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A reversal result in a unionized duopoly

*Discussion Paper n. 112*

2011
Discussion Paper n. 112, presentato: febbraio 2011

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Si prega di citare così:
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Abstract

This paper aims at investigating if the conventional wisdom, that a decrease in the degree of product differentiation (which implies increasing competition) always reduces firms’ profits, remains true in a differentiated duopoly model with decentralized, or firm-specific, monopoly unions. In this context, when product differentiation decreases, an important effect, termed “endogenous” or “union wage effect”, adds to the standard competition effect in affecting profits. Moreover, the union wage effect operates against the competition effect and, provided that unions are sufficiently wage-oriented, that is, they sufficiently prefer wages to employment, can actually reverse the conventional result under both Cournot and Bertrand competition. However, this is more likely to occur under competition à la Cournot.

Classificazione JEL: J43, J50, L13
Keywords: unionized duopoli, monopoly unions, product differentiation, profits
1 Introduction

A conventional wisdom in industrial economics suggests that a decrease in the degree of product differentiation always reduces firms’ profits by increasing the intensity of product market competition, irrespective of the mode of competition, i.e. irrespective of the fact that firms compete à la Cournot or à la Bertrand in the product market (e.g. Shy, 1995, pp. 138-140). The theoretical reason behind this result can be understood by referring to the standard differentiated duopoly model, due to Singh and Vives (1984), in which a decrease in the degree of product market differentiation diminishes total demand and induces firms to compete more aggressively. Under both quantity and price competition, this unambiguously leads to lower firms’ profits.

Whilst in the standard Singh and Vives’s (1984) model firms’ marginal production costs are assumed to be exogenously given, the growing literature on unionized oligopolies (see, e.g., the seminal works by Horn and Wolinsky (1988) and Dowrick (1989)) relaxes such assumption by admitting that (labor) costs are the outcome of a strategic game played between firms and unions before the former compete in the product market. This paper investigates if the conventional wisdom, that a decrease in the degree of product differentiation always reduces firms’ profits, remains true or can be reversed in a unionized duopoly model with decentralized, or firm-specific, monopoly unions.\(^1\)

Our main results can be summarized as follows. When firm-specific unions endogenously fix wages and product differentiation decreases, another important effect, that we will term endogenous or union wage effect, affects

\(^1\) Correa-Lopez and Naylor (2004) study a unionized duopolistic model to show that a “reversal result” can apply in relation to the ranking of Cournot and Bertrand equilibrium profits. Fanti and Meccheri (2011) introduce labor diminishing returns into the Correa-Lopez and Naylor (2004) framework and find that they tend to reinforce the mechanisms that contribute to the reversal result, making this event possible for a wider range of situations.
firms’ profits, together with the standard *competition effect*.\(^2\) Whilst the standard effect acts in reducing profits, the former always operates in the opposite direction, irrespective of the mode of competition. We highlight that, provided that unions are sufficiently wage-oriented (i.e. they sufficiently prefer wages to employment), the union wage effect can outweigh the competition effect, hence actually reverse the conventional result. Indeed, although the higher the degree of unions’ preferences towards wages with respect to employment, the higher the equilibrium wage, more wage-oriented unions can reduce wages more strongly when product differentiation decreases. Furthermore, the possibility for the “reversal result” to apply is higher when firms compete à la Cournot, instead of à la Bertrand, in the product market. Recently, Zanchettin (2006, section 4) also deals with the issue of this paper, i.e. the impact of the degree of product differentiation on equilibrium profits under Cournot and Bertrand competition.\(^3\) In particular, Zanchettin (2006) modifies the Singh and Vives (1984) original framework by allowing for a wider range of cost and demand asymmetry between firms, and finds that, under both modes of competition, the *efficient firm*’s profit and industry profits as a whole can decrease with the degree of product differentiation. Our paper differs from Zanchettin (2006) because we relax the Singh and Vives’s standard assumptions by introducing the role of unions in determining wages into the analysis, instead of the presence of asymmetric firms. As a consequence, our findings, as well as the economic mechanisms behind them, are different. Most notably, our results suggest that, when the reversal result applies, both firms’ profits increase with decreasing product differentiation (or

\(^2\) The union wage effect on profits relates to the fact, highlighted first by Dhillon and Petrakis (2002, section 4), that wages are decreasing in the degree of product substitutability. In this regard, as we will discuss below, the hypothesis of firm-specific unions play a role. Particularly, results would change qualitatively if unions were centralized (or coordinated) and the well known “wage-rigidity result” by Dhillon and Petrakis (2002) applies.

\(^3\) Zanchettin’s (2006) main purpose, however, is to compare, in a differentiated duopoly with asymmetric firms, Cournot and Bertrand equilibria.
increasing market competition), whilst in Zanchettin (2006) this only occurs with regard to the most efficient firm.

The remaining part of the paper is organized as follows. Section 2 describes the basic model and characterizes Cournot and Bertrand equilibria under exogenous wages. Section 3 introduces and analyzes the role played by (firm-specific) unions in determining wages. The main results on the relationship between the degree of product differentiation (or market competition) and profits under different modes of competition are presented and discussed in Section 4. Finally, Section 5 provides some concluding remarks.

2 The model

Following Singh and Vives (1984), we consider a model of differentiated product market duopoly, in which each firm sets its output, given predetermined wages, to maximize profits. Preferences of the representative consumer are given by:

\[ U(q_i, q_j) = \frac{1}{2} \left( q_i^2 + 2\gamma q_i q_j + q_j^2 \right) \]

where \( q_i \) and \( q_j \) denote outputs by firm \( i \) and \( j \), respectively, \( \alpha > 0 \), and \( \gamma \in (0, 1) \) denotes the extent of product differentiation, with goods assumed to be imperfect substitutes. In particular, notice that when \( \gamma \to 1 \), the products of the two firms tend to be undifferentiated, hence firms compete \textit{de facto} in the same market. At the other extreme, when \( \gamma \to 0 \), a monopoly tends to affirm in this market. Hence, the higher \( \gamma \), the higher the degree of competition in the product market. The derived product market demand for the representative firm \( i \) is linear and given by:

\[ U(q_i, q_j) = \alpha(q_i + q_j) - \frac{1}{2} \left( q_i^2 + 2\gamma q_i q_j + q_j^2 \right) \]

In fact, the following specification partly differs from Singh and Vives (1984) since we normalize to one the coefficients of the squared terms in the utility function (e.g. Correa-Lopez and Naylor, 2004; Zanchettin, 2006). This simplifies the following analysis without loss of generality.
Following standard assumptions in the literature, let assume that only labor input is used for production and that it exhibits constant returns, that is, \( q_i = l_i \), where \( l_i \) represents the number of workers employed by the firm \( i \) to produce \( q_i \) output units of the variety \( i \). Hence, the firm \( i \)'s profit can be written as:

\[
\pi_i = p_i q_i - w_i q_i
\]

where \( w_i \) is the per-worker wage paid by the firm \( i \), with \( w_i < \alpha \). In what follows, we will consider, according to the different modes of product market competition, the benchmark cases, in which wages are assumed to be exogenously given for firms. In Section 3, instead, we will introduce the role of unions in determining wages into the analysis.

### 2.1 Cournot competition with exogenous wages

Taking (2) and (3) into account, profit-maximization under Cournot competition leads to the following firm \( i \)'s best-reply function in the output space:

\[
q_i (q_j) = \frac{\alpha - \gamma q_j - w_i}{2}.
\]

As \( \gamma > 0 \), the best-reply functions are downward-sloping, that is, under the Cournot assumption, the product market game is played in strategic substitutes. From (4), and its equivalent for the firm \( j \), we can obtain, for given \( w_i \) and \( w_j \), the firm \( i \)'s output as:

\[
q_i (w_i, w_j) = \frac{(2 - \gamma) \alpha - 2 w_i + \gamma w_j}{4 - \gamma^2}
\]
and, by substituting (5) in (3), the firm $i$’s profit as:

$$\pi_i(w_i, w_j) = \frac{\left[\alpha(2 - \gamma) - 2w_i + \gamma w_j\right]^2}{(4 - \gamma^2)^2}.$$  

By assuming exogenous wages, we have $w_i = w_j = \bar{w}$, hence, by substituting in (6), we get equilibrium profits with exogenous wages as:

$$\pi_i = \pi_j = \pi_C = \frac{(\alpha - \bar{w})^2}{(2 + \gamma)^2}.$$  

where the subscript $C$ recalls that they are obtained under Cournot competition in the product market. As regards the object of this paper, it is easy to check from (7) that, according to the conventional wisdom, profits are positively correlated with the degree of product differentiation (which is decreasing in $\gamma$) or, in other words, are negatively correlated with the degree of market competition (which is increasing in $\gamma$).

### 2.2 Bertrand competition with exogenous wages

We consider now the case in which the product market game is characterized by price-setting behavior by firms, i.e. competition occurs à la Bertrand. From (2) and its counterpart for the firm $j$, we can write product demand for the firm $i$ as:

$$q_i(p_i, p_j) = \frac{\alpha(1 - \gamma) - p_i + \gamma p_j}{(1 - \gamma^2)}$$  

hence, using (3), the firm $i$’s profit is given by:
\( \pi_i = (p_i - w_i) \left[ \frac{\alpha(1 - \gamma) - \gamma p_j}{(1 - \gamma^2)} \right]. \)

From (9), the first-order condition for profit-maximization gives the firm \( i \)'s price choice, as a function of the price chosen by the firm \( j \), as:

\( p_i(p_j) = \frac{\alpha(1 - \gamma) + \gamma p_j + w_i}{2} \)

thus, for \( \gamma > 0 \), the Bertrand product market game is played in strategic complements. By substituting in (10) the corresponding equation for the firm \( j \) and solving for \( p_i \), we get the Bertrand equilibrium price for given wages, \( w_i \) and \( w_j \), as:

\( p_i(w_i, w_j) = \frac{\alpha[2 - \gamma(1 + \gamma)] + 2w_i + \gamma w_j}{4 - \gamma^2} \)

Hence, by substituting in (8), we get the sub-game perfect output as a function of wages as:

\( q_i(w_i, w_j) = \frac{\alpha(2 + \gamma)(1 - \gamma) - (2 - \gamma^2)w_i + \gamma w_j}{(4 - \gamma^2)(1 - \gamma^2)} \)

and, by using (11) and (9), the firm \( i \)'s profit as:

\( \pi_i(w_i, w_j) = \frac{\left[ \alpha(2 + \gamma)(1 - \gamma) - (2 - \gamma^2)w_i + \gamma w_j \right]^2}{\left[ (4 - \gamma^2)(1 - \gamma^2) \right]^2} \)

Again, by assuming exogenous wages, we have \( w_i = w_j = \bar{w} \), hence by substituting in (13), we get equilibrium profits:
where the subscript $B$ recalls that they are obtained under Bertrand competition in the product market. As in the Cournot case, it is easy to check from (14) that profits are positively correlated with the degree of product differentiation, i.e. equilibrium profits always increase when $\gamma$ decreases.

3 The unionized duopoly

A common feature of the standard literature is to implicitly assume that input markets are perfectly competitive, thus ignoring a possible role of the latter in determining the relationship between profits and the degree of market competition (i.e. product differentiation). In many cases, however, such assumption does not properly represent real world situations. For instance, labor markets are often unionized, as emphasized by the growing literature on “unionized oligopolies” (e.g. Horn and Wolinsky, 1988; Dowrick, 1989; Naylor, 1998, 1999; Correa-Lopez and Naylor, 2004; Brekke, 2004; Lommerud et al., 2005; Correa-Lopez, 2007). We join such literature by admitting that labor cost is no longer exogenously given, but it is the outcome of a two-stage strategic game played between each firm and a labor union.

Following the backward induction logic, in stage 2 (as already analyzed in Section 2), each firm decides, according to mode of product market competition its optimal level of output, which also implies its labor demand. In stage 1, instead, a firm-specific “monopolistic” union fixes wages.$^5$ As well known, union objectives are not necessarily dominated by wages. Following

$^5$ As well known, unionization structure may differ significantly around the world (e.g. Layard and Nickell, 1999; OECD, 1999). Indeed, firm-specific unions and decentralized wage setting are largely predominant in Japan and North America countries (see Iverson (1998) for an index on the degree of wage bargaining decentralization in different countries).
earlier works (e.g. Pencavel, 1985; Dowrick and Spencer, 1994; Petrakis and Vlassis, 2000), in order to derive tractable results for wage determination, we assume that the union $i$’s utility takes the Stone-Geary functional form $V_i = (w_i - w^o)^\theta l_i$, where $w^o$ is the reservation wage, while $\theta > 0$ is a parameter that represents the degree of the union’s orientation towards wages with respect to employment. In particular, a value of $\theta = 1$ gives the rent-maximizing case, in which unions place the same weight to wage and employment. Clearly, larger (smaller) $\theta$’s values imply that the union is more (less) concerned about wages and less (more) concerned about employment.

Since each firm-specific union concerns only about wages and employment of its own firm, recalling that $l_i = q_i$, the union $i$’s utility is given by:

$$V_i = (w_i - w^o)^\theta q_i.$$  \((15)\)

Furthermore, since both firms are unionized, unions’ choices take place simultaneously across firms, taking the other firm’s wage as given. Hence, by substituting (5), for the Cournot case, and (12), for the Bertrand case, in (15) and maximizing with respect to $w_i$, we get the sub-game perfect best-reply function in relation to the wage, $w_i(w_j)$, according to the type of competition in the product market as, respectively:

$$w_i(w_j)_C = \frac{w^o}{1 + \theta} + \frac{\theta[\alpha(2 - \gamma) + \gamma w_j]}{2(1 + \theta)}$$  \((16)\)

$$w_i(w_j)_B = \frac{w^o}{1 + \theta} + \frac{\theta[\alpha(2 + \gamma)(1 - \gamma) + \gamma w_j]}{(1 + \theta)(2 - \gamma^2)}.$$  \((17)\)

In symmetric sub-game perfect equilibrium $w_i = w_j = w$, hence, from (16) and (17), equilibrium wages in different competitive contexts are given by, respectively:

$$w_i = w_j = w^E_C = \frac{2w^o + \theta\alpha(2 - \gamma)}{2(1 + \theta) - \gamma\theta}.$$  \((18)\)
where the apex $U$ recalls that they are fixed by (firm-specific) unions.

Finally, in order to derive explicit equilibrium solutions for firms’ profits, we use (18), (19), (7) and (14) to obtain:

\[
\pi_i = \pi_j = \pi_C^U = \frac{4(\alpha - w^\circ)^2}{(2 + \gamma)^2[2(1 + \theta) - \gamma \theta]^2}
\]

\[
\pi_i = \pi_j = \pi_B^U = \frac{(2 - \gamma^2)(1 - \gamma)(\alpha - w^\circ)^2}{(2 - \gamma^2)(1 + \gamma)[2(1 + \theta) - \gamma \theta]^2}.
\]

4 Product differentiation, competition and profits

Referring to the results obtained in the previous section, we are now able to answer to the following issue: do the standard results with exogenous wages, that increasing competition (i.e. passing from product full differentiation to no differentiation) always decreases profits, holds true in the presence of decentralized monopoly unions in the labor market? In what follows, we will argue that, in both competition regimes, the answer to the question is not always positive, depending on the unions’ preferences towards wages.

In particular, in order to deeply analyze the issue, consider first that when, e.g., the degree of product market differentiation decreases (that is, product market competition becomes fiercer), two distinct effects affect firms’ profits: on the one hand, the direct effect (that we label as “competition effect”) of increasing market competition for a given labor input price, which, as shown in Section 2, is always profit-reducing;\(^6\) on the other hand, when wages are endogenously

\(^6\) It should be noted that changes in the degree of product differentiation affect total demand in a different way under Cournot and Bertrand competition (i.e. a different “demand expansion” effect, in the terminology used by Shaked and Sutton (1990),
determined by unions, there is also an indirect effect (that we term “endogenous” or “union wage effect”) operating via wages. Formally, we get:

\[
\frac{\partial \Pi}{\partial \gamma} = \frac{\partial \pi}{\partial \gamma} + \frac{\partial \pi}{\partial w} \frac{\partial w}{\partial \gamma}.
\]

In particular, when wages are exogenously given, the derivative of \( w \) with respect to \( \gamma \) is obviously zero, hence the endogenous wage effect is null. In such a case, only the standard competition effect operates and we get the standard result that, regardless of the mode of competition in the product market, profits always decrease with increasing competition. However, when wages are determined endogenously, \( \partial w/\partial \gamma \) may not be longer zero. In particular, under quite general conditions, Dhillon and Petrakis (2002) show that, whilst with centralized (or coordinated) unions a “wage rigidity result” applies, i.e. wages turn out to be same independently of the degree of product differentiation (as well as of other product market and bargaining institutional features),\(^7\) if wages are instead fixed by firm-specific (decentralized) unions, \( \partial w/\partial \gamma \) is not actually null.

**Lemma 1 (Dhillon and Petrakis, 2002).** Under both Cournot and Bertrand competition, when product differentiation decreases (i.e. market competition increases), the wage chosen in equilibrium by (firm-specific) unions decreases.

**Proof.** By differentiating (18) and (19) with respect to the degree of product differentiation \( \gamma \), we get:

---

\(^7\) This implies that, with centralized unions, the conventional result (i.e. equilibrium profits always decrease with decreasing product differentiation) can never be reversed.
The Lemma 1 above, which is obtained in our framework as a special case of the more general results provided by Dhillon and Petrakis (2002), makes sense and, intuitively, can be explained by the fact that, since unions are firm-specific, an increase of inter-firm competition in the product market also translates into an increase of inter-union competition. More exactly, when $\gamma$ increases (which also implies that employment tends to reduce for both firms), employment at a firm level becomes more sensitive with respect to wages and this drives firm-specific unions to undercut each other in wage setting in order to sufficiently preserve employment. Moreover, since with $\frac{\partial w}{\partial \gamma} < 0$ the union wage effect is positive,\(^8\) if the latter dominates the competition effect, the conventional finding on the relationship between profits and the degree of product market competition may be reversed.

Furthermore, it is also worthy to specify as the (negative) dynamics of wages with respect to $\gamma$ behaves according to the degree of unions’ orientation towards wages, that is in relation to $\theta$.

**Lemma 2.** Under both Cournot and Bertrand competition, there exists a threshold for the degree of unions’ orientation towards wages $\theta$, for which, when $\theta$ is lower than the threshold, the (positive) endogenous or union wage effect tends to be stronger as $\theta$ increases, whilst the reverse holds true when $\theta$

\(^8\) The sign of the (endogenous) union wage effect in (22) strictly depends on $\frac{\partial w}{\partial \gamma}$. Indeed, it is trivial to check from (7) and (14) that, as expected, profits are negatively correlated with wages (i.e. $\frac{\partial \sigma}{\partial w} < 0$) under both Cournot and Bertrand competition.
is higher than the threshold. In particular, under Cournot competition, the threshold is \( \theta = \frac{2}{2 - \gamma} \), whilst, under Bertrand competition, is \( \theta = \frac{2 - \gamma^2}{(2 + \gamma)(1 - \gamma)} \).

**Proof.** Under Cournot competition, by differentiating (23) in absolute value with respect to the degree of unions’ orientation towards wages \( \theta \), we get:

\[
\frac{\partial |w_c^U|/\partial \gamma}{\partial \theta} = \frac{2(\alpha - w^0)(2 - \theta(2 - \gamma))}{[2(1 + \theta) - \gamma \theta]} \geq 0 \iff 2 - \theta(2 - \gamma) \geq 0 \iff \theta \leq \frac{2}{2 - \gamma}.
\]

Instead, under Bertrand competition, by differentiating (24) in absolute value with respect to the degree of unions’ orientation towards wages \( \theta \), we get:

\[
\frac{\partial |w_B^U|/\partial \gamma}{\partial \theta} = \frac{(2 + \gamma^2)(\alpha - w^0)(2 - \gamma^2 - \theta(2 + \gamma)(1 - \gamma))}{[2(1 + \theta) - \gamma(\theta + \gamma(1 + \theta))]^2} \geq 0 \iff 2 - \gamma^2 - \theta(2 + \gamma)(1 - \gamma) \geq 0 \iff \theta \leq \frac{2 - \gamma^2}{2 + \gamma(1 - \gamma)}.
\]

□

Hence, although, as obvious, it always applies that the higher the degree of unions’ preferences towards wages with respect to employment (that is, the higher \( \theta \)), the higher the equilibrium wage,\(^9\) according to Lemma 2, when \( \gamma \) increases, more wage-oriented unions reduce wages more or less strongly according to the fact that the degree of unions’ preferences is or is not “sufficiently” low.\(^10\) More exactly, there is a “hump-shaped” relationship

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\(^9\) This can be easily verified by differentiating equilibrium wages of Eqs. (18) and (19) with respect to \( \theta \).

\(^10\) Notice that, under Bertrand competition, when \( \gamma \) is approaching 1, wages always reduce more slowly with \( \gamma \) when \( \theta \) increases. However, as it will be specified below (see, in particular, fn 11), this special case is not particularly relevant, since, when \( \gamma \to 1 \), the “reversal result” concerning the relationship between the degree of product differentiation and profits, trivially, can never apply.
between $\theta$ and $|\partial w^{U}/\partial \gamma|$. This can be explained by the fact that a change in wages takes place if changing the degree of product differentiation (or competition) changes the trade-off between wages and employment and this actually occurs especially when both wages and employment matter for unions, that is, for medium values of $\theta$.\(^{11}\) Instead, for low $\theta$'s values, wages are close to the reservation value $w^{o}$ and there is not much room for wage reductions. Hence, as $\theta$ increases, the larger is the wage reduction due to increased competition, i.e. $|\partial w^{U}/\partial \gamma|$. On the other side, for very high $\theta$'s values, unions have a strong preference for high wages and, even though there are considerable room for wage reductions, a change in $\gamma$ will only trigger small wage adjustments. Nevertheless, when $\theta$ is very high, even a marginal reduction in wages owing to increased competition can, under both Cournot and Bertrand competition, increase profits, as our main results, which are stated below, point out.

**Result 1 [Cournot].** Under Cournot competition, if unions are more wage-than employment-oriented, that is $\theta > 1$, there is always some degree of product differentiation $\gamma$ sufficiently large, for which firms’ profits increase for decreasing product differentiation (i.e. increasing market competition). Moreover, provided that $\theta > 1$, the higher $\theta$, the larger the range for $\gamma$’s values for which firms’ profits increase with $\gamma$.

**Proof.** By differentiating (20) with respect to the degree of product differentiation $\gamma$, we get:

$$
(27) \quad \frac{\partial \pi^{U}_{C}}{\partial \gamma} = -\frac{16(1-\gamma\theta)(\alpha - w^{o})^2}{(2 + \gamma)^3[2(1+\theta) - \gamma\theta]} > 0 \iff 1 - \gamma\theta < 0 \iff \theta > \theta^{c} = \frac{1}{\gamma}.
$$

\(^{11}\) The mechanism underlying such result is very similar to that produced by a merger, as discussed, for instance, in Lommerud et al. (2005).
Clearly, since $\bar{\theta}_C \rightarrow \infty$ when $\gamma \rightarrow 0$ and $\bar{\theta}_C \rightarrow 1$ when $\gamma \rightarrow 1$, $\theta > \bar{\theta}_C$ (hence, $\partial \pi^U_C / \partial \gamma > 0$) for some $\gamma$, if (and only if) $\theta > 1$. Furthermore, since $\bar{\theta}_C$ always decreases when $\gamma$ increases, the higher $\theta (> 1)$, the larger the range for $\gamma \in (0, 1)$, such that $\theta > \bar{\theta}_C$. □

Hence, under Cournot competition, the higher the degree of unions’ orientation towards wages and the lower the degree of product differentiation, the higher the probability that the endogenous wage effect (which operates in reducing wages and increasing profits when $\gamma$ increases) outweighs the competition effect, hence overturning the standard result that profits always decrease with increasing competition. This is because, as shown in Lemma 2, more wage-oriented unions reduce wages more strongly when product differentiation increases.

**Result 2 [Bertrand].** Under Bertrand competition, firms’ profits increase for decreasing product differentiation (i.e. increasing market competition) provided that unions are sufficiently more wage-than employment-oriented, that is $\theta$ is sufficiently large, and the degree of product differentiation $\gamma$ is neither too much small nor too much large. Moreover, (provided that $\theta$ is sufficiently large) the higher $\theta$, the larger the range for $\gamma$’s values, for which firms’ profits increase with $\gamma$.

By differentiating (21) with respect to the degree of product differentiation $\gamma$, we get:

$$\frac{\partial \pi^U_B}{\partial \gamma} = - \frac{2H(1+\theta)(2-\gamma^2)(\alpha - w^o)^2}{(2-\gamma)(1+\gamma)^2[2(1+\theta) - \gamma(\theta + \gamma(1+\theta))]}$$

where

$$H = \gamma^6(1+\theta) - \gamma^5(1+\theta) - \gamma^4(3+2\theta) + 2\gamma^3(2+\theta) + 4\gamma^2\theta - 4\gamma(1+\theta) + 4 .$$

It follows that:
Unfortunately, the nonlinearity of the expression for $\theta_B$ prevents us from using algebraic methods to derive a complete formal proof of Result 2 (notice, however, that $\theta_B \rightarrow \infty$ when $\gamma \rightarrow 0$ and when $\gamma \rightarrow 1$, hence $\theta > \theta_B$ is not possible when $\gamma$ is too much small and too much large). Nevertheless, we can refer to numerical simulations to provide an illustration of the content of such result, as graphically depicted by the Figure 1. In particular, the figure describes the behavior of the critical term in (28), $H$, for three different values of $\theta$, that is $\theta = 2, 3$ and 4.

\[
\frac{\partial \pi^U_B}{\partial \gamma} \geq 0 \iff H \leq 0 \iff \theta > \theta_B = \frac{\gamma^6 - \gamma^5 - 3\gamma^4 + 4\gamma^3 - 4\gamma + 4}{\gamma(-\gamma^3 + \gamma^4 + 2\gamma^3 - 2\gamma^2 - 4\gamma + 4)}.
\]

Fig. 1. Critical $H$ term (under Bertrand competition) behavior with respect to $\gamma$

and for different $\theta$s

When $\theta = 2$ (blue dotted line), since $H$ is positive for any $\gamma \in (0, 1)$, firms’ profits always decrease when $\gamma$ increases, that is, in line with Result 2, unions

\footnote{Obviously, when products tend to become perfect substitute ($\gamma \rightarrow 1$), profits tend to zero under price competition. Hence, necessarily, profits (strongly) reduce when $\gamma$ is approaching 1 thus the “reversal result” (i.e. $\theta > \theta_B$) cannot apply.}
place too little weight on wages for the standard result to be reversed. Instead, with \( \theta = 3 \) (red dotted-dashed line) and \( \theta = 4 \) (green solid line), the reversal result applies, but the range for the \( \gamma \)'s values for which this holds true depends on \( \theta \): the higher \( \theta \), the larger the range. In particular, according to the figure, when \( \theta = 4 \), \( H \) is negative, hence profits increase with \( \gamma \), for \( \gamma \in (0.265, 0.87) \), whilst, when, \( \theta = 3 \), this occurs for \( \gamma \in (0.38, 0.78) \).

Finally, from Results 1 and 2, we can also infer that the reversal result is more likely to occur under Cournot than under Bertrand competition. The following corollary, which derives from Results 1 and 2, formally states this finding.

**Corollary.** *The reversal of the relationship between product market differentiation and profits is more likely to occur when firms compete à la Cournot, instead of à la Bertrand, in the product market. Technically, for any \( \gamma \in (0, 1) \), \( \overline{\theta}_C < \overline{\theta}_B \).*

**Proof.** The corollary above straightforwardly follows from the comparison between critical thresholds under Cournot and Bertrand competition, that is \( \overline{\theta}_C \) and \( \overline{\theta}_B \):

\[
\Delta \overline{\theta} = \overline{\theta}_C - \overline{\theta}_B = \frac{1}{\gamma} - \frac{\gamma^6 - \gamma^5 - 3\gamma^4 + 4\gamma^3 - 4\gamma + 4}{\gamma(-\gamma^5 + \gamma^4 + 2\gamma^3 - 2\gamma^2 - 4\gamma + 4)} =
\frac{\gamma^2[2 + \gamma(-4\gamma + 2)]}{\gamma(-\gamma^5 + \gamma^4 + 2\gamma^3 - 2\gamma^2 - 4\gamma + 4)} < 0, \forall \gamma \in (0,1).
\]

\( \Box \)

Figure 2 provides a graphical analysis of the result highlighted in the corollary. In particular, it shows the behavior of the threshold values \( \overline{\theta}_C \) and \( \overline{\theta}_B \) according to \( \gamma \). From the figure clearly emerges as the curve related to \( \overline{\theta}_C \) always (i.e. for any \( \gamma \)) lies below that related to \( \overline{\theta}_B \), confirming that the reversal result is more likely to apply under Cournot competition. Also note that this
result can be found out by reading the graph in Figure 2 in a different (inverse) way too. In particular, we can fix a given $\theta$ on the vertical axis (instead of a given $\gamma$ on the horizontal axis) and look at which $\gamma$'s values the curves lie below the chosen $\theta$. In this perspective, it is easy to verify that: a) there exist some $\theta$'s values sufficiently low (e.g. $\theta = 2$), according to which, for some $\gamma$, the respective threshold is exceeded under Cournot competition, but it does not under Bertrand competition; and b) when $\theta$ is sufficiently high, such that the reversal result applies (for some $\gamma$) under both Cournot and Bertrand competition, the range of $\gamma$'s values, for which the threshold is “satisfied”, is always larger for Cournot competition.\textsuperscript{13}

\begin{figure}[h]
\centering
\includegraphics[width=\textwidth]{figure2.png}
\caption{Behavior of the threshold values $\tilde{\theta}_C$ (blue solid line) and $\tilde{\theta}_B$ (red dashed line) with respect to $\gamma$}
\end{figure}

\textsuperscript{13} More exactly, the range of $\gamma$'s values that satisfy the threshold under Bertrand competition is always a subset of the range of $\gamma$'s values that satisfy the threshold under Cournot competition.
5 Concluding remarks

In this paper we have investigated if the conventional wisdom, that a decrease in the degree of product differentiation always diminishes firms’ profits, holds true, under both Cournot and Bertrand competition, in a duopoly model with decentralized (firm-specific) monopoly unions. When product differentiation decreases, further than the standard competition effect (that always tends to reduce profits), another effect, which indirectly operates via wages, affects profits. In particular, an increase of inter-firm competition in the product market, owing to a decrease of product differentiation, also translates into an increase of inter-union competition and this drives unions to undercut each other in wage setting in order to sufficiently preserve employment. Moreover, if this “union wage effect” dominates the standard competition effect, the conventional finding on profits behavior according to the degree of product market competition is reversed. We have shown that this event can apply under both modes of competition (although it is more likely to occur under Cournot competition), provided that unions are sufficiently wage-oriented, that is, they sufficiently prefer wages to employment.

Our result calls for further analyses that are deferred to future research. In particular, extensions to other hypotheses concerning wage and employment determination in the presence of unionization (i.e. “right-to-manage” or efficient bargaining) deserve to be considered. Furthermore, whilst in this paper we have only concentrated on symmetric equilibrium, it would be particularly interesting to extend the analysis to different production technologies (e.g. convex cost functions) and asymmetric contexts by introducing some source of heterogeneity between firms and/or unions, such as different cost functions, different parameters in product market demands or heterogeneity in unions’ preferences towards wages.
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