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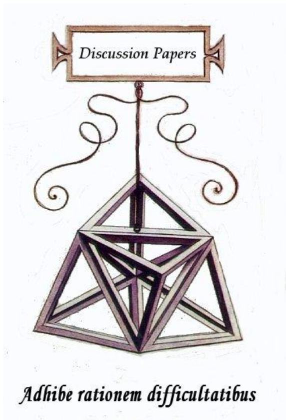

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Abstract

The effects of two environmental policy options for the reduction of pollution emissions, i.e. taxes and non-tradable quotas, are analyzed. In contrast to the prior literature this work endogenously takes into account the level of emissions before and after the adoption of the new environmental policy. The level of emissions is determined by solving the firm's profit maximization problem under taxes and fixed quotas. We find that the optimal adoption threshold under taxes is always larger than the adoption threshold under fixed quota, even in a setting characterized by ecological uncertainty and ambiguity - in the form of Choquet-Brownian motions - on future costs and benefits over adopting environmental policies.

Classificazione JEL: Environmental taxation; Non-tradable quotas; Optimal implementation time; Choquet-Brownian uncertainty; Real options

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1 Introduction

The choice of the appropriate policy instrument to control pollution and reduce environmental degradation caused by human activities has become a major concern in the public debate in the last decades. It is often indicated as a key factor to implement a more sustainable development worldwide. Environmental policy instruments are tools used by governments in order to prevent, reduce or mitigate harmful effects on terrestrial and marine ecosystems of accumulating greenhouse gases[1]. Examples of environmental policy instruments include: 1) tradable emissions permits (also known as cap and trade) and environmental taxes (also referred to as market-based instruments); 2) quotas, targets for cutting emissions and commands (also referred to as command-and-control instruments).

Market-based instruments, such as tradable permits of pollutants and environmental taxes, are rising up in the EU agenda. They aim to bring the environmental and health costs of economic activities into market prices and set a price on the use of natural resources like air, water and soil. Recent examples are the EU emission trading scheme and harmonized environmental taxation such as the Taxation of Energy Products Directive and the "Eurovignette" Directive for freight transport. Command-and-control instruments rely on prescribing rules and standards and using sanctions to enforce compliance with them. Command-and-control regulation requires polluters to meet specific emission-reduction targets and often requires the installation and use of specific types of equipment to reduce emissions[2]. An example is the Kyoto protocol on global warming. The major feature of the Kyoto protocol is that it sets binding targets for 39 industrialized countries and the European Union (i.e. the Annex I parties) for reducing greenhouse gas emissions. Emission quotas (also known as "assigned amounts") were agreed by each participating Annex 1 country, with the intention of reducing their average emissions during 2008-2012 to about 5 percent below 1990 levels[3].

As a consequence of a growing interest in the use of different types of policy instruments to control pollution, a large number of articles have been published in the environmental economics literature with the intent to investigate the rel-

[1] The principal greenhouse gases that enter the atmosphere because of human activities are: carbon dioxide (CO_2), methane (CH_4), nitrous oxide (N_2O), sulphur hexafluoride (SF_6), hydrofluorocarbons (HFCs) and perfluorocarbons (PFCs), among many others. See: http://www.epa.gov/climatechange/emissions/index.html.

[2] Source: European Environment Agency: http://www.eea.europa.eu/themes/policy/about-policy-instruments.

[3] The Kyoto protocol was initially adopted on 11 December 1997 in Kyoto and entered into force on 16 February 2005. Under the Kyoto Protocol, only the Annex I countries have committed themselves to national or joint reduction targets that range from a joint reduction of 8% for the European Union and other (central and eastern) European countries, to 7% for the United States, 6% for Japan, Canada, Hungary and Poland and 0% for New Zealand, Russia and Ukraine; moreover, 8% for Australia and 10% for Iceland. The United States is the only industrialized nation under Annex I that has not ratified the treaty and therefore is not bound by it. See the UN Climate Change web: http://unfccc.int/kyoto_protocol/items/3145.php , for further information

ative merits of price versus quantity instruments[4] to achieve reduction of greenhouse gases emissions. Weitzman (1974) initiated a discussion about the relative efficiency of alternative environmental policies in a simple analytical model characterized by uncertainty, second-best policy alternatives, and costly policy adjustments. The main implication for policy-makers is that taxes are more efficient when marginal benefits are relatively flat and quantity mechanisms are more efficient when benefits are relatively steep. Newell and Pizer (2003) confirm this result and show that taxes often generates the expected net benefits of quantity instruments when correlation of cost shocks across time, discounting, stock decay, and the rate of benefits growth are included. Hoel and Karp (2002) compare the effects of taxes and quotas for an environmental problem where the regulator and polluter have asymmetric information about abatement cost and environmental damage depends on a stock of pollutant. They find that taxes tend to dominate quotas and this effect is more pronounced for multiplicative uncertainty (Hoel and Karp, 2001). Although the so far cited papers account for uncertainty, they do not consider irreversibility[5]. Xepapadeas (2001) studies the behaviour of polluting firms regarding the expansion of abatement capital and location decisions in the presence of emission taxes, tradable permits and subsidies for the abatement investment under irreversibility. Insley (2003) studies the decision of an electric power utility regarding the abatement of sulfur dioxide (SO_2) emissions by installing a scrubber, assuming that SO_2 permit prices are stochastic and the construction process includes "time-to-build". Van Soest (2005) analyzes the impact of emission taxes and quotas on the timing of adoption of energy-saving technologies under irreversibility and stochastic arrival rate of the new technologies, and shows that: (i) increased environmental stringency (measured in tax and its equivalent in terms of quota) does not necessarily induce early adoption, and (ii) there is no unambiguous ranking of policy instruments in terms of the length of the adoption lag. Wirl (2006) investigates the implications of two different kind of irreversibilities (i.e. of CO_2 emissions and of stopping) on the optimal intertemporal accumulation of greenhouse gases in the atmosphere under uncertainty. He finds that an irreversible stopping of greenhouse gas emissions is never optimal and yields in the real option framework a less conservative emission policy, i.e. a later stopping, in comparison with the possibility to suspend emissions without sacrificing future fossil fuel uses. He then compares the effects of emission taxes and quantities on the optimal timing for policy adoption and shows that both policy instruments are equivalent in such a framework. Pindyck (2000) investigates how irreversibility and uncertainty influence the timing of policy planning and adoption. Pindyck (2002) generalizes Pindyck (2000) including two stochastic state variables: one captures uncertainty over environmental change (i.e., ecological uncertainty),

[4] For example, an emission cap and permit trading system is a quantity instrument because it fixes the level of emissions flow (quantity) and allows the price to vary. In contrast, emission taxes are a price instrument because the price is fixed and the level of emissions flow is allowed to vary according to economic activity.

[5] Arrow and Fisher (1974) first introduce the concept of quasi-option value in environmental economics where both uncertainty and restriction on reversibility of acts are assumed.

and the other uncertainty over the social costs of environmental damage (i.e., economic uncertainty). While Pindyck (2000, 2002) consider the optimal timing for a single environmental policy adoption, Goto et al. (2009) consider the choice of two alternative environmental policies under uncertainty. They do not discriminate between price and quantity mechanisms as other papers do, but simply index the two policies by 1 and 2 and find their ranking. The problem of the timing of policy intervention is also studied by Conrad (1997, 2000), Saphores and Carr (2000) and Nishide and Ohyama (2009).

Our paper is about the optimal timing of a new environmental policy in a framework where production causes pollution, firms are regulated either by environmental taxes or non-tradable quotas and the regulator is ambiguous over the economic effects of the policies[6]. In contrast to the prior literature we endogenously take into account the level of emissions before and after the adoption of the new environmental policy. In this context, the level of emissions is determined by solving the firm's profit maximization decisions under taxes and fixed quotas as in van Soest (2005). The regulator solves an optimal stopping problem in order to decide about the timing and ranking of the two policies. First, in Section 3 we assume that the future evolution of the stock of pollutant is deterministic, there is only uncertainty over the future economic net benefits of policy adoption and obtain a closed form solution for the thresholds. Next, in Section 4, we assume that there are both ecological uncertainty and ambiguity on the future costs and benefits over adopting environmental policies. As Asano (2010) emphasizes, the economic costs of policies aiming at reducing pollutants are not predictable and the perceived ambiguity of the government towards them may affect adoption timing in a non trivial way. Our main results are that *(i)* the optimal adoption threshold under taxes is always larger than the adoption threshold under fixed quota, even in this setting, and *(ii)* depending on the regulator's attitude towards ambiguity, uncertainty may increase or decrease the optimal timing of adopting the environmental policies.

Our model differs from the previous literature in various aspects. First, we extend and generalize the continuous-time model of environmental policy adoption in Pindyck (2000, 2002) to the case of two alternative environmental policy instruments. In contrast to Goto et al. (2009), who study the decision to implement two unspecified alternative environmental policies under economic uncertainty, we investigate the effects of environmental taxes and quotas on the optimal timing of emission reductions in a real option framework under ecological uncertainty and economic ambiguity. Second, in our paper ambiguity is modelled through Choquet-Brownian motions, rather that relying on the multiple-prior preferences which is based on the maximin criterion (see Gilboa and Schmeidler, 1989; Chen and Epstein, 2002), as has been done so far in the literature dealing with ambiguity and real options. A small number of papers have recently come out introducing ambiguity into real options (Miao and Wang, 2009; Trojanowska and Kort, 2007; Nishimura and Ozaki, 2007; Asano,

[6] As in van Soest (2005), we focus on taxes and non-tradable quotas and hence do not consider the dynamic incentives associated with a tradable permit system. See Requate (1998) for an interesting analysis of permits.

2010), although they are based on the maximin criterion, or the worst case scenario, and show that the impact of ambiguity on valuation and timing is often equivocal. Our approach employing Choquet-Brownian motions follows Kast and Lapied (2008), which differs from the literature so far, thereby avoiding limits inherent to the maximin criterion. In contrast to Asano (2010), who finds that an increase in ambiguity decreases the optimal timing of the environmental policy, we find that an ambiguity averse environmental regulator will delay the adoption of the new policies, while an ambiguity lover regulator will adopt them earlier than if neutral ambiguous. Third, differently from van Soest (2005) who analyzes the impact of taxes and quotas on the timing of firm investment decisions with respect to energy saving technologies, we investigate the environmental regulator's decision about the timing and ranking of the two environmental policies and also illustrate the effects of a change in attitude towards ambiguity on the value and timing of the two policies.

The paper is organized as follows. Section 2 describes the setup of the model. Section 3 solves the optimal stopping problem of the environmental regulator and finds the ranking of the two environmental policies when the stock of pollutant evolves deterministically and there is uncertainty over the future economic net benefits of policy adoption (Propositions 1 and 2). Section 4 introduces both ecological uncertainty and economic ambiguity and contains the most comprehensive result (Proposition 3). Numerical results are presented in Section 5. In particular, a detailed sensitivity analysis is shown as to deepen our understanding of the effects of environmental taxes and quotas on the optimal timing of emission reductions. Section 6 concludes the paper. All proofs are in the Appendix.

2 The model

We present a simple partial-equilibrium model to illustrate our arguments. A competitive industry consists of identical firms producing a homogeneous product. Output generates as a by-product environmental damages on society due to some externality. Let us consider a representative firm producing q units of output at each instant of time according to the simple production function[7]:

$$q\,(E) = \theta E^{a},$$

where E is the energy used as an input in the production process, a is the constant output elasticity, $0 < a < 1$, and θ is the parameter of energy efficiency. Let p be the fixed price of output and z the fixed unit cost of energy. The production of output generates pollutant emissions. Let M_t be a state variable denoting the stock of environmental pollutants, e.g. the average concentration of CO_2 or HFCs in the atmosphere. Let ξ_t be a flow variable that controls M_t. Emissions are assumed to be determined by the function: $\xi_t = \beta q_t$, where q_t is the amount of output produced at time t and β are emissions per unit of output. The evolution of M_t is given by (Pindyck, 2000, 2002):

[7] Time subscripts are suppressed when no confusion arises.

$$dM_t = (\xi_t - \delta M_t)\, dt, \quad M_0 = M \tag{1}$$

where $\delta \in (0,1)$ is the rate of natural decay of the stock pollutant over time, i.e., a fraction δ of the pollutant in the atmosphere diffuses into the ocean, forests, etc. In our model a policy involves a one-time reduction in ξ_t. Denote by τ_i the unknown adoption time of the new environmental policy i and assume that the dynamics of M_t changes after τ_i :

$$dM_t = \begin{cases} \left(\xi^N - \delta M_t\right) dt & \text{for } 0 \leq t < \tau_i \\ \left(\xi^A - \delta M_t\right) dt & \text{for } t \geq \tau_i \end{cases} \tag{2}$$

Here, superscripts N and A indicate the state of no-adoption and adoption of the new environmental policy, respectively. Until a policy is adopted ξ_t stays at the constant initial level ξ^N, while policy adoption implies a once-and-for-all reduction to a new permanent level ξ^A, with $0 \leq \xi^A < \xi^N$. Therefore, when the environmental regulator implements the new environmental policy, the level of emission flow is reduced by $\xi^N - \xi^A$[8]. In the next section we endogenously determine ξ^N and ξ^A by solving the firm's profit maximization problem under taxes and fixed quotas.

2.1 The firm's problem

Given the unknown adoption time of the new environmental policy, the firm has to decide about output production. The profit flow π in the absence of some policy intervention (for $0 < t < \tau_i$) is:

$$\pi^N = \max_E \{pq(E) - zE\}, \tag{3}$$

The value of E that maximizes the term within brackets is given by:

$$E = (ap\theta\, / z)^{1/(1-a)},$$

which leads to the following expression for π^N:

$$\pi^N = (1-a)\left(\frac{a}{z}\right)^{a/(1-a)} p^{1/(1-a)}\, \theta^{1/(1-a)} = \Psi^N \theta^\lambda,$$

where:

$$\Psi^N = (1-a)p^{1/(1-a)}\, (a\, / z)^{a/(1-a)}$$

and:

$$\lambda = 1\, / (1-a) > 1. \tag{4}$$

[8] This assumes that the speed of of pollution accumulation is reduced because of the policy implementation.

Profit maximization problem (3) allows us to determine the level of emissions ξ_t before the adoption of the new environmental policy:

$$\xi^N = \beta\theta E^a = \beta\theta^{1/(1-a)} \left(ap/z\right)^{a/(1-a)} . \tag{5}$$

Two environmental policy options will be considered here: the level of emission flow can be reduced either by setting a per-unit energy tax rate (ζ) or a non-tradable quota for energy use ($\bar{E}$). To make a comparison between the effect of taxes and quotas on the optimal adoption time, we suppose that, in the initial situation, energy use is the same under both regimes. Like van Soest (2005)[9], let us assume that the environmental regulator has determined the Pigouvian tax (ζ) that equates the marginal benefits and costs of pollution. The equivalent quota ($\bar{E}$) is assumed to be equal to the amount of energy that the firm will employ, given this tax rate. It is obtained by solving the profit maximization problem under taxes: $\max_E \{pq(E) - (z+\zeta)E\}$ .Thus,

$$\bar{E} = \left(ap\theta/(z+\zeta)\right)^{1/(1-a)} . \tag{6}$$

Using i ($i = T, Q$) to denote the policy regime i.e. taxes or non-tradable quotas, respectively, the firm's profit flow when the new environmental policy is adopted (for $t \geq \tau_i$) is:

$$\pi_i^A = \begin{cases} \max\limits_E \{pq(E) - (z+\zeta)E\} = \Psi_T^A \theta^\lambda & \text{if } \ i = T \\ pq(\bar{E}) - z\bar{E} = \Psi_Q^A \theta^\lambda & \text{if } i = Q \end{cases} , \tag{7}$$

where

$$\Psi_T^A = (1-a)p^{1/(1-a)} \left(a/(z+\zeta)\right)^{a/(1-a)} ,$$

$$\Psi_Q^A = \left[1 - za/(z+\zeta)\right] p^{1/(1-a)} \left(a/(z+\zeta)\right)^{a/(1-a)} ,$$

and $\bar{E}$ is given by (6)[10]. Notice that the optimal level of E for $i = T$ in (7) coincides with $\bar{E}$.

By (7) and (6) the level of emissions ξ_t after the adoption of the new environmental policy i can be obtained:

$$\xi^A = \beta\theta^{1/(1-a)} \left(ap/(z+\zeta)\right)^{a/(1-a)} .$$

Note that emissions reduce to the same level ξ^A under both regimes.

[9] See also Requate (1995) for further datails.

[10] It is easy to show that $\pi_T^A < \pi_Q^A$ for any $\zeta > 0$. This result still holds if we use the two inputs production function: $q(E,L) = \theta E^a L^b$, $a, b \geq 0$, $a + b < 1$, where L denotes the variable labour input and w is the cost per unit of input in L. According to Van Soest (2005), the instantaneous profit flow can be calculated as:

$$\pi_i^A = \begin{cases} \max_{E,L} \{p\theta E^a L^b - (z+\zeta)E - wL\} = \psi_T^A \theta^{\bar{\lambda}} \text{ if } i = T \\ \max_L \left\{p\theta \hat{E}^a L^b - z\hat{E} - wL\right\} = \psi_Q^A \theta^{\bar{\lambda}} \qquad \text{if } i = Q \end{cases}$$

3 The regulator's problem

In each period the regulator is supposed to decide whether to adopt the new environmental policy i or to postpone it to the next period.

Like Pindyck (2000, 2002), we assume that the flow of social costs (i.e. damages) associated with the stock variable M_t has the quadratic form:

$$D(X_t, M_t) = X_t M_t^2,$$

where X_t is a variable that stochastically shifts over time to reflect the damage due to the pollutant and is assumed to follow a geometric Brownian motion:

$$dX_t = \alpha X_t dt + \sigma X_t dW_t, \qquad X_0 = X \tag{8}$$

for constants $\alpha < r$, $\sigma > 0$. W_t is a standard Brownian motion and r denotes the risk-free rate of interest. The process X_t is assumed to capture economic uncertainty over future costs and benefits of policy adoptions. For example, changes in X_t might reflect the innovation of technologies that would reduce the damage from a pollutant, or demographic changes that would increase the social cost of M_t.

Let $B(X_t, M_t)$ denote the net benefit from emissions. If the regulator adopts the policy i, the net benefit, is:

$$B_i^A(X_t, M_t) = \pi_i^A - D^A(X_t, M_t), \text{ for } i = T, Q,$$

On the other hand, if the regulator never adopts the policy:

$$B^N(X_t, M_t) = \pi^N - D^N(X_t, M_t).$$

Let $K\left(\xi^N - \xi^A\right)$ be the cost of permanently reducing the emission flows which is given by[11]:

$$K\left(\xi^N - \xi^A\right) = k_1\left(\xi^N - \xi^A\right) + k_2\left(\xi^N - \xi^A\right)^2, \tag{9}$$

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where: $\bar{\lambda} = \frac{1}{1-a-b} > 0$,

$$\psi_T^A = (1-a-b)\left(p\left(\frac{a}{z+\zeta}\right)^a\left(\frac{b}{w}\right)^b\right)^{\frac{1}{1-a-b}},$$

$$\psi_Q^A = \left(1-b-\frac{za}{z+\zeta}\right)\left(p\left(\frac{a}{z+\zeta}\right)^a\left(\frac{b}{w}\right)^b\right)^{\frac{1}{1-a-b}},$$

$\hat{E}$ is calculated as: $\arg\max_E\left\{p\theta E^a L(E)^b - (z+\zeta)E - wL(E)\right\}$ and $L(E)$ is the optimal amount of labour used as a function of energy input.

[11] For simplicity, it is assumed that both instruments require the same gross investment cost (K).

with k_1, $k_2 \geq 0$. This cost is assumed to be completely sunk.

The objective of the regulator is to choose the optimal timing of adopting policy i that would reduce emissions to ξ^A such that the expected net present value function (using the discount rate r) of the difference between the net benefit $B(X_t, M_t)$ and the cost of policy adoption $K\left(\xi^N - \xi^A\right)$, is maximized:

$$W_i(X, M) = \sup_{\tau_i \in \mathcal{T}} \left\{ \mathbb{E} \int_0^{\infty} e^{-rt} B(X_t, M_t)\, dt - K\left(\xi^N - \xi^A\right) e^{-r\tau_i} \right\}, \text{ for } i = T, Q \tag{10}$$

subject to Eq. (2) for the evolution of M_t and Eq. (8) for the evolution of X_t. Here, $\mathcal{T}$ is the class of admissible implementation times relative to the filtration generated by the stochastic process X_t.

Applying the Dixit and Pindyck (1994) methodology[12], we can derive the optimal timing for the two environmental policies and the values to reduce emissions under the two environmental policies (see the Appendix). In particular, we can compute the two thresholds $\bar{X}_i$,$i = T, Q$, such that it is optimal to adopt policy i for $X > \bar{X}_i$, with:

$$\bar{X}_T = \left(\frac{\left(\Psi^N - \Psi_T^A\right)\theta^\lambda}{r} + K \right) \left(\frac{\kappa_1 \kappa_2 \kappa_3 \phi_1}{2\Theta\left(\phi_1 - 1\right)} \right),$$

$$\bar{X}_Q = \left(\frac{\left(\Psi^N - \Psi_Q^A\right)\theta^\lambda}{r} + K \right) \left(\frac{\kappa_1 \kappa_2 \kappa_3 \phi_1}{2\Theta\left(\phi_1 - 1\right)} \right),$$

where $\kappa_1 = r + 2\delta - \alpha$, $\kappa_2 = r + \delta - \alpha$, $\kappa_3 = r - \alpha$, ϕ_1 is the positive solution to the standard characteristic equation and $\Theta = \Theta(\xi^N, \xi^A, M, \kappa_3)$. From a comparison between $\bar{X}_T$ and $\bar{X}_Q$ and between the two values of the option to reduce emissions we can get the following main results:

Proposition 1 *The optimal adoption threshold under taxes is larger than the adoption threshold under fixed quota for any $\zeta > 0$.*

Proposition 2 *The value of the option to reduce emission under taxes is smaller than the value of the option to reduce emission under quota for any $\zeta > 0$.*

Proof. *of Propositions 1 and 2: in the Appendix.* ■

Thus, Propositions 1 and 2 provide us with a ranking between taxes and non-tradable quotas in this setting, in a non equivocal way. It is found that non-tradable quota are more conducive to early adoption than taxes. Regulators may care about early adoption: in this case, non-tradable quotas outperform taxes, i.e. they should be the preferred policy instrument.

[12] Real option methodology under taxation has been employed in several works, e.g. Agliardi and Agliardi (2008, 2009), Sarkar and Goukasian (2006), Wong (2007, 2010), although these papers do not deal with environmental issues.

4 Optimal timing for the new environmental policies under ecological uncertainty and Choquet-Brownian ambiguity.

In this section, we study the optimal timing of adopting the environmental policies in the presence of ecological uncertainty and Choquet-Brownian motion representing ambiguity[13]. By ambiguity (or Knightian uncertainty) we mean the fact that information is too imprecise to be summarized adequately by probabilities, as is often the case in many decision-making settings characterized by unpredictability. It seems to be particularly appropriate in the analysis of the regulator's optimal environmental policies, where the set of beliefs expands and various scenarios of climate change, future costs and benefits from adopting new policies need to be considered.

For the evolution of the stock of the environmental pollutant M_t we follow Pindyck (2002) and assume that M_t can be described by an Ornstein–Uhlenbeck process, that is:

$$dM_t = (\xi_t - \delta M_t)\, dt + \sigma_1 dW_t^1, \quad M_0 = M \tag{11}$$

with $\sigma_1 > 0$ and W_t^1 is a standard Brownian motion. The deterministic part of the process (11) is the same as before[14].

To analyze the effects of ambiguity on the value of adopting the environmental policies we adapt Kast, Lapied and Roubaud (2010) and assume that the regulator's beliefs are represented by "$c-$ignorance", so that the new geometric Brownian motion for X_t becomes[15]:

$$dX_t = (\alpha + m\sigma_2)\, X_t dt + s\sigma_2 X_t d\tilde{W}_t^2, \quad X_0 = X, \tag{12}$$

with $m = 2c - 1$, $s^2 = 4c(1-c)$ and c $(0 < c < 1)$ is the constant conditional capacity which summarizes the regulator's attitude toward ambiguity. Indeed, this representation is consistent both with ambiguity aversion $(c < \frac{1}{2})$ and an ambiguity lover regulator $(c > \frac{1}{2})$ [16]. The absence of ambiguity (or simple

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[13] A Choquet-Brownian motion is a distorted Wiener process, where the distortion derives from the nature and intensity of preferences towards ambiguity (see Kast, Lapied and Roubaud, 2010, Section 2.2, for more details on the construction of Choquet-Brownian processes).

[14] See Eq. (2) in Section 2

[15] We follow Kast, Lapied and Roubaud (2010) for the derivation of the modified process capturing economic uncertainty over future costs and benefits of policy adoptions. Specifically, let X_t follow a geometric Brownian motion: $dX_t = \alpha X_t dt + \sigma_2 X_t dW_t^2$, in the absence of ambiguity. This is the same as for the process X_t in the previous section. Within the framework of ambiguity in continuous-time of Kast, Lapied and Roubaud (2010), Eq. (12) is obtained by $dX_t = \alpha X_t dt + \sigma_2 X_t dW_t^2$ with $dW_t^2 = mdt + sd\tilde{W}_t^2$, where W_t^2 is a standard Brownian motion with mean $m = 2c - 1$ and variance $s^2 = 4c(1-c)$.

[16] For example, if the regulator is ambiguity averse $c < \frac{1}{2}$. Consequently, $0 < c < \frac{1}{2}$ implies $-1 < m < 0$ and $0 < s < 1$, and then $\alpha + m\sigma_2 < \alpha$ and $0 < s\sigma_2 < \sigma_2$. In other words, ambiguity aversion generates a reduction of the instantaneous mean and also of the volatility.

uncertainty) is included as a special case when $c = \frac{1}{2}$, so that $m = 0$ and $s = 1$. As usual, it is assumed that the no-bubble condition holds, where $\alpha + m\sigma_2 < r$. Moreover, $\sigma_2 > 0$ and $\tilde{W}_t^2$ is a standard Brownian motion which is assumed to be independent of W_t^1, i.e. $corr\left[W_t^1, \tilde{W}_t^2\right] = 0$.

As in the previous section, the flow of social costs associated with the stock variable M_t is the quadratic form:

$$D(X_t, M_t) = X_t M_t^2,$$

and the cost of permanently reducing the emission flows is given by (9).

4.1 Optimal environmental policies

In this subsection, we discuss the solution for the optimal stopping problem (10) subject to Eq. (11) for the evolution of M_t and Eq. (12) for the evolution of X_t. In order to solve the corresponding Hamilton-Jacobi-Bellman equations analytically and to provide the further characterization of the value of adopting the environmental policies, it is assumed as in Pindyck (2002) that the pollutant stock has a zero natural decay rate (i.e. $\delta = 0$)[17]. In other cases, numerical computations to solve the equations have to be performed.

Applying the Dixit and Pindyck (1994) methodology, we can derive the optimal timing for the two environmental policies and the values to reduce emissions under the two environmental policies (see the Appendix). In particular, in order to compare the effects of taxes and quotas on the optimal timing for policy adoption in the presence of ecological uncertainty and Choquet-Brownian ambiguity, we can compute the two thresholds X_T^* and X_Q^* , such that it is optimal to adopt policy i for $X > X_i^*$, as:

$$X_T^* = \left[\frac{\left(\Psi^N - \Psi_T^A\right)\theta^\lambda}{r} + K\right]\left(\frac{\phi\varpi^3}{2\Lambda\left(\phi - 1\right)}\right),$$

$$X_Q^* = \left[\frac{\left(\Psi^N - \Psi_Q^A\right)\theta^\lambda}{r} + K\right]\left(\frac{\phi\varpi^3}{2\Lambda\left(\phi - 1\right)}\right),$$

where $\varpi = (r - (\alpha + m\sigma_2))$, $\Lambda = M\varpi\left(\xi^N - \xi^A\right) + \left[\left(\xi^N\right)^2 - \left(\xi^A\right)^2\right]$ and ϕ is the positive solution ($\phi > 1$) to the standard characteristic equation. The following Proposition 3 provides the most comprehensive result and confirms that quotas outperform taxes if early adoption is our concern:

[17] The assumption that $\delta = 0$ is appropriate in the case that the pollutant M_t represents chlorofluorocarbon (CFCs) or methane (CH_4) that cause severe damage on the environment and do not depreciate at all once they are released into the atmosphere and the ocean.

Proposition 3 *In the presence of ecological uncertainty and Choquet-Brownian ambiguity: (1) the optimal adoption threshold under taxes is larger than the adoption threshold under fixed quota for any $\zeta > 0$, (2) the value of the option to reduce emission under taxes is smaller than the value of the option to reduce emission under quota for any $\zeta > 0$.*

Proof. *in the Appendix* ■

Remark 4 *In the special case when there is ecological uncertainty and no-ambiguity (i.e. when $c = \frac{1}{2}$) the optimal stopping boundary X_i^{**} (for $i = T, Q$), is given by:*

$$X_i^{**} = \left[\frac{\left(\Psi^N - \Psi_i^A\right) \theta^\lambda}{r} + K \right] \left(\frac{\gamma \kappa_3^3}{2\Upsilon (\gamma - 1)} \right),$$

and the variable A_i^{**} which enters the value option is given by:

$$A_i^{**} = \left(\frac{r(\gamma - 1)}{\left(\Psi^N - \Psi_i^A\right) \theta^\lambda + rK} \right)^{\gamma - 1} \left(\frac{2\Upsilon}{\gamma \kappa_3^3} \right)^\gamma e^{-\eta M},$$

where $\kappa_3 = r - \alpha$, $\Omega = \left(\xi^N + \xi^A\right) + M\kappa_3$, $\Upsilon = M\kappa_3 \left(\xi^N - \xi^A\right) + \left[\left(\xi^N\right)^2 - \left(\xi^A\right)^2\right]$, the exponent $\eta(M)$ is given by $\eta = \frac{\gamma \kappa_3}{\Omega}$ and

$$\gamma = \frac{\xi^N \kappa_3 \Omega - \frac{1}{2}\sigma_2^2 \Omega^2 + \alpha \Omega^2}{\sigma_2^2 \Omega^2 + \sigma_1^2 \kappa_3^2} \left[-1 + \sqrt{1 + \frac{2r\left(\sigma_2^2 \Omega^2 + \sigma_1^2 \kappa_3^2\right)}{\left(\xi^N \kappa_3 - \frac{1}{2}\sigma_2^2 \Omega + \alpha \Omega\right)^2}} \right].$$

Note that this result is similar to Pindyck (2002), although we consider the optimal timing of adopting two alternative environmental policies, i.e. taxes and non-tradable quotas, while Pindyck (2002) considers the optimal timing for a single environmental policy adoption.

In the next section some numerical results and a sensitivity analysis are presented to deepen our understanding of the effects of environmental taxes and quotas on the optimal timing of emission reductions and to analyze the effects of a change in perceived ambiguity.

5 Numerical application

In this section we provide some numerical applications to the optimal timing of adopting the environmental policies in the presence of ecological uncertainty and ambiguity. The change in perceived ambiguity will be examined as a deviation from the base case (or neutral case) of $c = 0.5$, which describes absence of ambiguity. In order to implement the analytical solutions and study their sensitivity analysis with respect to important value drivers we use Mathematica Programming. We use as much as possible the same parameter values as in Pindyck (2002) and Van Soest (2005). In particular, we assume that in the base case: $\alpha = 0$ (drift-rate of economic uncertainty in the absence of ambiguity), $r = 0.06$ (risk-free interest rate), $c = 0.5$ (absence of ambiguity), $\sigma_1 = 1.000.000$ (volatility of ecological uncertainty), $\sigma_2 = 0.05$ (volatility of economic uncertainty in the absence of ambiguity), $M = 10.000.000$ (tons), $k_1 = 1$ (proportional cost), $k_2 = 1.5$ (adjustment cost), $\beta = 1$ (emissions per unit of output), $\theta = 1$ (energy efficiency), $a = 0.65$ (output elasticity), $p = 1$ (price of output), $z = 0.2$ (cost of energy), $\zeta = 0.1$ (environmental tax).

Figure 1 shows the relation between the critical threshold $X_i^{**}(M)$ and the current pollutant stock M in the absence of ambiguity (i.e. when $c = \frac{1}{2}$). The effect of ecological uncertainty is investigated through two different curves. The dashed curve illustrates the sensitivity of the critical threshold under taxes, $X_T^{**}(M)$, with the current pollutant stock M, absent ambiguity, while the solid curve illustrates the sensitivity of the critical threshold under quotas, $X_Q^{**}(M)$, with the current pollutant stock M, absent ambiguity. We will consider current pollutant stock M in the range of $10 - 50$ Million tons. As we would expect the optimal adoption threshold under taxes is greater than the adoption threshold under fixed quota. As in Pindyck (2002), a larger M implies a larger social cost of environmental damage, and thus lower values of X at which it is optimal to adopt the policies.

Figure 2 shows the relation between the critical threshold $X_i^{**}(M)$ and the volatility σ_2 in the absence of ambiguity. The dashed curve illustrates the sensitivity of the critical threshold under taxes with the volatility σ_2 absent ambiguity, while the solid curve illustrates the sensitivity of the critical threshold under quotas with the volatility σ_2 absent ambiguity. We will consider values of the volatility σ_2 ranging from 0 to 1. As in Pindyck (2002), increases of economic uncertainty over future payoffs from reduced emissions increase the value of waiting, and raise the critical thresholds $X_T^{*}(M)$ and $X_Q^{*}(M)$.

Figure 3 shows the relation between the critical threshold $X_i^{**}(M)$ and the tax rate ζ in the absence of ambiguity. The dashed curve illustrates the sensitivity of the critical threshold under taxes with the tax rate ζ absent ambiguity, while the solid curve illustrates the sensitivity of the critical threshold under quotas with the tax rate ζ absent ambiguity. We will consider values of the tax rate ζ ranging from 0 to 1. A larger ζ affects the net benefit from reduced emissions and thus induces a larger optimal timing of adopting the environmental policies. Thus, more stringent environmental policies, measured in terms of higher tax rates, do not result in earlier adoption.

Finally, Figure 4 shows the effect of ambiguity on the optimal timing of adopting the environmental policies. The dashed curve illustrates the sensitivity of the critical threshold under taxes, $X_T^*(M)$, with the degree of c-ignorance, while the solid curve illustrates the sensitivity of the critical threshold under quotas, $X_Q^*(M)$, with the degree of c-ignorance. We will consider values of the degree of c-ignorance ranging from 0 to 1. A move down the interval $]0, 0.5[$ for c so that $m < 0$, $\alpha + m\sigma_2$ decreases and ϖ increases, is associated with an increase in ambiguity aversion and, correspondingly, an increase in the thresholds, that is, the regulator cannot pin down the possible scenarios of changes in the future and therefore becomes more cautious than before. A move up the interval $]0.5, 1[$ for c, so that $m > 0$, $\alpha + m\sigma_2$ increases and ϖ decreases, is associated with a more ambiguity lover regulator and, correspondingly, a decrease in the thresholds, that is, early adoption is stimulated. As in Kast, Lapied and Roubaud (2010) a larger c implies a lower value of the threshold X at which it is optimal to adopt the policies. In particular, an ambiguity averse environmental regulator (i.e. when $0 < c < 0.5$) will delay the adoption of the new policies, while an ambiguity seeker regulator (i.e. when $0.5 < c < 1$) will adopt them earlier than if he were ambiguity neutral (i.e. when $c = \frac{1}{2}$).

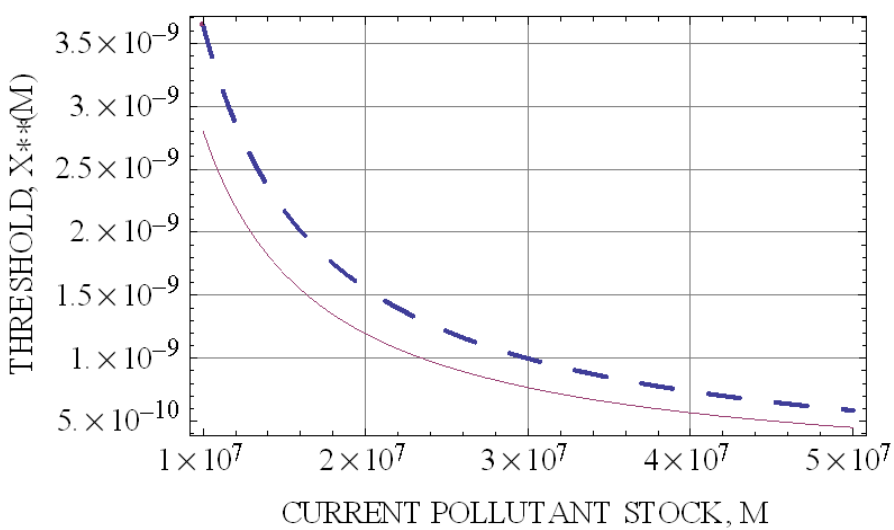

FIGURE 1: Relation between the critical threshold $X_T^{**}(M)$ and the current pollutant stock M absent ambiguity (dashed curve). Relation between the critical thresholds $X_Q^{**}(M)$ and the current pollutant stock M absent ambiguity (solid curve).

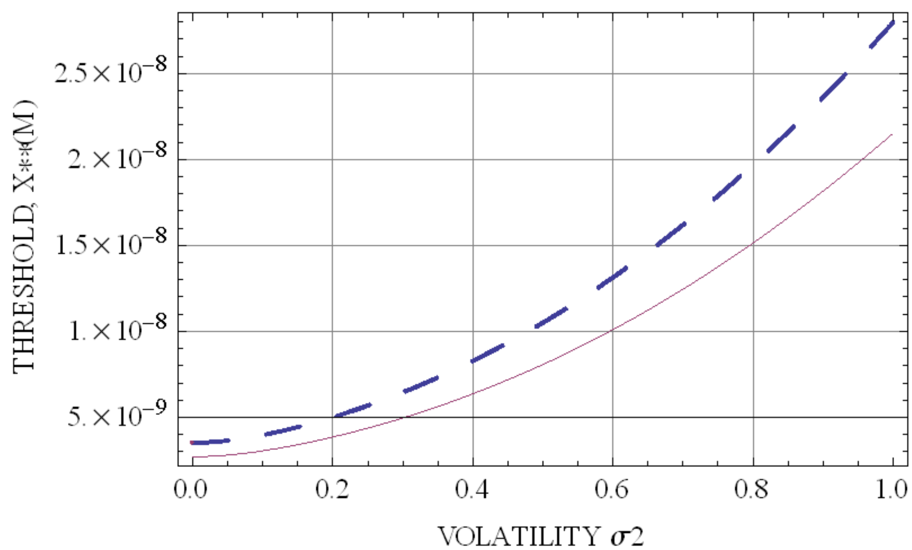

FIGURE 2: Relation between the critical threshold $X_T^{**}(M)$ and the volatility σ_2 absent ambiguity (dashed curve). Relation between the critical thresholds $X_Q^{**}(M)$ and the volatility σ_2 absent ambiguity (solid curve).

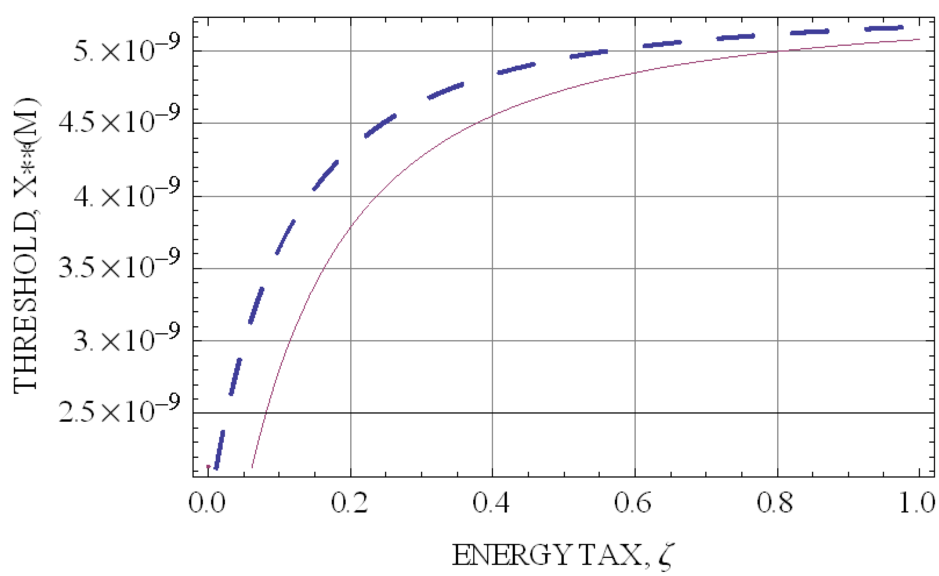

FIGURE 3: Relation between the critical threshold $X_T^{**}(M)$ and the energy tax ζ absent ambiguity (dashed curve). Relation between the critical thresholds $X_Q^{**}(M)$ and the energy tax ζ absent ambiguity (solid curve).

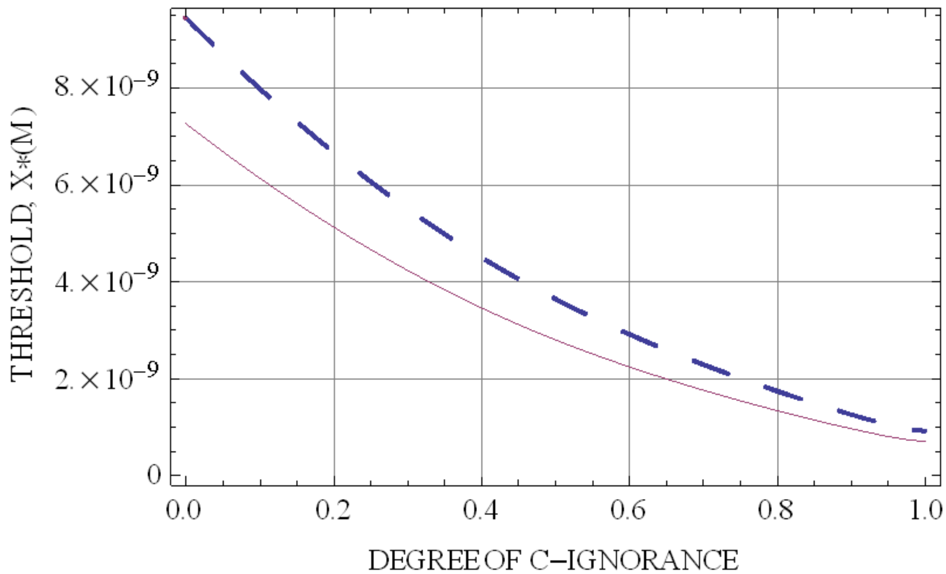

FIGURE 4: Relation between the critical threshold $X_T^*(M)$ and the degree of $c-$ignorance (dashed curve). Relation between the critical thresholds $X_Q^*(M)$ and and the degree of $c-$ignorance (solid curve).

6 Conclusion

The choice of environmental policy instruments has been the object of a substantial amount of attention in the recent debate, both in the literature and for policy prescriptions. Our paper contributes to the debate by discussing the impact of alternative policy instruments, i.e. taxes and quotas, on the optimal timing of emission reductions. A main conclusion is that the optimal adoption threshold under taxes is always larger than under quotas. Thus, in our model there is an unambiguous ranking of these policy instruments in terms of the adoption lags: if regulators wish to speed up the implementation of technologies reducing pollution emissions, then they may prefer quotas to taxes. This result is robust to various relevant parameters and changes in perceived ambiguity. Actually, a more stringent environmental policy, measured in terms of higher tax rates, further delays adoption. A more ambiguity averse regulator becomes more cautious in adopting the environmental policy options. Our sensitivity analysis provides a clue to regulators who are faced with environmental issues where economic costs and benefits cannot be forecasted and a lot of scenarios can be considered. Another alternative is to analyze the economic and policy consequences of catastrophic events (such as catastrophic climate change) and comprehensive damages, which should be fat tailed distributed, as Weitzman (2009) points out. Despite its adequacy to deal with extreme events, which are a realistic occurrence in environmental issues, a framework of fat-tailed cost benefit analysis of climate change within the real option approach will be analytically untractable in general and require numerical methods.

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7 Appendix

Proofs of Propositions 1 and 2.

In this Appendix we derive the optimal timing for the environmental policy i. We solve the problem by stochastic dynamic programming. Let $W_i^N = W_i^N(X,M)$ denote the value function for the non-adopt region (in which $\xi_t = \xi^N$). The corresponding Hamilton-Jacobi-Bellman equation is:

$$rW_i^N = B^N(X,M) + \left(\xi^N - \delta M\right)\frac{\partial W_i^N}{\partial M} + \alpha X\frac{\partial W_i^N}{\partial X} + \frac{1}{2}\sigma^2 X^2 \frac{\partial^2 W_i^N}{\partial X^2}.$$

It has the following general solution:

$$W_i^N(X,M) = A_{i1}X^{\phi_1} + A_{i2}X^{\phi_2} + \left[\frac{\Psi^N \theta^\lambda}{r} - \frac{XM^2}{\kappa_1} - \frac{2XM\xi^N}{\kappa_1\kappa_2} - \frac{2X\left(\xi^N\right)^2}{\kappa_1\kappa_2\kappa_3}\right] \tag{13}$$

where A_{i1} and A_{i2} are unknowns to be determined, $\kappa_1 = r+2\delta-\alpha$, $\kappa_2 = r+\delta-\alpha$, and $\kappa_3 = r-\alpha$. Here, ϕ_1 and ϕ_2 are the solution to the following characteristic equation:

$$\frac{1}{2}\sigma^2\phi(\phi-1)+\phi\alpha-r=0$$

and are given by:

$$\phi_1 = \frac{1}{2}-\frac{\alpha}{\sigma^2}+\sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} > 1$$

$$\phi_2 = \frac{1}{2}-\frac{\alpha}{\sigma^2}-\sqrt{\left(\frac{1}{2}-\frac{\alpha}{\sigma^2}\right)^2+\frac{2r}{\sigma^2}} < 0.$$

The term between the squared parentheses in (13) is a particular solution, which captures the expected net benefit from emissions in the case the environmental regulator has not adopted the policy and is calculated as[18]:

$$\mathbb{E}\left\{\int_0^\infty e^{-rt}B^N\left(X_t,M_t\right)dt\right\} =$$

$$= \int_0^\infty e^{-rt}\Psi^N\theta^\lambda dt - \int_0^\infty e^{-rt}Xe^{\alpha t}\left[\frac{\xi^N}{\delta}+\left(M-\frac{\xi^N}{\delta}\right)e^{-\delta t}\right]^2 dt$$

$$= \frac{\Psi^N\theta^\lambda}{r}-\frac{XM^2}{\kappa_1}-\frac{2XM\xi^N}{\kappa_1\kappa_2}-\frac{2X\left(\xi^N\right)^2}{\kappa_1\kappa_2\kappa_3},$$

where ξ^N is given by (5). Therefore, the parenthesis in (13) represents the fundamental term and the exponential terms account for the perpetual American option value.

Next, let $W_i^A = W_i^A(X,M)$ denote the value function for the adopt region (in which $\xi_t = \xi^A$). The corresponding Hamilton-Jacobi-Bellman equation is:

$$rW_i^A = B_i^A\left(X,M\right)+\left(\xi^A-\delta M\right)\frac{\partial W_i^A}{\partial M}+\alpha X\frac{\partial W_i^A}{\partial X}+\frac{1}{2}\sigma^2X^2\frac{\partial^2 W_i^A}{\partial X^2}.$$

When we consider a one-time reduction in ξ_t, there is no option term after pollutant emissions have been reduced to ξ^A, so the solution for $W_i^A\left(X,M\right)$ is given by:

$$W_i^A\left(X,M\right) = \frac{\Psi_i^A\theta^\lambda}{r}-\frac{XM^2}{\kappa_1}-\frac{2XM\xi^A}{\kappa_1\kappa_2}-\frac{2X\left(\xi^A\right)^2}{\kappa_1\kappa_2\kappa_3}.$$

[18] When $\xi_t = \xi^N$, the solution of Eq. (1) is given by: $M_t = \frac{\xi^N}{\delta}+\left(M_0-\frac{\xi^N}{\delta}\right)e^{-\delta t}$.

where the right-hand side captures the expected discounted value of $B_i^A(X_t, M_t)$ in the case the environmental regulator has adopted the policy i, and is calculated as:

$$\mathbb{E}\left\{\int_0^\infty e^{-rt} B_i^A(X_t, M_t)\, dt\right\} = \frac{\Psi_i^A \theta^\lambda}{r} - \frac{XM^2}{\kappa_1} - \frac{2XM\xi^A}{\kappa_1\kappa_2} - \frac{2X\left(\xi^A\right)^2}{\kappa_1\kappa_2\kappa_3}.$$

We know that the solutions for $W_i^N(X, M)$ and $W_i^A(X, M)$ must satisfy the following set of boundary conditions (see Pindyck 2000, 2002):

$$W_i^N(0, M) = 0, \tag{14}$$

$$W_i^A(0, M) = 0, \tag{15}$$

$$W_i^N(\bar{X}_i(M), M) = W_i^A\left(\bar{X}_i(M), M\right) - K, \tag{16}$$

and:

$$\frac{\partial W_i^N(\bar{X}_i(M), M)}{\partial X} = \frac{\partial W_i^A\left(\bar{X}_i(M), M\right)}{\partial X}. \tag{17}$$

Here, $\bar{X}_i(M)$ is a free boundary, which must be found as part of the solution, and which separates the adopt from the no-adopt regions. It is also the solution to the stopping problem (10):

$$\tau_i = \inf\left\{t > 0,\ X \geq \bar{X}_i(M)\right\}$$

Given M, the policy i should be adopted the first time the process X_t crosses the threshold $\bar{X}_i(M)$ from below. Boundary conditions (14) and (15) reflect the fact that if X_t is ever zero, it will remain at zero thereafter. Conditions (16) and (17) are the value matching and the smooth-pasting conditions, respectively. Conditions (14) and (15) imply:

$$W_i^N(0, M) = \frac{\Psi^N \theta^\lambda}{r}$$

and

$$W_i^A(0, M) = \frac{\Psi_i^A \theta^\lambda}{r}.$$

Accordingly, we disregard the negative root in order to prevent the value from becoming infinitely large when X_t tends to 0; thus, we set $A_{i2} = 0$ (see Dixit and Pindyck 1994). So (13) becomes:

$$W_i^N(X, M) = A_{i1}X^{\phi_1} + \frac{\Psi^N \theta^\lambda}{r} - \frac{XM^2}{\kappa_1} - \frac{2XM\xi^N}{\kappa_1\kappa_2} - \frac{2X\left(\xi^N\right)^2}{\kappa_1\kappa_2\kappa_3}. \tag{18}$$

The first term on the right-hand side of Eq. (18) is the value of the option to adopt policy i and reduce emissions to ξ^A, while the remaining terms represent the expected discounted value of $B^N(X_t, M_t)$.

The value matching condition (16) can be rearranged in the following manner:

$$A_{i1}\left(\bar{X}_i\right)^{\phi_1} = \frac{2\Theta\bar{X}_i}{\kappa_1\kappa_2\kappa_3} - \frac{\left(\Psi^N - \Psi_i^A\right)\theta^\lambda}{r} - K, \tag{19}$$

where:

$$\Theta = M\kappa_3\left(\xi^N - \xi^A\right) + \left[\left(\xi^N\right)^2 - \left(\xi^A\right)^2\right].$$

The smooth-pasting condition (17) yields:

$$A_{i1} = \frac{1}{\phi_1\left(\bar{X}_i\right)^{\phi_1 - 1}}\left(\frac{2\Theta}{\kappa_1\kappa_2\kappa_3}\right), \tag{20}$$

Plugging (20) into (19), we get the expressions reproduced in Section 3:

$$\bar{X}_i = \left(\frac{\left(\Psi^N - \Psi_i^A\right)\theta^\lambda}{r} + K\right)\left(\frac{\kappa_1\kappa_2\kappa_3\phi_1}{2\Theta\left(\phi_1 - 1\right)}\right). \tag{21}$$

Finally, substituting (21) into (20), we get:

$$A_{i1} = \left(\frac{r\left(\phi_1 - 1\right)}{\left(\Psi^N - \Psi_i^A\right)\theta^\lambda + rK}\right)^{\phi_1 - 1}\left(\frac{2\Theta}{\phi_1\kappa_1\kappa_2\kappa_3}\right)^{\phi_1},$$

Proof of Proposition 3.

Let us find a solution for the optimal stopping problem (10) subject to Eq. (11) for the evolution of M_t and Eq. (12) for the evolution of X_t. In this framework, the value function for the no-adopt region, $W_i^N(X, M)$ must satisfy the Hamilton-Jacobi-Bellman equation:

$$rW_i^N = B^N(X, M) + \left(\xi^N - \delta M\right)\frac{\partial W_i^N}{\partial M} + \left(\alpha + m\sigma_2\right)X\frac{\partial W_i^N}{\partial X} + \frac{1}{2}\left(s\sigma_2\right)^2 X^2\frac{\partial^2 W_i^N}{\partial X^2} + \frac{1}{2}\sigma_1^2\frac{\partial^2 W_i^N}{\partial M^2}, \tag{22}$$

and the value function for the adopt region, $W_i^A(X, M)$, must satisfy the Hamilton-Jacobi-Bellman equation:

$$rW_i^A = B_i^A(X,M) + \left(\xi^A - \delta M\right)\frac{\partial W_i^A}{\partial M} + (\alpha + m\sigma_2)X\frac{\partial W_i^A}{\partial X} + \frac{1}{2}(s\sigma_2)^2 X^2 \frac{\partial^2 W_i^A}{\partial X^2} + \frac{1}{2}\sigma_1^2\frac{\partial^2 W_i^N}{\partial M^2}.$$

The solutions for $W_i^N(X,M)$ and $W_i^A(X,M)$ must satisfy the following set of boundary conditions:

$$W_i^N(0,M) = 0, \tag{23}$$

$$W_i^A(0,M) = 0, \tag{24}$$

$$W_i^N(X_i^*(M), M) = W_i^A(X_i^*(M), M) - K, \tag{25}$$

$$\frac{\partial W_i^N(X_i^*(M), M)}{\partial X} = \frac{\partial W_i^A(X_i^*(M), M)}{\partial X}, \tag{26}$$

and:

$$\frac{\partial W_i^N(X_i^*(M), M)}{\partial M} = \frac{\partial W_i^A(X_i^*(M), M)}{\partial M}, \tag{27}$$

where $X_i^*(M)$ is the critical value of X at or above which the optimal environmental policies should be adopted. From now on, let us suppose for simplicity that the pollutant stock has a zero natural decay rate (i.e. $\delta = 0$).

We first calculate the expected discounted value of $B^N(X_t, M_t)$ and $B_i^A(X_t, M_t)$:

$$\mathbb{E}\left\{\int_0^\infty e^{-rt}B^N(X_t, M_t)\,dt\right\} =$$

$$= \int_0^\infty e^{-rt}\Psi^N\theta^\lambda dt - \int_0^\infty e^{-rt}Xe^{(\alpha+m\sigma_2)t}\left[M^2 + \left(\xi^N\right)^2 t^2 - \sigma_1^2 t + 2M\xi^N t\right]dt$$

$$= \frac{\Psi^N\theta^\lambda}{r} - \frac{XM^2}{\varpi} - \frac{2XM\xi^N}{\varpi^2} - \frac{2X\left(\xi^N\right)^2}{\varpi^3} - \frac{X\sigma_1^2}{\varpi^2},$$

and:

$$\mathbb{E}\left\{\int_0^\infty e^{-rt}B_i^A(X_t, M_t)\,dt\right\} = \frac{\Psi_i^A\theta^\lambda}{r} - \frac{XM^2}{\varpi} - \frac{2XM\xi^A}{\varpi^2} - \frac{2X\left(\xi^A\right)^2}{\varpi^3} - \frac{X\sigma_1^2}{\varpi^2}.$$

where $\varpi = (r - (\alpha + m\sigma_2))$. Therefore, the solutions $W_i^N(X,M)$ and $W_i^A(X,M)$ are given by:

$$W_i^N(X,M) = G_i(M)X^\phi + \frac{\Psi^N\theta^\lambda}{r} - \frac{XM^2}{\varpi} - \frac{2XM\xi^N}{\varpi^2} - \frac{2X\left(\xi^N\right)^2}{\varpi^3} - \frac{X\sigma_1^2}{\varpi^2},$$

and

$$W_i^A(X,M) = \frac{\Psi_i^A \theta^\lambda}{r} - \frac{XM^2}{\varpi} - \frac{2XM\xi^A}{\varpi^2} - \frac{2X\left(\xi^A\right)^2}{\varpi^3} - \frac{X\sigma_1^2}{\varpi^2},$$

where $G_i(M) = A_i^* e^{\Gamma M}$, $w_i^A = A_i^* e^{\Gamma M} X^\phi$ is the homogeneous solution to the partial differential equation (22) and $G_i(M)X^\phi$ is the value of the option to adopt policy i and reduce emissions to ξ^A.

Here, Γ and ϕ are the solutions to the following characteristic equation:

$$\frac{1}{2}(s\sigma_2)^2 \phi(\phi-1) + \frac{1}{2}\sigma_1^2\Gamma^2 + \xi^N\Gamma + (\alpha + m\sigma_2)\phi - r = 0, \tag{28}$$

which need to be found, together with A_i^* and $X_i^*(M)$ using the boundary conditions (23)-(27).

Boundary condition (25) implies:

$$A_i^* e^{\Gamma M}(X_i^*)^\phi = \frac{2\Lambda X_i^*}{\varpi^3} - \frac{\left(\Psi^N - \Psi_i^A\right)\theta^\lambda}{r} - K, \tag{29}$$

where:

$$\Lambda = M\varpi\left(\xi^N - \xi^A\right) + \left[\left(\xi^N\right)^2 - \left(\xi^A\right)^2\right].$$

The smooth-pasting condition (26) yields:

$$\phi A_i^* e^{\Gamma M}(X_i^*)^{\phi-1} = \frac{2\Lambda}{\varpi^3}, \tag{30}$$

and finally the smooth-pasting condition (27) yields:

$$A_i^* e^{\Gamma M}(X_i^*)^\phi = \frac{2X_i^*\left(\xi^N - \xi^A\right)}{\varpi^2\Gamma}. \tag{31}$$

Plugging (31) into (30) and after some algebra, we find:

$$\Gamma = \frac{\phi\varpi}{\left(\xi^N + \xi^A\right) + M\varpi}.$$

Then we plug (31) into (29) and obtain:

$$X_i^* = \left[\frac{\left(\Psi^N - \Psi_i^A\right)\theta^\lambda}{r} + K\right]\left(\frac{\phi\varpi^3}{2\Lambda(\phi-1)}\right). \tag{32}$$

By substituting (32) into (30), we obtain:

$$A_i^* = \left(\frac{r\left(\phi - 1\right)}{\left(\Psi^N - \Psi_i^A\right)\theta^\lambda + rK} \right)^{\phi - 1} \left(\frac{2\Lambda}{\phi \varpi^3} \right)^\phi e^{-\Gamma M}.$$

To find ϕ we plug Γ into (28) and solve the equation for ϕ. By the standard real options argument ϕ must be larger than 1. Setting $\Phi = \left(\xi^N + \xi^A\right) + M\varpi$, straightforward calculation yields:

$$\phi = \frac{\xi^N \varpi \Phi - \frac{1}{2}(s\sigma_2)^2 \Phi^2 + (\alpha + m\sigma_2)\Phi^2}{(s\sigma_2)^2 \Phi^2 + \sigma_1^2 \varpi^2} \left[-1 + \sqrt{1 + \frac{2r\left((s\sigma_2)^2 \Phi^2 + \sigma_1^2 \varpi^2\right)}{\left(\xi^N \varpi - \frac{1}{2}(s\sigma_2)^2 \Phi + (\alpha + m\sigma_2)\Phi\right)^2}} \right].$$