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Merger results under price competition and plant-specific unions

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**Merger results under price competition and plant-specific unions**

**Abstract**

This paper analyses the effects of a downstream merger in a differentiated duopoly under price competition and plant-specific unions. We show, in contrast with the preceding literature, that the standard welfare results may be reversed: a downstream merger may increase consumer surplus and overall welfare. In particular, this applies when unions are sufficiently wage-oriented and the market size is included in a certain intermediate range.

**Classificazione JEL:** D43, L13, J50

**Keywords:** mergers, social welfare, price competition, plant-specific unions
1 Introduction

A classical result in oligopoly theory is that in the standard model of differentiated duopoly with exogenous marginal costs (wages) Bertrand competition yields higher consumer surplus and total welfare at equilibrium than when duopolistic firms are merged. On the other hand the literature on unionised oligopolies has been rapidly increasing, focusing on the effects of unions’ behaviour on the nature and the outcome of product market competition as well as on the incentives for mergers (e.g., the early papers by Horn and Wolinsky (1988) and Dowrick (1989)). As regards the latter issue it has been subsequently shown by many authors (e.g., Brekke 2004, Lommerud et al. 2005 and Symeonidis 2010) that the presence of unions significantly alters the firms’ incentive to merge compared with a situation where wages are exogenously given and in particular that merger profitability depends crucially on the type of labour market unionisation: for instance in the case of plant-specific unions (i.e. wages are determined at each unit in the merged firm), the post-merger wage is reduced with respect to when firms are independent and thus the merger is more profitable than in the case with exogenous wages. Despite these several studies, the welfare effects are less clear as well as less investigated, in particular in the case of price competition. In fact Lommerud et al. (2005) focus mainly on Cournot competition while Brekke (2004) focuses on price and quality competition, and both works abstract from welfare analysis. Symeonidis (2010), on the contrary, addresses also the welfare issue assuming that wage is bargained between the merged firm and two independent rent-maximising unions.

While Symionides (2010) postulates only rent-interested unions, in this paper we extend such assumption by considering unions maximizing Stone-Geary utility functions, which are usually adopted by the literature for representing unions’ preferences (e.g., Pencavel 1991). Indeed, such unions’ preferences representation, differently from the assumption adopted in Symeonides (2010), permits to analyze different possible unions’ preferences towards wages with
respect to employment levels and this, as it will be shown, will prove to be crucial for obtaining our final results.

Focusing on the case of plant-specific unions, which are particularly interesting for the analysis of the merger’s outcomes in that after the realization of a merger unions again continue to only care about employment at its own plant (or unit) of the merged multiunit, we note that, while with quantity competition it has been well ascertained that a merger between downstream firms may raise consumer surplus and overall welfare, Symeonidis (2010, p. 230) states that “when competition is in prices [...] the standard welfare results are restored: a downstream merger always reduces consumer surplus and overall welfare.” In this paper we show, in contrast with the preceding literature, that the standard welfare results may be reversed even under price competition: a downstream merger may increases consumer surplus and overall welfare, in particular when unions are sufficiently wage-oriented and both the market size and the degree of product differentiation are included in a certain intermediate range. A condition why this result occurs is that unions may be sufficiently wage-oriented (while Symionides (2010), postulating only rent-interested unions, is unable to capture the effects of different unions’ preferences).

Despite the use of specific functional forms (i.e. linear demand, quadratic wage costs, Stone-Geary unions’ utility function), we have shown the rather general channels of transmission through which a downstream firms merger may be beneficial for consumers and for society as a whole. Our result is rather novel and may have evident anti-trust policy implications. Section 2 develops the model with plant-specific unions, while the general framework with exogenous wage costs is described in Appendix. Section 3 examines the equilibrium results both for a Bertrand industry and for a post-merger case when there exist plant-specific unions. Section 4 studies the merger welfare results in comparison with the pre-merger ones. Section 5 concludes.

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1 For instance see Symeonidis (2010, Prop. 2, p. 233).
2 Model

In this section we develop the basic model of a (pre-merger) Bertrand duopoly and a post-merger situation (presented in Appendix) with the introduction of plant-specific unions which are wage setting. In this respect, we follow a recently increasing literature which assumed that labour input costs are no longer exogenously given for firms, but they are the outcome of a strategic game played between each firm and its labor union. In particular input price (wage) paid by each downstream firm is the outcome of a strategic decision of its upstream supplier (labor union), as below described. The following rules of the game are applied: i) Stage 1: the firms decide to merge or not; ii) Stage 2: the plant-specific union independently sets wages; iii) Stage 3: the firms set price. The game is solved by backwards induction. Note that employment is indirectly determined at stage 3 of the game.

In particular in this section we examine whether and how the presence of plant-specific unions modifies the equilibrium outcomes derived in the standard basic model under wage costs exogeneity (see Appendix).

2.1 Pre-merger Bertrand equilibrium under plant-specific unions

Analyzing merger decision, a crucial aspect is connected with the role of unions with respect to the fact that they are either plant-specific or not. This is because, with plant-specific unions, labour input prices are determined at the plant level. Hence, the number of active unions is left unchanged by a merger and, after than a merger has been realized, unions only care about employment at its own plant (or unit) of the merged multiunit firm. As regards the plant-

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2 We assumed that firms’ workers are organised in unions with preferences over wage and employment and the unions unilaterally set the wage before employment is determined. This is a case of Monopoly Union, which, as known, belongs to the class of Right-to-Manage bargaining models.

3 By contrast, when unions are firm-specific, they also merge when the downstream firms merge, thus, in such a case, they care about the firm’s employment as a whole. Another case is when there is only a single centralised union which of course remains the same in the pre-merger and post-merger cases. In this section, we admit that labour
specific union’s behaviour, we admit that wages are fixed by “monopolistic” unions, which objectives, however, are not necessarily dominated by wages since also employment levels play an important role. But, given wages, the latter depend on firms’ choice about production. latter depend on firms’ choice about production.

In order to derive tractable results for wage determination, we assume, following many others works (e.g., Pencavel 1984, 1991; Dowrick and Spencer 1994; Petrakis and Vlassis 2000), that the generic union i’s utility takes the Stone-Geary functional form

\[ V_i^B = \left( w_i^B - w_j^B \right)^\theta L_i^B, \]

where \( w^0 \) is the reservation wage and \( \theta > 0 \) is a parameter that represents the degree of the union’s orientation towards wages, with respect to employment. In particular, a value of \( \theta = 1 \) gives the rent-maximizing case, whilst smaller \( \theta \)’s values imply that the union is less concerned about wages and more concerned about jobs. Since this does not produce any important changes on final results, we normalize to zero the reservation wage (\( w^0 = 0 \)). Hence, recalling that production technology exhibits constant returns to scale (i.e. \( L_i = q_i \)), the union i’ utility function in the no-merger game is given by:

\[ V_i^B = \left( w_i^B \right)^\theta L_i^B. \]

After substitution of (A6) in (1) as regards the union i (and its corresponding for the other union), the input price \( w_i \) is chosen by union i, which maximises (1), hence, we obtain the following wage reply function:

\[ w_i(w_j) = \frac{\theta\left[a(2-c^2) - c \right] + cw_j}{(2-c^2)(1+\theta)} \]

and in the symmetric pre-merger Bertrand-Nash equilibrium (\( w_i = w_j = w^B \)) we obtain the following union’s wage:

unions are plant-specific, while the analysis of these cases in which either unions are firm-specific or there is a central union are left for further research.
We are now in a position to derive explicit equilibrium solutions for the unionised Bertrand duopoly. Firm $i$’s Bertrand equilibrium output, price and profits, are, respectively:

\begin{align*}
q^B_i &= \frac{a(2 - c^2)}{(2 - c)(2 - c^2)(1 + \theta) - c\theta}(1 + c) \\
p^B_i &= \frac{a(1 - c)\left[\theta(4 - c^2) + 2 - c^2\right]}{(2 - c)(2 - c^2)(1 + \theta) - c\theta} \\
\pi^B_i &= \frac{a^2(1 - c)(2 - c^2)^2}{(2 - c)^2(2 - c^2)(1 + \theta) - c\theta}(1 + c).
\end{align*}

Instead, welfare outcomes are as follows:

\begin{align*}
CS^B &= \frac{a^2(2 - c^2)^2}{(2 - c)^2(2 - c^2)(1 + \theta) - c\theta}(1 + c) \\
\nu^B &= \frac{a(2 - c^2)\left[\frac{a\theta(1 - c)(2 + c)}{(2 - c^2)(1 + \theta) - c\theta}\right]^\theta}{(2 - c)(2 - c^2)(1 + \theta) - c\theta}(1 + c) \\
SW^B &= \frac{a(2 - c^2)\left[2(2 - c)(2 - c^2)(1 + \theta) - c\theta\right]\left[\frac{a\theta(1 - c)(2 + c)}{(2 - c^2)(1 + \theta) - c\theta}\right]^\theta + a(2 - c^2)(3 - 2c)}{(2 - c)^2(2 - c^2)(1 + \theta) - c\theta}(1 + c).
\end{align*}

2.2 Post-merger equilibrium under plant-specific unions.

In the post-merger game, the unions set their prices simultaneously, and noncooperatively, by maximising:
where \( w_i \) is the input price set by the independent union at plants \( i \). Taking (A14), it is easy to show that:

\[
(11) \quad w_i(w_j) = \frac{\theta[a(1-c)] + cw_j}{1 + \theta}
\]

and each plant specific union will set the following (equilibrium) wage:

\[
(12) \quad w_i = w_j = w^M = \frac{a\theta(1-c)}{1 + \theta(1-c)}.
\]

Therefore, given the labour price fixed by unions, the unionised firm post-merger equilibrium quantity, price and profits are, respectively:

\[
(13) \quad q^M = \frac{a}{1 + \theta(1-c)(1+c)}
\]

\[
(14) \quad p^M = \frac{a[1 + 2\theta(1-c)]}{2[1 + \theta(1-c)]}
\]

\[
(15) \quad \pi^M = \frac{a^2}{2(1+c)[1 + \theta(1-c)]^2}
\]

which also imply the following equilibrium welfare outcomes:

\[
(16) \quad CS^M = \frac{a^2}{4(1+c)[1 + \theta(1-c)]^2}
\]

\[
(17) \quad V^M = \frac{a[\theta(1-c)]^\gamma}{2(1+c)[1 + \theta(1-c)]}
\]
\[ SW^M = \frac{a \left[ 4(1 + \theta(1-c)) \left( \frac{a \theta(1-c)}{1 + \theta(1-c)} \right)^\beta + 3a \right]}{4(1+c)(1 + \theta(1-c))^2}. \]

### 3 Pre-merger and post-merger outcomes comparison

In this section we examine the implications of the presence of plant-specific unions on the equilibrium outcomes (in comparison with those derived under wage costs exogeneity in the Appendix).

Firstly, simple observations of (3) and (12) lead to the following:

**Remark:** Both pre and post-wages are increasing with the wage-orientation parameter and decreasing with the degree of horizontal differentiation.

Regarding the input price response to a merger, we get the following result:

**Result 1.** When downstream firms are price setting, independent unions set post merger wages lower than the pre-merger ones.

**Proof:** Since the wage differential (\(\Delta w\)) is given by:

\[ \Delta w = w^M - w^B = -\frac{ac\theta(1-c)(1+c)}{[(2-c^3)(1+\theta) - c\theta(1+c)][1 + \theta(1-c)]] < 0 \]

then Result 1 is proved.

\textit{Q.E.D.}

This result is similar to those obtained in a somewhat different context by Brekke (2004), Lommerud et al. (2005) and Symeonidis (2010) and also the intuition behind such results is similar: when wages are chosen at each firm-unit (or plant), a merger will lead to a decrease in wages for a twofold reason. First,
since there is a negative demand shifting effect, each plant-specific union has an incentive to reduce wages in order to dampen the negative employment effect. Secondly, it can be shown that the slope of the post-merger labour demand curve becomes flatter in a \((w-L)\)-space and thus the wage elasticity of labour demand increases, implying that labour demand is more responsive to wage differentials between the two unions. The intuition behind the wage elasticity change is that the merged firm is able to shift production between its two units (or plants) and this implies that the rivalry between the two plant-specific unions increases so pushing down the wage even further. The strength of the wage-reducing effect of intensified inter-union competition depends crucially on a very non linear interaction between the unions’ relative preferences for wage and employment \((\theta)\) and the degree of product substitutability \((c)\). In particular, an increase in the unions’ wage orientation initially tends to amplify the wage-reducing effect of a merger, but subsequently for further increase of such a preference for wages the wage reducing effect is dumped. In other words the relationship between the wage differential and the wage orientation parameter is U-shaped, as the following result shows:

**Result 2.** When unions are plant-specific, there exists a threshold value for \(\theta\), which is positively related to \(c\), below (beyond) which the differential between post- and pre-merger wages increases (decreases) when \(\theta\) increases.

*Proof:* By differentiating (19) with respect to we get:

\[
\frac{\partial(\Delta w)}{\partial \theta} = \frac{-ac(1-c)(1+c)\left[\theta^2(c^3-3c+2)+c^2-2\right]}{\left[(2-c^2)(1+\theta)-c\theta(1+c)\right]^2} \geq 0 \Leftrightarrow \theta \geq \frac{\sqrt{2-c^2}}{1-c}.
\]

Q.E.D.

Therefore the role played by the unions’ preferences is surprisingly twofold: a

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\(^4\) The reason is that, for a given wage, a merger leads to a lower quantity, which, in turn, lowers the demand for labourers.
large preference either for wages or employment tends to dampen the crucial wage reduction effect of a merger in an independently unionised labour market. From Result 2, it may be easily derived the following.

**Result 3.** *The higher is the product differentiation parameter (*c*), the larger is the threshold value of θ, beyond which the wage reducing effect of the merger is dumped.*

Results 2 and 3 are numerically illustrated in the following Figure 1.

![Figure 1](image)

**Fig. 1.** The differential between post and pre-merger wage for a varying unions’ wage orientation parameter with different product differentiation degrees 

\[ c = 0.25 \text{ (red)}, \quad c = 0.50 \text{ (black)}, \quad c = 0.75 \text{ (blue)}, \quad a = 5 \]

Figure 1 neatly also shows that the lower the product differentiation, the less the post merger wage is reduced with respect to the pre-merger one. Therefore it follows that a rather high product substitution is necessary for the reducing wage effect caused by the merger is very sizable.

The role played by the latter factor is rather intuitive: the lower is the product differentiation, more easily the merged firm is able to shift production between its two units and thus competition between the two plant-specific unions that supplies labourers is increased, which in turn causes a stronger reduction of the post-merger wage.
More in detail, Results 2 and 3 highlight that, when unions are plant-specific, the wage reduction due to an intensified inter-union competition after a merger depends crucially on a very nonlinear interaction between the unions’ relative preferences for wage over employment (θ) and the degree of product substitutability (c). In particular, an increase in θ initially tends to amplify the wage-reducing effect of a merger, but subsequently for further increase of θ over a given threshold, the wage reducing effect is dumped. Therefore, the role played by the unions’ preferences parameter is surprisingly twofold: a large preference either for wages or employment tends to dampen the crucial wage reduction effect of a merger. Moreover, the lower the degree of product differentiation, the lower the wage reduction produced by a merger. Hence, a rather high product substitutability is needed in order to make the wage reduction very sizable. In this regard, the role played by c is rather intuitive: the higher c (i.e. the higher the degree of product substitutability), the more easily the merged firm shifts production between its two plants. This increases competition between the two plant-specific unions, which, in turn, causes a stronger wage reduction after the merger has taken place.

Instead, the quantity differential (Δq) is given by:

\[
\Delta q = q^M - 2q^B = -\frac{ac(2 - c^2(1 - \theta) - c\theta)}{(1 + c)(2 - c)[(2 - c^2)(1 + \theta) - c\theta][(1 + \theta(1 - c))]}\]

from which the following result can be established:

**Result 4.** The post-merger quantity may be even larger than the pre-merger one depending on whether unions are sufficiently wage-oriented.\(^5\)

**Proof:** It is easily shown that the sign of (21) depends on both the levels of unions preference and product market differentiation parameters in the

\(^5\) Furthermore it must be noted that the critical threshold for θ (above which post-merger aggregate output is higher) is lower for intermediate c’s values, that is, when products are neither too much nor too little differentiated.
following way:

\[(22) \quad \Delta q > 0 \iff \theta > \frac{2 - c^2}{c(1-c)}.\]

Q.E.D.

This result is rather surprising and needs some explanation. First, we observe that when wage is exogenously given (see the comparison between (A7) and (A15) in the Appendix): 1) the Bertrand-Nash quantity is, as expected, always larger than the post-merger one, 2) both quantities are reduced, as expected, by increasing wage costs.\(^6\) Therefore, since we know from Result 1 that the post-merger wage is always lower and from Result 2 that the post merger wage reducing effect is amplified in an opportune range of intermediate values of the unions’ wage orientation parameter \(\theta\), then it follows that, when \(\theta\) is at least sufficiently close to the threshold value which maximises the wage reducing effect (i.e. it maximises the wage differential) and the product is scarcely differentiated – that is, the wage differential is very large as predicted by Result 2 and shown in Figure 1\(^7\) – it is logically possible that the post merger wage is so reduced that even the post-merger quantity becomes larger than the pre-merger one. In particular, such a result is more likely to occur, the higher the wage reduction associated to the merger, the latter depending, as affirmed by Results 1-3 and already discussed, on the interaction between \(\theta\) and \(c\).

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\(^6\) In particular, given wages, a merger implies a demand shifting effect, determining a reduction of output. This is due to the familiar effect of a merger, where merger participants coordinate their production volumes in order to internalise a negative externality (e.g. Lommerud et al., 2005). At the same time, however, plant-specific unions react to the merger by reducing wages and this (indirectly) operates in the opposite direction, by increasing (equilibrium) output.

\(^7\) For the sake of precision we note that the role played by the parameter \(c\) on the quantity differential is twofold: a direct effect according to which the Bertrand-Nash quantity is relatively raised more than the post-merger quantity by an increased product substitutability, and an indirect effect amplifying the relative wage costs differential in favour of the merger, so increasing relatively more the post-merger quantity than the Bertrand-Nash quantity. We have shown that for opportune values of the unions’ preference parameter the latter effect is dominating, so that the post-merger quantity becomes larger.
4 Welfare analysis

In this section we concentrate on welfare results. We compare, firstly, pre- and post-merger outcomes in relation to single welfare determinants and, secondly, we analyze overall social welfare under plant-specific unions. Our results will point out that there are circumstances where a merger is beneficial for society as a whole and circumstances where it is detrimental.

In relation to profits, we get:

\[
\Delta \pi = \Pi^U - 2\pi^\theta = \frac{a^2(\theta^2 P_1 + \theta P_2 + P_3)}{2[(2-c^2)(1+\theta) - c\theta]^2(1+c)(2-c)^2[1+\theta(1-c)]^2} > 0
\]

where

\[P_1 = (4c^6 - 11c^5 - 6c^4 + 37c^3 - 16c^2 - 24c + 16), \quad P_2 = (16 + 10c^4 + 16c^3 - 6c^5 - 28c^2 - 8c)\]

and \[P_3 = (c^5 - 4c^3 + 4c)\].

Result 5. In a duopoly with independent unions, a merger is always profitable (i.e. post-merger aggregate profits are always higher than pre-merger aggregate profits).

The intuition behind Result 5 is simple: as known, when wage is exogenously given the traditional result is that a merger is always profitable. Therefore, since we have shown that when independent unions are present the post-merger wage is always reduced (see Result 1), then it follows that, a fortiori with respect to the case of exogenous wage, post-merger profits are always higher. Moreover it is important to note that the profit differential, although it is always positive as just said in Result 5, displays a “humped” relationship with respect to the unions’ wage orientation parameter, that is such a differential is maximal for an opportune range of intermediate values of such a parameter, as Figure 2 below shows. This means that for large values of \(\theta\), the profitability of the merger tends to become negligible and this implies that the positive effect of the profits differential in favour of the post-merger situation on the overall social welfare
differential tend to strongly reduce for very high values of $\theta$.

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{fig2}
\caption{Profits differential for varying $\theta$
\[c = 0.75 \text{ (blue)}, c = 0.5 \text{ (black)}, c = 0.25 \text{ (red), } a = 1\]}
\end{figure}

Instead, the consumer surplus differential ($\Delta CS$) is:

\begin{equation}
\Delta CS = CS^c - CS^\theta = \frac{a^2 c \left[ \theta^2 c (3c^4 - 6c^3 - 5c^2 + 16c - 8) + 2\theta (2 - c^2)(4 + c^3 + c^2 - 6c) + (2 - c^2)(8 + c^3 - 4c^2 - 2c) \right]}{4 \left( (2 - c^2)(1 + \theta) - c\theta \right)^2 (1 + c)(1 + \theta(1 - c))^2}.
\end{equation}

From (24), the following result is established:

**Result 6.** The post-merger consumer surplus may be higher than the Bertrand-Nash one provided that unions are sufficiently wage oriented.

**Proof:** It is easy the see that there exists only one positive value of $\theta$ which is solution of (24). In particular, simple algebra reveals that:

\begin{equation}
\Delta CS \geq 0 \iff \theta \geq \frac{2 - c^2}{c(1 - c)}.
\end{equation}

\textit{Q.E.D.}
This is a rather unconventional result because consumers are benefited from a reduced market competition. Moreover, a straightforward analysis of the condition expressed by (25) shows that such an unconventional result is more likely when products are neither too much nor too little differentiated (i.e. the relationship between $\theta e c$ is inverted U-shaped). Furthermore, it is also noted that, although the consumer surplus reversal result occurs, for a given unions’ preference parameter, more likely when there exists an intermediate degree of product differentiation (and thus when products are scarcely differentiated a higher unions’ wage orientation is needed for having the reversal result), however in the latter case the “size” of the surplus consumer differential in favour of a post-merger situation is “magnified” (see Figure 3 below): this implies that the role played by the consumer surplus in determining the total welfare reversal result is larger when both unions are strongly wage–oriented and products tend to be sufficiently substitutes. Furthermore, the observation of (25) also suggests that our results would be in accord with Symeonidis (2010), i.e. if unions are wage-bill maximizing ($\theta = 1$) (that is, pre-merger consumer surplus is always higher than post-merger consumer surplus) but with unions relatively wage-interested the unconventional Result 6 may occur.

Fig 3. Consumer surplus differential for varying $\theta$ with different values of $c$

$[c = 0.3 \text{ (red)}, c = 0.5 \text{ (black)}, c = 0.8 \text{ (blue)}, a = 1.5]$
In relation to unions’ utility differential, we obtain:

\[(26)\]

\[
\Delta V = V^M - V^B = \frac{-2(1 + \theta(1 - c))(2 - c^2)\left[\frac{2 - c - c^2}{(2 - c^2)(1 + \theta) - c\theta}\right]^\theta + (2 - c)\left[(2 - c^2)(1 + \theta) - c\theta\left[\frac{1 - c}{1 + \theta(1 - c)}\right]^\theta\right]}{(2 - c)[(2 - c^2)(1 + \theta) - c\theta][1 + c](1 + \theta(1 - c))} < 0.
\]

**Result 7.** The aggregate unions’ utility is always lower under a firms’ merger than when firms are independent.

The effect of the firms’ merger on unions’ utility can be decomposed in two effects: a negative effect due to a reduced wage and an ambiguous effect due to the reduced (increased) employment for low (high) \(\theta\). However, even in the case of increased employment the balance of these two effects is such that the merger always reduces unions’ utility. It is important to observe the crucial role played by the market size parameter \(a\): if \(a < 1\) (\(> 1\)) then for increasing \(\theta\) such a differential tends to become negligible (large). This means that until \(a\) is less or not too larger than unity, the more wage oriented unions, the more equal their utilities become for increasing \(\theta\), under both market cases. On the contrary, for a sufficiently high value of \(a\) the utility differential tends to become, for increasing \(\theta\), more and more ample in favour of the Bertrand case. This implies that the case of large market size, especially jointly with a high unions’ wage orientation parameter, works for the traditional social welfare result, according to which pre-merger social welfare outperforms post-merger social welfare.

The crucial role played by the market size parameter and its interaction with \(\theta\) as well \(c\), clearly emerges from Figure 4. If \(a\) is low \((a < 1)\), the negative aggregate unions’ utility differential tends to become negligible as \(\theta\) increases. This means that the more wage-oriented unions, the more equal their utilities under both market cases. By contrast, if \(a\) is sufficiently high \((a > 1)\), for low
\( \theta \)’s values, aggregate unions’ utility differential increases with \( \theta \) while, for high \( \theta \)'s, it becomes more and more ample in favour of the Bertrand case when \( \theta \) increases (i.e. the relationship between aggregate unions’ utility and \( \theta \) is “inverted-U shaped”). Notice that, especially for the case with high \( a \)'s values, the degree of product differentiation also plays an important role. Particularly, the lower \( c \), the lower the unions’ utility differential in favour of the pre-merger case; with \( a > 1 \), this holds true, in particular, for high \( c \)'s values.

Finally we can study the behaviour of overall welfare. Particularly, the overall social welfare differential is given by:

(27)

\[
\Delta SW = SW^M - SW^B = \frac{a S_1 \left[ \frac{\alpha \theta (2 - c - c^2)}{(2 - c^2)(1 + \theta) - c \theta} \right]^\theta + ac S_2 + 4 S_3 \left[ \frac{\alpha \theta (1 - c)}{1 + \theta (1 - c)} \right]^\theta}{4(2 - c)^2 \left[ (2 - c^2)(1 + \theta) - c \theta \right]^2 (1 + c)(1 + \theta (1 - c))^2}
\]
where

\[ S_1 = (2 - c)(2 - c^2)(1 + \theta^2(1 - c)\theta + c^2 - 2)[1 + \theta(1 - c)]^2, \]

\[ S_2 = \theta^2(8c^6 - 21c^5 - 14c^4 + 63c^3 + 8c^2 - 92c + 48) + 2\theta(24 + 7c^4 + 12c^3 - 5c^2 - 6c^2 - 32c) \]

\[ + c(3c^4 - 4c^3 - 8c^2 + 16c - 8) \]

and \( S_3 = (2 - c)^2(1 + \theta^2(1 - c)\theta + c^2 - 2)[1 + \theta(1 - c)]. \)

Eq. (27) may have a priori any sign. The nonlinearity of such an expression prevents us from using algebraic methods to ascertain its sign. However, an exhaustive numerical analysis of (27) shows that, in contrast with the standard result of the preceding literature, there may be a welfare reversal. In particular the following holds:

**Result 8.** Post-merger social welfare may be higher than when firms are independent, under the following conditions: a sufficiently low degree of product differentiation as well as market size and sufficiently wage oriented unions (although in some cases not too much wage-oriented).

Some figures below numerically illustrates the occurrence of the welfare reversal result in accord with the content of the analytical Results 1-8.

Figure 5 shows that, given a sufficiently low product differentiation (\( c = 0.8 \)) and provided that \( a \) is less or slightly higher than unity, the social welfare reversal result occurs when unions are distinctly wage oriented (for instance, see the red line, corresponding to \( a = 0.2 \), when \( \theta > 1.6 \)) but in some cases if they are too much wage oriented the traditional result reappears (see the black line, corresponding to \( a = 1.05 \), when \( \theta > 9 \)). Figure 6 shows that for the special – but usual\(^9\) – case of unitary value of the market size parameter and for a sufficiently high wage orientation parameter’s value the welfare reversal result occurs in a certain intermediate range of the product differentiation degree parameter (e.g. a parametric range including values sufficiently high but less than one, as displayed, for example, by the red line, corresponding to \( \theta = 2.5 \),

\(^8\) Note that, from (27), in the special case of the so-called wage bill maximising unions (i.e. \( \theta = 1 \)) the reversal result cannot occur.

\(^9\) Indeed many noteworthy articles assume an unitary market size parameter (e.g. Brekke 2004).
where the reversal result occurs for $0.845 < c < 0.94$). Finally, Figure 7 displays the simultaneous effect of a joint variation of $\theta$ and $c$, again under an unitary market size parameter’s value, where, in line with Figures 5 and 6, the welfare reversal result occurs for couples of sufficiently high values of $\theta$ and $c$ (while the standard result is restored for $c$ very close to one).\textsuperscript{10}

\textbf{Fig. 5.} Welfare differential for varying $\theta$ with four different values of $a$ [$a = 0.2$ (red), $a = 0.5$ (blue), $a = 1.05$ (black), $a = 1.1$ (brown), $c = 0.8$]

\textsuperscript{10} Note that for $c = 1$ Bertrand competition collapses to the perfect competitive case and thus it is intuitive that the traditional welfare result is restored.
Fig. 6. Welfare differential for varying \( c \) with four different values of \( \theta \)

\[ \theta = 1 \text{ (blue)}, \ \theta = 2 \text{ (black)}, \ \theta = 2.5 \text{ (red)}, \ \theta = 3 \text{ (brown)}, \ a = 1 \]

Fig. 7. Welfare differential for jointly varying \( c \) and \( \theta \) with \( a = 1 \)

6 Conclusions
This paper analyses the effects of a downstream merger in a differentiated oligopoly when there are plant specific unions under price competition. We show, in contrast with the preceding literature, that the standard welfare results are reversed: a downstream merger may increases consumer surplus and overall welfare.

The result is rather counterintuitive and is driven by a somewhat complicated interaction between the three parameters representing market size, product differentiation and unions’ wage orientation. First, it is important the fact that independent unions moderate wage because the merged firm is able to shift production between its two units and thus rivalry between the two plant-specific unions that supply labourers is increased (and this occurs more easily when products are relatively high substitutes). As a consequence, when unions are distinctly interested to wages, quantity and price may be, rather counterintuitively, lower and higher, respectively, under a higher market competition. This is a reason why the consumer surplus may increase when competition is restricted. This also means that that post-merger profits are a fortiori higher under a plant-specific unionisation (due to the above mentioned wage moderation consequent to the unions’ rivalry). In order to better understand the working of the model with plant-specific unions, we note that in the standard case with exogenous wage there are two components of the overall social welfare: profits (higher in the post-merger case) and consumer surplus (higher under Bertrand competition), and it is well-known that the reduction of the latter, as a consequence of the merger, always dominates. In the case of unionised oligopoly there are three forces acting on the overall social welfare: profits (a fortiori higher in the post-merger case), consumer surplus (higher in post merger case than under Bertrand competition for sufficiently wage interested unions and low product differentiation), and unions’ utility\(^{11}\) (always higher under Bertrand competition, but with a very narrow differential when the market size is not too large and a very large differential when the market size is

\(^{11}\) As regards unions’ utility we note that wages are lower in the post-merger case, while the employment may be higher in the post merger case when, rather paradoxically, the unions are distinctly wage interested. However despite this ambiguity, the moderation wage effect always prevails and thus unions are beneficiated by Bertrand competition.
sufficiently large). Therefore the interaction between these three forces, as the present analysis has revealed, may trigger - for opportune values of the market size and the degree of product differentiation, on the one side, and for sufficiently wage interested independent unions on the other side - the social welfare reversal result. Our findings extend and complement those of Brekke (2004), Lommerud et al. (2005) and particularly Symeonidis (2010). In particular the latter author found that both consumer surplus and total welfare are lower under a downstream merger than when downstream firms are independent (Symeonidis, 2010, Prop. 4, p. 234), while the present paper has shown that if the unions’ objective is assumed to be more general and including wage (employment)-oriented preferences (differently from Symeonidis), then an unconventional positive social welfare effect of the merge may appear, with the consequent policy implications. Finally, in order to check the robustness of these results further research should investigate – again under price competition in the downstream market - alternative union structures such as a central union and firm-specific unions.

Appendix. Basic model with exogenous wages

A.1 Demand and costs structures

The model of the differentiated product market duopoly follows, as usual, the model of Singh and Vives (1984). In the product market game, as usual, each firm sets its output – given pre-determined wages – to maximize profits. Preferences of the representative consumer are given by:

(A1) \[ U(q_i, q_j) = a(q_i + q_j) - \frac{(q_i^2 + 2cq_iq_j + q_j^2)}{2}, \]
where $q_i$, $q_j$ denote outputs by firm i and j, respectively, $a > 0$, and $c \in (0, 1)$ denotes the extent of product differentiation with goods assumed to be imperfect substitutes.

The derived product market demand is linear and, for firm $i$ for example, is given by:

$$p_i(q_i, q_j) = a - cq_j - q_i.$$

As in the standard model labour is the sole productive input with a constant returns to labour technology, that is the two firms face the same constant marginal cost, $w$.

The following rules of the game are applied: i) Stage 1: the firms merge or not (the decision as regards whether to merge or not is assumed to be based on a payoff comparison with the no-merger benchmark equilibrium; ii) Stage 2: the firms set price. The game is solved by backwards induction.

### A.2 Pre-merger Bertrand equilibrium

In this section of the paper, we suppose that the product market game is characterized by price-setting behavior by firms. From (A2) and its counterpart for firm $j$, we can write product demand facing firm $i$ as

$$q_i(p_i, p_j) = \frac{a(1-c) - p_i + cp_j}{(1-c^2)},$$

Profits of firm $i$ are then given by:

$$\pi_i = p_i \left[ \frac{a(1-c) - p_i + cp_j}{(1-c^2)} \right] - w_i \left[ \frac{a(1-c) - p_j + cp_j}{(1-c^2)} \right].$$

From (A4), the first-order condition for profit-maximization gives the choice of price of firm $i$ in function of the price chosen by firm $j$:
\[(A5) \quad p_i(p_j) = \frac{a(1-c) + cp_j + w_i}{2}\]

and hence, for \(c > 0\), the Bertrand product market game is played in strategic complements.

Hence, substituting (A5) in (A3) the sub-game perfect output in function of wage rates, which is fixed by unions in the first stage of the game, is obtained:

\[(A6) \quad q_i(w_i, w_j) = \frac{(2 + c)(1-c)a - (2-c^2)w_i + cw_j}{(4-c^2)(1-c^2)}\]

Finally, equilibrium output, price and profits under Bertrand competition are given by:

\[(A7) \quad q^B = \frac{a - w}{(2-c)(1+c)}\]
\[(A8) \quad p^B = \frac{a(1-c) + w}{(2-c)}\]
\[(A9) \quad \pi^B = \frac{(1-c)(a-w)^2}{(2-c)^2(1+c)}\]

Instead, welfare equilibrium outcomes are given by:

\[(A10) \quad CS^B = \frac{(a-w)^2}{(2-c)^2(1+c)}\]
\[(A11) \quad SW^B = \frac{(3-2c)(a-w)^2}{(2-c)^2(1+c)}\]

\[A.3 \quad \textit{Post-merger equilibrium}\]

In the post-merger game, the merged firm, at stage 2 of the game, sets prices \(p_i\) and \(p_j\) to maximise:
\[ (A12) \quad \Pi^M = \pi_i + \pi_j = (p_i q_i - w_i q_i) + (p_j q_j - w_j q_j) \]

yielding the following outcome in terms of quantities:

\[ (A13) \quad q_i(w_i, w_j) = \frac{a(1-c) - w_i + cw_j}{2(1-c^2)}. \]

When a uniform labour price is exogenously given, we get \( w_i = w_j = w \), hence, the following post-merger equilibrium outcomes are derived:

\[ (A14) \quad q^M = \frac{a - w}{1 + c} \]
\[ (A15) \quad p^M = \frac{a + w}{2} \]
\[ (A16) \quad \pi^M = \frac{(a - w)^2}{2(1 + c)} \]
\[ (A17) \quad CS^M = \frac{(a - w)^2}{4(1 + c)} \]
\[ (A18) \quad SW^M = \frac{3(a - w)^2}{4(1 + c)}. \]

It is easy to see, by a simple comparison between (A11) and (A18) that the traditional welfare result holds for any \( c > 0 \), that is consumer surplus and total welfare are always higher under Bertrand competition than in the post-merger case.

References


