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**Fully-Funded and PAYG pension schemes facing with demographic changes**

*Discussion Paper n. 147*

2012
Discussion Paper n. 147, presentato: settembre 2012

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Abstract

Motivated by the recent population aging process as well as the tendency towards the substitution of PAYG with Fully Funded pension systems, we analyze the different features of both funded and unfunded pensions under the pressure of population aging. While virtually all previous work in this literature has predicted a reduction in pension benefits as well as a greater relative immunization of FF systems in the face of population aging, this paper shows that the former prediction only holds for specific assumptions relating to technology (i.e. sufficiently low capital shares), while the latter prediction is more likely to be reverted (i.e. the current dramatic aging could be more harmful (less beneficial) for FF pension systems than for PAYG pension systems).

Keywords  Demographic change; unfunded and funded pensions; OLG model

JEL Classification  J14; J18; O41
1. Introduction

In the recent years it is emerged a tendency towards the conversion of mature PAYG social security systems (PAYG from now onwards) into mandatory defined contributions systems managed in the capital market (Fully-Funded scheme, FF from now onwards). More than 20 countries have adopted the FF approach to social security.\(^1\) Other developed countries, such as Germany, Italy and UK, are moved in that direction.\(^2\) A confluence of multiple reasons may be considered responsible for this change: among others, (i) a more individualist, market oriented approach towards social security following mainstream economics and sovra-national organizations advices (e.g., Buchholz et al., 2008;\(^3\) (ii) an enhancement of efficiency, by reducing contribution rates and expanding labour productivity and capital markets; (iii) a transfer of some types of risks (e.g. financial) from the employer to the employed; (iv) a response to the economic effects of an aging population (e.g. financial pressures on the PAYG scheme caused by changes in fertility rates and/or life expectancies). In the usual OLG context,\(^4\) two results are well-known: first, a PAYG pension scheme causes a crowding out of private investment, thus reducing transitory and long run output and labour productivity, while a FF pension scheme eliminates such a crowding out and achieves the same result of a competitive economy without social security. This means that in the long run, given the standard realistic assumption of dynamically inefficient economies (under-saving), i) a PAYG pension scheme reduces welfare, and ii) the change of the intergenerational transfer implied by a change from a PAYG to a FF scheme always involves a Pareto deterioration.\(^5\)

Moreover many studies investigated financial and political risks corresponding with the two different pension schemes (e.g. Blake, 2000, 2003, 2008 for the UK and McHale, 2001; Shoven and Slavov, 2006 for the US). Other previous studies have accurately investigated the different intergenerational distributive features of different pension schemes (e.g. Smith, 1982; Thøgersen, 1998; Žamac, 2005; Wagener, 2003, 2004).

However what is very important in times of dramatic demographic changes, and that has been less widely considered is whether and how the two different pension schemes are affected by such demographic changes. As known, population aging is caused by a combination of increased longevity and reduced fertility. A common wisdom, in general based on an accounting point of view, maintains that aging undermines the sustainability of the PAYG pension scheme. For instance, Blake (2008, p. 357) notes that governments recognized “over the last-quarter century that


\(^2\) For instance, as regards the UK, Blake (2008, p. 342) notes that “by August 2006, 74% of private-sector company defined-benefit (DB) pension schemes in the United Kingdom had closed to new members and 41% had closed to additional contributions from existing members. At that time 67% of open, employer-sponsored pension schemes were defined-contribution (DC). In future, therefore, workers will have to look increasingly to the state or to a personal (or company) DC pension scheme to provide them with a retirement income.”

\(^3\) The World Bank has spent over $ 5.4 billion to promote the adoption of FF schemes (Buchholz et al., 2008).

\(^4\) For a recent critical survey on the OLG context see Weil (2008).

\(^5\) See Weil (2002) for a different opinion.
the combination of increasing longevity and declining fertility is fatal for the long-term viability of a pay-as-you-go state pension system without serious cutbacks in its generosity.” In contrast, a FF pension scheme seems to be less affected by such demographic events. For instance, Blake (2008, p. 357) again argues for a differential impact of demographic changes on the two schemes: “funded schemes are much less susceptible to (although not immune from) adverse demographic shocks than are pay-as-you-go schemes.”

In this paper we pose the following questions: (i) What are the different features of funded and unfunded pension schemes under the pressure of the population aging? (ii) Is it theoretically and realistically justified to believe that FF pension systems, facing with the current demographic changes, are relatively more immunized than PAYG ones? To answer these questions, we assume, as usual, Cobb-Douglas functional forms for obtaining closed form solutions and thus a clear-cut economic interpretation of the results, and we focus exclusively on the stationary state outcomes. Thus our paper provides an evaluation of how demographic changes induce qualitative effects on the two different pensions schemes, showing that, despite the accounting point of view, (i) FF benefits are not immune to fertility changes as well, (ii) both PAYG and FF benefits, in economies with sufficiently high capital shares, may, rather surprisingly, benefit both from (ii.1) the current fertility drop, in the same qualitative way, and (ii.2) from the current increase in the length of life. However in the latter case FF benefits seem to be harmed (beneficiated) more (less) than PAYG benefits. Therefore the suggestion is that the current dramatic aging (reduced fertility and increased longevity) could not only heavily also affect FF pension systems (in contrast with a conventional belief originated by an accounting point of view which would predict its “immunization”) but it could be more harmful for FF pension systems than for PAYG pension systems.

The remainder of the paper is organized as follows. Section 2 presents the basic model with the two different social security schemes and the corresponding steady state solutions, while Sections 3 and 4 discuss the effects of fertility and longevity on pensions, respectively. Section 5 summarises the results, while the concluding section provides a discussion about them.

2. The model

2.1. Individuals

Following Diamond (1965) we assume that: 1) young population $N_t$ grows at a constant rate $n$; 2) individuals live for two periods: youth (working time) and old-age (retirement time); 3) individuals belonging to generation $t$ have a homothetic and separable utility function defined over young-aged and old-aged consumptions, $c_{1,t}$ and $c_{2,t+1}$, respectively; 4) each young individual supplies inelastically one unit of labour in the labour market, and receives wage income at the competitive rate $w_t$; 5) during old-age agents are retired and live on the proceeds of their savings ($s_t$) plus the accrued interest at the rate $r_{t+1}$. Furthermore, each retiree is entitled to a pension benefit ($p_{t+1}$) financed at balanced budget through either a PAYG scheme or a FF scheme. Finally, we suppose old individuals survive to the retirement period with (constant) probability $0 < \pi < 1$. The existence of a perfect annuity market implies old survivors will benefit
not only from their own past saving plus interest, but also from the saving plus interest of those who have deceased.

To solve explicitly for the savings function, we assume that the representative individual born at time \( t \) has the following additive separable utility function:

\[
U_t = \ln(c_{1,t}) + \pi \gamma \ln(c_{2,t+1})
\]

and is faced with the following constrained program:

\[
\max_{\{s_t\}} U_t \quad \text{subject to the intra-temporal budget constraints}
\]

\[
c_{1,t} + s_t = w_t(1 - \theta)
\]

\[
c_{2,t+1} = \frac{(1 + r_{t+1})s_t}{\pi} + p_{t+1}
\]

where \( 0 < \theta < 1 \) is the contribution rate and \( 0 < \gamma < 1 \) is the subjective discount factor.

The solution of the individual’s maximisation program gives the following saving function:

\[
s_t = \frac{\pi \gamma w_t(1 - \theta)}{1 + \pi \gamma} - \frac{\pi p_{t+1}}{(1 + \pi \gamma)(1 + r_{t+1})}
\]

### 2.2. Fully-funded and pay-as-you-go pensions systems

In this section we present the balanced budget of the two pensions schemes and the corresponding temporary savings functions.

A) PAYG case

The government balances the PAYG social security budget in every period according to the following formula:

\[
\pi p_{s,t} = \theta w_t(1 + n),
\]

where the left-hand side represents the per capita social security expenditure and the right-hand side the per capita tax receipts.\(^6\) Inserting (4) into (3) to eliminate \( p \), the saving function chosen optimally by individuals becomes:

\[
s_{s,t} = \frac{\pi \gamma w_t(1 - \theta)}{1 + \pi \gamma} - \frac{(1 + n)\theta w_{t+1}}{1 + \pi \gamma}.
\]

B) FF case

The government (or a private pension fund) balances the Fully-funded social pensions budget in every period according to:

\[
\pi p_{FF,t} = \theta w_{t-1}(1 + r_t),
\]

where the left-hand side represents the pensions expenditure and the right-hand side the mandatory (or contractual) contribution receipts plus the matured interests. Such budget identity shows that a positive per capita pension benefit weighted by the probability to be alive at the retirement time (a positive left-hand side) must be exactly matched by the contribution rate multiplied for the wage rate perceived at the

\(^6\) Notice that, as usually assumed, agents act in an atomistic way and do not take the government budget into account when deciding on the savings path.
young age plus the accrued interest at the current market rate. Inserting (6) into (3) to eliminate \( p \), the saving function becomes:

\[
s_{FF,t} = \frac{w_t}{1 + \pi_t} \left[ \pi_t (1 - \theta) - \theta \right].
\]

(7)

2.3. Firms

As regards the production sector, we suppose firms are identical and act competitively. The (aggregate) constant returns to scale Cobb-Douglas technology of production is \( Y_t = AK_t \alpha L_t^{1-\alpha} \), where \( Y_t \), \( K_t \), and \( L_t = N_t \) are output, capital and the time-\( t \) labour input respectively, \( A > 0 \) represents a scale parameter and \( 0 < \alpha < 1 \) is the capital’s share on total output. Defining \( k_t := K_t / N_t \) and \( y_t := Y_t / N_t \) as capital and output per-capita respectively, the intensive form production function may be written as \( y_t = Ak_t^\alpha \). Assuming total depreciation of capital at the end of each period and knowing that final output is treated at unit price, profit maximisation leads to the following marginal conditions for capital and labour, respectively:

\[
r_t = \alpha Ak_t^{\alpha-1} - 1,
\]

(8)

\[
w_t = (1 - \alpha)Ak_t^\alpha.
\]

(9)

2.4. Equilibrium

Given the pensions budget (4) and (6) and knowing that population evolves according to \( N_{t+1} = (1 + n)N_t \), the market-clearing condition in goods as well as in capital markets is expressed by the following equalities: \( i) \ (1 + n)k_{t+1} = s_{SS,t} \) in the PAYG case, and \( ii) \ (1 + n)k_{t+1} = s_{FF,t} + \theta w_t \) in the FF case. Substituting out for savings according to Eqs. (5) and (7), respectively, exploiting (8) and (9), and assuming individuals are perfect foresighted, and solving for the steady state \( k_{t+1} = k_t = k^* \), we obtain the stationary per-capita capital stock in the PAYG and FF cases, respectively:

\[
k_{SS}^\ast(n, \pi) = \left\{ \frac{\pi_t (1 - \theta) \alpha (1 - \alpha) A}{(1 + n) [\alpha (1 + \pi_t) + \theta (1 - \alpha)]} \right\}^{\frac{1}{1-\alpha}}.
\]

(10)

\[
k_{FF}^\ast(n, \pi) = \left\{ \frac{\pi_t (1 - \alpha) A}{(1 + n) (1 + \pi_t)} \right\}^{\frac{1}{1-\alpha}}.
\]

(11)

2.5. Steady-state analysis

From Eqs. (10) and (11) it can easily be seen that the standard results of the neoclassical growth theory hold for both cases: \( i) \) an increased population growth will result in a reduced long-run stock of capital per person, and \( ii) \) an increased longevity will result in an increased long-run stock of capital per person. However we note that, although the qualitative effects of demographic changes on both capital stocks are the same, the quantitative ones are different, as it is easily viewed by the derivatives of both capital stocks with respect to \( n \) and \( \pi \). Therefore the steady-state analysis provides the following main results.
Result 1. As expected, the long run capital stock under PAYG pensions is lower than that under FF pensions. This straightforwardly derives by comparing Eqs. (10) and (11) (the formal proof is here omitted for brevity).

Result 2. By simply observing the two pension budgets, Eqs. (4) and (6), from an accounting point of view, we see that (i) both schemes seem to be affected in the same way by a change in longevity, while (ii) only the PAYG scheme seems to be influenced by a change in fertility.

However, in a general equilibrium economic context such as the present OLG model, even the factor prices are affected by demographic changes through the changes induced in the capital accumulation. To highlight the effects of endogenous factor prices, we rewrite the steady state pensions budget equations as:

\[
\pi p_{SS} = \theta w[k_{SS}(n, \pi)](1 + n). \tag{12}
\]

and

\[
\pi p_{FF} = \theta w[k_{FF}(n, \pi)](1 + r[k_{FF}(n, \pi)]). \tag{13}
\]

From Eqs. (12) and (13), and recalling the different quantitative responses of both capital stocks to demographic changes, it is evident that (i) even the FF pension scheme is affected by changes in fertility (as derived by the simple inspection of Eq. 13), and (ii) both pension schemes are affected in a substantially different way by demographic changes.

Result 3. Both FF and PAYG systems are affected in the same qualitative and quantitative way by demographic changes as regards the component due to the total contributions (i.e. \(\theta w\)), while as regards the component due to the “return”, which is given by the population growth rate and by the interest rate in the PAYG and in the FF schemes, respectively, demographic changes act in an asymmetric way: this is because the interest rate depends on the population growth rate as well as the longevity rate (as straightforwardly derived by a simple comparison of Eqs. 12 and 13). Therefore, from Eqs. (12) and (13) it is easy to see that a demographic change will cause a different variation of the two pension benefits.

In the next sections, we discuss separately the effects of the two components of the demographic change, i.e. fertility and longevity, on the two pension schemes.

3. Long-run effects of fertility rates on FF and PAYG pensions

Which are the effects of a lower fertility rate on pension payments? The accounting point of view (given by Eqs. 4 and 6) as well as a widespread common belief suggest that a drop in fertility rates, by reducing the number of future contributors to the PAYG system, requires – in order to keep balanced the PAYG pension budget, ceteris paribus as regards the contribution rates –, a corresponding reduction in future pensions per-pensioner while it should not affect or less affect the FF system. In what follows we will give a theoretical answer to the simple but important question raised above finding that, rather surprisingly, in the basic OLG model of neoclassical growth such a common belief may be reversed.

Totally differentiating Eqs. (12) and (13), with respect to \(n\) gives:
Eqs. (14) and (15) show the channels of transmission of the effects of fertility changes on pension benefits in the PAYG and FF cases, respectively. In particular, Eq. (14) reveals that the final effect of a positive change of population growth on long-run PAYG pension payments depends on two counterbalancing forces, and it appears to be ambiguous: (i) a positive (direct) effect which tends to decrease pensions owing to the reduced number of young-aged contributors to the PAYG system in the whole economy, and (ii) a negative (indirect) general equilibrium feedback effect which — owing to the negative effect of population growth on the long-run stock of capital per person — acts on pensions through the increased wage rate perceived by each young-aged contributor.

Eq. (15) shows that, while, in contrast with the PAYG case, there is no direct effect of fertility rates, also in the FF case the final effect of a raised population growth on long-run pension payments appears to be ambiguous and dependant on two counterbalancing forces, which are in this case both (indirect) general equilibrium feedback effects: (i) the same negative effect channelled through the increased wage (ii) a positive effect due to the increase in the returns of the pension contributions (i.e. the increase of the interest rate) induced by the negative effect of population growth on the long-run stock of capital per person.

To analyse which of the two forces ultimately dominates in both cases, we now combine Eqs. (4), (10) and (6), (11) to obtain the following steady-state pension formulas:

A) PAYG case

\[
\frac{dp_{SS}^*}{dn} = \frac{\partial p_{SS}^*}{\partial n} + \frac{\partial p_{SS}^*}{\partial w^*} \frac{\partial w^*}{\partial k^*} \frac{\partial k^*}{\partial n}, \text{ in the PAYG case, and}
\]

B) FF case

\[
\frac{dp_{FF}^*}{dn} = \frac{\partial p_{FF}^*}{\partial w^*} \frac{\partial w^*}{\partial k^*} \frac{\partial k^*}{\partial n} + \frac{\partial p_{FF}^*}{\partial r^*} \frac{\partial r^*}{\partial k^*} \frac{\partial k^*}{\partial n}, \text{ in the FF case.}
\]

From Eqs. (16) and (17) thus the following proposition holds:

**Proposition 1.** (1) Let \( \alpha < 1/2 \) hold. Then a reduced population growth always decreases pensions in both cases.

(2) Let \( \alpha > 1/2 \) hold. Then a reduced population growth always increases pensions in both cases.

**Proof.** The proof uses the following derivatives:
\[
\frac{\partial p_{ss}^*(n)}{\partial n} = 1 - 2\alpha \frac{\theta(1 - \alpha)}{1 - \alpha} \frac{\gamma(1 - \theta)\alpha(1 - \alpha)A}{\alpha(1 + \gamma) + \theta(1 - \alpha)} \left( \frac{\alpha}{1 - \alpha} \right)^{1 - \alpha} (1 + n)^{-\frac{a}{1 - a}}.
\]
\[
\frac{\partial p_{ff}^*(n)}{\partial n} = 1 - 2\alpha \frac{\alpha A^{1 - \alpha}}{1 - \alpha} \left( \frac{\pi\gamma(1 - \alpha)}{1 + \pi\gamma} \right)^{\frac{2a - 1}{\alpha}} \left( \frac{\alpha}{1 - \alpha} \right)^{1 - \alpha} (1 + n)^{-\frac{a}{1 - a}}.
\]

Therefore, if \( \alpha < 1/2 \), then \( \frac{\partial p_{ss}^*(n)}{\partial n} > 0 \) and \( \frac{\partial p_{ff}^*(n)}{\partial n} > 0 \). By contrast, \( \alpha > 1/2 \) implies \( \frac{\partial p_{ss}^*(n)}{\partial n} < 0 \) and \( \frac{\partial p_{ff}^*(n)}{\partial n} < 0 \). Q.E.D.

To sum up, when \( \alpha > 1/2 \),

fertility reductions improve, in contrast with a popular belief, the level of the pension benefits for both systems, that is, in such a case, the current fertility decline improves the returns of the FF system as well as the sustainability of the PAYG system.

4 Pensions and longevity in the long-run

In this section the sensitivity of pension benefits, displayed in eqs. (16) and (17), with respect to longevity changes is investigated. Therefore, the total derivative of Eq. (16) and (17) with respect to \( \pi \) gives:

\[
\frac{dp_{ss}^*}{d\pi} = \frac{dp_{ss}^*}{d\pi} + \frac{dp_{ss}^*}{dw} \frac{dw}{d\pi} + \frac{dp_{ss}^*}{dk} \frac{dk}{d\pi}.
\]

\[
\frac{dp_{ff}^*}{d\pi} = \frac{dp_{ff}^*}{d\pi} + \frac{dp_{ff}^*}{dw} \frac{dw}{d\pi} + \frac{dp_{ff}^*}{dr} \frac{dr}{d\pi} + \frac{dp_{ff}^*}{dk} \frac{dk}{d\pi}.
\]

Eqs. (18) and (19) reveal that the negative (direct) effect which tends to reduce pensions owing to the higher number of old-aged individuals, is only a partial effect and that the final effect of an increase in longevity on the long-run pension payment appears to be ambiguous, in that it depends on the interaction of the direct effect with (i) one counterbalancing force in the PAYG case, that is the positive (indirect) general equilibrium feedback effect which acts on pensions through an increased wage rate owing to an increased stock of capital per-capita, and (ii) two forces in the FF case, which are counterbalancing each other, that is (ii.1), the increased, similarly to the other case, wage effect and (ii.2) a negative (indirect) general equilibrium feedback effect which acts on pensions through reduced returns on the pension contributions (i.e. reduced interest rates) owing to an increased stock of capital per-capita. As for the PAYG case, the analysis of which of the two forces ultimately dominated has been provided by Fanti and Gori (2008) and therefore for such a case I will draw on that. In what follows I focus on the FF case and on the comparison with the PAYG case.

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7 As for the realism of such a case, see the next section.

8 Fanti and Gori (2008) have investigated the role of longevity changes on PAYG pension systems in a similar OLG model, but they abstracted from the effects of fertility changes as well as from the presence of FF pension systems.
From Eq. (17) we have the following proposition:

**Proposition 2.**

- (1) The FF pension benefit is always reduced by longevity increases if \( \alpha < 2/3 \) holds.
- (2) If \( 2/3 < \alpha < \alpha^o = \frac{2+\gamma}{3+\gamma} \) holds, then the pension benefit is an inverted U-shaped function of the rate of longevity and the pension benefit is maximised at \( \pi = \pi_p = \frac{3\alpha - 2}{\gamma(1-\alpha)} \).
- (3) Let \( \alpha^o < \alpha < 1 \) hold. Then an increase in longevity always increases pensions.

**Proof.** The proof uses the derivative \( \frac{\partial p^*(\pi)}{\partial \pi} \) and is obtained through simple algebraic manipulations (therefore it is omitted here for brevity). Q.E.D.

In order to compare the results for the two pensions scheme, we report the following result as regards the PAYG case (see Fantini and Gori, 2008, Proposition 1, p. 4): (1) an increase in longevity always reduces pensions if \( \alpha < 1/2 \), (2) the pension benefit is an inverted U-shaped function of the rate of longevity and pensions are maximised at \( \pi = \pi_p = \frac{(1-2\alpha)(\alpha + \theta(1-\alpha))}{\alpha(\alpha-1)\gamma} > 0 \) if \( 1/2 < \alpha < \overline{\alpha} := \frac{1+\gamma-3\theta + \sqrt{(1+\gamma-\theta)^2 + 4\theta}}{2[2(1-\theta) + \gamma]} > 0 \), (3) an increase in longevity always increases pensions if \( \overline{\alpha} < \alpha < 1 \).

Although an analytical comparison between the two results (Proposition 2 above and Fantini and Gori, 2008, Proposition 1, p. 4) may be cumbersome, it is easy to see that in the PAYG case \( \overline{\alpha} \) depends in a complicated way on the contribution rate with \( \lim_{\theta \to 0} \overline{\alpha} = \frac{1+\gamma}{2+\gamma} < \alpha^o \) and \( \lim_{\theta \to 1} \overline{\alpha} \equiv 1 > \alpha^o \), so that when the capital share is greater than 2/3 it is not possible to say whether an increasing longevity enhances the pension benefit more (less) in the FF case than in the PAYG case, since the answer depends on whether the contribution rate is sufficiently high (low). However, exhaustive numerical simulations suggest the following

**Result 4.** \( \alpha < \alpha^o \) always hold, and, furthermore, the difference between them is rather significant.\(^9\)

Overall, we note, on empirical grounds, that, while many countries have capital shares close to or larger than 0.50 (e.g. Italy, Spain, Japan, which, interestingly, are also among those countries showing higher longevity increases), it is rather unrealistic to observe capital shares larger than 2/3. In fact, drawing, for instance, on capital’s

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\(^9\) For instance by observing that for the most part of countries a contribution rate about 0.30 may be considered as sufficiently high and a commonly used level of the subjective discount factor is about 0.30 (e.g. de la Croix and Michel, 2002), we find that \( (\alpha^o = 0.697) < (\alpha^o = 0.5538) \).
share estimates by Jones (2003) and Rodriguez and Ortega (2006), we see that 1) according to Jones, countries such as Japan, Ireland, Italy, Portugal and Spain display capitals’ shares estimates higher than 1/2 (at least according to the measure without self employment correction); 2) according to Rodrigues and Ortega, about 88% of 111 countries show (by considering UNIDO data) a capital share larger than 0.5 (while by considering OCSE/SSIS and BSC data the percentage of countries with capital shares larger than 0.5 is lower but in any case significant)

Overall, we may conclude with the following Remark: in general, it is more likely that FF systems are negatively affected by longevity increases than PAYG systems and that in many countries having PAYG instead of FF pension schemes, the increase in the length of life may be beneficial for their pension benefits.

While Proposition 2 deals with the “qualitative” behaviour of the relationship between benefit payments and longevity rates in the two pension schemes, it is also important to investigate how longevity changes affect “quantitatively” the two pension schemes. In this respect the following result holds:

**Result 5. Increasing longevity always reduces (increases) the FF pension benefit more (less) than the PAYG one.**

**Proof.** By using the derivatives of eqs. (16) and (17) with respect to the longevity rates and comparing them, it is straightforward to see (the proof is omitted for brevity) that when longevity increases (i) for $\alpha < 1/2$ the PAYG benefit is reduced less than the FF benefit, (ii) for $1/2 < \alpha < 2/3$ the PAYG benefit is increased while the FF benefit is reduced, (iii) for $\alpha > 2/3$ the PAYG benefit increases more than the FF benefit. Q.E.D.

The above findings suggest, rather unexpectedly, that, on the one hand, increasing longevity may also increase pension benefits in both pension schemes (see Prop. 2), and, on the other hand, it is likely that the FF system is disadvantaged more than the PAYG system from the lengthening of life (see Results 4 and 5).

5. Population aging process and pension schemes.

Finally we note that, since the overall population dynamics results from fertility as well as mortality changes, (i.e. the population aging process is reinforced by both fertility and mortality reductions), then it rises the question of whether there is a trade-off between fertility reductions and longevity increases or not in determining pensions payments. The set of above mentioned results suggest the following summarizing result.

**Result 6. The effects of the current population aging process either penalize FF systems more than PAYG systems or advantage FF systems less than PAYG systems.**

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10 Apart from the reported empirical observations based on widespread statistical accounts, the plausibility of sufficiently high values of the capital shares may be also justified on more theoretical grounds, as exemplified by Chakraborty (2004, p. 124): “Given existing estimates of $\alpha$ in the US, a value greater than 0.5 may be rationalized by broadening the concept of capital. By including human capital, we would expect the share to be in the range (0.6 - 0.8) as in Mankiw et al. (1992), while incorporating organizational capital gives an estimate of 0.71 as in Chari et al. (1997).”

11 The disadvantage of FF systems with respect to the longevity increase is both in absolute terms, in the sense that the FF benefits are reduced when the PAYG benefits are increased, and in relative terms, in the sense that the FF benefits are always more reduced (less increased) than the PAYG benefits.
The line of reasoning behind this result is the following: A) since for the PAYG system: 
(i) for \( \alpha < 1/2 \) there is no trade-off: both fertility and mortality reductions are harmful for pensions benefits; (ii) for \( 1/2 < \alpha < \alpha^* \) there would be a trade-off, in that fertility reductions are beneficial, but longevity increases may be harmful for pension benefits; (iii) for \( \alpha^* < \alpha < 1 \) there is again no trade-off: both fertility and mortality reductions, that is the overall population aging process, are beneficial for pensions benefits; B) since for FF systems the three latter cases occur respectively (a) for \( \alpha < 2/3 \) (b) for \( 2/3 < \alpha < \alpha^e \), and (c) for \( \alpha^e < \alpha < 1 \), and we know that \( \alpha^* < 2/3 < \alpha^e \)(see Result 4); and C) since for FF systems not only the negative effects of longevity increases are possible under a larger a more plausible set of economic circumstances (see Proposition 2) but also are quantitatively more relevant (see Result 5), then we may argue that it is very likely that the population aging as a whole is more harmful for FF systems than for PAYG systems.

6. Conclusions

The objectives of the paper was to establish whether and how demographic changes (e.g. fertility and longevity changes) affect: (i) the FF pension scheme, and (ii) the two pension schemes (FF and PAYG) in a different way, with an usual toy-model for analysing pensions issues (i.e. Diamond, 1965).

As regards fertility changes, we find that (a) significant effects on the FF pension benefit, despite its apparent immunization from an accounting point of view, occur; (b) both FF and PAYG schemes are qualitatively affected in the same way; (c) the current fertility drop may increase, despite the common wisdom, pension benefits in both systems in economies with technologies sufficiently capital-oriented.

As regards the influence of longevity rates we find that (1) under technologies sufficiently capital-oriented, increasing longevity may even increase pension benefits, revealing the unexpected result that the wider the length of life, the more generous the FF pension payments may be;\(^{12}\) (2) although the pension formula of both pensions systems seems to be, from an accounting point of view, similarly affected by an increase in the length of life, the FF system is less immunised with respect to the increasing longevity than the PAYG system; this is because of dissimilar general equilibrium effects induced by longevity on the two pension budgets. In fact in economies in which the influence is negative for both systems (i.e. with capital shares \( \alpha < 1/2 \)) FF pension benefits are more damaged by the increasing length of life, while in the other economies (i.e. with capital shares \( \alpha > 1/2 \)) when longevity increases either FF benefits are reducing while PAYG benefits are increasing or FF benefits are increasing less than PAYG benefits.

These results suggest a twofold consideration: first, rather surprisingly both FF and PAYG benefits may be increased, under rather common values of capital shares, by both components (fertility and longevity) of the population aging; second, while fertility changes affect both pension schemes in the same qualitative way, the influence of longevity changes is rather different on the two pensions payments: FF benefits seems to be less immunised than PAYG benefits.

\(^{12}\) Notice that this result is new, while the result that a longevity increase may increase PAYG pension benefits has been shown by Fanti and Gori (2008).
Therefore, considering the overall population aging process dependant on both fertility reductions and longevity increases, this paper established that, in general, the current dramatic aging could be more harmful (less beneficial) for FF pension systems than for PAYG pension systems. These findings also constitute a policy warning about the responses of different pension schemes to the challenge issued by population aging.

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