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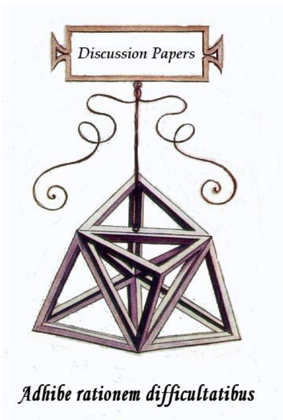

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Transition to Modern Growth: the Role of Technological Progress and Adult Mortality

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Davide Fiaschi -Tamara Fioroni

Transition to Modern Growth: the Role of Technological Progress and Adult Mortality

Abstract

This paper presents a model inspired to the Unified Growth Theory, where reductions in adult mortality, together with improvements in technological progress, are the deep causes of the transition from a Traditional (Malthusian) Regime to a Pre-modern Regime with accumulation of only fixed capital, and finally, to a Modern Regime, with the joint accumulation of fixed and human capital. A calibrated version of the model is able to reproduce the dynamics of UK economy in the period 1541-1914, both matching the periods of transition and the pattern of main macroeconomic variables. UK growth appears mainly due to technological progress before the half of nineteen century, while after the decline in adult mortality and factors accumulation played the major role. Finally, fertility decline along the nineteen century has only a marginal impact on growth because is more than balanced by the longer adult survival.

Classificazione JEL: O10, O40, I20
Keywords: Unified Growth Theory, Human Capital, Adult mortality, Nonlinear Dynamics, Endogenous Fertility, Industrial Revolution

Contents

I. Introduction

In literature there is no agreement on which are the main determinants of the extraordinary development in the last five centuries of Western economies and of the related phenomenon denoted Great Divergence (see Mokyr (1999)). The aim of this paper is to discuss how reductions in (adult) mortality, together with improvements in technological progress, affecting the accumulation of human and fixed capital, can be at the root of the long-run growth of Western countries, with the additional conjecture that the changes in both variables are driven by exogenous factors to economy (we refer to Section II. for a detailed discussion on this point).

We proposed a theoretical model of spirit of the Unified Growth Theory proposed by Galor (2005), augmented by the presence of adult mortality, of fixed (physical) capital accumulation, and of two technologies differing in their inputs, one with only land and unskilled labor, the other with fixed and human capital (see Hansen and Prescott (2002)). We then discuss the ability of a calibrated version of the model to reproduce the dynamics of main macroeconomic variables (GDP per worker, investment rate, structural change in output composition, interest rate, investment in education, fertility rate) of UK economy in the period 1541-1914, including the timing of transition across different regime (at the end of eighteen century for the transition from Malthusian to Pre-modern Regime, and at the half of the nineteen century for the transition from Pre-modern to Modern Regime). Finally, we estimate the contribution of different factors (technological progress, adult mortality, accumulation of fixed and human capital, workforce) to overall growth of UK. The latter appears mainly due to technological progress before the half of nineteen century, while after the decline in adult mortality and factors accumulation played the major role. The strong change in fertility in the nineteen century had a negligible impact on growth, because it appears more than balanced by the increase in adult mortality.

The overlapping generations model considers individuals potentially living for two periods (childhood and adulthood), with a subsistence consumption, and where the saving rate is an increasing function of wealth. Individuals arrive at adulthood with certainty and the risk of mortality occurs during adulthood. Individuals devote the first period of their life (childhood) to the acquisition of human capital (if any) and in the second period they allocate their income, given by the sum of their labor income and their (if any) bequest, between consumption and transfers to their offspring (their transfers are positive only when their income is over a certain threshold). The transfers (if positive) are invested in fixed capital and in the children's education in order to maximize the future income of children (see Galor and Moav (2004)).

An increase in adult survival has two opposite effects on transfers to the offspring: on the one hand, it raises the lifetime consumption of the parents reducing, given their income, the amount of their transfers; on the other hand, it increases their income (via an increase in their labour income) and hence their transfers.[1] At low level of income the first effect si more likely dominate the second one, therefore an increase in adult survival leads to a reduction in the amount of resources available to future generations, the opposite happens at high level of income. Empirical evidence discussed in Cervellati and Sunde, 2011 (see, in particular, their Figure 5) on the U-shaped relationship between life expectancy and the growth rate of per capita income supports this finding.[2]

The dynamics is characterized by three different regimes: (i) a *Traditional (Malthusian) Regime*, where output is produced only

[1] Many theoretical contributions find only a positive effects on growth of gains in life expectancy through various channels; see, for example, Cervellati and Sunde, 2005, Boucekkine et al., 2003, Soares, 2005, De la Croix and Licandro, 1999, Lagerlof, 2003, Weisdorf, 2004, and Bar and Leukhina, 2010). Some recent contributions argue for a positive effect on long-run growth of negative *shocks* to life expectancy (see Voigtlander and Voth (2013)).

[2] In particular, Cervellati and Sunde (2011) find a egative effect before the demographic transition (not necessarily related to the level of income), and a strong and positive effect after the demographic transition. On the other hand, Acemoglu and Johnson (2007) argue that improvements in life expectancy, rising population growth, have a negative effect on per capita GDP. Our paper may help to reconcile these (apparently) conflicting results adding some insights on the conditions under which improvements in life expectancy are good for the long-run growth of a country.

by a traditional technology, whose factors are unskilled labour and land; (ii) a *Pre-modern Regime*, where an increasing share of aggregate output is produced using fixed capital and unskilled labour in an industrial sector; and, finally, (iii) a *Modern Regime* where both fixed and human capital are used in the industrial sector. The transition from the Traditional to the Pre-modern, and, finally, to Modern Regime can be the result of an ongoing progress in the technological progress alone, or the joint effect of an increase in technological progress *and* adult survival. On the other hand, at low level of income (technological progress), a decrease in the adult survival can alone help the transition from a Traditional to a Pre-modern Regime (and viceversa).

The introduction of endogenous fertility does not substantially affect the main results of the paper, but just adds a possible self-reinforcing mechanism to the stability and transition from a regime to the other. The Traditional Regime assumes the typical characteristic of a Malthusian Regime (as it is denoted in the Unified Growth Theory), where increases in income are checked by increases in the population. Once a country escape from the Malthusian Regime the decreasing fertility with respect to income further boosts country's growth by increasing its accumulation of fixed and human capital. In the calibrated version of the model for UK the matching between simulated and observed fertility rate is the most problematic feature, suggesting that a more sophisticate theory is needed in line with the remark made by Easterlin (2004).

The paper is organized as follows: Section 2 discusses the related literature; Section 3 presents the model; Section 4 studies the transitions between regimes; Section 5 calibrate the model to UK economy for the period 1541-1914; and Section 6 concludes. Appendix gathers proof and other technical staff.

II. *Improvements in Technological Progress and the Decline in Adult Mortality*

The literature on the deep explanation(s) of long-run growth of Western countries in the last 500 years is enormous and still increasing.

Following a long tradition in economics dated to Joseph Schumpeter, Aghion and Howitt (2009) identify in the economic incentives behind the innovation activity (such as the profits for innovators) the main explanatory factors of country growth. Other scholars (see, e.g., Lucas, 1988), following a literature starting in 1960' (see, e.g., Cipolla (1962)), point to the accumulation of human capital. Still differently, other scholars point to the quality of institutions forged by environment and individual incentives (Acemoglu et al. (2001)).[3] Finally, some authors relate the increases in the stock of knowledge available in the economy to the size of population, so that they identify a direct causal relationship between the size (or the growth rate) of the population and technological progress (see Kremer (1993) and Galor (2005)). These explanations can also be viewed as complementary and to be applied to different periods and time scale of analysis (e.g., the Schumpterian theory to post First World War, the theory of knowledge accumulation to the very long long run).[4] Common to this literature is howver the search for an *endogenous* (mainly economic) explanation of countries' growth.

In the paper we take a different route, and consider the improvements in technological progress and adult survival as the result of some factors *exogenous* to economic sphere. We argue that this approach is the most suited to our scopes to explain the transition

[3]For example, Acemoglu et al. (2001) argument that the lower settler mortality in North America with respect to Center and Southern America has led to institutions more favourable to investment.

[4]But the prevalence of one or the other has crucial different policy implications. For example, if the quality of institutions is the key factor of development, then the adoption of Western institutions (e.g. democracy) is the main policy recommendation to poor countries; differently, if human capital is crucial the attention should be on all the factors favouring the accumulation of human capital (e.g., public expenditure in education). Finally, if incentives to innovate is crucial an efficient patent law system is needed.

of European countries, in particular UK, through different regimes since 1541 to 1914. As we will discuss below the debate is on the key source(s) of Industrial Revolution and of the adult mortality decline in the second half of nineteen century.[5] The dynamics of European economies after 1914, likely related to a more sophisticate theory of innovation and accumulation of human capital, and of economic factors affecting countries' health sector (in particular, the impact of public health systems), is out of the scope of the paper.

II.A. Improvements in Technology Progress

In the paper we adhere to the idea that Industrial Revolution is the result of the **cultural revolution** in the 17th century (Koyre, 1947, Cipolla, 1993, Rosenberg (1994), Mokyr (2002) Mokyr and 2010; Jacob and Stewart, 2004).

Starting from the perspective that " *... the origins of the Industrial Revolution reach back to that profound change in ideas, social structures, and value systems ...*"(see Cipolla (1993), p. 227) of the last part of Middle Age, we follow Mokyr (2002) in his idea that Industrial Revolution is mainly the result of a cultural revolution caused by the emergence of the new scientific method elaborated in the 17th century which particularly permeated the English society in the 18th and 19 centuries : "... the interconnections between the Industrial Revolution and those parts of Enlightenment movement that sought to rationalize and spread knowledge may have played a more important role than recent writings have given them credit for ... This would explain the timing of Industrial Revolution following the Enlightenment and - equally important - why it did not fizzle out like similar bursts of macroinvention in earlier times. It might also help explain why the Industrial Revolution takes place in western Europe ... "(see Mokyr, 2002, p. 29). The same conclusion is present in Jacob and Stewart (2004), who highlight the importance of the change in approach of individuals to scientific knowledge,

[5]Mokyr (1999) provides an extensive survey of the debate on the causes and nature of Industrial Revolution, while a survey on the causes of the decline in mortality can be found in Livi Bacci (2007).

which "become a centerpiece of Western culture, a partner with industry, ..." (see (Jacob and Stewart, 2004, p. 8)).

We are aware that other *more materialistic explanations* have been proposed (see, e.g, Cipolla, 1993, and Allen, 2009), but *discontinuity* is the major challenge for all these explanations (Clark (2007), p. 228)

Finally, Solow (2000, p. 97) discusses the pros and cons of the theories which aim to endogenize technological progress in contrast to the hypothesis of exogenous technological progress at the root of the traditional Solovian growth model. In particular, he argues how the development of a general and convincing framework is still to be reached, stressing how it is very reasonable to consider changes in technological progress as exogenous but not necessarily constant over time, and highlighting the importance to analyse how the economy reacts to these changes. The same issues is also discussed in (Acemoglu, 2008, p. 414), where the issue is posed in terms "wether innovation is mainly determined by scientific constraints and stimulated by scientific breakthroughs ... or whether is, at least in part, driven by profit motives". Acemoglu concludes in favor of a role of profits in the innovation (at least for developed countries after the second world war).[6]

II.B. The Decline in Mortality

Coherently with our perspective on technological progress decline in mortality beginning at the half of the nineteen century is assumed to have the same exogenous root of the *Industrial Revolution* (see Mokyr, 1991 and 2010; Preston (1975) and 1996; Easterlin (2004); Livi Bacci (2007); Ljungberg, 2013, Deaton (2006)). In particular, in the words of some of the most important scholars in the field "What the Mortality and Industrial Revolutions have in common is that they are both manifestations of the explosion in empirically based human knowledge, scientific and technological, that

[6]From another point of view McCloskey (1995) discuss how the attempt of many economists to endogenize the technological progress can be justify not in the empirical evidence in favor of such possibility, but in the current way to make research in the field of economics.

dates from the seventeenth century onward. " Easterlin (2004), p. 97; and, more precisely, "... the essential element in the gains was an enormous scientific breakthrough - the germ theory of desease. " Preston (1996), p. 6: Therefore, it is possbile to conclude that "[The dominant factors of] The mortality decline of the period since 1850 ... probably include social and cultural factors (methods of child rearing, personal hygiene, improved organization of markets, and so forth) in the first phase of transition ... " Livi Bacci (2007), p. 124.

The sources of mortality decline have been extensively debated among historical demographers, historians of medicine and economic historians (see Schofield et al., 1991). According to McKeown (1976), the principal cause of mortality decline in England from 1838 to the current days was the modern economic growth which, by increasing living standards, and particularly the nutritional level of population, inevitably increased resistance to infectious diseases.

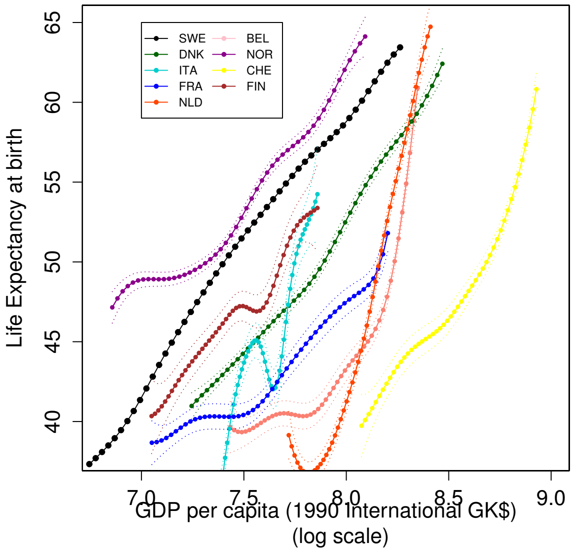
However, subsequent research has shown empirical evidence which contradicts McKeown's theory. As Livi Bacci (2007) asserts "This theory is countered by a number of considerations which make us look to other causes. In the first place, the link between nutrition and resistance to infection holds primarily in cases of severe malnutrition; and while these were frequent during periods of want, in normal years the diet of European populations seems to have been adequate. Second the latter half of the eighteenth century and the first decade of nineteenth, the period during which this mortality transition began, do not appear to have been such a fortunate epoch."(see Livi Bacci (2007), p.71). Moreover, Livi Bacci (2007) argues that the increase in longevity was caused principally by a reduction in young and infant mortality which occurred "not because of better nutrition, but because of improved child-rearing methods and better protection from the surrounding environment."(see Livi Bacci (2007) p.71).

In his 2004 book Robert Fogel emphasizes the strict relationship between better nutrition and mortality reduction; however, he finds evidence of a very limited or even opposing relationship between economic growth and improvement in nutritional status and health during the eighteenth and nineteenth centuries in both Europe and America[7]. In particular, Fogel points out that "The overall improvement in health and longevity during this period is less than might be expected from the rapid increases in per capita income indicated by national income accounts for most of the countries in question. More puzzling are the decades of sharp decline in height and life expectancy, some of which occurred during eras of undeniably vigorous economic growth. "(see Fogel (2004) p. 18).

With respect to more recent years, i.e. from 1900 to 1960, Preston (1975) finds that economic growth explains only $10-25$ percent of the increase in life expectancy whereas the remaining $75-90$ per cent of the growth in life expectancy is attributable to factors exogenous to countries' level of income. In particular, he emphasizes the crucial role of the widespread diffusion of medical innovation in reducing mortality: "It seems to have been predominantly broad-gauged public health programmes of insect control, environmental sanitation, health education and maternal and child health services that transformed the mortality picture in less developed areas, while it was primarily specific vaccines, antibiotics and sulphonamides in more developed areas. But the technologies were not, for the most part, indigenously developed by countries in either group. Universal values assured that health breakthroughs in any country would spread rapidly to all others where the means for implementation existed. The importance of exogenous, largely imported, health technology in the now-developed countries may have been underestimated for earlier periods as well."(see Preston (1975) p. 243).

Fig. 1 supports Preston's conclusion that factors not related to income explain the rise in life expectancy. The left panel of Fig. 1 shows that at the same level of per capita GDP corresponds very different levels of life expectancy, while the right panel of Fig. 1 shows that at a given year the differences in life expectancy across

[7]See also Deaton (2006) "In Britain, the United States, and much of Europe, there were periods in the nineteenth century when urbanization ran ahead of the rate of public health provision and population health deteriorated during periods of rapid economic growth."(Deaton, 2006, p. 111)



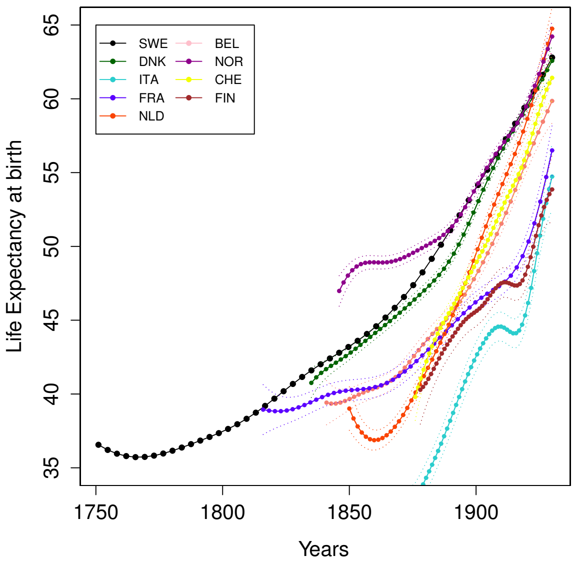


Figure 1: Life expectancy at birth versus per capita GDP. Per capita GDP is from Maddison Project Database (update 02/2013). Life expectancy at birth is from the Human Mortality Database (update 02/2013).

countries are low.

The left panel of Fig. 1 shows a marked increase in life expectancy since the end of nineteenth century, that is the period in which according to Easterlin (2004) the so-called Mortality Revolution started[8]. In particular Easterlin (2004)'s asserts "Since 1870, life expectancy at birth in many areas of the world has soared from values around 40 years or less to 70 years or more. The reduction in mortality has been accompanied by an associated improvement in health as the incidence of contagious disease has dramatically lessened. This lengthening of life and associated reduction in morbidity brought about by the "Mortality Revolution "has meant at least as much for human welfare as the improvement in living levels due to modern economic growth. Certainly the Mortality Revolution has substantially affected a much wider segment of the world's population. "(see Easterlin, 2004, p.84).

Easterlin (2004) argues that the rise in life expectancy crucially principal depends on the emergence and increasing importance of medical and technological innovations (see Easterlin, 2004, p. 86).[9]

III. The Model

The model is inspired by Galor and Moav (2004). Consider an economy populated by an overlapping generations of people who potentially live for two periods: childhood and adulthood. They live in childhood for sure but are subject to risk of dying during the adulthood. Denote the expected length of adulthood in period t by $p_t \in (0, 1)$, and the total number of adults at the begin of period t by L_t, the *actual aggregate labor supply* in the period t is equal to $p_t L_t$.

[8]The uniqueness of the development of scientific medicine in the nineteenth and twentieth centuries is also well documented in the history of medicine as, for example, by Dixon (1978), Watts (2003) and Porter (2006).

[9]In particular, he identifies three major breakthroughs which bring the mortality reduction: 1) new methods of preventing the transmission of disease, including clean water supply and education in personal hygiene; 2) new vaccines to prevent certain diseases which started in the 1890's; and 3) new drugs to cure infectious disease (antimicrobials) which started in the late 1930's (see Easterlin, 2004, p. 104 and also Deaton, 2006, p. 110).

III.A. Production

In every period, the economy produces a single material good, whose price is normalized to 1. Production may take place with two different technologies: a traditional technology that employs unskilled labour and land, and an industrial technology that employs fixed capital and skilled labour. While the traditional technology is always operating, the industrial technology, as we shall see below, will become available once technology has progressed enough (for the production structure we follow (Aghion and Howitt, 2009)). The traditional production function is given by:

$$Y_t^a = A_t^a (p_t L_t^a)^{1-\lambda} (T)^{\lambda}, \tag{1}$$

where A^a is a productivity parameter, pL_t^a is the actual amount of unskilled labour employed in the traditional sector in the period t and T is the quantity of land. The industrial production function is given by:

$$Y_t^m = A_t (p_t h_t L_t^m)^{1-\alpha} K_t^{\alpha}, \tag{2}$$

where $\alpha \in (0, 1)$, $A > 0$ is a technological parameter, and $p_t h_t L^m$ is the actual amount of skilled labour employed in the industrial sector given by the individual level of human capital and the actual labour force $p_t L_t^m$. As established below human capital increases with the resources invested in education and when these resources are zero $h_t(0) = 1$ and therefore the industrial sector employs unskilled labour.

When production is conducted using only the traditional technology the wage rate is given by:

$$w_t^a = (1 - \lambda) A_t^a p_t^{1-\lambda} (L_t^a)^{-\lambda} T^{\lambda}. \tag{3}$$

When industrial technology is operating the rate of return to capital r_t and the wage rate per efficiency unit of labor w_t^m are given by:

$$r_t = \alpha A_t p_t^{1-\alpha} \left(\frac{K_t}{h_t L_t^m} \right)^{\alpha - 1}; \tag{4}$$

$$w_t^m = (1-\alpha)\,A_t p_t^{1-\alpha}\left(\frac{K_t}{h_t L_t^m}\right)^{\alpha}. \qquad (5)$$

In the early stages of development production is conducted using the traditional technology while the industrial technology is latent since no fixed and human capital have been still accumulated. The economy will start employing the industrial technology together with the traditional technology when income will be sufficiently high. In particular, as it will be discussed below, the improvements in technological progress and adult survival will lead parents to leave a positive bequest to their children under the form of investments in fixed capital and education; this, in turn, will activate the industrial technology.

Total output is therefore given by:

$$Y_t = Y_t^a + Y_t^m. \qquad (6)$$

Workers are assumed to be perfectly mobile between the two sectors; therefore wages are equalized across sectors, i.e. $w_t^a = w_t^m h_t$. This implies that employment in the traditional sector is chosen in order to maximize profit (excluding the return to land), i.e. $L_t^a = \arg\max[A^a(p_t L_t^a)^{1-\lambda}(T)^{\lambda} - w_m h_t L_t^a]$ which solves for the following value:

$$L_t^a = \left[\frac{A_t^a p_t^{1-\lambda}(1-\lambda)}{w_t^m h_t}\right]^{1/\lambda} T. \qquad (7)$$

The amount of labor employed in the industrial sector is therefore:

$$L_t^m = L_t - L_t^a, \qquad (8)$$

where L_t is the size of working population. Assuming for simplicity that

$$\alpha = \lambda,$$

and that the productivity in the traditional sector has the same trend that productivity in the industrial sector, that is

$$A_t^a = \phi A_t,$$

with $\phi < 1$, the aggregate production function is given[10]:

$$Y_t = A_t p_t^{1-\alpha} L_t^{1-\alpha} \left(\tilde{T} + h_t^{\left(\frac{1-\alpha}{\alpha}\right)} K_t \right)^{\alpha}, \tag{12}$$

where $\tilde{T} = \phi^{1/\alpha} T$.

The *actual income per worker* in period t is therefore given by:

$$y_t = A_t p_t^{1-\alpha} \left(\frac{\tilde{T}}{L_t} + h_t^{\left(\frac{1-\alpha}{\alpha}\right)} k_t \right)^{\alpha}, \tag{13}$$

where $y_t \equiv Y_t / L_t$ and $k_t \equiv K_t / L_t$.

For future reference from Eq. (11) income per worker can be written as follows:

$$y_t = A_t p_t^{1-\alpha} \left(\frac{\tilde{T}/L_t}{l_t^a} \right)^{\alpha}, \tag{14}$$

where $1/l_t^a$ proxied for the accumulation of fixed and human capital.

III.B. Consumption and Total Transfers

In childhood individuals acquire education and make no decisions; in adulthood individuals work, have n_t children, possibly invest in their children's human capital, and save for the future wealth of their offspring.

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[10]Substituting Eq. (5) into Eq. (7) leads to:

$$L_t^a = \left[\frac{A_t^a p_t^{1-\lambda} (1-\lambda)(h_t L_t^m)^{\alpha}}{A_t p_t^{1-\alpha} (1-\alpha) h_t K_t^{\alpha}} \right]^{1/\lambda} T; \tag{9}$$

with $\lambda = \alpha$, it yields:

$$L_t^a = \frac{L_t^m \tilde{T}}{K_t h_t^{\frac{1-\alpha}{\alpha}}}. \tag{10}$$

Thus from Eq. (8) it follows that the labour share in the traditional sector is given by:

$$l_t^a = \frac{L_t^a}{L_t} = \frac{\tilde{T}/L_t}{k_t h_t^{\frac{1-\alpha}{\alpha}} + \tilde{T}/L_t}. \tag{11}$$

It follows that $\partial(L_t^a/L_t)/\partial h_t < 0$, $\partial(L_t^a/L_t)/\partial k_t < 0$ and $\partial(L_t^a/L_t)/\partial T > 0$.

To analyze adults behavior it is useful to conceptualize adulthood (of length p_t) as divided into time increments (for example years or months). At each time increments individuals (born in period $t-1$) allocate their income between consumption c_t and a transfer to their offspring b_t:

$$y_t = p_t c_t + p_t b_t. \tag{15}$$

where $p_t c_t$ is the actual consumption during the adulthood per individual, $p_t b_t$ is the actual bequest which each parent give to their children during their life.

The transfer $p_t b_t$, in turn, is allocated between the (actual) spending in children's education $p_t e_t$ and (actual) saving $p_t s_t$ for the future wealth of children:

$$pb_t = p_t s_t + p_t e_t. \tag{16}$$

The investment in education is devoted to increase children's human capital. In particular, each child with a total parental investment in education $p_t e_t$ receives an amount of $\bar{e}_t \equiv p_t e_t / n_t$ of investment in education and acquires:

$$h_{t+1} = h\left(\bar{e}_t\right) = \left(1 + D\bar{e}_t\right)^{\gamma}, \text{with } \gamma \in (0,1) \text{ and } D > 0, \tag{17}$$

efficiency units of human capital, where $h\left(0\right) = 1$, $h'\left(0\right) = \gamma D$ and $\lim_{\bar{e}_t \to \infty} h'\left(\bar{e}_t\right) = 0$ (Galor and Moav, 2004, 2006). Allowing for the case $\gamma \geq 1$ implies that human capital accumulation alone could generate a positive long-run growth. D is a scale parameter.

Individual preferences are defined over a consumption above a *subsistence level* $c^{\text{MIN}} > 0$ and the transfer to their children b_t. The expected utility function of altruistic individuals born in period $t-1$ is therefore:[11]

$$U = p_t[(1-\beta)\log(c_t) + \beta\log(b_t + \theta)], \tag{19}$$

[11] Following Rosen (1988) we assume the expected utility in the second period is given by the utility of state "life "given by the utility from consumption and the bequest to the children and the utility of state "death "given by M which is assumed to be equal to zero for simplicity:

$$U = p_t[(1-\beta)\log(c_t) + \beta\log(b_t + \theta)] + (1-p)M, \tag{18}$$

where $\beta \in (0,1)$ is the discount factor and $\theta > 0$ implies that children receive a positive transfer only when parent's income is sufficiently high (see Eq. (23) below).

Parents choose the level of consumption and the transfer to the offspring so as to maximize their expected utility, that is:

$$(c_t^*, b_t^*) = \arg\max_{c_t, b_t} \{p_t[(1-\beta)\log(c_t) + \beta\log(b_t + \theta)]\}, \tag{20}$$

subject to:

$$\begin{aligned} y_t &= p_t c_t + p_t b_t; \\ c_t &\geq c^{\text{MIN}}; \text{and} \\ b_t &\geq 0. \end{aligned}$$

Given the following condition on parameters, which ensures that, for low levels of income, the optimal consumption is increasing with respect to income while optimal bequest is zero (i.e. $y^{\text{MIN}} < y^C AP$):

$$c^{\text{MIN}} < \frac{(1-\beta)\theta}{\beta}, \tag{21}$$

the optimal levels of (actual) consumption and bequest are given as follows:[12]

$$c_t^* = \begin{cases} c^{\text{MIN}} & \text{if } y_t \in (0, y^{\text{MIN}}] \\ \dfrac{y_t}{p_t} & \text{if } y_t \in \left(y^{\text{MIN}}, y^{CAP}\right] \\ \dfrac{(1-\beta)(y_t + p_t\theta)}{p_t} & \text{if } y_t \in \left(y^{CAP}, \infty\right) \end{cases} \tag{22}$$

and:

$$b_t^* = \begin{cases} 0 & \text{if } y_t \in (0, y^{\text{MIN}}] \\ 0 & \text{if } y_t \in (y^{\text{MIN}}, y^{CAP}] \\ \dfrac{\beta y_t - \theta(1-\beta)p_t}{p_t} & \text{if } y_t \in (y^{CAP}, \infty) \end{cases} \tag{23}$$

[12]See Appendix A.

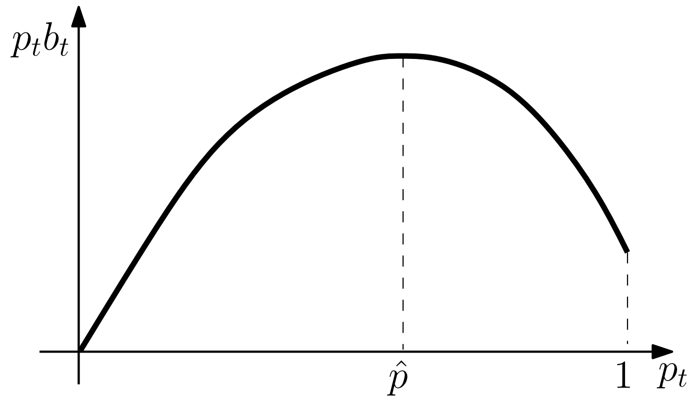


Figure 2: Bequest versus adult survival rate

where $y^{\text{MIN}} = p^{\text{MIN}} c^{\text{MIN}}$ and:

$$y^{CAP} = \frac{\theta(1-\beta)p^{CAP}}{\beta}, \tag{24}$$

where p^{CAP} is the adult survival in a period $t = CAP$.

III.C. Adult Survival and Bequest

Eq. (22) shows that a progress in adult survival in period t, when income is above subsistence level, increases actual consumption in adulthood, i.e. $\partial p_t c_t / \partial p_t = \partial y_t / \partial p_t + (1-\beta)\theta > 0$. By contrast, it has first an inverted U-shape relationship with actual bequest, pb_t.

The basic motivation behind this result is that the decline in adult mortality $1 - p_t$ has two opposing effects on bequest. On the one hand, higher longevity increases parents' consumption, thus reducing the overall transfers to their offspring; on the other hand, parents with longer working life experiment an increase in their income and in their transfers to offspring. When the initial level of income is sufficiently high, the latter effect always prevails, whereas at low levels of income holds the opposite. In particular, from Eq. (13) there exists a threshold level of adult survival rate denoted by $\hat{p}$ such that if $p_t < (>) \hat{p}$ the rise in adult survival rate positively (negatively) affects the total transfer to the offspring (see Fig. 2):

$$\hat{p} = \left[\frac{\beta(1-\alpha)A_t}{\theta(1-\beta)} \right]^{1/\alpha} \left(\frac{\tilde{T}}{L_t} + h_t^{\frac{1-\alpha}{\alpha}} k_t \right). \tag{25}$$

This threshold increases with the level of development of the country, i.e. with respect to A_t, h_t and k_t. On the other hand it decreases with respect to the size of workforce L_t.

III.D. Fixed and Human Capital

Eq. (16) shows that bequest is allocated between saving, i.e. accumulation of fixed capital, and education, i.e. accumulation of human capital. However, the economy begins to accumulate fixed capital only when parents are sufficiently rich (i.e. $y_t > y^{CAP}$, see Eq. (23)) to leave a positive transfer to their offspring, and to accumulate human capital for a still higher level of income (i.e. $y_t > y^{EDU}$, see Eq. (32) below).

The fixed capital stock in period $t+1$ is therefore given by the aggregate saving in period t:

$$K_{t+1} = L_t p_t s_t = L_t \left(p_t b_t - p_t e_t \right). \tag{26}$$

Adult population at time $t+1$, L_{t+1}, is given by:

$$L_{t+1} = n_t L_t, \tag{27}$$

where L_t is adult population and n_t is the fertility rate.

The capital/labour ratio is therefore equal to:

$$k_{t+1} = \bar{b}_t - \bar{e}_t, \tag{28}$$

where $\bar{b}_t \equiv p_t b_t / n_t$ and $\bar{e}_t \equiv p_t e_t / n_t$. In particular $\bar{b}_t$ is given as follows:

$$\bar{b}_t = \frac{\beta y_t - \theta(1-\beta)p_t}{n_t} \tag{29}$$

Parents choose the amount to invest in children's education in order to maximize the future income of offspring i.e. y_{t+1}. In the early stages of development, when the productivity in the industrial sector is relatively low with respect to the productivity in the traditional sector, individuals do not have incentive to invest in human capital of their children. However, as the level of industrial technology improves, human capital will be demanded and parents will have an incentive to invest in the human capital. Thus from Eqq. (13) and (17) it follows that:

$$e_t^* = \arg\max_{e_t \in [0, b_t]} \left[A_{t+1} p_{t+1}^{1-\alpha} \left(\frac{\tilde{T}}{L_{t+1}} + \left(1 + \frac{D p_t e_t}{n_t}\right)^{\frac{\gamma(1-\alpha)}{\alpha}} k_{t+1} \right)^{\alpha} \right], \tag{30}$$

where k_{t+1} is given by Eq. (28). Eq. (23) shows that the optimal level of education is positive only if income is sufficiently high, i.e.:

$$\bar{e}_t^* = \begin{cases} 0 & \text{if } y_t \in [0, y^{EDU}]; \\ \dfrac{\beta y_t - \theta(1-\beta)p_t - \tilde{b} n_t}{(1 + D\tilde{b})n_t} & \text{if } y_t \in (y^{EDU}, \infty) \end{cases} \tag{31}$$

where:

$$y^{EDU} \equiv \frac{\tilde{b} n^{EDU} + \theta(1-\beta)p^{EDU}}{\beta} \tag{32}$$

and:

$$\tilde{b} \equiv \frac{\alpha}{D(1-\alpha)\gamma}, \tag{33}$$

where p^{EDU} is the value of adult survival at the time period $t = EDU$.

From Eqq. (28) and (31) the capital-labor ratio in period $t+1$

is given by:

$$k_{t+1} = \begin{cases} 0 & \text{if } y_t \in \left[0, y^{CAP}\right]; \\ \dfrac{\beta y_t - \theta(1-\beta)p_t}{n_t} & \text{if } y_t \in \left(y^{CAP}, y^{EDU}\right]; \\ \left(\dfrac{\tilde{b}}{1+D\tilde{b}}\right)\left\{1 + D\left[\dfrac{\beta y_t - \theta(1-\beta)p_t}{n_t}\right]\right\} & \text{if } y_t \in \left(y^{EDU}, \infty\right). \end{cases} \quad (34)$$

IV. The Stages of Development

From Eqq. (23), (31) and (34) we can now characterize the dynamic of income per worker in period $t+1$ as follows:

$$y_{t+1} = \begin{cases} \dfrac{A_{t+1} p_{t+1}^{1-\alpha} \tilde{T}^{\alpha}}{(n_t L_t)^{\alpha}} & \text{if } y_t \in (y^{\mathrm{M}} \\ A_{t+1} p_{t+1}^{1-\alpha}\left[\dfrac{\tilde{T}}{n_t L_t} + \dfrac{\beta y_t - \theta(1-\beta)p_t}{n_t}\right]^{\alpha} & \text{if } y_t \in (y^{C} \\ A_{t+1} p_{t+1}^{1-\alpha}\left\{\dfrac{\tilde{T}}{n_t L_t} + \tilde{b}\left[\dfrac{n_t + D(\beta y_t - \theta(1-\beta)p_t)}{n_t(1+D\tilde{b})}\right]^{\frac{\gamma(1-\alpha)}{\alpha}+1}\right\}^{\alpha} & \text{if } y_t \in (y^{E} \end{cases} \quad (35)$$

The three ranges of y_t identify three distinct regimes: a *Traditional Regime*, i.e. $y_t \in (y^{\mathrm{MIN}}, y^{CAP})$, where production is conducted using traditional technology; a *Pre-modern Regime*, i.e. $y_t \in (y^{CAP}, y^{EDU})$, where output is the result of using fixed capital and unskilled labour in an industrial sector; and a *Modern Regime*, i.e. $y_t > y^{EDU}$ where both fixed and human capital are used in the industrial sector.

Simple calculations show a smoothing transition from the Premodern Regime to the Modern Regime that is $\lim_{y_t \to y^{EDU-}} \partial y_{t+1}/\partial y_t = \lim_{y_t \to y^{EDU+}} \partial y_{t+1}/\partial y_t$.

Assumption 1 *In the Traditional Regime, income is always sufficiently high to ensure an income at least equal to the subsistence level* $p^{\text{MIN}}c^{\text{MIN}}$*, that is (for technical details see Appendix A):*

$$A \geq A^{\text{MIN}}, \tag{36}$$

where:

$$A^{MIN} \equiv p^{\text{MIN}}c^{\text{MIN}} \left(\frac{p_t n_t L_t}{\tilde{T}}\right)^{\alpha}. \tag{37}$$

Proposition 1 states the conditions on under there exists one or more than one equilibria in the three regimes.

Proposition 1 *Suppose that Assumption 1 holds,* $L_{t+1} = L_t = L$ *(i.e.* $n_t = 1$*) and that adult survival is constant over time, i.e* $p_{t+1} = p_t = p$*; under some (not so restrictive) conditions on the model's parameters reported in Appendix B:*

- *if* $A \in \left[A^{MIN}, A^{TRA}\right)$*, then there exists one stable equilibrium in the Traditional Regime and possibly one unstable and one stable equilibrium in the Pre-modern Regime:*

$$A^{TRA} = \frac{\theta\left(1-\beta\right)}{\beta}\left(\frac{pL}{\tilde{T}}\right)^{\alpha} \tag{38}$$

- *If* $A \in \left[A^{TRA}, A^{PRE-MOD}\right]$*, then there exists one stable equilibrium in the Pre-modern Regime:*

$$A^{PRE-MOD} = \frac{\tilde{b} + \theta\left(1-\beta\right)p}{\beta p^{1-\alpha}\left(\frac{\tilde{T}}{L} + \tilde{b}\right)^{\alpha}}. \tag{39}$$

- *Finally, if* $A > A^{PRE-MOD}$ *there exists just one stable equilibrium in the Modern Regime.*

Proof. See Appendix B. ■

Fig. 3 provides a graphical exposition of the results contained in Proposition 1.

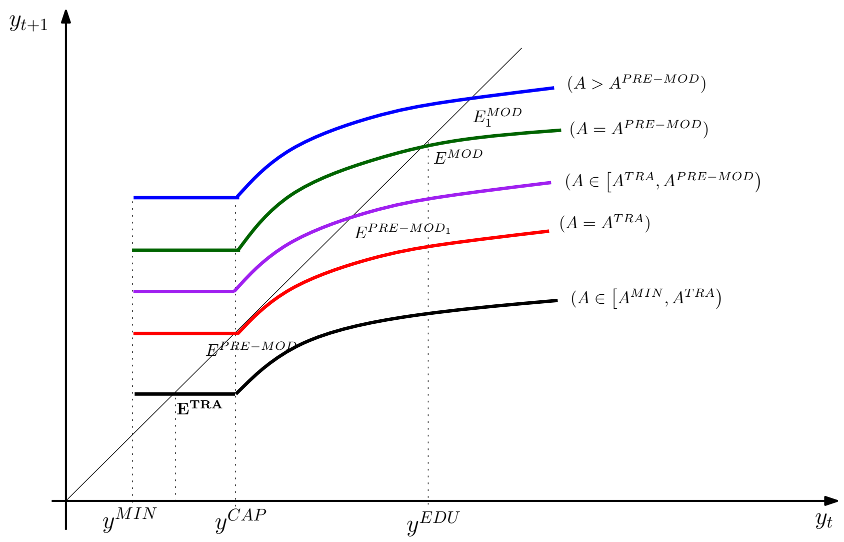


Figure 3: Stages of growth

At low level of A, i.e. above but around A^{MIN}, the only equilibrium of economy is in the Traditional Regime, i.e. E^{TRA}. When $A \geq A^{TRA}$ the equilibrium in the Traditional Regime disappears and the economy shows an equilibrium in the Pre-modern Regime. The level of A^{TRA} corresponds to the level of A such that equilibrium level of income is exactly equals to y^{CAP}.

The level of $A^{PRE-MOD}$ instead corresponds to the level of A such that the equilibrium in the Pre-modern Regime is at the bound of the range of no accumulation of human capital, that is the equilibrium level of income is equal to y^{EDU}.

Taken as constant the adult survival, as A increases over time the economy will pass through all three regimes. The transition from the Traditional to the Pre-modern Regime is driven by the increase in the traditional production, which allows to satisfy consumption and to make some transfers to offspring, the source of accumulation of fixed capital. The transition from Pre-modern Regime to Modern Regime is instead driven by the higher accumulation of fixed capital (transfers) generated by the increase of productivity in manufacturing sector (this also happen in traditional sector but the share of the latter on total output is declining in income): such accumulation increases the return to the investment in human capital, and,

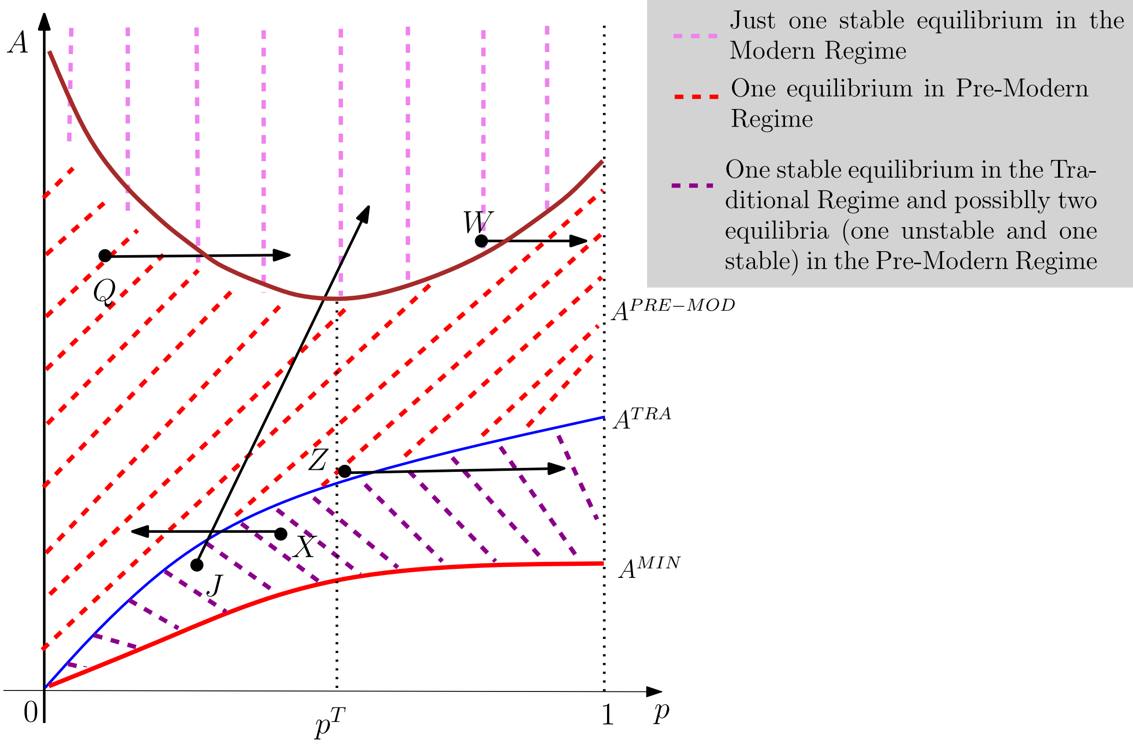


Figure 4: Dynamic across regimes

therefore, incentives such type of investment (see Robert E. Lucas (2004) for a similar argument).

IV.A. The Role of Mortality Rate in the Transition between Regimes

Below we analyze the impact of change in adult mortality on economic development. In the analysis population growth is assumed to be constant (i.e. $L_t = L$ and $n_t = 1$).

Fig. 4 characterizes the dynamic of income per-worker in the space (A, p). A^{MIN} and A^{TRA} are both increasing and concave with respect to p, while $A^{PRE-MOD}$ is decreasing until a given p^T and then increasing.[13]

Consider first the case of an economy with a low level of technological progress and a high mortality rate (i.e. point J in Fig. 4). The transition through the different regimes is driven by the simul-

[13] The level of $p^T = 1/\theta(1-\beta)D\gamma$.

taneous rise of technological progress and adult survival. This seems to be the growth path followed by the most of developed countries.

When technological progress does not change and its actual level is very low, the rise in longevity alone cannot allow the transition to the (Pre-)Modern Regime. Moreover, if the increase in adult survival is sufficiently high the economy regresses back to the Traditional Regime (this is the case of the trajectory starting from point Z.)

When the level of technological progress is not high enough and the increase in adult survival remains within a certain threshold, the economy can cross over to the Modern Regime (see the trajectory starting from the point Q). However, if the level of technological progress does not change, a further increase in longevity can push the economy back to the Pre-modern Regime (this is the case of the trajectory starting from point W). Finally, when the level of technological progress is very high, the increase in p alone allows for a change in regime.

As specified in Section III.C. the inverted U-shape relationship between adult survival and economic growth derives by the fact that advances in adult survival have two opposing effects on intergenerational transfers. The basic motivation underlying this result is the presence of diminishing returns at low levels of income: the rise in population, due to a decline in mortality, has a less than proportional effect on output because of the presence of the land. When adult survival rises above a certain threshold, at low levels of income, the rise in income is not sufficient to compensate the rise in consumption. On the other hand, at high levels of income economy accumulates fixed and human capital, and, therefore, the rise in adult survival always allows a level of income sufficiently high to compensate for the rise in consumption. In particular, for a sufficiently high income, the rise in longevity increases the return on investment in education and therefore the higher income perpetuates.

These results are in line with the empirical evidence discussed in Cervellati and Sunde, 2011 which show a non-linear relationship between life expectancy and economic growth. In particular, they show that this relationship is negative before the onset of demographic transition and strongly positive thereafter.

Finally, the path starting from the point X in Fig. 4 shows a scenario in which the rise in mortality, as for example because of an epidemic such as the Black Death, can have a positive effect on economic growth. In this case, the population reduction, increasing income per capita, can push the economy from the Traditional Regime to the Pre-modern Regime.

IV.B. Model with Endogenous Fertility

In the following we extend the model to include endogenous fertility. Individuals preferences are now defined over consumption, the transfer to their children b_t and the total number of children n_t of children who survive to the adulthood. Here we are assuming that parents derive utility from the number of surviving children to adulthood and not from the number of births (see Easterlin (2004) and Galor (2005)).

The optimal problem of parents is given as follows:

$$(c_t^*, b_t^*, n_t^*) = \arg\max_{c_t, b_t, n_t} \{p_t[(1-\beta)\log(c_t) + \epsilon \log(n_t) + \beta \log(b_t + \theta)]\}, \tag{40}$$

subject to:

$$\begin{aligned} y_t &= p_t c_t + p_t b_t + \delta n_t y_t; \\ c_t &\geq c^{\text{MIN}}; \\ b_t &\geq 0. \end{aligned}$$

where δ is the opportunity cost of raising children, that is the fraction of parents time required in order to raise each child ((Galor, 2005)).

Assuming that Condition (21) holds[14], the optimal levels of consumption, bequest and number of surviving children are given as

[14]Condition (21) ensures that $y^{\text{MIN}} < y^{SUB} < y^{CAP}$.

follows:[15].

$$c_t^* = \begin{cases} c^{\text{MIN}} & \text{if } y_t \in (y^{MIN}, y^{SUB}] \\ \\ \dfrac{(1-\beta)y_t}{(1-\beta+\epsilon)p_t} & \text{if } y_t \in (y^{SUB}, y^{CAP}] \\ \\ \dfrac{(1-\beta)(y_t+p_t\theta)}{(1+\epsilon)p_t} & \text{if } y_t \in (y^{CAP}, \infty) \end{cases} \tag{41}$$

and:

$$b_t^* = \begin{cases} 0 & \text{if } y_t \in (y^{\text{MIN}}, y^{CAP}] \\ \\ \dfrac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{p_t(1+\epsilon)} & \text{if } y_t \in (y^{CAP}, \infty) \end{cases} \tag{42}$$

and:

$$n_t^* = \begin{cases} \dfrac{y_t - p_t c^{\text{MIN}}}{\delta y_t} & \text{if } y_t \in (y^{\text{MIN}}, y^{SUB}] \\ \\ \dfrac{\epsilon}{\delta(1-\beta+\epsilon)} & \text{if } y_t \in (y^{SUB}, y^{CAP}] \\ \\ \dfrac{\epsilon(y_t+\theta p_t)}{\delta(1+\epsilon)y_t} & \text{if } y_t \in (y^{CAP}, \infty) \end{cases} \tag{43}$$

where $y^{\text{MIN}} = pc^{\text{MIN}}$ and:

$$y^{SUB} = \frac{p^{SUB} c^{\text{MIN}}(1-\beta+\epsilon)}{1-\beta}, \tag{44}$$

and:

$$y^{CAP} = \frac{\theta(1-\beta+\epsilon)p^{CAP}}{\beta}. \tag{45}$$

In the range $y_t \in (y^{SUB}, y^{CAP}]$ fertility reaches its maximum level. From Eq. (43), when income is sufficiently low that consumption is at subsistence level, fertility decreases with respect to adult survival; the opposite occurs when income is sufficiently high, i.e. $y_t > y^{CAP}$.

[15] See Appendix A

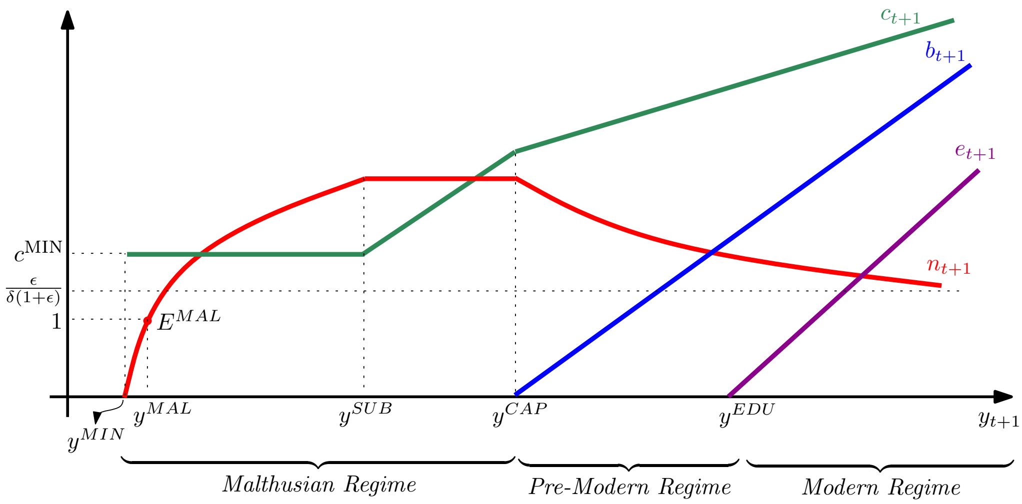


Figure 5: Optimal choices of individuals with endogenous fertility

The basic motivation of this result is that, when income is at subsistence level, as longevity increase parents have a lower number of children because there is a higher need to consume. On the other hand, if $y_t > y^{CAP}$, consumption is above the subsistence level and increasing longevity positively affects the number of children.

Fig. 5 makes clear that when income is sufficiently low, that is $y_t < y^{SUB}$ the per capita consumption is at subsistence level, the optimal choice for bequest is zero while the optimal number of children increases with respect to income. The Traditional Regime therefore assumes the typical characteristics of a Malthusian Regime.

The economy is in a Malthusian trap where production is conducted using only the traditional technology: any increase in per capita income (due to improvement in technological progress and/or adult survival) results in a surge of population; this increase, due to the diminishing returns to labour, leads a fall in per capita income; economy is therefore doomed to stagnate to a subsistence level in the long run.

Simple calculations show that in the range $y_t \in (y^{\text{MIN}}, y^{SUB}]$ there exists a unique stable equilibrium in which the population growth rate is zero (i.e. $n_t = 1$), that is:

$$y^{MAL} = \frac{p_t c^{\text{MIN}}}{1-\delta}, \tag{46}$$

which is, therefore, a Malthusian trap. In other words, the rise in technological progress increases the level of per-capita income at time t, i.e. $\partial y_t/\partial A > 0$, but if this rise fails to reach the level y^{SUB}, population increases and per capita income gradually falls back to the initial steady state equilibrium. In particular, the equilibrium level of adult population in the Malthusian Regime is given as follows, that is[16]:

$$L^{MAL} = \left[\frac{A_t p_t (1-\delta)}{pc^{\text{MIN}}} \right]^{1/\alpha} \tilde{T}, \tag{48}$$

When income increases (due to the increases in technological progress or mortality reduction) above the level y^{SUB}, this allows parents to escape from the subsistence level of consumption, thus consumption starts to increase and fertility rate becomes constant. However, the economy is still in the Malthusian Regime since parents do not have a sufficient level of income to leave a positive transfer to their children.

If income continues to increase, the constancy of the fertility rate ensures that, at a certain point, i.e. $y_t > y^{CAP}$, the economy moves into the Pre-modern Regime where parents start to devote a fraction of income for the future wealth of their children and the relationship between income and population growth becomes negative.

From Eqq. (42) and (43) the dynamic of income per worker can be characterized as follows:

[16]Eq. (48) follows from:

$$A_t p_t^{1-\alpha} \left(\frac{\tilde{T}}{L_t} \right)^{\alpha} = \frac{p_t c^{\text{MIN}}}{1-\delta} \tag{47}$$

$$y_{t+1} = \begin{cases} \frac{A_{t+1}p_{t+1}^{1-\alpha}\tilde{T}^{\alpha}}{L_t^{\alpha}} \left(\frac{\delta y_t}{y_t - p_t c^{\text{MIN}}}\right)^{\alpha} & \text{if } y_t \in (y^{\text{MIN}}, y^{SUB}]; \\ \frac{A_{t+1}p_{t+1}^{1-\alpha}\tilde{T}^{\alpha}}{L_t^{\alpha}} \left[\frac{\delta(1-\beta+\epsilon)}{\epsilon}\right]^{\alpha} & \text{if} y_t \in (y^{SUB}, y^{CAP}]; \\ A_{t+1}p_{t+1}^{1-\alpha} \left[\frac{\delta(1+\epsilon)y_t}{\epsilon(y_t+\theta p_t)}\right]^{\alpha} \left[\frac{\tilde{T}}{L_t} + \frac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{1+\epsilon}\right]^{\alpha} & \text{if } y_t \in (y^{CAP}, y^{EDU}]; \\ A_{t+1}p_{t+1}^{1-\alpha} \left\{\frac{\tilde{T}}{L_t}\frac{\delta(1+\epsilon)y_t}{\epsilon(y_t+\theta p_t)} + \tilde{b}\left[\frac{\epsilon(y_t+\theta p_t) + D(\beta y_t - \theta(1-\beta+\epsilon)p_t)\delta y_t}{\epsilon(y_t+\theta p_t)(1+D\tilde{b})}\right]^{\frac{\gamma(1-\alpha)}{\alpha}+1}\right\}^{\alpha} & \text{if } y_t \in [y^{EDU}, +\infty). \end{cases} \tag{49}$$

where:

$$y^{EDU} = \frac{\tilde{b}(1+\epsilon) + \theta(1-\beta+\epsilon)p^{EDU}}{\beta}. \tag{50}$$

The following Proposition states the conditions on under which we observe one or more than one equilibria in the three regimes.

Proposition 2 *Suppose that adult's survival is constant over time, i.e.* $p_{t+1} = p_t = p$, *then:*

• *If* $A \in \left[A^{MIN}, A^{TRA}\right)$, *then there exists at least one equilibrium in the Traditional Regime, where* A^{MIN} *is defined in Eq. (37) and:*

$$A^{TRA} = \frac{\theta(1-\beta+\epsilon)^{1-\alpha}}{\beta}\left(\frac{\epsilon p L_t}{\delta\tilde{T}}\right)^{\alpha}. \tag{51}$$

• *If* $A \in \left[A^{TRA}, A^{PRE-MOD}\right]$, *then there exists at least one equilibrium in the Pre-modern Regime:*

$$A^{PRE-MOD} = \frac{[\tilde{b}(1+\epsilon) + \theta(1-\beta+\epsilon)p]^{1-\alpha}[\epsilon(\tilde{b}+\theta p)]^{\alpha}}{\psi p^{1-\alpha}} \tag{52}$$

where $\psi = \beta\delta^{\alpha}(\tilde{T}/L + \tilde{b})^{\alpha}$.

• *If* $A > A^{PRE-MOD}$ *there exists at least one equilibrium in the Modern Regime.*

IV.C. Empirical Evidence on Fertility and Income

The inverted U-shaped relationship between fertility rate and income is typical of most economies. The left hand side of Fig. 6 depicts the general fertility rate [17], i.e. the number of births divided by the number of women aged 15-44, for some European countries in the period 1750 – 1920, The right hand side of Fig. 6 depicts the general fertility rate adjusted by the probability of surviving at 20. We are interested in the number of surviving children at 20 since parents maximize their utility with respect to the number of surviving children.

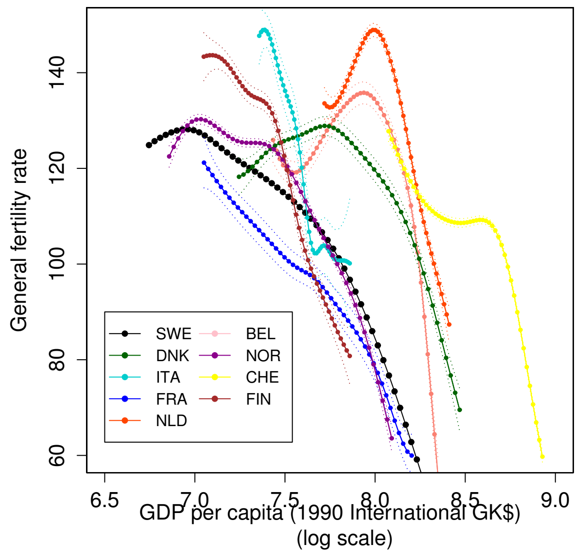
As depicted in Fig. 6 the general fertility rate, , for the most of countries, increases at low level of income and starts to decline when income is sufficiently high. However, in agreement with the theoretical predictions, the inverted U-shaped relationship between fertility and income is much more evident when we consider the number of surviving children.

A large body of the literature has developed theoretical and empirical models to analyze this path of fertility. Becker (1960)'s seminal work argues that the main reason behind the decline in fertility was the considerable increase in income which occurred as a result of the Industrial Revolution. In particular, as income increases the number of children in a household decreases because more affluent consumers tend to choose activities which require less time, instead children require a great deal of time and energy. Moreover, richer parents choose to have fewer children so that they can dedicate more time and resources to increase the "quality" of their offspring.

However, as is apparent from picture 6, the fact that the level of income which reverses the relationship between fertility and income differs across countries, suggests that there are also other specific country's factors which affect the relationship between fertility and income.

In this respect Clark's writes "Income, however, certainly cannot

[17] General fertility is an index of the rate of production of children, strongly correlated with the average number of children per women (see, for example, Livi Bacci, 2007)



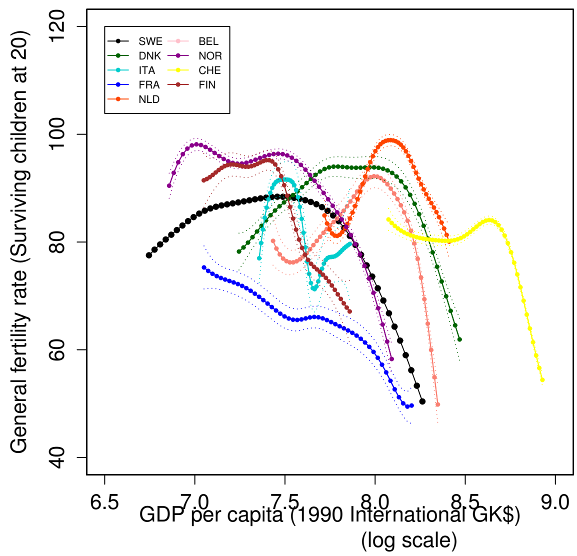


Figure 6: General Fertility rate versus Income. Europe: 1750-1930.Per capita GDP data are from Maddison project database (2013). General Fertility rate are our calculations from the Human Mortality Database

by itself explain the modern decline in fertility....Had income alone been determining fertility, the rich in the preindustrial world would already have been restricting their fertility "(see Clark, 2007, p. 293).

According to Clark the high fertility even within the rich, in the preindustrial period, could be either due to the absence of birth control or to the fact that, in the high mortality environment of the Malthusian era, people consciously had more children in the hope of achieving a desired family size of two or three surviving children and most particularly, in richer families, a surviving son.

Thus to explain the fertility decline we need to consider other factors beyond the income as conscious action to limit fertility, the reduction of child mortality and the increased social status of women (see among others Livi Bacci (2007), Easterlin, 2004).

V. A Quantitative Evaluation of the Model

The model is calibrated using the data for the U.K economy during the period $1591 - 1914$. In agreement with existing literature Malthusian Stagnation persisted until the end of the 18th century and the transition from the Malthusian regime to the Pre-modern Regime occurred at the beginning of the 19th century (see, among others, Galor, 2005, Clark, 2007 and Livi Bacci, 2007).

Thus, it is reasonable to assume that the transition from the Malthusian Regime to the Pre-modern Regime took place between 1770 and 1800 (Clark, 2007) and the transition from the Pre-modern to the Modern Regime took place between 1830 and 1860 (Cipolla, 1962).

As specified before $t = CAP$ denotes the year in which the transition from the Malthusian Regime to the Pre-modern Regime starts, that is $CAP = 1770$, and $t = EDU$ represents the year in which the economy enters the Modern Regime, that is $EDU = 1830$.

As depicted in Fig. 7 the growth rate of income per worker begins to increase significantly at the beginning of the 19th century. In Fig. 7 income per worker before 1851 is for England and Wales and after

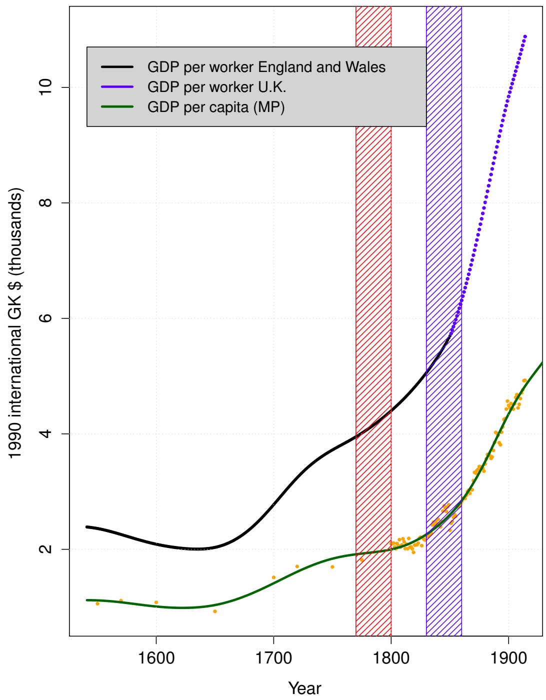


Figure 7: Transition from stagnation to growth: England 1591-1914.

1851 it also includes Scotland. In Appendix C we provide a detailed discussion on the calculation of the variables used in the analysis.

Moreover, we assume that the economy escapes from the subsistence level of consumption in 1760 and that there is Malthusian equilibrium in 1690 where, as shown in Fig. 7 GDP per worker is more or less constant.

The model is calibrated under the assumption that childhood has a length of 20 years and adulthood has a length of 50 years. This implies that the labor force in each period t, i.e. $p_t L_t$, is made up of people between the ages of 20 and 70.

Moreover, we assume that each time period has a length of 30 years.

Table 1 lists the observed parameter values and the observed variables used to calibrate the model. The calibrated parameter values used in the simulation are listed in Table 2 (for technical details on the parameter calculations see Appendix C).

Parameter	Definition	Value
$\lambda = \alpha$	Capital/Land share	0.4
r_{1914}	Interest rate in 1914	1.04
i_{1860}	Investment rate (1860)	0.09
$GDP.PC_{min}$	Minimum level of GDP per worker	0.73
l^a_{1914}	Labor share in the traditional sector (1914)	0.51
y per worker	Series of income per worker from 1591-1914	
Adult Survival	Series of adult survival from 1591-1914	

Table 1: Observed Parameters and Variables

Parameter	Definition	Value	
		Exogenous Fertility	Endogenous Fertility
β	Altruism factor	0.593	0.674
c^{MIN}	Subsistence consumption	7.653	7.653
θ	Bequest Parameter	26.786	15.632
γ	Return to education	0.428	0.884
$\tilde{T}$	$\phi^{1/\alpha}T$	47.389	47.389
D	Scale parameter	4.41	2.137
ϵ	Taste for children		0.465
δ	Time cost parameter		0.454

Table 2: Calibrated Parameter values

V.A. A growth Accounting Exercise

In this section we run a growth accounting exercise which allows us to analyze the specific contribution made by technological progress, mortality reduction and fixed and human capital accumulation to the transition from stagnation to growth in the UK economy. As specified in section 3.1, income per worker can be written as a function of the labor share in the traditional sector l^a_t which captures the contribution made by fixed and human capital to income per worker (see footnote 6):

$$y_t = A_t p_t \left(\frac{\tilde{T}/p_t L_t}{l^a_t} \right)^{\alpha}, \tag{53}$$

where $p_t L_t$ is the observed employment, p_t is the adult survival probability, that is the probability of surviving from 20 to 70. In the Appendix C we provide a detailed discussion on the calculations of all variables used in Eq. (53).

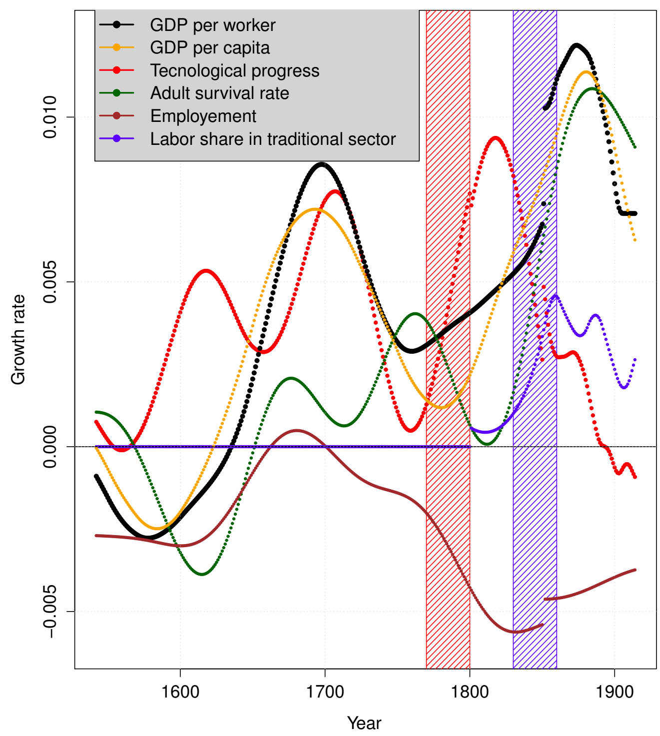


Figure 8: A growth accounting exercise

The specification in Eq. (53) is very useful because, while the data on fixed and human capital is not available, we do, however have data on labour share in the traditional sector. This, therefore enables us to compute the growth rate of technological progress as a residual:

$$g^A = g^y - g^p + \alpha g^{pL} + \alpha g^{l^a} \tag{54}$$

where $g^A \equiv A_{t+1}/A_t - 1$, $g^y \equiv y_{t+1}/y_t - 1$, $g^p_{t+1} \equiv p_{t+1}/p_t - 1$, $g^{pL} \equiv p_{t+1}L_{t+1}/p_tL_t - 1$ and $g^{l^a} \equiv l^a_{t+1}/l^a_t - 1$.

In Fig. 8 we plot, therefore, the single components of the growth rate of income per worker (black line). In particular, the red column shows the transition period from the Malthusian Regime to the Pre-modern Regime and the blue column represents the transition period from the Pre-modern Regime to the Modern Regime. The red line represents g^A, the green line g^p, the brown line $-\alpha g^{pL}$ and the blue line $-\alpha g^{l^a}$.

As is evident from Eq.(54) the growth rates of both technological progress and adult survival positively affect the growth rate of income per worker, while the growth rates of employment and labor share in the traditional sector have a negative effect. As depicted in Fig.8 in the early seventeenth century, the growth rate of income per worker is negative because the positive growth rate of technological progress is overshadowed by the negative growth rate of adult survival and the positive growth rate of population which, in turn, has a negative effect on the growth rate of income per worker.

There are two periods in which income per worker shows an increasing growth rate. The first occurs in the second half of the seventeenth century (this is well documented in Broadberry et al., 2010 and Broadberry et al., 2010b) and the second at the end of the nineteen century.

The main components which seem to explain the increase in the growth rate of income per worker in the second half of the seventeenth century is the decline in the growth rate of population, the increase in the growth rate of adult survival and the increase in technological progress (see Broadberry et al., 2010). The acceleration of the growth rate of income per worker at the end of the nineteen century is due to the acceleration in the growth rate of adult survival and fixed and human capital accumulation. The latter is captured by $-\alpha g^{l_a}$ (the blue line), which provides a measure of the structural change in the economy. Finally the decline in the growth rate of technological progress at the beginning of the twentieth century is well documented, see for example Antras and Voth (2003)).

In Fig. 9 we show the results of our calculations in the space (A, p). The parameter values used to calculate $A^{PRE-MOD}$ and A^{TRA} are shown in tables 1 and 2. Thus, as depicted in Fig. 9 the transition from the Traditional Regime to the Pre-modern Regime is characterized, in a first phase, by the increase in technological progress and the decrease in the adult survival probability and, in a second phase, by the joint increase of technological progress and adult survival. However, as is evident from both Figures 9 and 8

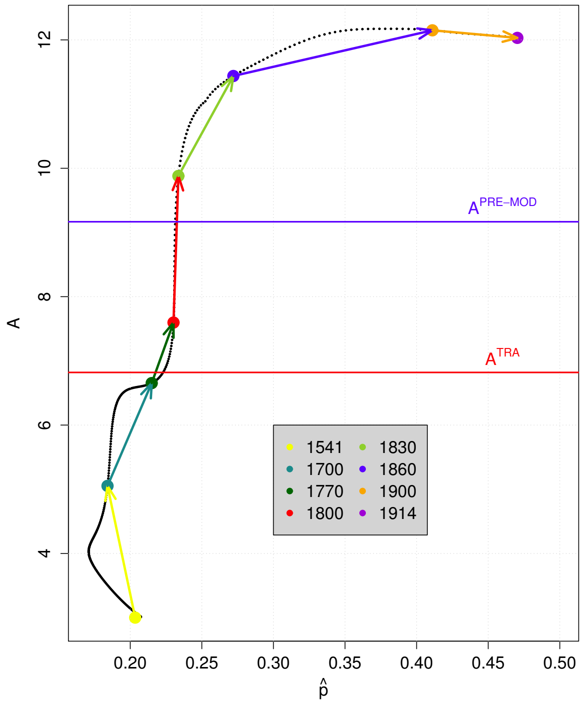


Figure 9: The Dynamic across Regimes: England and Wales 1591-1850; U.K. 1851-1914.

the increase in technological progress drove most of UK growth until 1851. On the other hand, the increases in adult survival and accumulation of fixed and human capital drove most of UK growth from 1851 to 1914.

V.B. Simulation

This section shows a simulation of the model to illustrate the mechanism described above and also the capability of the model to replicate the real patterns of long-term development in the U.K.

We first present the results of the simulation exercise when fertility is exogenous and then the results of the simulation when fertility is endogenous.

All variables in the initial period are chosen so that the simulation starts in the Traditional Regime.

V.B.i. Exogenous Fertility

Figure 10 shows the results of our simulation of income per worker given by Eq. (35) where we use the calibrated parameters in table 2 and the observed values for all exogenous variables i.e. n_t, p_t, A_t and L_t.

As is evident from Fig. 10 the simulated income per worker (red line) follows the observed income per worker (black line). The green points show the path of y^{CAP} and the blue points the path y^{EDU}.

Initially, income per worker is quite low for an extended period of time. At the end of the eighteen century it increases above the level y^{CAP} and parents begin to leave a positive transfer to their children (see Fig. 11). In the same period as depicted in Fig. 11 parent's consumption grows more slowly.

At a certain point (around 1850) income per worker increases above the level y^{EDU} and parents start investing in their children's education and in this way the economy enters the Modern Regime. Figure 12 compares the path of the simulated spending in children education with the observed primary enrollment rate.

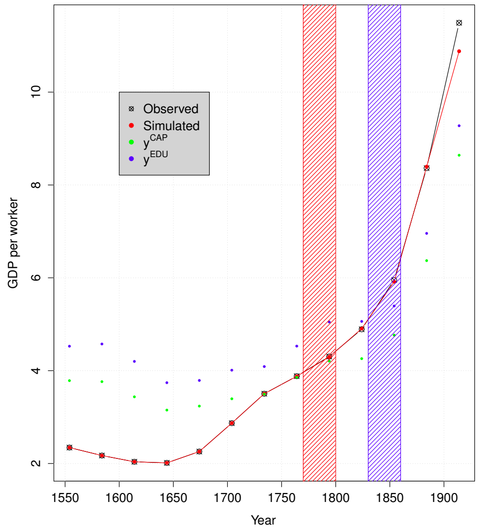


Figure 10: Observed vs Simulated Income per worker

Finally, in Fig.11 the orange curve shows the observed path of labour share in the traditional sector. Thus we can observe that the model can replicate the structural change in the economy as a larger fraction of individuals start investing in education.

Overall, we can conclude that the simulation when fertility is assumed exogenous demonstrates the models capability of explaining the transition from stagnation to growth.

V.B.ii. Endogenous Fertility

When we extend the model to include endogenous fertility, as depicted in Fig. 14 and 13 we find that the model reproduces only partially the observed path of income per worker and fertility behavior.

The issue is that we need a more complex model of fertility to allow us to include the differences between observed and optimal fertility behavior. This is due to possible natural and social constraints on theoretical maximum fertility (see Easterlin, 2004, Livi-Bacci, 200). Another possible solution could be to include an increasing marginal cost (in income and life expectancy) for child rearing.

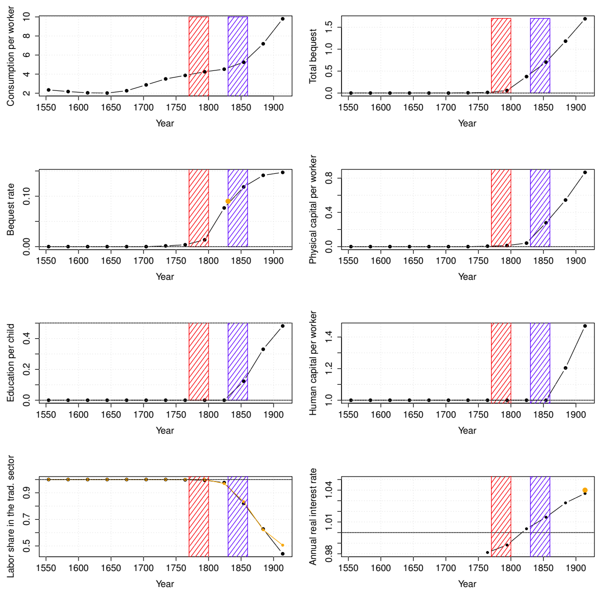


Figure 11: Simulation of the development process

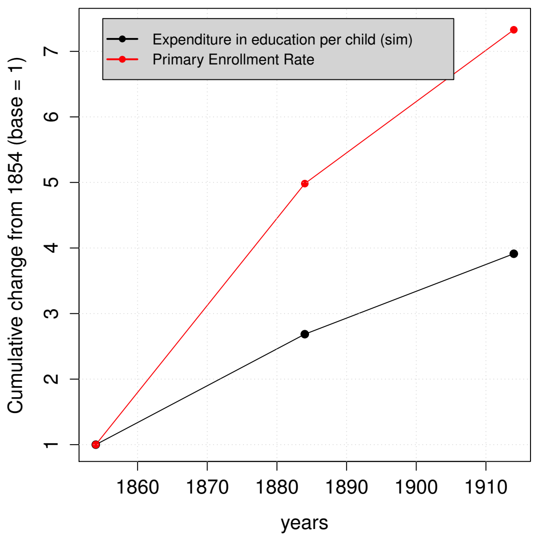


Figure 12: Expenditure in Education vs Primary Enrollment rate

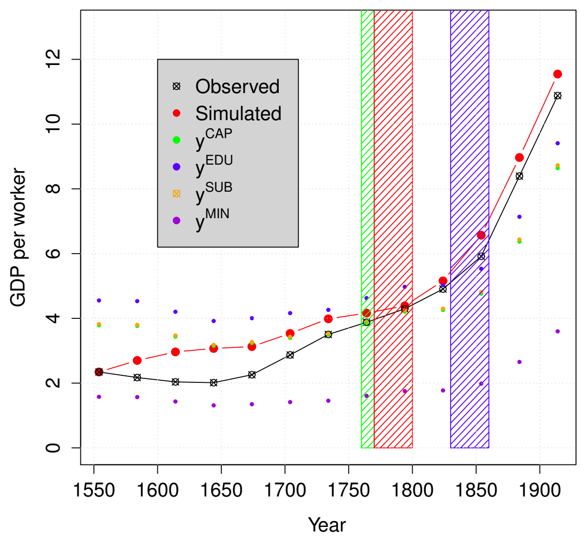


Figure 13: Observed vs Simulated Income per worker

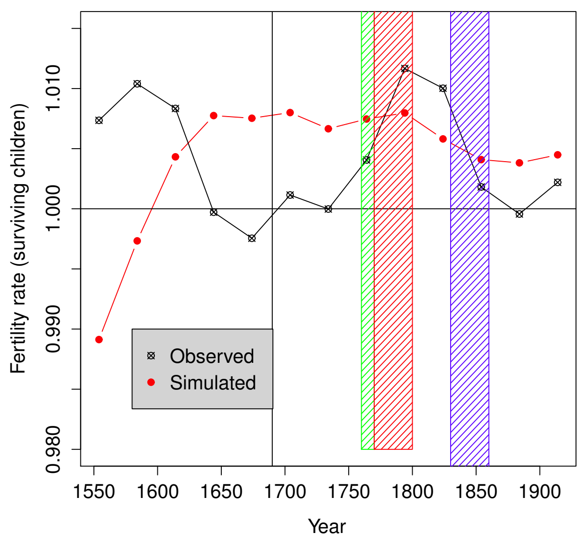


Figure 14: Observed vs Simulated Fertility

VI. Concluding Remarks

This paper contributes to the literature on the role of mortality reduction on economic growth by accounting for the differential effects of life expectancy during the various stages of economic development.

We find that the rise in technological progress always allows the transition from Malthusian (stagnant) Regime to a Modern Regime. However, the rise in longevity can have important effects on the transition. It has a positive effect on intergenerational transfer at high levels of income and a non-linear effect at low levels of income: this effect is positive if longevity remains below a threshold but becomes negative if longevity exceeds such threshold. The basic motivation underlying this result is the presence of the fixed factor land which leads to diminishing returns of labor in agriculture. Thus, if longevity increases above a certain threshold, at low levels of income, the rise in income is insufficient to compensate the rise in consumption. Reduction in intergeneration transfer, in turn reduces fixed capital accumulation, pushing the economy towards a Malthusian Regime. On the other hand, at high income levels, rising longevity do not has the same opposite effects. The rise in longevity, indeed, increasing the return on investment in education, stimulates investment in human capital and increases labour income. Thus the rise in income is sufficiently high to compensate for the increase in consumption, leading to higher intergenerational transfers. The presence of endogenous fertility dampens the effects of shocks on the economy but does not change qualitatively our results.

The quantitative exercise shows that the model presented in this paper, when fertility is exogenous, is able to reproduce the observed transition from stagnation to growth. By simulating the model, indeed, we show that the long-run behavior of income per worker follows the empirical evidence. By contrast, the model with endogenous fertility partially reproduces the observed path of income per worker and fertility. This result suggests the need of a more complex theory of fertility to take into account the possible natural and social constraints on the theoretical fertility.

Finally, the introduction of endogenous mortality should not affect the qualitative results of the paper but just adding a possible self-reinforcing mechanism to the transition from stagnation to growth.

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References

Acemoglu, D. (2008). *Introduction to Modern Economic Growth.* Princeton, USA: Princeton University Press.

Acemoglu, D. and S. Johnson (2007). "Disease and Development: the Effect of Life Expectancy on Economic Growth". *Journal of Political Economy 115*(61), 925985.

Acemoglu, D., S. Johnson, and J. A. Robinson (2001). "The Colonial Origins of Comparative Development: An Empirical Investigation". *American Economic Review 91*(5), 1369–1401.

Aghion, P. and P. Howitt (2009). *The Economicsof Growth* . Cambridge, Massachusetts: The MIT Press.

Antras, P. and H. J. Voth (2003). "Factor prices and productivity growth during the British industrial revolution". *Explorations in Economic History 40*, 5277.

Bar, M. and O. Leukhina (2010). "The role of mortality in the transmission of knowledge". *Journal of Economic Growth 15*(4), 291321.

Becker, G. (1960). *An Economic Analysis of Fertility.* Princeton, NJ: Princeton University Press.

Boucekkine, R., D. D. la Croix D., and O. Licandro (2003). "Early Mortality Declines at the dawn of modern growth". *Scandinavian Journal of Economics 105*(3), 401–418.

Broadberry, S., B. A. abd Klein, M. Overton, and B. van Leeuwen (2010). "British economic growth, 1270-1870". *Working Paper*.

Broadberry, S., B.M.S.Campbell, A. Klein, M. Overton, and B. van Leeuwen (2010). "English economic growth, 1270-1700". *Working Paper*.

Broadberry, S., B.M.S.Campbell, and B. van Leeuwen (2013). "When did britain industrialise? the sectoral distribution of the labour force and labour productivity in britain, 1381-1851?". *Explorations in Economic History 50*, 16–27.

Cervellati, M. and U. Sunde (2005). "Human Capital Formation, Life Expectancy and the Process of Development". *American Economic Review 95*, 1653–1672.

Cervellati, M. and U. Sunde (2011). "Life Expectancy and economic growth: the role of the demographic transition". *Journal of economic growth 16*, 99–133.

Cipolla, C. (1962). *The Economic History of World Population.* Harmondsworth, Penguin Books.

Clark, G. (2007). *A Farewell to Alms: A Brief Economic History of the World.* Princeton University Press.

De la Croix, D. and O. Licandro (1999). "Life Expectancy and Endogenous Growth". *Economic Letters* (65), 225–263.

Deaton, A. (2006). "the great escape: A review of robert fogels the escape from hunger and premature death, 17002100". *Journal of Economic Literature XLIV*, 106–114.

Dixon, B. (1978). *Beyond the Magic Bullet.* London: George Allen Unwin.

Easterlin, R. A. (2004). *The Reluctant Economist Perspectives on Economics, Economic History, and Demography.* Cambridge University Press.

Fogel, R. W. (2004). *The Escape from Hunger and Premature Death, 1700–2100. Europe, America, and the Third World.* Cambridge University Press.

Galor, O. (2005). "From Stagnation to Growth: Unified Growth Theory ". *in The Handbook of Economic Growth (A 'Classical' Prespective) 1*(Edited by Philippe Aghion and Steven Durlauf), 171–293.

Galor, O. and O. Moav (2004). "From Physical to Human Capital Accumulation: Inequality and the Process of Development". *Review of Economic Studies 71*, 1001–1026.

Galor, O. and O. Moav (2006). "Das Human Kapital: A Theory of the Demise of Class Structure". *Review of Economic Studies 73*, 85–117.

Hansen, G. and E. Prescott (2002). "Malthus to Solow ". *The American Economic Review 92*(4), 1205–1217.

Jacob, M. and L. Stewart (2004). *Pratical Matter*. Harvard, USA: Harvard University Press.

Kremer, M. (1993, August). Population Growth and Technological Change: One Million B.C. to 1990. *The Quarterly Journal of Economics 108*(3), 681–716.

Lagerlof, N. P. (2003). "From Malthus to Modern Growth: Can Epidemics Explain the three Regimes". *International Economic Review 44*, 755–777.

Livi Bacci, M. (2007). *A Concise History of World Population*. Malden, MA: Blackwell.

Lucas, R. J. (1988, July). On the mechanics of economic development. *Journal of Monetary Economics 22*(1), 3–42.

McCloskey, D. (1995). Once upon a time there was a theory. *Scientifica American February*, 25.

McKeown, T. (1976). *The Modern Rise of Population*. New York: Academic Press.

Mokyr, J. (1999). *Editor's Introduction: The New Economic History and the Industrial Revolution*. Boulder: Westview Press: The British Industrial Revolution: an Economic Perspective.

Mokyr, J. (2002). *The Gifts of Athena*. Priceton: Princeton University Press.

Porter, R. (2006). *The Cambridge History of Medicine*. Cambridge University Press.

Preston, S. (1975). "The Changing Relation between Mortality and Level of Economic Development". *Population Studies 29*(2), 231–248.

Robert E. Lucas, J. (2004). The Industrial Revolution: past and future. *Annual Report* (May), 5–20.

Rosen, S. (1988). "The Value of Changes in Life Expectancy". *Journal of Risk and Uncertainty 1*(3), 285–304.

Rosenberg, N. (1994). *Exploring the Black Box: Technology, Economics, and History*. Cambridge, UK: Cambridge University Press.

Schofield, R., D. Reher, and A. Bideau (1991). *The Decline of Mortality in Europe*. Oxford: Clarendon Press.

Soares, R. (2005). "Mortality Reductions, Educational Attainment and Fertility Choice". *American Economic Review 95*(3), 580–601.

Solow, R. (2000). *Growth Theory: an Exposition*. Oxford, UK: Oxford University Press.

Voigtlander, N. and H.-J. Voth (2013, Fall). Gifts of Mars: Warfare and Europe's Early Rise to Riches. *Journal of Economic Perspectives 27*(4), 165–86.

Watts, S. (2003). *Disease and Medicine in World History*. New York: Routledge.

Weisdorf, J. L. (2004). "From stagnation to Growth: Revisiting three historical regimes". *Journal of Population Economics 17*, 455–472.

Wrigley, E. and R. Schofield (1981). *Population History of England 15411871: A Reconstruction*. Cambridge, MA: The Harvard University Press.

A Optimal Choices

AA. Exogenous Fertility

The individual's maximization problem is given:

$$(c_t^*, b_t^*) = \arg\max_{c_t, b_t}\{p_t[(1-\beta)\log(c_t) + \beta\log(b_t+\theta)]\}, \tag{55}$$

subject to:

$$y_t = p_t c_t + p_t b_t;$$
$$c_t \geq c^{\text{MIN}};$$
$$b_t \geq 0.$$

The Lagrangian for problem (55) is given by:

$$\mathscr{L} = p\left[(1-\beta)\log\left(\frac{y_t - p_t b_t}{p_t}\right) + \beta\log(b_t+\theta)\right] + \lambda b_t + \mu\left(\frac{y_t - p_t b_t}{p_t} - c^{\text{MIN}}\right) \tag{56}$$

and the first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial b_t} = p_t\left[-\frac{(1-\beta)p_t}{y_t - p_t b_t} + \frac{\beta}{b_t+\theta}\right] + \lambda - \mu = 0.$$
$$\lambda b_t = 0$$
$$\mu\left(\frac{y_t - p_t b_t}{p} - c^{\text{MIN}}\right) = 0$$

Thus we can have different cases:

1. $c_t = c^{\text{MIN}}$ and $b_t = 0$.
2. $c_t > c^{\text{MIN}}$ and $b_t = 0$. Thus we have that $c_t = \frac{y_t}{p_t}$. This implies that $c_t > c^{MIN}$ if $y_t > p_t c^{MIN}$.
3. $c_t > c^{\text{MIN}}$ and $b_t > 0$. Thus solving the first order conditions we get:

$$c_t^* = \frac{(1-\beta)(y_t + p_t\theta)}{p_t} \tag{57}$$

$$b_t^* = \frac{\beta y_t - \theta(1-\beta)p_t}{p_t} \tag{58}$$

We do not consider the case $c_t = c^{\text{MIN}}$ and $b_t > 0$.

AB. Endogenous Fertility

When fertility is endogenous the agent's maximization problem is given:

$$(c_t^*, b_t^*, n_t^*) = \underset{c_t, b_t, n_t}{\arg\max} \{p_t[(1-\beta)\log(c_t) + \epsilon \log(n_t) + \beta \log(b_t + \theta)]\}, \tag{59}$$

subject to:

$$y_t = \delta y_t n_t + p_t c_t + p_t b_t;$$

$$c_t \geq c^{\mathrm{MIN}};$$

and

$$b_t \geq 0.$$

The Lagrangian for this optimization problem is given as follows:

$$\mathscr{L} = p_t \left[(1-\beta)\log c_t + \epsilon \log\left(\frac{y_t - p_t(c_t + b_t)}{\delta y_t}\right) + \beta \log(b_t + \theta)\right] + \lambda b_t + \mu\left(c_t - c^{\mathrm{M}}\right. \tag{60}$$

and the first order conditions are:

$$\frac{\partial \mathscr{L}}{\partial c_t} = \frac{(1-\beta)}{c_t} - \frac{\epsilon p_t}{y_t - p_t(c_t + b_t)} + \mu = 0. \tag{61}$$

$$\frac{\partial \mathscr{L}}{\partial b_t} = -\frac{\epsilon p_t}{y_t - p_t(c_t + b_t)} + \frac{\beta}{b_t + \theta} + \lambda = 0. \tag{62}$$

$$\lambda b_t = 0$$

$$\mu\left(c_t - c^{\mathrm{MIN}}\right) = 0$$

Thus we can have different cases:

1. $c_t = c^{\mathrm{MIN}}$ and $b_t = 0$. Thus given $c_t = c^{\mathrm{MIN}}$, from the budget constraint we get:

$$n_t^* = \frac{y_t - p_t c^{\mathrm{MIN}}}{\delta y_t} \tag{63}$$

2. $c_t > c^{\text{MIN}}$ and $b_t = 0$. Given that $\mu = 0$, we solve the first order condition (61) with respect to c_t:

$$c_t^* = \frac{(1-\beta)y_t}{(1-\beta+\epsilon)} \tag{64}$$

Substituting this solution into the budget constraint, the optimal number of children, is given by:

$$n_t^* = \frac{\epsilon}{\delta(1-\beta+\epsilon)} \tag{65}$$

3. $c_t > c^{\text{MIN}}$ and $b_t > 0$ Thus given $\mu = 0$ and $\lambda = 0$, from the first order conditions (61), (62) and the budget constraint we get:

$$c_t^* = \frac{(1-\beta)(y_t + p_t\theta)}{1+\epsilon} \tag{66}$$

$$b_t^* = \frac{\beta y_t - \theta(1-\beta+\epsilon)p_t}{1+\epsilon} \tag{67}$$

$$n_t^* = \frac{\epsilon(y_t + \theta p_t)}{\delta(1+\epsilon)y_t} \tag{68}$$

We do not consider the case $c_t = c^{\text{MIN}}$ and $b_t > 0$.

AC. Optimal Education

The maximization of Eq. (30) yields the following solution for the optimal education:

$$\bar{e_t}^* = \begin{cases} 0 & \text{if } \bar{b_t} \in [0, \tilde{b}]; \\ \dfrac{\bar{b_t} - \tilde{b}}{1 + D\tilde{b}} & \text{if } \bar{b_t} \in (\tilde{b}, \infty) \end{cases} \tag{69}$$

where:

$$\tilde{b} \equiv \frac{\alpha}{D(1-\alpha)\gamma}. \tag{70}$$

From Eqq.(29) and (69) the level of income such that $\bar{b}_t = \tilde{b}$, when fertility is exogenous, is given as follows:

$$y^{EDU} \equiv \frac{\tilde{b}n^{EDU} + \theta(1-\beta)p^{EDU}}{\beta} \tag{71}$$

When fertility is endogenous, y^{EDU} is calculated using Eq. (42):

$$y^{EDU} \equiv \frac{\tilde{b}(1+\epsilon) + \theta(1-\beta+\epsilon)p^{EDU}}{\beta}. \tag{72}$$

AD. Thresholds

From Eqq. (13) when production is conducted using traditional technology per-worker income at time $t+1$ is given by:

$$y_{t+1} = A_{t+1}p_{t+1}^{1-\alpha}\left(\frac{\tilde{T}}{L_{t+1}}\right)^{\alpha} \tag{73}$$

Thus, from Eq. (22), per capita income when production is conducted using traditional technology ensures a consumption at least equal to the subsistence level if:

$$Ap_{t+1}^{1-\alpha}\left(\frac{\tilde{T}}{L_t}\right)^{\alpha} \geq p^{\mathrm{MIN}}c^{\mathrm{MIN}} \tag{74}$$

which implies that:

$$A_{t+1} \geq A^{MIN} = \frac{p^{\mathrm{MIN}}c^{\mathrm{MIN}}}{p_{t+1}}\left(\frac{p_{t+1}L_{t+1}}{\tilde{T}}\right)^{\alpha}. \tag{75}$$

If p_t is constant over time we get:

$$A^{MIN} = c^{\mathrm{MIN}}\left(\frac{pL_{t+1}}{\tilde{T}}\right)^{\alpha}. \tag{76}$$

B Proof of Proposition 1 .

In the follows we assume that $p_{t+1} = p_t = p$, $L_{t+1} = L_t = L$, and $n_t = 1$.

From Eq.(35) if p_t is constant over time, it follows that:

$$\frac{\partial y_{t+1}}{\partial y_t} = \frac{A_{t+1}p^{1-\alpha}\alpha\beta}{n_t}\left[B(C+Qy_t-F)^{-\frac{\alpha}{\gamma}} + \tilde{b}(C+Qy_t-F)^{1-\gamma}\right]^{\alpha-1}, \tag{77}$$

where for simplicity we set $B = \tilde{T}/L$, $C = 1/(1+D\tilde{b})$ $Q = D\beta/(1+D\tilde{b})$, $F = \theta(1-\beta)p/(1+D\tilde{b})$.

From Eq.(77) it follows:

$$\lim_{y_t\to\infty}\frac{\partial y_{t+1}}{\partial y_t} = 0. \tag{78}$$

Given this condition, the economy shows one stable equilibrium in the Traditional Regime and possibly one unstable and one stable equilibrium in the Pre-modern Regime if the following conditions hold:

$$\lim_{y_t\to y^{CAP}} y_{t+1} \leq y^{CAP} \tag{79}$$

$$\lim_{y_t\to y^{EDU}} y_{t+1} \leq y^{EDU}. \tag{80}$$

- the first condition holds if $A_{t+1} < A^{TRA}$:

$$A^{TRA} = \frac{\theta(1-\beta)}{\beta}\left(\frac{pL}{\tilde{T}}\right)^{\alpha} \tag{81}$$

  where $A^{MIN} < A^{TRA}$ if assumption 21 holds.

- the second condition holds if:

$$A \leq A^{PRE-MOD} = \frac{\tilde{b} + \theta(1-\beta)p^{\alpha}}{\beta p^{1-\alpha}\left(\frac{\tilde{T}}{L_t} + \tilde{b}\right)^{\alpha}} \tag{82}$$

  where $\lim_{p\to 0} A^{PRE-MOD} = \infty$ and $\partial A^{PRE-MOD}/\partial p < 0$ if:

$$p < p^T = \frac{(1-\alpha)\tilde{b}}{\theta(1-\beta)\alpha}. \tag{83}$$

Some calculations show that, when, $A^{TRA} < A^{PRE-MOD}$ if:

$$\theta < \frac{\tilde{b}}{p(1-\beta)\left[\left(1+\tilde{b}\frac{L}{\tilde{T}}\right)^{\alpha}-1\right]} \tag{84}$$

An economy shows one stable equilibrium in the Pre-modern Regime if:

$$\lim_{y_t \to y^{CAP}} y_{t+1} \geq y^{CAP}, \tag{85}$$

$$\lim_{y_t \to y^{EDU}} y_{t+1} \leq y^{EDU}, \tag{86}$$

- The first condition holds if $A \geq A^{TRA}$
- The second condition holds if $A \leq A^{PRE-MOD}$

An economy shows one stable equilibrium in the Modern Regime if:

$$\lim_{y_t \to y^{CAP}} y_{t+1} \geq y^{CAP}, \tag{87}$$

$$\lim_{y_t \to y^{EDU}} y_{t+1} \geq y^{EDU}, \tag{88}$$

- The first condition holds if $A > A^{TRA}$
- The second and third conditions hold if $A > A^{PRE-MOD}$

C Quantitative Exercise

All variables are estimated using a nonparametric method.

Income per worker y_t is calculated using the data on income per capita i.e. $y_t^{p.c}$ from Maddison Project Database (2014) (see Fig. 15). Income per worker is calculated as:

$$y_t = \frac{y_t^{p.c} L_t^{TOT}}{p_t L_t}, \tag{89}$$

where L_t^{TOT} is the observed total population the data for which are taken, for the period 1541 to 1850 (England and Wales) from Wrigley and Schofield (1981) and for the period 1850 to 1914 (England, Wales and Scotland) from the International Historical Statistics.

The varible $p_t L_t$ is the employment which is calculated adjusting the labour force for the unemployment rate. The first data available for the unemployment rate is from 1855. Thus, the unemployment rate in the period 1541 to 1854 is supposed to be equal the average unemployment rate of the period $1855-1880$. From 1855 to 1914 we consider a smoothed unemployment rate. Fig.16 shows the results of our estimate for the unemployment rate.

The labour force, according to theoretical model is given by the people between the ages 20 and 70 and it is adjusted to take into account that some individuals does not belong to the labour force (15%) (see Fig. 16).

The labour share in the traditional sector is estimated using the data from Broadberry et al. (2013) for the period 1700 to 1851. For the period 1852-1914 the data are taken from the International Historical Statistics. In particular, we adjust this data to match labour share in our model which is set equal to one until t^{CAP}.

The probability of surviving from 20 t0 70 years is calculated as the ratio between the population 70 years old (i.e. Pop^{70}), in a given period (for example 1591) and the population 20 years old (i.e. Pop^{20}), 50 years before (for example 1541):

$$\hat{p} = \frac{Pop_{t+50}^{70}}{Pop_t^{20}} \tag{90}$$

The data for the period 1541 to 1821 are taken from Wrigley and Schofield (1981) and for the period 1841 to 1914 (England, Wales and Scotland) from the International Historical Statistics (see Fig. 17).

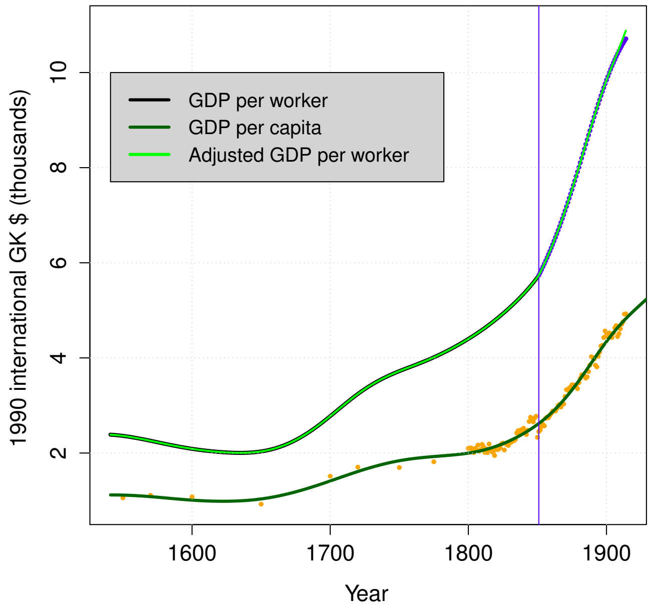

Figure 15: GDP per worker in England 1591-1914

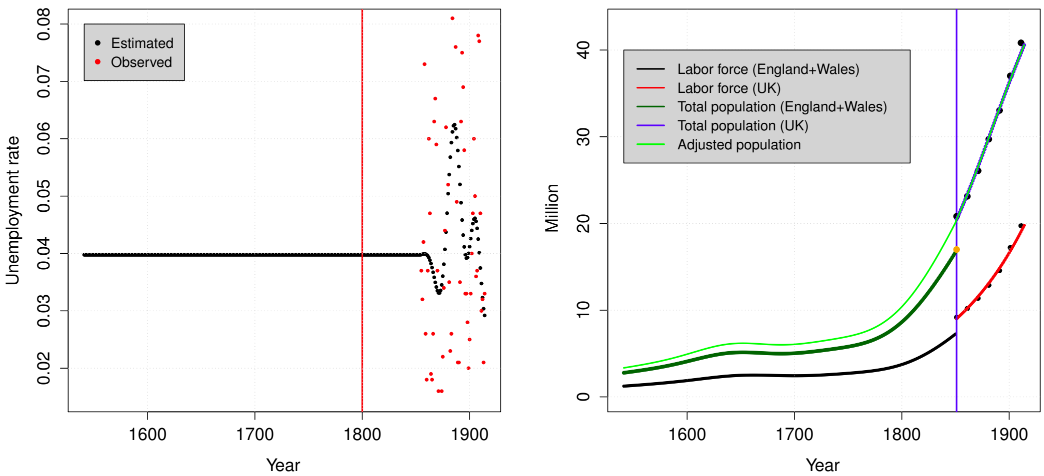

Figure 16: Unemployement and Labour Force UK 1541-1914

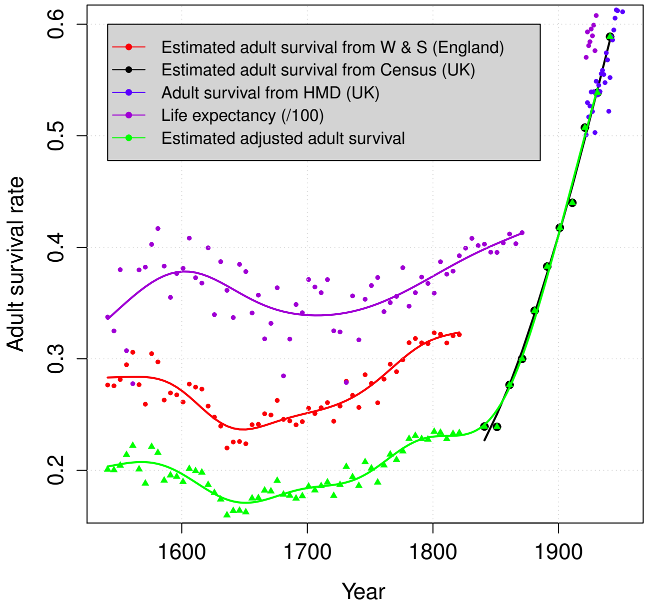

Figure 17: Adult Survival

CA. Parameters

Using the data on capital formation from the International Historical Statistics (2013) we can calculate the value of $\tilde{b}$. In particular from Eqq.(26) and (34) $\tilde{b}$ is a function of the capital share in the period $t.EDU$. In particular defining the capital share as i, $\tilde{b}$ can be calculated as follows :

$$\tilde{b} = \frac{iy_{tEDU}}{n_{tEDU}}. \tag{91}$$

where $i = 0.09$ is set equal its average value in the period $1840 - 1900$.

The value of $\tilde{b}$ is therefore used to compute $\tilde{T}$ which from Eq. (11) is given by:

$$\tilde{T} = \tilde{b}L_{t_{EDU}} \left(\frac{l^a_{tEDU}}{1 - l^a_{tEDU}} \right) \tag{92}$$

CA.i. Exogenous Fertility

From Eqq. (24) and (32) β can be calculated as follows:

$$\beta = \frac{\tilde{b}n^{EDU}}{y^{EDU} - \dfrac{y^{CAP}p^{EDU}}{p^{CAP}}}. \tag{93}$$

Thus from Eqq. (24) and (93) the parameter θ can be calculated as follows:

$$\theta = \frac{y^{CAP}\beta}{(1-\beta)p^{CAP}}. \tag{94}$$

To compute the parameter γ we use the value of the interest rate. In particular from Eqq. (4) and (13) we get:

$$h_{t+1} = \left[\frac{\left(\dfrac{y_{t+1}}{A_t p_{t+1}^{1-\alpha}}\right)^{1/\alpha} - \frac{\tilde{T}}{L_{t+1}}}{p_{t+1}(1-l_{t+1}^{a})\left(\dfrac{\alpha A_t}{r_{t+1}}\right)^{1/(1-\alpha)}}\right]^{\alpha}. \tag{95}$$

From eqq. (17), (31) and (70):

$$h_{t+1} = \left\{\frac{\gamma\tilde{b}(1-\alpha)n_t + \alpha[\beta y_t - \theta(1-\beta)p_t]}{\tilde{b}n_t[(1-\alpha)\gamma + \alpha]}\right\}^{\gamma}. \tag{96}$$

Thus from Eqq. (95) and (100) we compute the parameter γ.

CA.ii. Endogenous fertility

Using the threshold level of income y^{SUB} we have:

$$\epsilon = \frac{(1-\beta)(y^{SUB} - p^{SUB}c^{\mathrm{MIN}})}{p^{SUB}c^{\mathrm{MIN}}}. \tag{97}$$

From Eq. (45):

$$\theta = \frac{y_{CAP}\beta}{(1-\beta+\epsilon)p_{CAP}}. \tag{98}$$

Thus substituting Eqq.(97) and (98) into Eq.(50):

$$\beta = \frac{\tilde{b}y^{SUB}/p^{SUB}c^{\text{MIN}}}{y^{EDU} - y^{CAP}p^{EDU}/p^{CAP} + \tilde{b}(y^{SUB} - p^{SUB}c^{\text{MIN}})/p^{SUB}c^{\text{MIN}}} \tag{99}$$

To calculate the parameter γ we use Eq. (95) and the human capital production function given as follows:

$$h_{t+1} = \left\{ \frac{\gamma\tilde{b}(1-\alpha)n_t(1+\epsilon) + \alpha[\beta y_t - \theta(1-\beta+\epsilon)p]}{\tilde{b}(1+\epsilon)n_t[(1-\alpha)\gamma+\alpha]} \right\}^{\gamma}. \tag{100}$$

where n_t is given by Eq. (43).

Discussion Papers
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