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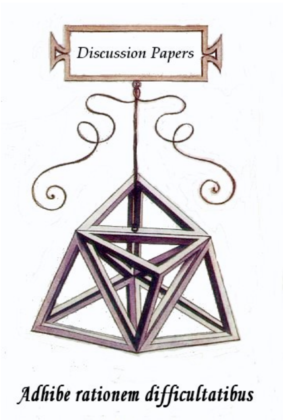

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Standardization of Credit Default Swaps Market

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Abstract

Standardization of credit derivatives was a necessary step towards a more transparent and better structured market, especially after recent financial turmoil. In this survey, we sum up the enhancements established by ISDA in 2009, focusing on vanilla instruments (Credit Default Swaps).
New contract features include changes in the cash flow and in post-default settlement mechanisms, where auctions are now provided; an exhaustive description of such features acts as a basis for quantitative analysis of this standard market. A rigorous depiction of the conversion mechanism, the ISDA CDS Standard model, is also provided.

Classificazione JEL: C60, G23, G28.
Keywords: Credit Default Swaps, Standardization, ISDA CDS Model, Upfront, Auction Settlement.

Contents

I. Introduction: The Credit Derivatives Market

The credit derivatives market has experienced rapid growth in the last decade, attracting several types of investors. It has also become the subject of intense debate, involving financial economists, institutions and regulators, as well as a large share of the public, especially after recent financial turmoil.
The birth of this market can be traced back to the early nineties, when the first vanilla instruments, namely, Credit Default Swaps (CDS), were created with the purpose of hedging credit risk exposures to given *Reference Entities*.
A CDS is a bilateral agreement between two counterparties, with an agreed date of expiration. One counterparty, the *Buyer*, pays a periodic coupon in order to receive protection against deteriorations in the creditworthiness of the Reference Entity that might cause permanent impairments to the value of its obligations. When a *Credit Event* affects the Reference Entity, the other counterparty, the protection *Seller*, bears the financial loss of the Buyer and partially refunds him up to a certain *Notional* amount N of the *Reference Obligations* issued by that Entity[1]; given this definition, a CDS contract could be likened to a traditional insurance contract: there are, however, at least two relevant differences.
First, stopping premium payments is typically sufficient to unwind an insurance contract, while, as for most of derivatives, closing out a CDS position means to sign another CDS contract and take the opposite side of the trade; second, there is no need for the Buyer to actually hold the obligations on which the CDS is written: he could be willing to take purely speculative positions by trading "naked" CDS.
Notwithstanding speculation, CDS are attractive as hedging instruments too: when competition increased across markets in the nineties, causing a relevant number of bankruptcies (for example, Enron and Worldcom), banks were forced to monitor and manage their credit portfolios more actively. CDS allowed credit risk to be managed separately from loan portfolios: by buying CDS protection, banks could mitigate the risk profile of their portfolios without altering their compositions. As an example, consider a small commercial bank who wants to hedge the credit risk of a corporate borrower to which the bank is already largely exposed; by transferring this credit risk to another bank through a CDS, the bank can keep lending to its customer, but avoids excessive concentration risks, reduces the resources committed to the borrower and frees capital for other investments.
CDS were originally highly tailored to the needs of the counterparties, which were free to privately agree on any of the clauses in these contracts, for example, the *Payment Dates* for the Buyer's premium or the different types of *Credit Events* leading to the *triggering* of the contract, that is, to the activation of the Seller's payment and the post-default settlement of the contract.

[1]Note that Reference Obligation of a given Entity includes a wide range of Obligations; previous to standardization, no limits were imposed to relevant Obligations for a transaction, as long as the Buyer found a counterparty willing to accept them.

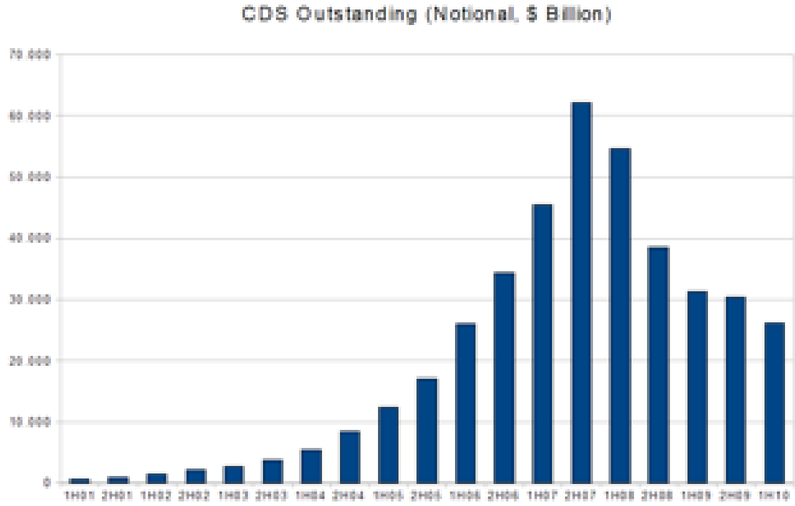


Figure 1: Semiannual evolution of CDS global market, 2001-2010. (*Source: Vause, 2010*)

Particular mention deserves the latter of these clauses: choosing different settlement methods is indeed strictly connected to the determination of the post-default value R of the Reference Obligation.
Cash Settlement provides a reimbursement equal to the difference between the par value and the post-default value of the Buyer's position. Because predicting ex-ante this *Recovery Rate* R was rather challenging for both counterparties, 73% of CDS contracts had *Physical Settlement* as a settlement clause until 2005, see Weistroffer (2009). Physical Settlement eliminates the problem of determining the final price R, because physical delivery of the Reference Obligation for a face value of N is performed in exchange for the Seller's post-default payment.
Figure 1 shows the impressive increase in the trading volume of CDS on a global scale, measured by the gross outstanding notional of the contracts, which reached its peak of nearly $60 trillions in the second half of 2007.
As observed in Vause (2010), the subsequent decline was not due to a decrease in the appeal of the CDS market, but rather to *trade compression* procedures aimed at reducing the outstanding notional, which will be described in what follows.[2]
Despite the benefits stemming from risk management and hedging procedures, recent crisis revealed several structural and operational shortcomings of credit derivatives market. In particular, because of the OTC nature of this market, relevant information about the real credit risk borne by protection Sellers was partly concealed, preventing regulators from collecting complete information on existing trades.

[2]Observe also that any CDS, since bilaterally traded, is double counted with this method so gross notional reflects past trades but provides little information on real credit risk bore by a dealer in this market. (Weistroffer, 2009)

Moreover, the bilateral nature of CDS contracts exposes them to *counterparty risk*, that is, the risk that one of the two parties does not fulfill its obligations to the other. Counterparty Risk is not independent from credit risk. As an example, consider a rise in the credit risk of the Reference Entity: this deterioration in creditworthiness weakens the Seller of protection, by increasing the likelihood that he will be asked to pay, and increases counterparty risk for the Buyer.[3]
The growth of the CDS market required the creation of a framework of greater legal certainty, capable of reducing the number of disputes and of facilitating supervision by market authorities. The main obstacle was the highly tailored nature of different contracts, self-assessed by the parties to each transaction.
Therefore, standardizing CDS contract was considered a necessary step towards a better regulation of the CDS market. A first attempt in this direction was made by the *International Swap and Derivatives Association (ISDA)*[4] through the *2003 Credit Derivatives Definition* (ISDA, 2003).
The financial crisis started in 2008 brought the CDS market to the attention of regulators again, leading to a substantial review of the 2003 Definitions: this took place in two stages, firstly with the March 2009 supplement (ISDA, 2009b), also known as the "*CDS Big Bang*", and subsequently with the July 2009 supplement (ISDA, 2009c), named "*CDS Small Bang*".
This paper focuses on this standardization process, exploring its main features, with a particular focus on the standardization of coupons and the introduction of *Upfront Payments* in CDS contracts. Before 2009, counterparties priced a CDS contract by agreeing on the annual coupon of the CDS itself (the *spread* S).
After the introduction of a standard coupon in 2009, counterparties started to price a CDS contract by agreeing on the Upfront payment, which represents the expected discounted value of the difference between the coupon that would have been agreed upon in the old regime and the standardized coupon; it is however still commonplace to quote a CDS price in terms of *conventional* spread, that is, the coupon that would be paid in an equivalent contract with zero Upfront.
In order to get the real cash flow of the contract, the Upfront correspondent to any given spread must be determined in the same way by all market participants. ISDA developed a toolkit, the *ISDA CDS Standard Model*, that provides a one-to-one mapping of these quotations (Upfront and conventional spread), based on standard no-arbitrage principles.
The paper is organised as follows: Section II describes the main characteristics of the *CDS Bangs*; section III explores the contract's triggering and post-default settlement. Section 4 explains the marking to market of CDS, their pricing and the conversion mechanism between different quotation systems. A rather detailed description of this mechanisms is then provided in the Appendix.

[3]There is also an indirect connection between credit and counterparty risk, due to the posting of collaterals, again Weistroffer (2009). Recent studies (European Central Bank, 2009) introduced also the concept of *wrong way risk* where the interaction between the two is reversed.

[4]ISDA is a private international association collecting more than 800 members including, among others, dealers, issuers and law firms.

II. Standardization: the CDS "Bangs"

The financial turmoil started in mid-2007 exacerbated many weaknesses of the financial system, having a widespread impact due to the interconnections across different markets. Several regulatory statements were specifically addressed to the CDS market: among them, the most influential was the President's Working Group[5] (PWG)'s *Policy Statement on Financial Markets*, dated 2008.
The main purpose of the document was to analyse the causes of the financial crisis, and to provide recommendations in order to "*..take the steps necessary to mitigate systemic risk, restore investor confidence, and facilitate stable economic growth*". Serious shortcomings in risk-management practices were revealed, that caused significant losses and balance-sheet pressures, these latter contributing in turn to the tightening of lending standards and terms, negatively impacting economic growth. Many key issues were listed: the inaccuracy and untimeliness of trade data submission, lack of robust procedures for the resolution of trade matching errors, major operational problems (counterparties miscommunication and increasing backlogs of unconfirmed trades) and uncertainties in post default settlement.
In order to solve these problems, the PWG proposed to create an infrastructure endowed with decisional power in any of the processing events over the lifetime of such contracts, to ensure transparency and coordination in determining relevant Credit Events for any transaction in the market; moreover, this infrastructure should also be responsible for determining a post-default value acknowledged by any investor. In order to achieve these goals, a precise ratification of relevant Credit Events was to be introduced. Moreover, in order to facilitate an electronic processing similar to that of an exchange board, it was deemed necesessary to reach a certain degree of standardization of the clauses that were formerly tailored to the needs of each couple of counterparties, involved in their specific transaction. Standardization of CDS contracts[6] is also a very cheap way to net out a large number of opposite positions with the same features (Entity, Maturity…), by offsetting the cash flows that these positions generate.
Netting out opposite positions can significantly reduce payment volumes so that cash shortages are less likely to cause a default; the total outstanding Notional as measured after netting out positions should also give a more efficient measure of credit exposures.
A response to the PWG's guidances was the introduction of two supplements to ISDA (2003), namely the *March 2009 Supplement* (ISDA, 2009b) , also known as the "*CDS Big Bang*"followed by the *July 2009 Supplement* (ISDA, 2009c) named "*CDS Small Bang*".

[5]The PWG was originally established to respond to the "Black Monday"of 1987, and gathers together several key representatives of US financial institutions.

[6]Note that standardization applies to the whole Credit Derivatives market; here however, we will deal only with single name vanilla instruments.

More than 2000 market participants, including banks as well as institutional investors, voluntarily adhered to *Big Bang* protocol (Amadei et al., 2011).
This latter, despite some changes to be globally applied, was specifically addressed to North American corporate contracts: the following *Small Bang* protocol was drawn mainly to introduce the same amendments for European Corporate and Western European Sovereign CDS, as well as to deal with the problem of credit Restructuring.
The main novelties introduced by the *Big Bang* contract are the following:

- introduction of *Determination Committees*;
- introduction of a dedicated *Central Clearing House*;
- introduction of an *Auction Settlement Method*;
- restrictions on *Restructuring* conventions;
- creation of a *Credit Event Backstop Date*;
- introduction of a *First Full Coupon* clause;
- introduction of a fixed coupon plus *Upfront* fees.

The *Small Bang* brought about these changes to European contracts; among the novelties of this second protocol, the most important was the hardwiring of an Auction mechanism for the Restructuring Credit Event. We briefly review these clauses following *Markit*'s reports (Markit, 2009b and Markit, 2009c).

II.A. Credit Derivatives Determination Committees

The *Big Bang* introduced five Determination Committees (DCs), one for each relevant region[7], and each of them having responsibilities with respect to that region; rules to determine the composition of a DC are explained in detail in Markit, 2009b: the final composition of each of the five is shown in Figure 2.
DCs are mainly addressed to harmonize industry and avoid misinterpretations as a Credit Event affects the Reference Entity. According to the 2003 rules, the potential occurrence of a Credit Event was determined through a private *Notice* delivered from one to the other party: this of course creates often disputes, both on relevance and timing of that Event. It could be also the case that triggering a CDS in any of these bilateral transactions led other backlogged transaction to demand for payment, creating unpredictable reactions in the market.

[7] Namely the Americas, Asia-ex Japan, Australia-New Zealand, EMEA (Europe,Middle-East and Africa) and Japan itslef.

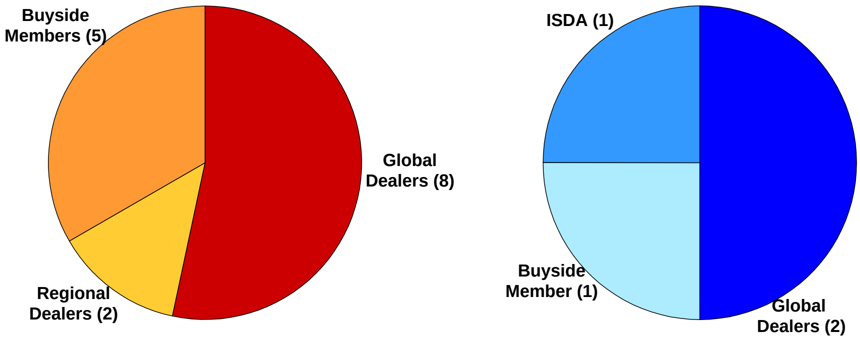


Figure 2: Composition of DCs: Voting and Non-voting members. (*Source: Markit, 2009b*)

The *Big Bang* was intended to rule out these problems introducing a simpler principle: any of the ISDA members, with the sponsor of a DC member, is now able to request the DC to be conveyed in order to take a decision on whether and when a specific Credit Event for the transaction occurred.
The date the request is bring forth is named *Credit Event Resolution Request Date*; the DC has then fifteen days to deliberate. If an eighty percent majority is not achieved, an external review can be demanded within an additional period of ten business days.
The *DC Credit Event/No Credit Event Announcement* is the day ISDA effectively takes its binding decision on target Credit Event. In case the Credit Event is announced, it will be the DC again to decide the terms of the Auction (see section II.C) and the set of *Deliverable Obligations*.
The presence of a Determination Committee is fundamental in order to introduce a Central Clearing House (section II.B): it standardizes the occurrence of the Credit Event and draws out the rules to determine the Final Price, so that any of the many positions of the CCH referred to the same Reference Obligation and Credit Event will be dealt with according to the same rules.

II.B. Central Clearing House

The introduction of a dedicated central counterparty in the credit derivatives market was a further step towards counterparty risk reduction.
Common feature to achieve this goal was the idea of *trade compression*, that is reducing the number of redundant contracts. This was at first achieved through operators, such as *CreditEx*, which collected multilateral information from the network of counterparties and, maintaining the same risk profile of each of the participants' positions, proposed a renewed set of trades that becomes compulsory for each of the parties agreeing to it.

Before the *CDS Bangs*, the CDS market looked like a complicated network of bilateral transactions, each of them possibly providing different clauses that were to be translated in a huge amount of stand-alone variables interacting in the same network, see Figure 3(a).
The introduction of these multilateral agreements urges coordination of market participants and the *Central Clearing House* is an improvement for this netting procedure, see Figure 3(b) and 3(c), and it also substitutes bilateral counterparty risk for the risk of its own failure. The drawback of such a market is that firms give up the opportunity of meeting their demands for specific products: moreover, clearing houses are not particularly efficient with products not very liquid, as is precisely the case with customized derivatives.[8]

II.C. Auction Settlement

Early in the life of CDS contracts, most of defaulted obligations were settled according to Physical Settlement, in order to avoid to forecast post-default values when marking the contract to the market. This system was coherent with the use of CDS as hedging instruments, but the growing interest of speculators in this market enhanced the number of Entities for which the outstanding Notional of CDS surpassed the outstanding debt they referred to.
When a Credit Event occurred, speculators on buy-side were forced to purchase obligations to be delivered in order to settle the transaction and thus receive the payment, creating artificial price pressures and distorting the market.
We quote, as an example, the *short squeeze* of the price of *Dana Corporation* bonds which followed its bankruptcy in March 2006, see Figure 4.
Cash Settlement was introduced in order to avoid these issues: payments owed to buyers due to triggered CDS did not request any delivery. The problem to be faced was to find a mechanism to set a transparent and trustworthy final price R that the whole market could use.
Following the recommendations of the PWG and in the same standardisation spirit, *Auction Settlement* is introduced in the *Big Bang* supplement; as pointed out by Markit (2009b), several auctions had been hold also before the *CDS Bangs*. The weakness was that participants were requested to sign separate protocols to adhere to any of the auctions, which was not particularly efficient compared to the hardwiring of the Auction methodology into standard contracts.
The main benefit of holding an Auction is to set a market price to be used to set *all* trades across the market. Physical delivery at different times could expose the Buyer-side counterparties to further profit or loss due to the investors scrambling to buy bonds, even if those same Buyers' positions remained flat.[9]

[8]See Stultz (2010) for a deeper analysis of the trade-off between central exchanges and OTC markets in this context.

[9]Physical Settlement was maintained in standard CDS as a *Fallback Settlement Method* in case the DC, due to an insufficient number of dealers willing to trade defaulted Obligations, decides to hold no Auction.

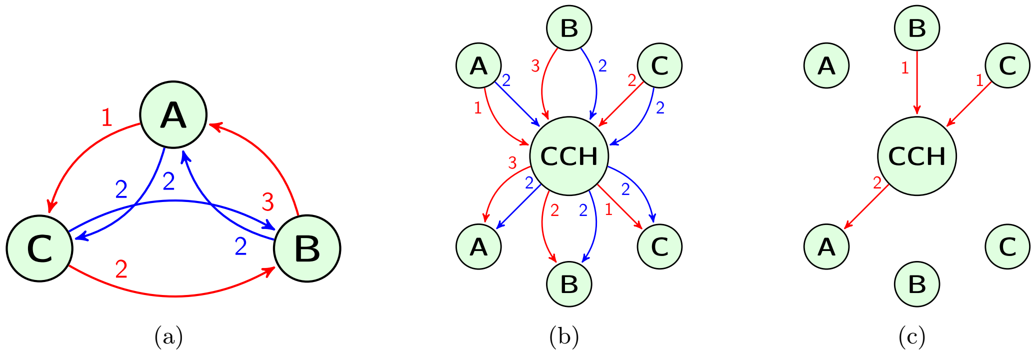


Figure 3: Trade compression through CCH: A, B, C are dealers and arrows represent protection sold from one entity to the other for a Notional equal to the number over it. Different colors refers to different Entities. Figure 3(a) shows a network of bilateral transactions processed to a central counterparty in 3(b) and then compressed 3(c). The outstanding Notional is reduced from 24 to 8. (*Source: Markit, 2009b*)

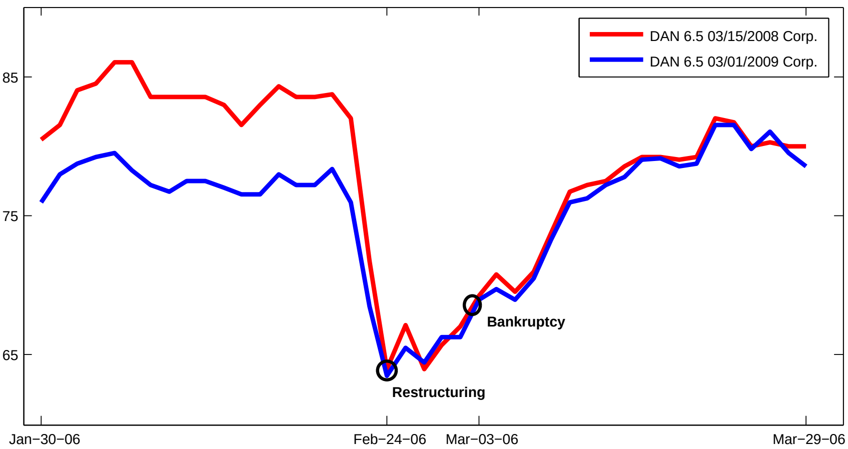


Figure 4: *Short squeeze* of Dana Corporation bonds at the turn of default. On February 24 Dana starts debt restructuring and on March 3 filed under Chapter 11 for Bankruptcy. After that, rush to buy bonds resulted in a sudden price rising of a yet defaulted obligation. (*Source: Amadei et al., 2011*)

II.D. Restructuring Clause Convention

Restructuring is the most tricky among Credit Events: it refers mainly to a change in the covenants of Reference Obligations, like delaying or change in the currency composition of payments. Although the grounds for a covenant breach leading to Restructuring can, and will be, precisely stated, they are considered a Credit Event only if related to deterioration of credit worthiness of the borrower. It could indeed happen, for example, that these changes are not disadvantageous for the Buyer, and yet he finds profit in triggering the CDS, or viceversa.
As an example, Markit (2009c) and Packer et al. (2005), in 2000 Conseco Finance restructured its bank debt to include new guarantees and increased coupons: these changes were not disadvantageous for the holders of these Obligations, and this fact was reflected by their price which was almost unaffected. Yet some banks on Buyer-side of transactions triggered the CDS delivering cheaper longer dated bonds and receiving this almost-par value for their restructured bonds.
According to ISDA, 2003, four different types of Restructuring clauses can be specified: the contracts are exchanged either with *Old Restructuring* (*Old R*), *Modified Restructuring* (*Mod R*), *Modified - Modified Restructuring* (*Mod-Mod R*) and *No Restructuring* (*No R*); the differences between these Restructuring clauses focus mainly on maturity and transferability of Deliverable Obligations.
Trading contracts with Restructuring obviously demand for additional premia: before 2009, CDS on North American Investment Grade typically traded with *Mod R* while North American High Yield traded with *No-R*; most European contracts instead provided *Mod-Mod R* clause.
The *Big Bang* ensured DCs the authority to hold Auctions to settle contracts after either a Failure to Pay or a Bankruptcy Event. It however prohibits explicitly from authorizing Auctions to settle trades after Restructuring Events: under the US jurisdiction, many Restructuring scenarios are filed as bankruptcy under "Chapter 11". On the contrary European jurisdiction separates Bankruptcy and Restructuring in a much more sharped way: as a result approximately 96% of European CDS contracts trade with Mod-Mod R.[10]
ISDA decided then to keep Restructuring clause in Europe, yet still an Auction mechanism was to be designed distinctively for such Events, and the CDS Small Bang is addressed exactly to tease out this problem; we will not enter into details, which are examined for example in Markit (2009b).
We only stress that the problem was that the combination of maturity limitation of deliverable obligations and maturity of CDS could require a too large number of different Auctions to settle all contracts, hardening price discovery and increasing operational risk as well as mispricing between one Auction and another.

[10] See again Markit (2009b).

As to deal with these issues, DC was granted with the power to aggregate different sets of Deliverable Obligations into *Maturity Buckets*, settling auctions only for those Buckets, in a way that multiple auctions are allowed but their number is limited; DC can also decide not to hold any Auction for a given Bucket[11] if it is likely that the Auction will be conducted on illiquid credits or is redundant across different Buckets.
If no Auction is held for a given Maturity Bucket, a *Movement Option* is provided, for which the Buyer can move to the closest following Maturity Bucket for which an Auction is being held and the Seller can move to the 30 years Maturity Bucket; if both exercise their Movement Option, the Seller prevails.
The last clause dealing with Restructuring is the so called *Use it or Lose it* feature: in case of Restructuring, a triggering deadline of five business days following the publication of the final list of Deliverable Obligations is established; this limitation is aimed to prevent protection Buyers not to trigger a CDS even if the Reference Obligation is traded below par value, in order to wait a subsequent Event and get a higher payout.
As a result of both *CDS Bangs*, North American standard CDS provides *No R* clause while European Corporate and Western European Sovereign trade with *Mod-Mod R*.

II.E. Credit Event Backstop Date

Before the *CDS Bangs*, CDS Protection started in most of the cases the first calendar day after the trade date; the introduction of a lookback period for Credit Events was deemed to be mandatory in order to reduce backlog of trades and allow the DC to announce (or "disannounce") a Credit Event without influencing the market through the time they spend in taking any decision.
In order to clarify this point, we follow Markit (2009b) with this simple example: assume an investor enters into a short position on a CDS with a given Maturity and a specific Reference Entity. One week later, in order to offset this position, he enters a long position on the same CDS with the same exact features.
Assume that DC has been convened within these two transactions' dates and later on they decide that a relevant Credit Event occurred within this rather small time interval: the two positions are not truly offset, since the investor must reimburse the Buyer because of first transaction but gets no money from the second, as the relevant Credit Event was timed before this latter.
In order eliminate this *Residual Stub Risk*, facilitating trade compression as well as the CCH's tasks, a *Credit Event Backstop Date* was introduced: if τ is the time of the Credit Event and t_0 is the *Trade Date*, the *Big Bang* provides that the Effective Date for protection will be the Backstop Date $t_0 - 60$, that is the Trade Date itself minus sixty calendar days, see Figure 5.

[11] Unless the so called *300/5* criterion applies, that is, if five or more Dealer Members of the DC are involved into more than 300 Transactions assigned to a Maturity Bucket are triggered by that Credit Event. In this case, an Auction will be automatically hold.

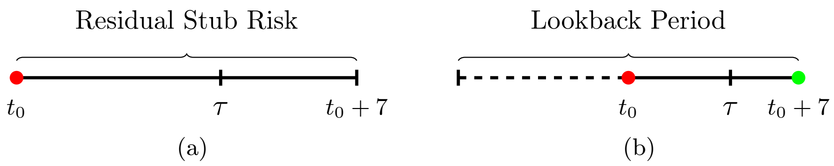


Figure 5: Protection before 5(a) and after 5(b) the *CDS Big Bang*. Red (green) dots shows the triggered short (long) positions. (*Source: Markit, 2009b*)

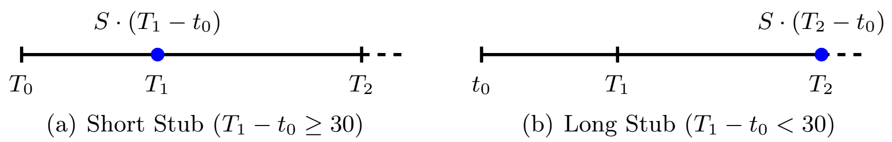


Figure 6: Cash Flow of Buyer's First Payment before the *CDS Big Bang* in case of Short Stub 6(a) and Long Stub 6(b). Blue dots point out payment dates while S is the annual coupon rate of the contract. (*Source: Markit, 2009b*)

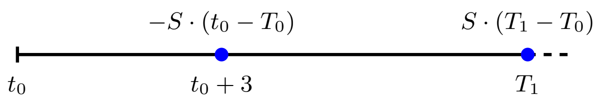


Figure 7: Cash Flow of Buyer's First Payment after *CDS Big Bang*, with Full First Coupon convention. The minus sign refers to a payment made by the Seller to the Buyer.
(*Source: Markit, 2009b*)

II.F. Payment Dates and Full First Coupon

As far as 2003, the dates when the Buyer paid the coupon to the seller were to be specifically agreed between the parties. The *Big Bang* explicitly standardised the CDS dates[12] defining them to be the 20th of March, June, September and December, business adjusted; those would also be the standard days of Maturity for contracts.
The first payment date is the first of these days following $t_0 + 1$: for example, trading a CDS with Maturity $T = 5$ years in January 2013 means the 20th of March 2013 will be the first payment date and the 20th of March 2018 will be the final date of the contract.
Observe that payments are postponed on the Buyer side: indeed, the coupon he pays on each Payment Date repays the Seller for protection offered in *previous* period; hence, for any trade set up off Payment Date we have to compute the payment amount and date for the residual time stub.
Since CDS Spreads are on yearly basis, in order to determine the first payment amount it is sufficient to determine the number of days in this first period; such amount is then retrieved as a fraction of the annual payment. It is still left to be established *when* this payment is due.
Before the *Big Bang*, the procedure to do so was rather tricky: the schedule of payments depended on whether the Trade Date occurred before or within 30 days from first payment date; in the first case the payment was accrued within a *Short Stub Period* going from t_0 to the first Payment Date T_1, and made on T_1; else, the Payment was accrued on a *Long Stub Period*, going from t_0 to the second Payment Date T_2, and made on this latter date, see Figure 6.
This mechanism clearly jars the request for standardisation, and mantains the complications in payments offsetting because of the strong dependance of cash flows on Trade Date. The proposed solution was to introduce a *Full First Coupon Payment*: that is, to set a payment on T_1 accrued on the whole period elapsing from previous standard CDS date T_0 to T_1.
In order to compensate the Buyer for the unprotected days he paid for, a *Riskless Accrued Premium*

$$S \cdot AP01 = S \cdot (t_0 - T_0)$$

is then owed by the Seller and paid three business days after t_0 (Figure 7).

II.G. Standardized Coupon

The coupon payment requested by the Seller should also be standardized, in order to match as many contract as possible in the central clearing procedure and facilitate trade compression: up to 2003, it was typical to trade CDS according to their annual coupon S, quoted in basis points per year.

[12] Although it was already common practice, to blend positions in CDS and bond market.

Since later 2008, it became common practice to trade CDS for highly distressed names and high yield names, with a fixed coupon (typically 500bp per year) and an attached Upfront payment to be made three business days after t_0.
Rennison et al. (2010) observe a fixed coupon coupled with an Upfront payment facilitates unwinds of contracts, as only a cash adjustment would be needed to exchange a CDS when a fixed cash flow would income in the future.
Moreover, Upfront payments make it easier for Sellers to deal with early default that triggered severe quantities of contracts, thus reducing counterparty risk by *pumping liquidity* in the market itself; moreover, this trading convention prevent many speculators to enter the market of CDS, attracted by free entrance.
The *Big Bang* standardizes coupon for North American contracts, either to 100 bp or 500 bp: the choice of the latter value was due to common market practices for highly volatile names; the other value was chosen to let several names trade near par with respect to previous quoted spread, avoiding to request a payment *from* the Seller to match the transaction.
The *Small Bang* addressed these issues to European Corporate and Western Sovereign CDS, but leaving a wider range of standard coupons, namely 25bp, 100bp, 300bp, 500bp, 750bp and 1000bp. One of the reasons for introducing such different coupons seems to be the cautiousness of customers to trade with large Upfront points.
Even if Upfront payment becomes in this way the real metric for CDS market, most dealers are still quoting the CDS Spread rather than their Upfront: thesef *Conventional* or *Par* Spreads are the value that sets to zero the expected cash flow of a CDS trading without any Upfront payment; such a computation requires a model that marks the CDS contract to the market.
Moreover, modeling hedging strategies that use CDS requires the analysis of their real cash flow, which *is not* the one provided by the par spread: any Spread curve should be converted to an Upfront curve and explored in this form.
In order to serve this conversion purpose, ISDA, developed a tool, known as the *ISDA-Markit CDS Standard Model.*[13] The code is available with open source license at *http://www.cdsmodel.com*; this tool allows investors to convert quotes in a unique standardized manner, in line with the purpose of the *Bangs*.
The ISDA-Markit converter will be deeply analyzed in Section 4 of this paper from a rather technical point of view; the appendix will be devoted to a detailed description of the converter's features in order to make it easier for it to be correctly implemented in any programming language.

[13]Markit is currently the administrator of this code, providing support and mantainance for the code, as well as further development.

III. The CDS Confirmation

This section is devoted to a brief but rather comprehensive review of the CDS Master Agreement together with the 2009 Supplements: we define the variables which are relevant in order to extrapolate the pricing equation of such derivatives straight out of the Confirmation itself and provide a description of the Auction methodology that are useful for a deeper exploration of Recovery Rates following default, see for example ICE, 2010.
The agreement between the two parties is legally defined as the *Confirmation* of the Credit Derivative transaction. The two parties are legally defined as *Fixed Rate Payer* (Buyer) and *Floating Rate Payer* (Seller), together agreeing on a third Reference Entity.[14] A *Notifying Party* (which could be either the Buyer or the Seller) is specified in the agreement, which is deemed to be responsible to communicate whether a relevant Credit Event for the transaction occurred; these latter clause lost most of their appeal since the introduction of Determination Committees, that can take binding decisions for both counterparties.
Three fixed dates are also specified in the contract: the *Trade Date* t_0, on which the contract is confirmed, the *Effective Date* t_0' on which protection starts and a *Scheduled Termination Date* T^*, to which we will simply refer to as *Maturity* of the contract[15]; considering the Backstop Date defined in previous section, for standardized contracts we have $t_0^{'} = t_0 - 60$.
The section is divided into four parts: at first we review the payments amounts and dates, in order to describe the cash flows of both counterparts; subsequently, we revisit the list of relevant Credit Events as well as the so called *triggering procedure*, that defines when a relevant Credit Event is deemed to occur.
We conclude the section with a description of the settlement procedure following a relevant Credit Event and define the *Termination Date* of the agreement.

III.A. Payments Amounts and Dates

The Confirmation specifies also the payment amounts and the dates on which they are due; the fixed one are, as the name suggest, those regarding the Buyer, while the protection Seller's payments are *floating*, as depending both on the relevant Credit Event and on the chosen Settlement Method.

- *Fixed Rate Payer Payment Dates*: we call set of dates $\{T_i\}_{i=1}^M$, adjusted to the agreed Business Day Convention, the set of coupon *Payments Dates*, that is the date when the Buyer pays the contract coupon to the Seller.

[14]The Credit Derivatives definitions ISDA (2003) supplied by ISDA (2009c) specifies also the rules to determine an eventual *Successor* to the Reference Entity, which in practice substitutes it both as an object of the contract and as a seller of the Reference Obligation. Since this aspect do not substantially modify our review, we drop it and refer the interested reader again to ISDA (2009b) and ISDA (2009c).
[15]Terminology here is misleading: T^* is not an exact date but rather the maturity of the contract, e.g. 2, 3, 5, 7, 10.. years.

Standard contracts provide Payments Dates to be the 20th of March, June, September and December, with T_1 being the earliest of these dates on or following $t_0 + 1$, T_0 the latest of these dates preceding $t_0 + 1$, and $T_M = T_1 + T^* = T_{4T^*+1}$, all dates adjusted following.
As any of the relevant Credit Event occurs, we can define the *Termination Date* $\bar{\tau}$, on which we shall come back later; the *Fixed Rate Payer Payment Dates* will be

$$\{(T_i \wedge \bar{\tau}) \cdot \mathbb{1}_{\{\bar{\tau} \geq T_i\}}\} \quad i = 1 \dots M \quad .$$

- *Fixed Rate Payer Calculation Period*: they are used in order to determine the effective amount to be paid to the Seller, according to the day count convention; namely, the first period is $[T_0, T_1 \wedge \tau]$, the intermediate are of the form $[T_{i-1}, T_i \wedge \tau)$ and the last is $[T_{M-1}, T_M \wedge \tau]$, where we define τ the *Event Determination Date* with respect to a relevant *Credit Event*, with the mathematical convention that $\tau = +\infty$ if no relevant Credit Event for target transaction occurs.[16] Notice that, when a Credit Event occurs, no more accrual days will be counted after it is determined to have happened on τ even if the payment referring to this last period will be made on $\bar{\tau} \geq \tau$.

- *Fixed Rate Count Fraction*: when a day count convention is specified[17], the Count Fraction is the number of days in each Calculation Period according to the rules determined by the selected convention.

- *Floating Rate Payer Calculation Amount*: it is the Notional N of the Reference Obligation for which the Buyer is buying protection.[18]

- *Fixed Rate*: it is the percentage of Notional that the Buyer owes to the seller on each Payment Date, usually expressed in *bps* per year; we refer to it as the *Spread* S of the CDS contract.

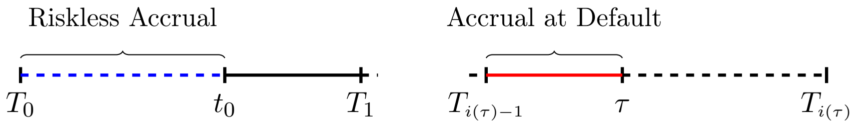
- *Fixed Amount*: it is the effective amount owed by the Buyer which will be eventually paid at each $\{T_i\}_{i=1}^{M}$, computed as $N \cdot S \cdot \Delta T_i$, where $\Delta T_i = T_i - T_{i-1}$ is measured according to the agreed Count Fraction.

- *Initial Payment*: it is the payment due from a party to the other that must be settle, if not differently specified, at time $t_0 + 3$, and we will refer to it as *Upfront Payment* U; by convention, we assume that the Buyer owns this payment to the Seller, but allow U to be negative in order to represent the opposite duty.

[16] We will also refer to each of these periods as the *Risky Accrual Periods*.

[17] North America Corporate and both European Corporate and Western Europe Sovereign CDS trade with ACT/360 count convention, with business day convention *Following*, see ISDA, 2009f and ISDA, 2009e; this means that the Spread refers to an year of *ACT*/360 days.

[18] More precisely, a percentage amount called *Reference Price* could be specified in the Confirmation, so that the Notional of the CDS is the Floating Rate Payer Calculation Amount times this percentage. Usually, and however if not conversely specified, the reference price is one hundred percent so we will not deal with this quibble in what follows.



(a) Inception (b) Credit Event

Figure 8: Accrual days at Inception 8(a) and in case of a Credit Event 8(b). (*Source: Markit, 2009b*)

Notice that, in order to mark the CDS to the market, we must offset from the Initial Payment the *Riskless Accrual Payment* $AP = S \cdot AP01$ owed by the Seller to the Buyer, computed by accruing the Fixed Rate S in the period $[T_0, t_0)$. Furthermore an *Accrual at Default* premium will be owed by the Buyer to his counterpart: indeed, looking to the definition of the Fixed Rate Payer Calculation Period, when Credit Events occur the last accrual period will be $[T_{i(\tau)-1}, \tau]$, where $T_{i(\tau)-1}$ is the last Payment Date preceeding τ, see Figure 8.

III.B. Credit Events

It is fundamental to be transparent in defining Credit Events that trigger the CDS: we briefly review the types of Credit Event that could be relevant for a Confirmation, while leaving to next section a rather detailed description of the triggering procedure; obviously, if any Credit Event is determined to have occurred on τ, it will be considered relevant as long as $\tau \leq T_M$.

- **Bankruptcy**
  The easiest Credit Event to define is Bankruptcy of the Reference Entity, as determined by its correspondent jurisdiction. We stress that if Reference Entity is dissolved but pursuant to amalgamation, consolidation or merging, such event *will not* be deemed to be a Bankruptcy Credit Event.

- **Obligation Acceleration/Obligation Default**
  A *Default Requirement* is specified in the Confirmation and we consider that *Obligation Acceleration/Default* happens if one or more obligations of the Entity, in an aggregate amount of not less than the Default Requirement, become due and payable before they would otherwise, as a consequence of a default event.
  Notice that we do not consider failure to pay under *every* Obligation of the Reference Entity as a Credit Event of this form; it could happen, for example, that one or more covenants underlying those obligations provide the measurement of some parameters that, if not in an established range, can be considered a breach on that covenant which in turn provides to an acceleration/default of those obligations.

- **Failure to pay**
  A *Payment Requirement* is specified in the Confirmation[19], and *Failure to Pay* occurs if Reference Entity fails to pay, when due, an aggregate amount of not less than the Payment Requirement on *any* of its obligations.
  We define the *Failure to Pay Date* as three days after a failure to pay occurred. It could be the case that a *Grace Period Extension* is applicable to the obligations, due either to the CDS or to the obligation itself.
  We refer the interested reader to ISDA, 2003 for further specification on this issue.

- **Repudiation/Moratorium**
  A *Repudiation* is a rejection or disaffirmation of a Governmental Authority of *any* of the obligations of the Reference Entity; a *Moratorium* is a delay in the repayment of *any* obligation of the Reference Entity as a consequence of a Governmental enforcement; an extension condition could be appliable also in this case. (ISDA, 2003)

- **Restructuring**: Restructuring occurs if changes in credit conditions are applicable and agreed by the Reference Entity and a sufficient number of the holders of such target Obligations, in a way that it binds all holders.
  It does include:

  - Reduction in rate/amount of interest payable or scheduled interest accruals of target obligation.
  - Reduction in amount of principal or premium payable at maturity by target obligation.
  - Postponement of a dates when any of the previous payments is due.
  - Change in the ranking of priority of *any* of Reference Entity's obligations, causing subordination of debt.
  - Change to a currency that is not a *Permitted Currency*.[20]

  As explained before, different types of Restructuring clauses are available, differing one to the other mainly on the maturities of deliverable obligations:

  - ***No R*** Restructuring does not trigger the CDS.
  - ***Old R*** Restructuring triggers the CDS, and limitations on deliverable Obligations are the least binding: namely, obligations with maximum maturity up to 30 years beyond the Restructuring Date are deliverable.

[19]For example, the coupon of a bond.
[20]Euro or any currency that has a local currency long-term debt with either maximum rating by any of Moody's, Standard&Poor's or Fitch or a G7 country, see section 4.7(a) of ISDA, 2009b.

- ***Mod R*** Restructuring triggers the CDS, but deliverable obligations must be fully transferrable (so most of the loans are excluded) and with a maturity not exceeding 30 months after the Restructuring Date.
- ***Mod Mod-R*** Restructuring triggers the CDS, but whatever be the debt to be delivered it shall not mature later than 60 months after Restructuring Date. Besides the difference upon Restructuring clauses, all standard CDS ensures protection against Failure to Pay, Bankruptcy and Repudiation/Moratorium.
  In next section, as describing the procedure for a CDS to be triggered, we will implicitly refer to standard contracts only, distinguishing them only through the Restructuring clause.

III.C. CDS Triggering

The Credit Derivatives Definition provides also conditions for which a Credit Event is considered to have happened, and specifies the terms under which the CDS is triggered.
The *Big Bang* supplement defines the *Credit Event Resolution Request Date* as the date when, with respect to a notice delivered to ISDA within the Delivery Period, the DC is requested to be convened to resolve whether and when a Credit Event relevant for a given Entity occurred; introducing also the *Small Bang*, the DC not only resolves the relevance and timing of the Credit Event, but the so called *DC Credit Event/No Credit Event Announcement* is exactly τ.
The decisional power granted to the DC is resumed in Section 1.8(c) of ISDA (2009b), with the sentence: "...no Event Determination Date will occur, and any Event Determination Date previously determined with respect to an event shall be deemed not to have occurred, if ... a DC No Credit Event Announcement occurs...".
Still some peculiar rules are applaible to each transaction, as counterparties can send private notices one to the other even if no Credit Event Request is delivered to ISDA, but the final decision is taken by the Committee only; further details are available in ISDA (2009b).
The Termination Date $\bar{\tau}$ is the date where post-default settlement of obligations and resolution of CDS covenants are settled; the distance $\bar{\tau} - \tau > 0$ depends on both Credit Event and Settlement Method, as well as on the type of Obligation targeted. We will not be more specific on that, as the topic is most suitable for traders than for academians; again we refer the interested reader to ISDA (2003).

III.D. Auction Settlement

The agreement provides that, in case a Credit Event occurs, counterparties will resolve their contractual obligations when the *Conditions to Settlement* are met, according to the agreed *Settlement Method*.[21]
Auction Settlement is now the standard for CDS contracts, so here we describe it in detail, being it the most pertinent within the context of standardization; details on the other methods can be found in ISDA (2003) and ISDA (2009b).
Conditions to Settlement will be deemed to be satisfied at the occurrence of an *Event Determination Date*, provided decision is not reversed before the *Auction Final Price Determination Date*, when post-default settlement is resolved.
Auction Settlement was introduced in the *Big Bang* Protocol by adding a brand new section to the Definitions, namely Section 12.
The rules specify that standard contracts are bound to Auction Settlement, and that the decision whether the Auction will be held, cancelled or that no auction will take place, is again in the hands of the DC.
DC decides the Auction date after a Credit Event is announced on τ and, on that date, an *Auction Final Price* R is determined through a two-step mechanism, which we will describe in what follows. Five days after the Auction has concluded, transactions are settled and Buyers receive from Sellers $(N \cdot (1 - R), 0)^+$; participants to the Auction include each global dealer Voting Member as well as regional dealer Voting Member of the relevant Convened DC, as well as any other institution that voluntarily submits to the DC Secretary (ISDA) a participating bidder letter and is approved by that same DC. Dealers not participating to the Auction will however settle their transactions at the Auction Final Price R.
Although any post-Auction settlement is resolved cash, bidders in the auction may submit a *Physical Settlement Request* to actually exchange Obligations at the post-Auction price. The Auction takes place in two stages: in the first the *Initial Market Mid Point* (IMM) is determined together with the *Open Interest* and eventual *Adjustment amounts*. Secondly, off-market limit orders are taken into consideration to match the Open Interest and the *Auction Final Price* is the output. We explain these steps following the numerical example in ICE, 2010.

[21] The agreement also specifies a set of *Deliverable Obligations* which can be legally derived in the place of the Reference Obligation; we will not analyze this aspect, for which we refer to Section 2 in ISDA (2009b).

Dealer	Bid	Offer
1	56	58
2	56.25	58.25
3	53.25	55.25
4	54	56
5	53.875	55.875
6	55	57
7	54.5	56.5
8	54.875	56.875
9	54.75	56.75
10	55	57

Dealer	Buy/Sell	Size($mm)
1	Buy	4
2	Sell	1
3	Buy	7
4	Buy	12
5	Sell	17
6	Buy	3
7	Sell	8
8	Sell	10
9	Sell	12
10	Buy	10
O.I.	Sell	12

Table 1: Two-way market (left table) for defaulted Obligation with ten dealers holding the Auction. Here the maximum bid/offer spread is set to 2% and Quotation size to $5 mm; the cap is 1% and we consider a minimum tick size of 1/8. Right table shows Physical Settlement Requests: offsetting positions gives us an Open Interest of $ 12mm to sell. (*Source: ICE, 2010*)

- **Step 1**:
Dealers involved in any Auction are always requested to supply a two-way market (bid/offer) for the defaulted Obligation. What is peculiarly arranged by the DC soon before the Auction date is a set of distinctive parameter for those markets: the bid/offer spread is pre-fixed as well as the *Quotation Size* for those prices. The *cap* for the Auction is usually defined as half of the bid/offer spread. It is also requested to participant to submit Physical Settlement Request, if any; nonetheless their willing to buy or sell, any of them will be committed to transact for a predetermined minimum amount (equal to the Quotation Size) at the Final Price R, which is determined by the Auction. In our example an Obligation with $100 par value is considered; inputs to this first step are the ones in Table 1 and 2.
There's a fifteen minutes time window to submit such inputs to *CreditEx* electronic platform; subsequently bids and offers are order in descending and ascending order, respectively, and crossing market positions are discarded. Then the remaining best half of the two way market is averaged and rounded up to the nearest tick in order to get the IMM, see Table 3.
As to discourage investors from manipulating prices, penalties are driven to participants submitting orders on the wrong side of the crossing market with respect to the Open Interest. For example, if an offer is lower then the IMM and the open interest suggest the contrary (i.e. the open interest is to buy, suggesting the price would have to go up so they shouldn't offer low) penalties will be given to low offers. The converse happen in our example, see again Table 3. Those dealers in the end will have to pay to ISDA an Adjustment Amount equal to the difference between their bid and the IMM times the Quotation Size.

Dealer	Bid
2	**56.25**
1	**56**
6	55
10	55
8	54.875
9	54.75
7	54.5
4	54
5	53.875
3	53.25

Dealer	Offer
3	55.25
5	55.875
4	56
7	56.5
9	56.75
8	56.875
6	57
10	57
1	58
2	58.25

Table 2: Sorted Bid and Ask. Grey shaded cells shows the crossing tranche of the markets, while blue shaded ones refers to the " best halves " of both markets which are averaged in order to get the IMM of 55.75$. Bold numbers are the off-market submissions on the wrong side of the market (here, the open interest is to sell) which will be penalized through the Adjustment Amounts. (*Source: ICE, 2010*)

- **Step 2**:
  A two-three hours stub is then left to the market so that it can digest these outputs; the second step is devoted to match the Open Interest and finally determine R. In our example, the Interest is to sell, so participants are allowed to set off-market limit orders to bid; with them, all the bids of the initial market are carried over at the Quotation Size, see Table 3. Crossing market bids are carried forward at the *lower* between the initial bid and the IMM (if Open Interest had been to buy, crossing offers would have been carried forward at the *higher* between the initial *offer* and the IMM).
  Off-market limit orders can be placed in any size, but in order to avoid large limit orders trying to manipulate the result, when the open interest is to sell, off-market limit orders with bids over IMM plus cap will be considered at cap, while if the open interest is to buy, offers below the IMM minus cap will be considered at IMM minus cap. Limit orders are finally ordered in descending (bids) or ascending (offers) order, and the Open Interest is matched by subtracting from it each limit order size, following that same order. If in this way the open interest is matched, the last price used to match will be R, whether if we run out of limit order the price (per unit) will be either 1 (if open interest is to buy) or 0 (if open interest is to sell), see Table 4. Previous description stresses out the importance of a precise model for the Recovery Rate R, which remains in fact one of the big deals in Credit Default Swaps quantitative modeling.
  Common practice is to assume Recovery to be known in advance so that the *Loss Given Default* per unit $LGD = (1 - R, 0)^+$ can be set as an input in the pricing equations driving such contracts; we shall explore in detail these and others aspects of CDS marking-to-market mechanism in next section.

Dealer	Bid	Bid Size
L1	57	2
L2	55	7
L3	54.75	8
L4	54	11
L5	52	3
2	55.75	5
1	55.75	5
6	55	5
10	55	5
8	54.875	5
9	54.75	5
7	54.5	5
4	54	5
5	53.875	5
3	53.25	5

Table 3: Limit orders in step 2: blue cells shows limits order off-market made by any of the dealers, with variable size; orange cells points out the crossing bids carried over at the IMM, the lower between their initial bid and the IMM. Light blue cells shows the in-market bids which are carried forward as they were in step 1. (*Source: ICE, 2010*)

Dealer	Bid	Bid Size
L1	**56.75**	2
1	55.75	5
2	55.75	5
L2	55	7
6	55	5
10	55	5
8	54.875	5
L3	54.75	8
9	54.75	5
7	54.5	11
L4	54	5
4	54	5
5	53.875	5
3	53.25	3
L5	52	3

Table 4: Determination of R. The first price has been modified since the highest off-market offer overcame the IMM plus the cap. Cells in light red are sufficient to offset the open interest; the red cell shows the final price R =55.75\$. (*Source: ICE, 2010*)

IV. CDS Pricing and Conversion

In order to get a conversion between running-spread and standard CDS, we first need a model to rely upon that marks the CDS to the market. The present value of the CDS is obviously contingent on determining default probabilities, which must be somehow implied from market quotes.
We derive a no-arbitrage based pricing formula for a CDS: a detailed analysis of the peculiar assumptions through which the formula is implemented in the *CDS Standard Model* is provided in the Appendix. First of all, we will assume that the Termination Date coincides with the Event Determination Date, that is post-default settlement is resolved the day that the relevant credit Event is deemed to have occurred: in this way, Accrual at Default and LGD will be exchanged exactly at τ; we consider negligible this friction[22] while discounting cash flows of the two counterparties at the present value (time t_0).
The protection Seller's cash flow is contingent on a Credit Event occurring before the Scheduled Termination Date T; given the pricing measure $\mathbb{Q}$, we define the *Survival Probabilities*

$$\mathbb{Q}(\tau \geq u) \tag{1}$$

as the probability that the (random) time τ occurs on or after time $u \geq t_0$.
The complete schedule of payments for the Seller takes into account both the Riskless Accrued Premium $S \cdot AP01$ and the LGD that will be paid when any of the relevant Credit Event occurs, see Figure 9.
Although the Effective Date of protection begins earlier than t_0, we assume that $t'_0 = t_0 + 1$ so that no Credit Event is known to have triggered the CDS at the time the two counterparts enters the contract (Markit, 2009a).
The discounted cash flow of the Seller is the stochastic process:

$$N \cdot (S \cdot AP01^{[t_0,T^*]} \cdot D_{t_0}(t_0+3) + LGD \cdot D_{t_0}(\tau) \cdot \mathbb{1}_{t_0 \leq \tau \leq T_M})$$

where $D_u(v)$ is the discount factor at time u for time v.
The Buyer's cash flow is composed by the Upfront payment, the discounted sum of future coupon payments and the Accrual at Default[23], see Figure 10.
Defining $T_{i(\tau)}$ as the first Payment Date following τ, we can write the discounted cash flow of the buyer at time T_0:

$$N \cdot \left(U^{[t_0,T^*]} \cdot D_{t_0}(t_0+3) + S \cdot \sum_{i=1}^{M} D_{t_0}(T_i) \cdot \Delta T_i \cdot \mathbb{1}_{\tau \geq T_i} + S \cdot D_{t_0}(\tau) \cdot (\tau - T_{i(\tau)-1}) \cdot \mathbb{1}_{t_0 < \tau < T_M} \right)$$

[22] As specified in Section III.A, latest payment date is on Terminatin Date, and this in principle affects discount factors.

[23] Notice that we have defined the last payment date as $T_M \wedge \tau$, so here we abuse of notation set T_M equal to T_{4T^*+1} and consider Accrual at Default.

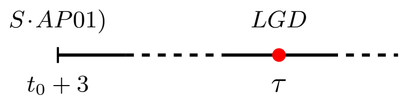


Figure 9: Single name CDS: Cash Flow of protection Seller

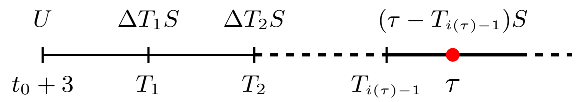


Figure 10: Single name CDS: Cash Flow of protection Buyer

In order to achieve the stochastic payoff at inception $\Pi(t_0)$, e.g. from the Buyer-side, we subtract these two cash flows: no-arbitrage arguments imply that the expected value of this payoff should be zero under $\mathbb{Q}$ and this builds up the pricing equation for the CDS contract.
We stress that, in what follows, it is implicitly assumed that:

- The discount factor process is independent of τ. We call $P_u(v) = \mathbb{E}^{\mathbb{Q}}(D_u(v))$;
- The LGD is constant and known, so that R is an input in the equation.

Dropping one of these hypotheses (or both) is one of the addresses of current research, and lies beyond the purpose of this paper; we further assume that the Notional is $N = 1$, in order to ease notations.[24]
Taking expectations *with respect to the pricing measure*, by no-arbitrage principle in the market implies

$$\mathbb{E}^{\mathbb{Q}}(\Pi(t_0)) = \pi(t_0) = 0.$$

This equation can be expanded as:

$$\begin{aligned} ProtLeg^{[t_0,T^*]}(R, P_{t_0}(\cdot), \mathbb{Q}) - PremLeg^{[t_0,T^*]}(S, P_{t_0}(\cdot), \mathbb{Q}) \\ + \big(S \cdot AP01^{[t_0,T^*]} - U\big) P_{t_0}(t_0 + 3) = 0 \end{aligned} \tag{2}$$

[24]However it is worth to remember that Upfront payment is usually quoted in percentage of Notional while Spread is quoted in basis points.

where the *Protection Leg* is defined as

$$ProtLeg^{[t_0,T^*]}(R, P_{t_0}(\cdot), \mathbb{Q}) = -LGD \cdot Prot01(P_{t_0}(\cdot), \mathbb{Q})$$

with

$$Prot01(P_{t_0}(\cdot), \mathbb{Q}) \quad = \quad \int_{t'_0}^{T_M} P_{t_0}(u) d\mathbb{Q}(\tau \geq u) \tag{3}$$

and the *Premium Leg*, including both future coupon payments and (expected) Accrual at default:

$$PremLeg^{[t_0,T^*]}(S, P_{t_0}(\cdot), \mathbb{Q}) = S \cdot (Prem01^{[t_0,T^*]}(P_{t_0}(\cdot), \mathbb{Q}) + ACC01^{[t_0,T^*]}(P_{t_0}(\cdot), \mathbb{Q}))$$

with

$$Prem01^{[t_0,T^*]}(P_{t_0}(\cdot), \mathbb{Q})) = \sum_{i=1}^{M} \Delta T_i P_{t_0}(T_i) \mathbb{Q}(\tau \geq T_i) \tag{4}$$

$$\begin{aligned} ACC01^{[t_0,T^*]}(P_{t_0}(\cdot), \mathbb{Q})) = & \int_{t_0}^{T_1} P_{t_0}(u)(u - T_1) d\mathbb{Q}(\tau \geq u) \\ & + \sum_{i=2}^{M} \int_{T_{i-1}}^{T_i} P_{t_0}(u)(u - T_{i-1}) d\mathbb{Q}(\tau \geq u). \end{aligned} \tag{5}$$

In the end, no arbitrage condition links the contract's variables through a non-linear implicit equation:

$$\pi(t_0) = \pi(t_0, T^*, P_{t_0}(\cdot), \mathbb{Q}, R, S, U) = 0$$

IV.A. Conversion Mechanism: the ISDA CDS Standard Model

In order to get the relation between U and S and proceed to the conversion, common practice is to choose R as input; modeling assumptions come out in order to derive the expected discount curve and the survival probabilities out of market quotes.
The proposed form for $P_{t_0}(\cdot)$ is the one of a discount curve obtained with a piecewise constant forward rate, computed through the LIBOR rates, which are locked at day $t_0 - 1$ for day t_0 and published by Markit, so that any investor is using exactly the same conversion mechanism (see the Appendix).
The ISDA standard CDS model assumes that the default probability before any time $u \geq t_0$ follows an exponential distribution with *constant* parameter λ:

$$\mathbb{Q}(\tau \geq u) = \exp(-\lambda(u - t_0)). \tag{6}$$

This model is also known as *flat hazard curve*[25] model; the parameter is not known and must be implied from the CDS market. We can thus write:

$$\pi(\lambda, S, U) = 0. \tag{7}$$

The conversion mechanism is based on the iterate application of zero-search method for equation (7) with respect to one of the three variables, depending on what is quoted on the market. Being t_0 the selected Trade Date, we proceed in two steps:

- From quoted Spread S_0 to upfront U^* plus standard coupon $\hat{S}$
  - Set $U = 0$ and $S = S_0$ in (7). Solve $\pi(\lambda, S_0, 0) = 0$ and retrieve λ_0^*.
  - Set $S = \hat{S}$ and $\lambda = \lambda_0^*$ in (7). Solve $\pi(\lambda_0^*, \hat{S}, U) = 0$ and retrieve U^*.
- From Upfront payment U_0 plus standard coupon $\hat{S}$ to par Spread S^*.
  - Set $U = U_0$ and $S = \hat{S}$ in (7). Solve $\pi(\lambda, \hat{S}, U_0) = 0$ and retrieve λ_0^*.
  - Set $U = 0$ and $\lambda = \lambda_0^*$ in (7). Solve $\pi(\lambda_0^*, S, 0) = 0$ and retrieve S.

Notice that in both cases we use market quotes (either Spreads or Upfronts) in order to imply the parameter λ; this means reading default probabilities in the market. Such an use of this parameter looks a bit strained: it should be better considered as a mere correspondence between Upfront and Spreads, playing a similar role to the mapping between implied volatility coming out of Black and Scholes equation(s).
Observe, for example, that this parameter depends on the chosen maturity T^*: that is, even if the CDS contract is exactly the same, a set of different default intensities can be implied using those different maturities. This means that within time intervals where more than one contract is current, a set of different default intensities for the same Entity is available, which is obviously inconsistent for modeling purposes (Beumee et al., 2009). At the moment, Markit has improved the CDS converter in a way such that piecewise constant hazard rate curves are obtainable from the market; moreover, it is now clear that not only a term structure but also a stochastic component should be considered for the *default intensity* λ, see Cont (2011) for a survey. ISDA-Markit 's modeling choices at least avoid the integrals to be approximated as a closed form is available under their assumptions, at least dropping one source of error in the computations.

[25]The choice of modeling default intensities without either a term structure and a volatility generates several theoretical controversies for which we refer to Beumee et al., 2009. Currently, many dealers bootstrap the hazard curve using standard CDS prices with different maturities.

Appendix: ISDA-markit CDS Converter Specification

Interest Rate Curve

In this section we provide a detailed description of the method ISDA uses to derive the discount curve to be used, following Markit (2010).
Set trade date equal to t_0, business adjusted. Observe that, except for Japan Reference Entities, no holidays are taken into account but Saturdays and Sundays: Standard CDS model uses LIBOR rates in order to imply the discount curve.
The conventional LIBOR rates to be used are the ones locked at (business) time $t_0 - 1$, daily updated on ISDA's website, so that anyone is marking to the market the CDS contract using the same discount curve.
Moreover, we assume that the *spot date* for these rates is $t_0 + 2$ business adjusted: in this way, the discount factor for time t_0 will be 1 at time $t_0 + 2$: this imply that the discount factor will be slightly greater than one at time t_0. We derive the discount curve up to 1 year directly from LIBOR Deposit Rates; we just have to consider the day count convention for these quotes, namely $ACT/360$.
LIBOR rates are available for $\{1, 2, 3, 6, 9, 12\}$ months deposit and $\{2, 3, 4, 5, 6, 7, 8, 9, 10, 12, 15, 20, 25\}$ year swap; we define $\{j_k\}_{k=1}^{19}$ the succession of these time distances, so that

$$t_{j_k} = t_0 + 2 + j_k$$

all dates business adjusted following.
In this section, we must be careful with the daily count convention, so we define $d(t, s)$ the distance between times t and u meausred with ACT/360 dcc and set

$$\Delta_{j_k} = d(t_{j_{k-1}}, t_{j_k}) \tag{8}$$

This sequence of dates defines a non-homogeneous timegrid

$$\mathcal{T}_1 = \{t_{j_k}\}_{k=0}^{19}$$

that we are going to use in order to construct the discount curve $P_{t_0}(\cdot)$.
Let $\hat{L}_{t_0}(t_{j_k})$, with $k \in 1 \dots 6$ be the conventional [26] deposit rate at t_0 for time t_{j_k}; we set:

$$P_{t_0}(t_{j_k}) = \frac{1}{1 + \hat{L}_{t_0}(t_{j_k}) \cdot \Delta_{j_k}}.$$

This implies a piecewise constant instantaneous forward rate:

$$f_{t_0}(t_{j_k}) = f_{j_k} = -\frac{1}{\Delta_{j_k}} \cdot \log \frac{P_{t_0}(t_{j_{k-1}})}{P_{t_0}(t_{j_k})}, \quad k = 1 \dots 6 \tag{9}$$

where f_{j_k} is the forward rate for time interval $[t_{j_{k-1}}, t_{j_k})$.

[26] Using locked Libor levels at $t_0 - 1$ as a proxy for trade date rates we are taking $L_{t_0-1}(u-1) = \mathbb{E}(L(t_0, u) | \mathcal{F}_{t_0-1}) = \hat{L}(t_0, u))$ as conventional rates for the conversion.

In order to imply discount factors and forward rates for longer time periods, we use the LIBOR Swap Rates $\hat{L}_{t_0}(t_{j_k})$, $k = 7, \dots 19$.
The Swap Rate is an annual coupon which is paid every 6 months with day count convention 30/360; define $t_h = t_0 + 2 + 6h$, with $h = 0, 1 \dots$ months, and $\delta_l = d'(t_{h-1}, t_h)$ the grid steps according to this convention.
The par-swap relationship implies, fixed a term $H \in \{1, 2 \dots 25\}$ years,

$$\sum_{h=1}^{H} \delta_h \hat{L}_{t_0}(t_h) P_{t_0}(t_h) = 1 \tag{10}$$

The discount curve is derived through an iterative process, assuming again a constant instantaneous forward rate between swap dates, that is, a log-linear interpolation of discount factors. For any $t \in (t_h, t_{h+1}]$

$$P_{t_0}(t) = \exp\left(\frac{d(t, t_{h+1})}{d(t_h, t_{h+1})} \log(P_{t_0})(t_h) + \frac{d(t_h, t)}{d(t_h, t_{h+1})} \log(P_{t_0}(t_{h+1})) \right) \tag{11}$$

Suppose for example that we want to calculate the two-year discount factor: we must discount the all four semiannual swap coupons to use the par-swap equation. We have computed the 6 months and the 1 year discount factors through the LIBOR quotes, but we do not have any quote to use for the 18 months factor. Using (11) with $h = 3$ (eighteen months), we just have one unknown in (10), namely $P_{T_0}(t_{j_7})$.
Notice that time fractions in parenthesis are computed as ACT/ACT days, which is coherent[27] with the first branch of the discount curve initially computed.
Iterating this process[28] leads to a complete set of forward rates $\{f_{j_k}\}$, so that the piecewise constant rate $f(u)$ can be written as

$$f(u) = f_{j_1} \cdot \mathbb{1}_{[t_0, t_{j_1}]} + \sum_{k=1}^{19} f_i \cdot \mathbb{1}_{(t_{j_{k-1}}, t_{j_k}]}(u), \quad u \geq t_0$$

This leads to a closed form for the discount factor:

$$P_{T_0}(u) = \exp\left(-\sum_{k=1}^{19} f_i \cdot d(u \wedge t_{j_{k-1}}, u \wedge t_{j_k}) \right), \quad u \geq t_0 \tag{12}$$

with which we can compute the discount curve on any time grid.

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[27](ACT/360)/(ACT/360)=ACT/ACT.

[28]The formula has to be rearranged when time lags between swap dates are greater than 1 year. However, piecewise constant forward rate always implies a single unknown in (10).

The Protection Leg

In order to ease notation, since in this section time distances are computed using ACT/365 daily count convention, we simply write $t-s$ implicitly assuming that this distance is computed as an actual day fraction of a non-leap year.
We rewrite (3) as

$$Prot01^{[t_0,T_M]}(\{f_{j_k}\},\lambda) = \int_{t_0+1}^{T_M} P_{t_0}(u)\lambda \exp(-\lambda(u-t_0))du.$$

The form (12) chosen for the Discount Curve, induces a closed form for the integral: let $\{f_{j_k}\}$ be the set of implied forward rates relevant for selected maturity T^*.
We define the *risky discount factor* $\Lambda_{j_k} = f_{j_k} + \lambda$, and set:

$$\mathcal{T}_1^* = \{t_{j_k}\}_{k=0}^{N^*}, \quad N^* = N(T^*) = \text{argmin}\{j_k : t_{j_k} \geq T_M\} \tag{13}$$

Using the set of correspondent $\{\Lambda_{j_k}\}$, we compute:

$$Prot01^{[t_0,T^*]}(\{f_{j_k}\},\lambda) = \lambda \sum_{i=1}^{N^*} \exp(-\lambda(t_{j_{k-1}} - t_0))P_{T_0}(t_{j_{k-1}})\frac{1-\exp(-\Lambda_{j_k}(t_{j_k}-t_{j_{k-1}}))}{\Lambda_{j_k}}.$$

The Premium Leg

In order to compute the Premium Leg, we set

$$\mathcal{T}_2 = \{T_i\}_{i=0}^{M} \tag{14}$$

where $\{T_i\}$ is the set of CDS Payment Dates, all business adjusted. Again, the assumption of a flat hazard curve together with a piecewise constant forward curve allows to explicitly solve the integral regardless of any approximation scheme.
In this case, we have:

$$Prem01^{[t_0,T^*]}(\{f_{j_k}\},\lambda) = \sum_{i=1}^{M}(T_i - T_{i-1})P_{t_0}(T_i)\exp(-\lambda(T_i - t_0))$$

$$\begin{aligned} ACC01^{[t_0,T^*]}(\{f_{j_k}\}(\cdot),\mathbb{Q})) = &\int_{t_0}^{T_1} P_{t_0}(u)(u-t_0)\exp(-\lambda(u-t_0))du \\ &+ \sum_{i=2}^{M}\int_{T_{i-1}}^{T_i} P_{t_0}(u)(u-T_{i-1})\exp(-\lambda(u-t_0))du. \end{aligned}$$

In order to explicitly compute $ACC01^{[t_0,T^*]}$, we define

$$\begin{aligned} \{t_h^1\}_{h=1}^{H_1} &= \Big(\mathcal{T}_1 \cup \mathcal{T}_2\Big) \cap [t_0, T_1] \\ \{t_h^i\}_{h=1}^{H_i} &= \Big(\mathcal{T}_1 \cup \mathcal{T}_2\Big) \cap [T_{i-1}, T_i] \quad i \geq 2 \end{aligned}$$

so that each of the intervals between standard coupon maturities is divided into subintervals on which the forward rate is constant and equal to $f_h^i \in \{f_{j_k}\}$.
Observe that $t_1^0 = t_0$ while, for $i \geq 2$, $t_0^i = T_{i-1}$ and $t_{H_i}^i = T_i$.
We have:

$$\begin{aligned} ACC01^{[t_0,T^*]}(\{f_{j_k}\},\lambda) =& \lambda \sum_{h=1}^{H_1} \int_{t_{h-1}^i}^{t_h^1} (u - t_0) \exp(-f_h^1(u - t_{h-1}^1) - \lambda(u - t_0))du \\ &+ \lambda \sum_{i=2}^{M} \sum_{h=1}^{H_i} \int_{t_{h-1}^i}^{t_h^i} (u - T_{i-1}) \exp(-f_h^i(u - t_{h-1}^i) - \lambda(u - t_0))du \end{aligned}$$

Again, we define the risky discount factors $\{\Lambda_h^i\} = \{f_h^i + \lambda\}$ and solve the integral, obtaining:

$$ACC01^{[t_0,T^*]}(\{f_{j_k}\},\lambda) = \lambda \sum_{h=1}^{H_1} \frac{P_{t_0}(t_{h-1}^1)\exp(-\lambda(t_{h-1}^1 - t_0))}{\Lambda_h^1} \cdot \Big(\frac{1}{\Lambda_h^1} + t_{h-1}^1 - t_0 -$$
$$\exp(-\Lambda_h^1(t_h^1 - t_{h-1}^1)) \cdot (\frac{1}{\Lambda_h^1} + t_h^1 - t_0)\Big) + \lambda \sum_{i=2}^{M} \sum_{h=1}^{H_i} \frac{P_{t_0}(t_{h-1}^i)\exp(-\lambda(t_{h-1}^i - t_0))}{\Lambda_h} \times$$
$$\Big(\frac{1}{\Lambda_h^i} + t_{h-1}^i - T_{i-1} - \exp(-\Lambda_h^i(t_h^i - t_{h-1}^i)) \cdot (\frac{1}{\Lambda_h^i} + t_h^i - T_{i-1})\Big)$$

where $P_{t_0}(\cdot)$ is computed as in (12).
Notice that also the coupon spread refers to a year of protection computed in ACT/360 day count convention; hence, the Protection Leg is

$$PremLeg^{[t_0,T^*]}(S,\{f_{j_k}\},\lambda) = \frac{365}{360} \cdot S \cdot \big(Prem01^{[t_0,T^*]}(\{f_{j_k}\},\lambda) + ACC01^{[t_0,T^*]}(\{f_{j_k}\},\lambda)\big).$$

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