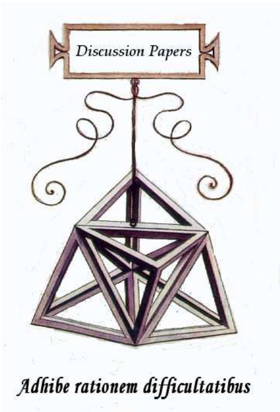
Discussion Papers

Collana di

E-papers del Dipartimento di Scienze Economiche – Università di Pisa



Luca Spataro - Luciano Fanti

From Malthusian to Modern fertility: When intergenerational transfers matter

Discussion Paper n. 163

2013

Discussion Paper n. 163, presentato: **maggio 2013**

Indirizzo degli Autori:
Luca Spataro
Dipartimento di Economia e Management, Università di Pisa (Italy). Via Ridolfi 10, 56124 Pisa, Italy. Tel: +39 0502216217 Fax: +39 0502216384. Email: l.spataro@ec.unipi.it.

Luciano Fanti
Dipartimento di Economia e Management, Università di Pisa, Via Ridolfi 10, 56124 Pisa, ITALY
e-mail: lfanti@ec.unipi.it

Si prega di citare così: Spataro L. and Fanti L. (2013), "From Malthusian to Modern fertility: When intergenerational transfers matter ", Discussion Papers del Dipartimento di Scienze Economiche – Università di Pisa, n. 163 (http://www.dse.ec.unipi.it/index.php?id=52).

Discussion Paper
n. 163

Luca Spataro - Luciano Fanti

From Malthusian to Modern fertility: When intergenerational transfers matter

Abstract

In a standard OLG model of a small open economy with logarithmic utility and endogenous fertility we show that the reversion of the relationship between fertility and wages (i.e. a transition from the Malthusian to the Modern fertility behaviour) may be possible in presence of intergenerational public transfers (i.e. public national debt or PAYG pensions). In fact, as known, the latter have been implemented mostly in the advanced Western Countries, where the fertility behavior reversion has mainly occurred. We show that such a reversion is more likely to occur in economies that are entailed with low interest rate, low costs for raising children and low degree of patience, and high preference for children.

J.E.L. classification: **D91, E62, H63, J13.**
Keywords: overlapping generations, endogenous fertility, savings, small open economy, public national debt, PAYG pension scheme, demographic transition.

1. Introduction

The process of economic development and in particular the so-called Demographic Transition have been the object of intense research in recent years.
According to Galor and Weil (1999) three distinct regimes have characterised the process of economic development: the 'Malthusian' Regime, the 'Post-Malthusian' Regime, and the 'Modern Growth' Regime.

The first and the latter regimes are those relevant (being the Post Malthusian one a regime that "fell between the two just described, [and] shared one characteristic with each of them." See Galor-Weil, 1999, p. 150) as regards the relation between changes in wages and changes in fertility, which is the relation of interest in this paper, in that the *Malthusian Regime* (*Modern Growth Regime*) is featured by a *positive (negative)* relationship between income per capita and population growth.

The economic reasons lying behind the transition from the Malthusian to Modern fertility behaviour have been largely investigated, as below surveyed. However less attention has been paid to the possible role played by the diffusion (starting from the nineteenth century) of intergenerational transfers (typically, pay-as-you-go, PAYG, pension systems and, similarly, public debt), mainly in the European countries which are exactly those in which the reversion of the fertility behaviour has been completed.[1]

In this paper, taking seriously into account the observation of Galor (2005, p. 45): "The simultaneous reversal in the significant upward trend in fertility rates among Western European countries suggests that a common economic force may have triggered the demographic transition in this region…" we investigate whether and how policies entailing redistributions among generations (i.e. pension systems and public debt) widely implemented in most Western European Countries might have played a role in the reversion of the fertility behaviour.

Therefore in this paper we try – in a context of small open OLG economies - an explanation which can add to the established explanations so far emerged in the literature. Since we analyse the determinants of the long-run relationship between fertility and wages in the presence of pension systems (public debt), then it is worth to note that, although such issues are not new in economic research, in general they have been so far analyzed separately.

In fact, several scholars have focused on the relationship between fertility and income although disregarding the role of intergenerational transfers (see, for example, Jones and Schoonbroodt 2010 and Renstrom and Spataro 2012 as for the role of technological shocks). Conversely other works focused on demographic issues with PAYG pension systems but abstracting from the fertility-wage relationship (e.g. van Groezen et al. 2003, Fenge and von Weiszacker 2010 and Fanti and Gori 2012).

As regards the fertility-wage relationship[2], some modern theories of fertility predict the reversion of Malthusian fertility, both because under suitable circumstances increasing wages lead to substitute quantity of children with their quality (see, e.g., Becker (1960), Becker and Tomes 1976) and because of the negative substitution effect of (female) wages on fertility, due to the potential increase of female participation (e.g. Mincer, 1966).

For instance a fertility transition in the models (set up in an overlapping generations context) of Becker et al. (1990), Tamura (1998), Lucas (1998), and Galor and Weil (1998) occurs as individuals begin to trade off quantity for quality. In Galor and Weil (1996) a demographic

---

[1] In fact Galor-Weil (1999, p.152) focus on "the experience of Europe and its offshoots, since they were the areas that went through the complete transition from the Malthusian Regime to modern growth".

[2] See Galor and Weil (1999) and Guinnane (2011) for an overview of several different theories of the demographic transition.

5

transition is generated through a difference in the endowments of men and women and a shift in comparative advantage.

More in general the literature proposed different mechanisms triggering the transition: for example 1) Becker and Barro (1988), in the context of a model of intergenerational altruism, show that increased (Harrod-neutral) technical progress brings upon a higher growth rate of consumption and a lower rate of fertility; 2) Jones (2001), by developing an idea-based growth model, introduces a third effect of increasing wages on fertility, in addition to the standard income and substitution effects - the subsistence consumption level effect – which is not traditionally present and is responsible for the emergence of the crucial feature of the demographic transition: in fact as the wage rises starting from low levels, the subsistence consumption level which the consumer is required to purchase gets cheaper, leading consumers to have more after-subsistence income to spend on both more children and more consumption, but as the wage gets sufficiently large this effect tends to vanish. 3) Galor and Weil (2000) argue that the positive effect of technical progress on the return to education and the feedback effect of higher education on technical progress brings upon a rapid decline in fertility accompanied by accelerated output growth; 4) Fanti and Gori (2007) attempt to provide a further explanation, complementary to those already existing in the literature, focusing on the effects of the unionization of the economies as a cause of the emergence of modern fertility behaviour in place of the Malthusian one.

As mentioned before, all these contributions overlook the analysis of the role that intergenerational transfers might have played in determining the transition of modern fertility behavior. In fact, there are a few exceptions. For example, Fanti and Spataro (2009) analyze the relationship between public debt and fertility in an OLG model with fixed costs for raising children, and show that the latter relation can be ambiguous.

Conversely, other works focused on demographic issues with PAYG pension systems but abstracting from the fertility-wage relationship (e.g. Cigno 1993, Zhang 1995 and, more recently, van Groezen et al. 2003, Fenge and von Weiszacker 2010 and Fanti and Gori 2012). However none of these works focus on the characteristics and on the possible reversion of the wage-fertility relationship.

In this paper we aim at filling this gap. We believe that our attempt is relevant for at least two reasons.

First, for theoretical purposes (to the best of our knowledge such an analysis in presence of intergenerational transfers has not been done so far). Second, for policy reasons, since the recent financial crisis has raised concern about the viability of sustained growth in presence of increasing levels of public debt and/or public pensions.

The main finding of the present paper is that while in the absence of intergenerational transfers the standard logarithmic OLG model would predict either a Malthusian fertility behaviour or even independence of fertility choices from wage (because with logarithmic utility and time-cost of childrearing – as assumed in the present paper - there is an exact balance between income and substitution effects),[3] when intergenerational transfers are introduced the relation between fertility and wage may become of Modern type, provided that some conditions on the love for children (sufficiently high) and rearing costs, interest rate, degree of patience (sufficiently low) hold. Noteworthy, we show that this result holds not only under constant public debt (i.e. public debt à la Diamond 1965) but also under a Defined Contribution pension system. Therefore our paper provides a novel, although partial, explanation which complements the other well established theories of the transition from a Malthusian to a Modern fertility behaviour.

[3] The one or the other outcome strictly depends on the assumption on the typology of children costs. In fact in another paper Fanti and Spataro (2009) have shown that when the cost of rearing children is fixed in terms of consumption goods the Malthusian Regime always occurs.

The work is organized as follows: in section 2 we lay out the model and in section 3 we carry out the analysis of the relationship between fertility (and savings) and public debt. Section 4 investigates such a relationship under a PAYG pension scheme. Conclusions will end the paper.

2. The model

In this work we extend a standard OLG model (Diamond, 1965) in order to entail endogenous fertility[4]. We imagine that individuals live for three periods (childhood, young adulthood, and old-age). In the first period individuals do not make any decisions. In the second period young adults are endowed with well-behaved preferences described by a utility function U. Such a function is defined over consumption in the second and third period of life (c_1, c_2) and on the number of children per adult (*n*), respectively, which are given birth by young adults. Moreover, in the second period individuals receive a salary *w* for their labor services (exogenously supplied) and decide how to split such an income over life-time consumption or on child-bearing. More precisely, we assume that each child costs a fixed fraction, $\tilde{q}$, of wage w[5]. Since we imagine that every single young adult can have children, it follows that the steady state population will be stationary or increasing if *n* is equal or bigger than *1* (thus *n-1* is the long run growth rate of the population).

2.1. Firms

We assume that firms run their business in a perfectly competitive environment and own a Constant-Returns-to-Scale production technology *F(K,L)* by which they transform physical capital *K* and labour *L* (which is identically equal to the young-age population *N*) into a consumption good *Y*. As a consequence, each firm hires capital and labour up to the point in which the cost of the last unit of input is equal to its marginal productivity. Hence, by defining $k=K/L$ the capital intensity, homogeneity of degree one of *F* yields $w = f(k) - f'(k)k$ and $r = f'(k)$, where *r* is the real interest rate, (in the case of absence of depreciation) or $r = f'(k) - 1$ (in the case of full depreciation) and f' indicates the derivative of *f* with respect to *k*. In this paper we focus on the case of small open economy, where the interest rate is given and wages and capital intensity are also fixed (although the latter are also functions of the production function parameters).

2.2. Government

We imagine that the government implements a redistributive policy between generations, by lump sum taxes or benefits. More precisely, we analyze the following two cases:

1) By following Diamond (1965), we assume that at each date *t* the government issues a non-negative amount B_t of national debt and finances it by partly rolling it over and partly by levying lump sum taxes upon the young adults, so that the dynamic equation of debt is:

$$B_{t+1}=B_tR_t-\tau_{1,t}N_{t-1} \qquad [1]$$

(where $R_t \equiv 1+r_t$, $\tau_{1,t}$ is the lump sum tax and $N_{t-1}=L_t$). In per worker terms we get:

---

[4] We adopt a standard method for endogeneizing fertility in OLG models, following, for instance, Galor and Weil, (1996), Strulik (1999, 2003).

[5] See for example Strulik (1999; 2003) and Boldrin and Jones (2002), who make the same assumption on the cost function. This function captures the modern view of a time-cost of childrearing in terms of forgone wages.

$b_{t+1}n_t=b_tR_t - \tau_{1,t}$; [2]

moreover, again by following Diamond (1965), we assume that government pursues the constancy of debt in per worker terms, such that:

$\tau_{1,t}=b(R_t-n_t)$. [3]

2) a PAYG pension system, according to which a lump sum contribution, τ_1, of the young generation of workers finances a lump sum pension benefit, τ_2, paid to the old generation.

In this case we have that, at the equilibrium:

$$\tau_{2,t}L_{t-1}=\tau_{1,t}L_t \quad [4]$$

Or, in per worker terms:

$$\tau_{2,t}=\tau_{1,t}n_{t-1} \quad [5]$$

2.3. Individuals

The young adult i faces the following maximization problem:

$$\max U\left(c_{1,t}^i, c_{2,t+1}^i, n_t^i\right)=z_1\log c_{1,t}^i+z_2\log, c_{2,t+1}^i+z_3\log n_t^i \quad [6]$$

Under public debt, the budget constraint is

$$c_{1,t}^i+\frac{c_{2,t+1}^i}{R_{t+1}}+w_t\tilde{q}n_t^i=w_t-\tau_{1,t}\,. \quad [7]$$

While, under a PAYG pension system, the latter becomes:

$$c_{1,t}^i+\frac{c_{2,t+1}^i}{R_{t+1}}+w_t\tilde{q}n_t{}^i=w_t-\tau_{1,t}+\frac{\tau_{2,t+1}}{R_{t+1}}\,. \quad [8]$$

We can realise the fact the two policies exert similar effects on the budget constraint and, hence, on the individual's behavior. In fact, under public debt, by using eq. [3], eq. [7] at the steady state is:

$$c_1^i+\frac{c_2^i}{R}+w\tilde{q}n^i=w+b(R-n) \quad [9]$$

while under a PAYG pension system, at the steady state, by using eq. [5], eq. [8] becomes:

$$c_1{}^i+\frac{c_2{}^i}{R}+w\tilde{q}n^i=w+\frac{\tau_1(n-R)}{R} \quad [10]$$

Note that we omit the superscript i for n in RHS both of eqs. [9] and [10], in that we assume, as usual, that individuals do not internalize the externality that their own fertility choices exert on the aggregate fertility rate and, hence, on the redistributive policies.

By comparing eqs. [9] and [10] we can see that in both cases, when $n>R$, the net life-time transfer comprised in either policies is positive, while, if $n<R$, such a net transfer is negative as a whole. The main difference is that, in the case of public debt, when $n>(<)R$ the lump sum tax on the young adult τ_1 is negative (positive),[6] while under a PAYG pension system τ_1 is positive in any case and the net life-time transfer is positive (negative). Thus, when $n>(<)R$ both public debt and pension benefits represent net positive (negative) wealth for individuals.

3. Steady state analysis in presence of public debt

We now start the analysis by focusing on the steady state equilibrium in case of constant per capita public debt.

3.1. Characterization of the solution

Under public debt issuing we the steady state solutions for s and n are:

$$s^* = z_2 w\tilde{q}\frac{w-bR}{w\tilde{q}v-bz_3} \tag{11}$$

and

$$n^* = z_3\frac{w-bR}{w\tilde{q}v-bz_3}; \tag{12}$$

with $v \equiv z_1 + z_2 + z_3$. In order for both s^* and n^* to be positive, it must be that:

$$\frac{w}{b} > \max\left[R, \frac{z_3}{\tilde{q}v}\right] \tag{13}$$

or

$$\frac{w}{b} < \min\left[R, \frac{z_3}{\tilde{q}v}\right] \tag{13'}$$

Moreover, by taking the derivative of [11] with respect to w, we get:

$$\frac{\partial s^*}{\partial w} = z_2\tilde{q}\frac{z_3 b(bR-2w)+w^2\tilde{q}v}{(w\tilde{q}v-bz_3)^2}, \tag{14}$$

[6] By passing, we note that with public debt young people face a “variable” lump-sum taxation. In section 4 we investigate the case of Defined Contribution PAYG pension system, in which young people face a “fixed” lump-sum taxation.

which has two roots for *w*:

$$w_{1,2}=\frac{bz_3}{\tilde{q}v}\left(1\pm\sqrt{1-\frac{\tilde{q}v}{z_3}R}\right). \qquad [15]$$

The latter roots are real only for $\frac{z_3}{\tilde{q}v}>R$. Hence, it follows that:

If $\frac{z_3}{\tilde{q}v}<R$, then:

$$\frac{\partial s^*}{\partial w}>0, n^*>0, s^*>0,\ \forall w \qquad [16]$$

If $\frac{z_3}{\tilde{q}v}>R$, then:

$$\frac{\partial s^*}{\partial w}>0, n^*>0, s^*>0,\ \forall w>w_1 \text{ or } w<w_2 \qquad [17]$$

However, since we are dealing with a long run OLG growth model and we expect, in line with observed historical data, that wages tend to increase in the long run,[7] we may drop case [13'] (which imposes *w* to be sufficiently low) and focus on case [13].

Furthermore, it seems reasonable to impose that, in the absence of public transfers, lifetime income $w(1-\tilde{q}n^*)$ is positive, such that, by [7] and [12]:

$$(1-\tilde{q}n^*)>0 \Leftrightarrow \frac{w}{b}>\frac{w^I}{b}\equiv\frac{\tilde{q}R-1}{\tilde{q}(z_1+z_2)}. \qquad [18]$$

Note that

$$R\underset{<}{\overset{>}{-}}\frac{w^I}{b}\Leftrightarrow R\underset{<}{\overset{>}{-}}\frac{z_3}{\tilde{q}v}. \qquad [19]$$

In the light of the above analysis, we can write the following:

Lemma 1: Sufficient for having $\frac{\partial s^*}{\partial w}, n^*, s^*>0$ is $\frac{w}{b}>\max\left[R,\frac{w_1}{b}\right]$, with $w_1=\frac{z_3 b}{\tilde{q}v}\left(1+\sqrt{1-\frac{\tilde{q}v}{z_3}R}\right)$ and $(z_1+z_2)>z_3$.

[7] We note that also in the small open economy context wages may steadily grow due to an exogenous labour productivity growth (here disregarded for simplicity. However, if explicitly considered, such extension would not affect the generality of the results). In this sense when we investigate how fertility changes with increasing wages, we may think that the latter grow due to exogenous technical progress (while the interest rate would be unaffected, keeping constant and equal to its international level. See Appendix A).

Proof: By focusing on case sub [13], we are left with $\frac{w}{b} > \max\left[R, \frac{z_3}{\tilde{q}v}\right]$. Moreover, if $R > \frac{z_3}{\tilde{q}v}$, by condition [16] and [19], then $\frac{w}{b} > R$ is sufficient condition.

If $R < \frac{z_3}{\tilde{q}v}$, then by condition [17] and [18] $\frac{w}{b} > \max\left[\frac{w_1}{b}, \frac{w^I}{b}\right]$ is sufficient. Finally, since, by eqs. [15] and [18] $w_1 \gtrless w^I \Leftrightarrow R \gtrless \frac{{z_3}^2 - (z_1 + z_2)^2}{\tilde{q}v}$, it descends that, by assuming $(z_1 + z_2) > z_3$, if $R < \frac{z_3}{\tilde{q}v}$, then $\frac{w}{b} > \frac{w_1}{b}$ is the sufficient condition[8].

In the reminder of the paper we will assume that conditions provided in Lemma 1 hold.

We now discuss the shape of the relationship between fertility, on the one side, and public debt and factor prices, on the other side, in a small open economy context.

3.2 The role of public debt and factor prices

As for public debt, the results of the steady-state comparative statics can be summarized as follows:

Result 1: $\frac{\partial s^*}{\partial b} \gtrless 0, \frac{\partial n^*}{\partial b} \gtrless 0 \quad \Leftrightarrow R \lessgtr \frac{z_3}{\tilde{q}v}$

Proof: by differentiating eqs. [11] and [12] with respect to b we get:

$$\frac{\partial s^*}{\partial b} = z_2 w^2 \tilde{q} \frac{z_3 - R\tilde{q}v}{(w\tilde{q}v - bz_3)^2} \quad \text{and} \quad \frac{\partial n^*}{\partial b} = z_3 w \frac{z_3 - R\tilde{q}v}{(w\tilde{q}v - bz_3)^2} \ .$$

The economic interpretation of such a result can be better appreciated through the following Lemma:

Lemma 2: $R \lessgtr \frac{z_3}{\tilde{q}v}$ iff $n \gtrless R$.

Proof: By exploiting eq. [12] we get $n - R = w\frac{z_3 - R\tilde{q}v}{wv\tilde{q} - bz_3}$, such that Lemma 2 follows.

As for the economic rationale behind the content of Result 1, recall that, by eq. [9], when $n > (<) R$ public debt is net positive (negative) wealth for individuals. Hence, an increase in debt brings about higher (lower) lifetime wealth, through which individuals increase (decrease) life-time consumption of goods, c_1 and c_2, and of children, n. Moreover, since higher public debt implies higher (lower) disposable income for the young adults, and given that the latter pursue higher (lower) consumption in both periods of their adulthood, increases in public debt entail also higher (lower) savings.

[8] The restriction $(z_1 + z_2) > z_3$ is set for the sake of simplicity and does not affect the generality of the results.

Finally, by mere observation of Result 1 we can also provide the following Corollary:

Corollary 1: Higher public debt may simultaneously increase (decrease) both savings and fertility if love for children is sufficiently low (high) and the interest rate, the cost for raising children and the intertemporal discount factor (z_2/z_1) are sufficiently high (low).

As regards the relationship between savings, fertility, and factor prices, preliminarily it is worth noting that, by observing eqs. [11] and [12], in the absence of public debt such a relationship is clear-cut: *s* is independent of *r* and is a positive function of *w*, while *n* is independent of any price.

However, things do change in presence of positive public debt. As for the role of the interest rate, it is easy to see that, when b>0,

$$\frac{\partial s^*}{\partial r} = -z_2 w\tilde{q}\frac{b}{w\tilde{q}v - bz_3} < 0 \text{ and } \frac{\partial n^*}{\partial r} = -z_3\frac{b}{w\tilde{q}v - bz_3} < 0\,. \qquad [20]$$

As for the role of wage, things interestingly change as well, and it can be shown that the following result holds:

Result 2: $\frac{\partial n^*}{\partial w}\frac{>}{<0}$ iff $R\frac{>}{<}\frac{z_3}{v\tilde{q}}$.

Proof: By derivating eq. [12] we get:

$$\frac{\partial n^*}{\partial w} = z_3 b\frac{\tilde{q}vR - z_3}{(w\tilde{q}v - bz_3)^2}\frac{>}{<}0 \text{ iff } R\frac{>}{<}\frac{z_3}{v\tilde{q}}$$

As regards the economic interpretation of the result above, firstly we note that the wage is both the labor income and a measure of the cost of having children ($w\tilde{q}$). Hence a wage increase raises both individuals' income and children costs, so entailing both an income and a substitution effect on the choice of the number of children. In the absence of pensions systems and with a logaritmic utility, as the present one, these two effects, as known, offset.

Moreover, we can show that:

Lemma 3: The income effect for *n* with respect to *w* is positive.
Proof: see Appendix B.

Hence, when *b* is positive, if *n>R* ($R < \frac{z_3}{v\tilde{q}}$), the increase in *w* implies that the negative substition effect prevails on the positive income effect. On the other hand, if *n<R* the income effect prevails on the substitution effect. Hence, in the first case where debt favours income of young adults, an increase in *w* is followed by a fertility decrease, while in the second case in which income of young adults is burdened by the presence of debt, when *w* increases fertility increases as well.

Hence, the following corollary holds:

Corollary 2: if $\tilde{q}$, z_2 / z_1, i.e. the individual's degree of patience and R, are sufficiently high (low) and z_3 is sufficiently low (high) then the relationship between *n* and *w* is positive (negative).

Finally, in order to check the robustness of the above results, in the next section we modify redistributive policy by focusing on the case of Defined Contribution scheme.

4. Defined Contribution pension scheme

In this section we develop the case of PAYG (i.e. Defined Contribution) pension scheme. First order conditions imply the following steady state solutions for *s** and *n**:

$$s^* = \frac{(w-\tau_1)(z_2 w\tilde{q}R - z_3\tau_1)}{w\tilde{q}vR - z_3\tau_1} \quad \text{and} \qquad [21]$$

$$n^* = z_3 \frac{(w-\tau_1)R}{w\tilde{q}vR - z_3\tau_1} \qquad [22]$$

In order for both *s** and *n** to be positive, it must be that:

$$\frac{w}{\tau_1} > \max\left[\frac{z_3}{z_2\tilde{q}R}, 1\right] \qquad [23]$$

Inequality [23] simply requires that, as usual in any OLG growth model, labour productivity and thus wages must be sufficiently high to allow for a feasible economic system (in line with Lemma 1 referring to the case of debt).

Moreover, by taking the derivatives of [21] and [22] with respect to all parameters, under condition [23], we get the following table:

Table 1. Comparative statics in presence of a Defined Contribution pension scheme

$\frac{\partial s^*}{\partial w}>0$	$\frac{\partial s^*}{\partial r}>0$	$\frac{\partial s^*}{\partial \tau_1}<0$	$\frac{\partial s^*}{\partial z_1}<0$	$\frac{\partial s^*}{\partial z_2}>0$	$\frac{\partial s^*}{\partial z_3}<0$	$\frac{\partial s^*}{\partial \tilde{q}}>0$
$\frac{\partial n^*}{\partial w}=?$	$\frac{\partial n^*}{\partial r}<0$	$\frac{\partial n^*}{\partial \tau_1}=?$	$\frac{\partial n^*}{\partial z_1}<0$	$\frac{\partial n^*}{\partial z_2}<0$	$\frac{\partial n^*}{\partial z_3}>0$	$\frac{\partial n^*}{\partial \tilde{q}}<0$

The ambiguity of sign of the relationship between fertility, on the one side, and wages and lump-sum pension contribution, on the other side, shown in Table 1 is investigated more in detail in the following:

$$\frac{\partial n^*}{\partial \tau_1} = z_3 R w \frac{z_3 - \tilde{q}vR}{(Rw\tilde{q}v - \tau_1 z_3)^2} \underset{<}{\overset{>}{-}} 0 \quad \Leftrightarrow R \underset{>}{\overset{<}{-}} \frac{z_3}{\tilde{q}v} \qquad [24]$$

$$\frac{\partial n^*}{\partial w} = z_3 R \tau_1 \frac{\tilde{q}vR - z_3}{(Rw\tilde{q}v - \tau_1 z_3)^2} \underset{<}{\overset{>}{-}} 0 \Leftrightarrow R \underset{<}{\overset{>}{-}} \frac{z_3}{\tilde{q}v} \qquad [25]$$

Some remarks are in order with respect to the above table: in comparison with the case of public debt, we note that now, differently from the previous case, savings are always decreased by higher lump-sum pension contributions. The reason is that higher contributions imply lower disposable income for young adults, and this in turn produces a crowing-out of savings, even if such a policy may leave individuals either richer or poorer. Moreover, differently from the case of public debt, now savings increase with the interest rate.

By contrast, fertility behaviour with respect to all the parameters are exactly the same in the case of debt and pensions.[9] In particular, focusing on our main objective, we may state the following noteworthy results (easily proved through eq. [24] and [25], respectively):

Result 3: higher pensions – in the form of Defined Contribution schemes – stimulate (reduce) fertility depending on whether love for children is sufficiently high (low), the interest rate, children costs and the degree of patience are sufficiently low (high).

Result 4: wage increases – in presence of Defined Contribution Pension schemes – stimulate (reduce) fertility depending on whether love for children is sufficiently low (high), the interest rate, children costs and the degree of patience are sufficiently high (low).

From both Results 3 and 4 we observe that pensions are fertility-reducing (augmenting) when families have a Malthusian (Modern) behaviour.

Moreover, the main comment in order here concerning the role of wages of fertility is the following: results 1 and 2 concerning the case of debt are equal to results 3 and 4 applying in the case of a Defined Contribution pension scheme.

Therefore, having shown that the same conclusions of our model in presence of public debt also apply in an economy entitled with a PAYG pension system, we are confident in the robustness of the following conclusion: the present paper predicts that, under a standard logarithmic utility, the introduction of pension systems (and public debt) may, under plausible circumstances (e.g. low international interest rates, preferences for old consumption, that is low degree of patience, time-costs of childrearing and high love for children), produce the main component of the demographic transition: fertility falls - instead of rising - as the wage rate rises. Alternatively, the presence of pension systems (public debt) allows for the passage from a Malthusian to a Modern fertility behaviour under some appropriate change in one or more of the above mentioned economic parameters.

5. Conclusions

In this paper we have analysed the behaviour of fertility in an OLG model of a small open economy in presence of intergenerational distribution policies. The main result is that an increase of wages can be beneficial (detrimental) for fertility when: i) the interest rate is sufficiently high (low), ii) the preference for children is sufficiently low (high) (relative to the preference for both own young and old consumption) and iii) the time-cost for childbearing is sufficiently high (low). This result provides a new possible channel – the implementation of intergenerational transfers policies - for the historically observed change - especially in advanced Western Countries - from a Malthusian to a Modern fertility behaviour.

[9] Given that the focus of the paper is on the fertility-wage relationship, we do not further pursue the comments on the different savings behaviours under the two different types of redistributive policy.

References

Barro, R. J. and Becker, G.S. (1989): “Fertility Choice in a Model of Economic Growth”, *Econometrica*, 57, 481-501.

Becker, G. S. (1960): ‘An economic analysis of fertility’, in NBER (ed.): *Demographic and Economic Change in Developed Countries*, Princeton University Press, Princeton, NJ.

Becker, G. S. and R. J. Barro (1988). “A Reformulation of the Economic Theory of Fertility”, *Quarterly Journal of Economics*, vol. 103 (1), February, pp. 1–25.

Becker G., K. M. Murphy, and R. Tamura, (1990). “Human Capital, Fertility and Economic Growth,” *Journal of Political Economy*, October, *98*, S12–S37.

Becker, G. S., and Tomes, N., (1976). "Child Endowments and the Quantity and Quality of Children," *Journal of Political Economy*, University of Chicago Press, vol. 84(4), 143-62.

Boldrin, M. and L. E. Jones (2002). “Mortality, Fertility and Saving in a Malthusian Economy”, *Review of Economic Dynamics*, vol. 5 (4), October, pp. 775–814.

Cigno, A. (1993): “Intergenerational transfers without altruism”, *European Journal of Political Economy*, 7: 505-18.

Diamond, P., (1965). “National debt in a neoclassical growth model”. *American Economic Review* 41, 1126-50.

Fanti, L. and Gori, L. (2007). “From the Malthusian to the Modern growth regime in an OLG model with unions”. *Economics Bulletin* 10, 1–10.

Fanti, L. and Gori, L. (2012). “Fertility and PAYG pensions in the overlapping generations model”. *Journal of Population Economics* (2012) 25:955-961.

Fanti, L. and Spataro, L. (2009). “Fertility and Public Debt”. Discussion Paper, Dipartimento di Scienze Economiche, n 89.

Fenge R. and von Weizsäcker J. (2010). “Mixing Bismarck and child pension systems: an optimum taxation approach”. *Journal of Population Economics* 23(2):805–823.

Galor O., Weil, D.N., (1996): The Gender Gap, Fertility and Growth. *American Economic Review* 86, 374-87.

Galor, O. and D. N. Weil, (1998). “Population, Technology, and Growth: From the Malthusian Regime to the Demographic Transition”. NBER Working Paper No. 6811.

Galor, O. and D. N. Weil (1999). “From Malthusian Stagnation to Modern Growth”, *American Economic Review*, vol. 89 (2), May, pp. 150–154.

Galor, O. and D. N. Weil (2000). “Population, Technology and Growth: From Malthusian Stagnation to the Demographic Transition and Beyond”, *American Economic Review*, vol. 90 (4), September, pp. 806–828.

Galor O. (2005). “From Stagnation to Growth: Unified Growth Theory”, *Handbook of Economic Growth*, P. Aghion and S. Durlauf (ed.), ed. 1, vol. 1, chapter 4, pp. 171-293, North-Holland, Amsterdam.

Guinnane, T.W. (2011): “The historical fertility transition: A guide for economists”, *Journal of Economic Literature*, 49:3, 589-614.

Jones C.I., (2001). “Was an Industrial Revolution Inevitable? Economic Growth Over the Very Long Run”, *Advances in Macroeconomics*, 1:2, n.1.

Jones L.,E. and Schoonbroodt, A., (2010): “Complements Versus Substitutes and Trend in Fertility Choice in Dynastic Models”, *International Economic Review*, 51:3, 671-99.

Lucas, R. E., (1998). “The Industrial Revolution: Past and Future,” University of Chicago mimeo.

Mincer, J. (1966): ‘Labour force participation and unemployment: a review of recent evidence’, in Gordon, R. A. and Gordon, M. S. (eds): *Prosperity and Unemployment*, Wiley, New York.

Spataro, L. and Fanti, L. (2011): “The Optimal level of Debt in an OLG Model with Endogenous Fertility”. *German Economic Review*, 12(3), 351–69.
Renstrom, T. and Spataro, L. (2012). “Population Growth and Technological Progress: a Pure Welfarist Approach”, Durham University Business School Working Papers Series, n. 8.
Strulik, H., (1999). “Demographic transition, stagnation, and demoeconomic cycles in a model for the less developed economy”. *Journal of Macroeconomics* 21, 397-413.
Strulik, H., (2003). “Mortality, the Trade-off Between Child Quality and Quantity, and Demoeconomic Development”. *Metroeconomica* 54, 499-520.
Tamura, R., (1998). “From Agriculture to Industry: Human Capital and Specialization,”. Clemson University, mimeo.
van Groezen B., Leers T. and Meijdam L. (2003). „Social security and endogenous fertility: pensions and child allowances as Siamese twins”. *Journal of Public Economics* 87(2):233–251.
Zhang, J. (1995): “Social Security and Economic Growth”. *Journal of Public Economics* 58:185-213.

Appendix A

Here we show a possible interpretation of the wage changes analyzed in this work. We assume, as usual, a Cobb-Douglas technology, such that $y = Gk^h$ (where $G>0$ is a constant index of technology, y is output per worker and h is the weight of capital in the production function as well as the distributive share of capital), and assume full depreciation of capital, then, we have that, at any time t, the market equilibrium implies:

$$1 + r_t = hG\left(k_t^*\right)^{h-1} \Rightarrow k_t^* = \left[\frac{hG}{1+r_t}\right]^{\frac{1}{1-h}} \qquad [A.1]$$

$$w_t^* = \left(k_t^*\right)^h (1-h)G = (1-h)G^{\frac{1}{1-h}}\left[\frac{h}{1+r_t}\right]^{\frac{h}{1-h}} = (1+r_t)\frac{1-h}{h}k_t^* \qquad [A.2]$$

It is easy to see that, in a small open economy, wages may increase under positive improvements of the exogenous technical progress, G, in that, by equations [A.1] and [A.2], $\frac{\partial w^*}{\partial G} = (1+\bar{r})\frac{1-h}{h}\frac{\partial k^*}{\partial G} > 0^*$. Note that the interest rate is not affected by the changes in G in that, in small open economy, it is equal to the international interest rate.

Appendix B

Proof of Lemma 3:

Recall that, by eq. [15], Lemma 1 implies that when $R > \frac{z_3}{v\tilde{q}}$ then $\frac{w}{b} > R$, and when $R < \frac{z_3}{v\tilde{q}}$, then $\frac{w}{b} > \frac{w_1}{b}$.

Moreover, since, by Slutsky equation:

$$\frac{\partial n}{\partial w} = \frac{\partial n}{\partial w}^{s} - \frac{\partial n}{\partial M}\frac{\partial M}{\partial w} \qquad \text{[B.1]}$$

Where the second object at the RHS of [B.1] is the income effect (IE) and M is the equivalent variation of income. By equation [29] it is easy to show that $M = w(\tilde{q}n - 1)$, such that:

$$\frac{\partial M}{\partial w} = (\tilde{q}n - 1)$$

Moreover, since $\frac{\partial n}{\partial M} = \frac{z_3}{w\tilde{q}v - bz_3} > 0$, we can conclude that

$$sign[IE] = sign(1 - \tilde{q}n)$$

which is positive under eq. [18] and Lemma 1.