Financial Constraints and Firm Dynamics *

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Abstract

This paper analyzes the effect of financial constraints (FCs) on firms' dynamics. We measure FCs with an official credit rating which captures availability and cost of external resources. We find that FCs undermine firm growth, induce anti-correlation in growth shocks and reduce the dependence of growth volatility on size. FCs also associate with asymmetries in growth rates distributions, preventing potentially fast growing firms from seizing attractive growth opportunities, and further deteriorating the growth prospects of already slow growing firms. The sub-diffusive nature of the growth process of constrained firms is compatible with the distinctive properties of their size distribution.

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1 Introduction

Firms' ability to access external financial resources is well known to represents a factor that may influence several dimensions of firm dynamics, as the links between financial and operational activities of firms involve many types of investment decisions, pertaining to, for instance, entry and survival in a market, job creation and destruction, innovative activity, or internationalization patterns.

Within the vast body of literature studying the many possible relationships between finance and firms' dynamics, a well developed tradition of empirical studies seek to identify the effect of financing problems on the size-growth trajectories of firms (for reviews, see Whited, 2006; Fagiolo and Luzzi, 2006; Oliveira and Fortunato, 2006). A first major issue in this identification rests in the intrinsically difficult task of measuring financial constraints (FCs). Even when data allow to assess whether banks or other financial institutions refuse a loan or charge particularly high interest rates to a given firm, the observation of these factual occurrences may not be enough. Indeed financial or any other type of constraints are likely to be anticipated, and therefore likely to affect the behaviour of firms mainly through managerial counter-factual considerations. Missing requests for new loans can follow from an actual lack of necessity to obtain external finance, as much as they can be ascribed to anticipated refusals, with consequent exclusive reliance of the firm on internal resources.

In facing this difficulty, researchers have followed different approaches to distinguish constrained from unconstrained firms. The debate about which particular measure to use, originated in the literature on financing constraints to firms' investment (Fazzari et al., 1988; Kaplan and Zingales, 2000), is still ongoing. A first strategy is to start from "hard" data, i.e. from standard business register datasets, and then sort firms into constrained or unconstrained groups according to the relative ranking in the cross-sectional distribution of some variable which is ex ante supposed to strongly correlate with the need, availability and cost of external finance. Examples include age, size, cash flow, leverage, availability of collateral, interest coverage, payout ratio, and cash flow sensitivity of cash. Some authors suggest instead to use classifications based on the construction of multivariate index measures of FCs. The advantage is that these indexes summarize several aspects of firm financial structure into a single indicator, and allow to capture different degrees of FCs, avoiding a simple binary categorization (Kaplan and Zingales, 1997; Whited and Wu, 2006; Musso and Schiavo, 2008).

The most common alternative consists of classifications based on survey data. Surveys typically involve managers or entrepreneurs, who are asked to make a self-assessment about whether firms have been rationed or not, about whether the cost and the amount of granted loans were in line with the expectations and needs, and, more generally, about the difficulties they have faced in accessing financing from banks or other institutions (Winker, 1999; Angelini and Generale, 2008; Campello et al., 2010).

None of the proposed approaches are without their pitfalls. On the one hand, proxies based on hard data, both univariate or multivariate, inevitably give an indirect measure of FCs, as the implicit assumption is that the poor records of firms with respect to the chosen variables get translated into banks' or investors' unwillingness to grant credit. This assumption appears particularly problematic when the analysis is exclusively based on exogenous variables, like age, or structural and extremely persistent variables, like availability of collateral. On the other hand, survey based measures, which are seemingly closer to answer the question whether a firm has actually been constrained or not, are well known to suffer from misreporting and sample selection bias, whose effect is difficult to quantify. Moreover, by collecting the opinion of the credit seeker about their own financing conditions, survey data look at the demand side of credit relations. Given the strong informational asymmetries characterizing capital markets, however, it is the opinion of the credit supplier on the credit seeker that, more plausibly, determines access to finance.

In this paper we follow a different approach: we proxy FCs through a credit rating measure. Credit ratings, similarly to multivariate indexes of FCs derived from hard data, are built to account for wide range of potential sources of financial problems, also including qualitative factors. In our opinion, the key advantage of credit ratings is that, by their very nature, they account for the "opinion [of credit suppliers] on the future obligor's capacity to meet its financial obligations" (Crouhy et al., 2001, p.51). They thus reflect financial markets' evaluation of the credit quality of a firm (Whited, 1992; Almeida et al., 2004), this way getting closer to measure whether external finance is actually an option for a particular firm. Indeed, the use of ratings meet those requirements identified as desirable for a measure of financial constraints that have motivated the introduction of multivariate indexes of FCs (Cleary, 1999; Lamont et al., 2001). First, they result from a multivariate score, thus summarizing a wide range of dimensions of firm performance. Second, they are updated in every year, thus allowing for the identification of time effects. Third, the graduation of scores attributed by credit ratings to the different firms allow to distinguish among different degrees of difficulty in accessing external funds, and thus does not force the researcher to work with a binary categorization of constrained versus non-constrained firms. An additional desirable feature of credit ratings is related to the external nature of the assessment provided by ratings. Assessing the overall risk of a firm through balance sheets variables indeed requires to control for the availability of business opportunity, in order to avoid the endogeneity generated by the co-determination of debt and business expansion. The traditional way of doing this is to condition the analysis on variable based on a measure of equity, like Tobin's Q. In general, however, measures of equity are very difficult to obtain, especially when the price of the ownership is not available. In practice, the difficulty of measuring equity limits the applicability of this strategy only to publicly traded firms. Conversely, credit ratings by definition balance between financial exposure and business opportunities. They do so by summarizing a large number of quantitative and qualitative indicators, and thus result in a more reliable and plausibly direct measure. In particular, we use an official credit rating issued by an independent institution and available for all the firms in the dataset. The official source, the high reliability and the widespread use of this specific rating strongly support its role as an actual benchmark for the lending decisions of banks and financial institutions.

Besides the contrasting views on the identification of valid measures of FCs, the major limitation of the existing studies rests in the very limited scope of the proposed empirical analysis. The common approach is to check the significance of the selected FC measure in a standard firm growth regression, either by directly including the FC proxy among the regressors, or by modeling FCs as dummy variables indicating that a firm belongs to some specific FC category. The generally accepted finding is that FCs negatively affect the growth prospects of firms, and that this effect is stronger for younger and smaller firms (see Angelini and Generale, 2008).[1] This kind of specifications, however, can only identify location-shift effects in the conditional distribution of growth rates, which is accounted for by a statistically significant correlation of average growth rates with the FC proxy, or by observed deviations in expected growth rates between the classes of constrained and unconstrained

[1]These findings are in line with the recent theoretical literature on financing and growth models (see Cooley and Quadrini, 2001; Albuquerque and Hopenhayn, 2004; Clementi and Hopenhayn, 2006), largely based on the models of industrial dynamics in Jovanovic (1982) and Hopenhayn (1992).

firms. In spite of the general agreement that FCs hamper firm growth, there is no reason why this should exclusively translate into a negative shift of the average growth rate. In fact, various pieces of evidence make this shift hypothesis appear rather simplistic. Firstly, the overwhelming evidence that FC problems affect several dimensions of firms' strategies, such as investment/divestment in fixed capital (Fazzari et al., 1988; Devereux and Schiantarelli, 1990; Bond et al., 2003) and in working capital (Fazzari and Petersen, 1993), wages (Michelacci and Quadrini, 2009), cash management policies (Campello et al., 2010), inventory demand (Kashyap et al., 1994), or R&D and innovation strategies (Hall, 2002; Brown et al., 2009), clearly supports that the role played by FCs is complex and structured. Secondly, recent qualitative evidence on firms' reactions to the current financial crisis (see Campello et al., 2010) suggests that firms might undertake heterogeneous responses to FC problems: some firms react by abandoning investment projects, despite their perceived potential, while other firms, especially those which are already experiencing poor growth dynamics, tend to display a much higher propensity to sell off productive assets as a way to generate funds. Complex interactions within firms together with potential heterogeneous responses across firms advance the conjecture that a primarily, and yet unexplored effect of FCs is to induce differences in different quantiles of the (conditional) growth rates distribution.

To account for the many possible channels through which FCs can affect firm growth, in this paper we extend the usual autoregressive growth model and focus on the distributional properties of firm growth rates. We introduce a parametric specification of the heteroskedasticity of growth rates and we allow for asymmetries in growth shocks across firms subject to different strength of FCs. The first extension is motivated by the robust empirical observation that smaller firms experience more volatile growth patterns (among others, see Hymer and Pashigian, 1962; Amaral et al., 1997; Bottazzi and Secchi, 2005). Such heteroskedasticity is typically viewed as a factor to wash away in obtaining consistent estimates (Hall, 1987; Evans, 1987; Dunne et al., 1988). Conversely, we consider it as part of the phenomenon under study, and we want to understand if FCs have a role in explaining the relationship between volatility of growth and size. Our second extension, that is the assessment of possible asymmetries, is pursued by investigating the extent to which FCs affect the overall shape of growth rates distribution, a topic so far largely neglected (see Fagiolo and Luzzi, 2006, for the only exception we are aware of).

Our framework and empirical analysis enable to bridge the short run distributional properties of firm growth dynamics with the observed differences in the long run behaviour of the firm size distribution (FSD) of constrained and non-constrained firms. Exploring such differences in FSD is of recent interest and the evidence is controversial. Cabral and Mata (2003) found that the evolution of the FSD is determined by firms ceasing to be financially constrained, while Fagiolo and Luzzi (2006) and Angelini and Generale (2008) concluded that FCs are not the main determinant of FSD evolution. At least part of the explanation for such seemingly contrasting evidence may come from the different proxies of FCs employed in these studies. Cabral and Mata (2003) measure FCs with age, assuming that younger firms are more constrained, while Fagiolo and Luzzi (2006) and Angelini and Generale (2008) adopt reported cash flow and survey-based measure of FCs, respectively.

Using a large panel of Italian manufacturing firms, our analysis reveals that FCs do affect the process of firm growth through multiple channels. In the short term, FCs reduce the average firm growth rate, induce anti-correlation in growth shocks and reduce the strength of the dependence of volatility of growth rates on size. In addition, we also find asymmetric effect on growth rates distribution. On the one hand, FCs prevent attractive growth opportunities from being seized by constrained but yet potentially fast growing firms. This effect is particularly strong for younger firms. At the same time, and especially among older firms, FCs tend to be associated with a further depression in the growth prospects of already slow-growing firms. These effects are consistent with the distinctive features of the size distribution of more severely constrained firms obtained through a snapshot analysis on cross-sectional data.

The paper is organized as follows. Section 2 describes the data, introduces our measure of FCs and provides a first descriptive account of the relevance of the FC phenomenon. Section 3 analyzes the role of FCs in affecting the age profile of the FSD. In Section 4 we develop our baseline framework and derive the hypotheses guiding our empirical investigations and the interpretation of results. Section 5 presents the main results of our analysis of FC effects on the patterns of firm growth, also investigating the effects of FCs on the firm growth rates distribution. Section 6 tests the robustness of the findings with respect to a set of potentially relevant determinants of size-growth dynamics and firms' financing decisions. In Section 7 we summarize our findings and conclude.

2 Financing constraints: definition and basic facts

We employ a large database of Italian firms maintained by the Italian Company Account Data Service (Centrale dei Bilanci, CeBi-CERVED). CeBi was founded as a joint agency of the Bank of Italy and the Italian Banking Association in the early 1980s to assist in supervising risk exposure of the Italian banking system. Today it is a private company owned by major Italian banks, which continue to exploit its services in gathering and sharing information about firms. The long term institutional role of CeBi ensures high levels of data reliability, substantially limiting measurement errors. The dataset is of a business register type, collecting annual reports for virtually all *limited liability* firms. Available for the present study is an unbalanced panel of $161,297$ firms active in manufacturing over the period 2000-2003. Considering this sector, our data account for about 45% of total employment and about 65% of aggregate value added over the years of observation.[2] Moreover, the data replicate pretty well the distributional profile of firm size in the overall population of Italian manufacturing.[3] For each firm, we were able to access a subset of the original list of variables included in the dataset. We compute age from year of foundation, and proxy firm size through real Total Sales. The preference for Total Sales over Number of Employees is because in our data, due to the Italian accounting rules, employment figures are reported in the notes accompanying financial statements, and are therefore likely to be affected by less reliable updates. For small firms especially, a mistake of even few units of personnel in employment reports may produce a huge error in the measurement of employment growth rates.

As a measure of FCs we adopt the credit rating index that CeBi produces for all the firms included in the dataset. In general, CeBi ratings share with ratings issued by international agencies the advantage to incorporate capital markets' and investors' views on whether access to external finance is worth to a firm or not. In addition, CeBi ratings possess several desirable feature when compared with Moody's or Standard & Poor's. First, they give an assessment of the overall quality of a firm, rather than judge the quality of a single liability, i.e a specific bond or commercial paper

[2]These shares are computed with respect to National Accounts data by sector of activity, as reported by Eurostat. Pistaferri et al. (2010) report similar figures. They also report that the CeBi database contains approximately the 7% of all Italian manufacturing firms.

[3]For 2003, the annual report of the Italian Statistical Office (ISTAT, 2005) provides the following distribution: 82% of firms has less then 10 employees; 15% has 10-to-49 employees; 2% has 50-to-249 employees, and 1% has more than 250 employees. In our data there is a very mild overrepresentation of medium-larger firms: 78% of firms has less then 10 employees; 13% is in the 10-249 size class; 8% has 50-to-249 employees and 1% has more than 250 employees.

of a company. Second, they are available for all the firms included in the dataset, while credit files from international rating institutions bias the scope of analysis towards a much less representative sub-sample of firms. Third, the CEBI index is perceived as an official rating, due to the long lasting relationship of CEBI with the Italian banking and credit systems. This consideration motivates the heavy reliance of banks on CEBI ratings, and the tight link between the CEBI index and the availability and cost of external finance: the probability that a firm with poor CeBi rating receives credit is hugely reduced, if not zero. This is strongly supported by the empirical evidence in Pistaferri et al. (2010). Moreover (Panetta et al., 2009) show that a bad CeBi rating has a strong association with higher cost of credit.

The CeBi index is a score ranking firms in 9 categories of creditworthiness: 1-high reliability, 2-reliability, 3-ample solvency, 4-solvency, 5-vulnerability, 6-high vulnerability, 7-risk, 8-high risk, and 9-extremely high risk. The ranking is purely ordinal. We define three classes of firms subject to different degrees of financial constraints: Non Financially Constrained (NFC), Mildly Financially Constrained (MFC) and Highly Financially Constrained (HFC), corresponding respectively to firms rated from 1 to 4, 5 to 7, and 8 to 9. Since the CeBi index is updated at the end of each year, it is the rating in $t-1$ that is relevant for credit suppliers when they have to decide whether to provide credit in year t. Therefore, the assignment to the three classes is based on one-period lagged values of the rating. Together, this choice also reduces the simultaneity issue potentially arising in regression analysis.[4]

Table 1 shows descriptive statistics.[5] According to our definition financing problems appear to represent a significant phenomenon: about 10% of the whole sample is affected by severe difficulties in raising external resources (i.e. are HFC firms), while almost half of our sample (47%, cfr. the MFC class) faces less severe, but still significant problems. This is in partial contrast with a result

[4]In order to check the sensitivity of our results to the adopted classification, we also considered three alternative assignments. First, we divided firms according to their ratings in $t-2$, accounting for the possibility that ratings are available with some delay to banks and financial institutions. In the second procedure, firms were assigned to FC classes on the basis of the worst rating displayed over the sample period. Finally, we restricted the analysis to firms that did never change their financial status over the whole time window (i.e., based on their ratings in the different years, they always fell in the same FC class). Our main conclusions were not affected by the choice of the assignment procedure, though. All the results are available upon request.

[5]Nominal sales are deflated via 3-digit sectoral production price indexes made available by the Italian Statistical office, base year 2000. A basic cleaning procedure to remove a few outlying observations is applied (see the appendix for details). Reported results refer to pooled data over 2001-2003, as one year is lost due to use of 1-year lagged FC status.

Table 1: FINANCIAL CONSTRAINTS BY AGE CLASSES

	Whole Sample		Non Financially Constrained		Mildly Financially Constrained		Highly Financially constrained	
Firm's age (years)	Number of obs.	Size: mean (median)	Number of obs. (percentage of age class)	Size: mean (median)	Number of obs. (percentage of age class)	Size: mean (median)	Number of obs. (percentage of age class)	Size: mean (median)
0-4	54,171	1.683	15,412	1.637	27,923	1.903	10,836	1.184
		(0.523)	(28.5)	(0.432)	(51.5)	(0.653)	(20.0)	(0.370)
5-10	57,485	2.945	23,780	3.705	35,215	3.032	8,070	1.539
		(0.807)	(41.4)	(0.822)	(61.3)	(0.981)	(14.0)	(0.408)
11-20	80,643	6.193	37,821	7.089	36,783	5.703	6,039	3.560
		(1.519)	(46.9)	(1.637)	(45.6)	(1.647)	(7.5)	(0.500)
21-30	45,128	8.776	24,405	9.536	18,527	8.464	2,196	2.967
		(2.549)	(54.1)	(2.663)	(41.1)	(2.756)	(4.9)	(0.598)
31-∞	25,989	21.592	14371	22.090	10,131	18.930	1,487	34.911
		(3.937)	(55.3)	(4.359)	(39.0)	(4.117)	(5.7)	(1.239)
Total	272,996	6.429	115,789	8.046	128,579	5.586	28,628	3.674
		(1.226)	(42.4)	(1.485)	(47.1)	(1.305)	(10.5)	(0.436)

Size as real sales, millions of euro - Pooled data over 2001-2003.

reported in Angelini and Generale (2008) on a smaller Italian dataset. Secondly, confirming a robust finding in the literature, FCs seem more relevant among young and small firms: almost 20% of young firms are HFC, and, moreover, the median size of HFC firms is, in all age classes, almost one third smaller as compared to the other FC classes. However, FCs are pervasive, affecting firms of different sizes and ages: among older firms, about 6% are in the HFC class, and their mean size is comparable with the mean size of similarly aged firms in the other two FC classes.[6]

3 Financing constraints and age profile of the firm size distribution

Figure 1 reports kernel estimates of the empirical density of real sales by age.[7] Results broadly confirm the basic stylized facts observed in previous studies, where size is proxied with employment: the FSD is right-skewed and both the mode and the width of the distribution increase with age. This visual impression is confirmed by a Fligner-Policello test for stochastic dominance. The FSD of older firms dominates those of younger firms, meaning that a firm randomly drawn from the group of older firms is, with a probability significantly higher than 50%, bigger than a firm randomly extracted from the group of younger firms.[8]

However, from the graphical analysis alone it is difficult to provide a precise statement on the validity of a second common piece of evidence reported in the literature, i.e. that the degree of FSD skewness diminishes with age. Available studies tend to agree on this point, although Angelini and Generale (2008) report that the FSD appears to be more symmetric when using sales, instead of number of employees. To provide a quantitative assessment of this issue, we consider the Asymmetric Exponential Power (AEP) distribution. This family copes with asymmetries and leptokurtosis, at the

[6]The very high mean found within HFC old firms, $34,911$, is explained by the presence of a quite large firm (actually the largest in the dataset) which is old and HFC over the sample period. The mean size falls to $14,257$ if we exclude this single firm from the sample.

[7]Here as well as throughout the work, estimates of densities are obtained using the Epanenchnikov kernel with the bandwidth set using the optimal rule described in Silverman (1986).

[8]This test is presented in Fligner and Policello (1981) and can be interpreted as a test of stochastic dominance in the case of asymmetric samples. A pair-wise comparison of the distribution in Figure 1 confirms significant differences, with very small p-scores (less than 10^{-6} in all cases).

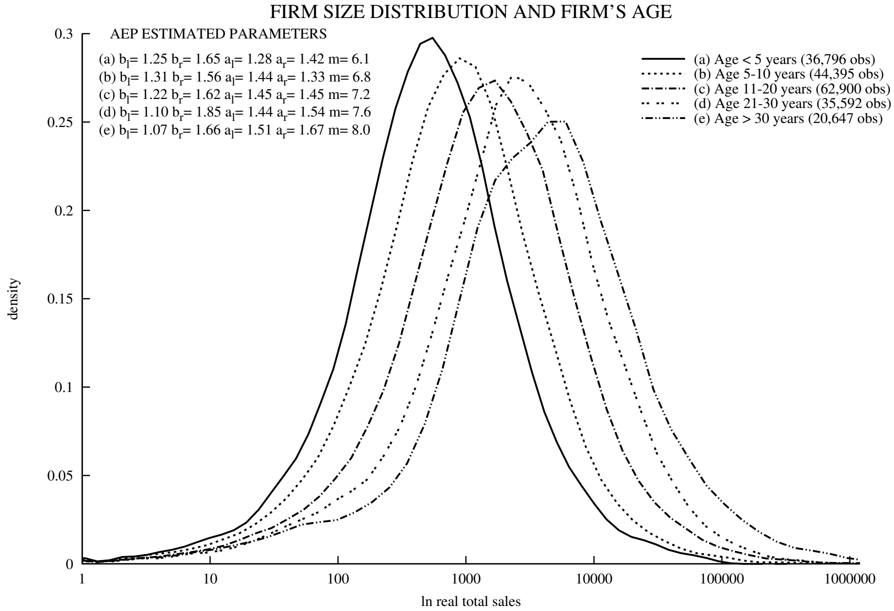

Figure 1: FSD and Age. Pooled data over 2001-2003.

same time allowing for a continuous variation from non-normality to normality. The AEP density

$$f_{\mathrm{AEP}}(x;\mathbf{p}) = \frac{1}{C}\ e^{-\left(\frac{1}{b_l}\left|\frac{x-m}{a_l}\right|^{b_l}\theta(m-x)+\frac{1}{b_r}\left|\frac{x-m}{a_r}\right|^{b_r}\theta(x-m)\right)}\quad, \tag{1}$$

where $\mathbf{p} = (b_l, b_r, a_l, a_r, m)$, $\theta(x)$ is the Heaviside theta function and $C = a_l A_0(b_l) + a_r A_0(b_r)$ with $A_k(x) = x^{\frac{k+1}{x}-1}\ \Gamma\left(\frac{k+1}{x}\right)$, is characterized by 5 parameters. Two positive shape parameters, b_r and b_l, describe the tail behavior in the upper and lower tail, respectively. Two positive scale parameters, a_r and a_l, are associated with the width of the distribution above and below the modal value, which is captured through the location parameter m.

Maximum Likelihood estimates of the AEP parameters are reported in Figure 1 (corresponding standard errors are always smaller than 0.05). They reveal two different patterns in the degree of FSD skewness, arising respectively in the right- and left-hand side of the distribution. The left tail becomes fatter as age increases (b_l decreases while a_l slightly increases), meaning that among relatively smaller firms size differences are bigger for older firms. Age does not seem to play a relevant role, conversely, in shaping the right-hand side of the distribution (b_r is basically the same in all age classes), apart from a moderate increase of the support (a_r increases with age).[9]

[9]Notice that the Extended Generalized Gamma distribution applied in Cabral and Mata (2003), which possesses only one shape parameter, would not have allowed to independently account for the different behaviors observed in the two

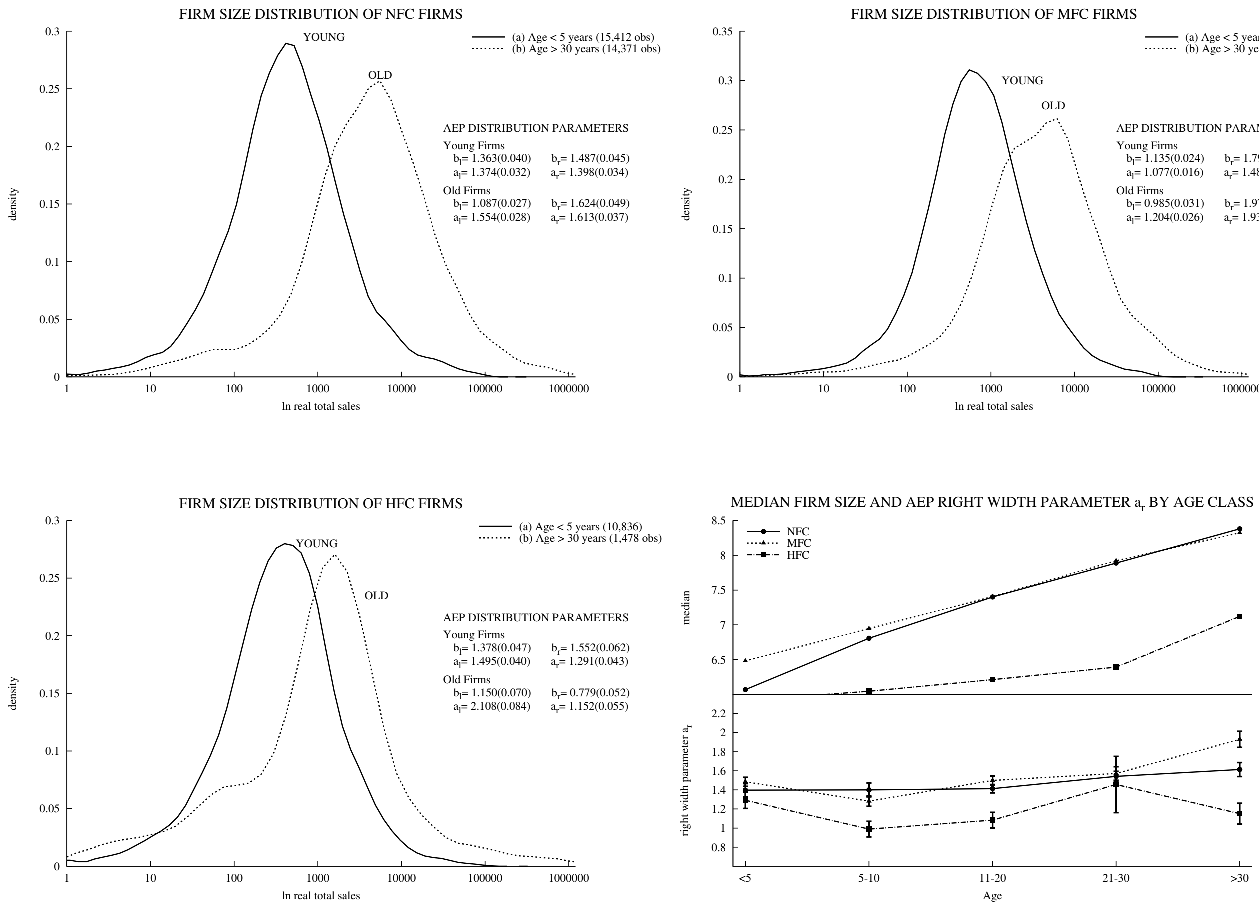

Figure 2: FSD, Age and Financial Constraints. Pooled data over 2001-2003.

We then ask whether disaggregation into FC classes can help explaining the asymmetric effect that age seems to exert on the properties of the FSD. Figure 2 reports kernel estimates of the FSD for firms in the different FC classes, directly comparing young (less than 5 years) and old (more than 30 years) firms in each class.[10]. Since we cannot follow cohorts of firms in our data, a comparison across firms of different age is the only way to have a clue on the relationship between size, age and financial constraints. The results (top left and right, and bottom left panels) suggest that the size-age profile of NFC and MFC firms share similar distributional properties, while the FSD of HFC firms display distinctive features. The difference essentially concerns the intensity of the effect that age exerts on the location and variance of the size distribution. When comparing young and old firms within each class, we observe that the increase in both location and variance induced by firm aging is much milder among HFC firms than in the other two classes. This is confirmed by the

tails.

[10]Other age classes are not reported for the sake of clarity.

results in the bottom-right plot. Here we proxy location and width of the FSD, respectively, with the median size and the estimates of the right width AEP parameter, a_r, and then report how these two indicators vary by age and FC class. Both measures are very similar across all the FC classes when we consider young firms. Then, as age increases, it is possible to identify two diverging trends, one common to NFC and MFC firms, and a second specific to the group of HFC firms. The median size of NFC and MFC firms increases more than tenfold from young to old firms, while the median size of HFC firms increases only by a factor of 5. Similarly, the estimates of a_r reveal that FSD dispersion increases significantly with age for NFC and MFC firms, while the increase is much more modest for HFC firms. The existence of such diverging patterns is also supported by the estimates of b_r, the parameter describing the right tail behavior. Indeed from very similar values for young firms ($\sim$1.5, $\sim$1.8 and $\sim$1.6 for the NFC, MFC and HFC classes respectively), the estimated coefficients diverge when old firms are considered: old NFC and old MFC firms display values of b_r closer to 2, and hence approximately more consistent with a Gaussian distribution, while for old HFC firms the estimated b_r drops from 1.6 to 0.8.

In summary, we observe that, irrespectively of the FC class, the left tail of the size distribution of older firms is fatter than the size distribution of young firms, so that the same age-related effect on the left tail is observed in the aggregate (c.f. Figure 1). Concerning the right tail, when financial constraints are weak we observe a progressive tendency toward the Gaussian shape for older firms, while an opposite tendency emerges in the HFC class, so that the overall effect of age on the aggregate right tail is weak. Moreover, while the distribution of young firms is similar across different FC classes, clear-cut differences appear when older firms are considered. This fact suggests a certain degree of persistence in FC classes.[11] Indeed if the probability of a firm to belong to a FC class at a given age were independent of its past growth process, the FC decomposition of the FSD would not reveal stronger differences among the older firms than among the younger ones. Moreover, the effect exerted by financial conditions seems to extend beyond a simple shift in the mean, as testified by the age profile of the estimated parameters. In the next section we propose a framework designed to capture the different effects plausibly at the basis of the interaction between age, financial constraints

[11]Due to the short time window of our database we cannot directly test the persistence in firm financial conditions over long span of times. The analysis of transition matrix between FC classes reveals a significant persistence. The average 1-year probability to remain in the same class is 81.03% for NFC firms, 75.51% for MFC firms, and 57.88% for HFC firms.

and firm growth, which has been revealed by the snapshot analysis of the size distribution.

4 Analytical Framework

We start from the phenomenological model of industrial dynamics based on the classical work by Gibrat (1931). Let s_t be the logarithm of firm size at time t. The simple integrated process

$$s_t = s_{t-1} + \epsilon_t \tag{2}$$

with *iid* distributed shocks ϵ_t, often referred to as the "Law of Proportionate Effect", has been shown to yield a good first order description of the observed dynamics of firm size (see among others Mansfield, 1962; Kumar, 1985; Hall, 1987). In order to account for the various effects of FCs on firm growth, we consider the following generalized version of the model

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\ s_{t-1} + \sigma_{FC}(s_{t-1})\epsilon_{FC,t}\ . \tag{3}$$

Here λ captures an autoregressive component in the (log) levels of firm size, σ is a function describing the heteroskedastic structure of the process and ϵ are assumed to be independent of size, and we also allow for FC-class dependent parameters and shocks. The inclusion of an AR(1) structure accounts for the fact that smaller firms are often reported to grow faster (see Lotti et al., 2003, for an in-depth review of the empirical literature).[12] The function σ introduces a dependence of the standard deviation of growth shocks on size, which has been reported in a large number of empirical studies. The common finding is that volatility is higher for smaller firms, and that the relationship displays an exponential decrease (see the discussion and references in Bottazzi and Secchi, 2005).

Separate estimation of equation (3) by FC class drops the requirement of orthogonality between an FC proxy explicitly included among the regressors and the distribution of residuals ϵ.[13] At the

[12] The AR(1) specification can be replaced with a more general linear model. For the present discussion the 1-lag structure is sufficient. We checked that the inclusion of further lags does not generate significant modifications in the estimates of λ.

[13] Since our results below confirm the presence of significant differences in the distribution of residuals across the three FC classes, estimation of equation (3) on the entire sample with the direct inclusion of FC dummies is likely to produce biased point estimates.

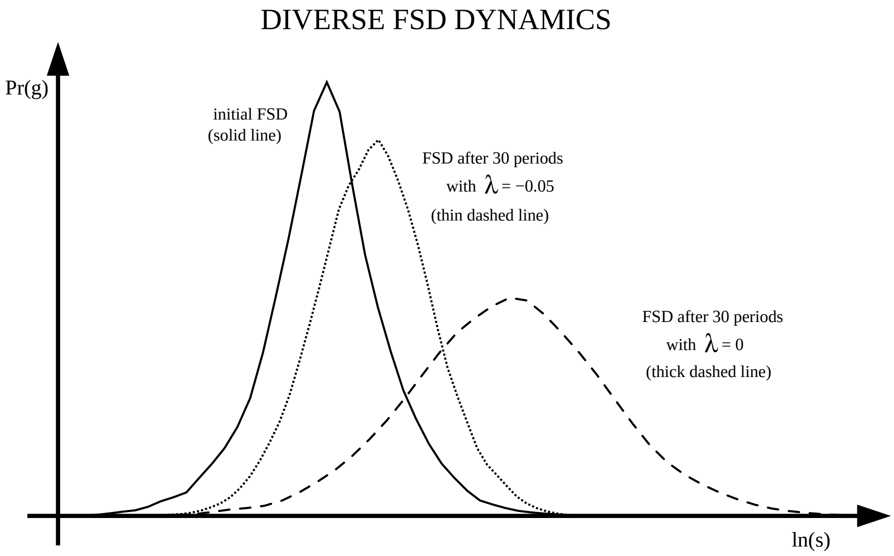

Figure 3: Evolution of the FSD for two different values of the autoregressive coefficient λ in equation (3). In the simulations we consider $15,000$ firms and we set $M_\epsilon = 0.25$ and $V_\epsilon = 0.5$.

same time, by introducing FC class specific coefficients, the framework allows FCs to produce an effect through four different channels: on the drift term c, on the autoregressive term λ, on the heteroskedasticity term $\sigma(s_{t-1})$ and on the properties of the distribution of growth shocks ϵ. Let us outline the economic interpretation of these channels, and the predictions that can be made.

First, differences in c across FC classes provide information on the effect of FCs on the central tendency of the distributions, i.e. on the aforementioned location-shift effects across constrained or non-constrained firms. This is the kind of effect captured by the standard growth regression models traditionally proposed in the literature. Under the plausible conjecture that FCs reduce the set or the amount of growth opportunities seizable by constrained firms, the prediction is that the group of most severely constrained firms has the lowest estimated c.

Second, the coefficient λ is related to the long-term dynamics of the evolution of size. Too see how, let us neglect, for the sake of simplicity, the FC subscript and the heteroskedasticity correction, and let the mean and variance of the size distribution at time t be M_{s_t} and V_{s_t}, respectively. Under the hypothesis of a constant λ, their evolution from $t = 0$ to $t = T$ is given by

$$M_{s_T} = (1+\lambda)^T M_{s_0} + \frac{(1+\lambda)^T - 1}{\lambda} M_\epsilon \ , \quad V_{s_T} = (1+\lambda)^{2T} V_{s_0} + \frac{(1+\lambda)^{2T} - 1}{(1+\lambda)^2 - 1} V_\epsilon$$

where M_ϵ and V_ϵ are the mean and variance of the shocks ϵ.[14] When $\lambda = 0$, as in the benchmark Gibrat's model, we have a diffusion process: the time evolution of s_T follows a unit root process (discrete Brownian motion) asymptotically diverging to a log-normal FSD with variance and mean increasing proportionally to T. Conversely, when $\lambda < 0$ the process is sub-diffusive and the FSD converges in probability to a stationary distribution with finite variance $V_\epsilon/(1-(1+\lambda)^2)$. The empirical analysis in Section 3 shows that the FSD for the HFC class does not display any tendency toward a Gaussian shape when older firms are considered. This suggests that $\lambda < 0$ may be the case for more severely constrained firms. Indeed Figure 3 shows that even small differences in the value of λ can quickly produce significantly different FSD shapes.

Third, differences in σ across FC classes capture an heteroskedasticity effect, reflecting the possibility that FCs also produce changes in the way the variability of growth rates depends on size. The often found reduction of growth rate volatility with size has been interpreted as a portfolio effect (Bottazzi and Secchi, 2005): since larger firms are typically more diversified than small firms (in terms of products, lines of business, plants) they can balance negative and positive shocks hitting their single branches (at least if the various activities are weakly correlated). According to this interpretation, we can conjecture that FCs, by reducing the range of attainable new growth opportunities, also reduce the diversification advantage of bigger firms. We therefore expect to observe weaker heteroskedasticity effects within the group of the most severely constrained firms.

Finally, concerning the possible effects of FC on the empirical distribution of growth shocks, we can sketch some predictions based on the qualitative findings in Campello et al. (2010). In Figure 4 the solid line corresponds to a Laplace distribution of growth shocks (a "tent" on a log-scale). This distribution represents a natural benchmark, because invariably observed in empirical data across different countries and at different levels of sectoral aggregation (cfr. Stanley et al., 1996; Bottazzi and Secchi, 2006).[15] The dashed line describes the possible distributional effects that could plausibly emerge under the influence of binding FCs. One effect is a "pinioning " effect: FCs prevent firms that face potentially good growth opportunities from actually seizing some of them (beyond a certain 'hit FC' threshold), thus forcing these firms to abandon or postpone some profitable investment projects.

[14]See the Appendix for a formal derivation.

[15]A first attempt to explain the emergence of this stylized fact, based on the idea of dynamic increasing returns, is presented in Bottazzi and Secchi (2006).

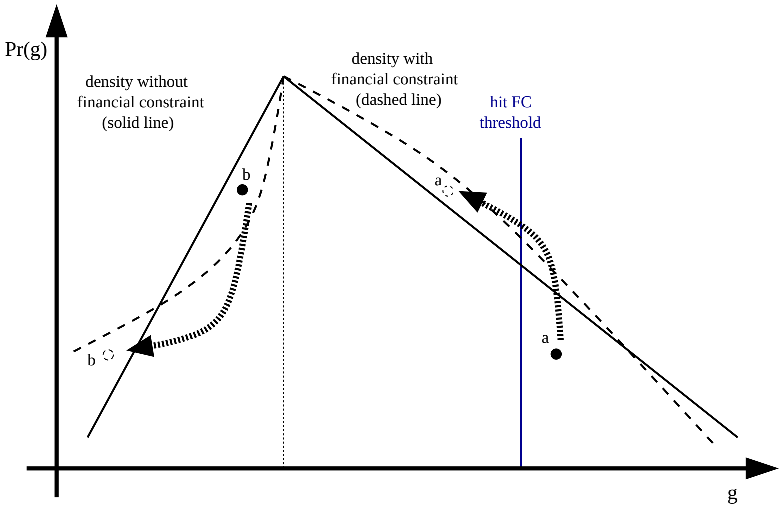

Figure 4: Possible effects of financing constraints on the growth rates distribution.

Although positive growth is still attainable in the presence of FCs, these firms would have enjoyed much higher growth records, if not hit by FCs. Such an effect would imply a slimming down of the right tail of the growth shocks distribution (cfr. 'case a' in Figure 4). Another possibility is that FCs associate with a "loss reinforcing" effect. This predicts that firms who are already facing losses in market shares will experience a further deterioration in their poor growth rates in the presence of credit constraints problems, for example because they are forced to sell productive assets and divest activities, thus ultimately facing a reduction in revenues. This effect would be reflected in a shift of mass from the left-hand part of the density towards the bottom extreme, generating a fatter left tail (cfr. 'case b' in Figure 4).

5 Main results

A preliminary step in estimating equation (3) involves modeling heteroskedasticity. We characterize $\sigma_{FC}(s_{t-1})$ starting from the data. We consider the standard definition of growth rates in terms of log-differences of size

$$g_{i,t} = s_{i,t} - s_{i,t-1} \quad , \tag{4}$$

and then, for each FC class, we plot the standard deviation of g computed within different bins (quantiles) of the log-size distribution against the average log-size of the bin. Figure 5 reports results

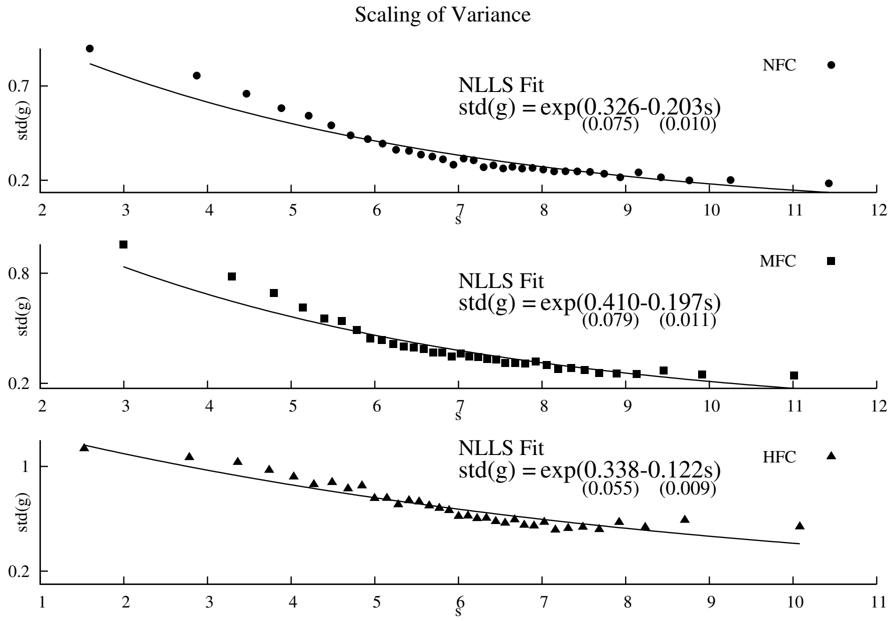

Figure 5: Empirical relation between the standard deviation of growth and firm size, by FC classes. Pooled data over 2001-2003.

obtained with 35 size bins. The whole procedure is very robust in terms of choice of the number of bins. Scatter plots of the data tend to agree with previous studies, finding that the relationship displays an exponential decrease. This is confirmed, for all FC classes, by the Non-Linear Least Squares estimates reported in the graphs. It is also worth noticing that the relationship does not depend on age. In fact, within each FC class, we do not observe any statistically significant difference in the estimated relation when considering young versus old firms.[16]

Taking this evidence into account, we insert an explicit exponential heteroskedasticity term, $\sigma_{FC}(s_{t-1}) = \exp(\gamma_{FC} \cdot s_{t-1})$, into our baseline model

$$s_t - s_{t-1} = c_{FC} + \lambda_{FC}\, s_{t-1} + \exp(\gamma_{FC} \cdot s_{t-1})\epsilon_{FC,t} \quad . \tag{5}$$

A further important modeling issue concerns an appropriate treatment of the distribution of residuals. As mentioned, previous studies have documented that the distribution of growth shocks, once heteroskedasticity has been properly modeled, is well approximated by a Laplace distribution. A first choice would therefore be to allow for Laplacian residuals, via Least Absolute Deviation (LAD) estimates. However, following the discussion in Section 4, we are also interested into possible

[16]Results available upon request.

asymmetries in the distribution of growth shocks. Accordingly, we resort to maximum likelihood estimates of equation (5) with Asymmetric Laplace residuals (ALAD estimates).[17]

Table 2 presents the results (cfr. Model 1) obtained in each FC class. A first notable finding concerns the cross-class patterns in the autoregressive components. The estimated λ is barely significant and practically zero for both NFC and MFC firms. This suggests that an integrated process can represent a good approximation for the evolution of size in these two classes. Conversely, the estimate of λ is significantly negative for HFC firms (about -0.03, roughly four times bigger, in absolute value, than in the MFC class and two orders of magnitude bigger than in the NFC class). This reveals that strong FCs give rise to sizeable deviations from the Gibrat's benchmark.[18]

The patterns in the constant terms are in line with expectations: average growth rate is positive for non constrained firms, while statistically equal to zero in the other two classes. Confirming intuition and standard results in the literature, FC problems reduce the average growth rate.

The estimates of the γ coefficients, confirming the graphical investigation reported in Figure 5, reveal the clear-cut role of FCs in explaining the heteroskedasticity of growth shocks. For NFC and MFC firms the estimated value is very close to -0.20 (which is strikingly similar to those reported in other studies on different data). This means that, in these two classes, the standard deviation of growth rates among largest firms (say, those firms with $s_{t-1} \simeq 10$), is approximately three times smaller than the standard deviation among small firms (say, those firms with $s_{t-1} \simeq 4$). Instead, within HFC firms the estimated γ is about -0.16, implying a smaller reduction in growth dispersion when moving from small to big firms, as compared to the other two classes (growth dispersion among larger firms is only about twice smaller than among smaller firms). This is once again in accordance with the intuition that FCs create a threshold effect, reducing the span of growth opportunities that constrained firms can access. According to the aforementioned "portfolio theory" interpretation, the implication is that the diversification advantage of bigger firms is considerably reduced by the effect of FCs.

Finally, the estimates of a_l and a_r suggest a relatively symmetric distribution of residuals. How-

[17]This corresponds to assume that the error term follows an AEP distribution with $b_l = b_r = 1$, and with a_l and a_r estimated from data.

[18]If one is ready to accept the persistence in financial conditions over relatively long spans of time that we have indirectly inferred is Section 3, this result is sufficient to explain the lack of Gaussianization in the right tail of the FSD observed among the HFC firms (cf. Figure 2 above).

Table 2: REGRESSION ANALYSIS[a]

	FC CLASS	Main Estimates Model 1	Robustness checks Model 2A	Model 2B
	NFC			
γ		-0.222*(0.001)	-0.207*(0.001)	-0.208*(0.001)
c		0.009*(0.001)	0.017*(0.001)	0.015*(0.001)
λ		-0.0007*(0.0003)	-0.008*(0.001)	-0.008*(0.001)
ln(Age$_{i,t}$)			-0.023*(0.001)	-0.023*(0.001)
ln(ASSETS$^{b}_{i,t-1}$)			0.021*(0.001)	0.020*(0.001)
ln(GOM$^{b}_{i,t-1}$)			0.002(0.001)	0.002(0.001)
a$_l$, a$_r$		0.211, 0.190	0.202, 0.177	0.202, 0.177
Number of observations		117,871	109,995	109,995
	MFC			
γ		-0.220*(0.001)	-0.205*(0.001)	-0.205*(0.001)
c		-0.011*(0.001)	0.003*(0.001)	-0.007*(0.001)
λ		-0.0076*(0.0003)	-0.018*(0.001)	-0.018*(0.001)
ln(Age$_{i,t}$)			-0.040*(0.001)	-0.040*(0.001)
ln(Assets$^{b}_{i,t-1}$)			0.028*(0.001)	0.027*(0.001)
ln(GOM$^{b}_{i,t-1}$)			0.009*(0.001)	0.010*(0.001)
a$_l$, a$_r$		0.243, 0.240	0.231, 0.225	0.230, 0.225
Number of observations		131,276	122,417	122,417
	HFC			
γ		-0.161*(0.002)	-0.143*(0.002)	-0.143*(0.002)
c		-0.013*(0.003)	0.023*(0.003)	0.014*(0.003)
λ		-0.030*(0.001)	-0.052*(0.002)	-0.052*(0.002)
ln(Age$_{i,t}$)			-0.125*(0.003)	-0.127*(0.003)
ln(Assets$^{b}_{i,t-1}$)			0.066*(0.003)	0.064*(0.003)
ln(GOM$^{b}_{i,t-1}$)			0.019*(0.002)	0.021*(0.002)
a$_l$, a$_r$		0.478, 0.470	0.453, 0.415	0.453, 0.414
Number of observations		29,775	25,541	25,541

[a] ALAD estimates, standard errors in parenthesis.
[b] ASSETS is proxied with Net Tangible Assets. Gross Operating Margin(GOM) has been transformed to avoid negative numbers.
[c] $\ln(Age)$, $\ln(ASSETS)$, $\ln(GOM)$ are in Z-scores.
[*] Significantly different from zero at 1% level. Standard errors obtained using sandwich estimator.

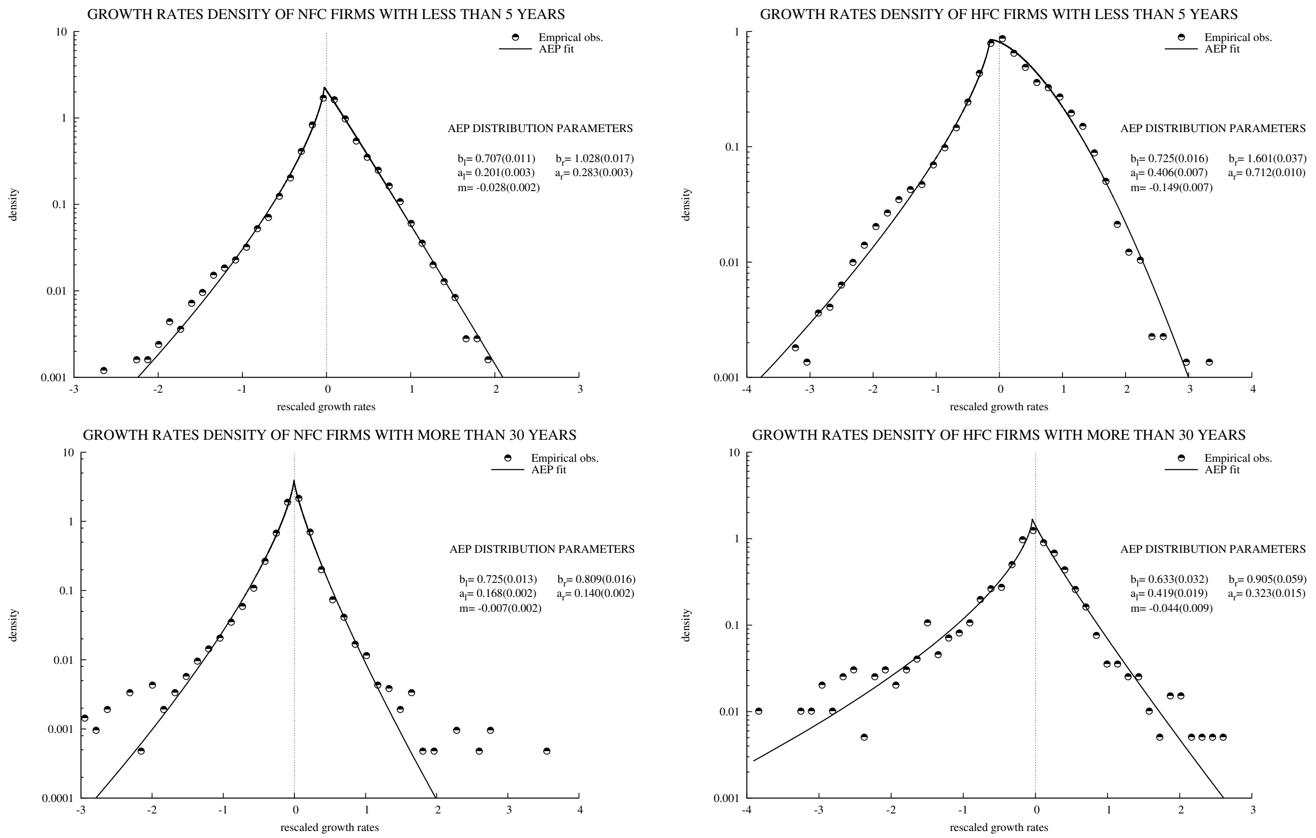

Figure 6: Growth rates distributions and financial constraints. Pooled data over 2001-2003.

ever, the ALAD estimation assumes an exact Laplace shape (i.e., b_l=b_r=1). In order to provide a more general assessment of the possible presence of asymmetry it is worthwhile investigating the structure of the residuals, also with respect to different age classes. This is done in Figure 6 where we show kernel estimates of the empirical distributions of the residuals for young-NFC firms (top-left), young-HFC firms (top-right), old-NFC firms (bottom-left), and old-HFC firms (bottom-right).[19] Differences in tail behavior are quantified by an AEP fit (solid line). The estimates of the coefficients (b_l, b_r, a_l, a_r) are reported in each panel. A comparison across the estimates confirms the tent-shape approximation. However, the age-class disaggregation reveals apparent differences in the shape of the shocks distributions. The very presence of such a sizeable effect is an interesting finding *per se*. Recall that in fact location-shift and variance-shift effects due to FCs are already captured in the regression through c and σ, respectively. Thus, what remains in the residuals is only the result of asymmetric tail effects induced by FCs. Let us first focus on young firms (compare the two top

[19]The distributions of MFC firms are not presented here to keep the figures more readable. The results (available upon request), substantially replicate the findings obtained for NFC firms, and thus do not affect the main conclusions of our reasoning.

panels in Figure 6). If we move from NFC to HFC firms, we observe a clear-cut slimming down of the right tail: there is a leftward shift in probability mass from the right tail to the central part of the distribution (b_r increases from about 1.03 for NFC firms to almost 1.60 for the HFC class). Correspondingly, the right width parameter a_r also shows a clear-cut increase (from about 0.28 to about 0.70). In contrast, the left tails of the two distributions do not display any significant difference (both a_l and b_l are similar across NFC and HFC firms). The picture changes completely when we consider old firms (see the bottom panels). In this case the differences between NCF and HFC firms are stronger in the left tail. HFC firms have a fatter left tail, suggesting that FCs produce a shift in probability mass towards the left tail: b_l decreases from 0.73 to almost 0.63.[20] Overall, these findings are in line with the existence of the two types of FC effects described in Section 4, and also suggest that such effects operate differently on different age classes. The "pinioning" effect of FCs mainly affects young firms, while older firms are those mostly affected by the "loss reinforcing" effect of FCs.

6 Robustness checks

In this section we present a series of robustness checks which validate our main results and reveal further interesting insights about the role the FC exert on firm dynamics.

A first issue relates to sample censoring. Although the time span covered in the data is relatively short, and thus exit is not frequent, it is known that exit rates tend to be higher among smaller firms. To the extent that smaller firms are more likely to be financially constrained, this could induce a spurious difference in the estimated value of λ across the three different FC classes. To check for this effect we split the sample in size classes (defined according to Eurostat definitions based on number of employees) and re-estimate our baseline equation (5) within each size class. Results in Table 3 confirm the presence of a size effect: irrespectively of the FC class, the value of λ gets reduced when larger firms are considered and becomes identically unsignificant for larger firms. At the same time, however, the effect of FC is the same in each size class: when the value of λ is significantly different from zero, it becomes smaller for more constrained firms. Thus, our main

[20] There is also an effect on the right side of the supports, qualitatively similar to that noted across young firms, and resulting in a fatter right tail for NFC firms. For old firms, however, the effect is very mild.

Table 3: REGRESSION ANALYSIS BY SIZE CLASSES [a,b]

	Micro(0-9)	Small(10-49)	Medium(50-249)	Large(250+)
NFC				
γ	-0.2520(0.0014)	-0.1196(0.0045)	-0.1591(0.0052)	-0.1098(0.0104)
c	0.0056(0.0009)	0.0095(0.0010)	0.0009(0.0011)	-0.0058(0.0024)
λ	-0.0115(0.0006)	-0.0114(0.0013)	-0.0064(0.0013)	-0.0024(0.0022)
Obs.	74192	20044	12131	2006
MFC				
γ	-0.2407(0.0015)	-0.1707(0.0042)	-0.1704(0.0056)	-0.2307(0.0131)
c	-0.0171(0.0009)	-0.0117(0.0013)	-0.0142(0.0015)	-0.0290(0.0040)
λ	-0.0220(0.0007)	-0.0308(0.0015)	-0.0102(0.0017)	-0.0057(0.0034)
Obs.	92144	18998	10165	1278
HFC				
γ	-0.1715(0.0029)	-0.2219(0.0136)	-0.1995(0.0192)	-0.1339(0.0213)
c	-0.0067(0.0031)	-0.1264(0.0080)	-0.0366(0.0121)	-0.0916(0.0217)
λ	-0.0736(0.0023)	-0.1272(0.0085)	-0.0630(0.0116)	-0.0141(0.0128)
Obs.	22672	1737	662	138

[a] ALAD estimates of Equation (5), standard errors (via sandwich estimator) in parenthesis.
[b] Size classes in terms of number of employees, according to EUROSTAT definition.

conclusion that FCs tend to induce a sub-diffusive growth pattern is largely confirmed.

A further major concern is that our baseline framework in equation (5) can clearly leave out important variables which are likely to play a role in size-growth dynamics and represent confounding factors which might induce biased estimates. We thus need to enlarge the set of explanatory variables considered.

Panel estimates with firm fixed effects, which would help to control for unobserved time-invariant heterogeneity, are limited by both the relatively short time dimension of the data, and by the lack of non-linear and non-Gaussian extensions of panel GMM-like methods required by our framework.[21] However, with available data we can control for several important factors. Firstly, the inclusion of firm age is mandatory, given the high correlation of age with size, and the significant effects that age has on the distributional properties of both size and growth. Secondly, we need to control for the two crucial dimensions which interact with external FCs in determining the overall amount

[21] We nonetheless explored maximum likelihood estimates of equation (5) with firm fixed effects and Gaussian residuals, pooling across FC classes and adding two dummy indicators for firms belonging to MFC and HFC class. The HFC dummy coefficient turned out negative and significant ($\sim -0.012, S.E. 0.0023$), suggesting that the explanatory power of ratings is not washed away by the inclusion of fixed effects.

of financial resources available to a firm, namely availability of internally generated resources and availability of collateral. The rationale behind the inclusion of a proxy for collateral is that, as predicted by theory and confirmed by evidence (Angelini and Generale, 2008), the availability of hard capital can ease the access to external financing. We measure collateral via the the stock of Net Tangible Assets (labeled ASSETS). Further, we proxy internal resources with the logarithm of Gross Operating Margin (GOM, equivalent to the EBIDTA), thus yielding a measure of the profit margin generated by the operating activities of a firm.[22] Given the relatively high frequency of negative GOM in the sample (about 30%), negative GOM values were transformed to 1 before taking logs. In fact, for the purposes of our analysis, negative and null operating revenues can be considered equivalent, as in both cases there is a need for the firm to completely rely on external resources in financing its operations.[23]

We run a preliminary Granger causality test between firm growth rates and FC. We estimate two regression models. In the first model we use dummy variables distinguishing whether a firm belongs to HFC class or not. In the second model we directly use the original risk-rating values as defined in the database. Both models are augmented by the controls discussed above (age, GOM and ASSET, plus lagged size). In both specifications, pooling over all the sample, we find that while past FC status Granger-causes growth, past growth does not Granger-causes FC status. This result supports our choice to use lagged values of ratings as proxy for FC to growth.

Then, we move to our main robustness analysis, adding the controls to our baseline specification. We first perform Maximum Likelihood ALAD estimates of the following extended model

$$\begin{aligned} s_t - s_{t-1} = c_{_{FC}} + \lambda_{_{FC}}\ s_{t-1} + \beta_{1_{FC}}\ \ln(age_t) + \beta_{2_{FC}}\ \ln(GOM_{t-1}) + \\ \beta_{3_{FC}}\ \ln(ASSETS_{t-1}) + \exp(\gamma_{_{FC}}\ s_{t-1})\epsilon_{t_{FC}} \end{aligned} \quad (6)$$

where both GOM and ASSETS enter with a 1-period lag, at least partially accounting for simultaneity issues concerning these variables, and we again model heteroskedasticity via an exponential

[22]The use of GOM implies, by definition, that we do not consider the cash flow generated by non operating earnings and losses. These should not be very relevant, however, since we are working with manufacturing firms. Moreover, due to the limited data availability, we cannot consider the cash flows absorbed by taxes. Assuming, as a first approximation, a constant tax rate, this would amount to a constant and consequently irrelevant shift in the value of our regressor.

[23]As done for size, both GOM and ASSETS were deflated with appropriate sectoral price indexes, at the 3-digit level of industry disaggregation.

correction.[24] Finally, in order to ease comparison of coefficients' magnitudes, the control variables enter as Z-scores.

Results are reported in Table 2 above (cfr. Model 2A). The most notable change induced by the inclusion of controls is that deviations from the Gibrat's benchmark of $\lambda = 0$ are now observed in all the FC classes. As frequently reported in studies exploring augmented Gibrat's regression, additional regressors absorb part of the size coefficient. However, the estimates of λ across the FC classes reproduce the pattern previously obtained from our baseline model: the autoregressive coefficient has a much lower value for the HFC class, thus confirming that the negative impact of size on growth rates is stronger for financially constrained firms. Estimates of the heteroskedasticity parameter γ are basically unaffected by the addition of further regressors and confirm the patterns emerging from the simplest specification.

In general, the coefficients on the added covariates present interesting cross-class differences. Age displays a negative and significant coefficient in all classes, in agreement with the expectation that on average older firms grow less than younger firms. The magnitude increases with the strength of FCs, however, thus revealing that the detrimental role of age is stronger among HFC firms. It should also be noted that age is the regressor with the highest coefficient in absolute value. Next, concerning the role of ASSETS, we find a positive and significant coefficient, bigger for HFC firms: the availability of hard capital as collateral becomes more beneficial for growth when FC are stronger. Similarly, the availability of internal resources has a positive association with growth only when some degree of FCs is present (GOM is not significant for NFC, positive and significant for MFC and HFC). However, even when significant, the magnitudes of GOM coefficients are negligible in practical terms, suggesting that internal resources play (if any) a second order role compared to other regressors.[25]

A further check concerns the possible role of sector-specific dynamics. It is well known that a firm's dependence on external financing varies across industrial sectors (Rajan and Zingales, 1998),

[24] Concerning the use of a GMM-SYS estimator, standard Sargan/Hansen tests confirm that the time span of the database is too short to identify a valid set of instruments among past levels and past differences of the covariates.

[25] We also estimated the model with two alternative definitions of the GOM variable, both aiming at including negative GOM in the regression. First, using the ratio GOM/ASSETS yields unsignificant coefficient for this further variable, and only results into a moderate and homogenous reduction of the scaling parameter across classes. Second, we introduced a dummy taking value 1 when GOM is positive, and 0 otherwise: this turned significant in all classes, but without affecting the other coefficients. Results are available upon request.

Table 4: Growth Rates Distributions – Robustness checks

	AEP Parameters			
	b_l	b_r	a_l	a_r
YOUNG (age < 5)				
NFC	0.737(0.013)	0.989(0.017)	0.196(0.003)	0.262(0.003)
HFC	0.717(0.016)	1.574(0.038)	0.385(0.007)	0.659(0.010)
OLD (age > 30)				
NFC	0.724(0.013)	0.842(0.017)	0.160(0.002)	0.140(0.002)
HFC	0.657(0.035)	1.043(0.073)	0.407(0.019)	0.340(0.016)

[a] AEP fit of residuals from Equation (6), Pavitt class dummies also included. Standard errors (via sandwich estimator) in parenthesis.

so that it is likely that firms operating in different industries would display, on average, a different degree of exposure to FC problems. There is also evidence (Hall, 2002) that such sectoral differences in modes of financing, and thus differential exposure to FCs, are very likely to vary depending on the sources and procedures of the innovation activity of firms. In order to control for these industry-wide factors, we re-estimate equation (6) adding dummy variables which corresponds to the classical Pavitt taxonomy of sectoral patterns of innovation (Pavitt, 1984). The results (cfr. Model 2B in Table 2) are clearly in line with previous estimates: all the coefficients remain unchanged in practical terms.[26]

Finally, we also investigate whether the distributional properties of growth shocks are affected by the inclusion of the new regressors. To this purpose we perform AEP estimates of the empirical distribution of residuals of Model 2B, by FC classes and separately for young and old firms. Note that location-shift effects due to age are captured by the age coefficient in the regression, and also recall that (as shown in Section 5) age does not have any residual effect on the variance of growth rates,

[26]We also explored a further specification considering 2-period lags of size, ASSETS and GOM. This allows for a check of varying effects over time, and provides a further control for possible endogeneity of covariates at $t-1$. The estimates of λ retain their signs and magnitudes, again displaying negligible values for NFC firms and then increasingly negative as FCs become stronger. Second lag coefficients of GOM and ASSETS absorb part of the first lag effects of these variables. The most noticeable difference compared to the estimates presented in Table 2 is a significant reduction in the age coefficient, whose magnitude becomes comparable with that of the other regressors and across FC classes. Also notice that estimates of the extended model in equation (6) by size classes confirm the minor role of censoring in driving the findings: patterns in λ reproduce the estimates reported in Table 3 above. All the results are available upon request.

once controlling for size. Therefore, distributional differences in the residuals of Model 2B across age classes point toward additional effects of age in the tails. The estimates of AEP parameters, reported in Table 4, are not significantly different from those obtained with the simplest model specification (apart from a small increase in the b_l parameter for HFC firms).

Overall, our main conclusions remain valid to the inclusion of other firm level relevant determinants of size-growth dynamics, and remain unchanged when we also control for differences in sectoral patterns of innovation.

7 Conclusion

CeBi credit ratings represent a good measure of access to external resources. They summarize several dimensions of a firm's financial conditions and allow to capture different degrees of credit problems, thus improving upon the rather strict binary distinction between constrained versus non-constrained firms often adopted in the literature. They are heavily relied upon by banks and investors and represent a key ingredient in lending decisions. Using CeBi ratings to build a proxy for financial constraints, we extended the typical autoregressive linear model of size-growth dynamics by including a parametric description of heteroskedasticity and by providing a more flexible and robust characterization of growth shocks. Our results shows that the effects of FC on firm growth are sizeable and operate through several channels. Firstly, FCs magnify the negative effect of size on expected growth rates: the lower average growth rate that typically characterizes large versus small firms becomes even lower when FCs are presents. This is consistent with the age profile of the firm size distribution of constrained and unconstrained firms. For older firms, the FSD of non constrained firms possesses a Gaussian shape, while the FSD of financially constrained firms is more peaked. This is the typical signature of the sub-diffusive nature of the growth process associated with a negative autoregressive coefficient. Since our measure of FCs varies over time, the fact that we identify significant differences in the size distribution of different FC classes suggests a relatively high degree of persistence across the different groups. This is an interesting aspect of the FC phenomenon, which we cannot however test directly, given the relatively short temporal span of our data.

A further effect of FCs is on the relationship between firm size and variance of growth rates.

Larger firms are well known to generally display a lower variability in their growth rates. This observation has been related to a portfolio effect: larger firms tend to be more diversified, and thus, to the extent that the different activities are weakly related, diversification produces a lower volatility in aggregate growth rates. FCs seem to reduce the ability of larger firms to exploit their diversified structure. Indeed for more severely constrained firms, the negative relationship between growth rates variability and size is weaker than for unconstrained firms.

Furthermore, once the autoregressive structure and the heteroskedasticity effects are controlled for, our model reveals that FCs have an additional, asymmetric effect on the tails of the growth rates distribution. We are able to identify a "loss reinforcing" effect: firms who are already witnessing a reduction in sales, see their performance worsened in the presence of FCs. This is plausibly the results of activity dismissal and divestment. At the same time, however, firms experiencing positive growth rates, if hit by FCs, are likely to see their growth potentials depressed. In fact, credit problems generate a ”pinioning ” effect which prevents constrained firms from fully seizing the available growth opportunities. The economic consequences of these two effects are different. While the loss reinforcing effect can reflect a beneficial market selection mechanism, generating, at least in the long run, a more efficient reallocation of productive resources, the pinioning effect is plausibly a simple waste of good growth opportunities. The fact that the pinioning mechanism is more common across younger firms is not unexpected and is compatible with the presence of frictions and inefficiencies in the capital market.

According to our credit rating based measure of financial constraints, the problem of credit rationing is widespread and affects a much larger population of Italian manufacturing firms than what suggested by previous predictions obtained from survey-based measures (Angelini and Generale, 2008, see). This difference can be explained either by a self-selection bias in the population of respondents which is known to often affect survey data, or by admitting the possibility that not all firms with poor credit ratings were actually to be considered financially rationed. However, this consideration does not weaken the conclusions of our analysis. On the contrary, the fact that we still observe significant differences among the FC classes, notwithstanding the possible use of a somewhat loose proxy of FCs, represents a strong proof of the existence of a real economic effect. The adoption of a more stringent measure of FC would change the results in the direction of an even cleaner identification of this effect.

Finally, it is worth asking if our measure of FCs can also be considered as a proxy for the overall availability of financial resources, capturing at the same time difficulties in accessing external finance as well as shortage of internal financial resources. We tend to believe it can, as indeed internal resources constitute the best guarantee to potential lenders that firms are able to sustain the due interest payments. As a result, firms with sound financial conditions and reasonable levels of profits are almost automatically assigned high ratings, while the shortage of internal resources, whether generated by poor operating performances or by unsound financial conditions, is very likely to be punished with bad ratings. In any case, our conclusions are still valid even when we explicitly add a control for the availability of internal resources. Indeed, while profit margins are associated with a positive shift in the average growth rate, both the pinioning and the loss reinforcing effects of FCs remain unchanged, as does the reduced ability of larger and financially constrained firms to exploit diversification economies.

In summary, we have shown that FC problems do have relevant effects on the operating activities of firms. In order to identify these effects, however, one has to do more work than just relying upon standard linear regression framework. FC effects are indeed manifold and impact on several aspects of firm growth dynamics, ranging well beyond a shift in the expected growth rates.

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8 APPENDIX

8.1 Cleaning anomalous observations

We removed a few anomalous data from our sample. Cleaning was performed using Total Sales as a reference variable. For each firm, a missing value was inserted, in the place of the original value of Total Sales, when the latter lay outside the interval

$$[Median(TS_i)/10; Median(TS_i) * 10] \quad , \tag{7}$$

where the median is computed over the years for which data are available for firm i. Table 5 shows yearly descriptive statistics computed before and after the cleaning. It is apparent that the procedure does not introduce any relevant change to the data.

Table 5: TOTAL SALES[a] DESCRIPTIVE STATISTICS

BEFORE CLEANING FILTER								
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5700.82	1014.00	48730.09	57.89	4894.16	1.00	5634948.00	109689.00
2001	5972.90	1011.00	73679.67	141.82	29897.12	1.00	17547260.00	113405.00
2002	5804.92	973.00	67304.35	146.66	32359.62	1.00	16484840.00	116084.00
2003	5639.77	953.00	64724.22	147.42	32317.38	1.00	15803760.00	115777.00
AFTER CLEANING FILTER								
Year	Mean	Median	Std. Dev.	Skewness	Kurtosis	Min	Max	Obs.
2000	5754.55	1046.00	47700.57	58.99	5192.76	1.00	5634948.00	107250.00
2001	5878.64	1025.00	69435.93	159.48	37224.24	1.00	17547260.00	112036.00
2002	5806.96	992.00	67093.95	150.02	33371.72	1.00	16484840.00	113849.00
2003	5688.46	981.00	65417.79	147.67	32063.94	1.00	15803760.00	111810.00

[a] Nominal Total Sales in thousands of Euro.

8.2 Asymptotic behavior of the autoregressive process

Start from the model of firm size evolution as described in (3), where the shocks ϵ are independent and identically distributed according to a probability density f with mean c. Let s_0 be the initial size of the firm. By dropping the heteroskedastic term (i.e. setting $\sigma(s_t) = 1$) for simplicity, and by recursive application of (3), the size after T time steps, s_T, can be written as the weighted sum of T independent random variables

$$s_T = (1+\lambda)^T \, s_0 + \sum_{\tau=0}^{T-1} (1+\lambda)^\tau \, \epsilon_{t-\tau} \ .$$

Consider the cumulant generating function of the size at time T, $\tilde{g}_{s_T}$, defined as the logarithm of the Fourier transform of the unconditional distribution

$$\tilde{g}_{s_T}(k) = \log \mathrm{E}[e^{iks_T}] \ .$$

Due to the i.i.d. nature of the shocks it is immediate to see that

$$\tilde{g}_{s_T}(k) = \tilde{g}_{s_0}((1+\lambda)^T k) + \sum_{\tau=0}^{T-1} \tilde{f}((1+\lambda)^\tau k)$$

where $\tilde{g}_{s_0}$ and $\tilde{f}$ are the cumulants of the initial size distribution and of the shocks distribution, respectively. As a consequence, if the initial size distribution and the shocks distribution possess the cumulant of order n, C^n, then the size distribution at time T also possesses it, and thus, with obvious notation

$$C^n_{s_T} = \frac{d^n}{dk^n} \tilde{g}_{s_T}(k) \bigg|_{k=0} = (1+\lambda)^{nT} \, C^n_{s_0} + \frac{(1+\lambda)^{nT} - 1}{(1+\lambda)^n - 1} \, C^n_\epsilon \ .$$

Equation (4) in Section 4 directly follows by noting that the mean and the variance are the first and second cumulants, respectively: $M = C^1$ and $V = C^2$.