Cross-ownership and unions in a Cournot duopoly: when profits reduce with horizontal product differentiation

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Luciano Fanti

Department of Economics, University of Pisa
Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy

e-mail address: lfanti@ec.unipi.it
tel.: +39 050 22 16 369
fax: +39 050 22 16 384

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Cross-ownership and unions in a Cournot duopoly: when profits reduce with horizontal product differentiation

Luciano Fanti*

Department of Economics, University of Pisa, Via Cosimo Ridolfi, 10, I–56124 Pisa (PI), Italy

Abstract Motivated by the widespread presence both of decentralised unions and cross-participation at ownership level (for instance in Japan and US), this paper aims at investigating whether the conventional wisdom that a reduction in the degree of product differentiation (which increases competition) always reduces firms’ profits, remains true in a differentiated duopoly extended with both equity participation of one firm in another firm and risk-neutral (or risk-averse) decentralised monopoly unions. We show that such a common wisdom, while it holds when either unions or cross-ownership separately exist, is reversed for a fairly low degree of product differentiation and large percentage of cross-ownership when both unions and cross-ownership are in existence: this is because the interaction between the reduced employment due to cross-ownership and the moderation of wage claims due to the menaces for employment, both magnified by increasing product market competition, works to increase profits more than how the latter are reduced by a higher competition.

Keywords Cross-ownership; Duopoly; Unions

JEL Classification D43; L13; L4; J51

* E-mail address: lfanti@ec.unipi.it; tel.: +39 050 22 16 369; fax: +39 050 22 16 384.
1. Introduction

Equity participation of one firm in another one (i.e., cross-participation at ownership level) is widely present in several industries, especially if relation-specific investments exist. For instance, in the automobile industry there are many examples of partial ownership of rivals (see Alley, 1997, as regards the Japanese and the U.S. industries,1 and Barcena-Ruiz and Oilazola, 2001, as regards French industry, where Renault acquired a 36.8% equity stake in Nissan Motor in 1999). In particular cross-ownership is widely observed in the Japanese industrial structure (e.g. Osano, 1996, 2011), which has been object of several studies, aiming to provide explanations of why partial ownership arrangements are formed: for instance Berglof and Perotti (1994) argue that they facilitate cooperation and mutual monitoring (e.g. as to possible deviant managers) among keiretsu firms, Osano (1996) shows that they may solve managerial myopia problem under the menace of hostile takeovers, Aoki (1988) suggests that they allow for an improvement in efficiency of automobile producers (which tend to hold minority equity ownership in their downstream suppliers) by enhancing the cooperation between them and the suppliers. Another interesting example of the acquisition of participation in rival firms concerns the Japanese steel industry.2 As regards the cross-shareholdings in the U.S. industrial structure, the relevant feature in general is that a small and young technologically advanced firm sells a minority equity stake to a large old firm endowed of well established abilities in the areas of production and distribution, with self-evident reciprocal gains (e.g. Pisano, 1989, as regards the U.S. biotechnology industry).3

As is known, another stylised fact is the existence of trade unions in oligopolistic sectors (which are, of course, those interested or, at least, more interested than competitive sectors, to the partial cross-ownership phenomenon): for instance Booth (1995, p. 95) observes: “It appears to be an empirical regularity that imperfections in the labor market are correlated with imperfections in the product market”. Moreover, unionisation structure may differ significantly around the world (e.g. Layard and Nickell, 1999: OECD, 1999): for instance, Iverson (1998) provides an index on the degree of wage bargaining decentralisation in different countries. We note that in

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1 Another illustrative example (Barcena-Ruiz and Oilazola, 2001) is given by the French firm Renault, which acquired a 36.8% equity stake in Nissan Motor in 1999 (Renault Presse, 10/20/99).
2 As reported by Gilo and Speigel (2003, p. 3): “In the early 90’s, Japanese Nippon Steel and Korean Pohang Iron, two of the worlds’ largest steelmakers, held 0.5% ownership stakes in each other. They increased these stakes to 1% in the late 90’s and recently planned to increase them to 3%. In November 2002 Nippon Steel has reached an agreement with two of its main rivals in Japan, Sumitomo Metal Industries and Kobe Steel, according to which Nippon and Sumitomo will each own about 2% of Kobe while Kobe will acquire about 0.3% of Nippon (see “Nippon Steel, Posco Extend Partnership: Steel World’s Largest Producers Put Historical Animosities Behind Them and Increase Shareholdings,” Financial Times, August 3, 2000, Companies & Finance: Asia-Pacific, 23; ”Japanese Steelmaker to Trade Stakes,” The Daily Deal, November 15, 2002, M&A). Likewise, Japan’s second largest producer, Kawasaki Steel Company, purchased a minority stake in Korean Dongkuk Steel Company, while holding (at the time) a 40% stake in American steelmaker Armco (see Dongkuk Enters Strategic Alliance with Kawasaki,” Financial Times, August 6, 1999, Companies & Finance: Asia-Pacific, 26).
3 Of course it must be noted that also big established U.S. industries are interested to the partial cross-ownership between them. For instance, Gilo and Spiegel (2003, p. 2) report that 1) Microsoft acquired in August 1997 approximately 7% of the nonvoting stock of its historic competitor in the PC market, that is Apple, and in June 1999 it took a 10% stake in Inprise/Borland Corp. which is one of its main rivals in the software applications market; 2) Gillette acquired 22.9% of the nonvoting stock and approximately 13.6% of the debt of Wilkinson Sword, one of its largest rivals in the wet shaving razor blade market.
Japan and North American countries firm-specific unions and decentralised wage settings are largely predominant.

A recent growing literature on “unionised oligopolies” (e.g. Horn and Wolinsky, 1988; Dowrick, 1989; Naylor 1999; Haucup et al., 2001; Correa-Lopez and Naylor, 2004; Fanti and Meccheri, 2011a,b) argue that labor cost is no longer exogenously given, but it is the outcome of a two-stage strategic game played between each firm and a labor union.

We adopt this approach to tackle the following issue. A tenet in industrial economics suggests that a decrease in the degree of product differentiation in the standard differentiated duopoly model (e.g. Dixit, 1979; Singh and Vives, 1984), implies a reduction in the market demand, increases the intensity of product market competition and thus always reduces firms’ profits (see, e.g., Shy, 1995, pp. 138-140).

This paper aims at answering to the question whether and how such a tenet remains true in a differentiated duopoly model characterised by the presence of partial cross-ownership arrangements and decentralized monopoly unions.

The main result is that the common wisdom, while it always holds when either unions or partial cross-ownership separately exist, is reversed for a fairly low product differentiation and a large percentage of cross-ownership when both unions and cross-ownership are present.

Our study is, at the best of our knowledge, novel in that existing analyses of cross-ownership in oligopolies\(^4\) neglect trade unions behaviours, whereas existing studies on unionised oligopolies neglect the presence of cross-ownership.\(^5\)

The remaining part of the paper is organized as follows. Section 2 describes the basic model with cross-ownership under “competitive” wages. Section 3 introduces and analyses the behaviour of decentralised (rent-maximising) unions in determining wages. Section 4 analyses equilibrium outcomes and discusses the main results on the relationship between the degree of product differentiation (or market competition) and profits, depending on the roles played by the percentage of cross-ownership and by unions’ behaviour. Finally, Section 5 provides some concluding remarks.

2. The “benchmark” model with “competitive” labour markets

We consider a single industry consisting of two firms, 1 and 2, which produce a homogeneous good. There are two shareholders, A and B. Firm 1 is completely owned by shareholder A, who owns a participation also in firm 2. Therefore firm 2 is jointly owned by the two shareholders, with shareholder B having the majority of shares and thus also the control of firm 2. We denote by \(h\ (0<h<1/2)\) the fraction of shares that shareholder A has in firm 2. Shareholders are assumed to maximize their total profit, which means that the objective function of shareholder A is

\(^4\) Several studies analysed the effects of cross-ownership on, for instance, \(i)\) the managerial incentives in a managerial delegation duopoly (Macho-Stadler and Verdier, 1991); \(ii)\) the incentives of firms to engage in tacit collusion (Reitman, 1994), specifically under symmetric (Gilo et al., 2006) or asymmetric costs (Gilo et al., 2008); \(iii)\) the incentives to acquire cost-saving production technologies (Bacena-Ruiz and Oliazola, 2007); \(iv)\) the level of privatization in a mixed duopoly (Pal, 2010); \(v)\) the strategic alliances under the possibility of reallocation of corporate resources (Osano, 2011). However, the previous literature has not analysed the role played by the partial cross-ownership on the relationship between the degree of product market competition and profitability in a unionised duopoly.

\(^5\) An exception is Fanti (2011), who, however, different from the present paper, studies a duopoly with a homogenous product and focus only on welfare issues.
\[\pi_a = \pi_1 + h\pi_2\]  
\hspace{1cm} (1.1)  
while the objective function of shareholder B is  
\[\pi_b = (1 - h)\pi_2,\]  
\hspace{1cm} (1.2)  
We assume, following an established literature (Horn and Wolinsky (1988), Dowrick (1989), Naylor (1999), Correa-Lopez and Naylor (2004) that: i) labour is the sole productive input; ii) there is a constant returns to labour technology, so that  
\[q_i = L_i\]  
\hspace{1cm} (2),  
where \(q_i\) represents output, \(L_i\) the units of labour employed by the ith firm and thus \(q_i\) also represents the employment of firm \(i\). As a consequence, the two firms face the constant marginal cost given by the wage per unit of labour, \(w_i\). Therefore, profits of firm \(i\) can be written as  
\[\pi_i = p_iq_i - w_iq_i, \quad i = 1, 2\]  
\hspace{1cm} (3)  
where \(w_i\) denotes the wage paid by firm \(i\) and is assumed to capture all short-run marginal costs.

As regards the determination of the product market demand, following an established literature (see, e.g., Dixit, 1979; Singh and Vives, 1984; Qiu, 1997; Hackner, 2000; Correa-López and Naylor, 2004; Gosh and Mitra, 2010; Fanti and Meccheri, 2011a), we assume that preferences\(^*\) of the representative consumer over \(q\) are given by:

\[U(q_i, q_j) = a(q_i + q_j) - \frac{1}{2}(q_i^2 + q_j^2 + 2dq_iq_j),\]  
\hspace{1cm} (4)

where \(a > 0\) is a parameter that captures the size of the market demand and \(-1 < d < 1\) represents the degree of horizontal product differentiation. Now, some clarifications on the parameter \(d\) are in order. If \(d = 0\), then goods of variety 1 and 2 are independent (i.e. each firm behaves as if it were a monopolist in its specific market); if \(d = 1\), then goods 1 and 2 are perfect substitutes, i.e. homogeneous; \(0 < d < 1\) captures the case of imperfect substitutability between goods. The degree of substitutability increases, or equivalently, the extent of product differentiation decreases as the parameter \(d\) raises; a negative value of \(d\) instead implies that goods 1 and 2 are complements, while \(d = -1\) reflects the case of perfect complementarity.

The inverse demand functions of goods 1 and 2 that come from the maximisation by the representative consumer of Eq. (4) subject to the budget constraint \(p_1q_1 + p_2q_2 + y = M\) (where \(y\) is the numeraire good\(^\dagger\) and \(M\) denotes the consumer’s exogenously given income), are the following:

\[p_1(q_1, q_2) = a - q_1 - dq_2,\]  
\hspace{1cm} (5.1)

\[p_2(q_1, q_2) = a - q_2 - dq_1.\]  
\hspace{1cm} (5.2)

From (1.1), (1.2), (5.1) and (5.2), under profit-maximisation, firm \(i\)’s best-reply function is

\[q_1 (q_2) = \frac{(a - dq_2(1 + h) - w_i)}{2}\]  
\hspace{1cm} (6)

\[q_2 (q_1) = \frac{(a - dq_1 - w_i)}{2}\]  
\hspace{1cm} (7)

\(^*\) The important feature of such preferences is that they generate a system of linear demand functions.

\(^\dagger\) In this class of model it is implicitly assumed that, separately from the duopoly under study, there exist a competitive sector that produces the numeraire good \(y\).
As \( h > 0 \), by assumption, the best-reply functions are downward-sloping, that is, under the Cournot assumption, the product market game is played in strategic substitutes. From (6) and (7) we obtain equilibrium output (respectively, by firm \( i \), given \( w_i \) and \( w_j \)):

\[
q_i = \frac{a(2-d(1+h)) - 2w_i + dw_i(1+h)}{4 - d^2(1+h)}
\]

(8)

\[
q_2 = \frac{a(2-d) - 2w_j + dw_j}{4 - d^2(1+h)}
\]

(9)

It is easy to observe that, under cross-ownership, equilibrium output by firm 1 is lower than equilibrium output by firm 2 as the former firm internalises the fact that both firms compete in quantities and thus the latter one is “more aggressive”.

In the absence of firm-specific unions, firms pay the same wage, namely the “competitive” or reservation wage, i.e. \( w_1 = w_2 = w^\circ \). Therefore from (3), (5), (8), (9), the equilibrium profit is derived as follows:

\[
\pi_1 = \frac{(a - w^\circ)^2[h(1+d) + d - 2]dh(d-1) + d - 2}{(4 - d^2(1+h))^2}
\]

(10)

\[
\pi_2 = \frac{(a - w^\circ)^2(2-d)^2}{(4 - d^2(1+h))^2}
\]

(11)

\[
\pi_A = \pi_1 + h\pi_2
\]

(12.1)

\[
\pi_B = (1-h)(a - w^\circ)^2(2-d)^2
\]

(12.2)

\[
\Pi = \pi_1 + \pi_2
\]

(13)

It is easy to see that: the profits of firm 1, shareholder A and B and total industry are harmed by an increase in the degree of product substitutability, \( d \), (i.e. \( \frac{\partial \pi_1}{\partial d}, \frac{\partial \pi_A}{\partial d}, \frac{\partial \Pi}{\partial d} < 0 \)). However, profits of firm 2 and thus of shareholder B may increase with \( d \), for sufficiently high values of \( d \) (\( \frac{\partial \pi_2}{\partial d} < 0 \iff d < d^\circ \)). Therefore, although shareholders A and B may have divergent interests to differentiate products for a certain interval of the differentiation degree, industry profits always benefit from the product differentiation: thus the “benchmark” model with “competitive” labour markets gives the result that the higher the product differentiation, the higher total profits are: this result is expected since the product differentiation implies a “reduction” in the product market competition.

3. The union’s wage setting

\(^a\) The proofs of the sign of the derivatives are straightforward and thus omitted here for economy of space.
Following the well-established static unionised oligopoly literature mentioned above, we assume that the cost of production of the $i$th firm (i.e., the wage per unit of labour, $w_i$) is no longer exogenous while being the outcome of a strategic decision of its upstream supplier (labour union), as described below. In particular, following the backward induction logic, in stage 2 (as already analysed in Section 2), each firm competes on the differentiated product market choosing its optimal level of output (i.e., employment). In stage 1, instead, a firm-specific “monopolistic” union fixes wages. We assume that unions are identical and each union is of the utilitarian type, i.e. the union maximises the sum of individual workers’ utilities. As is known, union objectives are not necessarily dominated by wages. In order to derive analytical tractable results for the wage, we assume – following, amongst many others, Pencavel (1984, 1985), Dowrick and Spencer (1994), and Petrakis and Vlassis (2000) –, that the union determines the wage by maximising the following Stone–Geary objective function:

$$V = (w - \omega^\theta) L,$$

where $w_i$ is the union’s wage, $\omega$ is the reservation or competitive wage, $L$ is the labour employed by the firm and $\theta > 0$. A value of $\theta = 1$ gives the rent-maximising case (i.e., the union seeks to maximise the total rent), or alternatively defines the risk-neutrality of unionised workers. A value of $\theta < 1$ can be thought of as the representative member’s relative rate of risk aversion, (provided that union membership is fixed and all members are identical - see, e.g., Oswald, 1982; Pencavel, 1991; Booth, 1995; Petrakis and Vlassis, 2000).

By recalling that $q_i = L_i$, the firm-specific (decentralised) union $i$’s objective Eq. (14) can then be written as follows:

$$\max_{\{w_i\}} V_i = (w_i - \omega^\theta) q_i,$$

where $q_i$ (that is the firm’s output for any given level of wages) is given by eqs. (6) and (7), respectively.

In particular, as regards the firm 1’s union, the maximisation of

$$\max_{\{w_1\}} V_1(w_1 - \omega^\theta) q_1,$$

after substitution of eq. (6) in (16), obtains

$$w_1(q_1) = \frac{2\omega^\theta + \theta dw_1(1 + h) + a(2 - d(1 + h))}{2(1 + \theta)}$$

and, as regards the firm 2’s union, the maximisation of

$$\max_{\{w_2\}} V_2 = (w_2 - \omega^\theta) q_2$$

after substitution of eq. (7) in (18), obtains

$$w_2(q_1) = \frac{2\omega^\theta + \theta dw_2 + a(2 - d)}{2(1 + \theta)}$$

Eqs. (17) and (19) define the sub-game perfect best-reply function in wages of union–firm pair $i$ under the assumption of a non-cooperative Cournot–Nash equilibrium in the product market. In the sequel, for simplicity, we analyse the case of rent-maximising unions ($\theta = 1$) (in appendix the case of risk-averse unionised workers will be shortly investigated as well).

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9 When $\omega = 0$ the unions aims to maximise the wage bill.
4. Equilibrium outcomes and the relation between profits and product market competition.

4.1 Equilibrium wages, production and profits.

In sub-game perfect equilibrium wages are

\[ w_1 = \frac{dh(a(d + 2) - 2w^o) + (d + 4)(-a(d - 2) - 2w^o)}{d^2(1 + h) - 16} \]  \( (20) \)

\[ w_2 = \frac{d^2 ah + (d + 4)(-a(d - 2) - 2w^o)}{d^2(1 + h) - 16} \]  \( (21) \)

Finally, the sub-game perfect equilibrium quantities - after substitution of (20) in (8) and of (21) in (9) - and profits - after substitution of (20) in (10) and of (21) in (11) - are given by

\[ q_1 = \frac{2(a - w^o) \left[ d^2(1 + h) + 2d(1 + h) - 8 \right]}{4 - d^2(1 + h) \left[ d^2(1 + h) - 16 \right]} \]  \( (22) \)

\[ q_2 = \frac{2(a - w^o) \left[ d^2(1 + h) + 2d - 8 \right]}{4 - d^2(1 + h) \left[ d^2(1 + h) - 16 \right]} \]  \( (23) \)

\[ \pi_1 = \frac{4(a - w^o)^2 \left[ B + 2A \left[ h^2 d^3 + dh(d^2 + d - 4) + d^2 - 4 \right] + A \right]}{4 - d^2(1 + h) \left[ B - 8 - 4 \right]^2} \]  \( (24) \)

\[ A = 2 \left[ dh(d - 1) + d - 2 \right]; \quad B = \left[ d^2(1 + h) - 4 \right] \]

\[ \pi_2 = \frac{4(a - w^o)^2 \left[ B + 2(d - 2) \right]}{4 - d^2(1 + h) \left[ B - 8 - 4 \right]^2} \]  \( (25) \)

\[ \Pi = \frac{4(a - w^o)^2 \left[ d^4 h^3 + d^2 h^2 (d^2 + d - 4) + dh(d^2 + 4d - 4)(d^2 - 4) + 2(d^2 - 4)^2 \right] + 2C + 4D}{(4 - d^2(1 + h))^2 \left[ (d^2(1 + h) - 4) - 12 \right]^2} \]

\[ C = d^4 h^3 + d^2 h^2 (3d^2 - 2d - 4) + 2dh(d^2 + 3d - 2)(d - 2) + 2(d - 2)^2; \]

\[ D = d^2 h^2 (d - 1) + d^2 h(d - 2) + 2(d - 2)^2 \]  \( (26) \)

4.2 The relation between profits and product market competition.

The analytical investigation of (26) is of course cumbersome. In order to establish whether the common belief that profits are reduced by a reduced product differentiation (i.e. by a fierce competition) holds, we have to calculate the derivative of (26) with respect to the degree of differentiation \( d \), that is:
\[
\left. \frac{\partial \Pi}{\partial d} \right|_{a=1} = \frac{4(a - w^2)^2 \left[- A_6 d^6 - A_5 d^5 - A_4 d^4 + A_3 d^3 - A_2 d^2 + A_1 d - A_0 \right]}{(4 - d^2(1 + h))^3 \left[(d^2(1 + h) - 16) \right]^3}
\]

\[
A_6 = 3h(1 + h)^2 (h^2 + 2h + 1); \quad A_5 = 8(1 + h)^2 (h^3 + 4h^2 + 4h + 1); \quad A_4 = 20(1 + h)(2 - h^2 + 3h);
\]

\[
A_3 = 2h(1 + h)(3h^2 + 12h^2 + 20h + 16); \quad A_2 = 32(-2h^3 + 17h^2 + 36h + 19); \quad A_1 = 32(-2h^3 + 15h^2 + 36h + 19);
\]

\[
A_0 = 38(-h^2 + 5h + 6); \quad A_1 = 512(-3h^2 + 6h + 14); \quad A_0 = 2048(2 - h);
\]

(27)

By a straightforward application of the Descartes’ Theorem and knowing that \( a_0 < 0 \) and \( a_8 < 0 \), the following lemma holds:

**Lemma 1**: Since there are four changes of sign in the coefficients of the polynomial equation of eight degree in (27), there are four positive roots (if real) for \( d \) (i.e. \( d_1, d_2, d_3, d_4 \)), and profits are increasing (decreasing) with \( d \) for values included between \( d_1 < d < d_2, d_3 < d < d_4 \) (for values included between \( -1 < d < d_1, d_2 < d < d_3 \) and for values \( d_4 < d \)).

It is important to ascertain the number of roots which are, if any, included in the economically meaningful domain \( d \in (-1,1) \). Since the numerator of Eq. (27) is a polynomial equation of eight degree in the variable \( d \) with all its coefficients being non-numeric, the exact formulas for its roots are not tractable. However, by using numerical simulations, we are able to determine whether it is possible the counter-intuitive result that profits are increasing with \( d \) or not (i.e. at least \( -1 < d_1 < 1 \) or not). It is easy to numerically ascertain that the root \( d_1 \) is monotonically decreasing with \( h \) and in particular \( 1 > d_1 > 0.5 \) for \( 0 < h < 0.5 \) (e.g. \( d_1 = 0.75 \) when \( h=0.20 \), and \( d_1 = 0.57 \) when \( h=0.40 \)), while the other roots are never included in the interval of interest (i.e. \( d \in (-1,1) \)).

**Result 1**: The higher both \( h \) and \( d \), the more likely industry profits are increasing with \( d \).

Result 1 is clearly illustrated by Fig. 1.
Fig. 1. Industry Profit for a varying product differentiation (d) with different percentages of cross-ownership, h: h=0.10 (blue line), h=0.20 (black line), h=0.30 (red line), h=0.40 (brown line), h=0.50 (green line). (a=2, w°=0.5, 0=1).

Fig. 2 shows that also for all the single profits (i.e., of firm 1, firm 2, shareholder A and shareholder B) result 1 holds true: the higher both h and d, the more likely all the single profits are increasing with d.
Fig. 2. **Single Profit for a varying product differentiation (d): firm 1 (blue line), firm 2 (black line), shareholder A (red line), shareholder B (brown line).** \( a=2, w^*=0.5, h=0.4, \theta=1 \).  

**Lemma 2:** 1) in the absence of unions and in the presence of cross-ownership, profits are, as expected, always decreasing with increasing \( d \) (i.e. with fiercer product competition); 2) in the presence of unions, but in the absence of cross-ownership, again profits are, as expected, always decreasing with increasing \( d \).

**Proof:** the part 1) of the lemma is straightforwardly proven by the negative sign of the derivative of eqs. (10) and (11) with respect to \( d \) (omitted here for brevity); the part 2) by showing that when \( h=0 \) the industry profit (eq. 26) and its derivative w.r.t \( d \) (eq. 27) boil down to, respectively:

\[
\Pi = \frac{8(a - w^*)^2}{(2 + d)^2(4 - d)^2} \tag{26.1}
\]

\[
\left. \frac{d\Pi}{d^d} \right|_{\theta=1} = -\frac{32(1 - d)(a - w^*)^2}{(2 + d)^3(4 - d)^3} < 0 \tag{27.1}
\]

Lemma 2 is clearly illustrated by Figs. 3 and 4.
Fig. 3. Profits for a varying product differentiation (d) when unions are absent: firm 1 (blue line), firm 2 (black line), shareholder A (red line), shareholder B (brown line), industry (green line). ($a=2$, $w^0=0.5$, $h=0.3$, $\theta=0$).
Fig. 4. Firms’ profit for a varying product differentiation (d) when unions are present, without cross-ownership: firm 1 (blue line), firm 2 (black line), shareholder A (red line), shareholder B (brown line), industry (green line). (a=2, w°=0.5, h=0, θ=1).

Therefore from Result 1 and Lemma 2, the following Remark holds:

**Remark 1:** Since neither with the sole presence of unions nor with the sole presence of cross-ownership the common belief may be reversed in a standard Cournot duopoly, then it is the interaction between unionisation and cross-ownership which determine the possibility of the counterintuitive result.

In particular, the reason why the result occurs may be explained through a twofold effect triggered by an the increased product substitutability: the first one acts on firm-specific unions’ behaviour and the second one on the firms’ behaviour when partial cross-ownership does exist. As to the first effect, the intuition is that, since unions are firm-specific, an increase of inter-firm competition in the product market also translates into an increase of inter-union competition: more exactly, when the product substitutability increases, employment at a firm level becomes more sensitive with respect to wages and this drives firm-specific unions to undercut each other in wage setting in order to sufficiently preserve employment. This effect has already been stressed by Fanti and Meccheri (2011b). However, if only risk-neutral as well as risk-averse unions exist, then the reduction in wages can never dominate the reduction in profits due to the increased product competition, as shown by Fanti and Meccheri (2011b), and thus the conventional finding on the relationship between profits and the degree of product market competition may not be reversed. As to the second effect, we argue that, when there is cross-ownership, the total demand of labour (i.e.
employment and output) is reduced by an increasing cross-ownership\textsuperscript{10} and thus in
equilibrium the wage undercutting by firm-specific unions will result higher than in
the case without cross-ownership (ceteris paribus as regards the target of preserving
employment). Moreover, since the higher the product substitutability, the higher the
reduction in total employment caused by an increasing cross-ownership\textsuperscript{11}, then the
equilibrium wage undercutting will result higher when the product substitutability is
higher. This twofold negative effect on the total employment (and thus on equilibrium
wage claims) due to an increasing cross-ownership, on the one hand, and an increasing
product substitutability on the other hand, explains why, surprisingly, profits tend to
be positively linked with a fiercer product competition to the extent that both the
percentage of cross-ownership and the degree of product substitutability tend to be
sufficiently high.

5. Concluding remarks

Motivated by the widespread presence both of decentralised labour unions and cross-
participation at ownership level (for instance in Japan as well as in U.S.), this paper
investigated if the conventional wisdom that a decrease in the degree of product
differentiation (which implies an increasing competition) always reduces firms’ profits,
remains true in a differentiated duopoly model extended with both equity
participation of one firm in another firm and risk-neutral (or risk-averse)
decentralized monopoly unions (which are rather widespread, for instance, both in
Japan and U.S.).

The main result is that the common wisdom, while it holds when there are
separately either unions or cross-ownership, is reversed for a sufficiently low product
differentiation and a sufficiently large percentage of cross-ownership when both
unions (either risk-neutral or risk-averse) and cross-ownership are present. This is
because the interaction between the effect of an increasing percentage of cross-
ownership which reduces the employment and the moderation of wage claims due to
such menaces for employment, both magnified by an increasing product market
competition, works for enhancing profits more than how the latter are reduced by a
higher competition. We are aware of the fact that our analysis is limited to those
industries where unions are wage-setters as well as firm-specific. While we cannot
explore the robustness of our results under different modes of labor market
organization (e.g. centralised unions, centralised employers’ associations, bargaining
on wage and employment, and so on) in more detail here, we believe it provides an
interesting topic for future research.

\textsuperscript{10} More precisely, employment tends to strongly reduce for firm 1 and slightly increase for firm 2, but in
the overall employment is always reducing with an increasing percentage of cross-ownership and,
moreover, such reducing effect is increasing with an increasing product substitutability (as resulting by
the first derivatives of eqs. (8), (9) · and of their sum · w.r.t. \( h \) and by the second cross-derivatives w.r.t.
\( d \), respectively, which are omitted here for brevity).

\textsuperscript{11} A negative relationship between employment and increasing product substitutability is a standard
result in a differentiated Cournot duopoly, and it is easily shown that it also holds in the presence of
cross-ownership by the derivatives – omitted here for brevity – of eqs. (8), (9) · and of their sum · w.r.t.
\( d \). However in the present model in which there is partial cross-ownership, such a negative relationship
is magnified by an increasing percentage of cross-ownership (as resulting by the second cross-
derivatives w.r.t. \( h \), again omitted here for brevity), that is the higher \( h \) and \( d \), the lower the demand of
labour is.
References


**Appendix**

In this Appendix in order to evaluate the robustness of the results of the main text, we perform a numerical investigation of the case of risk-averse workers (i.e. $\theta < 1$).\(^\text{12}\)

In sub-game perfect equilibrium wages, quantities and profits are, respectively:

\[
 W_i = \frac{d^2h(a(d\theta + 2) - 2w^\circ) + (d\theta + 2(1+\theta))(-a\theta(d - 2) - 2w^\circ)}{d^2\theta^2(1+h) - 4(\theta + 1)^2} \tag{A.1}
\]

\(^{12}\)As noted by Petrakis and Vlassis (2000, p. 266) “alternatively, $\theta$ denotes the representative union member’s elasticity of substitution between wages and employment.”
\[ w_2 = \frac{\theta^2 d^2 ah + (d\theta + 2(1 + \theta))(-a\theta(d - 2) - 2w^\theta)}{d^2 \theta^2 (1 + h) - 4(\theta + 1)^2} \]  
(A.2)

\[ q_1 = \frac{2(a - w^\theta)[d^2 \theta(1 + h) + 2d(1 + h) - 4(1 + \theta)]}{4 - d^2(1 + h) - d^2 \theta^2 (1 + h) - 4(1 + \theta)^2} \]  
(A.3)

\[ q_2 = \frac{2(a - w^\theta)[d^2 \theta(1 + h) + 2d - 4(1 + \theta)]}{4 - d^2(1 + h) - d^2 \theta^2 (1 + h) - 4(1 + \theta)^2} \]  
(A.4)

\[ \pi_1 = \frac{4(a - w^\theta)^2[\theta B + 2A]\left[\theta [d^2d^3 + dh(d^2 + d - 4) + d^2 - 4] + A\right]}{4 - d^2(1 + h)^2[\theta^2 B - 8\theta - 4]^2} \]  
\[ A = 2[dh(d - 1) + d - 2]; \quad B = [d^2(1 + h) - 4]\]  
(A.5)

\[ \pi_2 = \frac{4(a - w^\theta)^2[\theta B + 2(d - 2)]^2}{4 - d^2(1 + h)^2[\theta^2 B - 8\theta - 4]^2} \]  
(A.6)

\[ \Pi = \frac{4(a - w^\theta)^2[\theta^2 [d^5h^3 + d^2h^2(d^2 + d - 4) + dh(d^2 + 4d - 4)(d^2 - 4) + 2(d^2 - 4)^2] + 2\theta C + 4D]}{4 - d^2(1 + h)^2[\theta^2(d^2(1 + h) - 4) - (8\theta + 4)]^2} \]

\[ C = d^4h^3 + d^2h^2(3d^2 - 2d - 4) + 2dh(d^2 + 3d - 2)(d - 2) + 2(d - 2)^2; \]
\[ D = d^2h^2(d - 1) + d^2h(d - 2) + 2(d - 2)^2 \]  
(A.7)

The analytical investigation of (A.7) is of course even more cumbersome of the corresponding (26) in the main text, so that we offer numerical results.

**Result A.1:** Also when unionised workers are risk-averse, profits may be reduced by a higher product market competition, provided that \( h, d \) and the risk-aversion parameter \( \theta \) are sufficiently high.

Fig. A1 clearly displays that for values of the percentage of cross-ownership larger than about 0.25 and for sufficiently high values of \( d \), even if workers are sufficiently risk-averse with a parameter \( \theta=0.75 \), industry profits benefit from an increased product market competition (e.g. for a cross-participation in firm 2 of about 50% profits increase for \( d>0.75 \)).
Fig. A1. Industry Profit for a varying product differentiation ($d$) with different percentages of cross-ownership, $h$, with a unions’ risk-aversion parameter $\theta=0.75$: $h=0.10$ (blue line), $h=0.20$ (black line), $h=0.30$ (red line), $h=0.40$ (brown line), $h=0.50$ (green line). ($a=2$, $w^o=0.5$).