in wage units.

The accelerationist evolution law for the investment expenditure in (B.1) suggests that in each period $\widehat{I}_t$ is assumed to be proportional to the lagged change in output plus an autonomous component. As a consequence, assuming that expectations are the main driver of the investment expenditure, this means that profit earners' beliefs improve (get worse) whenever they observed an increase (decrease) in output.

In the remainder of this sub-section, I provide a computational experiment aimed at checking the dynamic properties of the model economy described in section 2 under the hypothesis that investment expenditure follows the accelerator principle. Specifically, I implement the Shimer's (2005) procedure to the artificial series generated by the model economy under the assumption that $\widehat{I}_t$ follows the second-order process in (B.1) while TFP is stochastic.

The process in (B.1) is calibrated as follows. First, in order to have an investment expenditure that fluctuates around its social optimum level I set $\widehat{I}_a = \frac{\alpha}{1-\alpha} L_S$. Thereafter, I calibrate the acceleration parameter in order to avoid explosive dynamics. Specifically, I set $d = 0.10$. Using the remainder of the parameters' configuration in table 2 and implementing the Shimer's (2005) procedure, I derive the numerical results in table B.1

(standard errors in parenthesis).

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.029 (0.001)	0.087 (0.004)	0.310 (0.017)	0.021 (0.003)
Quarterly autocorrelation		−0.082 (0.071)	−0.076 (0.072)	−0.077 (0.072)	0.862 (0.063)
Correlation matrix	$\ln U$	1	−0.998 (0.000)	−0.998 (0.000)	−0.143 (0.050)
	$\ln V$	–	1	0.999 (0.000)	0.141 (0.049)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.141 (0.050)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions:	$K_0 = 32.331$; $K_1 = 32.328$	$\widehat{I}_0 = 0.403$; $\widehat{I}_1 = 0.404$	$A_0 = 1.000$; $A_1 = 1.001$		

Table B.1: Investment accelerator and supply shocks

The numerical results for a model economy with investment accelerator and stochastic productivity shocks suggest the following conclusions. First, the suggested setting replicates the ranking of the observed standard deviations of all the variables: labor market tightness (real wage) is the most (less) volatile series. Second, the model matches the actual sign of the correlation coefficients of all the variables. Again, it is worth remarking the negative correlation between unemployment and recruiters (Beveridge curve) and the pro-cyclicity of real wage rates. Third, the model with investment accelerator understates the volatility of unemployment and recruiters but provides standard deviations for labor market tightness and real wage rates quite close to their observed counterpart. Finally, expect for real wage rates, the model predicts counter-factual negative autocorrelation coefficients though not statistically significant.

5.4 Chaotic Investments

The last option for the evolution of the investment expenditure in wage units that I test is a first-order quadratic deterministic process such as

$$\widehat{I}_t = a\widehat{I}_{t-1}\left(b - \widehat{I}_{t-1}\right) \tag{C.1}$$

where a and b are positive constants.

As suggested in an influential paper by May (1976), the process in (C.1) is suited to describe phenomena with the tendency to growth (shrink) at low (high) levels of $\widehat{I}_t$. Therefore, assuming that expectations are the main driver of the investment expenditure, this would mean that profit earners have the self-fulfilling belief that periods of "bull" markets are necessarily followed by periods of "bear" markets and vice versa. Moreover, it is well possible to find pairs $\{a, b\}$ such that $\widehat{I}_t$ follows a very complicated (chaotic) dynamics.[33]

In the remainder of this sub-section, I provide a computational experiment aimed at checking the dynamic properties of the model economy described in section 2 under the hypothesis that investment expenditure follows a chaotic process. Specifically, I implement the Shimer's (2005) procedure to the artificial series generated by the suggested model economy under the assumption that $\widehat{I}_t$ follows the deterministic quadratic process in (C.1) parameterized in order to display a chaotic dynamics while TFP is stochastic.

A pair $\{a, b\}$ such that the process in (C.1) evolves according to a chaotic dynamics can be found as follows. First, exploiting the result in (28), b is set at the level $\frac{\alpha}{1-\alpha}$. In this way, $\widehat{I}_t$ is automatically constrained in its meaningful interval. Second, given the value of α in table 2 and the value of b fixed in such a manner, I find a value of a that delivers deterministic fluctuations by drawing the corresponding bifurcation diagram.[34]

[33] An interesting application of chaos theory to investments and economics is provided by Peters (1994).

[34] This can be easily done by using, for example, the computational package E&F CHAOS (e.g. Diks et al. 2008).

See figure C.1.

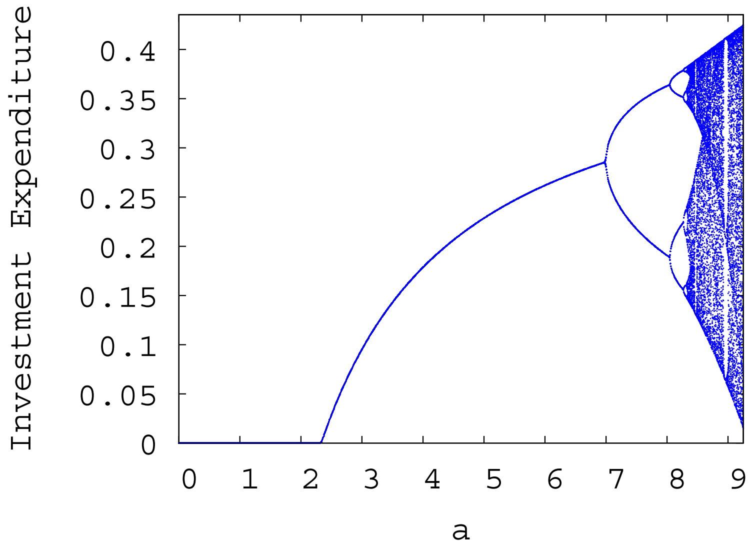

Figure C.1: Bifurcation diagram

The bifurcation diagram in figure C.1 shows that, for example, $a = 8.4$ is a value that delivers a-periodic deterministic oscillations.[35] Using the remainder of the parameters' configuration in table 2 and implementing the Shimer's (2005) procedure, I derive the numerical results in table C.1 (standard errors in parentheses).

The numerical results for a model economy with chaotic investments and stochastic productivity shocks suggest the following conclusions. First, all the uncertainty of the artificial series rely in the real wage rates and their covariance with the other involved variables. Obviously, this is due to the fact that unemployment and recruiters are driven by a deterministic process while real wage rates are affected by the stochastic AR(1) process for TFP. Second, the suggested setting replicates the ranking of the observed standard deviations of all the variables: labor market tightness (real wage) is the most

[35] According to the proposed calibration, the maximum of the non-linear map for $\widehat{I}_t$ is found at the 90% of $\frac{\alpha}{1-\alpha}$.

(less) volatile series. Third, the model matches the actual sign of the correlation coefficients of all the variables. Again, it is worth noting the negative correlation between unemployment and recruiters (Beveridge curve) and the pro-cyclicity of real wage rates. Fourth, the model with chaotic investments tends to overstate the volatility of all the variables but it understates the relative volatility of labor market tightness with respect to real wage rates. Finally, the model predicts counter-factual negative autocorrelation coefficients for all the variables.

		$\ln U$	$\ln V$	$\ln\left(\frac{V}{U}\right)$	$\ln\left(\frac{w}{p}\right)$
Standard deviation		0.564 (0.000)	0.751 (0.000)	0.798 (0.000)	0.159 (0.003)
Quarterly autocorrelation		−0.920 (0.000)	−0.843 (0.000)	−0.847 (0.000)	−0.769 (0.032)
Correlation matrix	$\ln U$	1	−0.961 (0.000)	−0.963 (0.000)	−0.952 (0.002)
	$\ln V$	–	1	0.999 (0.000)	0.991 (0.002)
	$\ln\left(\frac{V}{U}\right)$	–	–	1	0.991 (0.002)
	$\ln\left(\frac{w}{p}\right)$	–	–	–	1
Initial conditions: $K_0 = 30.345$; $\widehat{I}_0 = 0.3736$; $A_0 = 1.000$					

Table C.1: Chaotic investments and supply shocks

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